Undue Diligence

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Abstract

Modern financial markets increasingly rely on complex financial products. These products often change hands even though the buyers acquire little information about the underlying structure of the financial asset. The greater the complexity of the asset structure, the more opaque it tends to be in the sense that acquiring information about the structure is difficult and thus more costly. But are these opaque assets socially beneficial?

To address this question, we construct a environment in which agents trade assets that have random returns. The buyer of the asset has the opportunity to inspect the asset, at some cost, to assess its fundamental value. In short, the buyer chooses to perform due diligence or not prior to accepting the asset. We use a mechanism design approach to determine when it is socially optimal to have opaque assets and when it is not. We characterize the set of allocations that satisfy sequential participation constraints for both buyers and sellers of the assets.

1 Introduction

Most people consider information as a good, in the sense that more should always be preferred to less. This view is undoubtedly correct from the individual perspective. Since at least Hirshleifer (1971), however, economists have known that more information can sometimes be detrimental from a social perspective. This is perhaps not too surprising from what we know can follow in second-best environments (Lipsey and Lancaster, 1956).

The Hirshleifer perspective leads us to re-examine the issue of nondisclosure practices in the financial sector. Consider, for example, the fact that banks do not always disclose the market value of their assets. Consider as well the fact that bank regulators do not disclose their assessments of individual banks and that central banks do not disclose the identity of those agencies making use of the discount window. In the shadow-banking sector, some of the private assets that circulated as collateral in repo exchanges were criticized as being "too complicated and opaque"–properties that almost seemed purposely designed to discourage due diligence. The question is whether practices like these possess some underlying efficiency rationale.

We think that the answer to this question has some bearing on what we understand to be good properties for exchange media to possess (by exchange media, we include the collateral objects that circulate in the repo market). One desirable property of a monetary instrument is that it passes easily from hand to hand with little or no due diligence. Indeed, the need to inspect an asset before accepting as payment literally destroys its liquidity value.

Gary Gorton calls this property "information insensitivity." U.S. dollars and U.S. Treasury debt are good examples of this property. That is, little if any resources are devoted to inspecting the underlying properties of U.S. debt when it is accepted in payment (or used as collateral in a short-term lending arrangement). This is not the same thing as saying that U.S. debt is risk-free. What it means is that no one has an incentive to generate private information about an asset (the way that is commonly done for equity, for example). Historically, the private sector seems to go to great length to manufacture "informationally insensitive" debt. Private banknotes in the past, for example, were designed as senior claims to bank assets (in addition to being made redeemable on demand for specie). More recently, the AAA rated tranches of MBS that circulated as collateral in the repo market were arguably designed to be "informationally insensitive"–even though this latter example turned out to be a spectacular failure.

Let us now describe what we do in a little more detail. We construct an environment where, because of a limited commitment friction, an asset is necessary to facilitate intertemporal exchange. The return to the asset is stochastic. More importantly, the expected return is stochastic–conditional forecasts are rationally updated as new information arrives (according to a separate, exogenous stochastic process). This new information–or *news*, for short–has, by construction, zero social value.

We assume that society is in possession of an "information switch" that can turn the news flow on or off. Turning the switch off corresponds to the nondisclosure of (privately valuable) information. In our second-best world (limited commitment), bad news events lead to credit crises, with tightening debt constraints. This is because bad news depresses asset prices and hence contracts the market value of collateral. If news is not disclosed, asset prices do not react, and economic welfare is improved. Hence, it is optimal in the environment described thus far to have an "opaque" asset circulate as an exchange medium.

We extend this basic environment in an important and realistic way by assuming that individuals possess their own information gathering technology. If the cost of operating this technology is small enough, individuals will have an incentive to generate information about the asset in the event that the news is not disclosed (information switch is off). We call such an activity "undue diligence" because while information gathering of this sort is privately beneficial, it is socially detrimental.

Our main findings are as follows. If the cost of gathering information is sufficiently high for individuals, then the nondisclosure of information is socially optimal (information switch off). This is because news has no social value and is potentially detrimental, but individuals have no incentive to acquire it when it is privately too costly to do so.

If, on the other hand, the cost of gathering information is sufficiently low, then information disclosure is socially optimal (information switch on). This is because releasing information prevents wasteful information gathering activities. If individuals have a private incentive to acquire information, society may as well make it freely available. This corresponds to rendering an exchange medium perfectly transparent. The result is the occasional credit crunch, but this is the constrained efficient solution.

To sum up in one sentence, this paper advocates institutions to disseminate information only when information acquisition costs for individuals are sufficiently small. Transparent exchange media are needed when it is necessary to prevent undue diligence.

2 The environment

2.1 Basics

Our basic framework is the quasilinear model developed by Lagos and Wright [1]. Time is discrete and the horizon is infinite. Each time period is divided into two subperiods, labeled *day* and *night*.

The economy is populated by two types of agents, labeled b and s and referred to as 'buyers' and 'sellers,' respectively. Agents are infinitely-lived and there is an equal measure (a continuum) of each agent type. Agent types are defined by their characteristics at night. Let $q \in \mathbb{R}_+$ denote output at night. Type b agents want to consume this output, while type s agents have an ability to produce it. Let $u(q)$ denote the utility associated with consumption and $-h(q)$ the utility associated with production. We apply the standard restrictions; $u'' < 0 < u'$ and $h', h'' > 0$. In addition, assume $u(0) = h(0) = 0$, $u'(0) = \infty$ and $h'(0) = 0$. There is a unique $0 < q^* < \infty$ satisfying $u'(q^*) = h'(q^*)$.

During the day, all agents can produce general goods y with a constant returns to scale production technology where one unit is produced with one unit of labor generating one unit of disutility. Consumption of general goods yields utility $U(y) = y$ where negative values are interpreted as production. All agents have a common discount factor $0 < \beta < 1$. The objectives, for type b

and s agents respectively, are given by:

$$W_b = E_0 \sum_{t=0}^{\infty} \beta^t \left[y_{b,t} + u(q_{b,t})\right] \quad (1)$$

$$W_s = E_0 \sum_{t=0}^{\infty} \beta^t \left[y_{s,t} - h(q_{s,t})\right] \quad (2)$$

Agents are centrally located during the day, but disperse to form random pairwise meetings at night. Agents of different types meet each other with probability one at night. The match process is such that individuals will (almost surely) never meet each other again.[1]

There is a single asset in the economy; a "Lucas tree." The asset generates a stochastic flow of nondurable output $z_t \geq 0$ at the beginning of each day. For simplicity, this dividend is assumed to take on two values, $0 \leq z^l < z^h < \infty$. Let $\pi \equiv \Pr\left[z_t = z^h\right]$ denote the probability of a high dividend, and let $\bar{z} \equiv \pi z^h + (1-\pi) z^l$ denote the unconditional expected level of the dividend flow.

Feasibility at night requires $q_{b,t} = q_{s,t} = q_t \geq 0$ for all t. Restricting attention to stationary allocations, and assuming that buyers and sellers are treated symmetrically within their type, the resource constraints in the day are given by:

$$y_s^h + y_b^h = z^h \quad (3)$$

$$y_s^l + y_b^l = z^l \quad (4)$$

Defining $\bar{y}_s = \pi y_s^h + (1-\pi) y_s^l$ and $\bar{y}_b = \pi y_b^h + (1-\pi) y_b^l$, these resource constraints imply

$$\bar{y}_s + \bar{y}_b = \bar{z} \quad (5)$$

An *allocation* (stationary) is denoted by the list $\left(q^l, q^h, y_s^l, y_s^h, y_b^l, y_b^h\right)$. A *feasible allocation* is denoted by $\left(q^l, q^h, y_s^l, y_s^h\right)$ since it is an allocation that satisfies $q \geq 0$, (3), and (4).

We distinguish between two types of allocations; one in which q is state-contingent (separating) and one in which it is not (pooling). A *pooling allocation* is an allocation where $q^l = q^h = q$. In a pooling allocation, agents do not know the dividend state but they can calculate the expected payoff $\bar{y}_s$ and $\bar{y}_b$ since they know y_s^l, y_s^h, y_b^l, y_b^h and the probability distribution π. To reduce notation, we denote a pooling allocation by the tuple $(q, \bar{y}_s)$. A *separating allocation* is an allocation with $q^l \neq q^h$. In any separating allocation, agents know the dividend state.

Finally, an *efficient allocation* is a feasible allocation that satisfies $q^l = q^h = q^*$, where q^* satisfies $u'(q^*) = h'(q^*)$. It is then clear that an efficient allocation is always a pooling allocation.

[1] This assumption rules out bilateral intertemporal trading relationships.

This basic environment will subsequently be modified to incorporate an inspection technology; see Section 4.

3 Incentive-feasible allocations

An *incentive-feasible* allocation is a feasible allocation that satisfies a set of sequential participation (SP) constraints. We now describe these sets for the pooling allocations and then for the separating allocations.

3.1 Incentive-feasible pooling allocations

Because sellers incur sacrifice (production) at night, and buyers incur sacrifice in the day, there are two relevant SP constraints to consider. Consider a feasible pooling allocation $(q, \bar{y}_s)$. The SP constraint for a seller at night is given by:

$$-h(q) + \beta(\bar{y}_s + V_s) \geq \beta^T V_s \tag{6}$$

where

$$V_s = \frac{-h(q) + \beta\bar{y}_s}{1-\beta} \tag{7}$$

Here, V_s is the continuation value for a seller who follows a recommended pooling allocation. The parameter T governs the severity of the punishment associated with noncompliance. Setting $T = \infty$, for example, implies perpetual ostracism. Combining (6) and (7) yields,[2]

$$-h(q) + \beta\bar{y}_s \geq 0 \tag{8}$$

Let $S \equiv \{(q, \bar{y}_s) : -h(q) + \beta\bar{y}_s \geq 0\}$ denote the set of allocations $(q, \bar{y}_s)$ that induce SP for the seller. Given the assumed properties for h, the set S is clearly non-empty and strictly convex.

The SP constraint for the buyer in the day is given by,

$$y_b^i + u(q) + \beta W_b \geq \beta^T W_b \text{ for } i \in \{l, h\} \tag{9}$$

where

$$W_b = \frac{\bar{y}_b + u(q)}{1-\beta} \tag{10}$$

Here, W_b is the continuation value for a buyer who follows a recommended pooling allocation.

[2] To see this, use (7) to replace V^s in (6) to get

$$[-h(q) + \bar{y}_s]\left(1 + \frac{\beta - \beta^T}{1-\beta}\right) \geq 0.$$

Since $\left(1 + \frac{\beta-\beta^T}{1-\beta}\right) > 0$, we get (8).

Lemma 1 *If a feasible pooling allocation $(q, \bar{y}_s)$ satisfies the sequential participation constraint for the buyer (9), it also satisfies the inequality*

$$\bar{y}_s \leq \bar{z} + u(q) \tag{11}$$

Proof. A buyer is just willing to accept a pooling allocation if $y_b^h + u(q) + \left(\beta - \beta^T\right) W_b = 0$ in state h and $y_b^l + u(q) + \left(\beta - \beta^T\right) W_b = 0$ in state l.[3] Thus, to derive the set of pooling allocations that make him indifferent between participating and not participating, we can set $y_b^l = y_b^h = \bar{y}_b = -u(q) - \left(\beta - \beta^T\right) W_b$. If $y_b^l = y_b^h = \bar{y}_b$, he is accepting if

$$\bar{y}_b + u(q) + \beta W_b \geq \beta^T W_b \tag{12}$$

This condition also applies if $y_b^l \neq y_b^h$. Why is this? Consider a pooling allocation with $y_b^l \neq y_b^h$ and associated expected value $\bar{y}_b = \pi y_b^h + (1 - \pi) y_b^l$. If condition (9) holds for $i = l, h$, then (12) will also hold. If, on the other hand, condition (9) is violated for $i = l, h$, then there is no combination of (y_b^l, y_b^h) that yield the same expected value $\bar{y}_b$ and satisfies (12). Finally, if condition (9) holds for only one $i = l, h$ and (12) holds then it is possible to transfer the reward in the slack state to the tight state without altering $\bar{y}_b$. In particular, one may as well set $y_b^l = y_b^h = \bar{y}_b$.

Thus, we can set $y_b^l = y_b^h = \bar{y}_b$ without loss in generality. Then conditions (12) and (10) imply $\bar{y}_b + u(q) \geq 0$. Finally, replacing $\bar{y}_b = \bar{z} - \bar{y}_s$, we get (11). ∎

Definition 2 *An incentive-feasible pooling allocation is a feasible allocation $(q, \bar{y}_s)$ that satisfies the SP constraints (8) and (11).*

Let $B \equiv \{(q, \bar{y}_s) : \bar{y}_s \leq \bar{z} + u(q)\}$ denote the set of allocations $(q, \bar{y}_s)$ that induce SP for the buyer. Given the assumed properties of u, the set B is clearly non-empty and convex.

The set of incentive-feasible pooling allocations is given by $S \cap B$; which is clearly a non-empty and compact set. Together, conditions (8) and (11) imply that any incentive-feasible pooling allocation $(q, \bar{y}_s)$ must satisfy,

$$\frac{1}{\beta} h(q) \leq \bar{y}_s \leq \bar{z} + u(q) \tag{13}$$

Definition 3 *Let $\Psi(q) \equiv \beta[\bar{z} + u(q)] - h(q)$ and define $q_m > 0$ by $\Psi(q_m) = 0$.*

Note that the quantity $q_m > 0$ is unique since Ψ is strictly concave with $\Psi(0) = \bar{z}$.

[3] Lemma 1 also applies for a separating allocation since the dividende state is observed at the beginning of the day.

Proposition 4 *An incentive-feasible pooling allocation* $(q, \bar{y}_s)$ *exists for any* $q \in [0, q_m]$.

Proof. Given the properties of $\Psi(q)$, there is an unique maximizer $\hat{q}$ that solves $\beta u'(\hat{q}) - h'(\hat{q}) = 0$. It follows that Ψ is increasing in $q \in (0, \hat{q})$ and decreasing for $q > \hat{q}$. For q_m, the incentive feasible allocation is unique and satisfies $(q, \bar{y}_s) = [q_m, \bar{z} + u(q_m)]$. For any $0 \leq q < q_m$, condition (13) implies $h(q) < \beta[\bar{z} + u(q)]$ and so there exists a continuum of incentive-feasible pooling allocations $(q, \bar{y}_s)$ with $\bar{y}_s \in \left[\frac{1}{\beta}h(q), \bar{z} + u(q)\right]$. The expected transfer $\bar{y}_s$ only affects how the surplus is shared. ■

The quantity q_m is the maximal value of q such that the SP constraints of buyers and sellers hold. Whether q^* belongs to this set depends on parameters. Evidently, if $q^* \leq q_m$, the first-best quantity can be implemented. Condition (13) suggests that we can choose parameters β and $\bar{z}$ so that the following condition holds:

$$\beta^{-1}h(q^*) = \bar{y}_s^* = \bar{z} + u(q^*) \tag{14}$$

That is, condition (14) identifies parameters such that the efficient allocation is just implementable; i.e., it leaves both seller and buyer on their respective SP constraints. In what follows, we will use this parameterization as a benchmark and refer to it as assumption [**A1**].

[**A1**] $\beta = \beta^* \equiv \frac{h(q^*)}{\bar{z}+u(q^*)}$ (The efficient allocation is just implementable.)

It is clear that if $\beta \geq \beta^*$, then q^* is implementable.

3.2 Incentive-feasible separating allocations

Consider a feasible separating allocation $\left(q^l, q^h, y_s^h, y_s^l\right)$. Let $\bar{h}\left(q^l, q^h\right) \equiv \pi h\left(q^h\right) + (1-\pi) h\left(q^l\right)$ and $\bar{u}\left(q^l, q^h\right) \equiv \pi u\left(q^h\right) + (1-\pi) u\left(q^l\right)$, where we sometimes omit the arguments and only write $\bar{h}$ and $\bar{u}$. In a separating allocation agents know the dividend state at night, but not in the day.

The set of state-contingent SP constraints for the seller at night are given by:

$$-h\left(q^i\right) + \beta\left(y_s^i + V_s\right) \geq \beta^T V_s,\ i = l, h \tag{15}$$

where $V_s = (1-\beta)^{-1}\left(-\bar{h} + \beta\bar{y}_s\right)$ is the continuation value for a seller who follows a recommended separating allocation.

The set of state-contingent SP constraints for the buyer in the day are given by:

$$y_b^i + \bar{u} + \beta W_b \geq \beta^T W_b \text{ for } i \in \{l, h\} \tag{16}$$

where

$$W_b = \frac{\bar{y}_b + \bar{u}}{1-\beta} \tag{17}$$

Here, W_b is the continuation value for a buyer who follows a recommended separating allocation. Note that $\bar{u}$ replaces $u(q)$ in (16) because the buyer does not yet know the state that will prevail at night.

Lemma 5 *If a feasible separating allocation (q^l, q^h, y_s^l, y_s^h) satisfies the sequential participation constraint for the buyer (16), it also satisfies the inequality*

$$\bar{y}_s \leq \bar{z} + \bar{u} \tag{18}$$

Proof. A buyer is just willing to accept a separating allocation if $y_b^h + \bar{u} + \left(\beta - \beta^T\right) W_b = 0$ in state h and $y_b^l + \bar{u} + \left(\beta - \beta^T\right) W_b = 0$ in state l.[4] Thus, to derive the set of separating allocations that make him indifferent between participating and not participating, we can set $y_b^l = y_b^h = \bar{y}_b = -\bar{u} - \left(\beta - \beta^T\right) W_b$. If $y_b^l = y_b^h = \bar{y}_b$, he accepts if

$$\bar{y}_b + \bar{u} + \beta W_b \geq \beta^T W_b \tag{19}$$

This condition also applies if $y_b^l \neq y_b^h$. Why is this? Consider a pooling allocation with $y_b^l \neq y_b^h$ and associated expected value $\bar{y}_b = \pi y_b^h + (1-\pi) y_b^l$. If condition (9) holds for $i = l, h$, then (12) will also hold. If, on the other hand, condition (9) is violated for $i = l, h$, then there is no combination of (y_b^l, y_b^h) that yield the same expected value $\bar{y}_b$ and satisfies (12). Finally, if condition (9) holds for only one $i = l, h$ and (12) holds then it is possible to transfer the reward in the slack state to the tight state without altering $\bar{y}_b$. In particular, one may as well set $y_b^l = y_b^h = \bar{y}_b$.

Thus, we can set $y_b^l = y_b^h = \bar{y}_b$ without loss in generality. Then conditions (12) and (10) imply $\bar{y}_b + \bar{u} \geq 0$. Finally, replacing $\bar{y}_b = \bar{z} - \bar{y}_s$, we get (18). ■

Definition 6 *An incentive-feasible separating allocation is a feasible allocation $\left(q^l, q^h, y_s^l, y_s^h\right)$ with $q^l \neq q^h$ that satisfies the SP constraints (15) and (18).*

We want to characterize the set of incentive-feasible separating allocations. To do this, we assign zero surplus to the buyer. In this manner, the seller can be allocated maxiumum incentive to produce output (q^l, q^h) at night. Consequently, we set

$$\bar{y}_s = \bar{z} + \bar{u} \tag{20}$$

in what follows.

[4] Lemma 1 also applies for a separating allocation since the dividende state is observed at the beginning of the day.

Furthermore, the resource constraints then imply $y_s^l = z^l + \bar{u}$ and $y_s^h = z^h + \bar{u}$. We can use (20) to write the state-contingent SPs for the seller as follows:

$$-h\left(q^h\right) + \beta\left(\bar{u} + z^h\right) \geq \left(\beta^T - \beta\right) V_s \quad (21)$$

$$-h\left(q^l\right) + \beta\left(\bar{u} + z^l\right) \geq \left(\beta^T - \beta\right) V_s \quad (22)$$

Let

$$\begin{aligned} S^h &\equiv \left\{(q^l, q^h) : -h\left(q^h\right) + \beta\left(\bar{u} + z^h\right) - \left(\beta^T - \beta\right) V_s \geq 0\right\} \\ S^l &\equiv \left\{(q^l, q^h) : -h\left(q^l\right) + \beta\left(\bar{u} + z^l\right) - \left(\beta^T - \beta\right) V_s \geq 0\right\} \end{aligned}$$

denote the set of quantities (q^l, q^h) that induce SP for the seller in states l and h, respectively. Given the assumed properties for $h(q)$ and $u(q)$, the set S^l and S^h are clearly non-empty and strictly convex. The set of incentive-feasible quantities is given by $S^l \cap S^h$; which is clearly a non-empty and compact set.

3.3 Constrained-efficient allocation

In this section, we characterize the constrained-efficient allocation. To do so, we need to compare the welfare properties of the pooling and separating allocations. It is clear that for $\beta \geq \beta^*$, an incentive-feasible pooling allocation with $q^l = q^h = q^*$ exists. In this case, the constrained-efficient allocation is a pooling allocation with $q^l = q^h = q^*$. When $\beta < \beta^*$ the nature of the constrained-efficient allocation is less obvious.

We use the discounted expected utility to compare the welfare properties of different allocations. For any allocation, our welfare criterion is

$$W\left(q^l, q^h\right) \equiv \left[\bar{u}(q^l, q^h) - \bar{h}(q^l, q^h) + \bar{z}\right](1 - \beta)^{-1} \quad (23)$$

Proposition 7 *The constrained-efficient allocation is a pooling allocation with*

$$q^l = q^h = \begin{cases} q^* & \text{if} \quad \beta \geq \beta^* \\ q_m & \text{if} \quad \beta < \beta^* \end{cases}$$

Proof. For $\beta \geq \beta^*$, an incentive-feasible pooling allocation with $q^l = q^h = q^*$ exists. Hence, the constrained-efficient allocation is a pooling allocation with $q^l = q^h = q^*$. From Lemma 1, if $\beta < \beta^*$ the constrained-efficient pooling allocation satisfies

$$W(q_m, q_m) \equiv [u(q_m) - h(q_m) + \bar{z}](1 - \beta)^{-1},$$

where q_m satisfies

$$\beta[\bar{z} + u(q_m)] - h(q_m) = 0.$$

From (21) and (22), the constrained-efficient separating allocation satisfies

$$-\bar{h} + \beta\left(\bar{u} + \bar{z}\right) \geq 0. \tag{24}$$

To see this, multiply both sides of (21) by π and both sides of (22) by $1 - \pi$. Then, add the two equations and rearrange to get

$$-\bar{h} + \beta\left(\bar{u} + \bar{z}\right) \geq \left(\beta^T - \beta\right) V_s$$

Next, replace V_s and rearrange to get

$$\left[-\bar{h} + \beta\left(\bar{u} + \bar{z}\right)\right]\left(1 + \frac{\beta - \beta^T}{1 - \beta}\right) \geq 0.$$

Since $1 + \left(\beta - \beta^T\right)(1 - \beta)^{-1} > 0$, condition (24) follows immediately.

Concavity of $u(q)$ and convexity of $h(q)$ imply that any separating allocation satisfies

$$-h\left(\bar{q}\right) + \beta\left[u\left(\bar{q}\right) + \bar{z}\right] > -\bar{h} + \beta\left(\bar{u} + \bar{z}\right) \geq 0,$$

where $\bar{q} = \pi q^h + (1 - \pi) q^l$. The term on the LHS of the strict inequality is equal to zero at $\bar{q} = q_m$. But this implies that in any separating allocation, we must have $\bar{q} < q_m$. Welfare in any separating allocation satisfies

$$\left[u(\bar{q}) - h(\bar{q}) + \bar{z}\right](1 - \beta)^{-1} > \left(\bar{u} - \bar{h} + \bar{z}\right)(1 - \beta)^{-1}.$$

Then, since $\bar{q} < q_m$ we have

$$\left[u(q_m) - h(q_m) + \bar{z}\right](1 - \beta)^{-1} > \left[u(\bar{q}) - h(\bar{q}) + \bar{z}\right](1 - \beta)^{-1} > \left(\bar{u} - \bar{h} + \bar{z}\right)(1 - \beta)^{-1}.$$

Thus, we have just proved that if $\beta < \beta^*$ the constrained-efficient allocation is a pooling allocation with $q = q_m$. ■

The results to this point suggest that nondisclosure is optimal (pooling is efficient). Several other papers have similar results (cite). But they abstract from information acquisition. While information here has no social benefit, it does possess private benefit. And one would expect that agents might want to acquire this information, if it is not too costly to do so.

To explore this, we endow sellers with information acquisition technology and explore the implications. In particular, does nondisclosure continue to be socially desirable?

4 Information acquisition

Imagine now that there is an information acquisition technology available to each agent at the beginning of each night. The utility cost $\gamma \geq 0$ is incurred if the technology is operated. Operating the technology generates information about the dividend to be realized the next day. For simplicity, assume that the signal thus received reveals the future dividend with exact precision. Assume that it is in no way possible to communicate this information publicly.

Except for actions relating to information acquisition, individual trading histories are observable. Thus, whether an agent chooses to operate the information acquisition technology, together with what he learns from its operation, remains private information. To be specific, we are assuming that nothing related to information acquisition (including the information itself) can be communicated to the public (though communication within a match remains feasible).

In this environment, information acquisition constitutes a social waste. The information has no social benefit and there is an acquisition cost. Nevertheless, there may exist private incentives to acquire information. Moreover, its acquisition may inhibit *ex ante* efficient trades from occurring. What is the intuition for this?

Consider a incentive-feasible pooling allocation with $y_s^l < y_s^h$. Note that the constrained-efficient allocation under [A1] is one such allocation. For any such allocation, the seller is asked to produce q at night, essentially, in exchange for promised (expected) utility $\bar{y}_s$ the next day. One property of this allocation is that, *ex post*, the seller faces a greater reward in the high-dividend state. Consequently, it is conceivable that the seller might not want to produce at level q if he knew beforehand that the asset is expected to generate a low yield. If the cost of acquiring this information is sufficiently low, then he will have an incentive to acquire it for the purpose of rejecting the trade in the event of "bad news."[5]

4.1 Incentive-feasible pooling allocations with inspection technology

The incentive to acquire information will be absent if and only if,

$$-h(q) + \beta\left(\bar{y}_s + V_s\right) \geq -\gamma + \pi\left[-h(q) + \beta\left(y_s^h + V_s\right)\right] + (1-\pi)\beta^T V_s \tag{25}$$

The LHS denotes the utility payoff from accepting the recommended pooling allocation without information acquisition. The RHS denotes the utility payoff associated with information acquisition. Gathering information entails the direct cost γ to the seller. The recommendation is then accepted only in the event of "good news." That is, it can never be optimal to gather costly information and then not condition future behavior on it. Note that rejecting the

[5]Note that the buyer has no incentive to acquire information. Why is this? Loosely, the buyer does not have to worry about the credibility of a daytime reward (the way the seller must). The buyer will, in general, have to be punished in the day and he can deal with his participation choice at the beginning of the day, after productivity is realized.

recommendation entails the punishment $\beta^T V_s$.

Combining (7) with (25), the "no inspection condition" (NIC) can be rewritten as follows,

$$\gamma \geq (1-\pi)\left\{-\left[\beta y_s^l - h(q)\right] - \left(\frac{\beta-\beta^T}{1-\beta}\right)\left[\beta \bar{y}_s - h\left(q\right)\right]\right\} \tag{26}$$

Thus, the greater the expected period surplus accruing to the seller $[\beta \bar{y}_s - h\left(q\right)]$, the easier it will be to dissuade him from gathering information. Furthermore, the greater the period surplus in the l-state accruing to the seller $\left[\beta y_s^l - h(q)\right]$, the easier it will be to dissuade him from gathering information.

Definition 8 *An incentive-feasible pooling allocation that does not promote inspection by the seller is a feasible allocation $(q, \bar{y}_s)$ that satisfies the SP constraints (8) and (9), and the NIC (26).*

It is clear that the set of incentive-feasible allocations that do not promote inspection by the seller is a subset of $S \cap B$ since we added condition (26). The question is whether it is a strict subset. The answer is yes if the inspection cost is sufficiently low. To see this, we can rewrite (26) along the same lines as the SP constraint of the seller. That is, we smooth the daytime reward (punishment) for the buyer, so that $y_b^l = y_b^h = \bar{y}_b$. This is without loss in generality since, for a given q, the right-hand side of (26) is minimized by choosing the largest transfers y_s^l and y_s^h that are consistent with the buyer's SP constraint. As before, this implies that $y_s^l = z^l - \bar{y}_b = z^l - (\bar{z} - \bar{y}_s)$. Replacing y_s^l in (26) and rearranging yields

$$\bar{y}_s \geq \beta^{-1} h\left(q\right) + \beta^{-1}\left(\frac{\bar{\gamma}-\gamma}{\Omega_T - \pi}\right) \tag{27}$$

where $\Omega_T \equiv 1 + (1-\pi)\left(\beta - \beta^T\right)(1-\beta)^{-1} \geq 1$ and $\bar{\gamma} \equiv \pi\left(1-\pi\right)\beta\left(z^h - z^l\right)$.

Let $I \equiv \left\{(q, \bar{y}_s) : \bar{y}_s - \beta^{-1} h\left(q\right) - \beta^{-1}\left(\frac{\bar{\gamma}-\gamma}{\Omega_T - \pi}\right) \geq 0\right\}$ denote the set of allocations $(q, \bar{y}_s)$ that induce no inspection by the seller. Given the assumed properties for h, the set I is clearly non-empty and strictly convex.

Proposition 9 *If $\gamma < \bar{\gamma}$, $I \subset S$ and $I \cap B \subset S \cap B$*

The proof of Proposition 1 is obvious since the seller's SP constraint is $\bar{y}_s \geq \beta^{-1} h\left(q\right)$. Hence, if $\gamma < \bar{\gamma}$, $I \subset S$ which immediately implies that $I \cap B \subset S \cap B$.

From Proposition 1, if $\gamma < \bar{\gamma}$, an incentive-feasible pooling allocation that does not promote inspection by the seller is in $I \cap B$. Hence, conditions (11) and (27) imply that any incentive-feasible pooling allocation that does not promote inspection by the seller must satisfy,

$$\beta^{-1}h\left(q\right)+\beta^{-1}\left(\frac{\bar{\gamma}-\gamma}{\Omega_T-\pi}\right)\leq\bar{y}_s\leq\bar{z}+u(q) \tag{28}$$

Definition 10 *Define* $\Psi_1\left(q\right)\equiv\bar{z}+u(q_0)-\beta^{-1}h\left(q_0\right)-\beta^{-1}\left(\frac{\bar{\gamma}-\gamma}{\Omega_T-\pi}\right)$ *and let* q_0 *denote the maximum* q *satisfying* $\Psi_1(q_0)=0$.

Lemma 11 *If* $\gamma<\bar{\gamma}$ *there exists an incentive-feasible pooling allocation for any* $q\in[0,q_0]$ *that does not promote inspection by the seller.*

Proof. Note that $\Psi_1\left(q\right)$ has an unique maximum at $\hat{q}\in(0,q_0)$ satisfying $\beta u'\left(\hat{q}\right)-h'\left(\hat{q}\right)=0$. Therefore $\Psi_1\left(q\right)$ is increasing over $q<\hat{q}$ and decreasing over $q>\hat{q}$. At q_0, the incentive-feasible pooling allocation that does not promote inspection by the seller is unique and satisfies $\left(q,y_s^l,y_s^h\right)=[q_0,\bar{z}+u(q_0)]$. For any $0\leq q<q_0$, $\frac{1}{\beta}h(q)<\bar{z}+u(q)$ and so there exists a continuum of incentive-feasible allocations that do not promote inspection by the seller which only differ in the expected transfer $\bar{y}_s$. ■

The quantity q_0 is the maximal value of q that is incentive-feasible and does not promote inspection by the seller. Whether q^* belongs to the set $[0,q_0]$ depends on parameters. If $q^*\leq q_0$ then the first-best quantity is obviously implementable. Note that condition (28) suggests that we can choose parameters β and $\bar{z}$ so that the following condition holds:

$$\beta^{-1}h\left(q^*\right)+\beta^{-1}\left(\frac{\bar{\gamma}-\gamma}{\Omega_T-\pi}\right)=\bar{y}_s^*=\bar{z}+u(q^*) \tag{29}$$

That is, condition (29) identifies parameters such that the efficient allocation is just implementable (it leaves the buyer on his SP constraint) and is just sufficient to discourage inspection by the seller. It is then clear that q^* is implementable iff

$$\beta\geq\left[\frac{h(q^*)+\left(\frac{\bar{\gamma}-\gamma}{\Omega_T-\pi}\right)}{\bar{z}+u(q^*)}\right]\equiv\beta^{**}$$

Note that $\beta^{**}>\beta^*$ if $\gamma<\bar{\gamma}$ and $\beta^{**}=\beta^*$ if $\gamma=\bar{\gamma}$ since $\Omega_T-\pi>0$. Thus, if $\beta\in[\beta^*,\beta^{**})$ and $\gamma<\bar{\gamma}$, then q^* is implementable in the absence of an inspection technology while it is not implementable if sellers have access to such a technology. Furthermore, if we impose condition [A1], then for any $\gamma<\bar{\gamma}$ the efficient quantity is not implementable since it violates the NIC (26).

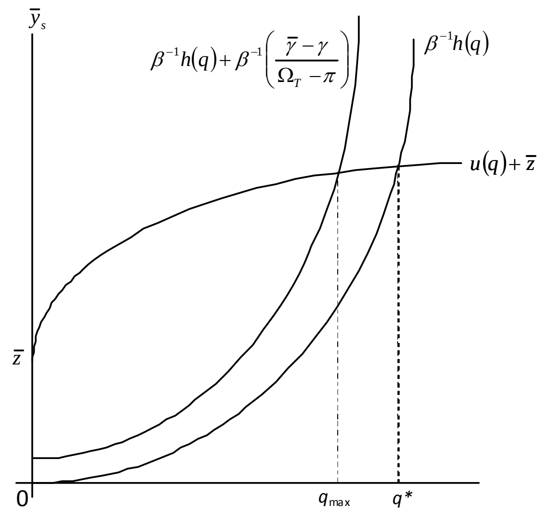


Figure 1: Incentive-feasible pooling allocations when $\gamma < \bar{\gamma}$.
change $q_{\max}$ to q_0 and mention that $q^* = q_m$

4.2 Constrained-efficient pooling allocation

In this section, we continue to restrict our attention to incentive-feasible pooling allocations that do not promote information acquisition. We impose condition [A1]. Recall that [A1] implies that the efficient quantity q^* is just implementable when $\gamma = \bar{\gamma}$ and it is not when $\gamma < \bar{\gamma}$.

Definition 12 *Let q_1 satisfy $\beta\left[\bar{z} + u(q_1)\right] - h\left(q_1\right) - \max\left[0, \frac{\bar{\gamma}-\gamma}{\Omega_T-\pi}\right] = 0$.*

Note that when $\gamma < \bar{\gamma}$, then q_1 satisfies $\Psi_1(q_1) = 0$; so that $q_1 = q_0$. When $\gamma \geq \bar{\gamma}$, then q_1 satisfies $\Psi(q_1) = 0$; in which case, $q_1 = q_m$. Moreover, recall that $q_m = q^*$ under [A1]. It follows that,

Lemma 13 *Under [A1], the constrained-efficient pooling allocation that does not promote inspection by the seller satisfies*

$$(q, \bar{y}_s) = [q_1, \bar{z} + u(q_1)] \tag{30}$$

4.3 Constrained-efficient separating allocations

We are now ready to characterize the constrained-efficient separating allocation. To this end, let $(q^l, q^h) = (q_0^l, q_0^h)$ be the quantities that solve (21) and (22) at equality.

Lemma 14 *The quantities* (q_0^l, q_0^h) *are unique and implement the allocation*

$$\left(q^l, q^h, y_s^l, y_s^h\right) = \left[q_0^l, q_0^h,\ z^l + \bar{u}(q_0^l, q_0^h),\ z^h + \bar{u}(q_0^l, q_0^h)\right] \tag{31}$$

where $q_0^l < q_0^h$ *solve*

$$-h\left(q_0^h\right) + \beta\left[\bar{u}(q_0^l, q_0^h) + z^h\right] = 0 \tag{32}$$

$$-h\left(q_0^l\right) + \beta\left[\bar{u}(q_0^l, q_0^h) + z^l\right] = 0 \tag{33}$$

Proof. The quantities q^l and q^h solve (21) and (22) at equality. That is,

$$-h\left(q^h\right) + \beta\left(\bar{u} + z^h\right) = \left(\beta^T - \beta\right) V_s \tag{34}$$

$$-h\left(q^l\right) + \beta\left(\bar{u} + z^l\right) = \left(\beta^T - \beta\right) V_s \tag{35}$$

Such as solution exists and is unique. To see this, multiply (34) by π and (35) by $1 - \pi$, and add the two equations to get

$$-\bar{h}(q_0^l, q_0^h) + \beta\left[\bar{u}(q_0^l, q_0^h) + \bar{z}\right] = \left(\beta^T - \beta\right) V_s.$$

Since $V_s = -\bar{h}(q_0^l, q_0^h) + \beta\left[\bar{u}(q_0^l, q_0^h) + \bar{z}\right](1-\beta)^{-1}$, we get $(1-\beta) V_s = \left(\beta^T - \beta\right) V_s$, which implyies that $V_s = 0$; i.e.,

$$-\bar{h}(q_0^l, q_0^h) + \beta\left[\bar{u}(q_0^l, q_0^h) + \bar{z}\right] = 0$$

Finally, setting $V_s = 0$ in (34) and (35) yields (32) and (33). It is then straightforward to show that (32) and (33) have a unique solution. (Do we wish to elaborate? Do we need conditions on z_l?) ■

It is tempting to suspect that the quantities (q_0^l, q_0^h) are the welfare maximizing quantities among the set of all incentive-feasible separating allocations. In fact, this is true in many cases. For example, one can show that it is true when $T = 1$. However, it is not true in general for two reasons.

First, one can have $q_0^l < q^* < q_0^h$. In this case, in can be optimal to reduce q_0^h even when it might involve reducing q_0^l too. Second, even when $q_0^l < q_0^h < q^*$, there are cases where one can reduce q_0^h and increase q_0^l and attain a better allocation. Finally, note that under [A1] it can never be the case that $q^* < q_0^l < q_0^h$ since in any separating allocation $\bar{q} < q_1 = q^*$.

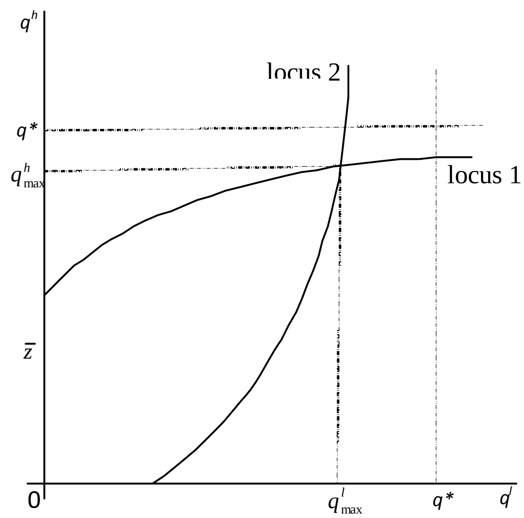


Figure 2: Separating equilibrium
Change $q_{\max}$

4.4 Constrained efficient allocation with inspection technology

For the following comparison, we do not need to know the exact details of the constrained-efficient separating allocation. Since the set $S^l \cap S^h$ is non-empty and compact, we know that such an allocation exists; we denote it by,

$$\left(q^l, q^h, y^l_s, y^h_s\right) = \left[q^l_*,\, q^h_*,\, z^l + \bar{u}(q^l_*, q^h_*),\, z^h + \bar{u}(q^l_*, q^h_*)\right] \tag{36}$$

For the purpose of comparison, it is sufficient to know that the quantities (q^l_0, q^h_0) will never yield a strictly higher level of welfare than the constrained-efficient quantities (q^l_*, q^h_*).

We have two candidates for the constrained-efficient allocation. Either it is (30) or (36). Welfare for the constrained-efficient pooling allocation is

$$W\left(q_1, q_1\right) = \left[u(q_1) - h(q_1) + \bar{z}\right](1-\beta)^{-1}$$

Using (28) at equality, we can write this as follows:

$$W\left(q_1, q_1\right) = \beta^{-1}\left[h(q_1) + \frac{\bar{\gamma} - \gamma}{\left(\Omega_T - \pi\right)(1-\beta)}\right] \tag{37}$$

Welfare for the constrained-efficient separating allocation is

$$W\left(q^l_*, q^h_*\right) = \left[\bar{u}(q^l_*, q^h_*) - \bar{h}(q^l_*, q^h_*) + \bar{z}\right](1-\beta)^{-1}$$

In what follows we use the fact that

$$W\left(q_*^l, q_*^h\right) \geq W\left(q_1^l, q_1^h\right) = \left[\bar{u}(q_1^l, q_1^h) - \bar{h}(q_1^l, q_1^h) + \bar{z}\right](1-\beta)^{-1}$$

Using (34) and (35), we can write $W\left(q_1^l, q_1^h\right)$ as follows:

$$W\left(q_1^l, q_1^h\right) \equiv \bar{h}(q_1^l, q_1^h)\beta^{-1} \tag{38}$$

Proposition 15 *Under [A1], there exists a critical value* $0 < \hat{\gamma} < \bar{\gamma}$*, such that if* $\bar{\gamma} \geq \gamma \geq \hat{\gamma}$*, the constrained-efficient allocation is (30) and if* $\gamma \leq \hat{\gamma}$*, the constrained-efficient allocation is (36).*

Proof. From Lemma 3, if $\gamma = \bar{\gamma}$, we have $q_1 = q^*$ so that the constrained-efficient allocation is the pooling allocation with $q^l = q^h = q^*$.

Next note that $W(q_1, q_1)$ is strictly increasing in $\gamma < \bar{\gamma}$. To see this, note that from (28) at equality we get

$$\frac{dq_1}{d\gamma} = -\frac{1}{\left[\beta u'(q_1) - h'(q_1)\right](\Omega_T - \pi)} > 0$$

since $\beta u'(q_1) - h'(q_1) < 0$. Then,

$$\frac{dW(q_1, q_1)}{d\gamma} = \left[u'(q_1) - h'(q_1)\right]\frac{dq_1}{d\gamma} > 0$$

since $q_1 < q^*$ if $\gamma < \bar{\gamma}$. Hence, there exists a unique value of $\gamma < \bar{\gamma}$, denoted by $\hat{\gamma}$, such that $W(q_1, q_1) = W\left(q_*^l, q_*^h\right)$ because $W\left(q_*^l, q_*^h\right)$ is independent of γ.

We now establish that $\hat{\gamma} > 0$. From (28), at $\gamma = 0$, q_1 satisfies

$$\beta\pi\left(z^h - z^l\right) = -h(q_1) + \beta\left[u(q_1) + \bar{z}\right].$$

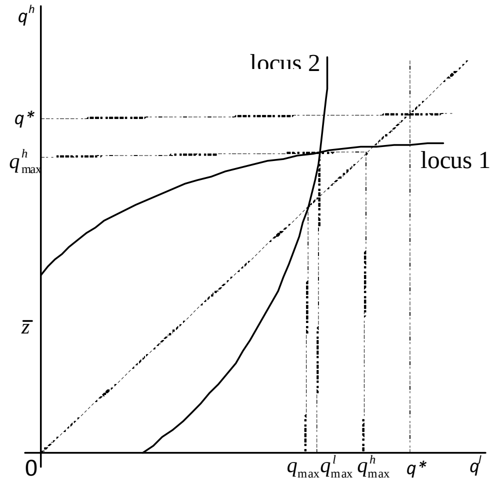
Next, consider (35) at equality. Setting $q^l = q^h = q$ and rearranging yields

$$\beta\pi\left(z^h - z^l\right) = -h(q) + \beta\left[u(q) + \bar{z}\right].$$

Thus, the quantities $q^l = q^h = q_1$ satisfy (35) at equality. Then, (32) and (33) imply that

$$q_1 < q_1^l < q_1^h. \tag{39}$$

This, can be also seen from the Figure 3.



Then, (39) immediately implies that $W\left(q_1^l, q_1^h\right) > W\left(q_1, q_1\right)$. Finally, since $W\left(q_*^l, q_*^h\right) \geq W\left(q_1^l, q_1^h\right)$, it follows that $\hat{\gamma} > 0$. ■

The gist of this Proposition is that if γ is sufficiently low, the economy is better off knowing the true state. In contrast, if the inspection cost are sufficiently large, then ignorance is optimal.

4.5 Extensions

In the previous section we have compared an economy with positive inspection cost γ to an economy that has no inspection cost $\gamma = 0$. We have asked the question whether the constrained-efficient allocation is in the former or the later. In some sense we have looked at an economy where sellers have inspection costs $\gamma \geq 0$ and the planner must decide of whether to reveal the true state and implement the constrained-efficient allocation (36) or hide the state and implement the constrained-efficient allocation such that sellers have no incentive to inspect (30).

We can also look at the a different question. Suppose the planer has no knowledge of the state. In this case he faces a different problem. If he wants to implement a state-contingent allocation, he needs to take into account the incentives of the sellers to sacrifice disutility γ in order to learn the state. In what follows we look at this question...... I have stopped here.

We have already derived the state-contingent SPs for the seller. Let us now derive conditions for which the seller is willing to inspect for $\gamma > 0$. The seller has tree deviations to consider. First, he does not inspect and produces q^h. This will be detected by the planner in the low state and punished. Second, he does not inspect and produces q^l. This will be detected and punished in the high state. Third, he does not inspect and produces $q = 0$. He is willing to inspect if[6]

$$-\gamma - \bar{h} + \beta \bar{y}_s + \beta V^s \geq -h(q^h) + \pi \left(\beta y_s^h + \beta V^s\right) + (1-\pi)\beta^T V^s \text{ and} \tag{40}$$

$$-\gamma - \bar{h} + \beta \bar{y}_s + \beta V^s \geq -h(q^l) + (1-\pi)\left(\beta y_s^l + \beta V^s\right) + \pi \beta^T V^s \tag{41}$$

$$-\gamma - \bar{h} + \beta \bar{y}_s + \beta V^s \geq \beta V^s \tag{42}$$

where

$$V_s = \left[\frac{-\bar{h} + \beta \bar{y}_s - \gamma}{1-\beta}\right].$$

The left-hand side of each inequality is the expected payoff of the proposed equilibrium strategy prior to inspection. The right-hand side are the expected payoffs of the three deviations, respectively.

As always, by making the buyers indifferent, we can write the three conditions as follows

$$(1-\pi)\left[h(q^h) - h(q^l) + \beta\left(\bar{u} + z^l\right) - \left(\beta^T - \beta\right)V^s\right] \geq \gamma \text{ and}$$

$$(1-\pi)\left[h(q^l) - h(q^h) + \beta\left(\bar{u} + z^h\right) - \left(\beta^T - \beta\right)V^s\right] \geq \gamma$$

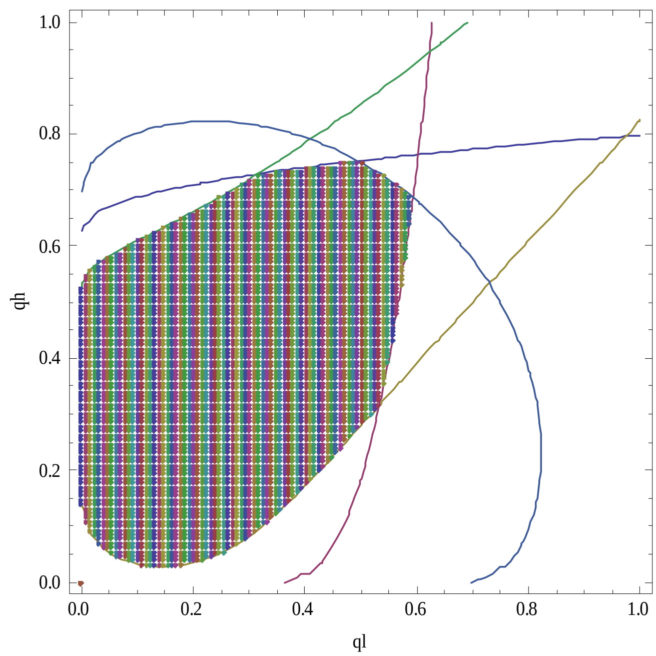
$$-\bar{h} + \beta\left(\bar{u} + \bar{z}\right) - \left(\beta^T - \beta\right)V^s \geq \gamma$$

[6] Dave: there are more possible deviations. For example: don not inspect and produce $q > 0$ with $q \neq q^l, q^h$. All these deviations make the deviator strictly worse off than any of the proposed three deviations.

Definition 16 *Let* $I \equiv \left\{ (q^l, q^h) : \textit{such that } (21), (22), (40), (41), \textit{ and } (42) \textit{ hold} \right\}$ *denote the set of allocations* $\left(q^l, q^h\right)$ *that induce SP for buyers and sellers and inspection by sellers.*

It is very hard to characterize this set analytically. All five inequalities can be binding in $\left(q^l, q^h\right)$ space. There is no way that we can reduce the characterization of this set by eliminating some redundant condition.

I can however program it and look at it for various examples. Here is one:



Note that $(q^l_{\max}, q^h_{\max})$ as defined above is not part of the set. It is in fact never part of the set if $\gamma > 0$. This can be shown analytically. In all my examples, the welfare maximizing allocation is the (q^l, q^h) that solves (22) and (42) at equality.

5 Appendix

References

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