Fiscal Policy, Default Risk
and Euro Area Sovereign Bond Spreads

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Abstract
This paper develops an arbitrage-free affine term structure model of potentially defaultable sovereign bonds to model a cross-section of six euro area government bond yield curves. We make use of the coexistence of a common monetary policy under European Monetary Union, which determines the short end of the yield curve that is common to all countries, and decentralized debt policies which drive expected default probabilities and thereby spreads at the long end. The factors of our term structure model are observable macroeconomic variables, including measures of government solvency. When applying this model to yield curves of six EMU member countries over the period January 1999 to March 2010, we find strong evidence for a break in the relationship between the fiscal variable and the default intensities in 2008. Despite using no latent factors, our model produces an excellent fit to both yield levels and spreads. For highly indebted countries, following the break the sensitivity of spreads to the fiscal variable rises sharply.

JEL classification: E6, H6.

Keywords: Government debt, affine term structure models, default risk, yield spreads, fiscal projections.

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1 Introduction

Recent years have not been lacking in drama in financial markets. At the time of this writing, government bond markets in the euro area are at the center of attention amidst intense speculation about the possibility of default by one or more member countries of the euro area. Indeed, yield spreads between government bonds of euro area countries and German government bonds of comparable maturity have experienced a striking regime shift sometime during early 2008. For example, during the period January 1999 to September 2008 the yield spread (in our data set, which we will describe in detail below) of 10-year government bonds for France, Italy and Spain over German yields of the same maturity averaged 5.1, 23.6 and 10.5 basis points respectively, with standard deviations of 6.6, 11.7 and 11.5 basis points. During the period from October 2008 to March 2010, these averages rose to 23.7, 86.9 and 62.5 basis points respectively, and their standard deviations rose sharply. (Developments in some euro area government bond markets since then make even these spreads appear modest.) It is therefore crucial to understand what determined the spreads before and after this evident break, and what might have caused the break.

In this paper, we use recent advances in term structure modeling to explain the evolution of euro area sovereign yield spreads, with the goal of understanding the role of macroeconomic variables and especially of fiscal policies in determining yield spreads at all maturities, both before and since the onset of the crisis. Specifically, we jointly model the zero-coupon yield curves of government bonds of six euro area countries within an affine term structure model of potentially defaultable bonds, using only macroeconomic variables as factors. To estimate this model, we make use of a rich data set of government bond yields at many maturities of those six countries that covers the period from the beginning of stage three of European Monetary Union (EMU) in January 1999 to the spring of 2010. The probabilities of default perceived by investors are linked to the same macroeconomic fundamentals that drive yields. Our key findings are that a small set of macroeconomic variables can explain the term structures of all six countries remarkably well, and that the governments’ debt

\footnote{From the perspective of academic research, the events discussed above play out against the backdrop of intense efforts over the past decade to arrive at a better understanding of the macroeconomic determinants of asset prices in general, and of the linkages between the term structure of interest rates and the macroeconomy in particular. Gürkaynak and Wright (2010) provide an up-to-date survey of research on the term structure from a macroeconomic perspective.}
Service commitments have a significant influence on yields, which was minuscule before the spring of 2008 and has since had a major effect on perceived default probabilities.

Spreads among euro area government bonds since the beginning of stage three of European Monetary Union have attracted substantial attention in the literature. We discuss the related literature in the following subsection. Euro area yield spreads are particularly interesting because they allow us to study the macroeconomic determinants of term premia and default risk premia. Under the assumption (maintained throughout our study) that the probability of a country leaving the euro is considered nil, expectations of future short-term interest rates are identical across countries and exchange-rate risk is not priced. Hence our data set allows us to focus on term and default risk premia and their relation to the common monetary policy on one hand and country-specific fiscal policies on the other.

The literature in this area has mostly focused on regressions of yield spreads of other euro area members vis-à-vis Germany at certain maturities on country-specific variables such as fiscal variables, proxies for liquidity (such as size of the outstanding debt), proxies for time-varying risk aversion (as captured by private credit spreads) etc. We share with this literature the focus on fiscal variables (in addition to macroeconomic determinants of the common short-term interest rate) as explanatory variables. We depart from these earlier studies by estimating a multicountry affine term structure model, thereby using the entire cross-sectional information in the term structure by imposing the restrictions implied by ruling out arbitrage across maturities and borrowers, and by allowing for the interaction between macroeconomic variables and prices of risk.

Within the finance literature on the term structure, our paper is the first application of an affine term structure model of defaultable bonds to euro area yield curves that uses macroeconomic variables as factors. Linking the term structures to macroeconomic variables is challenging for several reasons. Because of the still limited sample since the introduction of the euro, we rely on monthly data, which is a compromise between the higher-frequency yield data and the lower-frequency macro data. Even then, since we are particularly interested in fiscal developments, using national accounts data interpolated to monthly frequency is problematic because of smoothing. We therefore rely in our estimation on the Kalman filter to infer missing observations of fiscal variables rather than a more mechanical interpolation routine.

Another major challenge is the clear evidence for a structural break in the relationship
between macroeconomic variables and spreads sometime during the first half of 2008. We model this break as having occurred in the parameters linking the macroeconomic states to default intensities. We focus on these parameters as source of instability because we are interested in understanding why the spreads of certain issuers rose sharply without evidence of a large increase in term premia for (arguably) risk-free issuers. Because this break dwarfs any evidence for instability in this relationship that may have existed prior to 2008, modeling this break as a regime switch would likely lead to the conclusion that there was only one regime shift, in early 2008, and thus to a near-diagonal transition matrix so as to generate nearly-unforecastable regime switches. We instead chose to model the break as exogenous, cognizant that doing so means that after the break agents price assets as if they expect the new regime to last forever – an assumption that we feel is justified given our choice of sample, but of course not for the indefinite future.

Our sample for yields starts with the beginning of Stage Three of EMU in January 1999 (with Greek yields starting shortly after EMU accession in 2001) and ends in March 2010. We are reluctant to extend the sample period beyond this point for two reasons. First, our term structure model of multiple, potentially defaultable issuers (with Germany assumed to be free of default risk so as to normalize spreads relative to German benchmark yields) assumes that each issuer is not bearing risk of other issuers defaulting. From the spring of 2010 on, this assumption becomes questionable in light of the support extended through the European Financial Stability Facility. Second, from May 2010 on the ECB intervened in government bond markets under its Securities Markets Programme, which reportedly had large effects on yields for certain issuers that are unrelated to the macroeconomic determinants we are interested in.

We first conduct some preliminary analysis to decide on both the most relevant fiscal variable to include and the most likely break point between macroeconomic variables and spreads. Our term structure model of multiple defaultable issuers that we then estimate conditional on our choices of fiscal variable and break date fits yields for all countries and a wide range of maturities impressively well, despite the fact that we do not use latent factors in our term structure model and restrict ourselves to two euro area-wide and for each country one fiscal factor. Measurement error standard deviations are on average across countries and maturities only about 20 basis points. We model the break as occurring in the linear relationship between the factors and the default intensities that drive spreads.
There is strong evidence for such a break, with the sensitivity of spreads to especially the fiscal factors increasing sharply during the post-break sample.

In the following subsection we review related literature. In section 2 we present some exploratory results using OLS regressions, which help us to determine the states to include in the term structure model as well as the break date. Section 3 describes the affine term structure model. Section 4 presents our results. Section 5 offers conclusions. Details on the data and the model specification are in appendices.

1.1 Relation to the literature

As mentioned earlier, a large empirical literature has studied government bond spreads in the euro area since the beginning of the common monetary policy in 1999 with the goal of identifying the determinants of term and risk premia in the absence of exchange-rate risk. Many of these studies rely on regressions of yield spreads at certain maturities on candidate explanatory variables. A common finding in this literature, beginning with Codogno et al. (2003) and Bernoth et al. (2006) and including more recent studies such as Manganelli and Wolswijk (2009), Haugh et al. (2009) and Schuknecht et al. (2010), is that euro area sovereign yield spreads seem to strongly comove. Principal component analysis regularly reveals that the first principal component accounts for more than 80% in the total variation of yield spreads. This finding suggests that a common factor, frequently interpreted as time-varying risk aversion of international investors that affects all yield spreads through the repricing of given country-specific risk characteristics, is the dominant force, making it difficult a priori to identify the role of country-specific variables such as fiscal policies in the determination of spreads. Laubach (2010), however, presents evidence that the strength of comovement among yield spreads varies substantially over time and has weakened since 2009.

If we assume that investors assign zero probability to the event of a member country leaving the euro, yield spreads can be explained as compensation for either liquidity risk or default risk. How to distinguish between these two interpretations of spreads has been a source of disagreement in the literature. Although since the eruption of the Greek fiscal crisis in November 2009 it seems plausible that default risk has been the dominant market concern, the relative importance of liquidity versus default risk is less clear during the first ten years of EMU. In their early study based on four years of monthly data, Codogno
et al. (2003) concluded that “the risk of default is a small but important component of yield differentials” while liquidity factors seemed to be of lesser importance. But it seems difficult to explain persistent positive spreads of government bonds of AAA-rated countries over Bunds as being driven by factors other than liquidity risk.

Several recent studies conclude that the importance of liquidity risk seems to vary over time with proxies of international investor risk aversion. Beber et al. (2009), using intraday European bond quotes from the period April 2003 to December 2004, find that differences in credit quality among countries play a major role, but that “in times of market stress, investors chase liquidity, not credit quality.” By contrast, Favero et al. (2010) conclude that the interaction between liquidity demand and risk is negative. They attribute the difference between their results and those of Beber et al. to the fact that Beber et al. “control for country-specific risk but do not consider aggregate risk factors.” In pooled regressions of quarterly spread data for ten euro area countries including an interaction term between a proxy for risk aversion and the volume of bonds outstanding (as proxy for liquidity) as well various fiscal variables to account for credit risk, Haugh et al. (2009) find a significant role for liquidity in line with the sign of Beber et al., with liquidity (or lack thereof) making a large contribution to the spreads of Irish and Finnish government bonds in late 2008 and early 2009.2 While we do not deny that liquidity risk may in some instances and for some countries (those with small size of debt outstanding relative to euro area sovereign debt overall) have a sizeable role to play, we interpret the results from this literature as pointing more consistently to an important role of credit risk factors emanating from public finances, and therefore concentrate on those. This view is furthermore vindicated by the fact that we mainly focus on countries whose debt market is generally considered large and liquid, as it is the case for the biggest four euro-area countries or for a largely indebted but nevertheless “core” EMU-country like Belgium. Admittedly, this claim is weaker concerning the last country in our panel, Greece, but we think that it would be difficult to argue that concerns about fiscal sustainability in Greece were not the key driver of the surge in Greek bond yields over the past two years.

We depart from the literature discussed so far by using a no-arbitrage term structure model so as to exploit the information contained in the entire maturity spectrum of yield

\footnote{Aßmann and Boysen-Hogrefe (2010) model time-varying risk aversion as a latent variable and conclude, similar to Beber et al. (2009) and Haugh et al. (2009), that liquidity matters in times of stress.}
spreads. Not only can we multiply manifold the number of observations used in the analysis, we can also sharpen the conclusions regarding the determinants of yield spreads by estimating their effects on bonds of different maturities. The “essentially affine” class of term structure models that we use was first proposed by Duffee (2002) as a special case of affine term structure models. Beginning with the work of Ang and Piazzesi (2003), a growing literature has explored the role of macroeconomic variables as factors. Lemke (2008) estimates a model with only observable macroeconomic factors for German bond yields during the euro area. For the U.S., Dai and Philippon (2006) and Laubach (2010) include fiscal variables as factors among the variables.

In order to study the role of default risk in determining yield spreads, we employ the extension of affine term structure models to defaultable bonds proposed by Duffie and Singleton (1999). Geyer et al. (2004) provide an early application of such a model to euro area spreads, without, however, including macroeconomic variables as factors. More recently, Monfort and Renne (2011) generalize this model to account for regime switching and both default risk and liquidity factors and apply this model to jointly model a swap yield curve, ten sovereign yield curves and a German agency yield curve, using latent factors. Unlike our study, they do not focus on breaks in the default intensities but instead allow for a wider range of possible parameter shifts, including changes in factor volatilities. Doing so helps to mitigate the problem of estimating a near-diagonal transition matrix discussed above. This problem would arguably be more severe in our case, which is why we abstract from regime switching.

Lastly, our results have implications for the long empirical literature on the effects of fiscal policy on interest rates (e.g. Ardagna et al. 2007). For the euro area, Faini (2006) provides evidence that fiscal policy (as measured by the deficit/GDP and debt/GDP ratios) significantly affects the level of average euro area long-term yields, with significant spillover effects from one country’s fiscal policy stance to the euro-area wide level and very small

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3 This use of macroeconomic variables as factors is not uncontroversial. Duffee (2009) and Joslin et al. (2010) have pointed to the importance of unspanned macro risks, i.e. that current macroeconomic variables cannot be recovered from current yields, but that macroeconomic variables nonetheless can affect future yield curves through their impact on expected future short-term interest rates. See Gürkaynak and Wright (2010) for further discussion.

4 Amato and Luisi (2006) is to our knowledge the first use of an affine term structure model of defaultable bonds with macroeconomic variables as factors, but applied to U.S. corporate bond spreads.
effects on spreads. Our results confirm his findings during the period prior to the break in early 2008, but also document the increase in spread sensitivities since then.

2 Fiscal sustainability and euro-area sovereign bond yields: preliminary evidence

Affine term structure models rely on the assumption that linear relations hold between bond prices or yields and the observable macro factors that drive the yield curve. Hence, as a first pass, simple OLS regressions can provide us with useful insights about the set of variables that are likely to span the curve of each country, as shown in Dai and Philippon (2006). Following these authors, we present in this section the results of regressions of bond yields of several maturities on measures of the fiscal stance and usual macroeconomic controls for each of the six countries in our sample. The idea is that if, for any given country, a given fiscal variable fails to explain significantly bond yields of different maturities in simple reduced-form regressions, then there is no point including this variable as a factor in our more sophisticated (and heavily constrained) no-arbitrage multicountry term structure model.

For each of the six countries, we use monthly observations of government bond yields at 2, 5 and 10 years maturities, that we regress on the 1-month risk free short term rate, a monthly indicator for the position in the domestic business cycle, domestic HICP inflation and four alternative measures of national fiscal imbalances. A data appendix details the sources and methodology for the zero-coupon yields used throughout. The short term rate is measured using prices of 1-month OIS swaps rather than euro area money market rates, which have been obviously comprising a certain amount of premia for credit and liquidity risks since the start of the financial crisis in August 2007. For each country, the monthly national business cycle indicator is constructed as the first principal component of sectoral activity indices taken from Eurostat’s business surveys.

The appropriate choice of the most relevant measure of fiscal imbalances at the national level is less clear. Previous studies frequently consider the deficit to GDP or the debt to GDP ratios, or forecasts thereof (see e.g. Codogno et al., 2003, for the euro area and Laubach, 2009, Dai and Philippon, 2006, for the US). Bernoth et al. (2006) argue that debt service (defined as the ratio of gross interest payments to current government revenue)
is more appropriate when trying to assess the impact of fiscal balances on euro area bond yields, if only because governments have less incentive to manipulate it than other measures that are used officially to monitor whether national fiscal positions meet the obligations set out by the Stability and Growth Pact. Furthermore, Haugh et al. (2009) find that both fiscal deficit and debt service help to explain a substantial part of cross-sectional variations in euro area bond yields during the recent crisis. It can be also noted that debt service is routinely monitored by bond markets participants in order to gauge the sustainability of issuing countries’ fiscal imbalances.

Either one of these three measures suffers potentially from an endogeneity problem. In practice, as long as the average maturity of countries’ debt is not too short, so that the share of total debt that needs to be refinanced each period is small, the contemporaneous effect of changes in interest rates on either the deficit/GDP ratio or the debt service ratio is rather modest. The primary deficit/GDP ratio obviously does not suffer from this potential problem of reverse causality. As a consequence, we consider here four alternative measures of fiscal imbalances: total fiscal and primary deficit to GDP, the debt to GDP ratio as well as the debt service to income ratio. We take the corresponding series from the OECD Economic Outlook database. OECD data are provided on a semi-annual basis with quarterly frequency. For the needs of the preliminary regressions conducted in this section, we interpolated these quarterly series using simple cubic splines. Note however that, in the subsequent estimation of our affine term structure model, missing observations of the fiscal variable are dealt with in a more satisfying manner using the Kalman filter, as detailed in section 4.1 below. Figure 2 shows the debt service ratios as interpolated to monthly frequency with the Kalman filter.

As evidenced by Figure 1, the level and dynamics of euro area spreads seem to have undergone a structural break at some point during the year 2008, a move that has been generally commented as reflecting a repricing of country-specific risk, notably credit risk that would have been neglected prior to the 2007- crisis. We thus added a break in the constant and in the regression coefficients of the fiscal variable in our regressions. In order

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5Gross debt issuance in 2010 ranged from between 8 and 10 percent of GDP for Germany, France and Spain, to nearly 17 percent for Italy and Greece. An increase in the spread of 100 bps for a country that needs to refinance debt in the amount of 10 percent of GDP would add in the same year at most 0.1 percent of GDP to the deficit. Only for Greece, which at the end of our sample was facing spreads around 300 bps that have since risen to 900 bps, would the endogeneity problem be serious.
to limit the arbitrariness inherent to the selection of any exogenous break, we had our choice guided by a simple statistic: the average $R^2$ over regressions for all countries and maturities, conditional on a given break date. Figure 4 shows the resulting statistic for each of the candidate four fiscal variables when the break date is allowed to vary between the end of 2006 and the end of 2009. Two facts emerge from this exercise. First, the explanatory power of debt to GDP and debt service is higher than that of the deficit to GDP ratios. Second, we reach at least local maxima of the average $R^2$ for both debt to GDP and debt service regressions at the end of the first and fourth quarters of 2008. On this basis, we finally chose October 2008 as our preferred break date, which leaves enough observations available after the break for estimation purposes.

The results of these regressions can be summarized as follows. We first note that, whatever the fiscal variable, the sensitivity of longer term yields to the short term rate and the share of variance explained by macroeconomic factors decreases with maturity, consistent with the results of others in the yield curve literature (see e.g. Ang and Piazzesi, 2003). Overall, the crisis dummy, which takes the value one from October 2008 onward, as well as domestic activity, the fiscal variable and the interaction term standing for non-linear effects of fiscal variables in crisis times all turn out to be significant. By contrast, domestic inflation is generally not significant. In the analysis that follows, we are therefore omitting inflation from the state vector.

Due to sign conventions in the construction of fiscal variables, we should expect a negative sign for coefficients of deficit measures (i.e. more negative fiscal balances should imply higher yields accounting for larger risk premia), and a positive sign for the debt ratio and debt service measures. The deficit variable enters significantly but with the wrong sign for four out of six countries, although the sign is reversed in crisis times as expected. Conversely, debt to GDP turns out to be negatively correlated with French and German yields, even in crisis times, which may be interpreted as signalling these two “core” countries as relative safe havens in times of heightened risk perception. Last but not least, the debt service to income variable is positively correlated with yields in all countries in quiet times, while pushing yields of “core” versus more “peripheral” countries into opposite directions in crisis times (respectively downward and upward). Overall, based on these preliminary results, we decided to use the ratio of debt service to government income as our best measure of fiscal imbalances in the following.
3 An affine term structure model of defaultable bonds

3.1 Dynamics of the pricing factors under the historical measure

Let us denote respectively by $r_t$ and $x_t$ the one-period rate –or short-term rate– and the European business cycle indicator. These factors, together with the country-specific fiscal variables $f_{i,t}$, constitute the set of pricing factors. These factors are stacked in a vector $X_t = [x_t \ r_t \ f_{1,t} \ \ldots \ f_{N,t}]'$. The vector of factors depends on its lagged values and is affected by a vector $\varepsilon_t$ of idiosyncratic shock. Accordingly, its dynamics follows a VAR($p$):

$$X_t = \mu_X + \Phi_1X_{t-1} + \ldots + \Phi_pX_{t-p} + \Sigma X\varepsilon_t$$

where the $\varepsilon_t$’s are i.i.d. $N(0, I)$. For the sake of parsimony, we assume that the matrix $\Sigma_X$ is diagonal, which implies that the innovations of the factors are orthogonal. Moreover, additional constraints are imposed on the auto-regressive matrices $\Phi_i$. First, the short term rate $r_t$ is assumed not to respond to past values of the fiscal variables $f_{i,t}$. Therefore, the dynamics of the short term rate –or the reaction function of the central bank– is:

$$r_t = \mu_r + (\theta_1x_{t-1} + \rho_1r_{t-1}) + \ldots + (\theta_p x_{t-p} + \rho_p r_{t-p}) + \sigma_r \varepsilon_{r,t}.$$ 

Second, we assume that the European business cycle depends on an aggregated fiscal stance that is equal to the weighted sum of the country-specific fiscal stances. The countries’ weights $\{W_j\}_{j \in [1,N]}$ are proportional to the GDP of the respective countries at the end of 2007. Third, in order to keep the number of parameters within reasonable limits, we assume that the parameters defining the dynamics of the $N$ fiscal variables are not country-specific, except for the standard deviations of the innovations. That is, for any country $j$:

$$f_{j,t} = \mu_f + (\kappa_1x_{t-1} + \zeta_1r_{t-1} + \rho_1f_{j,t-1}) + \ldots + (\kappa_p x_{t-p} + \zeta_p r_{t-p} + \rho_p f_{j,t-p}) + \sigma_{f,j} \varepsilon_{f,t}.$$ 

Formally, these constraints imply the following form for the matrices $\Phi_i$ and $\Sigma$:

$$\Phi_i = \begin{bmatrix}
\alpha_i & \beta_i \\
\theta_i & \rho_i \\
\kappa_i & \zeta_i \\
\vdots & \vdots \\
\kappa_i & \zeta_i
\end{bmatrix}, \quad \Sigma = \begin{bmatrix}
\sigma_x & 0 & \ldots & 0 \\
0 & \sigma_r & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \sigma_{f,N}
\end{bmatrix}.$$
It will prove convenient in the pricing framework that follows to turn the model into its VAR(1) representation. To that end, let us define a new state vector $F_t$ in which the vectors from $X_t$ to $X_{t-p+1}$ are stacked. The dynamics of $F_t$ are given by

$$F_t = \mu + \Phi F_{t-1} + \Sigma \varepsilon_t,$$

with

$$\mu = \begin{bmatrix} \mu_X \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_p \\ \text{Id} & 0 & 0 & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & \text{Id} & 0 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma_X & 0 \\ 0 & 0 \end{bmatrix}.$$  

3.2 Dynamics of the factors under the risk-neutral measure

It is well-known that the existence of a positive stochastic discount factor is equivalent to the absence of arbitrage opportunities (see, e.g., Hansen and Richard, 1987). Following, amongst many others, Ang and Piazzesi (2003), we postulate the following form for the stochastic discount factor $m_{t,t+1}$:

$$m_{t,t+1} = \exp(-r_t) \frac{\xi_{t+1}}{\xi_t}$$

where $\xi_t$ follows a log-normal process defined by:

$$\xi_t = \xi_{t-1} \exp \left( -\frac{1}{2} \lambda_t' \lambda_{t-1} - \lambda_t' \varepsilon_t \right)$$

with $\lambda_t = \lambda_0 + \lambda_1 F_t$. Under these assumptions, it can be shown that the dynamics of the pricing factors under the risk-neutral measure $Q$ is defined by:

$$F_t = \mu^* + \Phi^* F_{t-1} + \Sigma^* \varepsilon^*_t$$

where the $\varepsilon^*_t$’s are i.i.d. $\mathcal{N}(0, I)$ and with:

$$\mu^* = \mu - \lambda_0 \Sigma$$

$$\Phi^* = \Phi - \lambda_1 \Sigma.$$  

3.3 Bond pricing

Let us denote by $P(t,h)$ the price at time $t$ of a risk-free zero-coupon bond of residual maturity $h$. This price is given by:

$$P(t,h) = E(m_{t,t+1} \ldots m_{t,t+h}) \quad \text{or} \quad P(t,h) = \mathbb{E}^Q(\exp(-r_t - r_{t+1} - \ldots - r_{t+h-1})).$$
To price bonds subject to credit risk, we introduce default intensities—or hazard rates—for each country. The default intensity of country $j$, denoted by $s_{j,t}$, reflects credit risk embedded in the bonds issued by this country. If recovery rates were nil, the default intensity at time $t$ would be the default probability of the considered debtor at that period. However, recovery rates are strictly positive processes. Therefore, the hazard rates $s_{j,t}$ should be more rigorously termed as "recovery-adjusted default intensities" (see, e.g. Monfort and Renne, 2011).

Duffie and Singleton (1999) show that defaultable bonds can be priced using the same machinery than for risk-free bonds by simply replacing the short-term risk-free rate $r_t$ by the default-adjusted short-term rate $r_t + s_{j,t+1}$. Formally, denoting by $P_j(t, h)$ the price at time $t$ of a bond of residual maturity $h$ issued by country $j$, we have:

$$P_j(t, h) = E^Q(\exp\left[-(r_t + s_{t+1}) - \ldots - (r_{t+h-1} + s_{t+h})\right]).$$

Appendix B shows that bond prices are exponential affine in the factors $F_t$ when the hazard rates are affine in the same factors. That is:

$$P_j(t, h) = \exp(A_{j,h} + B_{j,h}F_t)$$

where the vectors $A_{j,h}$ and $B_{j,h}$ are obtained by applying recursive formulas. The continuously compounded yield, denoted by $y_{j,h,t}$ and defined by $-\log(B_j(t, h))/h$, are given by:

$$y_{j,h,t} = \overline{A}_{j,h} + \overline{B}_{j,h}F_t$$

with $\overline{A}_{j,h} = -A_{j,h}/h$ and $\overline{B}_{j,h} = -B_{j,h}/h$.

### 3.4 Introducing a break in the hazard rates

The modeling approach is completed by the introduction of a break at time $\tau$. The period posterior to $\tau$ corresponds to the crisis period. We assume that the break concerns the specifications of the hazard rates $s_{j,t}$ and that the stochastic discount factor as well as the historical dynamics of the pricing factors $F_t$ are not affected by that break. Underlying is the assumption that there is more inertia in the specifications of the dynamics of the factors than in the specifications of the default intensities. Then, the main changes implied by the

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6Intuitively, with a constant recovery rate of $R$, the recovery-adjusted default intensity $s_{j,t}$ would be approximately equal to $(1 - R)\tilde{s}_{j,t}$ where $\tilde{s}_{j,t}$ is the default probability of country $j$ at time $t$. 

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crisis in terms of bond pricing would regard the way the investors form expectations about the default probabilities of the countries.

In that context, the hazard rates are given by:

\[
s_{j,t} = I(t < \tau) \times \left[ \gamma_{j,0}^{bb} + \gamma_{j,1}^{bb} F_t \right] + I(t \geq \tau) \times \left[ \gamma_{j,0}^{pb} + \gamma_{j,1}^{pb} F_t \right]
\]

where the \(bb\) and \(pb\) subscripts respectively stand for “before break” and “post break”.

Further, we assume that such a break in the future is considered impossible before the break date: whereas agents’ expectations regarding future values of the hazard rates \(s_{j,t+h}\) –for any horizon \(h\)– are based on the vectors of parameters \(\gamma_{j,0}^{bb}\) and \(\gamma_{j,0}^{bb}\) until time \(\tau - 1\), these expectations get based on the \(\gamma_{j,0}^{pb}\)’s and on the \(\gamma_{j,1}^{pb}\)’s from time \(\tau\) onwards. To put it differently, the break we model was unforeseeable and is considered as permanent. In terms of bond pricing, the presence of the break implies the existence of two sets of vectors \(\{A_{bb}^{j,h}, B_{bb}^{j,h}\}_{j,h}\) and \(\{A_{pb}^{j,h}, B_{pb}^{j,h}\}_{j,h}\). Wheras the former were used to price bonds before the break, the latter are used to price the bonds after the break.

Our assumption that the break only applies to the parameters \(\gamma\) of the hazard rates, but not for example to the parameters \(\lambda\) of the market prices of risk has the interpretation that during the early parts of 2008, for given values of the factors \(F_t\) the probability of default perceived by investors increased. Investors did not become more risk averse (in the sense that the price of risk did not change), but simply reassessed the “quantity of risk” that was to be priced. In contrast to Monfort and Renne (2011) we also do not allow for a break in the volatilities of the factors. Given that in our model these factors are observed macroeconomic variables instead of latent, and that our post-break sample spans only two years, it would be difficult to estimate a change in volatilities on such a short sample with any confidence.

4 Estimation and results

4.1 Estimation

The estimation is based on two steps. In the first one, we estimate the parameters that enter the historical dynamics of the factors, namely \(\Theta_1 = [\alpha', \beta', \theta', \rho', \kappa', \zeta', \omega', \sigma']\), where \(\alpha\), for instance, is equal to \([\alpha_1, \alpha_2, \ldots, \alpha_p]'\) and \(\sigma\) is the vector \([\sigma_1, \ldots, \sigma_N]'\). In the second step, the parameter defining the risk-neutral dynamics of the factors, namely \(\Theta_2 = \)
\([\alpha^*, \beta^*, \theta^*, \rho^*, \kappa^*, \zeta^*, \omega^*]'\), as well as the \(\gamma\)'s are estimated. German yields are considered as risk-free.

In the first step we estimate the historical dynamics of \(F_t\) given by equation (1). The model does not involve unobservable, or latent, variables. However, given that the frequency of the fiscal variable is lower than the monthly one retained for the estimation, we have to deal with missing data. This problem is overcome using the Kalman filter, which provides us with the log-likelihood of the model defined by (1).

The second step of the estimation is based on a non-linear least square procedure (see, e.g. Moench, 2008). The procedure consists in minimizing the sum of pricing errors across time, countries and maturities. Specifically, let us denote by \(y_{j,h,t}^o\) the observed continuously-compounded zero-coupon yield of maturity \(h\) of country \(j\). The model-based counterpart of \(y_{j,h,t}^o\) is a function of \(\Theta_2\) and \(F_t\).\(^7\) The function of \(\Theta_2\) that we want to minimize is the following expression:\(^8\)

\[
\sum_{h,t} \left( y_{1,h,t}(\Theta_2, F_t) - y_{1,h,t}^o \right)^2 + \sum_{j>1,h,t} \left( y_{j,h,t}(\Theta_2, F_t) - y_{1,h,t}(\Theta_2, F_t) - (y_{j,h,t}^o - y_{1,h,t}^o) \right)^2.
\]

Error on German yields Error on spreads vs. Germany

Whereas we optimize the fit of the German yields, we look for the best fit of the spreads vs. Germany for other countries (credit spreads hereinafter). This is done in order to favour the fit of the credit spreads by compelling the hazard rates to reflect differences in credit risk amongst countries.

The computation of the first-step covariance matrix of the estimates is based on the Hessian. Since we only use observable factors to model the yields, the pricing errors are subject to heterogeneity and auto-correlation. To cope with this, the computation of the second-step covariance matrix uses Newey-West (1987) heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimators (see Appendix C).

4.2 Discussion

Figure 3 plots the observed yields of German bonds with maturities of 1 to 10 years against the yields simulated with our baseline model (in the following called model III; alternative

\(^7\) Actually, the model-based yield also depends on the \(\sigma\)'s. However, in that second step of the estimation, we suppose that these are fixed to their first-step estimated value.

\(^8\) The minimization is carried out in Matlab using iteratively a Newton algorithm and a Nelder-Mead simplex algorithm until reaching convergence.
model versions will be introduced below). Similarly, Figures 4 and 5 show the observed and estimated bond spreads against Germany for the remaining five euro area countries and two maturities (5 and 10 years). Overall, the model does a reasonably good job in tracking both the absolute level of yields of German government bonds and the level of spreads of other countries vis-à-vis Germany. As appears on the last row of Table 2, the standard deviation of the measurement errors over all countries and maturities amounts to a modest 20 basis points, which compares favourably with the usual fit of affine term structure macrofinance models. This is all the more remarkable because we restrict the state space of factors spanning the six yield curves to observable factors only, while most studies in this literature also incorporate latent factors in the model (see e.g. Ang and Piazzesi, 2003, Dai and Philippon, 2005, Rudebusch and Wu, 2008). Considering all countries and maturities, the model principally fails to capture a spike in yields in mid-2002 (which is not obviously related to any monetary policy decision neither to a particular fiscal event at that time), as well as a hump in the levels of long term spreads over 2001-2002. Note however that the fit to observed spreads is particularly good from 2003 on, which means that the model provides quite an appropriate tool to analyze the changes in bond risk pricing that were brought along by the 2007 financial turmoil.

The two columns of Tables 1 and 2 labelled “Model III” show the estimates of the parameters of the default intensities $s_{j,t}$ for our baseline model, while contrasting the two sub-periods before and after the assumed break date of October 2008. Focusing first of the sensitivities of default intensities to the fiscal variable, $\gamma^f_j$, we first find that the domestic debt service variable stands out as a significant determinant of spreads for most countries, and with the expected positive sign. Our findings also confirm the widespread intuition that the crisis brought about a rise in the magnitude of these sensitivities to fiscal sustainability measures, which we view as a major contribution of this study. Indeed, while they come out as negligible on the pre-crisis period, they consistently rise with the crisis, magnifying afterwards the impact of aggravated national fiscal imbalances on intra-EMU spreads. For instance, whereas before the crisis a one point increase in the Italian debt service ratio would have induced only a tiny change in the compensation required for the increased default risk (by some 5 basis points), the required compensation after the crisis would have been more than five times larger, by some 26 basis points.9

---

9 The lack of precision of some of the post-break estimates is due to the short post-break sample of 18
Regarding other parameters, we also find that the crisis was associated with a significant increase in the average level $\gamma_0$ of default intensities for all countries, witnessing an upward shift in the required compensation for default risk in both highly indebted and peripheral countries. By contrast, the sensitivities of default intensities to either the euro area business conditions or the short term interest rate ($\gamma^x_j$ and $\gamma^r_j$ respectively) come out as weak and generally non significant.

Our baseline model is one in which we already imposed a number of restrictions on the $\gamma$ parameters. These restrictions are by and large validated by a series of Wald tests, looking first at parameter estimates obtained with an unrestricted model (denoted model I hereafter), then at alternative, nested specifications. Tables 3 to 6 presents the results of tests of various restrictions imposed to the unrestricted model I. The first column gives the $p$ values of a test of the null of no break in the parameters, considering them all together or by successive blocks for each variable. The successive columns provide block-by-block tests of the null that the default price coefficients are common across all five countries or equally null, considering in turn the pre- and post-break periods. Results of the tests confirm that a break in each of the $\gamma$’s is required by the data, and that both $\gamma_0$ and $\gamma^f_j$ are non-zero and different across countries in each sub-period. In contrast, the tests validate the null of zero values for the sensitivities to the cycle and the monetary policy rate on the pre-crisis period.

Based on the selection of model III, we finally ask to what extent the change in spreads evident in Figure 1 are explained by changes in macroeconomic factors holding the $\gamma$ parameters fixed at their pre-break values, and to what extent the change in spreads reflects the change in the $\gamma$ parameters. Letting $\bar{S}_j$ denote the average of country $j$’s 5-year spread vis-a-vis Germany during the post-break period, and $S_j$ the value of the spread just prior to the break date, we can decompose the change in country $j$’s spread as

$$\bar{S}_j - S_j = A_{pb}^j - A_{DE}^j + (B_{pb}^j - B_{DE}^j)\bar{F} - \left[A_{bb}^j - A_{DE}^j + (B_{bb}^j - B_{DE}^j)F_{08:09}\right] = \Delta A_j + (B_{pb}^j - B_{DE}^j)(\bar{F} - F_{08:09}) + (B_{pb}^j - B_{bb}^j)F_{08:09}$$

where $\Delta A_j = A_{pb}^j - A_{bb}^j$, and $A_{DE}, B_{DE}$ are the same before and after break. Table 7 provides the results. As can be seen there, the bulk of the change in spreads is attributed months.
to changes in the $\gamma$ parameters. The only important contribution from changes in macroeconomic conditions comes from the sharp reduction in short-term interest rates during the post-break period, which works in the direction of lowering spreads, all else equal.

## 5 Conclusion

In this paper we take a first step towards exploring the information contained in yields across sovereign issuers, maturities and time to obtain a better understanding of the macroeconomic determinants of euro area sovereign yield spreads through the lens of an arbitrage-free term structure model. Our main results at this preliminary stage are that a small number of macroeconomic variables can explain the evolution of yields at various maturities and across time and issuers impressively well, that there is a significant relationship between debt service as a fraction of tax revenues and yield spreads, and that this relationship has undergone a major break in 2008.

In future work we plan to expand the analysis in several directions. One important direction is to model the information set of investors yet more carefully, by relying on real-time data (which we have from the OECD for a subset of vintages during our sample) and on survey expectations of fiscal and other variables as noisy measurements of model-implied investors’ expectations. Doing so would at least partially address the problem of “fiscal foresight,” whereby frequently investors have more information about future fiscal policies than a simple VAR model that an econometrician would use to proxy for investors’ expectations (Leeper et al., 2008). A second direction is a more detailed analysis of the contributions of the various factors to yield spreads over time.
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Table 1: Estimates of default intensities parameters (first part), for alternative model specifications. Student-t are reported below the parameter estimates (in parenthesis). The bottom line presents the average standard deviation of the measurement errors (in percentage points), across countries and maturities.
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Table 2: Estimates of default intensities parameters (continued), for alternative model specifications. Student-t are reported below the parameter estimates (in parenthesis). The bottom line presents the average standard deviation of the measurement errors (in percentage points), across countries and maturities.
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Table 3: Wald tests – this table presents p-values of Wald tests performed using Model I’s parameter estimates and the covariance matrix of these parameters.

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Table 4: Wald tests – this table presents p-values of Wald tests performed using Model II’s parameter estimates and the covariance matrix of these parameters.

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Table 5: Wald tests – this table presents p-values of Wald tests performed using Model III’s parameter estimates and the covariance matrix of these parameters.
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Table 6: Wald tests – this table presents p-values of Wald tests performed using Model IV’s parameter estimates and the covariance matrix of these parameters.

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<td>0.5510</td>
<td>0.4255</td>
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Table 7: Decomposition of changes in spreads during the post-break period
A Data

In this section, we describe the data that we used throughout the paper. In particular, we detailed the procedure that we used to get government bond yields at different maturities.

For the activity data, we rely on monthly sectoral business survey data published by the DG ECFIN of the European Commission. For each country, our national business cycle indicator is the first principal component that we extract from confidence indicator surveys: we include in this computation construction, consumer, industrial, retail and services sectors of the national business cycle indicator. The Euro area business cycle indicator is then computed on the basis of the GDP-weighted average of this national business cycle indicator (normalized by the total weight of GDP for the six European countries included in our sample). The short term rate is measured using prices of 1 month OIS. Inflation data are the domestic harmonized consumer price index (HCPI).

For the fiscal data, we rely on OECD data from Economic Outlook published in May 2010 (n°87). We use the deficit to GDP, primary deficit to GDP, debt to GDP and debt service to income receipt ratio. These quarterly fiscal data are revised data. In the preliminary regressions presented in section 2, we interpolated these quarterly series using simple cubic splines in order to deal with monthly frequency. In the affine term structure model developed in sections 3 and 4 we used the Kalman filter procedure in order to deal with monthly frequency for the fiscal data in a more sophisticated way as described in section 4.1.

B Derivation of the recursive formulas

Let $P_j(t,h)$ denote the price, at time $t$, of a zero-coupon bond issued by country $j$ with a residual maturity of $h$ periods. Assume that, for a given $h \geq 1$, there exist some matrices $A_{j,1}, \ldots, A_{j,h-1}$ and $B_{j,1}, \ldots, B_{j,h-1}$ that are such that, for any period $t$ and any maturity $n \in \{1, \ldots, h-1\}$, $B_j(t,n) = A_{j,n} + B_{j,n}F_t$. Naturally, the latter formula is valid only if country $j$ has not defaulted before $t$ (otherwise, we would have the trivial prices $P_j(t,n) = 0$ for any maturity $n$).

Let us consider the price of a $h$-period bond at time $t$. This price is given by:

$$P_j(t,h) = \exp(-r_t)E^Q(P_j(t+1,h-1))$$

$$= \exp(-A_1 - B_1F_t)E^Q(P_j(t+1,h-1))$$

$$= \exp(-A_1 - B_1F_t)E^Q[\mathbb{I}(d_{j,t+1} = 0) \times \{A_{j,h-1} + B_{j,h-1}F_{t+1}\}]$$

where $d_{j,t}$ is a default indicator which is equal to 1 if country $j$ has defaulted at or before $t$ and is equal to 0 otherwise. We have:
\begin{align*}
    P_j(t, h) &= \exp(-A_1 - B_1 F_t) E^Q \left[ E^Q (\mathbb{1}(d_{j,t+1} = 0) \times \exp \{ A_{j,h-1} + B_{j,h-1} F_{t+1} \} | F_{t+1}) \right] \\
    &= \exp(-A_1 - B_1 F_t) E^Q \left[ \exp \{ A_{j,h-1} + B_{j,h-1} F_{t+1} \} E^Q (\mathbb{1}(d_{j,t+1} = 0) | F_{t+1}) \right] \\
    &= \exp(-A_1 - B_1 F_t) E^Q \left[ \exp \{ A_{j,h-1} + B_{j,h-1} F_{t+1} \} \exp (-s_{j,t+1}) \right] \\
    &= \exp(-A_1 - B_1 F_t) E^Q \left[ \exp \{ A_{j,h-1} - \gamma_0 + (B_{j,h-1} - \gamma_1) F_{t+1} \} \right] \\
    &= \exp(-A_1 - B_1 F_t) E^Q \left[ \exp \{ A_{j,h-1} - \gamma_0 + (B_{j,h-1} - \gamma_1) (\mu^* + \Phi^* F_t + \epsilon^*_t) \} \right] \\
    &= \exp(-A_1 - B_1 F_t + A_{j,h-1} - \gamma_0 + (B_{j,h-1} - \gamma_1) (\mu^* + \Phi^* F_t) + \\
    &\quad \frac{1}{2}(B_{j,h-1} - \gamma_1) \Sigma \Sigma' (B_{j,h-1} - \gamma_1)' \right).
\end{align*}

Therefore, with

\begin{equation*}
\begin{cases}
    A_{j,h} = -A_1 + A_{j,h-1} - \gamma_0 + (B_{j,h-1} - \gamma_1) \mu^* + \frac{1}{2}(B_{j,h-1} - \gamma_1) \Sigma \Sigma' (B_{j,h-1} - \gamma_1)' \\
    B_{j,h} = -B_1 + (B_{j,h-1} - \gamma_1) \Phi^*,
\end{cases}
\end{equation*}

we have $P_j(t, h) = A_{j,h} + B_{j,h} F_t$. Hence, we have shown how to compute recursively the $A_{j,h}$’s and $B_{j,h}$’s.

\section*{C \ Computational of the covariance matrix of the estimators}

In this appendix, we present the methodology used to compute the covariance matrix of the parameter estimates obtained by minimization of the sum of squared pricing errors. This refers to the second step of estimation presented in 4.1. The parameters are the entries of vector $\Theta_2$, that parameterizes the risk-neutral dynamics as well as the hazard rates of the different countries. As explained in 4.1:

\[ \Theta_2 = \arg \min_{\theta} \sum_{j,h,t} (y_{j,h,t}^o - y_{j,h,t}(\theta, F_t))^2. \]

This estimator must satisfy the first-order conditions:

\[ \sum_{j,h,t} \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} (y_{j,h,t}^o - y_{j,h,t}(\Theta_2, F_t)) = 0, \]

where the left-hand side of the previous equation is of dimension $k \times 1$ (the dimension of vector $\Theta_2$). Deriving the Taylor expansion of the previous equation in a neighborhood of the limit value $\Theta_2$, and
multiplying by $\sqrt{T}$ leads to:

\[
0 \simeq \sqrt{T} \sum_{j,h,t} \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} (y_{j,h,t}^\rho - y_{j,h,t}(\Theta_2, F_t)) + \\
\sqrt{T} (\Theta_2 - \Theta_2) \left[ \sum_{j,h,t} \frac{\partial^2 y_{j,h,t}(\Theta_2, F_t)}{\partial \theta \partial \theta'} (y_{j,h,t}^\rho - y_{j,h,t}(\Theta_2, F_t)) \\
- \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} \left( \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} \right) \right].
\]

Since $E(y_{j,h,t}^\rho - y_{j,h,t}(\Theta_2, F_t)) = 0$, we have

\[
\frac{1}{T} \sum_{j,h,t} \frac{\partial^2 y_{j,h,t}(\Theta_2, F_t)}{\partial \theta \partial \theta'} (y_{j,h,t}^\rho - y_{j,h,t}(\Theta_2, F_t)) \overset{a.s.}{\to} 0.
\]

Therefore:

\[
\sqrt{T} (\Theta_2 - \Theta_2) \simeq \left[ \frac{1}{T} \sum_{j,h,t} \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} \left( \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} \right) \right]^{-1} \times \\
\frac{1}{\sqrt{T}} \sum_{j,h,t} \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} (y_{j,h,t}^\rho - y_{j,h,t}(\Theta_2, F_t)).
\]

Hence, the asymptotic distribution of $\sqrt{T} (\Theta_2 - \Theta_2)$ is given by $\hat{J}^{-1} \hat{I} \hat{I}^{-1}$ where:

\[
\hat{J}^{-1} = \left[ \frac{1}{T} \sum_{j,h,t} \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} \left( \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} \right) \right]^{-1}.
\]

As regards $\hat{I}$ – that is the covariance matrix of $\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \gamma_t$ with $\gamma_t = \sum_{j,h} \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} (y_{j,h,t}^\rho - y_{j,h,t}(\Theta_2, F_t))$ –, we use the Newey-West (1987) HAC estimator. This estimate is given by:

\[
\hat{I} = \sum_{i=-(T-m+1)}^{i=T-m-1} \kappa \left( \frac{i}{m} \right) \hat{cov}(\hat{\gamma}_t, \hat{\gamma}_{t+i})
\]

where $\hat{\gamma}_t = \sum_{j,h} \frac{\partial y_{j,h,t}(\Theta_2, F_t)}{\partial \theta} (y_{j,h,t}^\rho - y_{j,h,t}(\Theta_2, F_t))$ and where $\hat{cov}$ denoting the sample covariance matrix. We use the Bartlett kernel $\kappa(x) = 1 - |x|^\alpha$.\(^{10}\)

\(^{10}\)The kernel bandwidth $m$ is taken equal to 6 (one semester) in our analysis, which is of the order of magnitude of $T^{1/3}$ (that is usually seen as a “rule-of-thumb” guide for this value).
Figure 1: 10-year sovereign spreads against Germany

Figure 2: Debt service-to-revenues ratio
Figure 3: Short rate projections: Consensus Forecasts vs. model

Figure 4: Actual vs. fitted yields: Germany
Figure 5: Actual and fitted 5-year sovereign spreads

Figure 6: Actual and fitted 10-year sovereign spreads
Figure 7: Decomposition of fitted yields: Germany

Figure 8: Decomposition of fitted yields: Spain