Sequential versus Static Screening: An equivalence result

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Abstract

We show that the sequential screening model is equivalent to the standard static screening model. We use this insight to shed new light on the relation between static and dynamic screening models.

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1 Introduction

Recent years have witnessed an increased interest in dynamic adverse selection models in which agents receive novel private information over time.¹ These models have not only been successfully applied to important issues such as dynamic pricing or dynamic optimal procurement, they also raise interesting conceptual questions to what extent insights from static models are robust and extend to dynamic environments.

In this manuscript, we consider the most basic dynamic adverse selection model, the so-called sequential screening model, and show that this model can be equivalently represented as a static screening model. In a sequential screening model, a seller offers a single good for sale, but in contrast to a static environment, the buyer initially has private information only about the distribution of his valuation, and he fully learns his valuation only after contracting has taken place.² More specifically, we show that every sequential screening model corresponds to a specific static screening model as described in Fudenberg and Tirole (1991) so that any solution of one problem is also a solution to the other. Reversely, we identify a class of static screening models that each correspond to an appropriate sequential screening model.

Key in establishing the connection between the sequential and the static model is to explicitly allow for the use of stochastic contracts in the static model. Intuitively, stochastic contracts enter the picture, because in the sequential model the terms of trade depend on the buyer's valuation that realizes ex post and are, from an ex ante perspective, therefore stochastic. Our insight is that the induced distribution of terms of trade can be replicated by a stochastic contract in the static model so that a party's expected utility in the sequential model, where the expectation is taken with respect to the buyer's future valuation, coincides with its expected utility in the static model, where the expectation is taken with respect to the uncertainty generated by the stochastic contract.

It is worth mentioning that our result is *not* implied by a general principle, such as, for example, Pontryagin's maximum principle which states that a dynamic optimization problem can be reduced to *some* static problem subject to *some* constraints. Rather, the insight of our paper is more specific and, therefore, more surprising: the sequential screening model can be represented

¹E.g., Baron and Besanko (1984), Courty and Li (2000), Battaglini (2005), Dai et al. (2006), Esö and Szentes (2007a, b, 2013), Inderst and Peitz (2012), Hoffmann and Inderst (2011), Krähmer and Strausz (2011, 2014a, b), Nocke et al. (2011), Boleslavsky and Said (2013), and Pavan et al. (2014).

²The sequential screening model was introduced by Courty and Li (2000). For a textbook treatment, see Chapter 11 in Börgers, 2015.

as a very specific static model, namely exactly as the familiar standard principal agent adverse selection model.

Having shown that a sequential screening model can be represented as a static screening model, we identify, in a second step, the counterparts of well–understood features of the static screening model in the corresponding sequential screening model, such as the single-crossing condition, the conditions that imply the optimality of static contracts, or the characterization of incentive compatibility.

The paper is organized as follows. The next section introduces the two models and derives our two main results. Section 3 discusses our result, and Section 4 concludes.

2 Sequential versus Static Screening

To show our main result that the sequential screening model corresponds to a static screening model and vice versa, we begin by describing the two models.

2.1 The sequential screening problem

This subsection considers the sequential screening model of Courty and Li (2000). There is a buyer (the agent, he) and a seller (the principal, she), who has a single unit of a good for sale. The buyer's valuation of the good is $x \in [0, 1]$ and the seller's opportunity costs are $c \ge 0$. The terms of trade specify the probability $q \in [0, 1]$ with which the good is exchanged and an unconditional payment $t \in \mathbf{R}$ from the buyer to the seller. The parties are risk neutral and have quasi-linear utility functions. That is, the seller's profit equals payments minus her expected opportunity costs, t - cq, and the buyer's utility equals his expected valuation minus payments, xq - t. Each party's reservation utility is normalized to 0.

There are three periods. At the contracting stage in period 1, no party knows the buyer's true valuation, but the buyer privately knows that his valuation x is distributed according to distribution function $G(x|\theta)$ on the support [0,1]. While the buyer's *ex ante type* θ is his private information, it is commonly known that θ is drawn from the distribution $F(\theta)$ with support [0,1]. In period 2, after the buyer has accepted the contract, the buyer privately observes his true valuation x. We refer to x as the buyer's *ex post type*. Finally, in period 3, the contract is implemented. We allow the seller's opportunity costs $c = c(\theta, x)$ to depend on the buyer's types.

The seller's problem is to design a contract that maximizes her expected profits. By the revelation principle for sequential games (e.g., Myerson 1986), the optimal contract can be found in the class of direct and incentive compatible contracts which, on the equilibrium path, induce the buyer to report his type truthfully. Formally, a *direct contract*

$$\gamma^{d} \equiv \{ (q^{d}(\hat{\theta}, \hat{x}), t(\hat{\theta}, \hat{x})) | (\hat{\theta}, \hat{x}) \in [0, 1]^{2} \}$$
(1)

requires the buyer to report an ex ante type θ in period 1, and an ex post type x in period 2. A contract commits the seller to a *selling schedule* $q^d(\hat{\theta}, \hat{x})$ and a *transfer schedule* $t(\hat{\theta}, \hat{x})$.

If the buyer's true ex post type is x and his period 1 report was $\hat{\theta}$, then his utility from reporting \hat{x} in period 2 is

$$\tilde{u}(\hat{x}|\hat{\theta}, x) \equiv xq^d(\hat{\theta}, \hat{x}) - t(\hat{\theta}, \hat{x}).$$
⁽²⁾

We denote the buyer's period 2 utility from truth-telling by

$$u(\theta, x) \equiv \tilde{u}(x|\theta, x). \tag{3}$$

The contract is *incentive compatible in period 2* if it gives the buyer an incentive to announce his ex post type truthfully:

$$u(\theta, x) \ge \tilde{u}(\hat{x}|\theta, x) \quad \forall \hat{x}, \theta, x.$$
(4)

If the contract is incentive compatible in period 2, the buyer announces his ex post type truthfully no matter what his report in the first period.³ Hence, if the buyer's true ex ante type is θ , then his period 1 utility from reporting $\hat{\theta}$ is

$$\tilde{U}^{d}(\hat{\theta}|\theta) \equiv \int_{0}^{1} u(\hat{\theta}|x) dG(x|\theta).$$
(5)

We denote the buyer's period 1 utility from truth-telling by

$$U^{d}(\theta) \equiv \tilde{U}^{d}(\theta|\theta). \tag{6}$$

The contract is *incentive compatible in period 1* if it induces the buyer to announce his ex ante type truthfully:

$$U^{d}(\theta) \ge \tilde{U}^{d}(\hat{\theta}|\theta) \quad \forall \hat{\theta}, \theta.$$
(7)

³Because the buyer's period 2 utility is independent of his ex ante type, a contract which is incentive compatible in period 2 automatically induces truth–telling in period 2 also off the equilibrium path, that is, if the buyer has misreported his ex ante type in period 1.

Finally, an incentive compatible contract is *ex ante individually rational* if it yields the buyer at least his outside option of zero:

$$U^{d}(\theta) \ge 0 \quad \forall \theta. \tag{8}$$

We say a contract is *feasible* if it is incentive compatible in both periods and ex ante individually rational.

The following lemma is a standard result in monopolistic screening, and we therefore omit the proof.

Lemma 1 A contract γ^d satisfies the period 2 incentive compatibility constraints (4) if and only if i) $u(\theta, x)$ is absolutely continuous in x; ii) $q^d(\theta, x)$ is increasing in x; and iii) $\partial u(\theta, x)/\partial x = q^d(\theta, x)$ for almost all x.

Since u is absolutely continuous in x, we may use integration by parts to rewrite the agent's period 1 utility as

$$\tilde{U}^{d}(\hat{\theta}|\theta) = \int_{0}^{1} u(\hat{\theta}, x) dG(x|\theta) = \int_{0}^{1} q^{d}(\hat{\theta}, x) [1 - G(x|\theta)] dx + u(\hat{\theta}, 0).$$
(9)

The seller's payoff from a feasible contract is the difference between aggregate surplus and the buyer's utility. That is, if the buyer's ex ante type is θ , the seller's conditional expected payoff, conditional on θ , is

$$W^{d}(\theta) = \int_{0}^{1} [(x - c(\theta, x))q^{d}(\theta, x) - u(\theta, x)] dG(x|\theta).$$
(10)

Using (9), we can rewrite the seller's payoff as

$$W^{d}(\theta) = \int_{0}^{1} \left[x - c(\theta, x) - \frac{1 - G(x|\theta)}{g(x|\theta)} \right] q^{d}(\theta, x) dG(x|\theta) - u(\theta, 0).$$
(11)

The seller's problem is therefore to find a selling schedule q^d and utility levels $u(\cdot, 0)$ for the buyer's lowest ex post type that solves the following maximization problem:

$$\mathcal{P}^{d}: \max_{q^{d}(\theta, x), u(\theta, 0)} \int_{0}^{1} W^{d}(\theta) dF(\theta) \quad \text{s.t.}$$

$$q^{d}(\theta, x) \text{ increasing in } x, \ q^{d}(\theta, x) \in [0, 1],$$

$$U^{d}(\theta) \ge \tilde{U}^{d}(\hat{\theta}|\theta),$$

$$U^{d}(\theta) \ge 0.$$

2.2 A general static screening problem

We now specify a general static screening problem that is based on the formulation in Fudenberg and Tirole (1991), but explicitly allows for stochastic contracts. In particular, we consider a principal and a privately informed agent who can trade some quantity $x \in [0, 1]$. An allocation specifies a, possibly stochastic, quantity x to be traded and a transfer $t \in \mathbf{R}$ from the agent to the principal. Before the principal offers a contract, the agent privately learns his *type* $\theta \in [0, 1]$, which is drawn from a distribution $F(\theta)$ with support [0, 1]. Given a type θ and an allocation (x, t), the principal's utility is $S(\theta, x) + t$, and the agent's utility is $V(\theta, x) - t$. Hence, as in Fudenberg and Tirole (1991), our specification allows for arbitrary quasi-linear utility functions, including the interdependent value case where the principal's utility depends directly on the agent's type.

Applying the revelation principle, the principal offers the agent a direct contract

$$\gamma^{s} = \{ (q^{s}(\hat{\theta}, x), t(\hat{\theta})) | \hat{\theta} \in [0, 1] \}.$$
(12)

We explicitly allow the principal to propose a contract with a stochastic quantity schedule. Therefore, $q^s(\theta, x)$ represents a cumulative distribution function (cdf) with the interpretation that, if the agent reports θ , then the probability that the quantity traded is at least x is $q^s(\theta, x)$. Consequently, $q^s(\theta, x)$ is positive and increasing in x. It will be convenient to define the unit interval as the domain of $q^s(\theta, \cdot)$ so that $q^s(\theta, 1) = 1$. We explicitly allow for mass points so that $q^s(\theta, \cdot)$ is not necessarily continuous. In particular, if q^s has a mass point in x = 0, then $q^s(\theta, 0) > 0$.

Hence, a contract γ^s yields an agent with type θ who reports $\hat{\theta}$, the expected utility

$$\tilde{U}^{s}(\hat{\theta}|\theta) \equiv V(\theta,0)q^{s}(\hat{\theta},0) + \int_{0}^{1} V(\theta,x)dq^{s}(\hat{\theta},x) - t(\hat{\theta}).$$
(13)

The first term on the right hand side accounts for the possible mass point in x = 0, and the integral is the expectation over the remaining mass and corresponds to the (Riemann–Stieltjes) integral with respect to the function $q^s(\hat{\theta}, \cdot)$. We denote agent type θ 's expected utility from truth–telling by

$$U^{s}(\theta) \equiv \tilde{U}^{s}(\theta|\theta). \tag{14}$$

A contract is feasible if it is incentive compatible, that is,

$$U^{s}(\theta) \ge \tilde{U}^{s}(\hat{\theta}|\theta) \quad \forall \hat{\theta}, \theta \tag{15}$$

and individually rational, that is,

$$U^{s}(\theta) \ge 0 \quad \forall \theta. \tag{16}$$

The principal's expected utility from a feasible contract is

$$W^{s}(\theta) \equiv S(\theta, 0)q^{s}(\theta, 0) + \int_{0}^{1} S(\theta, x)dq^{s}(\theta, x) + t(\theta).$$
(17)

Consequently, an optimal contract (q^s, t) in the static principal agent problem solves

$$\mathcal{P}^{s}: \max_{q^{s}(\cdot,\cdot),t(\cdot)} \int_{0}^{1} W^{s}(\theta) dF(\theta) \quad \text{s.t.}$$

$$q^{s}(\theta, x) \text{ increasing in } x, \ q^{s}(\theta, x) \in [0, 1], q^{s}(\theta, 1) = 1,$$

$$U^{s}(\theta) \ge \tilde{U}^{s}(\hat{\theta}|\theta),$$

$$U^{s}(\theta) \ge 0,$$

where the first constraint expresses the fact that $q^{s}(\theta, \cdot)$ is a cdf on [0, 1].

2.3 Equivalence result

We are now in the position to show our main result and formalize the sense in which both models are equivalent. We first argue that any sequential screening model corresponds to an appropriately defined static screening problem. Before stating this result, note that any selling schedule q^d in the sequential model corresponds to a stochastic trading schedule in the static model.⁴

Proposition 1 Suppose $(q^d, u(\cdot, 0))$ is a solution to \mathscr{P}^d . Then $(q^s, t(\cdot)) = (q^d, -u(\cdot, 0))$ is solution to \mathscr{P}^s , where

$$V(\theta, x) = \int_{x}^{1} 1 - G(z|\theta) dz, \qquad (18)$$

$$S(\theta, x) = \int_{x}^{1} (z - c(\theta, x))g(z|\theta) - [1 - G(z|\theta)]dz.$$
(19)

Proof of Proposition 1: We show that for $t(\theta) = -u(\cdot, 0)$, and for *V* and *S* defined in (18) and (19):

$$\tilde{U}^{s}(\hat{\theta}|\theta) = \tilde{U}^{d}(\hat{\theta}|\theta), \quad and \quad W^{s}(\theta) = W^{d}(\theta).$$
 (20)

This implies that \mathcal{P}^s and \mathcal{P}^d are equivalent and thus the solutions coincide.

⁴In the case that the dynamic schedule q^d is strictly smaller than 1 at x = 1, we may, without loss of generality, replace it with the schedule that coincides with q^d for all $x \in [0, 1)$ and equals 1 for x = 1.

To see (20), observe that $V(\theta, x)$ is a decreasing function in x with $dV = -(1 - G(x|\theta))dx$. Hence, we can write (9) as a Riemann–Stieltjes integral with respect to V:

$$\tilde{U}^{d}(\hat{\theta}|\theta) = -\int_{0}^{1} q^{d}(\hat{\theta}, x) dV(\theta, x) + u(\hat{\theta}, 0).$$
(21)

Applying integration by parts for Riemann-Stieltjes integrals, we obtain

$$\tilde{U}^{d}(\hat{\theta}|\theta) = -q^{d}(\hat{\theta}, x)V(\theta, x)\Big|_{0}^{1} + \int_{0}^{1} V(\theta, x)dq^{d}(\hat{\theta}, x) + u(\hat{\theta}, 0)$$
(22)

$$= q^{d}(\hat{\theta}, 0)V(\theta, 0) + \int_{0}^{1} V(\theta, x)dq^{d}(\hat{\theta}, x) + u(\hat{\theta}, 0)$$
(23)

$$= q^{s}(\hat{\theta},0)V(\theta,0) + \int_{0}^{1} V(\theta,x)dq^{s}(\hat{\theta},x) - t(\hat{\theta})$$
(24)

$$= \tilde{U}^{s}(\hat{\theta}|\theta), \tag{25}$$

Q.E.D.

where in the second line, we have used that $V(\theta, 1) = 0$, and in the third line we have used the definitions of $q^s(\hat{\theta}, x)$ and $t(\hat{\theta})$ in the statement of the proposition.

The proof that $W^{s}(\theta) = W^{d}(\theta)$ is analogous.

Next, we state a reverse of Proposition 1 which specifies conditions under which a static screening problem corresponds to a sequential screening problem. To state this result, note that any stochastic trading schedule q^s corresponds to a selling schedule in the sequential model.

Proposition 2 Suppose $(q^s, t(\cdot))$ is a solution to \mathcal{P}^s . If

$$V_x(\theta, 0) = -1; \quad V_x(\theta, 1) = 0; \quad V_{xx} \ge 0,$$
 (26)

then $(q^d, u(\cdot, 0)) = (q^s, -t(\cdot))$ is a solution to \mathscr{P}^s , where

$$G(x|\theta) = V_x(\theta, x) + 1, \tag{27}$$

$$c(\theta, x) = x - \frac{V_x(\theta, x)}{V_{xx}(\theta, x)} + \frac{S_x(\theta, x)}{V_{xx}(\theta, x)}.$$
(28)

Proof of Proposition 2: Observe first that $G(x|\theta)$ as defined in (27) is a cumulative distribution function by the properties in (26). Next, the same argument as in the proof of Proposition 1 imply that the solutions coincide if (18) and (19) hold. Using (27) and (28), this, however, follows from a straightforward computation. Q.E.D.

3 Application of equivalence result

3.1 Single-Crossing

It is well–known that in the static screening problem, a key role is played by the so-called single– crossing condition⁵ which requires that the agent's marginal utility with respect to the allocation is monotone in his type. In our formulation, this means that V_{θ_x} is of constant sign, or equivalently, that the derivative V_x is monotone in θ . The single-crossing condition ensures that the so-called "first order approach" is valid, that is, it is sufficient to solve a "local problem" which requires only that truth-telling is a local optimum.

Using Proposition 1, we can pinpoint precisely the counterpart of the single-crossing condition in the sequential screening problem. Since in the static counterpart of the sequential problem, the agent's utility function is $V(\theta, x) = \int_x^1 1 - G(z|\theta) dz$, the single-crossing condition means that $V_{x\theta} = \partial G(x|\theta)/\partial \theta$ is of constant sign. In other words, the collection of conditional distributions $\{G(x|\theta)|\theta \in [0,1]\}$ is ordered in the sense of first order stochastic dominance (FOSD).

Indeed, Courty and Li (2000) demonstrate that if the conditional distributions can be FOSD ranked, then the sequential screening problem can be solved by the first order approach. Our result illuminates the deeper reason for this. Reversely, Proposition 1 also makes clear that a sequential screening problem where the conditional distributions cannot be FOSD ranked, corresponds to a static screening problem without single-crossing, as discussed, e.g., in Araujo and Moreira (2010). One approach in such a case is to identify subdomains of the space of types and allocations on which the single-crossing condition holds and to provide conditions under which the solution falls into one such domain. In fact, for the alternative specification in which the conditional distributions are ordered in a sense of second order stochastic dominance (SOSD), Courty and Li (2000) do provide a solution to the sequential screening problem. Their approach is precisely to show that the solution they identify for the local problem falls into a domain where their SOSD implies the FOSD ranking, that is, where the single-crossing condition holds.

3.2 Characterization of optimal contracts

Proposition 1 is potentially useful for applications of sequential screening models because it implies that the solution to any sequential screening problem can be found by solving the corresponding static problem. Notice, however, that solving the static problem might not be straightforward

⁵This condition is also referred to as "sorting", "constant sign", or "Spence-Mirrlees" condition.

because our equivalence requires that we explicitly allow for *stochastic* selling schedules $q^s(\theta, x)$ in the static problem. In contrast, most treatments of the static principal agent problem, including Fudenberg and Tirole (1991), characterize optimal contracts only within the restricted class of *deterministic* selling schedules. Proposition 1 is therefore most useful under conditions where the corresponding static problem is known to exhibit a deterministic solution. We proceed by discussing such conditions.

In our framework, a selling schedule $q^s(\theta, x)$ for the static problem is deterministic if it corresponds to a degenerate distribution function which places mass 1 on a distinct quantity $x^s(\theta)$. This means that there exists a function $x^s : [0, 1] \rightarrow [0, 1]$ such that

$$q^{s}(\theta, x) = \mathbf{1}_{[x^{s}(\theta), 1]}(x), \tag{29}$$

where $\mathbf{1}_A(x)$ expresses the indicator function, which equals 1 if $x \in A$ and 0 otherwise. Thus, we can identify a deterministic schedule $q^s(\theta, x)$ with its associated function $x^s(\theta)$. Proposition 1 therefore implies the following corollary.

Corollary 1 If there is a solution to \mathscr{P}^s that exhibits a deterministic selling schedule $x^s(\theta)$, then there is a solution to \mathscr{P}^d which exhibits the selling schedule q^d that is equal to the step function $q^d(\theta, x) = \mathbf{1}_{[x^s(\theta), 1]}(x)$.

Strausz (2006) explicitly addresses the question when a deterministic contract is optimal in a static principal agent problem and shows that this is the case if a deterministic solution to the "local" version of problem \mathcal{P}^s , where only the local incentive constraints are imposed, automatically satisfies all omitted global constraints.⁶

In a context where the single-crossing condition holds, this is the case when the optimal deterministic solution to the local problem displays a schedule $x^{s}(\theta)$ which is monotone in the type θ , that is, it does not involve "bunching". Applying standard steps (see, e.g., Fudenberg and Tirole, 1991), it can be shown that the schedule $x^{s}(\theta)$ is the point-wise maximizer of the virtual surplus function

$$Z(\theta, x) = S(\theta, x) + V(\theta, x) - \frac{1 - F(\theta)}{f(\theta)} V_{\theta}(\theta, x).$$
(30)

⁶Even though Strausz (2006) formally derives this result for discrete type's θ , the result can be extend to continuous type spaces under the usual conditions which ensure that the envelope theorem holds (see for instance Pavan et al. 2014).

Therefore, if $x^{s}(\theta)$ is monotone, it follows from Strausz (2006) that $x^{s}(\theta)$ is also optimal in the larger class of stochastic selling schedules.

We now apply these considerations to the sequential screening problem. By Proposition 1, the virtual surplus of the corresponding static principal agent problem rewrites as

$$Z(\theta, x) = \int_{x}^{1} (z - c(\theta, z))g(z|\theta)dz + \frac{1 - F(\theta)}{f(\theta)} \int_{x}^{1} G_{\theta}(z|\theta)dz.$$
(31)

An interior maximizer $x^{s}(\theta)$ satisfies the first-order condition $Z_{x}(\theta, x^{s}(\theta)) = 0$, or, equivalently,

$$\phi(\theta, x^{s}(\theta)) = 0, \tag{32}$$

where

$$\phi(\theta, x) = x - c(\theta, x) + \frac{1 - F(\theta)}{f(\theta)} \frac{G_{\theta}(x|\theta)}{g(x|\theta)}.$$
(33)

Therefore, if ϕ is monotone in both arguments, $x^{s}(\theta)$ is monotone, and Corollary 1 implies that $q^{d}(\theta, x) = \mathbf{1}_{[x^{s}(\theta), 1]}(x)$ is a solution to the sequential screening problem.

Indeed, the monotonicity of ϕ in both arguments is precisely the regularity condition that Courty and Li (2000) identify to show that the solution to (32) represents the optimal selling schedule in the sequential screening problem \mathcal{P}^d .

3.3 Characterization of Incentive Compatibility

An appealing property of the static screening problem, both aesthetically and practically, is that when the single-crossing condition holds, the incentive compatibility constraints (14) can be *characterized* in terms of monotonicity of the selling schedule and a "revenue equivalence" formula which pins down the marginal utility of the agent as a function of the selling schedule alone. Crucially, this characterization holds only for the class of deterministic contracts with selling schedules as in (29) and says that a contract is incentive compatible if and only if $x^{s}(\theta)$ is monotone in θ and delivers the agent the marginal utility (with respect to θ) $U^{s'}(\theta) = V_{\theta}(\theta, x^{s}(\theta))$.

Proposition 1—or rather, the proof of Proposition 1, which establishes that the agent's expected utility in the sequential and the static models coincide—implies that in the sequential model, period 1 incentive compatibility (7) can, for deterministic contracts, be characterized in a corresponding manner. More specifically, a deterministic contract with selling schedule $q^d(\theta, x) = \mathbf{1}_{[x^s(\theta), 1]}(x)$ in the sequential model is period 1 incentive compatible if and only if

 $x^{s}(\theta)$ is monotone in θ , that is, q^{d} is monotone in both arguments, and the agent's marginal utility is $U^{d'}(\theta) = V_{\theta}(\theta, x^{s}(\theta)) = \int_{x^{s}(\theta)}^{1} 1 - G_{\theta}(z|\theta) dz$.⁷

For stochastic contracts, it is well–known that an analogous characterization is not available. In particular, while incentive compatibility does still pin down the agent's marginal utility, it does *not* imply that the schedules q^s or q^d are monotone in both arguments. In fact, as Strausz (2006) demonstrates, the benefit of using stochastic contract lies precisely in the leeway they provide to relax monotonicity.

4 Conclusion

We show that the sequential and the standard static screening model are essentially equivalent and and we we show that a number of salient features of the sequential model correspond to well–understood features of the static model. A question that we pursue in the future is to what extent this equivalence extends to richer dynamic models with a longer time horizon and multiple trading periods.

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