Subprime Borrowers, Securitization and the Transmission of Business Cycles^{*}

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Abstract

One of the roots of the recent global financial crisis has been seen in the design of hybrid subprime mortgage contracts leading to high sensitivity of these types of loans to changes in housing prices. The market of subprime loans, especially in the last years preceding the crisis, has been highly financed by securitization, and subprime securitization was made one of the scapegoats for the Great Recession. This paper investigates whether the securitization can have a positive effect on the economy. The formal setup is a theoretical macroeconomic model with different types of borrowers and banks acting as financial intermediaries, in which households and entrepreneurs borrow against housing collateral. It is shown that due to interbank linkages, the existence of subprime securitization may have either a stabilizing or a destabilizing effect on the economy, depending on who is the final buyer of securitzed assets. This paper investigates the importance of deleveraging conducted by banks in the face of binding capital constraints for aggregate economic development, and provides a theoretical explanation for the negative correlation between subprime defaults and commercial lending observed for U.S. banks during the Great Recession.

Keywords: Subprime Borrowers, Securitization, Financial Intermediation, Great Recession **JEL-Classification**: E32, E44, G01, G13, G21, R21

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1 Introduction

The 2007-2009 crisis, labeled as the Great Recession, has been the longest and the most severe post-war recession in the U.S. The crisis drew the attention of economists towards such subjects as bubbles, the role of financial intermediaries in the economy, as well as various aspects of mortgage markets. A common point of departure for researchers analyzing the Great Recession is often the relatively small subprime mortgage market in the U.S. that may have been one of the roots of the prolonged downturn. Globalized financial markets and mortgage derivatives enabled the domestic housing market crisis to spread to other countries and continents. This paper investigates potential sources of the amplification mechanism during the recent crisis in the U.S. market. I focus on the design of hybrid subprime mortgages that were a combination of fixed-rate and adjustable-rate contracts allowing hybrid mortgages to have a short-term character, and their importance for business cycles. Moreover, I discuss the role of securitization, a process that transfers the underlying risk from loan originators to investors through the creation of securities backed by pooled mortgages, in financing subprime loans. Since the subprime mortgage crisis in the U.S. closely preceded the Great Recession, I want to investigate how these two events are linked. Specifically, I analyze different securitization scenarios to see under which conditions the securitization of subprime loans would have a positive impact on the responses of the economy to negative shocks.

This paper presents a calibrated model in a linear New-Keynesian Dynamic Stochastic General Equilibrium (DSGE) framework that builds on models with credit frictions, particularly collateral constraints. The focus is on the role of subprime mortgages and securitization in the recent crisis, and the importance of the bank lending channel in the presence of binding capital The model incorporates some aspects of financial modeling (mortgage-backed constraints. securities, MBS) into a standard macroeconomic framework, which is the main contribution of the paper. Four different versions of the model are compared: a baseline model without securitization, two models with securitization in which only non-financial agents buy securitized assets, and a model with securitization in which financial intermediaries acquire asset-backed securities. I leave aside the modeling of the portfolio decisions of agents, as answering the question of who was buying how much of the securitized assets and why is beyond the scope of this paper. The aim of the exercise is much more humble; assuming that securitization took place and securitized products were bought by different agents in the economy, I want to investigate whether there is any difference in the reaction of the economy to different shocks, depending on who is the ultimate bearer of the subprime risk.

In my analysis, I focus on two shocks: monetary and preference. The monetary shock is modeled as an exogenous increase in the nominal interest rate set by the central bank, that in the current setup equals to the interest rate on deposits. It is important to understand the response of the model economy to such a change in monetary policy, as the period of rising interest rates (starting from July 2004 and ending in August 2007) shortly preceded the outbreak of the Great Recession in the U.S. Moreover, it is a shock that is usually discussed in the macroeconomic literature, which makes the predictions generated by the present model comparable with other papers in that field. Secondly, I discuss the impact of a negative preference shock in the economy, designed as an exogenous change in the demand for housing stock experienced by households. This may capture - in reduced form - a regulatory or taxation reform that makes investment in housing less attractive to households. To my knowledge, directly prior to the crisis no such regulation or taxation change took place, but a housing preference shock can be seen as a useful way of designing a shock that has a direct impact on housing prices. A negative housing preference shock leads to a fall in housing prices, which is the event that I am most concerned about in this paper for two main reasons: first, because the developments on the housing market played a crucial role in the Great Recession, and second, because they are related to the default behavior of adjustable-rate mortgages that I model in this paper.

The results show that the specific design of subprime mortgage contracts alone, which were highly sensitive to changes in housing prices, did not amplify the U.S. business cycle - it merely led to a redistribution effect between subprime borrowers and lenders. However, the securitization of subprime mortgages may have caused an amplification through the balance sheet effects of banks that were holding the securitized products. If MBS were held by non-banks, securitization would have had a positive effect of risk-spreading, leading to a smoother response of output to different shocks. Securitization itself thus cannot be blamed for the severity of the crisis. This is consistent with Jaffee et al. (2009) (p.71) who conclude: "The financial crisis occurred because financial institutions did not follow the business model of securitization. Rather than acting as intermediaries by transferring the risk from mortgage lenders to capital market investors, they became the investors. They put 'skin in the game'".

The results of this paper support the thesis that in principle, securitization, even of the 'dangerous' subprime risk, makes sense, because different market participants have different investment horizons and may be better able to bear the credit risk than the originator. Ideally, securitized products would end up in the portfolios of institutions such as pension funds that can cushion short-term losses better than financial intermediaries. The problem occurs if financial institutions themselves engage in such transactions, because they mainly rely on short-term funding. The present model shows that, if banks facing capital constraints buy MBS tranches, which lose value in the downturn, the capital constraint gets tighter, so the whole intermediation process is disrupted. Through the deleveraging process, lending to other agents in the economy declines, causing a credit crunch, partial termination of production and a fall in output. The model demonstrates the relevance of this process in a general equilibrium framework and offers a theoretical explanation for the negative correlation between subprime defaults and commercial lending observed for U.S. banks during the crisis. It is important to note that, although my paper is motivated by the events in the subprime securitization market and hence, I model specifically the securitization of adjustable-rate mortgages, the main mechanism through which securitization impacts the economy in the model is the balance sheet dynamics of financial intermediaries. Therefore, the model is also applicable to securitization of different types of assets, not only mortgages.

The present paper relates to three main strands of the literature. It is an extension of Iacoviello (2005) that relies on the seminal paper by Kiyotaki and Moore (1997). In both

models, the importance of collateral constraints and the imperfect enforcement of lenders' rights that lead to the establishment of a certain loan to value ratio are emphasized. Iacoviello (2005) focuses on loans backed by real estate, which makes his model a natural starting point for my exercise investigating the role of subprime securitization. I extend the model by adding the banking sector and considering the securitization of subprime loans. The balance sheet effects discussed in my paper resemble dynamics occurring in Iacoviello (2014) that models the consequences of an exogenous fall in banks' equity. The second strand of literature important for my paper is mainly represented by Adrian and Shin (2010) and Adrian and Shin (2011) that focus on the balance sheets of financial intermediaries and the empirical properties of the behavior of banks. Lastly, the empirical evidence on the recent crisis delivers many insights. The present paper mainly relies on a comprehensive study of Gorton (2008), who describes in detail the subprime mortgage market in the U.S. and the securitization of subprime mortgages. Another important reference is Gorton and Souleles (2007) who describe the basics of the securitization process. Hellwig (2009) also delivers an extensive descriptive analysis of the events leading to the Great Recession.

When it comes to the modeling of securitization in a general equilibrium model, to my knowledge only three attempts have been made, and all of them focus on the problem of the asymmetric information. Faia (2011) models the secondary market for bank loans in a model with solid micro-foundations in which several economic agents face a moral hazard problem. On the one hand, capital producers that obtain funds from banks may choose to exert low effort, which undermines the success probability of their project, but provides them with a private benefit. On the other hand, the incentive to monitor the projects decreases for bankers, once a secondary market for loans exists. Faia (2011) concludes that the existence of secondary markets amplifies the dynamics of macro variables. Hobijn and Ravenna (2010) model securitization in a setup with banks that have access to costly screening which provides them with information about the credit score of the borrowers. Borrowing households are either honest or dishonest, which leads to a default event. Hobijn and Ravenna (2010) demonstrate that securitization reduces the equilibrium interest rates, and the decline is most pronounced for riskier, subprime borrowers who gain the most from the securitization process. The authors examine the response of financial variables, such as interest rate spreads, to a monetary and financial shock and conclude that with securitization the reaction of financial variables is amplified in comparison to a standard New-Keynesian model. Lastly, Kuncl (2014) analyzes the role of asymmetric information in the secondary loan market, in a setup in which firms with profitable investment opportunities sell the cash-flows from their projects to firms with low or no investment opportunities. Although all three papers deal with securitization, the focus and modeling devices applied in these papers differ considerably from the setup in this paper. Firstly, I focus on the real estate market, which is not described in any of the discussed papers. Secondly, in this paper, the intermediation role of banks (absent in Kuncl, 2014) plays an important part, as well as the interbank market. Finally, while information asymmetry is at the heart of analysis of the other three papers and gives them a microeconomic flavor, in my paper it appears only indirectly through the existence of borrowing and capital constraints.

Why is it important to consider recent developments in a general equilibrium macro framework when the finance and microeconomics literature deliver a fairly good description of economic agents' incentives and amplification processes caused by financial frictions? The general equilibrium macroeconomic setup is especially useful for examining the positive aspects of securitization through inter-market linkages. To show why securitization may have a positive impact on the overall economy, I explicitly model the interbank sector. When distinct financial intermediaries are connected through loan and deposit contracts (i.e. assets of one banking institution correspond to liabilities of another banking institution), the changes in the balance sheet of one of them will automatically lead to changes in the balance sheet of the second intermediary. Securitization of subprime loans releases the pressure on the subprime loan originators' balance sheets, which, through the interbank market, has a positive effect on the balance sheets of other financial intermediaries in the economy, since they finance subprime lenders with deposits. This positive aspect of securitization is present in all versions of the model with securitization that I consider. However, the overall impact of securitization on the economy depends on other endogenously arising processes. It turns out that the effect may be negative, if the deleveraging effect, present in the model with banks investing in MBS, is stronger than the positive effect of securitization. Moreover, deleveraging may lead to a vicious circle of falls in asset prices and further deleveraging (Adrian and Shin, 2010), leading even to instability of the system, if capital constraints imposed on banks are very low. Low capital contraints lead to higher leverage and subsequently, more pronounced deleveraging, when a negative shock hits the economy. It is important to note that, even if in the present model some decisions and constraints are exogenously imposed on agents in the economy, their responses to shocks are endogenous, and by calibrating the model to U.S. data, one can measure and assess the strength of these reactions. Comparing different versions of the model with securitization enables me to further conduct counterfactual analysis and determine how the U.S. economy could have evolved after the initial shocks, had people followed the intended business model of securitization. The results suggest that in this case the maximum quarterly output loss in the U.S. economy during the Great Recession would have amounted to 10% of that observed in the data.

The model presented in the paper is complex, as it incorporates agents differing in their impatience level, two types of bankers, as well as diverse collateral constraints. Yet, the main message of the paper is simple - binding collateral constraints faced by financial intermediaries may lead to disruptions in the lending market and may amplify losses from an exogenous negative shock, leading to a decline in output. The understanding of the deleveraging of banks' balance sheets is crucial for the analysis of the presented general equilibrium model. Readers who are not familiar with the importance of binding capital constraints for the balance sheet dynamics of banks may find the plain numerical example presented in Appendix A helpful. In what follows, I describe the peculiarities of the subprime market (Section 2.1) and some empirical relations between the MBS and commercial loans observed in the data during the crisis (Section 2.2), which will make the interpretation of chosen assumptions and modeling devices easier. Section 3 presents the baseline model and Section 4 is its extension with securitization. The main results are presented in Section 5. Section 6 presents sensitivity analysis and discusses an extension to the model in which I introduce impatient prime borrowers into the model economy who may borrow long-term, which reflects the existence of fixed-interest rate mortgages in real life. The main conclusions of the paper are summarized in Section 7.

2 Stylized Facts

2.1 Subprime Mortgage Market

The subprime mortgage market became one of the scapegoats of the Great Recession in the United States. However, some commentators (see Liebowitz, 2009) point to the fact that subprime borrowers themselves are not to blame, but rather adjustable rate mortgages (particularly hybrid mortgages) that led to disruptions in both the subprime and prime mortgage markets. This section provides evidence on the foreclosures and delinquencies¹ in the U.S. mortgage market, as well as a short review of empirical facts that help to address this comment.

It is remarkable that the events in the subprime mortgage market are important for the understanding of the roots of the crisis, because subprime borrowing accounts for only a small percentage of the whole mortgage market (the share of subprime originations is depicted in Figure 1). Although there is no exact definition of a subprime borrower or market, there are certain features common to all subprime loan contracts. A prime mortgage in the U.S. is usually collateralized and has a fixed interest rate for 30 years. Subprime borrowers often can provide neither collateral, nor income (so called "NINJAs" - No Income, No Job or Assets, see Jovanović, 2013). The down-payment rate in the case of prime borrowers is usually higher than in the subprime case. However, the difference is not as overwhelming as one may expect. Amromin and Paulson (2010) provide detailed data on loan to value (LTV) ratios for both groups of borrowers in the years 2004-2007. In the case of prime mortgages, the average LTV ratio ranged from 74.89% to 77.75%, while in the case of subprime mortgages, it ranged from 79.63% to 80.69%. The biggest difference between these two groups has been noted in the FICO score, which measures the creditworthiness of borrowers and is used by lenders to determine the credit risk. In the case of prime borrowers it ranged from 706 to 715, while in the case of subprime borrowers, it ranged from 597 to 617 (the FICO score ranges from 300 to 850, with the higher, the better). Subprime borrowing was thriving thanks to a common belief that housing prices will rise on average. Indeed, until the recent crisis the U.S. market did not experience a countrywide decrease in housing prices since the 1930s.

Since subprime borrowers often do not have well-documented assets or income, it poses a challenge to create a loan contract that will enable them to pay the installments. The solutions to this problem were hybrid adjustable rate mortgages of type 2/28 or 3/27, in which the first period's (2 or 3 years) interest rate was fixed and the rest (28 or 27 years respectively) was varying. The shift from the fixed interest rate to the adjustable one occurred at a previously

 $^{^{1}}$ A delinquent loan is a loan with a delay of payment of at least 30 days. The total delinquency rate takes into account all past-due categories (30, 60, 90 days and over), but excludes loans in the foreclosure process.

specified reset date. As Kliff and Mills (2007) note, before the outbreak of the crisis, these hybrid mortgages made up about two thirds of all ARM (adjustable rate mortgage) originations and were basically short-term fixed rate mortgages that converted into an adjustable rate mortgage after the initial period. Gorton (2008) explains how this kind of contract can be interpreted as a short-term contract. The initial interest rate depended on the loan to value ratio, which in turn depended on changes in house prices. When house prices were rising, households were able to refinance and repay the debt at the reset date, and in even some cases, extract equity from homes. When house prices were falling, the LTV ratio was rising, followed by an increase in the interest rate at the reset date, so that many households were not able to repay the contracted installment, or even defaulted. Amromin and Paulson (2010) provide evidence of a high sensitivity of defaults to changes in home prices among subprime borrowers already in years before the crisis, compared to a very low sensitivity among prime borrowers (for 2004: -0.183 for subprime borrowers and -0.00166 for prime borrowers). Short-term characteristics of subprime loans as well as their high sensitivity to housing prices observed in the data enable me to model the subprime loan contract as a one-period contract with the possibility of default linked to changes in house prices.

How do developments on the subprime mortgage market relate to the economic performance of the U.S.? Figure 1 presents subprime loans originations as a share of the total market, nonagency² securitization activity (RMBS - residential mortgage backed securities - and securities based on home equity loans), as well as the real GDP growth rate.



Figure 1: Subprime market and real GDP (annual data)

The peak of subprime originations coincided with the peak in non-agency securitization

²Agency securities are securities that are either issued or guaranteed by federal agencies and government sponsored enterprises, such as Ginnie Mae, Fannie Mae or Freddie Mac. Non-agency securities are securities issued by private companies and lack the explicit or implicit guarantee of the U.S. government.

activities and both of them almost dried out in 2008 (further data not available). This reflects the fact that securitization was the main financing method for subprime originations. The majority of subprime mortgages were pooled together and sold in the financial market as MBS, which were often a base for a further securitization instrument - a collateralized debt obligation (CDO).³ Subprime originations peaked in 2006, while the 4th quarter of 2006 denotes the peak in the U.S. house price index (USSTHPI). The developments in the housing and mortgage market led the changes in U.S. GDP growth. According to the NBER, the last recession started in December 2007 (4th quarter) and ended in June 2009 (2nd quarter). Thus, the data supports the thesis that the recession was linked to the housing market, similar to other recent crisis episodes in industrialized economies (Reinhart and Rogoff, 2009). My paper investigates a potential transmission mechanism through which changes in the housing market affect U.S. GDP growth.

As noted before, the distinguishing feature of subprime mortgages was their hybrid character. However, prime borrowers also take out ARM loans. Examining foreclosures and delinquencies data (exclusive of loans in the foreclosure process) enables me to address the question of whether the subprime market or the ARM market was decisive for the GDP developments. Figure 2 depicts the share of mortgages entering the foreclosure process in the U.S., both for subprime and prime borrowers, taking into account ARM and FRM (fixed rate mortgages).



Share of ARM and FRM (%) starting foreclosures in the U.S.

Figure 2: Foreclosures

Figure 2 reveals that the fraction of foreclosures is the highest among ARMs, but it is clear that the fraction of subprime foreclosures was higher and prime foreclosures only followed the developments in the subprime market. An interesting observation can be drawn from comparing Figure 2 with Figure 3, which presents delinquencies for the same type of loans. The peak in

³The ratio of securitized subprime/Alt-A mortgages rose from 46% in 2001 to 93% in 2007 (Geithner, 2011, p.11). Alt-A mortgages are mortgages with characteristics that places them between prime and subprime mortgages.



Percent share of delinquent subprime and prime mortgages in the U.S.

Figure 3: Delinquencies

loan delinquencies occurs visibly later than the peak in foreclosures, which partially results from governmental actions in the U.S. aimed at reducing the share of foreclosures in order to stop declines in house prices. In 2009, the Home Affordable Modification Program was launched, which "is designed to help financially struggling homeowners avoid foreclosure by modifying loans to a level that is affordable for borrowers now and sustainable over the long term".⁴ The increasing rate of delinquencies, even when foreclosures already started to fall, suggests that banks and financial institutions that were exposed to subprime risk, were holding the assets on their balance sheets. Notably, although ARM delinquencies are much higher than FRM delinquencies for both types of borrowers, in the case of subprime borrowers, the FRM delinquencies are almost as high as delinquencies on the hybrid loans, and much higher than any delinquencies observed for prime borrowers. Thus, Liebowitz (2009) noting that not subprime loans but rather ARM loans caused the mortgage crisis is to some extent right, but making either subprime borrowers only or adjustable rate mortgages only the scapegoats of the crisis is both wrong. In what follows, the focus of this paper is put on hybrid subprime mortgages, the subcategory of ARMs. Their market almost vanished after the crisis. However, ARMs still exist within and outside the U.S. despite the drop in the share of the market (see Moench et al., 2010).

2.2 MBS and Commercial Loans Holdings by Banks

As securitization was the main financing method for subprime originations, the majority of subprime mortgages were pooled together and sold in the financial market as MBS. The

⁴https://www.hmpadmin.com/portal/programs/hamp.jsp

bonds or pass-through securities (called so because the monthly loan payments are passed through to the holders of security) were then sold to pension funds, banks, investment funds and personal investors. The securitization of subprime loans might have made the whole financial system vulnerable to housing prices, which is much less the case when financial intermediaries only securitize prime loans, whose value does not depend so much on the condition of the housing market. Moreover, it is important to stress that securitization is not equal to loan sales. A sold loan is no more marketable than the loan itself, whereas securitization creates a new quality through various credit enhancements.⁵ Loans are sold in a secondary market, whereas securitization creates a new primary market. That is why Gorton (2008) calls the chain of securitized subprime securities a chain of many primary markets. At the first stage, securitization is often conducted via a special purpose vehicle (SPV) that exists only for the purpose of securitization, is set up by the originator, and does not even have any employees. The securitization process includes repackaging many assets, including car or student loans into derivative securities consisting usually of three tranches: senior, mezzanine and equity, with the latter being the most risky one. The process of tranching is the most important credit enhancement of securitized products, without which it would be difficult to explain the demand of investors for the product. Individual pricing and payoff structures of distinct tranches provide incentives for the acquisition - e.g. senior tranches were usually given an A rating by rating agencies, which made them a perfect asset for banks wanting to loosen their capital constraint. The residential mortgage backed securities (RMBS) played the biggest role in the securitization market just before and after the recent financial crisis. Consequently, in my model, I concentrate on RMBS. The specific design of SPVs enables me to model the securitization process without introducing a new agent into the model economy.

The present model is calibrated to the U.S. economy, as it has been the root of the financial crisis. The paper emphasizes the importance of financial intermediation for the production process that is financed by bank loans. It is a well-known fact that opposite to European markets where banks are an important source of credit to firms (bank-based system), the U.S. is characterized by a market-based system, i.e. firms resort more to corporate bonds and stock financing rather than bank loans. Although banks played a less and less important role over time in the financing of non-financial businesses in the U.S.⁶, bank loans still provide around 12% of funds to non-financial corporations. This is a considerable share and the bank lending channel presented in this paper may be one of the explanations for the size and length of the Great Recession.

In order to understand the crisis it is informative to look not only at the balance sheets of non-financial businesses and their funding sources, but also at the balance sheets of U.S. banks. In the following, I will focus on the asset side of banks, with a special emphasis on commercial real estate loans (modeled in the paper) and MBS holdings. Figure 4 presents

⁵Credit enhancement includes: tranching of the risk of loss, over-collateralization, guarantee by an insurance company. Discussed further in Gorton and Souleles (2007).

⁶Detailed data on the balance sheets of nonfinancial businesses are available in the Flow of Funds Tables of the Federal Reserve Board, Financial Accounts - Z.1, for flows see F.101, for levels: L.101, http://www.federalreserve.gov/apps/fof/FOFTables.aspx.

the fraction of MBS holdings, commercial real estate loans and commercial loans (all loans to firms, including real estate) in all bank assets over time. The graph is generated using data for large domestically-chartered commercial banks that are a good proxy for all U.S. banks and are chosen due to the better availability of data. A detailed data description is available in Appendix B. It is visible that the fractions of MBS and commercial real estate loans went into opposite directions from ca. 2007. The negative correlation between the fractions can be also observed while taking into account total commercial loans, not only the real estate ones. An analogous graph for lending levels that exhibits the same pattern can be found in Appendix C (Figure 17).

Recalling Figure 3, one can see that the divergence in the fractions of MBS and commercial real estate loans is preceded by a large surge in subprime default rates that started in 2006 and coincided with the beginning of the fall in housing prices. The fraction of MBS and commercial real estate loans were approximately equal when expressed as a percentage of U.S. bank assets until the subprime default rates began to increase. Only with an increase in subprime delinquencies did the fraction of total MBS rise and the fraction of commercial real estate loans fall, suggesting that securitized assets experiencing a fall in value may have crowded out lending to entrepreneurs.



Figure 4: Composition of large banks' balance sheets

We should thus observe a negative correlation between the fraction of commercial real estate loans on the asset side of the bank and the subprime default rates. The correlation is clearly negative for the period 2006-2010, as can be inferred from Figure 5. The graph presenting the correlation for the whole sample period 1998-2013 is given in Appendix C (Figure 18). No visible correlation can be observed in the pre-and post-crisis data.

Negative correlation does not imply any causal effects, but the present paper offers an explanation for the empirical facts. Rising subprime default rates lead to a fall in the value of subprime loans or securitized products backed by these loans. This puts a strain on banks' balance sheets and forces them to deleverage, which reduces lending to firms. Why does the fraction of held MBS increase during the crisis despite the rising default rates on these securities? The banks, even if they wanted, could not sell the toxic assets as the market for them dried out when the scale of the crisis was made public: subprime MBS suddenly became illiquid. The omnipresent illiquidity prompted the Federal Reserve to introduce some of its programs aimed at increasing liquidity⁷, but the first acquisitions of toxic assets in 2009 were focused on guaranteed agency mortgages whose boom and bust was less pronounced than developments in the non-agency mortgage market. In fact, it turns out the the role of government is decisive for the shape of the graph shown in Figure 4. If one takes the agency and non-agency MBS held by banks separately into account, it turns out that the agency MBS holdings were going up, while the non-agency MBS holdings stabilized at the peak of the crisis (which confirms the illiquidity hypothesis) and started to experience a persistent decline at the end of 2009, which is depicted in Figure 19 presented in Appendix C. The Figure 20, also presented in Appendix C, shows the agency and non-agency MBS holdings as a fraction of total assets.

The present paper does not model the portfolio decision of banks - it assumes that in a world with securitization they hold MBS on their balance sheet. It is a strong assumption, but not so unrealistic given the empirical observations just discussed. In some states of the world, like a financial crisis leading to the disruption of market liquidity, banks may indeed have no choice but to keep some assets on their balance sheets.

Subprime default rate vs. fraction of commercial loans in the U.S. (1Q2006-4Q2010), large banks





Source: Thomson Reuters Datastream and FRB data

⁷http://www.marketwatch.com/story/fed-starts-program-to-buy-illiquid-mortgage-assets

3 The Benchmark Model

The model economy is inhabited by a continuum of households that differ in their degree of impatience. All households offer labor services to entrepreneurs producing intermediate output. Households consume final goods and derive utility from housing services. Patient households save in the form of deposits kept at commercial banks that grant loans to entrepreneurs and offer loans on the interbank market. In the baseline version of the model, it is assumed that all impatient borrowers have subprime characteristics: they borrow from a subprime lender against housing collateral (an extension involving the existence of prime borrowers who may borrow for long-term and do not default on their loan obligations is presented in section 6.2 and Appendix G). The collateral constraints faced by borrowers determine the amount they can borrow from the bank, while bankers set the interest rates on loans, taking into account different borrowing constraints and default probabilities. The debt contracts in the economy are written in nominal terms, as in Iacoviello (2005). The financial connections of the agents are shown in Figure 6. There is a central bank in the economy implementing a Taylor rule and choosing the interest rate on deposits. Retailers, who produce a final good out of the intermediary good, are the source of nominal stickiness in the economy.



Figure 6: Financial connections in the benchmark model

3.1 Patient Households: Savers

The problem of patient households ('savers') is identical to the one in Iacoviello (2005) with one difference: instead of providing loans to households and entrepreneurs, they save in the form of one-period deposits held at banks. Patient households consume, work and accumulate housing. Their optimization problem and the First Order Conditions (FOCs) are presented in Appendix D.1.

3.2 Impatient Households: Subprimers

Impatient households are borrowers in the model economy. The feature that distinguishes them from impatient households modeled in Iacoviello (2005) is that they may default on their loan obligation, with the default rate sensitive to house prices, which reflects the adjustable-rate feature observed in the data.

Impatient subprime households have the following utility function:

$$\max_{b_t^{Sub}, h_t^{Sub}, L_t^{Sub}} E_0 \sum_{t=0}^{\infty} \beta^{Sub, t} \left(\log c_t^{Sub} + j_t \log h_t^{Sub} - \frac{L_t^{Sub} \eta^{Sub}}{\eta^{Sub}} \right).$$
(1)

The budget constraint of the impatient subprime household is:

$$c_t^{Sub} + q_t (h_t^{Sub} - h_{t-1}^{Sub}) + (1 - \delta_{s,t}) R_{s,t-1} b_{t-1}^{Sub} / \pi_t = b_t^{Sub} + w_t^{Sub} L_t^{Sub},$$
(2)

where $R_{s,t}$ is the nominal interest rate on subprime loans and

$$\delta_{s,t} = \delta_s - \phi_{s,h}(q_t - Q) \tag{3}$$

(δ_s denotes the positive steady state value of default rate, Q is the steady state value of housing prices, $\phi_{s,h}$ denotes subprimers' default sensitivity to house price changes) is the default rate on loans. The dependence on house prices is chosen to capture the high sensitivity of the hybrid subprime mortgage contract to changes in housing prices and its gamble characteristics.⁸ Subprime lenders bet on an increase in house prices because they may then expect a lower than predicted default rate and thus, faster repayment of the loan.⁹

Impatient households may borrow against the future value of their housing collateral:

$$R_{s,t}b_t^{Sub} \le m^{Sub}E_t(q_{t+1}\pi_{t+1})h_t^{Sub}.$$
(4)

The FOCs of subprime borrowers are presented in Appendix D.2.

⁸Forlati and Lambertini (2011) consider a model with risky mortgages and endogenous default rate arising from idiosyncratic shocks to households housing investment, which is also a proxy for modeling the negative home equity and its consequences. However, in their model firms do not borrow capital from financial intermediaries, so one important transmission channel of the crisis is excluded.

⁹Given the formulation in 3, theoretically, when a large shock occurrs, the default rate can turn negative. However, the positive steady state rate of default, as well as the fact that in a loglinearized model only shocks of a small amplitude can be considered, prevent this from happening in the current setup.

It is important to note that, although the collateral constraint of subprime borrowers does not refer to their possible default, the interest rate paid on their subprime loans includes the default premium. They pay a higher interest rate reflecting their ex ante probability of default. The subprime interest rate is determined by the subprime lenders' optimization problem, see equation 14.

3.3 Entrepreneurs

The problem of entrepreneurs is similar to that in Iacoviello (2005) with the exclusion of capital accumulation and investment conducted by firms.¹⁰ They produce intermediate output priced at P_t^w , using housing stock and labor provided by households, and sell it to retailers. They borrow short-term to cover their expenditures, facing a collateral constraint analogous to the one faced by households. Their optimization problem and the FOCs are presented in Appendix D.3.

3.4 Retailers

The problem of retailers is identical to that in Iacoviello (2005): they are the source of price stickiness in the economy. I present the equations concerning the retailer sector in Appendix D.4.

3.5 Bankers

3.5.1 Commercial Bankers

Commercial bankers collect deposits from patient households and issue loans to entrepreneurs. They also provide interbank loans for subprime lenders that operate as a bank.¹¹ Commercial bankers maximize utility from their consumption (as in Iacoviello, 2014):

$$\max_{c_{b,t}} E_0 \sum_{t=0}^{\infty} \beta_b^t (\log c_{b,t}), \tag{5}$$

where β_b is assumed to be lower than the discount factor of patient households (necessary condition for the capital constraint to be binding - see Iacoviello, 2014).

The budget constraint of bankers is:

$$c_{b,t} + \frac{R_{d,t-1}d_{t-1}}{\pi_t} + bb_t + b_{e,t} = d_t + \frac{R_{b,t-1}bb_{t-1}}{\pi_t} + \frac{R_{e,t-1}b_{e,t-1}}{\pi_t},$$
(6)

 $^{^{10}}$ Capital and investment were part of the model in the earlier version of the paper, Grodecka (2013), and their inclusion does not change the results qualitatively, so for simplicity reason they were left out from this analysis.

¹¹The distinction between commercial and subprime bankers is not necessary for the benchmark version of the model, but becomes important once securitization is introduced into the model economy. The evidence from the U.S. suggests that there were several banks and financial intermediaries that specialized specifically in the subprime market.

where bb_t denotes interbank lending and $R_{b,t}$ is the interbank interest rate.

The banker's balance sheet looks as follows:

Assets	Liabilities
Interbank loans: bb_t	Deposits d_t
Loans to entrepreneurs: $b_{e,t}$	Equity eq_t

Thus, a Basel-type capital constraint, given exogenously, has the form:

$$\tau \le \frac{bb_t + b_{e,t} - d_t}{\chi^{Intb}bb_t + \chi^{Firm}b_{e,t}},\tag{7}$$

where $\chi^{Intb} < \chi^{Firm}$ are risk weights of assets and τ denotes an equity ratio set by a regulator. The condition states that the ratio of equity (defined as assets minus deposits) to risk weighted assets has to exceed some exogenously chosen number.

The FOCs of the bankers' problem determine the interest rates paid on deposits and different types of loans (G_t denotes the Lagrangian multiplier on the capital constraint):

w.r.t. bb_t

$$\frac{1}{c_{b,t}} = \beta_b E_t \left(\frac{R_{b,t}}{c_{b,t+1} \pi_{t+1}} \right) + (1 - \tau \chi^{Intb}) G_t, \tag{8}$$

w.r.t. $b_{e,t}$

$$\frac{1}{c_{b,t}} = \beta_b E_t \left(\frac{R_{e,t}}{c_{b,t+1} \pi_{t+1}} \right) + (1 - \tau \chi^{Firm}) G_t, \tag{9}$$

w.r.t.
$$d_t$$

$$\frac{1}{c_{b,t}} = \beta_b E_t \left(\frac{R_{d,t}}{c_{b,t+1} \pi_{t+1}} \right) + G_t.$$
(10)

The interpretation of equations 8 to 10 is crucial for understanding the main result of the paper. The equations without considering the Lagrangian multiplier on the capital constraint represent typical Euler equations, saying that the banker must be indifferent between consuming one unit of consumption today, and lending one unit today and consuming it tomorrow. The capital constraint of bankers introduces a wedge between the cost and marginal gain from lending. Its bindingness influences the bankers' decisions between consumption and borrow-ing/lending and gives rise to the process of deleveraging. This results in a shrinking balance sheet in the face of a negative shock, as bankers are impatient and prefer to consume rather than raise equity or increase their lending.

3.5.2 Subprime Lenders

Subprime lenders operate as financial intermediaries that collect deposits from the interbank market and issue subprime loans.

Their optimization problem is:

$$\max_{c_{bb,t}} E_0 \sum_{t=0}^{\infty} \beta_{bb}^t (\log c_{bb,t}), \tag{11}$$

s.t.

$$c_{bb,t} + b_t^{Sub} + R_{b,t-1}bb_{t-1}/\pi_t = bb_t + R_{s,t-1}(1 - \delta_{s,t})b_{t-1}^{Sub}/\pi_t$$
(12)

I assume that banks hold a reserve for future losses, taking into account the ex ante (steady state) default rate. The subprime banker's balance sheet is:

Assets	Liabilities
Loans to subprime borrowers: b_t^{Sub}	Interbank deposits bb_t
Loss reserve $-\delta_s b_t^{Sub}$	Equity eq_t

Thus, a Basel-type capital constraint, given exogenously, has the form:

$$\tau^{Sub} \le \frac{(1-\delta_s)b_t^{Sub} - bb_t}{\chi^{Sub}(1-\delta_s)b_t^{Sub}},\tag{13}$$

where the risk weight on subprime loans is denoted by χ^{Sub} .

The FOCs of the subprime bankers' problem (GG_t denotes the Lagrangian multiplier on the capital constraint of subprime lenders) are:

w.r.t. b_t^{Sub}

$$\frac{1}{c_{bb,t}} = \beta_{bb} E_t \left(\frac{R_{s,t}(1 - \delta_{s,t+1})}{c_{bb,t+1}\pi_{t+1}} \right) + (1 - \tau^{Sub} \chi^{Sub})(1 - \delta_s) GG_t,$$
(14)

w.r.t. bb_t

$$\frac{1}{c_{bb,t}} = \beta_{bb} E_t \left(\frac{R_{b,t}}{c_{bb,t+1} \pi_{t+1}} \right) + GG_t.$$

$$\tag{15}$$

Equation 14 determines the interest rate paid on subprime loans and makes clear that when pricing the subprime loan, the subprime lender takes into account the default probability of the borrowers. As a consequence, the steady state interest rate on subprime loans is higher than that of loans with a zero default probability.

3.6 Central Bank

The central bank implements a Taylor type interest rate rule (identical to Iacoviello, 2005). It is assumed that the interest rate set by the central bank equals the interest rate paid on deposits (disregarding reserve requirements):

$$R_{d,t} = (R_{d,t-1})^{r_R} E_t \left(\pi_{t-1}^{1+r_\pi} \left(\frac{Y_{t-1}}{Y} \right)^{r_y} \bar{rr} \right)^{1-r_R} e_{R,t}.$$
 (16)

3.7 Market Clearing Conditions

I assume that real estate is fixed in the aggregate, which guarantees a variable price of housing. The market clearing condition for the housing market is:

$$1 = h_t^{Savers} + h_t^{Sub} + h_{e,t}.$$
(17)

The goods market clearing condition is given by:

$$Y_t = c_t^{Savers} + c_t^{Sub} + c_{e,t} + c_{b,t} + c_{bb,t}.$$
(18)

The market clearing conditions for labor are defined by equations 23 and 33 for the patient households' labor supply and demand, and by equations 26 and 34 for the impatient subprime households. The lending to different agents is determined through their collateral constraints, while the market clearing conditions for the loan and deposits markets are given by the capital constraints of the bankers (equation 7 and 13).

4 Model with Securitization of Subprime Loans

The data provides evidence for the importance of securitization in subprime lending. The majority of subprime loans have been securitized, first in the form of a RMBS, which often was a building block of CDO structures. Usually, different subprime borrowers have different default probabilities, so securitization may be a way to average the risk on subprime exposure. In the present model, all subprime borrowers have the same default rate, which can be interpreted as a default rate representing the mean of the aggregate distribution over all subprime borrowers, who differ in their default sensitivity at an individual level. Typically, an MBS structure consists of three tranches: senior, mezzanine and equity. To simplify the computation, I assume that the model's RMBS consists only of two tranches: senior and equity.¹² Figure 7 illustrates the payoff functions of investors in the RMBS.

The security is a pass-through security, which means that the nominal loan proceeds are redistributed to the MBS investors. The smaller the loss on the underlying loan portfolio (determined by the default rate), the larger is the payoff of equity tranche investors. The size of the equity tranche, determined by the parameter f, called in the CDO jargon the attachment point, defines the maximum risk exposure of equity tranche investors. If there is a loss on the underlying loan portfolio, the equity tranche investors get the difference between the size of

¹²Gorton (2008) argues that subprime securitization differs from the securitization of other assets because the tranche sizes are not fixed. There is dynamic tranching as a function of excess spread and prepayments, so the whole structure is sensitive to house prices. At the beginning of the existence of a subprime MBS, the equity tranches are usually very thin and along with repayments of the subprime loans they reach their target level. However, if house prices decline from the very beginning, the equity tranche remains very thin and thus senior tranche holders are subject to a very large subprime risk (that was the case for MBS issued in 2006 and later). This works as another amplification mechanism in the design of subprime security. In the version of the present model in which different tranches are bought by different agents, presented in Appendix E, it is assumed that tranche sizes are fixed from the beginning. Including varying tranche sizes in the model would amplify the effects of shocks in the economy.



Figure 7: A two-tranche MBS (face value written in nominal terms)

the equity tranche and the loss. However, if the loss exceeds the size of the tranche, the equity tranche investors simply get nothing from their investment, and the senior tranche investors begin to suffer. Their payoff function is a minimum function. They either get back the tranche size, or the difference between the face value of the MBS and the loss (in the case where losses are bigger than the size of the equity tranche). $P_{s,t} = min(S_t - fS_t, S_t - Loss_t)$ denotes the payoff of senior tranche buyers, and $P_{e,t} = max(fS_t - Loss_t, 0)$ denotes the payoff of equity tranche buyers, where the principal of the MBS is (in real terms) $S_t = R_{s,t-1}b_{t-1}^{Sub}/\pi_t$, and loss equals $\delta_{s,t}S_t$. Independent of the outcome, the cash flows distributed to investors always equal the cash flows from subprime loans (including losses), which is illustrated in Table 1:

	Scenario	
	Loss is bigger than the equity tranche	Loss is smaller than the equity tranche
	$\delta_{s,t}S_t > fS_t$	$\delta_{s,t} S_t < f S_t$
Payoff of equity tranche holder	0	$fS - \delta_{s,t}S_t$
Payoff of senior tranche holder	$S_t - \delta_{s,t} S_t$	$S_t - fS_t$
Sum of payoffs	$S_t - \delta_{s,t} S_t$	$S_t - \delta_{s,t} S_t$

Table 1: MBS payoffs - two scenarios

The characteristics of the MBS presented in Table 1 makes the inclusion of securitization in the benchmark model straightforward. In what follows, I assume in each version of the model with securitization that the same investor buys both the senior and the equity tranche of the MBS, in practice acquiring the whole cashflow from loan proceeds. It is also possible to assume that different tranches are bought by different investors. I consider this case in Appendix E that explains how equity and senior tranche payoffs resemble payoffs from investment in European options and presents the characteristics of the chosen approximation of the maximum and minimum functions (the logistics function), which are functions with a kink (see Figure 21). The qualitative results of my analysis do not change irrespective of the fact whether different agents buy different tranches (results with the use of the logistics function presented in Appendix E) or one agent buys both tranches (presented in the main part of the paper). In what follows, I present results for three different models with securitization: in the first version, the entrepreneur invests in the loan proceeds; in the second version, it is the patient household that acquires the MBS claims; and in the third version, commercial bankers are investors in securitized assets. Why might commercial bankers buy claims on MBS? One reason may be the diversification of their credit risk and the exposure to a different credit market. Also, they may be as optimistic as subprime borrowers are, and believe that housing prices will continue to rise. Moreover, senior tranches usually have the highest possible rating, so the risk weight on them is very low and the purchase has a positive impact on the balance sheet of banks. The regulatory capital arbitrage is the reason why subprime lenders may want to conduct securitization and why commercial bankers may want to buy certain tranches, as described in Jones (2000). Why might patient households and entrepreneurs buy MBS tranches? For them, this investment is just another possibility to smooth their consumption.

I assume that certain agents in the economy invest in MBS securities, and I do not model their decision as a portfolio choice decision, which allows me to use the first order approximation to solve the model.¹³ For answering the research question of this paper this approach is sufficient, as I do not aim to explain how the securitized assets were distributed among the investors.

Securitization changes the capital costraint faced by originations of the subprime loans, as due to the repackaging and sale of the assets, they may remove part of the risk from the balance sheet. In the case of entrepreneurs and patient households who buy MBS tranches, their budget constraint changes to include the new asset acquired, and the FOC with respect to the new asset determines its price. When commercial bankers invest in MBS tranches, apart from a changed budget constraint, the capital requirement of the bankers also changes in order to include the new asset into the balance sheet of the investor. Since these changes are not substantial relative to the baseline model, I discuss their impact on specific model equations in Appendix F.

5 Calibration and Results

5.1 Solution Method and Calibration

The model is log-linearized around the steady state. The log-linearized equations present variables in the form of percent deviations from the steady state, which makes the interpretation of model variables easier. All equations describing the model (also shock processes) are given in Appendix D.5.¹⁴ I calibrate the model using parameter values from the literature, as well as empirical papers (see Table 2).

Following Iacoviello (2005), I assume that patient households have the highest discount

¹³For the determination of the portfolio choice, higher-order solutions have to be used, as under the first order approximation, the equilibrium portfolio is not determined (Devereux and Sutherland, 2010).

¹⁴A list of the log-linearized equations for the extended version of the model (including capital and investment, as well as impatient prime borrowers), may be found in the previous working paper version of this model, Grodecka (2013).

Description	Parameter	Value
Discount factor of patient households	β	0.995
Discount factor of impatient households	β^{Sub}	0.93
Discount factor of entrepreneurs	γ	0.96
Discount factor of commercial bankers	β_b	0.97
Discount factor of subprime lenders	β_{bb}	0.95
Weight on housing services	J	0.09
Loan to value entrepreneurs	m	0.99
Loan to value subprime households	m^{Sub}	0.99
Labor supply aversion	$\eta^{Savers} = \eta^{Sub}$	2
Housing share in production function	ν	0.15
Steady state gross markup	X	1.05
Patient households wage share	α	0.87
Probability fixed price	heta	0.55
Capital adjustment costs	ϕ	2
Risk weight of interbank loans	χ^{Intb}	0.2
Risk weight on commercial loans	χ^{Firms}	1.5
Risk weight of commercial and subprime loans	χ^{Sub}	4.5
Commercial bankers capital requirement	au	0.13
Subprime lenders capital requirement	$ au^{Sub}$	0.2
Subprimers' default sensitivity to house price changes	ϕ_{sh}	0.183
Steady state subprime default rate	δ_s	0.05
Weight of policy response to interest rate	r_R	0.73
Weight of policy response to inflation	r_{π}	0.27
Weight of policy response to output	r_y	0.13
Autocorrelation of preference shock	ρ_j	0.95
Standard deviation of preference shock	σ_{ε_i}	1
Standard deviation of monetary shock	σ_{ε_R}	1
Tranche retention by banks	t	0.01

Table 2: Calibrated parameters

factor, followed by entrepreneurs and both types of bankers. The most impatient agents in the economy are subprime borrowers. The choice of discount factors assures that the collateral constraints in the model are always binding. The parameter J controls the stock of residential housing over annual output in the steady state, J = 0.09 fixes this ratio around 150%, which is in line with the data from the Flow of Funds accounts (table B.100, row 4). The LTV ratios for firms and subprime borrowers are set at 0.99, which is a high value, but is consistent with the literature (Iacoviello, 2014). Parameter η is chosen to fix the Frisch labor supply elasticity at 1. The chosen value lies between the estimates provided by microeconomic studies (0-0.54) and by macroeconomic studies (2-4) (see Peterman, 2012). The steady state gross markup is a value taken from Iacoviello (2005). The patient households' wage share of 0.87 corresponds to the conclusions of Jappelli (1990) who finds that 19% of U.S. families are rationed in credit markets and they account for 12.7% of total wage income. The value of 0.55 for the parameter θ describing the price rigidity is consistent with the evidence of Dhyne et al. (2006) who show that the average price duration in the United States equals 6.7 months.

Parameters describing the risk weights of different types of loans are based on U.S. regu-

lations of the Federal Deposit Insurance Corporation (Code of Federal Regulations - Title 12: Banks and Banking, 12 CFR Appendix A to Part 325 - Statement of Policy on Risk-Based Capital). Interbank loans have the lowest risk weight, followed by the risk weight on commercial loans (the factor for risky loans has been applied). The risk weight on subprime loans has a very high value, which is consistent with the Expanded Guidance for Subprime Lending Programs,¹⁵ stating "that an institution would hold capital against subprime portfolios in an amount that is one and a half to three times greater than what is appropriate for non-subprime assets of a similar type". The capital ratio for commercial bankers corresponds to the average regulatory capital to risk-weighted assets for the United States before the crisis, reported in the FRED database.¹⁶ The capital ratio for subprime lenders is higher than for commercial bankers, which again, corresponds to the Expanded Guidance for Subprime Lending Programs: "Institutions with subprime programs affected by this quidance should have capital ratios that are well above the averages for their traditional peer groups or other similarly situated institutions that are not engaged in subprime lending. (...) institutions that underwrite higher-risk subprime pools, such as unsecured loans or high loan-to-value second mortgages, may need significantly higher levels of capital, perhaps as high as 100% of the loans outstanding depending on the level and volatility of risk".

The sensitivity of subprime households to housing price changes has been chosen according to the pre-crisis data. Over time, the sensitivity changed, but on average one can assume that it did not exceed 20% (Amromin and Paulson, 2010). The subprime default rate is chosen to be 5% in the steady state. According to the data presented in Demyanyk and Hemert (2011), in the decade preceding the crisis, the default rate on subprime hybrid loans oscillated around 10%. However, usually when a household defaults on its mortgage, the bank seizes and sells the property, receiving some foreclosure value. The present model does not have this feature, thus the steady state default rate is half of that in the data. Also, a higher steady state default rate would result in an unreasonably high steady state value for the interest rate on subprime loans. The Taylor rule coefficients are taken from Iacoviello (2005). The shocks are assumed to be persistent, with the autocorrelation coefficient equal to 0.95. I consider a 1 percent shock in each case. For the parameters governing the securitization process, evidence suggests that on average, retention of securitized assets is higher in Europe than in the U.S. Whereas originators usually held around 5% of issued securities in Europe, the retention rate was often at 0% and rarely exceeded 1% for MBS in the U.S. Retention percentages for CDOs and ABS (Asset Backed Securities) were usually higher, but in the years 2002-2009, on average they did not exceed 7% (Global Financial Stability Report, October 2009, p. 100-107).

 $^{^{15} \}rm http://www.federalreserve.gov/boarddocs/srletters/2001/sr0104a1.pdf <math display="inline">^{16} \rm Series~DDSI05USA156NWDB$

5.2 Model Dynamics

I consider two shocks: monetary and preference.¹⁷ The monetary shock is defined as an exogenous increase in the interest rate set by the central bank and can be interpreted as a move of the central bank that is inconsistent with the usually applied Taylor rule. The negative preference shock represents a change in the preference for housing among households. This may capture - in reduced form - a regulatory or taxation reform that makes investment in the housing market less attractive to households (regulatory reforms allowing for a large range of mortgage products could have led to a positive preference shock in the U.S., see Temkin et al., 2002).

The introduction of subprimers' default rate sensitive to housing prices has only a negligible impact on impulse response functions to shocks in the baseline model without securitization. The varying default rate, particularly, the rising default rate after a negative shock leading to a fall in housing prices, is a positive wealth effect from the subprimers' perspective - they may repay less than contracted. Feeling wealthier, subprime borrowers will reduce their labor supply when compared to the case where the default rate does not vary, which drives output down. For subprime lenders, the rising default rate represents a negative wealth effect, because they do not get back all the contracted loan installments. Suffering losses on their loan portfolio, subprime lenders face a tighter capital constraint. They will reduce their lending to subprime borrowers and raise the interest rate on subprime loans, but their consumption will also go down. The described redistribution effect and balance sheet effect have a negative effect on overall consumption, and more responsive housing prices affect other borrowers in the economy who use housing stock as collateral for their loans. However, the subdivision of the banking sector into the subprime and the commercial segments prevents the negative developments in the subprime market from spreading to other sectors of the economy, especially the production sector which is unaffected by subprimers' defaults and no significant effect on the aggregate output can be observed.

A more interesting comparison is given in Figure 8 which presents the impulse responses for output of the benchmark model (solid line) and three versions of the model with securitization. Impulse responses are presented as percent deviations from the steady state. The dashed green line shows the responses of the model in which entrepreneurs buy MBS tranches, the dotted magenta line presents the second version of the model with securitization, in which patient households buy MBS tranches, whereas the dashed-dotted red line shows the responses of the model in which commercial bankers buy MBS tranches. In the case of both shocks in the model, in which patient households or entrepreneurs acquire claims on subprime loans, the output response is smaller than in the benchmark case. While looking at Figure 8, it is important to note that the model with securitization in which patient households buy MBS claims leads to a relatively worse output performance compared to the version in which entrepreneurs become the investors of new assets. This is due to the special role patient households play in the model

¹⁷An earlier working paper version presenting this model (Grodecka, 2013) includes also a technology and an inflation shock. Monetary and preference shocks are the most important in explaining the main transmission mechanism, so I only focus on them.



Note: All impulse responses are measured in percentage deviations from steady state

economy: they are the source of commercial bankers' deposits, and their savings behavior affects the aggregate balance sheet of the economy. In the model in which they invest in MBS, they have less resources to save in form of deposits, and so this version of the model with securitization leads to less lending than the version in which firms acquire subprime loan proceeds.

Due to securitization, the capital constraint of subprime lenders becomes relatively looser (they hold less assets decreasing in value on their balance sheets; the numerical example discussing the relation between the value of assets and the bindingness of the capital constraint is given in Appendix A) and their consumption is less responsive to shocks than in the benchmark model. As subprime lenders' liabilities (interbank deposits) are assets of commercial bankers, securitization, by enabling subprime lenders to sell toxic assets, will protect their balance sheets from shrinking in the case of a negative shock. The mechanism of interbank linkages is presented in Figure 9, which shows balance sheets of the subprime lender and the commercial lender (balance sheets do not necessarily have to be of the same size, as depicted in Figure 9). Before a negative shock, the balance sheets have a size depicted by the solid black line. After a negative shock, the overall lending decreases, but the deleveraging effect is different depending on who is the ultimate bearer of the securitized risk.

Through the interbank linkages, a larger (relative to the benchmark without securitization) subprime balance sheet leads, ceteris paribus, to a larger commercial bankers' balance sheet, and thus more potential lending to firms. Of course, buying claims on MBS tranches changes the budget constraints of the investors and has impacts on their consumption, but they can absorb losses on MBS through working and saving (patient households) or borrowing (entrepreneurs). The overall effect of securitization is positive, because the risk is spread among different agents in the economy. This is the way securitization was expected and is supposed to work.

However, another possibility was also considered - that commercial bankers buy MBS proceeds. If securitized assets are bought by commercial bankers, there is an amplification of the output response after shocks. The amplification occurs not only in comparison to the version of



Figure 9: Interconnected balance sheets of financial intermediaries in the model

the model in which securitized products are bought by savers and entrepreneurs, but also with respect to the benchmark model without securitization. What is the reason for this amplified contraction? All the effects occur through the balance sheets of both types of bankers. Issuing MBS makes the capital constraint of subprime lenders looser (in the case of a negative shock), whereas it tightens the capital constraint of commercial bankers because they hold the MBS (that is declining in value after a negative shock because of the increasing default rate) on their balance sheets. To reduce the tightness of the constraint, commercial bankers may either reduce their consumption or lending (a similar mechanism occurs in Iacoviello, 2014). In the present model, they do both.

When a negative shock hits the economy and commercial bankers buy MBS tranches, their capital constraint gets tighter and they reduce the lending to entrepreneurs who finance housing stock purchases with loans from the bank. As the housing stock is a production factor, output in the economy goes down more than without securitization. When non-banks buy MBS tranches, there is no loss on the balance sheet of commercial bankers and the securitization has an overall positive effect. In the benchmark case, entrepreneurs are relatively unaffected by the defaults in the subprime sector. When commercial bankers engage in securitization, a more direct link is created between the production sector and the subprime mortgage market, so that entrepreneurs suffer from losses in the subprime portfolio more than in the benchmark case. These dynamics are visible in Figures 10 and 11 which present chosen model variables after a monetary shock and the preference shock. From Figures 10 and 11 it is visible that commercial bankers become buyers of MBS, the entrepreneurial borrowing and housing stock are considerably lower than in the benchmark case and in the case where only patient households and entrepreneurs buy MBS. Also the aggregate balance sheet represented by the overall lending sector confirms the intuition presented in Figure 9. Due to a negative shock, the lending goes down in all of the



Figure 10: Impulse responses of models with and without subprime securitization



considered models, but the strength of this effect differs. The fact that bankers face a capital constraint is crucial for obtaining the above result.



Figure 11: Impulse responses of model versions with and without subprime securitization



Apart from considering the impulse response functions, one can also have a look at the

model's theoretical moments. Table 3 presented below shows the standard deviations of the main variables of interest for the benchmark model and the three versions of the model with securitization. For the purpose of the table, I denote the model with securitization in which entrepreneurs buy MBS as Sec1, the model in which patient households buy MBS by Sec2 and the model in which commercial bankers buy MBS as Sec3. I normalize the standard deviations of the benchmark model to 1 and present the standard deviations for the other models in relation to the benchmark, so the numbers presented in columns 3-5 have a percentage interpretation. A number smaller than 1 means that a given variable is less volatile relative to the benchmark model without securitization, while a number larger than 1 denotes larger volatility.

	Standard deviation			
Variable	Benchmark	Sec1	$\operatorname{Sec2}$	Sec3
Output	1	0.503	0.795	1.697
Aggregate lending	1	0.442	0.790	2.541
Nominal interest rate	1	0.485	0.668	1.441
House prices	1	0.252	0.212	1.775
Entrepreneurial borrowing	1	0.378	0.791	2.491
Entrepreneurial housing stock	1	0.464	0.933	2.553

Table 3: Simulated moments of chosen variables

In case of each variable, the standard deviation of the model in which commercial bankers buy MBS (Sec3) is considerably larger than in the benchmark case without securitization. In the case of the model where entrepreneurs (Sec1) and patient households (Sec2) buy MBS, the opposite is the case: both models exhibit a much smaller volatility of considered variations relative to the benchmark. Notably, the model with patient households as MBS investors (Sec2) demonstrates larger variable volatility than the model with entreprenuers as investors. Thus, the simulated moments of the economy confirm the intuition provided by the analysis of impulse response functions: the effects of securitization may be either stabilizing or destabilizing depending on the final buyer of securitized assets, and they are most positive in the model in which entrepreneurs invest in MBS tranches.

5.3 Crisis Experiment

How do the model's predictions relate to the housing prices and output fall observed in the data during the Great Recession? To answer this question, I take into account the seasonally adjusted USSTHPI series¹⁸ and real GDP (available from the Bureau of Economic Analysis). The raw data exhibits a trend in both cases. In order to make the data comparable to the model outcomes presented as percentage change from the steady state, I use the HP-filter to calculate the trend and cyclical component of both series and express the cyclical component as percentage deviations from the trend. Figure 12 presents the percent deviations from trend observed in the data for real GDP (upper panel) and housing prices (lower panel) in the U.S.

 $^{^{18}\}mathrm{The}$ series has been adjusted using the X-12-ARIMA program.



in the years 1975-2013. The gray bars indicate NBER recessions. The last recession started in December 2007 (4th quarter) and ended in June 2009 (2nd quarter).

The analysis of data reveals that the cyclical component of housing prices fell below zero (steady state) between the 4th quarter of 2007 and the 1st quarter of 2008 (and crosses the zero-line from below for one period in the 2nd quarter of 2008), while the cyclical component of GDP turned negative two quarters after housing prices fell, in the 3rd quarter of 2008. Notice that the time when the cyclical component turns negative does not coincide with the peak of GDP and housing prices, as in both cases, the peaks represent positive cyclical divergence from the steady state. Using a log-linearized DSGE model as an analysis tool, I can by construction only look at the deviations from the steady state - before the exogenous shock occurs, the economy is at the steady state. After the cyclical component of house prices turns negative, it reaches a low of -3.81% in the 4th quarter of 2009. The low of the GDP cyclical component occurs earlier, even if the fall itself starts later, and it takes the value of -2.91%, experiencing a relatively fast recovery afterwards (while the cyclical component of housing prices shows a W-shaped pattern).

To investigate how the predictions of my stylized model correspond to the dynamics observed in the data, I calibrate the housing preference shock in three considered models to get an initial fall in housing prices of 1.2627%, as this has been the deviation from the trend in the first two quarters when the cyclical component of housing prices turned negative. Figure 13 shows the



Figure 13: Crisis experiment

Note: All impulse responses are measured in percentage deviations from steady state

results of this exercise, presenting the results for three models and the data series (starting from the 3rd quarter of 2008, when the cyclical component of GDP turns negative and the cyclical component of housing prices falls below zero for the second time) for the first 10 quarters after the shock.

The model, especially its version with banks investing in MBS does a good job replicating the hump-shaped response of output after the negative preference shock. The initial fall of housing prices of 1.2627% leads to a relatively fast come-back of housing prices to the steady state in the two models with securitization in which non-financial agents buy claims on MBS. The baseline model predicts a maximum drop in housing prices of 2.17% and the model with securitization in which bankers invest in MBS shows a drop of 4.16%, which is closest to the data (3.81%). When it comes to the output response, unsurprisingly the model with securitization in which commercial bankers engage in the acquisition of MBS tranches, comes closest to the data, generating an output fall of 2.65% (compared to 2.91% in the data). The deviations from trend observed in the data are larger than the ones generated by the model, but having in mind Figure 12, one may recall that the fall in prices and GDP started from an above-trend level, which may have given more impetus to the variables.

Given the simplicity of the model and the fact that the model is not explicitly estimated to match the data, the comparison of the model and data series is more than satisfying. The version of the model in which bankers buy MBS generates impulse response functions similar to the output behavior in the data, which suggests that the model may highlight an important amplification mechanism that played a role in the crisis. A comparison of the red dashed-dotted line in Figure 13 with the other lines allows for conducting a simple counterfactual exercise and

shows how output could have evolved if there was no securitization or if securitized products were bought by final buyers other than commercial bankers. In both cases, the fall in output would be lower and the recovery would be faster. Specifically, the maximum output fall in the model with securitization and entrepreneurs as investors corresponds to only ca. 10% of the maximum output fall in the model with bankers as investors. Assuming that in reality the latter case occurred and applying this number to the output fall in the data, we get a maximum output fall of 0.29% instead of 2.91% observed in the data, corresponding to the case in which only non-financial investors (entrepreneurs) buy MBS tranches.

6 Sensitivity Analysis and Extension

6.1Sensitivity Analysis

In order to test the model's robustness, I compute a sensitivity analysis with respect to the housing share in the production function, commercial bankers' capital ratio and tranche retention by subprime lenders. The results are presented as the difference between the IRFs of the benchmark model (solid blue line in all graphs) and the model with securitization in which bankers are the investors, after a monetary shock.¹⁹ The larger the difference, the larger the negative effect compared to the economy without securitization.

Figure 14 presents the differences for different values of housing share in the production function. The larger the housing share, the stronger the negative effects of securitization on housing prices and output. This is an intuitive result: given that entrepreneurial housing stock falls in response to the negative shock, if it is a relatively more important factor of production (ν is larger), output will experience a larger drop.



Difference between model without securitization and with (banks investing), monetary shock

Figure 14: Sensitivity w.r.t. ν

¹⁹Results for the preference shock are qualitatively the same.



Figure 15: Sensitivity w.r.t. τ

From the policymaker's point of view, it is important to examine the effects of increasing regulation in the banking market. Could more strict regulations, i.e. higher capital ratios and higher tranche retention rates protect the economy from large output falls, analogous to those that occurred during the Great Recession? Figure 15 presents the sensitivity analysis w.r.t. different capital ratios, and Figure 16 presents results for different tranche retention rates.

Figure 15 shows that, as capital ratios for commercial bankers increase, the difference between the baseline model and the model with bankers as investors in securitized assets falls. This suggests that, given the existence of equity constraints, their higher value is better for the economy, as it reduces deleveraging effects and the fall in housing prices and output. When it comes to imposing higher retention rates on subprime lenders, Figure 16 suggests that such a macroprudential policy is less effective than determining the level of capital ratios. Higher retention rates lead to smaller differences between the baseline and the 'bad securitization' model, but the effects are quantitatively negligible even for tranche retention rates as high as 50%. This can hinge on the fact that the subprime lending sector is more regulated in the first place. Higher capital ratios for subprime lenders and high risk weights on subprime loans significantly reduce the leverage of the subprime sector as compared to the commercial banking sector, so introducing stricter regulations has a relatively smaller marginal impact on the behavior of the economy.



Figure 16: Sensitivity w.r.t. t

6.2 Extension

In the baseline version of the model, I consider only subprime impatient borrowers that constitute 100% of all households' borrowing. As discussed in Section 2.1, subprime hybrid contracts were in reality a minor part of the U.S. mortgage market, which was dominated by prime fixed-interest rate contracts. In an extension of the model, I consider the existence of impatient prime borrowers that do not default on their loans and have access to long-term contracts. Equations describing the optimization problem of the prime borrower are presented in Appendix G. I assume that prime borrowers, unlike subprime borrowers, have access to loans offered by commercial bankers. In this version of the model, I calibrate the subprimers' share in the market to reflect the average share observed in the data in the pre-crisis years: 20%. I also assume that prime borrowers have access to 4-period contracts.²⁰

It turns out that adding impatient prime borrowers to the model reduces the volatility of the model's variables. Prime households that take out fixed interest rate loans are not as sensitive to changes in housing prices and interest rates as the subprimers, which makes their borrowing less responsive to shocks. As subprimers are now only a subset of borrowing households, in the extended model, their default leads to less disruptions. The effects are quantitatively less strong than in the presented baseline model, but the qualitative results remain the same. In addition to the deleveraging effect w.r.t. lending to firms, the extended version of the model features also a reduction in lending to prime households in the model with bankers as investors in the securitization market. The graphs presenting the behavior of main variables of interest for the model with prime borrowers are included in Appendix G.

²⁰In the earlier version of the model also 2-period subprime and 6-period prime loans have been considered. The results do not change qualitatively in this case and the quantitative impact is limited.

7 Conclusion

In this paper, I analyze the importance of the specific design of subprime contracts and the securitization of subprime loans in generating cyclical fluctuations in the U.S. in a New-Keynesian model. The evidence suggests that the existence of subprime borrowers alone cannot account for the amplification of the responses of output and housing prices to different shocks in the economy. This paper also gives an answer to the question whether the securitization of subprime loans could be a factor that amplified the response of the economy to negative shocks, like the one we observed during the Great Recession. It turns out that the effects of securitization of subprime loans depend on who is the final buyer of securitized assets. If households and entrepreneurs purchase MBS tranches, securitization has a positive overall effect on the economy, spreading the subprime risk among different agents. Facing a negative shock and losses on securitized portfolios, these agents adjust their labor supply and saving decisions (patient households) or borrowing (entrepreneur) so as to cushion the effects of the exogenous disturbances. The positive effects of securitization arise thanks to an interconnected banking sector in which changes in the balance sheet of one financial intermediary have an impact on the balance sheets of other financial intermediaries in the economy through interbank loan contracts. However, if financial intermediaries (that are the source of credit to firms in the economy) purchase MBS tranches, the negative effects of securitization prevail. This results in a bigger contraction of output after a negative shock when compared with the case where non-banks buy MBS tranches or without securitization. The positive risk-sharing aspect of securitization is mostly suppressed in this situation, because the capital constraint on the side of banks is a source of additional financial frictions. The counterfactual exercise conducted in this paper suggests that if financial institutions followed the intended business model of securitization, the maximum quarterly output loss in the U.S. economy during the Great Recession would have been much smaller and shorter-lasting compared to that actually experienced.

The results of the paper are in line with narrative explanations of the crisis provided by Hellwig (2009) and Jaffee et al. (2009). It is shown that securitization per se cannot be blamed for the crisis, because it may have a positive impact on the economy if the securitized products are bought by agents that do not play the role of a financial intermediary in the economy. Obviously, it may be that unless there was the possibility of securitization, the bankers would not issue as many subprime loans as they did in the first place. The present paper deals, however, with the possible transmission mechanism in an economy with subprime borrowers and securitization, rather than the reasons for the existence of the subprime market and the subprime securitization with their incentive problems.

The presented setup addresses several important questions of policymakers, like the burden of regulations in the economy. It turns out that raising capital ratios is an effective method of reducing negative deleveraging effects, while imposing higher tranche retention rates on subprime lenders is relatively less efficient, as they are already more regulated and the marginal effect of additional regulation is comparatively small. Moreover, the paper's results suggest that the segmentation of the banking sector and avoiding interbank linkages between banks operating in different segments may be a good way of preventing the negative spillovers of credit defaults in the economy. This may not only reflect the separation of the subprime and prime loans segments, but also the separation of commercial and investment banking, which was the case in the United States for several decades due to the Glass-Steagall Act of 1933. The separation between commercial and investment banks was abolished at the beginning of the new century, and it might have been one of the causes of the widespread crisis, as the current paper shows. Thus, from the point of view of the policymaker, it is crucial to ensure that banks disclose all information about their assets, even those hidden from the balance sheet that may give a hint about potential linkages between different banking sectors and branches.

The model operates in a closed-economy setup, however it is easy to imagine that the two banking sectors presented in the model represent financial intermediaries of two different countries.²¹ If toxic assets generated in country A are sold to commercial banks in country B, country A is basically able to transfer all the default risk and losses to country B, which will suffer from a recession due to the engagement in the international financial market (country A will remain practically intact). This narrative can be easily adopted to partially explain what happened during the recent financial crisis. The U.S. was the country issuing toxic assets and it was selling them to foreign investors, transferring the subprime risk from the country to the international market. This is why, e.g. many European banks, municipalities etc., had problems when the defaults in the U.S. subprime market started, and the crisis spread around the world. In reality, not only did the international buyers of RMBS suffer from losses, but the U.S. economy experienced a recession as well (thus the country A from our example did not remain intact). This is partially due to the fact that U.S. banks also engaged in the acquisition of toxic assets. Also, other factors, such as labor market developments in the U.S. played a role, which are, however, not considered in this model.

To sum up, this paper combines the macroeconomic framework with financial economics, presenting one important channel that may have played a role in the amplification of the recent crisis in the U.S. economy. It provides evidence that financial intermediaries and the constraints they are facing are an important feature of macroeconomic models.

 $^{^{21}}$ Kollmann et al. (2011) investigate the role of bank capital requirements in the international context, modeling a global bank subject to loan default shocks.

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A Banks' Capital Constraints and Deleveraging

One of the major results of this paper hinges on the fact that the financial intermediaries face collateral constraints, forcing them to maintain a certain equity to assets ratio. Following Adrian and Shin (2010), this section focuses on the balance sheet effects of capital constraints faced by bankers that are crucial to understand the effects of securitization in the model presented in this paper. Consider a simplified balance sheet of a financial intermediary:

Assets	Liabilities
Loans $L = 100$	Deposits D=90
	Equity $E = 10$

The ratio of equity to assets is given by 10/100 = 10%. Assume that these 10% correspond to the capital constraint set by the regulator. The leverage ratio is given by the inverse of the capital ratio, i.e. 100/10 = 10. The capital constraint is always binding, i.e. the banker will avoid holding excess equity which would lower his leverage ratio, and I assume that equity adjusts first to changes on the asset side. Consider a scenario in which the value of assets falls by 1%. The balance sheet looks in this case as follows:

Assets	Liabilities
Loans $L = 99$	Deposits D=90
	Equity $E=9$

At first, the capital ratio falls to 9/99 = 9.09% and leverage increases to 99/9 = 11. A banker trying to maintain a constant equity ratio will try to bring leverage down to the previous level and he can do so in two ways. He may choose to deleverage, i.e. reduce lending from depositors, which would lead to a shrinking balance sheet, since as D falls, loans L also have to fall. To reduce leverage to 10, the banker has to reduce deposits and assets by D solving the following equation: (99 - D)/9 = 10. Thus, borrowing from depositors has to be reduced by 9 units which results in the following balance sheet:

Assets	Liabilities
Loans $L = 90$	Deposits D=81
	Equity $E=9$

After this operation, the capital ratio and leverage are back to the original levels. The initial fall in asset prices of 1% resulted in a fall in lending by 10%. Alternatively, the banker may choose to raise new equity, which would result in an expansion of the balance sheet. To determine how much equity needs to be raised we solve for E in equation (99+E)/(9+E) = 10. The bank needs to raise 1 additional unit of equity to bring the leverage down to the previous level, which results in the same balance sheet as before the fall in asset prices.

Which of the two alternative ways of reducing leverage is mostly chosen by financial intermediaries that face a fall in their asset prices? Unfortunately, the first one is more common, i.e. deleveraging leading to shrinking balance sheets and sales of assets. It is unfortunate, as the initial fall in prices may lead to sales of assets, which may further drive their prices down, leading to a vicious circle. Why do banks tend to adjust their deposits rather than equity? As Adrian and Shin (2011) document, the equity of financial intermediaries behaves in many cases like a pre-determined variable and it is relatively sticky, which may be explained by possible non-pecuniary benefits to bank owners (new equity leads to dilution of the value of stakes of the insiders, loss of the control over shares). In the stock market context, raising new equity through issuing new shares may be difficult in times of falling asset prices. Thus, even if theoretically a financial intermediary facing an increased leverage has two options to cope with that situation, a fall in asset prices often leads to a contraction in the balance sheet.

This mechanism is crucial for understanding the behavior of bankers in the paper. Facing losses on subprime MBS, bankers decide to reduce their lending to productive firms, worsening the effects of the initial negative shock in the economy. They could raise new equity, but this would mean that they have to reduce their current consumption. As they are modeled as impatient agents maximizing their consumption, a shortcut that enables me to capture the stickiness of the equity observed by Adrian and Shin (2011), they choose to deleverage rather than cut consumption.

Can higher capital ratios reduce the deleveraging effect? Consider an economy in which there is a capital ratio of 20%. Then, the balance sheet looks as follows:

Assets	Liabilities
Loans $L = 100$	Deposits D=80
	Equity $E=20$

In this case, a fall of asset prices by 1% will require an adjustment by D units: 19/(99-D) = 20%. D=4, i.e. the initial fall of 1% will lead to shrinking of the balance sheet by overall 5 units, which is 5% of its overall size, as opposed to 10% when the capital ratio was 10%. Thus, a higher capital requirement reduces the negative deleveraging effects.

How does the risk-weighting of assets (present in Basel regulations and in this paper) change the considerations about changes in the size of the balance sheet? Until now, it was assumed that the risk-weight of the assets is 1. What if it were twice as big or two times smaller? Consider the second case and assume that L=100 and the required capital ratio is 10%. The risk weighting enables us to keep only 5 units of equity E (E/0.5*L=10%), and finance the lending mostly with deposits D=95, which means that our leverage increases to 20 (10 in the base case without risk-weighting). What happens if asset prices fall by 1%? The capital ratio becomes 4/0.5 * 99 = 8.08%. To bring the capital ratio up to 10%, the banker will reduce its borrowing and lending by D: 4/0.5 * (99 - D) = 10%. The solution is D=19. That means that the initial fall of asset prices by 1% leads to the overall decline in lending by 20%. Introducing the risk weights influences the strength of the bankers' response in the event of asset price decline. A risk weight smaller than 1 increases the leverage of the financial intermediary and exacerbates the deleveraging process, whereas a risk weight larger than 1 will have stabilizing effects.

The basic analysis of banks' balance sheets suggests that higher capital ratios and risk weights higher than 1 (100%) may help reduce the negative effects of a fall in asset prices in the economy. Will the dynamic general equilibrium model confirm these conclusions? It turns out that yes - binding capital constraints may indeed lead to large amplification effects in the face of a crisis, and higher capital constraints reduce the deleveraging effect.

B Data Description

To produce the Figures 5 and 18 and compare them to the data for all U.S. chartered banks, I use monthly data provided by the Federal Reseve in Table H.8 Assets and Liabilities of Commercial Banks in the United States.²²The data on MBS holdings and data on commercial real estate loans begins in 10.1996 for large commercial banks. If I wanted to use data for all commercial banks, the data on MBS holdings starts in 07.2009 and the data on commercial real estate loans begins in 06.2004. Since I am interested in a longer perspective, I use data for large banks as a proxy for all U.S. banks. The total assets of the large domesticallychartered commercial banks constituted in years 1985-2013 around 56% to 68.5%, with a falling tendency over time. The developments in the fractions of total commercial loans and commercial real estate loans follow similar patterns. In period 06.2004-06.2013 the correlation coefficient between the fraction of total commercial loans in large domestically-chartered banks and all banks stood at 91.14%. For the fraction of commercial real estate loans, the coefficient in the same period was even larger: 95.38%. Thus, the conclusions remain relevant for the whole banking sector in the U.S., even though some of the graphs in this paper are only prepared using data for large domestically-chartered banks. For total consumer loans, the corresponding correlation coefficients are significantly lower: for the fraction of consumer real estate loans we obtain a correlation of 61.34%, while for the fraction of total consumer loans the correlation is 59.02%.

I use the following series:

For large domestically chartered commercial banks:

Commercial and industrial loans: B1023NLGAM

Commercial real estate loans:B1219NLGAM

Commercial loans: B1023NLGAM + B1219NLGAM

Treasury and agency securities: Mortgage-backed securities (MBS): B1301NLGAM

Other securities: Mortgage-backed securities: B1303NLGAM

Total MBS holdings: B1301NLGAM+ B1303NLGAM

Consumer real estate loans: B1027NLGAM + B1220NLGAM

Other consumer loans: B1029NLGAM

Total consumer loans: B1029NLGAM+ B1027NLGAM + B1220NLGAM

Total assets:B1151NLGAM

²²Available on http://www.federalreserve.gov/releases/h8/current/.

For all commercial banks:

Commercial and industrial loans: B1023NCBAM Commercial real estate loans: B1219NCBAM Commercial loans: B1023NCBAM+ B1219NCBAM Treasury and agency securities: Mortgage-backed securities (MBS): B1301NCBAM Other securities: Mortgage-backed securities: B1303NCBAM Total MBS holdings: B1301NCBAM+ B1303NCBAM Consumer real estate loans: B1027NCBAM+B1220NCBAM Other consumer loans: B1029NCBAM Total consumer loans: B1029NCBAM+ B1027NCBAM+B1220NCBAM Total assets: B1151NCBAM

C Additional Graphs



Commercial loans and total MBS holdings, large U.S. banks

Figure 17: Commercial loans holdings in the U.S., large banks



Subprime default rate vs. fraction of commercial loans in the U.S. (1Q1998-1Q2013), large banks

Figure 18: Subprime default rates and commercial loans in the U.S.

Source: Thomson Reuters Datastream and FRB data



MBS holdings, large U.S. banks

Source: FRB Table H.8 Assets and Liabilities of Commercial Banks

Figure 19: MBS holdings, large U.S. banks



MBS holdings as % of total bank assets, large U.S. banks

Source: FRB Table H.8 Assets and Liabilities of Commercial Banks

Figure 20: MBS holdings as fraction of total bank assets, large U.S. banks

D Baseline Model Equations

D.1 The Optimization Problem of the Patient Household

All equations and constraints are written in real terms. Patient households maximize the utility function given by:

$$\max_{b_t^{Savers}, h_t^{Savers}, L_t^{Savers}, L_t^{Savers}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t^{Savers} + j_t \log h_t^{Savers} - \frac{L_t^{Savers} \eta^{Savers}}{\eta^{Savers}} \right), \tag{19}$$

where c_t^{Savers} denotes the consumption of the final good, j_t is the marginal utility of housing subject to random disturbances (following Iacoviello, the disturbance is common to patient and impatient households, and is a proxy for a housing demand or housing preference shock), h_t^{Savers} is the housing stock held by savers, L_t^{Savers} denotes labor supply of patient households.

The budget constraint of the patient household in real terms is:

$$c_t^{Savers} + q_t(h_t^{Savers} - h_{t-1}^{Savers}) + d_t = R_{d,t-1}d_{t-1}/\pi_t + w_t^{Savers}L_t^{Savers} + F_t,$$
(20)

where d_t denotes deposits, $R_{d,t}$ is the interest rate paid on deposits, F_t are profits from retailers (redistributed only to patient households), $w_t^{Savers} L_t^{Savers}$ is labor income, $q_t = Q_t/P_t$ denotes the real housing price, $\pi_t = P_t/P_{t-1}$ is inflation.

The First Order Conditions (FOCs) are:

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w.r.t. d_t

$$\frac{1}{c_t^{Savers}} = \beta E_t \left(\frac{1}{c_{t+1}^{Savers} \pi_{t+1}} \right) R_{d,t},\tag{21}$$

w.r.t. h_t^{Savers}

$$\frac{q_t}{c_t^{Savers}} = \beta E_t \left(\frac{q_{t+1}}{c_{t+1}^{Savers}}\right) + \frac{j_t}{h_t^{Savers}},\tag{22}$$

w.r.t. L_t^{Savers}

$$w_t^{Savers} = L_t^{Savers\eta^{Savers}-1} c_t^{Savers}.$$
(23)

D.2 FOCs of the Impatient Subprime Household

The FOCs are (λ_t^{Sub} is the Lagrangian multiplier on the borrowing constraint): w.r.t. b_t^{Sub}

$$\frac{1}{c_t^{Sub}} = \beta^{Sub} E_t \left(\frac{(1 - \delta_{s,t}) R_{s,t}}{c_{t+1}^{Sub} \pi_{t+1}} \right) + \lambda_t^{Sub} R_{s,t},$$
(24)

w.r.t. h_t^{Sub}

$$\frac{q_t}{c_t^{Sub}} = \beta^{Sub} E_t \left(\frac{q_{t+1}}{c_{t+1}^{Sub}} + \lambda_t^{Sub} m^{Sub} q_{t+1} \pi_{t+1} \right) + \frac{j_t}{h_t^{Sub}},\tag{25}$$

w.r.t. L_t^{Sub}

$$w_t^{Sub} = L_t^{Sub\eta^{Sub}-1} c_t^{Sub}, \tag{26}$$

D.3 The Optimization Problem and the FOCs of the Entrepreneur

The utility function of the entrepreneur is:

$$\max_{b_{e,t},h_{e,t},L_t^{Savers},L_t^{Sub}} E_0 \sum_{t=0}^{\infty} \gamma^t \log(c_{e,t}),$$
(27)

where $b_{e,t}$ is the borrowing of firms, $h_{e,t}$ denotes their housing stock, L_t is the labor of households, and $c_{e,t}$ denotes firms' consumption.

The production function is:

$$Y_t = h_{e,t-1}^{\nu} L_t^{Savers^{\alpha(1-\nu)}} L_t^{Sub^{(1-\alpha)(1-\nu)}},$$
(28)

where ν denotes the housing share in the production function and the parameter α controls for patient households' labor share in the production function.

The entrepreneurs' budget constraint is:

$$\frac{Y_t}{X_t} + b_{e,t} = c_{e,t} + q_t(h_{e,t} - h_{e,t-1}) + \frac{R_{e,t-1}}{\pi_t} b_{e,t-1} + w_t^{Savers} L_t^{Savers} + w_t^{Sub} L_t^{Sub},$$
(29)

where $R_{e,t-1}$ is the nominal interest rate on loans between period t-1 and t, and X_t is the markup of final over intermediate goods.

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Entrepreneurs face a borrowing constraint:

$$R_{e,t}b_{e,t} \le mE_t(q_{t+1}h_{e,t}\pi_{t+1}). \tag{30}$$

The FOCs of the entrepreneur are (denote by $\lambda_{e,t}$ the Lagrangian multiplier on the borrowing constraint):

w.r.t $b_{e,t}$

$$\frac{1}{c_{e,t}} = \gamma E_t \left(\frac{R_{e,t}}{c_{e,t+1}\pi_{t+1}} \right) + \lambda_{e,t} R_{e,t}, \tag{31}$$

w.r.t. $h_{e,t}$

$$\frac{q_t}{c_{e,t}} = E_t \left[\frac{\gamma}{c_{e,t+1}} \left(\nu \frac{Y_{t+1}}{X_{t+1}h_{e,t}} + q_{t+1} \right) + \lambda_{e,t} m q_{t+1} \pi_{t+1} \right],$$
(32)

w.r.t. labor:

$$w_t^{Savers} = \frac{\alpha(1-\mu-\nu)Y_t}{X_t L_t^{Savers}},\tag{33}$$

$$w_t^{Sub} = \frac{(1-\alpha)(1-\mu-\nu)sY_t}{X_t L_t^{Sub}}.$$
(34)

D.4 Retailers

Retailers acquire intermediate goods produced by the entrepreneurs at price P_t^w , then differentiate them into $Y_t(z)$ (retailers of mass 1 are indexed by z) and sell at price $P_t(z)$. The aggregate output index is given by:

$$Y_t^f = \left(\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} \,\mathrm{d}z\right)^{\frac{\varepsilon}{\varepsilon-1}},\tag{35}$$

where $\varepsilon > 1$. The price index is given by:

$$P_t = \left(\int_0^1 P_t(z)^{1-\varepsilon} \,\mathrm{d}z\right)^{\frac{1}{1-\varepsilon}}.$$
(36)

Retailers can change their sale price every period with probability $1 - \theta$. A fraction θ stays unchanged every period. The reset price of the retailer is denoted by $P_t^*(z)$ and $Y_{t+k}^*(z) = \left(\frac{P_t^*(z)}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$ is the corresponding demand.

The retailer maximizes the following equation:

$$\sum_{k=0}^{\infty} \theta^k E_t \Biggl\{ \Lambda_{t,k} \Biggl(\frac{P_t^*(z)}{P_{t+k}} - \frac{X}{X_{t+k}} \Biggr) Y_{t+k}^*(z) \Biggr\},$$
(37)

where $\Lambda_{t,k} = \beta^k \left(\frac{c_t^{Savers}}{c_{t+k}^{Savers}}\right)$ is the patient household relevant discount factor, $X_t = \frac{P_t}{P_t^w}$ is the markup of final over intermediate goods and X denotes the steady state value of the markup.

The aggregate price level evolution is given by:

$$P_t = (\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}.$$
(38)

Combining the last two equations and log-linearizing leads to the following formulation of a forward-looking Phillips curve

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \kappa \hat{X}_t, \tag{39}$$

where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$ and hatted variables denote percent deviations from the steady state.

D.5 All Baseline Model's Equations

1. Aggregate demand block

$$Y_t = c_t^{Savers} + c_t^{Sub} + c_{e,t} + c_{b,t} + c_{bb,t}$$
(40)

$$\frac{1}{c_{b,t}} = \beta_b E_t \left(\frac{R_{b,t}}{c_{b,t+1}\pi_{t+1}}\right) + (1 - \tau \chi^{intb})G_t \tag{41}$$

$$\frac{1}{c_{b,t}} = \beta_b E_t \left(\frac{R_{e,t}}{c_{b,t+1}\pi_{t+1}}\right) + (1 - \tau \chi^{firm})G_t \tag{42}$$

$$\frac{1}{c_{b,t}} = \beta_b E_t \left(\frac{R_{d,t}}{c_{b,t+1}\pi_{t+1}}\right) + G_t \tag{43}$$

$$\frac{1}{c_t^{Savers}} = \beta E_t \left(\frac{1}{c_{t+1}^{Savers} \pi_{t+1}}\right) R_{d,t} \tag{44}$$

$$\frac{1}{c_{e,t}} = \gamma E_t (\frac{R_{e,t}}{c_{e,t+1}\pi_{t+1}}) + \lambda_{e,t} R_{e,t}$$
(45)

$$\frac{1}{c_t^{Sub}} = \beta^{Sub} E_t(\frac{(1-\delta_{s,t})R_{s,t}}{c_{t+1}^{Sub}\pi_{t+1}}) + \lambda_t^{Sub} R_{s,t}$$
(46)

$$\frac{1}{c_{bb,t}} = \beta_b E_t \left(\frac{R_{s,t}(1-\delta_{s,t+1})}{c_{bb,t+1}\pi_{t+1}}\right) + (1-\tau^{Sub}\chi^{Sub})(1-\delta_s)GG_t$$
(47)

$$\frac{1}{c_{bb,t}} = \beta_b E_t(\frac{R_{b,t}}{c_{bb,t+1}\pi_{t+1}}) + GG_t \tag{48}$$

2. Aggregate supply

$$Y_t = h_{e,t-1}^{\nu} L_t^{Savers^{\alpha(1-\nu)}} L_t^{Sub^{(1-\alpha)(1-\nu)}}$$
(49)

$$w_t^{Savers} = \frac{\alpha (1 - \mu - \nu) Y_t}{X_t L_t^{Savers}}$$
(50)

$$w_t^{Sub} = \frac{(1-\alpha)(1-\mu-\nu)sY_t}{X_t L_t^{Sub}}$$
(51)

$$\hat{\pi_t} = \beta E_t \hat{\pi_{t+1}} - \kappa \hat{X_t} \tag{52}$$

3. Housing market block

$$1 = h_t^{Savers} + h_t^{Sub} + h_{e,t} \tag{53}$$

$$\frac{q_t}{c_t^{Savers}} = \beta E_t(\frac{q_{t+1}}{c_{t+1}^{Savers}}) + \frac{j_t}{h_t^{Savers}}$$
(54)

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$$\frac{q_t}{c_t^{Sub}} = \beta^{Sub} E_t \left(\frac{q_{t+1}}{c_{t+1}^{Sub}} + \lambda_t^{Sub} m^{Sub} q_{t+1} \pi_{t+1}\right) + \frac{j_t}{h_t^{Sub}}$$
(55)

$$\frac{q_t}{c_{e,t}} = E_t \left[\frac{\gamma}{c_{e,t+1}} \left(\nu \frac{Y_{t+1}}{X_{t+1}h_{e,t}} + q_{t+1} \right) + \lambda_{e,t} m q_{t+1} \pi_{t+1} \right]$$
(56)

4.Borrowing constraints

$$R_{s,t}b_t^{Sub} = m^{Sub}E_t(q_{t+1}\pi_{t+1})h_t^{Sub}$$
(57)

$$R_{e,t}b_{e,t} = mE_t(q_{t+1}h_{e,t}\pi_{t+1}).$$
(58)

$$\tau = \frac{bb_t + b_{e,t} - d_t}{\chi^{Intb}bb_t + \chi^{Firm}b_{e,t}}$$
(59)

$$\tau^{Sub} = \frac{(1 - \delta_s)b_t^{Sub} - bb_t}{\chi^{Sub}(1 - \delta_s)b_t^{Sub}}$$
(60)

$$\delta_{s,t} = \delta_s - \phi_{s,h}(q_t - Q) \tag{61}$$

5. Budget constraints/ evolution of state variables

$$c_t^{Savers} + q_t(h_t^{Savers} - h_{t-1}^{Savers}) + d_t = R_{d,t-1}d_{t-1}/\pi_t + w_t^{Savers}L_t^{Savers} + F_t$$
(62)

$$\frac{Y_t}{X_t} + b_{e,t} = c_{e,t} + q_t(h_{e,t} - h_{e,t-1}) + \frac{R_{e,t-1}}{\pi_t} b_{e,t-1} + w_t^{Savers} L_t^{Savers} + w_t^{Sub} L_t^{Sub}$$
(63)

$$c_{b,t} + \frac{R_{d,t-1}d_{t-1}}{\pi_t} + bb_t + b_{e,t} = d_t + \frac{R_{b,t-1}bb_{t-1}}{\pi_t} + \frac{R_{e,t-1}b_{e,t-1}}{\pi_t},$$
(64)

$$c_{bb,t} + b_t^{Sub} + R_{b,t-1}bb_{t-1}/\pi_t = bb_t + R_{s,t-1}(1-\delta_{s,t})b_{t-1}^{Sub}/\pi_t$$
(65)

6.Shock processes and monetary policy rule

$$\ln j_t = \rho_j \ln j_{t-1} + \varepsilon_{j,t,} \tag{66}$$

$$R_{d,t} = (R_{d,t-1})^{r_R} E_t (\pi_{t-1}^{1+r_\pi} (\frac{Y_{t-1}}{Y})^{r_y} \bar{rr})^{1-r_R} \varepsilon_{R,t}.$$
(67)

E The option characteristics of the tranching problem

The payoffs of equity and senior tranche holders resemble payoffs from investment in European options. A European option is a financial instrument that gives the holder the right (but not the obligation) to buy (in case of a call option) or to sell (in case of a put option) the underlying asset at a certain price (reference price, called also strike) at a certain period in time (expiration date of the option). Buying a call option or writing (selling) a put option, the economic agent bets on the increase of the underlying asset price. By selling a call option (having a short call position) or buying a put option (having a long put position), the investor bets on the fall in the underlying asset price. The holder of an equity tranche of MBS gets payoffs equal to the ones from a long put position - he invests in the hope that the default rate



Figure 21: Option position of MBS investors

(which can be interpreted as the underlying asset) will decrease. Also, investing in a senior tranche of an MBS is profitable when the default rate decreases. Note that

$$P_{s}, t = \min(S_{t} - fS_{t}, S_{t} - \delta_{s,t}S_{t}) = S_{t}(1 - f) - \max(S_{t}\delta_{s,t} - fS_{t}, 0) = S_{t}(1 - \delta_{s,t}) - \max(fS_{t} - \delta_{s,t}S_{t}, 0)$$
(68)

Thus, the payoff of the senior tranche can be rewritten as having a long position in the face value of the tranche and a short call position, or a long position in the cash flows from subprime loans and a short put. Notice that in the case of the equity tranche and the senior tranche payoff, the face value of the MBS, S_t , can be factored out. The underlying asset for the investors of MBS tranches is the default of subprime loans $\delta_{s,t}$, whereas the exercise price of the options they trade equals f (the attachment point of senior tranche). Figure 21 visualizes the profit (on the vertical axis) of investing in a short call and long put position depending on the default of subprime loans (horizontal axis). The lower the default, the higher the profit of investors (or the lower the loss).

After a shock, payoffs are realized and it is known whether the loss was bigger than the size of the equity tranche. Thus, the investors get a well-known proportion of subprime cashflows. However, while deciding about investing in the next period, they take into account the expected future value of payoffs to evaluate the amount of money they want to pay for the given tranche. Note that while evaluating the expected payoff of tranches, $E_t(Loss_t) = E_t(\delta_{s,t+1}S_{t+1})$ is unknown, because the default rate is a jump variable. Thus, an appropriate expression for $E_t[min(S_{t+1} - fS_{t+1}, S_t - \delta_{s,t+1}S_{t+1})]$ and $E_t[max(fS_{t+1} - \delta_{s,t+1}S_{t+1}, 0)]$ is needed. As noted before, in both cases the $E_t[S_{t+1}]$ can be factored out. However the uncertainty remains with respect to the development of $E_t[\delta_{s,t+1}]$. One can use the Black-Scholes formula to evaluate payoffs, but this requires certain assumptions that cannot be made here (stable volatility of default rate, risk-free interest rate). There is a simple method to smoothly approximate a function with a kink, like the ones drawn above. The logistic function provides an approximation of maximum and minimum functions, which makes the solution tractable.²³ The maximum and

 $^{^{23}}$ Actually, the logistic function is used in one of the financial methods of estimating the value of securitized products. In finance, apart from the Black-Scholes formula and copula methods for option pricing, neural networks have been used to price options (that have a logistic function in the solution) at least since the



The maximum function and its logistics approximation (f=0.2)

Figure 22: Logistics function as an approximation of the maximum function

minimum payoffs can be thus approximated with a logistic function: $E_t[max(f - \delta_{s,t+1}, 0)] \approx E_t[f - \frac{\delta_{s,t+1} - f}{1 + e^{(\delta_{s,t+1} - f)}}]$, whereas $E_t[max(\delta_{s,t+1} - f, 0)] \approx E_t[-f - \frac{\delta_{s,t+1} + f}{1 + e^{(-\delta_{s,t+1} + f)}}]$. Figure 22 provides a visualization of the approximation by the logistics function. The solid line presents the maximum function, while the dotted line its approximation. The x-axis corresponds to different values of δ - subprimers' default rate.

Equation 68 shows three analogous representations of the payoff that goes to senior tranche holders. From this representation, one can see that the minimum function can be rewritten in such a way that only one approximation with the logistic function has to be made to find the expected payoffs of both tranche holders (for a long put). Having rewritten expected payoffs using the approximating function, one can log-linearize the conditions determining the behavior of the price of MBS tranches and consumption of the agents engaged in the transaction.

In what follows, I show that the solution of the model with the use of the logistics function to approximate the maximum functions denoting investors' payoffs yields qualitatively the same results as the easier model presented in the main part of the paper, even though the approximation by the logistics function is not very exact near the model's steady state ($\delta = 0.05$ presented by the vertical dashed green line on the Figure 23).

To investigate whether the engagement of commercial banks in the securitization process could be one of the factors amplifying the negative results of different shocks in the economy, I consider two cases. In the first case, I assume that the generated MBS tranches are bought by patient households (because they are more patient, they acquire claims on the senior tranche) and entrepreneurs (because of their degree of impatience, they are more prone to acquire claims on the equity tranche). In the second case, I assume that the commercial bankers buy the senior tranche of MBS and the entrepreneurs invest in the equity tranche (one could also assume that the commercial bankers buy both the equity and the senior tranche, which would be a more

publication of Hutchinson et al. (1994).



Figure 23: Logistics function as an approximation of the maximum function

extreme case and would lead to quantitatively stronger results). In both cases, subprime lenders retain a vertical fraction t of the issued security (equivalent to retaining a percentage t of cash flows).²⁴

E.1 First Version: Patient Households and Entrepreneurs Invest in MBS Tranches

In the first version of the model with securitization of subprime loans, patient households invest in the senior tranche, and entrepreneurs in the equity tranche.

The budget constraints of investors change and a new term describing investment in the derivative security appears. First, denote the payoff of the senior tranche $E_t[min(S_{t+1} - fS_{t+1}, S_{t+1} - \delta_{s,t+1}S_{t+1})]$ as $MBS_{s,t}$ and the price of the senior tranche by $p_{s,t}$. Then, the budget constraint of the patient household is (remember that subprime lenders retain portion t of every tranche):

$$c_t^{Savers} + q_t (h_t^{Savers} - h_{t-1}^{Savers}) + d_t + (1-t)p_{s,t}MBS_{s,t} = R_{d,t-1}d_{t-1}/\pi_t + w_t^{Savers}L_t^{Savers} + F_t + (1-t)MBS_{s,t-1}.$$
(69)

In each period, the patient household gets revenue from investing in the senior tranche and buys a claim on future proceedings from investment in MBS. The FOCs of prime households do not change, but there is a new equation determining the price of the new claim:

$$\beta \frac{1}{c_{t+1}^{Savers}} = p_{s,t} \frac{1}{c_t^{Savers}}.$$
(70)

Analogously, denote the terms describing the investment in the equity tranche $E_t[max(fS_{t+1} -$

²⁴In general, the literature discusses three main types of retention: vertical slice retention, horizontal slice retention, and an equivalent exposure of the securitized pool, discussed further in Geithner (2011). In the present model's case, vertical slice retention generates the same payoff for the bank as equivalent exposure.

 $\delta_{s,t+1}S_{t+1}, 0$] as $MBS_{e,t}$ and $max(fS_t - \delta_{s,t}S_t, 0)$ as $MBS_{e,t-1}$ and the price of the equity tranche by $p_{e,t}$. Then, the budget constraint of the entrepreneur is:

$$\frac{Y_t}{X_t} + b_{e,t} + (1-t)MBS_{e,t-1} = c_{e,t} + q_t(h_{e,t} - h_{e,t-1}) + \frac{R_{e,t-1}}{\pi_t}b_{e,t-1} + w_t^{Savers}L_t^{Savers} + w_t^{Sub}L_t^{Sub} + (1-t)p_{e,t}MBS_{e,t}.$$
 (71)

The FOC w.r.t to the new claim is:

$$\gamma \frac{1}{c_{e,t+1}} = p_{e,t} \frac{1}{c_{e,t}}.$$
(72)

The long-put approximation is given by the logistics function:

$$P_t = E_t[max(f - \delta_{s,t+1}, 0)] \approx E_t[f - \frac{\delta_{s,t+1} - f}{1 + e^{(\delta_{s,t+1} - f)}}]$$
(73)

Along with the optimization problems of agents investing in the security, the problem of subprime lenders also changes in the wake of securitization of subprime loans. They have now to include only the retained proportion of subprime loans in their balance sheet:

Assets	Liabilities
Loans to subprime borrowers: tb_t^{Sub}	Interbank deposits bb_t
Loss reserve $-t\delta_s b_t^{Sub}$	

Thus, a Basel-type capital constraint, given exogenously, is now given by:

$$\tau^{Sub} \le \frac{t(1-\delta_s)b_t^{Sub} - bb_t}{\chi^{Sub}t(1-\delta_s)b_t^{Sub}}.$$
(74)

The budget constraint of subprime lenders changes. Note that when it comes to the transfer of already realized cashflows, it holds that:

 $(1-t)[min(S_t-fS_t, S_t-\delta_{s,t}S_t)+max(fS_t-\delta_{s,t}S_t, 0)] = (1-t)[S_t(1-\delta_{s,t})] = (1-t)[R_{s,t-1}b_{t-1}^{Sub}(1-\delta_{s,t})/\pi_t]$. Yet, in the case of claims purchases on future proceedings, this shortcut cannot be made because the prices of both tranches differ, since the agents that buy them have different discount factors. Thus, the budget constraint of the subprime lender is:

$$c_{bb,t} + b_t^{Sub} + R_{b,t-1}bb_{t-1}/\pi_t - (1-t)[p_{s,t}MBS_{s,t} + p_{e,t}MBS_{e,t}] = bb_t + tR_{s,t-1}(1-\delta_{s,t})b_{t-1}^{Sub}/\pi_t.$$
 (75)

The prices of the tranches are determined by equations 84 and 80.

E.2 Second Version: Commercial Bankers and Entrepreneurs Invest in MBS Tranches

In the second version of the model with securitization, commercial bankers invest in the senior tranche, whereas entrepreneurs, as in the first case, buy claims on the equity tranche. The problem of the entrepreneurs does not change with respect to the version of the model when patient households and entrepreneurs buy MBS tranches. The budget constraint of commercial bankers changes, as well as their balance sheet and capital constraint. I assume here that the risk weight on the senior tranche is as high as in the case of interbank deposits (since it is highly rated), whereas the risk weight on the equity tranche equals the risk weight of subprime loans.

The commercial bankers' balance sheet is:	
Assets	Liabilities
Interbank loans : bb_t	Deposits d_t
Loans to entrepreneurs: $b_{e,t}$	Equity eq_t
MBS security - senior tranche: $(1-t)MBS_{s,t}$	

Thus, a Basel-type capital constraint, given exogenously, has the form:

$$\tau \le \frac{bb_t + b_{e,t} + (1 - t)MBS_{s,t} - d_t}{\chi^{Intb}bb_t + \chi^{Firm}b_{e,t} + \chi^{Sub}(1 - t)MBS_{s,t}}.$$
(76)

The budget constraint of commercial bankers is now:

$$c_{b,t} + R_{d,t-1}d_{t-1}/\pi_t + bb_t + b_{e,t} + (1-t)p_{s,t}MBS_{s,t} = d_t + R_{b,t-1}bb_{t-1}/\pi_t + R_{e,t-1}b_{e,t-1}/\pi_t + (1-t)MBS_{s,t-1}.$$
 (77)

New FOC:

w.r.t.
$$MBS_{s,t}$$

$$\beta_b \frac{1}{c_{b,t+1}} = p_{s,t} \frac{1}{c_{b,t}}.$$
(78)

The problem of subprime lender is analogous to the case where patient households and entrepreneurs buy MBS tranches.

E.3 Results

For calibration it is assumed that the attachment point f = 0.2. The attachment point of the senior tranche corresponds to the data average (Hull and White, 2010). Figure 24 shows the results of the baseline model (blue solid line) and the two models with securitization in which different agents buy MBS tranches (green dashed line - entrepreneurs buy the equity tranche, patient households buy the senior tranche; red dotted-dashed line - entrepreneurs buy the equity tranche, commercial bankers buy the senior tranche) after the monetary shock, and



Figure 24: Impulse responses of model versions with and without securitization, monetary shock

Figure 25 shows the impulse response functions of chosen variables after the preference shock. The slightly different solution method from the one presented in the main part of the paper does not affect qualitatively the results: when commercial bankers engage in the acquisition of the MBS, the securitization has a destabilizing effect on the economy, while when only non-financial agents in the economy buy MBS tranches, the securitization has a positive effect on lending and output.



Figure 25: Impulse responses of model versions with and without securitization, preference shock

F Three Versions of the Model with Securitization

F.1 First Version: Entrepreneurs Invest in MBS Tranches

In the first version of the model with securitization of subprime loans, entrepreneurs buy both the senior tranche and the equity tranche.

The budget constraint of investors changes and a new term describing investment in the derivative security appears. First, denote the payoff from the investment $(S_{t+1} - \delta_{s,t+1}S_{t+1})$ as $MBS_{e,t}$ and the price of tranches by $p_{e,t}$. Then, the budget constraint of the entrepreneur is (remember that subprime lenders retain portion t of every tranche):

$$\frac{Y_t}{X_t} + b_{e,t} + (1-t)MBS_{e,t-1} = c_{e,t} + q_t(h_{e,t} - h_{e,t-1}) + \frac{R_{e,t-1}}{\pi_t}b_{e,t-1} + w_t^{Savers}L_t^{Savers} + w_t^{Sub}L_t^{Sub} + (1-t)p_{e,t}MBS_{e,t}.$$
 (79)

The FOC w.r.t to the new claim is:

$$\gamma \frac{1}{c_{e,t+1}} = p_{e,t} \frac{1}{c_{e,t}}.$$
(80)

Along with the optimization problems of agents investing in the security, the problem of subprime lenders also changes in the wake of securitization of subprime loans. They have now to include only the retained proportion of subprime loans in their balance sheet:

Assets	Liabilities
Loans to subprime borrowers: tb_t^{Sub}	Interbank deposits bb_t
Loss reserve $-t\delta_s b_t^{Sub}$	

Thus, a Basel-type capital constraint, given exogenously, is now given by:

$$\tau^{Sub} \le \frac{t(1-\delta_s)b_t^{Sub} - bb_t}{\chi^{Sub}t(1-\delta_s)b_t^{Sub}}.$$
(81)

The budget constraint of subprime lenders changes. Recall that when it comes to the transfer of already realized cashflows, it holds that:

 $(1-t)[min(S_t - fS_t, S_t - \delta_{s,t}S_t) + max(fS_t - \delta_{s,t}S_t, 0)] = (1-t)[S_t(1-\delta_{s,t})] = (1-t)[R_{s,t-1}b_{t-1}^{Sub}(1-\delta_{s,t})/\pi_t].$ The budget constraint of the subprime lender is:

$$c_{bb,t} + b_t^{Sub} + R_{b,t-1}bb_{t-1}/\pi_t - (1-t)p_{e,t}MBS_{e,t} = bb_t + tR_{s,t-1}(1-\delta_{s,t})b_{t-1}^{Sub}/\pi_t.$$
 (82)

The price of the tranches is determined by equations 80.

F.2 Second Version: Patient Households Invest in MBS Tranches

Analogously to the problem described in the previous subsection, when patient households acquire MBS claims, their budget constraint changes. First, denote the payoff from the investment $(S_{t+1} - \delta_{s,t+1}S_{t+1})$ as $MBS_{s,t}$ and the price of tranches by $p_{s,t}$. Then the budget constraint of the patient households looks as follows:

$$c_t^{Savers} + q_t (h_t^{Savers} - h_{t-1}^{Savers}) + d_t + (1-t)p_{s,t}MBS_{s,t} = R_{d,t-1}d_{t-1}/\pi_t + w_t^{Savers}L_t^{Savers} + F_t + (1-t)MBS_{s,t-1}.$$
(83)

In each period, the patient household gets revenue from investing in the senior tranche and buys a claim on future proceedings from investment in MBS. The FOCs of prime households do not change, but there is a new equation determining the price of the new claim:

$$\beta \frac{1}{c_{t+1}^{Savers}} = p_{s,t} \frac{1}{c_t^{Savers}}.$$
(84)

The problem of subprime lender is analogous to the case where entrepreneurs buy MBS tranches.

F.3 Third Version: Commercial Bankers Invest in MBS Tranches

In the third version of the model with securitization, commercial bankers invest in subprime loan proceeds. The budget constraint of commercial bankers changes, as well as their balance sheet and capital constraint. I assume here that the risk weight on both tranches in this version of the model is as high as in the case of interbank deposits, reflecting the fact that securitized assets were highly rated.

In this case, the payoff from the investment $(S_{t+1} - \delta_{s,t+1}S_{t+1})$ is denoted $MBS_{s,t}$ and the price of tranches by $p_{b,t}$

The commercial bankers' balance sheet is:

Assets	Liabilities
Interbank loans : bb_t	Deposits d_t
Loans to entrepreneurs: $b_{e,t}$	Equity eq_t
MBS security - senior tranche: $(1 - t)MBS_{s,t}$	

Thus, a Basel-type capital constraint, given exogenously, has the form:

$$\tau \le \frac{bb_t + b_{e,t} + (1-t)MBS_{s,t} - d_t}{\chi^{Intb}bb_t + \chi^{Firm}b_{e,t} + \chi^{Int}(1-t)MBS_{s,t}}.$$
(85)

The budget constraint of commercial bankers is now:

$$c_{b,t} + R_{d,t-1}d_{t-1}/\pi_t + bb_t + b_{e,t} + (1-t)p_{b,t}MBS_{s,t} = d_t + R_{b,t-1}bb_{t-1}/\pi_t + R_{e,t-1}b_{e,t-1}/\pi_t + (1-t)MBS_{s,t-1}.$$
 (86)

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New FOC:

w.r.t. $MBS_{s,t}$

$$\beta_b \frac{1}{c_{b,t+1}} = p_{b,t} \frac{1}{c_{b,t}}.$$
(87)

The problem of the subprime lender is analogous to the case where entrepreneurs and patient households buy MBS tranches.

G Extension: Impatient Prime Borrowers

Impatient prime borrowers consume, work and demand real estate.

They maximize a lifetime utility function given by

$$\max_{b_t^{Prime}, h_t^{Prime}, L_t^{Prime}} E_0 \sum_{t=0}^{\infty} \beta^{Primet} (\log c_t^{Prime} + j_t^{Prime} \log h_t^{Prime} - \frac{L_t^{Prime} \eta^{Prime}}{\eta^{Prime}})$$
(88)

The budget constraint of the impatient household looks as follows:

$$c_t^{Prime} + q_t(h_t^{Prime} - h_{t-1}^{Prime}) + 1/T \sum_{j=1}^T \frac{R_{T,t-j}b_{T,t-j}^{Prime}}{\prod_{i=0}^{j-1} \pi_{t-i}} = b_{T,t}^{Prime} + w_t^{Prime} L_t^{Prime}, \qquad (89)$$

where $b_{T,t}$ is a loan contract with maturity T purchased at time t.

The setup differs from the Iacoviello (2005) version, because it is assumed that impatient prime households have access to more than one-period loans.²⁵ Their borrowing in period t depends on the expected value of housing in period t+T and the amount of outstanding debt. Figure 4 shows an example of loan installments in this setup for T=2, two-period contracts (in nominal terms). Total interest cost is due in equal fractions in every period (as in Calza et al. (2013)). This assumption aims to capture the characteristics of a prime mortgage contract in the U.S., which is characterized by a fixed interest rate over a longer time period. It also distinguishes prime borrowers from subprime ones who have only access to short-term, oneperiod loans.

They face a borrowing constraint (household commits to repay debt at time t + T):

$$R_{T,t}b_{T,t}^{Prime} \le m^{Prime}E_t(q_{t+T})h_{t+T-1}^{Prime}\prod_{j=1}^T \pi_{t+j} - 1/T\sum_{j=1}^{T-1}\frac{R_{T,t-j}b_{T,t-j}^{Prime}}{\prod_{i=0}^{j-1}\pi_{t-i}},$$
(90)

The FOCs are $(\lambda_t^{Prime}$ is the Lagrangian multiplier on the borrowing constraint):

²⁵This issue has been addressed by Calza et al. (2013) who show that the variable-rate mortgage structure magnifies the responses of consumption and residential investment to monetary policy shock, whereas a contract in which the rate is fixed for T=2 periods dampens the impulse response of considered variables. Unlike in Calza et al. (2013), in the present model, borrowing in each period depends not only on the future value of housing prices, but also on the outstanding debt from previous periods.



Figure 26: Installment payments of the prime borrower in the case of two-period contracts (in nominal terms)

w.r.t. b_t^{Prime}

$$\frac{1}{c_t^{Prime}} = E_t (1/T \sum_{j=1}^T \beta^{Primej} \frac{R_{T,t}}{c_{t+j}^{Prime} \prod_{i=0}^{j-1} \pi_{t+1-i}}) + \lambda_t^{Prime} R_{T,t} + E_t (1/T \sum_{j=1}^{T-1} \lambda_{t+j}^{Prime} \beta^{Primej} \frac{R_{T,t}}{\prod_{i=0}^{j-1} \pi_{t+1-i}}),$$
(91)

w.r.t. h_t^{Prime}

$$\frac{q_t}{c_t^{Prime}} = E_t \left(\beta^{Prime} \frac{q_{t+1}}{c_{t+1}^{Prime}} + \beta^{Prime^{1-T}} \lambda_{t+1-T}^{Prime} m^{Prime} q_{t+1} \prod_{i=0}^{I-1} \pi_{t+1-i}\right) + \frac{j_t}{h_t^{Prime}}, \qquad (92)$$

w.r.t. L_t^{Prime}

$$w_t'' = L_t^{Prime\eta^{Prime}-1} c_t^{Prime}, (93)$$

w.r.t. λ_t^{Prime}

$$R_t b_t^{Prime} = m^{Prime} E_t(q_{t+1} \prod_{j=1}^T \pi_{t+j}) h_t^{Prime} - 1/T \sum_{j=1}^{T-1} \frac{R_{T,t-j} b_{T,t-j}^{Prime}}{\prod_{i=0}^{j-1} \pi_{t-i}}.$$
(94)

For computation of the extended version, I assume that impatient prime borrowers differ from impatient borrowers in the following characteristics: their LTV ratio is lower ($m^{Prime} = 0.75$), they have access to 4-period loans, the risk-weight on their loans is lower than for subprime loans ($\chi^{Prime} = 0.5$), and they borrow from commercial bankers.



Output response to shocks, model with prime borrowers

Figure 27: Output responses of model versions with and without securitization

As stated in the main part of the paper, the existence of prime borrowers does not change the conclusions from the model. The output response, presented in Figure 27, is less negative in the case where bankers invest in MBS compared to the baseline model without prime borrowers, because in this version of the model subprime borrowers constitute only a subsection of households' borrowing.

The logic behind the contraction of balance sheets applies also to the extended model, both for the preference and the monetary shock, presented respectively in Figure 28 and 29. Apart from the effect of securitization on entrepreneurial borrowing and housing stock, in the model with prime borrowers, we observe also changes in prime borrowers' housing and borrowing responses.



IRF to preference shock, model with prime borrowers

Figure 28: Impulse responses of model versions with and without securitization



IRF to monetary shock, model with prime borrowers

Figure 29: Impulse responses of model versions with and without securitization