# Behavioral Economics: Solution to Problem Set 7

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**Exercise 1:**

**Time-consistent agent:** The problem that a time-consistent agent faces is simply to maximze his lifetime-utility by choosing $e_1$ and $e_2$, that is

$$U^{TC} = -\frac{1}{2}(e_1)^2 - \frac{1}{2}(e_2)^2 + \left[2(e_1+e_2) - \frac{1}{2}(e_1+e_2)^2\right] \rightarrow \max_{e_1,e_2}! \, .$$

The corresponding first-order conditions are given by

$$\frac{\partial U^{TC}}{\partial e_1} = -e_1 + 2 - (e_1+e_2) \stackrel{!}{=} 0$$

$$\frac{\partial U^{TC}}{\partial e_2} = -e_2 + 2 - (e_1+e_2) \stackrel{!}{=} 0$$

Obviously a time-consistent agent prefers effort-smoothing in the sense that he chooses $e^{TC} = \frac{2}{3}$ in both periods. Hence, a time-consistent agent exhibits a total effort of $2e^{TC} = \frac{4}{3}$.

**Naive agent:** When choosing his effort level in the first period, a naif agent believes that he is going to behave like a time-consistent agent in period 2, that is, he believes he is going to choose his effort level in $t = 2$ according to the following maximization problem:

$$\max_{e_2} \; -\frac{1}{2}e_2^2 + \left[2(\hat{e}_1+e_2) - \frac{1}{2}(\hat{e}_1+e_2)^2\right]$$

The corresponding first-order condition is given by

$$-\, e_2^{TC} + 2 - (\hat{e}_1 + e_2^{TC}) = 0$$

$$\Rightarrow \; e_2^{TC}(\hat{e}_1) = 1 - \frac{\hat{e}_1}{2}$$

Hence, in period 1 a naif agent faces the following optimization problem:

$$\max_{e_1} \; -\frac{1}{2}e_1^2 - \beta\frac{1}{2}\left(1-\frac{e_1}{2}\right)^2 + \beta\left[2\left(e_1+\left(1-\frac{e_1}{2}\right)\right) - \frac{1}{2}\left(e_1+\left(1-\frac{e_1}{2}\right)\right)^2\right]$$

Here, the first-order condition reads as follows:

$$-\, e_1^N + \beta(2 - (e_1^N + \left(\frac{e_1^N}{2}\right))) = 0$$

$$\Rightarrow e_1^N = \frac{2\beta}{2+\beta}$$

With this effort choice in period 1, the actual problem of a naif agent in period 2 takes the following form:

$$\max_{e_2} \; -\frac{1}{2}e_2^2 + \beta\left[2\left(\frac{2\beta}{2+\beta} + e_2\right) - \frac{1}{2}\left(\frac{2\beta}{2+\beta} + e_2\right)^2\right]$$

Differentaiting with respect to $e_2$ yields the following first-order condition:

$$\begin{aligned} -\, e_2^N + \beta\left(2 - \left(\frac{2\beta}{2+\beta} + e_2^N\right)\right) &= 0 \\ \Rightarrow e_2^N &= \frac{4\beta}{(1+\beta)(2+\beta)} \end{aligned}$$

Now it is straightforward to establish the following results: A naif agent

- increases his efford from period to period, $e_1^N < e_2^N$,
- works less than a time-consistent agent in period 1, $e_1^N < e^{TC}$,
- exhibits a lower total effort than a time-consistent agent, $e_1^N + e_2^N < 2e^{TC}$.

$$e_1^N = \frac{2\beta}{2+\beta} < \frac{2}{1+\beta}\frac{2\beta}{2+\beta} = e_2^N$$

$$\left[\beta = 1 \Rightarrow e_1^N = \frac{2}{3} \text{ and } \frac{de_1^N}{d\beta} = \frac{4}{(2+\beta)^2} > 0\right] \;\Rightarrow\; e_1^N < e^{TC}$$

$$e_1^N + e_2^N = \frac{2\beta(1+\beta) + 4\beta}{(1+\beta)(2+\beta)} < \frac{4}{3} = 2e^{TC} \Leftrightarrow 4 > \beta^2 + 3\beta$$

**Sophisticated agent:** In contrast to a naive agent, a sophisticated agent correctly predicts his second-period behavior when choosing his effort level in the first period, that is he knows that he is going to choose his effort level in $t = 2$ according to the following maximization problem:

$$\max_{e_2} \; -\frac{1}{2}e_2^2 + \beta\left[2(\hat{e}_1 + e_2) - \frac{1}{2}(\hat{e}_1 + e_2)^2\right]$$

The corresponding first-order condition is given by

$$\begin{aligned} -\frac{1}{2}e_2^S + \beta\left[2 - (\hat{e}_1 + e_2^S)\right] &= 0 \\ \Rightarrow\; e_2^S(\hat{e}_1) &= \frac{\beta(2 - \hat{e}_1)}{1+\beta} \end{aligned}$$

Hence, in period 1 a sophisticated agent faces the following optimization problem:

$$\max_{e_1} \; -\frac{1}{2}e_1^2 - \beta\frac{1}{2}\left(\frac{\beta(2-\hat{e}_1)}{1+\beta}\right)^2 + \beta\left[2\left(e_1 + \frac{\beta(2-\hat{e}_1)}{1+\beta}\right) - \frac{1}{2}\left(e_1 + \frac{\beta(2-\hat{e}_1)}{1+\beta}\right)^2\right]$$

Here, the first-order condition takes the following form:

$$
\begin{aligned}
&-e_1^S + \frac{\beta^3}{(1+\beta)^2}(2-e_1^S) + \beta\left[\frac{2}{1+\beta} - \left(\frac{1}{1+\beta}e_1^S + \frac{2\beta}{1+\beta}\right)\frac{1}{1+\beta}\right] = 0 \\
&\Rightarrow e_1^S = \frac{2\beta(1+\beta^2)}{\beta^3+\beta^2+3\beta+1}
\end{aligned}
$$

With $e_1^S$ determined, $e_2^S$ is given by

$$
e_2^S = \frac{2\beta(1+\beta)}{\beta^3+\beta^2+3\beta+1}
$$

Now we can easily compare the effort choices of a sophisticate agent to those of a naive agent and a time-consistent agent: A sophisticate agent

- increases his efford from period to period, $e_1^S < e_2^S$,
- works less than a time-consistent agent more than a naive agent in period 1, $e_1^N < e_1^S < e^{TC}$,
- exhibits a lower total effort than a time-consistent agent and a higher total effort than a naive agent, $e_1^N + e_2^N < e_1^S + e_2^S < 2e^{TC}$.

$$
\begin{aligned}
e_1^N = \frac{2\beta}{2+\beta} < \frac{2\beta(1+\beta^2)}{\beta^3+\beta^2+3\beta+1} = e_1^S \quad &\Leftrightarrow \quad (1-\beta)^2 > 0 \\
e_1^S = \frac{2\beta(1+\beta^2)}{\beta^3+\beta^2+3\beta+1} < \frac{2}{3} = e^{TC} \quad &\Leftrightarrow \beta < 1
\end{aligned}
$$

$$
e_1^N + e_2^N = \frac{2\beta(1+\beta)+4\beta}{(1+\beta)(2+\beta)} < \frac{2\beta^3+2\beta^2+4\beta}{\beta^3+\beta^2+3\beta+1} = e_1^S + e_2^S \quad \Leftrightarrow \beta(1-\beta)^2 > 0
$$

$$
e_1^S + e_2^S = \frac{2\beta^3+2\beta^2+4\beta}{\beta^3+\beta^2+3\beta+1} < \frac{4}{3} = 2e^{TC} \Leftrightarrow 2\beta(1+\beta) < 4
$$

**Long-Run Preferences:** Following O'Donoghue and Rabin (1999, 2005) we use people's long-run preferences as welfare criterion.

**Definition 1** *A person's long-run preferences are given by* $U_0(e_1, e_2) \equiv -c(e_1) - c(e_2) + g(e_1 + e_2)$.

Long-run preferences reflect a person's preferences when asked from a prior perspective when she has no option to indulge immediate gratification. To formalize this long-run perspective, it is assumed that there is a (fictitiuos) period 0 where a person has no decision to make.[1]

[1] Another possibility would be to apply the Pareto criterion, where one outcome is deemed better than another if and only if the person views it as better at all points in time. A discussion of these two welfare criteria for hyperbolic discounters is provided in O'Donoghue and Rabin (2005).

The long-run utilities for the three types are as follows:

$$U_0^{TC} = -\frac{1}{2}\left(\frac{2}{3}\right)^2 - \frac{1}{2}\left(\frac{2}{3}\right)^2 + 2\frac{4}{3} - \frac{1}{2}\left(\frac{4}{3}\right)^2 = \frac{4}{3}$$

$$U_0^N = -\frac{1}{2}\left(\frac{2\beta}{2+\beta}\right)^2 - \frac{1}{2}\left(\frac{4\beta}{(1+\beta)(2+\beta)}\right)^2 + 2\frac{2\beta(1+\beta)+4\beta}{(1+\beta)(2+\beta)} - \frac{1}{2}\left(\frac{2\beta(1+\beta)+4\beta}{(1+\beta)(2+\beta)}\right)^2 = \frac{8\beta(3+2\beta+\beta^2)}{(1+\beta)^2(2+\beta)^2}$$

$$U_0^N = -\frac{1}{2}\left(\frac{2\beta(1+\beta^2)}{\beta^3+\beta^2+3\beta+1}\right)^2 - \frac{1}{2}\left(\frac{2\beta(1+\beta)}{\beta^3+\beta^2+3\beta+1}\right)^2 + 2\frac{2\beta^3+2\beta^2+4\beta}{\beta^3+\beta^2+3\beta+1} - \frac{1}{2}\left(\frac{2\beta^3+2\beta^2+4\beta}{\beta^3+\beta^2+3\beta+1}\right)^2 = \frac{4\beta(\beta^4+2\beta^3+3\beta^2+4\beta+2)}{(\beta^3+\beta^2+3\beta+1)^2}$$

**Exercise 2:**

a) If a **time consistent** person does one of the two projects, $A$ or $B$, he works in the first and in the second period. His intertemporal utility in period 1 when he does project $A$ is $U_1^{TC}(A,1,2) = -4 - 22 + 40 = 14$. If the TC does project $B$ his utility is $U_1^{TC}(B,1,2) = -8 - 4 + 25 = 13$. When the TC does neither project $A$ nor project $B$ his utility is zero. Hence, a TC starts project $A$ in the first period and finishes it in the second period.

b) In *period 3* a hyperbolic discounter with $\beta = 1/2$ will never complete a project, $-45 + \frac{1}{2}(40) < 0$ and $-15 + \frac{1}{2}(25) < 0$. Consequently, whatever the person has done in the two previos periods the hyperbolic discounter chooses not to work in the third period.

   In *period 2* a **sophisticated** person correctly predicts her behavior in period 3. Therefore, when her first period self has not started a project she will not start a project in period 2 because she realizes that she will not finish the project. Suppose she started project $A$ in the first period, then she can either complete the project in the second period or she does nothing. The corresponding utilities are:

$$U_2^S(A,1,2) = -22 + \frac{1}{2}(40) = -2 < 0 = U_2^S(\text{not working})$$

   Given a sophisticate started project $A$ in the first period she will not finish the project, neither in period 2 nor in period 3. Now, suppose the first period sophisticate worked on project $B$, then she can either finish the project in period 2 or she does nothing. Note that she will not finish project $B$ in the third period.

$$U_2^S(B,1,2) = -4 + \frac{1}{2}(25) = 8,5 > 0 = U_2^S(\text{not working})$$

Therefore, a sophisticate works in period 2 only if the first period self started project $B$. In this case she completes the project in the second period.

*Period 1:* A sophisticate correctly predicts her future behavior, therefore she knows that she will complete a project in the future only when she now starts project $B$.

$$U_1^S(B,1,2) = -8 + \frac{1}{2}(-4+25) = 2,5 > 0 = U_1^S(\text{not working})$$

To sum up, a sophisticated hyperbolic discounter does project $B$. She starts the project in the first period and finishes it in the second period.

c) *Period 1*: A **naif** believes that he will behave as a TC in the future. Therefore there are three possible szenarios: (i) he starts project $A$ in $t = 1$ and finishes it in $t = 2$, his utility is then $U_1^N(A,1,2) = -4 + (1/2)(-22+40) = 5$, (ii) he starts project $B$ in $t = 1$ and finishes the project in $t = 2$, $U_1^N(A,1,2) = -8 + (1/2)(-4+25) = 2,5$, (iii) he enjoys his life in period 1 and works on project $B$ in period 2 and 3 $U_1^N(B,1,2) = 0 + (1/2)(-4-15+25) = 3$. In the first period a naive hyperbolic discounter starts project $A$.

*Period 2*: Note that the naif can not procrastinate finishing the project, because even a TC will not finish project $A$ in period 3. When he finishes project $A$ in period 2 his utility is $U_2^N(A,1,2) = -22 + (1/2)(40) = -2 < 0 = U_2^N(\text{not working})$. Instead of not working he can also start project $B$ in period 2, his utility then is $U_2^N(B,2,39 = -4 + (1/2)(-15+25) = 1$. Thus, a naif does not complete project $A$ in period 2, however, he starts a new project $B$.

*Period 3*: Finishing one of the two started projects leads to a negative utility, $-45 + (1/2((40) < 0$ and $-15 + (1/2)(25) < 0$. Therefore, a naif does not work in the third period. To sum up, a naif starts project $A$ and project $B$ in the first, respectively, second period but does not finish one.

d) **Intuition:** In period 1 a hyperbolic discounter prefers to work on project $A$ in the first two periods. A sophisticate, who is fully aware of her own self control problems, recognizes that she will not complete the project in period 2 and therefore she will never finish that project (Pessimism Effect). A naif in contrary beliefs that he will behave time consistent in the future. A naif wrongly predicts that he will finish the project in period 2. In period 2, however, due to the present biased a naif does not complete project $A$. Still being overly optimistic a naif starts project $B$ in the second period but he does not finish it in period 3. To sum up, naifs tend to start projects when costs are low but maybe never finish these projects. A sophisticate, who is pessimistic about her future behavior, is less likely to start a project. When a sophisticate has started a project, however, she will complete the project.

**Exercise 3:**

**a)** Behavior of a TC

- **Old age:** a TC chooses hit when hooked and he refrains when unhooked.
- **Middle age:** the utilities of a TC given he is unhooked are

$$U_2^{TC}(hit|hooked) = -8 - 23 = -31$$
$$U_2^{TC}(ref|hooked) = -25 + 0 = -25$$

  In the middle age a TC chooses refrain when hooked. TC's utilities when he is unhooked from hitting and refraining, respectively, are:

$$U_2^{TC}(hit|unhooked) = 10 - 23 = -13$$
$$U_2^{TC}(ref|unhooked) = 0 + 0 = 0$$

  A TC always refrains in his middle age.

- **Youth:**

$$U_1^{TC}(hit) = 14 - 25 + 0 = -11$$
$$U_1^{TC}(ref) = 0 + 0 + 0 = 0$$

  TCs refrain throughout their lives.

**b)** Behavior of a naive person

- **Youth:** In the youth a naif believes that he will refrain in the middle age and in the old age. Thus, the percieved utilities of a naif in the first period are

$$U_1^N(hit) = 14 + \frac{1}{2}(-25) = 1.5$$
$$U_1^N(ref) = 0 + \frac{1}{2}(0) = 0$$

  A naif chooses hit in the youth

- **Middle age:** A naif believes that he will hit in the old age when he hits today and that he will refrain in the old age when he refrains today. The utilities of a naif in the middle age are

$$U_2^{TC}(hit|hooked) = -8 + \frac{1}{2}(-23) = -19.5$$
$$U_2^{TC}(ref|hooked) = -25 + \frac{1}{2}(0) = -25$$

  In the middle age a naif chooses hit.

- **Old age:** Clearly, a naif hits in the old age. To sum up, naifs choose to hit throughout their lives.

**c)** Behavior of a sophisticated person

- **Old age:** a sophisticate chooses hit when hooked and he refrains when unhooked.

- **Middle age:** the utilities of a sophisticate given he is unhooked are

$$U_2^S(hit|hooked) = -8 + \frac{1}{2}(-23) = -19.5$$
$$U_2^S(ref|hooked) = -25 + \frac{1}{2}(0) = -25$$

  In the middle age a sophisticate chooses hit when hooked. Sophisticate's utielties when unhooked from hitting and refraining, respectively, are:

$$U_2^S(hit|unhooked) = 10 + \frac{1}{2}(-23) = -1.5$$
$$U_2^S(ref|unhooked) = 0 + \frac{1}{2}(0) = 0$$

  A sophisticate refrains in the middle age when she has refrained in the youth (is unhooked).

- **Youth:**

$$U_1^S(hit) = 14 + \frac{1}{2}(-8 - 23) = -17$$
$$U_1^S(ref) = 0 + \frac{1}{2}(0 + 0) = 0$$

  Sophisticates refrain throughout their lives.

**d) Intuition:** In their youth, sophisticates correctly recognize that hitting now means also hitting in both middle age and old age, whereas refraining now means refraining in both middle age and old age. Since even in their youth sophisticates percieve always hitting to be worse than always refraining, they choose to refrain in their youth. In contrast, a naif mispredicts in his youth the behavior of his future selfs. A naif believes that he will refrain in both middle age and old age no matter what he does today. Importantly, in their youth hyperbolic discounters would most like to hit now and then refrain thereafter. But sophisticates recognize that they are not able to refrain in the middle age when they hit in the youth, therefore sophisticates refrain in their youth in order to induce good behavior in the future (i.e., because of the incentive effect).

The result that sophistication makes addiction less likely highly depends on the nonstationary utilities assumed in the exercise. In stationary environments, where the temptation to consume the addictive good depends on past consumption but not on the specific date, sophistication exacerbates the tendency to get addicted. In stationary environments only the pessimism effect is operative.