Quantifying the Shadow Economy: Measurement *with* Theory*

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Abstract

We construct a dynamic, general equilibrium model of tax evasion where agents choose to report some of their income. Unreported income requires using a payment method that avoids recordkeeping – cash. Trade using cash to avoid taxes is the 'shadow economy' in our model. We then calibrate our model using money, interest rate and GDP data to back out the size of the shadow economy for a sample of 30 countries and compare our estimates to traditional ad hoc estimates. Our results generate reasonably larger estimates for the size of the shadow economy than exist in the prevailing literature.

JEL Codes: E4, E5.

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1 Introduction

There is a vast literature that studies the shadow, or underground, economy (see Schneider and Enste (2000) for a review). While the definition of the shadow economy is subject to debate, a standard taxonomy attributes most of this activity to being the result of either 1) trade in illegal goods and services, or 2) tax evasion.[1]

The key question in this literature is how large is the shadow economy? Answering this question requires measuring the activity in the shadow economy. This is hard to do since the point of trading in this economy is to avoid detection. Therefore researchers have to employ indirect methods to tease out estimates of the size of the shadow economy. These methods include surveys of citizens, discrepancies in national income accounting, money demand estimation and electricity use.

Estimates for the shadow economy in OECD countries range from 5% of official GDP to 27% while developing economies are much higher, ranging from 25% of official GDP to around 70%. While there is considerable uncertainty around these estimates, if they are remotely accurate, then studying the shadow economy would appear to be of first-order importance for economists studying business cycle behavior, optimal fiscal policy and development.

However, a survey of this literature reveals a disturbing observation – none of the empirical estimates are obtained using a rigorous theoretical model. This observation has been made before. In his paper "Quantifying the Black Economy: 'Measurement without Theory' Yet Again?", Thomas (1999) forcefully critiques this literature for not using economic theory to derive estimates of the shadow economy or the implications of those estimates. To quote

> "A large number of economists have sought to estimate the size of the black economy, but often without giving any explicit reasons for why this exercise is worth undertaking. It seems that a large black economy is 'a bad thing', as it may undermine people's willingness to pay taxes and a government's ignorance of the size of the black economy may lead to the imposition of incorrect macroeconomic policies. However, how large is large? What is important, the absolute level of the black economy, its relative size or its rate of change over time? Suppose that a committee of wise and learned economists, after much thought and consultation, informs the government that in their collective judgement the size of the black economy in the United Kingdom in 1998 corresponds to 12.56% of GNP. What should the government do? Should it behave differently if the estimate were 22.56%? *Without a theoretical framework, we have no way to answer these questions.*" (Thomas, p. F381 emphasis added)

[1] See Feige (1989, 1994) for more on this definition.

A careful inspection of the literature since the publication of Thomas's paper shows that almost no progress has been made in using theory to guide our measurement of the shadow economy.[2] Why is this? It could be that the size of the shadow economy is a macroeconomic issue yet most of the behavior of the shadow economy involves microeconomic decision-making on tax evasion, illegal activities and the like. It could be that monetary exchange is at the heart of trade in the shadow economy and this requires a dynamic, general equilibrium model to understand how aggregate currency demand is driven by individual decisions to evade taxes. Maybe the literature has given up trying to construct such a theoretical framework because it is too difficult or complex of an undertaking to capture all of these elements of informal activity.

But such an attitude is surprising to a modern macroeconomist. Why? Because modern macroeconomics builds dynamic, general equilibrium models with microfoundations and calibrate them to the data. While this methodology is standard in macroeconomics, to date, it has not been done in the shadow economy literature.[3] Our objective in this paper is to do just that.

We define the shadow economy as cash transactions done solely to evade taxes. The formal economy consists of all reported income. We do not include tax evasion done via accounting mismeasurement nor do we make a distiction between legal and illegal goods – all goods are legal in our economy. Evasion of taxes is the only illegal activity. Clearly, illegal activities are an important component of the shadow economy however we have chosen to ignore it. The reason for doing so is twofold. First, we do not want to get bogged down in a discussion of why some goods or services are illegal. Second, sorting out legal from illegal goods in the international data is a quantitative nightmare. As a result, one should take our estimates as a lower bound on the size of the shadow economy.

To conduct our analysis, we use the Lagos-Wright (2005) search theoretic model of money. The LW model is convenient for two reasons. The centralized-decentralized trading structure works well for capturing the idea that some trades are easily measured (those in centralized markets) while others are more easily hidden (those in decentralized meetings). Second, the quasi-linear preference structure allows us to control the distribution of money balances over time.

The main difference from LW is that we assume there are no information frictions that

[2] By structural, we mean a fully specified dynamic, general equilibrium model with optimizing agents – not a structural econometric model.

[3] There are several papers that have used dynamic general equilibrium models to study the shadow economy [see Koreshkova (2005), Amaral and Quintin (2006), Aruoba (2009)] but none of them use the models to estimate the size of the shadow economy. In fact, some like Koreshkova (2005), use prevailing estimates of the shadow economy to calibrate the the size of the shadow economy in their models.

make money essential for trade. There is a record-keeping technology, communication of trading histories and enforcement that allow exchange to be conducted through credit. Therefore, all trade can be conducted with credit. However, if agents use credit then the transaction is recorded and reported to the government who can enforce payment of income taxes. On the other hand, in decentralized meetings, monetary transactions are not recorded or reported to the government, which allows agents to evade income taxes.

The question is will agents use credit and be part of the formal economy or will they participate in the shadow economy by using money to evade taxes? While cash allows agents to evade taxes it is not costless to do so – money can be taxed via inflation. Thus, agents must decide whether to pay the inflation tax or the income tax (or some combination of the two) and this in turn determines the size of the shadow economy.

Our key theoretical results are as follows. First, the size of the shadow economy is endogenous and depends on the rate of inflation, the marginal tax rate and how the tax savings from using cash are split between buyers and sellers. Second, *distortionary* taxation is the reason our shadow economy exists. If the government finances spending with lump-sum taxes then credit is used to facilitate all trade and the shadow economy disappears.[4] Third, with distortionary taxation, the shadow economy exists as long as the inflation rate is not 'too high' relative to the income tax rate. If inflation is high enough, agents resort to credit, pay the income tax and all trade is in the formal economy. The critical inflation rate is a function of the tax rate, buyer's bargaining power and the extent of trading frictions in the shadow economy.

Turning to our quantitative results, we show that standard money demand estimates can be used to back out the size of the shadow economy. While this sounds similar to existing currency demand approaches to quantifying the size of the shadow economy, our framework is an improvement to the prevailing literature since we do not require the assumption of having no shadow economy in a base year. Moreover, since cash is the only means of payment that leaves no trace, increases in currency demand deposits that are due largely to a slowdown in demand deposits are not attributed to an increase in the shadow economy as is typically assumed in the empirical literature.[5] As a benchmark comparison of estimates, we contrast them with those in Schneider and Enste (2000) which thoroughly canvasses the literature on this issue. We report the results for 20 countries and find estimates varying from 2% for the U.S. to over 400% for Russia. Our estimates also tend to be larger for most countries than those reported in Schneider and Enste. However, due to the usual problem

[4] This is a critical distinction from Aruoba's (2009) model – if distortionary taxes are eliminated, he still has a shadow economy. Thus, his model is not about tax evasion but rather illegal goods.

[5] We refer the reader to Gillian Garcia (1978), Park (1979), and Feige (1996) for a discussion of the impact of this channel for the estimates of the shadow economy for the United States.

of estimating money demand curves in conjunction with the short time series we have for some of our countries, we are suspicious as to the robustness of our estimates. However, our main objective is to provide a methodology for using theoretical models to guide the process of quantifying the shadow economy rather than deriving definitive estimates.

The structure of the paper is as follows. Section 2, contains the environment and policy actions. In Section 3, we construct an equilibrium for our economy. In Section 4, we show existence and uniqueness of the equilibrium and characterize it. It also contains some examples and extensions of the basic model. Section 6 contains the calibration procedure and our empirical estimates of the size of the shadow economy. Section 7 concludes. All proofs are in the Appendix.

2 The Model

The Environment Time is discrete and each period is divided into two subperiods. There is a generic good that can be produced and consumed in each subperiod. This good is perishable across subperiods. As in Rocheteau and Wright (2005), there is a continuum of agents of measure 1 who are divided into two groups of equal size, called buyers and sellers. Buyers wish to consume during the first subperiod but cannot produce and while sellers can produce in the first subperiod but do not wish to consume. In this subperiod, agents meet pairwise in a decentralized market denoted DM and each buyer is matched with a seller with probability σ. Buyers get utility $\varepsilon u(q)$ from consuming q units of the good where ε is an idiosyncratic preference shock with distribution $\mathcal{F}(\varepsilon)$ and compact support $[\underline{\varepsilon}, \overline{\varepsilon}]$. We assume $u'(q), -u''(q) > 0$ and $u(0) = 0$. Sellers incur utility cost $c(q)$ from producing q units of the good with the following properties $c'(q) > 0$, $c''(q) \geq 0$ and $c(0) = 0$.

In the second subperiod all agents consume and goods are traded in a centralized Walrasian market denoted CM. Agents can also sell labor to competitive firms and are paid w per unit of labor supplied. Both sets of agents get utility $U(x)$ from consuming x units of the good and incur disutility cost $-h$ from supplying h units of the good. Time is discounted from the CM to the DM at rate $\beta_C = \beta < 1$ and from the DM to the CM at rate $\beta_D = 1$. Firms in the CM can produce one unit of output per unit of labor used in production. It then follows that $w = 1$. Although we assume that there are different utility and production costs across the two sub-periods, none of our results would change if we assumed $U(x) = u(q)$ and $c(q) = q$.

In the CM, agents take prices parametrically. Let P_{CM} denote the money price of goods. It is more convenient to use $\phi = 1/P_{CM}$ which is the CM goods price of money. In the DM, terms of trade for pairwise meetings are determined by proportional bargaining. This entails distributing a fraction θ of the match surplus to the buyer, and fraction $1-\theta$ to the seller. Since buyers cannot produce in the DM some form of payment is needed to entice sellers to trade. We assume that individual trading histories can be costlessly recorded and communicated to other agents. We also assume promises of repayment can be enforced. Consequently, credit is a feasible form of payment in both the DM and the CM. It is important to stress that our assumptions imply that financial markets are fully developed and efficiently operated. Hence, our results are not driven by incomplete financial markets.

We also assume there is a fiat object called money that can be used for payment in either market. The aggregate stock of fiat currency per capita is given by M_t and grows at the gross rate of $\gamma = 1+\pi$ implying $M_t = \gamma M_{t-1}$. Monetary injections occur in the CM and as payment for goods and services. Let $z = M/P_{CM} = \phi M$ denote real balances in the CM. Finally, for notational purposes we drop the t subscript and denote time as -1 for $t-1$, $+1$ for $t+1$ and so on.

Fiscal Policy We assume that the government uses distortionary and lump-sum taxes to finance a constant stream of government spending, G, in the CM. The government imposes a linear tax rate τ on labor income that can be observed and uses lump-sum taxes T as needed to balance the budget. All taxes are paid in the CM even if the income was generated in the DM. At this point, we do not need to assume that money is issued by the government. Agents may choose to use another object as a medium of exchange, e.g. a foreign currency that is not controlled by the local government. But as a useful starting point, we will assume that government-issued fiat money is the monetary object in our economy.

Regarding the government's ability to observe incomes, we assume that all labor income generated in the CM is reported to the government by firms. Thus, regardless of whether wage payments are made in cash or with credit, income is observed and thus can be taxed. However, income earned by sellers in the DM may or may not be observed by the government depending on the form of payment used. For illustration, suppose in a DM trade, a buyer pays with a combination of cash and credit. The seller extends a loan of size ℓ to the buyer and this is reported to the government, who treats the recorded transaction ℓ as taxable income. However, whatever portion of the transaction that is done with money is not recorded and so there is nothing to report to the government. Furthermore, we assume the government cannot observe agent's money holdings in the CM. Consequently, cash income earned by sellers is unobservable and cannot be taxed.

The government budget constraint is

$$G = \tau H + \tau L + T + \phi\left(M - M_{-1}\right)$$

where H is aggregate labor income in the CM, L the reported income of all sellers in the DM and T is lump-sum tax revenue. The last term is real seigniorage.

3 Markets and trades

3.1 CM

Buyers During the centralized market buyers also choose how much to consume and work but also how much money to carry to the next period's decentralized market. These choices are made before ε_{+1} is realized in the next DM. Loan payments also must be settled. Hence, at the beginning of the centralized market the problem of a representative buyer holding z units of real balances and outstanding real loans ℓ (a liability) is denoted by:

$$W(z,\ell) = \max_{x,h,z_{+1}} \{U(x) - h + \beta V(z_{+1})\} \tag{1}$$
$$s.t. \;\; x = (1-\tau)h + z - \ell - \gamma z_{+1} - T$$

where $\gamma = \phi/\phi_{+1}$ is the inflation rate in the CM from period t to the $t+1$. This problem can be rewritten as

$$W(z,\ell) = (1-\tau)^{-1}(z-\ell-T) + \max_{x,z_{+1}} \left\{U(x) - (1-\tau)^{-1}x - (1-\tau)^{-1}\gamma z_{+1} + \beta V(z_{+1})\right\}.$$

The first-order conditions yield

$$U'(\hat{x}) = (1-\tau)^{-1} \tag{2}$$
$$\beta V'(z_{+1}) \leq \gamma(1-\tau)^{-1} \quad (= 0 \;\; if \;\; z_{+1} > 0) \tag{3}$$

and the envelope conditions are $W_z = (1-\tau)^{-1}, W_\ell = -(1-\tau)^{-1}$. The first best allocation would satisfy $U'(x^*) = 1$ so the presence of distorting labor taxes lowers consumption since $\hat{x} < x^*$.

Sellers During the centralized market sellers choose consumption and how much labor to supply. It is straightforward to show that seller's will not take money balances into the next DM since they have no need for it. Let the CM value function for a seller be denoted $W^s(z,-\ell)$ where $z \equiv \phi m$ and ℓ are his holdings of real balances and loans extended (an asset) measured in units of the CM good. Hence, the value function of a representative seller at the beginning of the CM is given by:

$$W^s(z,\ell) = \max_{x,h} \{U(x) - h + \beta V^s\} \tag{4}$$
$$s.t.\;\; x = (1-\tau) h + z + [1-\tau(\ell)]\,\ell - T.$$

where V^s is the value function entering the next DM for a seller and

$$\begin{aligned} \tau(\ell) &= \tau \;\text{ if } \ell \geq 0 \\ \tau(\ell) &= 0 \;\text{ otherwise.} \end{aligned}$$

This function taxes income earned via issuing credit but does not subsidize borrowing by sellers. The idea is to tax income and not financial transactions unrelated to the generation of income. Substituting out for h using the budget constraint yields

$$W^s(z,-\ell) = (1-\tau)^{-1}\{z + [1-\tau(\ell)]\,\ell - T\} + \max_x \left\{U(x) - (1-\tau)^{-1} x + \beta V^s\right\}$$

The first-order conditions yield $U'(\hat{x}) = (1-\tau)^{-1}$ and the envelope conditions $W^s_z(z,\ell) = (1-\tau)^{-1}, W^s_\ell(z,\ell) = (1-\tau)^{-1}[1-\tau(\ell)]$. The envelope conditions show that cash has a higher value in the CM to a seller than income received as a loan repayment.

3.2 DM

In the decentralized market buyers observe their idiosyncratic realization of ε and with probability σ are randomly matched with a seller. Terms of trade are given by proportional bargaining with threat points given by no trade. The seller has to decide whether or not to offer credit to the buyer. If it is extended, the buyer decides whether to use it or not. As we show below, the buyer will always use credit if it is offered. If credit is used, the value paid for with credit is recorded as taxable income for the seller. If cash is used for any part of the transaction, it is not recorded and is part of the shadow economy. The first best allocation is the quantity q^*_ε solving $\varepsilon u'(q^*_\varepsilon) = c'(q^*_\varepsilon)$.

In a match with a buyer of type ε, the seller produces q_ε for the buyer. The first best allocation is the quantity q^*_ε solving $\varepsilon u'(q^*_\varepsilon) = c'(q^*_\varepsilon)$. The buyer gives the seller d_ε units of real balances and receives a loan of size ℓ_ε. The buyer's surplus is

$$\begin{aligned} \mathcal{S}^b_\varepsilon &\equiv \varepsilon u(q_\varepsilon) + W(z - d_\varepsilon, \ell_\varepsilon) - W(z,0) \\ &= \varepsilon u(q_\varepsilon) - (1-\tau)^{-1}(d_\varepsilon + \ell_\varepsilon), \end{aligned}$$

while the sellers surplus equals:

$$\begin{aligned}\mathcal{S}^s_\varepsilon &\equiv -c(q_\varepsilon) + W^s(d_\varepsilon, \ell_\varepsilon) - W^s(0,0) \\ &= -c(q_\varepsilon) + (1-\tau)^{-1} d_\varepsilon + (1-\tau)^{-1}[1-\tau(\ell)]\,\ell_\varepsilon.\end{aligned}$$

The total surplus in a match of type ε is given by:

$$\mathcal{S}_\varepsilon = \varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau(\ell)(1-\tau)^{-1}\ell_\varepsilon.$$

It is obvious from this expression that using credit, ceteris paribus lowers the match surplus. The reason is the seller has to pay taxes on this income which lowers the net gains from trade. Thus, by lowering the amount of credit extended by one unit, the seller saves $\tau/(1-\tau)$ units of labor in the next CM. This creates extra surplus for the buyer and seller to split.

With proportional bargaining, the buyer gets the fraction $\theta\mathcal{S}_\varepsilon$ while the seller gets $(1-\theta)\mathcal{S}_\varepsilon$. Thus we have

$$\begin{aligned}\mathcal{S}^b_\varepsilon &= \theta\left[\varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau(\ell)(1-\tau)^{-1}\ell_\varepsilon\right] \\ \mathcal{S}^s_\varepsilon &= (1-\theta)\left[\varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau(\ell)(1-\tau)^{-1}\ell_\varepsilon\right].\end{aligned}$$

The buyer's surplus can be rearranged to obtain

$$d_\varepsilon = (1-\tau)\left[(1-\theta)\varepsilon u(q_\varepsilon) + \theta c(q_\varepsilon)\right] - \left[1-\theta\tau(\ell)\right]\ell_\varepsilon.$$

For $\ell > 0$, we have $|\partial\ell_\varepsilon/\partial d_\varepsilon| = (1-\theta\tau)^{-1} > 1$ for $\theta > 0$. Bringing in one less unit of real balances increases the loan amount by more than one unit. This is the way in which the seller must be compensated for extending credit.[6] Typically, by using an extra unit of cash rather than credit, a buyer saves principal and interest on a loan, $1+i$. The implicit interest here is thus given by

$$i_{DM} = \frac{\theta\tau}{1-\theta\tau}. \tag{5}$$

The implicit interest rate is increasing in both the tax rate and the buyer's bargaining power. The tax rate effect is clear – the higher is τ the more costly it is for the seller to extend credit to the buyer. Therefore he charges a higher rate of interest. What is less clear is why the interest rate is increasing in the buyer's bargaining power. One's intuition would be that it should go down. The reason is as follows: As θ increases, the buyer can extract more q from the seller. Since the money holdings are given in the match, the seller has to give a

[6] Alternatively, reducing the loan amount more than 1-for-1 is the way the seller compensates the buyer for bringing in an additional unit of money.

bigger loan to the buyer which imposes a tax liability on the seller. In order to compensate the seller, the implicit interest rate that the buyer pays must therefore go up.

The seller faces the following problem:

$$\max_{q_\varepsilon, d_\varepsilon . \ell_\varepsilon} -c(q_\varepsilon) + (1-\tau)^{-1} d_\varepsilon + (1-\tau)^{-1} [1-\tau(\ell)] \ell_\varepsilon$$

$$s.t. \quad 0 \leq d_\varepsilon \leq z, \qquad 0 \leq \ell_\varepsilon$$

$$\theta \left[\varepsilon u(q_\varepsilon) - c(q_\varepsilon) - \tau(\ell)(1-\tau)^{-1} \ell_\alpha\right] \leq \varepsilon u(q_\varepsilon) - (1-\tau)^{-1}(d_\varepsilon + \ell_\varepsilon).$$

The seller chooses how much output to produce and how the buyer should pay for it subject to the constraint that the buyer receives no less than $\mathcal{S}^b_\varepsilon$. The solution to this problem is as follows. For $z \geq z^*_\varepsilon \equiv (1-\tau)[(1-\theta)\varepsilon u(q^*_\varepsilon) + \theta c(q^*_\varepsilon)]$ we have

$$\begin{aligned} d_\varepsilon &= z^*_\varepsilon, \quad \tau(\ell) = \ell_\varepsilon = 0 \\ \varepsilon u'(q^*_\varepsilon) &= c'(q^*_\varepsilon) \end{aligned}$$

For $\tilde{z}_\varepsilon < z < z^*_\varepsilon$, where $\tilde{z}_\varepsilon$ is defined below, we obtain

$$\begin{aligned} d_\varepsilon &= z, \quad \tau(\ell) = \ell_\varepsilon = 0 \\ z &= (1-\tau)[(1-\theta)\varepsilon u(\hat{q}_\varepsilon) + \theta c(\hat{q}_\varepsilon)] \end{aligned}$$

where $\hat{q}_\varepsilon < q^*_\varepsilon$ and is increasing in z. In both cases, no credit is extended so the entire value of the trade is in the shadow economy.

For $0 \leq z < \tilde{z}_\varepsilon$ we have

$$\begin{aligned} d_\varepsilon &= z \quad \tau(\ell) = \tau, \\ \ell_\varepsilon &= (1-\theta\tau)^{-1}\{(1-\tau)[(1-\theta)\varepsilon u(\tilde{q}_\varepsilon) + \theta c(\tilde{q}_\varepsilon)] - z\} \\ \varepsilon u'(\tilde{q}_\varepsilon) &= (1-\tau)^{-1} c'(\tilde{q}_\varepsilon) \end{aligned} \tag{6}$$

where $\tilde{q}_\varepsilon < \hat{q}_\varepsilon$ and $\tilde{z}_\varepsilon \equiv (1-\tau)[(1-\theta)\varepsilon u(\tilde{q}_\varepsilon) + \theta c(\tilde{q}_\varepsilon)]$. Note that the critical values, z^*_ε and $\tilde{z}_\varepsilon$ differ across buyer types. It is straightforward to show that both are monotonically increasing in ε. In this last case, some part of the trade is recorded in the formal sector while the cash portion goes into the shadow economy.

The nature of this solution is that if the buyer has sufficiently high real balances, he acquires the first-best quantity and pays with cash. No credit is used and the transaction is not recorded. If the buyer's real balances are somewhat lower (below z^*_ε) the seller chooses not to extend credit and accepts only cash. However, rather than the first-best quantity he produces something less. Again, the transaction is not recorded. Finally if real balances are low enough, then the seller takes all of the cash and gives the buyer enough credit to acquire $\tilde{q}_\varepsilon$. The portion of the transaction involving credit is recorded and the seller pays taxes on that earned income in the CM.

There are several key observations from this solution. First, for $z < \tilde{z}_\varepsilon$ the quantity traded, $\tilde{q}_\varepsilon$, is independent of how much money is exchanged. This means that even if the buyer has no cash, trade still occurs in the DM via the use of credit. Second, the critical values for money balances, z^*_ε and $\tilde{z}_\varepsilon$, are functions of the buyer's preference parameter. For high ε buyers, the first-best quantity is much larger so z^*_ε is larger as well. The reverse is true for low ε buyers. Hence, for a given amount of real balances, a buyer may get the first-best quantity using only cash if he has a low preference shock whereas he gets $\tilde{q}_\varepsilon < q^*_\varepsilon$ and pays with cash and credit if he has a high preference shock.

3.3 Optimal money buyer's money holdings

Buyers must choose the optimal amount of real balances to carry from the CM to next period's DM. The key tradeoffs of this intertemporal choice are the cost of carrying money (given by the inflation rate) vis-a-vis the expected benefit of using money for trades in the DM. Specifically, the buyer's intertemporal optimization is:

$$\max_z -(\gamma-\beta)z + \beta\sigma\theta \int_{\underline{\varepsilon}}^{\varepsilon^*(z)} \left[\varepsilon u(q^*_\varepsilon(z)) - c(q^*_\varepsilon(z))\right] d\mathcal{F}(\varepsilon) + \tag{7}$$

$$+\beta\sigma\theta \int_{\varepsilon^*(z)}^{\tilde{\varepsilon}(z)} \left[\varepsilon u(\widehat{q}_\varepsilon(z)) - c(\widehat{q}_\varepsilon(z))\right] d\mathcal{F}(\varepsilon) +$$

$$+\beta\sigma\theta \int_{\tilde{\varepsilon}(z)}^{\bar{\varepsilon}} \left[\varepsilon u(\tilde{q}_\varepsilon(z)) - c(\tilde{q}_\varepsilon(z)) - \tau(\ell)(1-\tau)^{-1}\ell(z)\right] d\mathcal{F}(\varepsilon).$$

Here, the function $\varepsilon^*(z)$ is a value such that all decentralized trades with preference shock lower than $\varepsilon^*(z)$ are not constrained. In turn, $\widetilde{\varepsilon}(z)$ captures the lowest value of the preference shock such that the DM bargaining problem requires a positive loan. The properties of the proportional bargaining solution derived before imply functions $\varepsilon^*(z)$ and $\widetilde{\varepsilon}(z)$ are well defined, increasing, and satisfy $\widetilde{\varepsilon}(z) \geq \varepsilon^*(z)$ for all z.[7] Finally, note that the nominal interest rate on a bond traded from the CM to the next CM would pay $1+i = (1+r)(1+\pi)$ where π is the inflation rate from today's CM to tomorrow's CM. It is straightforward to show that $i = (\gamma-\beta)/\beta$.

The tradeoffs faced by the buyer can be easily seen by computing how a marginal increase

[7] Our appendix provides a formal proof for these results.

in real holdings affects the buyer's intertemporal objective. The derivative of the buyer's objective (7) with respect to real balances yields

$$-i\beta + \beta\sigma\theta \int_{\varepsilon^*(z)}^{\tilde{\varepsilon}(z)} \frac{[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{[(1-\theta)\,\varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)]} d\mathcal{F}(\varepsilon) \tag{8}$$
$$\beta\sigma\theta\tau\,(1-\tau)^{-1}\,(1-\theta\tau)^{-1} \int_{\tilde{\varepsilon}(z)}^{\bar{\varepsilon}} d\mathcal{F}(\varepsilon).$$

This expression is fairly intuitive. The first term is the cost of bringing money into the DM. The derivative of the second term of the buyer's objective is zero because the expected payoff of unconstrained trades does not change by bringing more money. The second term in (8) is the expected increase in the buyer's surplus that results from bringing more money to constrained trades that do not use credit. The last term in (8) reflects the fact that bringing more money lowers the size of loans in transactions where credit is used, and thus the tax extracted from the match. This tax savings is partially passed to the buyer, as dictated by the bargaining solution.

4 Equilibrium

For the reminder of this paper we will focus our attention on symmetric stationary equilibria. Symmetry requires all similar agents to undertake the same actions. We say that a stationary equilibrium is monetary when buyers carry a strictly positive amount of real balances from the centralized market to next period's decentralized market.

Stationary equilibria is an income tax rate, τ, and a collection of sequences of lump-sum transfers, prices, money holdings, and time-invariant allocations of consumption and hours worked at the centralized market, and terms of trade functions for the DM,

$$\{T_t, \phi_t, x_t^b, h_t^b, x_t^s, h_t^s, m_t\}, q_\varepsilon, \ell_\varepsilon, d_\varepsilon,$$

such that: (a) The money holdings, consumption and hours worked allocations for the buyers are optimal taking as given the tax rate, lump-sum transfers and terms of trade functions; (b) the consumption and hours worked allocations solve the seller's problem x_t^s, h_t^s; (c) the money demanded by buyers equals the money supply; (d) equilibrium prices ϕ_t grow at the same rate as the money supply; (e) the government's budget constraint is balanced.

The main theoretical results on the shadow economy are summarized by the following proposition and corollary:

Proposition 1 *A unique stationary equilibrium exists. Further, there are three classes of equilibria: (i) Buyers hold enough money so that all trades in the DM are done using cash; (ii) There is a high enough inflation rate,* $\tilde{\gamma} \equiv \beta\left[1 + \sigma\theta\tau(1-\theta\tau)^{-1}\right]$*, such that buyers hold no money and thus all DM transactions are based on credit;(iii) There are intermediate inflation rates such that buyers carry a positive amount of money. If credit is used in a match, it is used simultaneously with money.*

The key point of this proposition is that the size of the shadow economy hinges on the inflation rate relative to the critical inflation rate $\tilde{\gamma}$ which in turn depends on the labor tax rate and other key parameters.

The derivative of the buyer's objective (8) and Proposition 1 imply the following relationships.

Corollary 2 *For a given parameterization: (i), a higher inflation rate lowers the money holdings of buyers thus increasing the measure of trades where credit is used; (ii) higher income taxes increase the return of money and thus lower the measure of trades where credit is used.*

For very low inflation rates, the inflation tax on cash is small and buyers are willing to carry more cash to get better terms of trades from sellers. This means the shadow economy is relatively large. As inflation increases, the inflation tax that buyers must incur rises as well. As a result, they carry less cash and rely on some credit to help finance their purchases from sellers. Finally, for sufficiently high inflation rates, buyers would have to bear a high inflation tax to help sellers evade taxes. Since they do not pay the income taxes, they are not willing to carry cash into the DM, which drives the size of the shadow economy to zero. This captures a common intuition that high inflation allows governments to tax the shadow economy and drive agents into the formal economy. Hence, according to our model, increasing inflation results in a smaller informal sector, while increasing taxes increases the size of the informal sector.

For sufficiently high inflation rates, our model predicts that the shadow economy disappears, However, we are assuming that there is not another currency available to conduct transactions. If agents had to use domestic currency to trade in the shadow economy then the government could just inflate at high enough rates to drive everyone into the formal economy. However, in reality, if a government tried this, agents could easily switch to a foreign currency to conduct trades. Thus, currency subsitution puts an upper bound on how much the government can inflate to tax the shadow economy.

Note that the larger is τ the larger is $\tilde{\gamma}$. This also is true for an increase in θ. Both parameters make money more valuable when trading which increases the real demand for money and thus the quantity of goods exchanged. Consequently, $\hat{q}_\varepsilon$ is traded over a wider range of real balances. This has the effect of crowding out formal (credit) trades.

It should be stressed that the buyer does not pay the income taxes associated with the credit transaction. Thus, he would always prefer to use credit. It is the seller who benefits from the cash transaction. So the seller must induce the buyer to bring cash into the DM by sharing the tax saving with him by charging him a lower price for cash relative to credit. Note that if we have seller-take-all, $\theta = 0$, then $\tilde{\gamma} = \beta$ (or the Friedman rule, $i = 0$) is the only monetary equilibrium. In short, if the seller does not share any of the tax savings associated with cash, the buyer will not bring any in and uses credit as a means of payment.

4.1 Examples

Homogeneous buyers In order to understand how heterogeneity affects the model, suppose $\mathcal{F}(\varepsilon)$ is degenerate. We have

$$(1-\tau)\,V'(z) = \begin{cases} 1 & \text{if } z \geq z_\varepsilon^*, \\ 1 + \frac{\sigma\theta[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{(1-\theta)\varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)} & \text{if } \tilde{z}_\varepsilon \leq z \leq z_\varepsilon^*, \\ 1 + \sigma\theta\tau\,(1-\theta\tau)^{-1} & \text{if } z \leq \tilde{z}_\varepsilon. \end{cases}$$

The first-order condition for z in the CM yields the following solutions for q_ε

$$\begin{aligned} q_\varepsilon &= q_\varepsilon^* && \text{for } \gamma = \beta, \\ i &= \tfrac{\sigma\theta[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{(1-\theta)\varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)} && \text{for } \beta < \gamma \leq \tilde{\gamma}, \\ q_\varepsilon &= \tilde{q}_\varepsilon < q_\varepsilon^* && \text{for } \tilde{\gamma} \leq \gamma. \end{aligned} \tag{9}$$

The goods price of money is then

$$\begin{aligned} \phi &= M^{-1}(1-\tau)\left[(1-\theta)\,\varepsilon u(\hat{q}_\varepsilon) + \theta c(\hat{q}_\varepsilon)\right] && \text{for } \beta < \gamma \leq \tilde{\gamma}, \\ \phi &= z = 0 && \text{for } \gamma > \tilde{\gamma}. \end{aligned}$$

For $\gamma \leq \tilde{\gamma}$ agents only use money to trade while for $\gamma > \tilde{\gamma}$ all buyers resort to credit to pay the sellers. In short, no monetary equilibrium exists. This means all trade in the DM is in the shadow economy or it is all in the formal sector. Thus, in order to have an equilibrium where there is a mix of formal and informal trade in the DM, we need a non-degenerate distribution over ε.

Two state example Consider the follow 2-point distribution $\varepsilon \in \{\underline{\varepsilon}, \overline{\varepsilon}\}$ where $\overline{\varepsilon}$ occurs with probability λ. Conjecture that the spread between these two values is small enough so that $z^*_{\underline{\varepsilon}} > \tilde{z}_{\overline{\varepsilon}}$. We have the unique solutions for q_ε

$$
\begin{array}{lll}
q_{\underline{\varepsilon}} = q^*_{\underline{\varepsilon}}, \qquad q_{\overline{\varepsilon}} = q^*_{\overline{\varepsilon}} & & \text{for } \gamma = \beta, \\
q_{\underline{\varepsilon}} = q^*_{\underline{\varepsilon}}, \qquad \frac{\gamma-\beta}{\beta} = i = \frac{\sigma\lambda\theta[\varepsilon u'(\hat{q}_{\overline{\varepsilon}})-c'(\hat{q}_{\overline{\varepsilon}})]}{(1-\theta)\varepsilon u'(\hat{q}_{\overline{\varepsilon}})+\theta c'(\hat{q}_{\overline{\varepsilon}})} & & \text{for } \beta < \gamma \leq \gamma_1, \\
\left.\begin{array}{l}
\frac{\gamma-\beta}{\beta} = \frac{(1-\lambda)\sigma\theta\left[\varepsilon u'(\hat{q}_{\underline{\varepsilon}})-c'(\hat{q}_{\underline{\varepsilon}})\right]}{(1-\theta)\varepsilon u'(\hat{q}_{\underline{\varepsilon}})+\theta c'(\hat{q}_{\underline{\varepsilon}})} + \frac{\lambda\sigma\theta[\varepsilon u'(\hat{q}_{\overline{\varepsilon}})-c'(\hat{q}_{\overline{\varepsilon}})]}{(1-\theta)\varepsilon u'(\hat{q}_{\overline{\varepsilon}})+\theta c'(\hat{q}_{\overline{\varepsilon}})} \\
0 = (1-\theta)\left[\overline{\varepsilon} u(\hat{q}_{\overline{\varepsilon}}) - \underline{\varepsilon} u(\hat{q}_{\underline{\varepsilon}})\right] - \theta\left[c(\hat{q}_{\underline{\varepsilon}}) - c(\hat{q}_{\overline{\varepsilon}})\right]
\end{array}\right\} & & \text{for } \gamma_1 < \gamma < \gamma_2, \\
q_{\overline{\varepsilon}} = \tilde{q}_{\overline{\varepsilon}} \qquad \frac{\gamma-\beta\left[1+\lambda\sigma\theta\tau(1-\theta\tau)^{-1}\right]}{\beta} = \frac{\sigma\theta(1-\lambda)\left[\varepsilon u'(\hat{q}_{\underline{\varepsilon}})-c'(\hat{q}_{\underline{\varepsilon}})\right]}{(1-\theta)\varepsilon u'(\hat{q}_{\underline{\varepsilon}})+\theta c'(\hat{q}_{\underline{\varepsilon}})} & & \text{for } \gamma_2 \leq \gamma < \tilde{\gamma}, \\
q_{\overline{\varepsilon}} = \tilde{q}_{\overline{\varepsilon}}, \qquad q_{\underline{\varepsilon}} = \tilde{q}_{\underline{\varepsilon}} & & \text{for } \tilde{\gamma} \leq \gamma
\end{array}
$$

where γ_1 is derived from the following two equations

$$
\begin{aligned}
0 &= (1-\theta)\left[\overline{\varepsilon} u(\bar{q}_{\overline{\varepsilon}}) - \underline{\varepsilon} u(q^*_{\overline{\varepsilon}})\right] - \theta\left[c(q^*_{\overline{\varepsilon}}) - c(\bar{q}_{\overline{\varepsilon}})\right] \\
\gamma_1 &= \beta + \beta\frac{\sigma\lambda\theta\left[\varepsilon u'(\bar{q}_{\overline{\varepsilon}}) - c'(\bar{q}_{\overline{\varepsilon}})\right]}{(1-\theta)\,\varepsilon u'(\bar{q}_{\overline{\varepsilon}}) + \theta c'(\bar{q}_{\overline{\varepsilon}})}.
\end{aligned}
$$

The first equation yields a value $\bar{q}_{\overline{\varepsilon}}$ associated with the low ε money balances just binding while the second comes from the FOC. Similarly, we obtain γ_2 from

$$
\begin{aligned}
0 &= (1-\theta)\left[\overline{\varepsilon} u(\tilde{q}_{\overline{\varepsilon}}) - \underline{\varepsilon} u(\bar{q}_{\underline{\varepsilon}})\right] - \theta\left[c(\bar{q}_{\underline{\varepsilon}}) - c(\tilde{q}_{\overline{\varepsilon}})\right] \\
\gamma_2 &= \beta + \beta\left[\frac{\sigma\theta\,(1-\lambda)\left[\varepsilon u'(\bar{q}_{\underline{\varepsilon}}) - c'(\bar{q}_{\underline{\varepsilon}})\right]}{(1-\theta)\,\varepsilon u'(\bar{q}_{\underline{\varepsilon}}) + \theta c'(\bar{q}_{\underline{\varepsilon}})} + \lambda\sigma\theta\tau\,(1-\theta\tau)^{-1}\right].
\end{aligned}
$$

As before, for $z \leq \tilde{z}_{\underline{\varepsilon}}$ for $\gamma > \tilde{\gamma}$ no monetary equilibrium exists; only a credit equilibrium exists. We now have a range of inflation rates such that money and credit trades coexist; those for $\tilde{z}_{\underline{\varepsilon}} \leq z \leq \tilde{z}_{\overline{\varepsilon}}$. For z in this range, the high ε buyers do not have enough cash so they acquire $\tilde{q}_{\overline{\varepsilon}}$ with a combination of cash and credit.

Figure 1 shows the different possible equilibria. For $\gamma = \beta$ we get the first best. For $\gamma_1 \leq \gamma \leq \gamma_2$ we have an equilibrium where $\hat{q}_{\overline{\varepsilon}} < q^*_{\overline{\varepsilon}}$ and $\hat{q}_{\underline{\varepsilon}} < q^*_{\underline{\varepsilon}}$. In this range, again, all trade in the DM is in the shadow economy. For $\gamma_2 \leq \gamma \leq \tilde{\gamma}$ we have coexistence of money and credit meaning some of the trades are in the shadow economy and some in the formal economy. The high ε buyer is using both cash and credit to acquire $\tilde{q}_{\overline{\varepsilon}}$ while the low ε buyer continues to use only cash. Finally above $\tilde{\gamma}$ both buyers are using credit and all DM trade

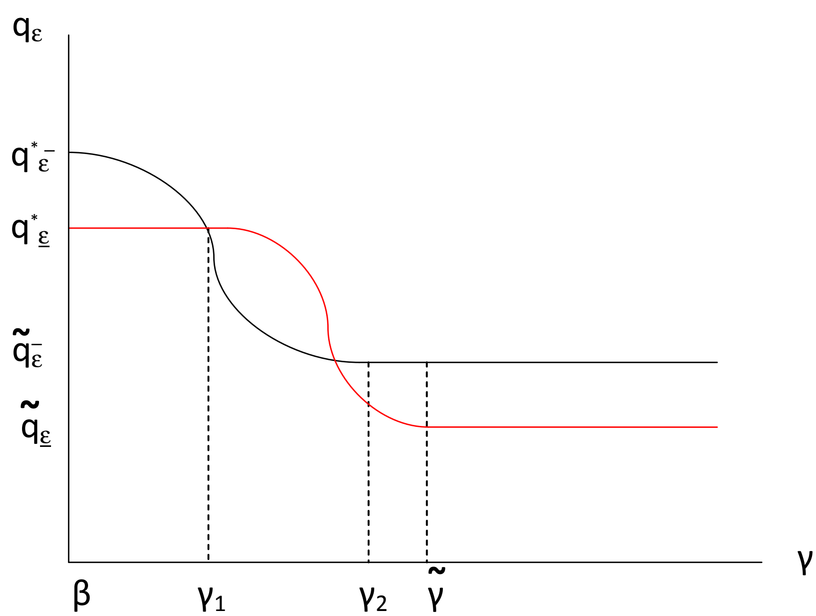

Figure 1: 2 Types of Buyers

is in the formal economy.

Productivity Differentials It is often argued that small firms tend to be in the shadow economy while larger firms tend to operate in the formal economy. We can study this case by assuming that sellers differ by their productivities (either permanent or temporary). We interpret high productivity sellers to be large 'firms' since the can produce a large amount of output at a relatively low marginal cost. Low productivity firms do the opposite.

Assume that sellers' utility cost of producing is given by $c(q, \alpha)$ where α is a productivity parameter with $c_\alpha(q, \alpha), c_{q\alpha}(q, \alpha) < 0$. Consider a 2-point distribution $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$. In this case, we can redo the bargaining solutions and derive the surpluses as before. We can show existence of equilibrium as before: for $\gamma = \beta$ we get the first-best allocation and for $\gamma > \tilde{\gamma}$ we have the credit-only equilibrium. For inflation rates between thos values, we can show that a monetary equilibrium exists but uniqueness is difficult to prove without further restrictions (such as imposing buyer-take-all, $\theta = 1$). For a 2 point distribution, linear utility and a CES cost function, we obtain a unique equilibrium that can be characterized in Figure 2.

For $\beta \leq \gamma \leq \gamma_2$ we have a money only equilibrium where all DM trade is in the shadow economy. For $\gamma_2 \leq \gamma \leq \tilde{\gamma}$ we have coexistence of trade in the formal and informal sectors. The high α sellers accept cash and extend credit to let the buyer acquire $\tilde{q}_{\bar{\varepsilon}}$ while the low

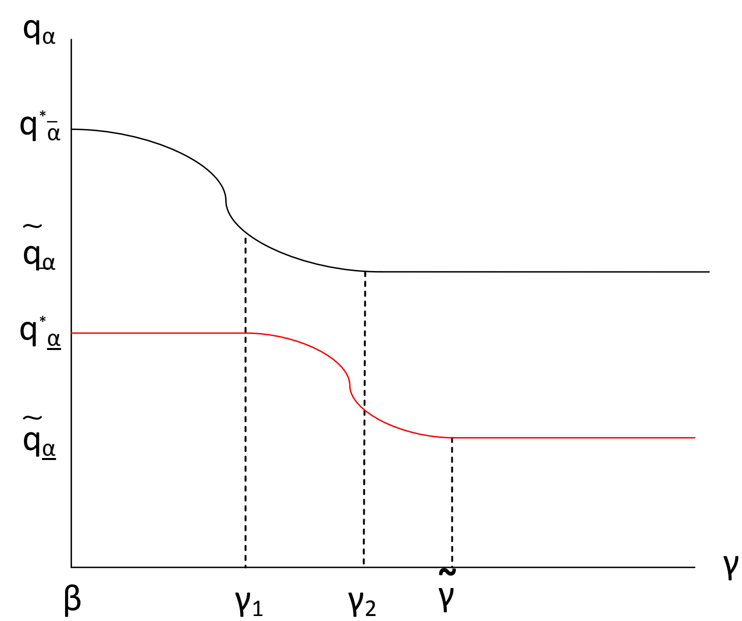

Figure 2: 2 Types of Sellers

α seller continues to accept only cash. Finally at $\tilde{\gamma}$ both sellers extend credit to the buyer and above that only the credit equilibrium exists meaning all trade is recorded in the formal economy.

In the α-model, for $\gamma_2 \leq \gamma \leq \tilde{\gamma}$ large producers (the high productivity sellers) use credit while low productivity sellers are paid in cash. This is consistent with empirical evidence that large firms tend to operate in the formal economy while small firms are the ones most likely to produce solely for cash in the informal sector.

5 Measuring the Size of the Shadow Economy

The size of the shadow economy is typically measured as a percentage of formal GDP. We do the same in order to compare our estimates to the existing literature. Let s_I denote the size of the informal sector measured as

$$s_I = \frac{P_I Y_I}{P_F Y_F}$$

where $P_I Y_I$ is nominal GDP in the shadow economy and $P_F Y_F$ is measured nominal GDP. In our model, $P_I Y_I$ is equal to cash spent in the DM by buyers who have a match. As a benchmark, consider the economy with homogeneous buyers. We showed that, in this case, all trade in the DM is done in cash and is not recorded in formal GDP. The measure of buyers with a successful match is given by σ. Thus, $P_I Y_I = \sigma M$ which implies

$$s_I = \frac{P_I Y_I}{P_F Y_F} = \frac{\sigma M}{P_F Y_F}. \tag{10}$$

We can obtain an estimate of $M/P_F Y_F$ directly from the data once a time interval and monetary aggregate are chosen. All that remains to be done is obtain a calibrated value of σ.[8]

5.1 Calibration

Let the equilibrium relationship for money balances be given by $M = L(i) P_F Y_F$ where $P_F Y_F$ is measured GDP and $L(i)$ is an arbitrary function of the nominal interest rate.[9] Thus measured GDP consists of output in the CM. Letting $P_F Y_F = P_{CM} Y_{CM}$ we have

$$L(i) = \frac{M}{P_{CM} Y_{CM}} = \frac{z}{Y_{CM}}.$$

The interest elasticity of money balances is then given by

$$\epsilon_i = \frac{dM}{di}\frac{i}{M} = L'(i) P_{CM} Y_{CM} \frac{i}{M} = \frac{L'(i)}{L(i)} i.$$

Real CM output in our model is given by $Y_{CM} = x^*$ where x^* solves $U'(x^*) = (1-\tau)^{-1}$. Assume CM preferences are given by $U(x) = B \ln x$ which gives us $x^* = (1-\tau) B$. Thus

$$L(i) = \frac{z}{(1-\tau) B}.$$

where real balances z implicitly depend on i with

$$L'(i) = \frac{1}{(1-\tau) B}\frac{dz}{di}.$$

[8] Our strategy follows Lucas (2000) and Lagos and Wright (2005) by using data to derive a money demand curve. Intuitively, this approach gives us two numbers: 1) the interest elasticity (or semi-interest elasticity) of money demand and 2) average money balances at the average nominal interest rate. The first is a 'slope' measurement and the second is a 'level' measurement. We then construct a similar type of money demand from our theoretical model and use these empircal values to pin down parameters in the theoretical money demand curve.

[9] This formulation implicitly assumed an income elasticity of one for money demand.

The interest elasticity is

$$\epsilon_i = i\frac{L'(i)}{L(i)}\frac{dz}{di} = \frac{i}{z}\frac{dz}{di}. \tag{11}$$

From the bargaining solution, real balances are given by

$$z = (1-\tau)\left[(1-\theta)\,u(q) + \theta c(q)\right] \tag{12}$$

Recall from (9) that

$$i = \frac{\sigma\theta\left[\varepsilon u'(q) - c'(q)\right]}{(1-\theta)\,\varepsilon u'(q) + \theta c'(q)} \qquad \text{for } \beta < \gamma \leq \tilde{\gamma}. \tag{13}$$

For $\tilde{\gamma} \leq \gamma$ the size of the shadow economy is zero. Assume that inflation is below this cutoff. Totally differentiating (12) and (13) yields

$$\frac{dz}{di} = \frac{dz}{dq}\frac{dq}{di} = \frac{(1-\tau)\left[(1-\theta)\,u'(q) + \theta c'(q)\right]^2}{\left[\sigma\theta - i\,(1-\theta)\right]u''(q) - (i\theta + \sigma\theta)\,c''(q)}. \tag{14}$$

Using (12) and (14) in (11) yields

$$\epsilon_i = \frac{i}{(1-\tau)\left[(1-\theta)\,u(q) + \theta c(q)\right]}\frac{(1-\tau)\left[(1-\theta)\,u'(q) + \theta c'(q)\right]^2}{\left[\sigma\theta - i\,(1-\theta)\right]u''(q) - (i\theta + \sigma\theta)\,c''(q)}.$$

Following Lagos and Wright (2005) assume the following functional forms

$$u(q) = \frac{(b+q)^{1-\rho} - b^{1-\rho}}{1-\rho} \qquad c(q) = q.$$

Using (13) with $b \to 0$ we obtain

$$q = \left[\frac{\sigma\theta - i\,(1-\theta)}{(\sigma+i)\,\theta}\right]^{1/\rho}$$

With these preferences, the interest elasticity reduces to

$$\epsilon_i = \frac{i\left[\frac{\sigma}{\sigma\theta - i(1-\theta)}\right]^2}{-\rho\left[\frac{\sigma\theta - i(1-\theta)}{(\sigma+i)\theta}\right]^{1/\rho}\left[\left(\frac{1-\theta}{1-\rho}\right)\frac{(\sigma+i)}{\sigma\theta - i(1-\theta)} + 1\right](\sigma+i)\,\theta}.$$

In addition to σ we have two other parameters in this expression, θ and ρ, that need to be pinned down. It is common to either use mark-up data on prices to pin down θ or to impose buyer-take-all ($\theta = 1$). Since we cannot get reliable data on mark-ups across countries, we choose to impose $\theta = 1$. Furthermore, Waller (2011) shows that balanced growth in this class of quasi-linear models requires $\rho = 1$ (log utility in DM) so we impose this restriction on DM preferences. Consequently, we are left with

$$q = \frac{\sigma}{\sigma + i} \qquad z = \frac{(1-\tau)\,\sigma}{\sigma + i}.$$

Finally, we have

$$\sigma = -\frac{i}{\epsilon_i} \tag{15}$$

$$L(i) = \frac{M}{P_{CM}Y_{CM}} = \frac{\sigma}{(\sigma + i)\,B}. \tag{16}$$

We need to pin down two parameters, σ and B. We estimate standard money demand regressions to obtain empirical estimates of the money demand elasticity, $\hat{\epsilon}_i$. With those estimates we use the average nominal interest rate over the sample period, $\hat{\imath}$, and (15) to get a calibrated value $\hat{\sigma}$. Since $M/P_{CM}Y_{CM}$ can be interpreted as the inverse of velocity, we use the time averaged value of velocity and use it as an empirical value for $\hat{L}(i)$. Using $\hat{\imath}$, $\hat{\sigma}$ and $\hat{L}(i)$ we can back out B from (16) which simply ensures logical consistency in the model.

6 Estimates of the Shadow Economy

7 Data

To calibrate σ we need to estimate money demand equations for the subset of countries we study. This is a daunting task for several reasons. First, we have to confront all of the problems and issues of estimating money demand functions in a world of changing financial and payment structures. Second, while there are many problems obtaining robust estimates of money demand elasticities for the U.S., the problem is even worse when looking at international data with developing nations. Finally, there are serious data issues when trying to construct a consistent measure of money, inflation and interest rates across a wide sample of countries. Despite these problems, we proceed down this path in order to illustrate our methodolgy for quantifying the size of the shadow economy.

We estimate a typical money demand equation given in (17), where the variables m, y, and i are the logarithms of real money, real output, and nominal interest rate and Δ indicates the first difference, according to the Cochrane-Orcutt procedure.

$$\Delta m_t = \beta_y \Delta y_t + \beta_i \Delta i_t - \rho \beta_y \Delta y_{t-1} - \rho \beta_i \Delta i_{t-1} + \rho \Delta m_{t-1} + \nu_t. \tag{17}$$

All data is from the IMF/IFS. The series specifiers are given in table **??**. In general the sample of countries considered is given by data availability (there has to be data for four different time series' over the same time horizon). For each country, the sample is determined by the intersection of the time horizon of each variable. We use nominal GDP and the GDP deflator, seasonally adjusted when possible. For the interest rate, we considered the money market rate (Fed funds rate equivalent) as provided by the IMF/IFS.

The biggest issue, somewhat surprisingly, is that the data on M_0 varies widely due to differing M_0 definitions across countries. One way to deal with this is to use IFS data that is constructed using the answers from a common survey of central banks that asks for specific asset positions. Hence, assuming reliable answers by the central banks, it measures the exact same thing for about 100 countries. We use this data set to construct Table 1 below. Countries with non-negative interest rate elasticities have been removed which reduces the number of countries substantially. We also exclude Euro-zone economies since the data is post European monetary union. We have very few Latin American countries in our sample because of the instability in money demand estimates for most of these countries.

One reason that we get a large number of non-negative elasticity estimates. is that the IFS time series starts in 2001 implying a very small sample size per country. So, we looked at other measures of M_0 that where reported by central banks to the IMF/IFS to expand the set of countries. These are not consistent definitions of M_0 across countries but we use it anyway. Shadow economy estimates for these countries are contained in Table 2. In the appendix we list the average interest rates, average values of M_0/GDP and our estimates of the interest elasticity of M_0 for both the consistent and inconsistent measures of M_0 (Tables 3 and 4 respectively).

7.1 Results

The results of our estimation yield the following sizes for the shadow economy for those countries for which we obtained non-negligible interest elasticity estimates. The reason is that for countries with interest elasticities close to zero, our calibrated values of σ get very large and the size of the shadow economy is thousands of times larger than the formal economy. We exclude these estimates. Since our main goal is to get some idea of the quantitative magnitudes coming out of methodology, we report our estimates for a selected set of economies in Tables 1 and 2. For comparison, we contrast our estimates to those of

Schneider and Enste (2000), who give a range of estimates for countries based on electricity use and currency demand estimates. The superscript a denotes estimates of from Table 2 in SE, b corresponds to estimates in Table 4 of SE, c denotes estimates from Table 5 and d denotes those from Table 6.

Table 1: Estimates of the Shadow Economy (Consistent M_0)

	GPW	SE (2000)
Country	s_I	s_I
Bolivia	73.8	65.6^b
Canada	6.8	$10\text{-}13^d$
Georgia	380	$28\text{-}40^a$
Iceland	31.6	NA
Indonesia	43.2	NA
Japan	26.2	$8\text{-}10^a$
Latvia	222.4	$20\text{-}27^a$
Macao	83.0	NA
Mauritius	56.2	20^b
Mexico	52.0	$40\text{-}60^a$
Morocco	115.6	$39\text{-}45^a$
Philippines	59.8	$38\text{-}50^a$
Poland	50.3	$20\text{-}28^a$
Romania	238.0	$16\text{-}31^c$
Russia	203.8	$20\text{-}27^a$
South Africa	11.0	9^b
Sweden	29.4	$13\text{-}23^a$
Thailand	288.0	70^a
Turkey	81.4	NA
United States	2.4	$8\text{-}10^a$

Table 2: Estimates of the Shadow Economy (Inconsistent M_0)

	GPW	SE (2000)
Country	s_I	$\mathbf{s}_I$
Australia	24.3	10-15[d]
Hong Kong	38.8	13[b]
S. Korea	48.9	38-50[a]
Kyrgyzstan	156.1	14-36[c]
New Zealand	30.4	9[d]
Norway	119.7	13-23[a]
Singapore	26.6	13[a]
Switzerland	18.2	8-10[a]
Tunisia	19.7	39-45[a]
United Kingdom	19.9	13-23[a]

Several interesting observations arise. First, we obtain higher estimates of the shadow economy compared to those reported in Schneider and Enste. Second, our U.S. estimate is very low while Norway, New Zealand are very high. Fourth, our estimates for countries in the former Soviet Union and Eastern Europe are substantially higher. Lastly, for 7 of the 30 countries our estimates are close to or within the ranges reported in Schneider and Enste, which suggests that there is some consistency in approaches. The differences in the others may well be due to different time periods studied. In the end, our estimates suggest the shadow economy is a more important issue than previously considered and has serious implications for public finance, labor and monetary policy in a large number of countries.

Our benchmark model is very simple and to illustrate the method we imposed restrictions on key parameters to reduce the calibration exercise to estimating a single parameter. However, one could calibrate the other parameters θ and ρ as well as introducing heterogeneity across buyers or sellers. This would require other data targets for the calibration. Given our experience with the difficulty in obtaining consistent data constructions for M_0 this is left to future work.

8 Conclusion

Modern macroeconomics used dynamic general equilibrium models as laboratories for quantitative analysis. We employ this methodology to tackle a quantitative issue that heretofore lacks theoretical foundations – quantifying the size of the shadow economy. We construct a dynamic monetary model where agents choose to engage in formal or informal trades based on incentives to evade taxes. We are able to construct a theoretical measure of the size of the shadow economy which we then calibrate to money demand data. Our estimates of the size of the shadow economy are on average higher than those reported in the literature yet a non-trivial fraction are consistent with prevailing estimates. While there clearly is more quantitive work that could be done using our framework, we believe our quantitative analysis opens up new doors for research in this area.

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Appendix

Bargaining solution.

$$\max_{d_\varepsilon, q_\varepsilon . \ell_\varepsilon} -c\left(q_\varepsilon\right) + (1-\tau)^{-1}\phi d_\varepsilon + (1-\tau)^{-1}\left[1-\tau\left(\ell\right)\right]\phi \ell_\varepsilon$$

$$s.t. \quad 0 \leq d_\varepsilon \leq m, \qquad 0 \leq \ell_\varepsilon$$

$$\varepsilon u(q_\varepsilon) - (1-\tau)^{-1}\phi\left(d_\varepsilon + \ell_\varepsilon\right) \geq \theta\left[\varepsilon u(q_\varepsilon) - c\left(q_\varepsilon\right) - \tau\left(\ell\right)(1-\tau)^{-1}\phi \ell_\varepsilon\right].$$

where

$$\phi \ell_\varepsilon = \left[1-\theta\tau\left(\ell\right)\right]^{-1}\left\{(1-\tau)\left[(1-\theta)\varepsilon u(q_\varepsilon) + \theta c(q_\varepsilon)\right] - \phi d_\varepsilon\right\}$$

Let $\phi\lambda_m$ denote the Lagrangian multiplier on the upper bound on d_ε, $\phi\lambda_\ell$ be the multiplier on non-negative lending and λ_s be the multiplier on the buyer's surplus constraint. We can ignore the lower bound on d_ε for now. The FOC are

$$\begin{aligned}
d_\varepsilon &: \; 0 = (1-\tau)^{-1} - \lambda_m - \lambda_s (1-\tau)^{-1} \\
q_\varepsilon &: \; 0 = -c'\left(q_\varepsilon\right) + \lambda_s\left[(1-\theta)\varepsilon u'\left(q_\varepsilon\right) + \theta c'\left(q_\varepsilon\right)\right] \\
\ell_\varepsilon &: \; 0 = (1-\tau)^{-1}\left[1-\tau\left(\ell\right)\right] + \lambda_\ell - (1-\tau)^{-1}\lambda_s\left[1-\theta\tau\left(\ell\right)\right]
\end{aligned}$$

For $q_\varepsilon > 0, \lambda_s > 0$.
Case 1: $\lambda_m = \lambda_\ell = 0$. In this case we have

$$\begin{aligned}
d_\varepsilon &: \; \lambda_s = 1 \\
q_\alpha &: \; 0 = \varepsilon u'(q_\varepsilon) - c'\left(q_\varepsilon\right) \\
\ell_\varepsilon &: \; \lambda_s = \frac{1-\tau\left(\ell\right)}{1-\theta\tau\left(\ell\right)}
\end{aligned}$$

The first and last expressions require $\tau\left(\ell\right) = 0$ and thus $\ell = 0$. Thus the solution has $\ell = \tau\left(\ell\right) = 0$ and $q_\varepsilon = q_\varepsilon^*$ with $d_\varepsilon^* = m_\varepsilon^* \equiv \phi^{-1}(1-\tau)\left[(1-\theta)\varepsilon u(q_\varepsilon^*) + \theta c(q_\varepsilon^*)\right] \leq m$.Case 2: $\lambda_m = 0, \lambda_\ell > 0$. In this case we have $\ell = \tau\left(\ell\right) = 0$ and

$$\begin{aligned}
d_\varepsilon &: \; \lambda_s = 1 \\
q_\varepsilon &: \; 0 = \varepsilon u'(q_\varepsilon) - c'\left(q_\varepsilon\right)
\end{aligned}$$

which yields the same solution as before. So if $\lambda_m = 0$, then $\ell = \tau\left(\ell\right) = 0$. In short, a buyer would never take out a loan and keep some cash.
Case 3: $\lambda_m > 0, \lambda_\ell > 0$. In this case we have $\ell = \tau\left(\ell\right) = 0$, $d_\varepsilon = m$ and q_ε solves $\phi m =$

$(1-\tau)\left[(1-\theta)\,\varepsilon u(q_\varepsilon)+\theta c(q_\varepsilon)\right].$
Case 4: $\lambda_m>0, \lambda_\ell=0$. In this case we have $d_\varepsilon=m$ and

$$\begin{aligned} q_\varepsilon &: \; 0=-c'\left(q_\varepsilon\right)+\lambda_s\left[(1-\theta)\,\varepsilon u'\left(q_\varepsilon\right)+\theta c'\left(q_\varepsilon\right)\right] \\ \ell_\varepsilon &: \; 0=(1-\tau)^{-1}\left[1-\tau\left(\ell\right)\right]-(1-\tau)^{-1}\lambda_s\left[1-\theta\tau\left(\ell\right)\right] \end{aligned}$$

$$\begin{aligned} q_\varepsilon &: \; 0=\left[1-\tau\left(\ell\right)\right]\varepsilon u'(q_\varepsilon)-c'\left(q_\varepsilon\right) \\ \ell_\varepsilon &: \; \lambda_s=\left[1-\tau\left(\ell\right)\right]/\left[1-\theta\tau\left(\ell\right)\right] \end{aligned}$$

and ℓ_ε is given by

$$\phi\ell_\varepsilon=\left[1-\theta\tau\left(\ell\right)\right]^{-1}\left\{(1-\tau)\left[(1-\theta)\,\varepsilon u(q_\varepsilon)+\theta c(q_\varepsilon)\right]-\phi m\right\}.$$

If $\ell_\varepsilon=0$ then $\tau\left(\ell\right)=0$ and we have

$$\begin{aligned} \varepsilon u'(q_\varepsilon^*) &= c'\left(q_\varepsilon^*\right) \\ \phi m &= (1-\tau)\left[(1-\theta)\,\varepsilon u(q_\varepsilon^*)+\theta c(q_\varepsilon^*)\right] \end{aligned}$$

but this can only be satisfied if

$$\phi m=m_\varepsilon^*\equiv\phi^{-1}\left(1-\tau\right)\left[(1-\theta)\,\varepsilon u(q_\varepsilon^*)+\theta c(q_\varepsilon^*)\right].$$

If this does not hold then we must have $\ell_\varepsilon>0$ and $\tau\left(\ell\right)=\tau$. As a result we have

$$\varepsilon u'(\tilde{q}_\varepsilon)=(1-\tau)^{-1}c'\left(\tilde{q}_\varepsilon\right)$$

and

$$\phi\ell_\varepsilon=(1-\theta\tau)^{-1}\left\{(1-\tau)\left[(1-\theta)\,\varepsilon u(\tilde{q}_\varepsilon)+\theta c(\tilde{q}_\varepsilon)\right]-\phi m\right\}$$

In this case we see that $\tilde{q}_\varepsilon$ is independent of m so $\ell_\varepsilon>0$ requires $m<\phi^{-1}\left(1-\tau\right)\left[(1-\theta)\,\varepsilon u(\tilde{q}_\varepsilon)+\theta c(\tilde{q}_\varepsilon)\right]\equiv\tilde{m}_\varepsilon$.

Demand for Money. Here, we derive the properties of the cut-off functions $\varepsilon^*(z)$ and $\tilde{\varepsilon}(z)$.We start by showing the money holdings required to obtained the uncsontrained allocation are increasing in the preference shock ε. For it, consider the solution q_ε^* to:

$$\varepsilon u'(q_\varepsilon^*)=c'(q_\varepsilon^*)$$

$$\frac{\partial q_\varepsilon^*}{\partial \varepsilon} = -\frac{u'}{\varepsilon u'' - c''}$$

Hence, if costs are convex, the previous derivative is always positive. For the buyer to capture a proportion θ of the total surplus, we must have:

$$z_\varepsilon^* \equiv (1-\tau)\left[(1-\theta)\,\varepsilon u(q_\varepsilon^*) + \theta c(q_\varepsilon^*)\right].$$

Finally,

$$\frac{\partial z_\varepsilon^*}{\partial \varepsilon} = (1-\tau)[(1-\theta)(u(q_\varepsilon^*) + \varepsilon u'(q_\varepsilon^*)) + \theta c'(q_\varepsilon^*)]\frac{\partial q_\varepsilon^*}{\partial \varepsilon} > 0.$$

Observe also that $\varepsilon^*(z)$ can be defined implicitly, at each z, as the solution to

$$z = (1-\tau)\left[(1-\theta)\,\varepsilon^* u(q_{\varepsilon^*}^*) + \theta c(q_{\varepsilon^*}^*)\right].$$

Since z_ε^* is increasing in ε it follows that $\varepsilon^*(z)$ in increasing in z. ■

To understand why $\tilde{\varepsilon}(z) \geq \varepsilon^*(z)$, consider a shock ε_0 larger than, but close enough to $\varepsilon^*(z)$. If credit is going to be used then the surplus obtained has to be larger than what is attainable with money only. This is true because, other things equal, loans reduce the total surplus. What is attainable with money only (in a constrained trade)? In these types of trades output is given by

$$z = (1-\tau)\left[(1-\theta)\,\varepsilon_0 u(\hat{q}_{\varepsilon_0}) + \theta c(\hat{q}_{\varepsilon_0})\right].$$

Furthermore, $\hat{q}_{\varepsilon_0}$ must converge to q_ε^* as $z \to z_{\varepsilon_0}^*$. Hence, the surplus attainable with money only when ε_0 approaches $\varepsilon^*(z)$ from above, is also close to the optimal one. What is attainable with loans? Observe that output under a credit trade is given by

$$\widetilde{\varepsilon} u'(\tilde{q}_{\widetilde{\varepsilon}}) = (1-\tau)^{-1}\, c'(\tilde{q}_{\widetilde{\varepsilon}}), \tag{18}$$

because of the tax wedge, $\tilde{q}_{\widetilde{\varepsilon}} < \hat{q}_{\varepsilon_0}$. Thus, a trade with loans attains a strictly lower output, involves a strictly positive loan and results then in a lower surplus. It follows that trades with loans can only occur for shocks strictly higher than $\varepsilon^*(z)$. ■

Buyer's objective. Consider now the derivative of the objective function evaluated at strictly positive money holdings:

$$(\beta - \gamma) + \beta\sigma\theta\tau\,(1-\tau)^{-1}\,(1-\theta\tau)^{-1}\int_{\tilde{\varepsilon}(z)}^{\bar{\varepsilon}} d\mathcal{F}(\varepsilon) +$$

$$+\beta\sigma\theta\int_{\varepsilon^*(z)}^{\tilde{\varepsilon}(z)}\frac{[\varepsilon u'(\hat{q}_\varepsilon)-c'(\hat{q}_\varepsilon)]}{[(1-\theta)\,\varepsilon u'(\hat{q}_\varepsilon)+\theta c'(\hat{q}_\varepsilon)]}d\mathcal{F}(\varepsilon)+$$

$$+\beta\sigma\theta f(\varepsilon^*)\frac{\partial\varepsilon^*(z)}{\partial z}\left[\varepsilon^* u(q^*_{\varepsilon^*}(z))-c(q^*_{\varepsilon^*}(z))-(\varepsilon^* u(\widehat{q}_{\varepsilon^*}(z))-c(\widehat{q}_{\varepsilon^*}(z)))\right]+$$

$$+\beta\sigma\theta f(\hat{\varepsilon})\frac{\partial\hat{\varepsilon}(z)}{\partial z}\left[\hat{\varepsilon}u(\widehat{q}_{\hat{\varepsilon}}(z))-c(\widehat{q}_{\hat{\varepsilon}}(z))-(\hat{\varepsilon}u(\tilde{q}_{\hat{\varepsilon}}(z))-c(\tilde{q}_{\hat{\varepsilon}}(z))-\tau(\ell)(1-\tau)^{-1}\ell(z))\right].$$

The last two terms of this derivative take into account that bringing more money changes the types of trades the buyer may face. Specifically, higher money holdings allow more unconstrained trades to occur (the previous to last term), which obviously reduces the number of constrained trades. Higher real holdings also increases the return of constrained trades, which simultaneously reduces the set of trades where credit is employed. These last two terms of the derivative of the buyer are, nevertheless, equal to zero. The previous to last term is zero because it evaluates the surplus exactly at the shock where the constraint starts binding. At that point the two surpluses are thus equal to each other. The last term is zero because it evaluates surpluses exactly at the point where buying with credit is equivalent to buying under a constrained trade. ■

Uniqueness of equilibrium. For ease of presentation, we start with the case where inflation is not too high, namely, $(\beta-\gamma)+\beta\sigma\theta\tau(1-\tau)^{-1}(1-\theta\tau)^{-1}>0$.

(a) For all money holdings $0<z<z_0$ where z_0 is the largest holding such that $\tilde{\varepsilon}(z_0)=\underline{\varepsilon}$. Because of the above assumption, the derivative of the buyer's objective function is positive. Optimal money holdings must be higher than z_0.

(b) for money holdings $z_0<z<z_1$, where z_1 is the highest value of money holdings such that $\varepsilon^*(z_1)=\underline{\varepsilon}$ the derivative of the objective is

$$(\beta-\gamma)+\beta\sigma\theta\tau(1-\tau)^{-1}(1-\theta\tau)^{-1}\int_{\tilde{\varepsilon}(z)}^{\bar{\varepsilon}}d\mathcal{F}(\varepsilon)+$$

$$\beta\sigma\theta\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}(z)}\frac{[\varepsilon u'(\hat{q}_\varepsilon)-c'(\hat{q}_\varepsilon)]}{[(1-\theta)\,\varepsilon u'(\hat{q}_\varepsilon)+\theta c'(\hat{q}_\varepsilon)]}d\mathcal{F}(\varepsilon).$$

We know $\varepsilon u'(\hat{q}_\varepsilon)-c'(\hat{q}_\varepsilon)>0$. But since $\tilde{\varepsilon}(z)$ is increasing in z the second term in the sum is less than one. The sign of this derivative depends on the specific functional forms and parameterizations chosen. It is easy to impose additional regularity conditions such that $\frac{[\varepsilon u'(\hat{q}_\varepsilon)-c'(\hat{q}_\varepsilon)]}{[(1-\theta)\varepsilon u'(\hat{q}_\varepsilon)+\theta c'(\hat{q}_\varepsilon)]}$ is decreasing in real money holdings. Under these conditions it suffices to check the value of this derivative at z_1. If it is negative, then there is a unique zero for the derivative of the buyer's objective in the $z_0<z<z_1$ range.

(c) Finally, for money holdings $z>z_1$ we have $\tilde{\varepsilon}(z)>\varepsilon^*(z)>\underline{\varepsilon}$. The derivative of the objective is

$$(\beta - \gamma) + \beta\sigma\theta\tau\,(1-\tau)^{-1}\,(1-\theta\tau)^{-1}\int_{\tilde{\varepsilon}(z)}^{\bar{\varepsilon}} d\mathcal{F}(\varepsilon) +$$

$$\beta\sigma\theta\int_{\varepsilon^*(z)}^{\tilde{\varepsilon}(z)} \frac{[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{[(1-\theta)\,\varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)]} d\mathcal{F}(\varepsilon).$$

We now show that as z increases to $z^*_{\bar{\varepsilon}}$ then the last two terms of the function vanish. This proves the objective is decreasing near $z^*_{\bar{\varepsilon}}$. Hence, the derivative of the objective, if positive at z_1 must have a unique zero in the range $z_1 < z < z^*_{\bar{\varepsilon}}$ whenever $\frac{[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{[(1-\theta)\varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)]}$ is decreasing in z.

Uniqueness of equilibrium holds generally because the results in Wright (2010) can be applied to our model even when $\frac{[\varepsilon u'(\hat{q}_\varepsilon) - c'(\hat{q}_\varepsilon)]}{[(1-\theta)\varepsilon u'(\hat{q}_\varepsilon) + \theta c'(\hat{q}_\varepsilon)]}$ is not decreasing in real money holdings. Equilibrium for high inflation rates, that is, when $(\beta - \gamma) + \beta\sigma\theta\tau\,(1-\tau)^{-1}\,(1-\theta\tau)^{-1} < 0$ imply the objective in decreasing in money holdings up to $z = z_1$. Computing the optimum requires thus a direct comparison of the surplus obtained with money holdings higher than z_1 and the surplus obtained when buyers carry zero money holdings and all transactions are based on credit. ■

Table 3 Data for shadow economy estimates in Table 2.

Country	*Sample*	$\hat{\imath}$	*Velocity*	$\hat{\epsilon}$
Bolivia	Q4 2001 - Q3 2009	8.5841	1.7634	-0.065923
Canada	Q4 2001 - Q3 2010	6.983	30.09	-0.034319
China,P.R.:Macao		4.4709	3.9209	-0.013745
Georgia	Q1 2001 - Q4 2007	22.4843	3.3693	-0.017559
Iceland	Q4 2001 - Q3 2010	12.401	5.5021	-0.071336
Indonesia	Q1 1997 - Q3 2010	13.9408	3.3505	-0.096313
Japan	Q4 2001 - Q3 2010	5.5608	4.9234	-0.043127
Latvia	Q1 2001 - Q3 2010	7.832	1.9231	-0.018309
Mauritius	Q4 2001 - Q3 2010	8.6057	2.3308	-0.065743
Mexico	Q4 1985 - Q3 2010	30.8255	29.3572	-0.020202
Morocco	Q4 2001 - Q4 2009	6.0299	0.98054	-0.053208
Philippines	Q4 1985 - Q4 2008	12.0721	2.9532	-0.068409
Poland	Q4 1996 - Q3 2010	15.6368	3.2452	-0.095698
Romania	Q4 2001 - Q3 2010	35.1549	3.709	-0.039838
Russian Federation	Q4 2000 - Q3 2009	31.747	2.1928	-0.071025
South Africa	Q4 2001 - Q3 2010	9.3932	21.0922	-0.04203
Sweden	Q1 1995 - Q3 2004	8.2515	7.2107	-0.038893
Thailand	Q4 2001 - Q3 2010	7.9905	2.4156	-0.011487
Turkey	Q4 2001 - Q3 2010	50.8825	4.2728	-0.14624
United States	Q1 1959 - Q4 2010	5.5864	17.1322	-0.13673

Table 4 Data for shadow economy estimates in Table 3.

Country	*Sample*	$\hat{\imath}$	*Velocity*	$\hat{\epsilon}$
Australia	Q3 1969 - Q4 2010	8.2663	4.5683	-0.074468
Hong Kong	Q4 1990 - Q4 2010	3.5711	1.7984	-0.051137
S. Korea	Q4 1976 - Q3 2010	10.5771	4.1039	-0.05271
Kyrgyzstan	Q1 2000 - Q4 2010	14.9351	2.108	-0.045377
New Zealand	Q2 1987 - Q4 2010	9.0384	11.2564	-0.026454
Norway	Q4 1971 - Q3 2009	8.2835	4.3842	-0.015789
Singapore	Q1 2003 - Q3 2010	4.377	2.1922	-0.075167
Switzerland	Q4 1975 - Q4 2010	2.4789	2.2242	-0.061206
Tunisia	Q1 2000 - Q4 2007	7.6727	2.3984	-0.16271
United Kingdom	Q1 1972 - Q4 2010	7.1609	5.1623	-0.069809