A MODEL OF PRICE DISCRIMINATION UNDER LOSS AVERSION AND STATE-CONTINGENT REFERENCE POINTS

JUAN CARLOS CARBAJAL AND JEFFREY C. ELY

Abstract. We study optimal price discrimination by a monopolist who faces a continuum of consumers with heterogeneous tastes and reference-dependent preferences. A consumer's total valuation for product quality is composed of an intrinsic consumption valuation, which is affected by a privately known state signal, and a gain-loss valuation that depends on deviations of purchased quality from a state-contingent reference quality level. Buyers are loss-averse, so that deviations from their reference levels are evaluated differently depending on whether they are gains or losses. As Kőszegi and Rabin (2006), we let gains and losses relative to the reference point be evaluated in terms of changes in the consumption valuation, but we differ in terms of how comparisons take place. In particular, because purchasing decisions take place after the state signals have been observed, each buyer evaluates consumption outcomes relative to his state-contingent reference quality level. We consider different ways in which reference quality levels are formed, capturing this process by a reference plan, and derive optimal contract menus for monotone reference consumption plans. The novel effects for product line design due to the interaction between loss aversion and incomplete information are thoroughly studied, with special emphasis on self-confirming contract menus, in which the quality levels offered by the firm coincides with the reference quality levels. We characterize the firm's unique preferred self-confirming contract menu and specify conditions under which it also constitutes the consumers' preferred self-confirming menu.

1. Introduction

Traditionally, the design of the product line has been seen as an attempt from firms to improve margins by discriminating among consumers with different willingness to pay. In recent years, a growing literature points out various other ways in which contract menus may affect consumers' choices. Kamenica (2008) argues that some context effects —e.g., the compromise effect described by Simonson (1989)— can be explained by the informational content a product line conveys to uninformed buyers: a firm introduces a "premium loss leader" to manipulate consumers' beliefs about the product characteristics and increase the demand for less expensive goods. But even in settings where consumers have less problems assessing their intrinsic valuation for a good, context effects may arise if buyers care about comparisons between different available options in the product line —as in Orhun (2009)— or comparisons between available options and subjective beliefs about consumption outcomes that act as reference points. In the latter case, the contract menu

Date: October 2013.

Key words and phrases. Price discrimination, product line design, loss aversion, reference-dependent preferences, reference consumption plan, self-confirming contracts.

This paper was previously circulated as *Optimal contracts for loss-averse consumers*. We believe the new title describes better the contents and message of the paper. We wish to thank Jürgen Eichberger, Gabriele Gratton, Paul Heidhues, Fabian Herweg, Heiko Karle, Jihong Lee, Konrad Mierendorff, Ludovic Renou, Antonio Rosato, Klaus M. Schmidt and seminar participants at ANU, CORE, Maastricht University, Humboldt University–Berlin, Universities of Zurich, Heidelberg, Munich and New South Wales, NASM–ES 2012, AETW 2012, SITE 2012, and AMES 2013 for helpful comments. Early parts of this work was conducted while Carbajal was visiting the Department of Economics, University of Bonn; he wishes to thank Benny Moldovanu for his gracious hospitality. Research assistance was provided by Alexandra Bratanova and Kim Nguyen. Financial support from the Australian Research Council (Carbajal) and the National Science Foundation (Ely) is gratefully acknowledged.

not only responds to but also shapes those beliefs. This may be in place in the design of a luxury brand product line whose most exclusive items serve as what some commentators refer to as "anchors". Thus, a premium leader is not introduced to manipulate consumers' beliefs about the product characteristics, but instead to manipulate consumers' subjective expectations about their own future consumption.[1]

To help fix some relevant ideas, consider the following hypothetical situation. An executive of an multinational company is planning to buy himself a gift to celebrate a successful year before the company announces the next round of senior staff relocation. He has decided to spend up to a certain portion of his bonus in expensive personal accessories, and is aware that potential destinations include Chicago and Lima. After spending some time looking at catalogues, reviews and advertisements, he is convinced that nothing but the exclusive platinum Grand Complications Patek Philippe watch would be fashionable in Chicago, but the less expensive and more practical stainless steel Aquanaut Patek Philippe watch would be appropriate for Lima. By the time he manages to do his shopping, the company has already announced he is to move to Chicago, and therefore his own expectations are to purchase the platinum watch. Buying the stainless steel model instead may leave a feeling of disappointment, as our executive will compare it with what he was expecting to consume in the event of being transferred to Chicago.

The key feature of the above example is the presence of different states that influence both consumers' willingness to pay for a certain good and their subjective expectations of future consumption. A reasonable conjecture is that these expectations can be manipulated, to some degree, by the design of the product line, and firms may find profitable to do so when consumers compare their actual purchasing decisions with their expected consumption levels. How shall a firm specify its optimal contract menu under these circumstances? To investigate these issues, we propose a model of monopoly price discrimination when consumers' preferences are heterogeneous and exhibit loss aversion. In our model, presented in Section 2, there is a state parameter, lying in an interval of the real line, that affects both consumers taste for quality and their expectations of future consumption.[2] To be more precise, we assume that a consumer's purchasing decision is determined by his intrinsic taste for product quality and, in addition, by comparisons between the offered quality level and a reference quality level. Moreover, both the willingness to pay for quality and the reference quality level are determined by the realization of the state parameter θ, which we also refer to as the consumer's type. Thus, after observing his type, a potential customer enters the market anticipating a certain reference consumption level and experiences gains or losses relative to his state-specific consumption utility according to whether his chosen quality exceeds or falls short of the state-contingent reference point. The monopolist's optimal product line design takes consumers' reference dependence into account and reflects the interaction between loss aversion and the traditional rent extraction and incentive compatibility tradeoff.

The way we allow the state parameter to determine willingness to pay for quality is standard (so, for instance, the single-crossing condition is satisfied). To model the interaction between the state parameter and the reference point, we assume the existence of a *reference plan*; i.e., a weakly monotone increasing function mapping types into reference quality levels. This way, a higher type signifies a higher willingness to pay *and* a higher

[1]Christina Binkley, reporting for *The Wall Street Journal* (The Psychology of the $14,000 Handbag, August 9, 2007), quotes "retail-consulting guru Paco Underhill" explaining how this strategy dates back to the 17th century: "You sold one thing to the king, but everyone in court had to have a lesser one."

[2]Our model builds on classic monopoly pricing models under asymmetric information developed by Mussa and Rosen (1978) and Maskin and Riley (1984).

reference quality level.[3] We then derive the optimal contract menu for any such reference plan. Our approach enables comparative statics analysis of the offered product line as well as monopoly profits arising from changes in the level and shape of the reference plan, for example due to targeted advertising, fashion and product shows, and so forth. Importantly, we assume that consumers can always shun the firm's offers and opt instead to buy an inexpensive substitute good of minimal quality in a secondary market: if our executive is relocated to Lima and, despite his expectations, decides against the $14,000 stainless steel watch, he nonetheless purchases the $500 knockoff. Why would a consumer feel a loss when he opts for the outside option? Because the realization of a particular state, and the consequent anticipation of consumption, creates what Ariely and Simonson (2003) called a "pseudo-endowment effect".

In Section 3 we analyze the benchmark model in which the realization of the state parameter is observed by the monopolist. We show how loss aversion invites upward distortions in the offered quality levels relative to the loss-neutral complete information case. This occurs, in particular, when buyers enter the market with high reference quality levels. In this case, the first-best quality would fall short of the reference level and the loss-averse consumer is willing to pay a premium to reduce the associated loss. The monopolist exploits this by increasing both quality and price until either these marginal gains are exhausted or the offered quality hits the reference level, shutting down any further gains. In particular, over a wide range of states, profit maximization implies matching reference qualities exactly. A similar logic drives the comparative statics results under complete information. If the monopolist is able, say via advertising or from historical product offerings, to inflate consumers' reference levels, then the effect would magnify upward distortions in the product line and increase revenue. While an empirical demonstration of inefficiently high quality offers by firms may be challenging,[4] our predictions here could be used to indirectly test for loss aversion in the laboratory: everything equal, loss-averse consumers will respond to higher reference consumption points differently than loss-neutral consumers.[5]

In Section 4 we turn to our full model in which the realization of the state parameter is privately known to each consumer and unobservable by the monopolist. A contract menu is now feasible only if it satisfies the self-selection incentive compatibility and participation constraints. We study the novel ways in which these constraints interact with consumption loss aversion. Two new effects emerge, compared to the complete information benchmark. First, the marginal profitability of increased quality is reduced due to incentive issues familiar from traditional models of monopoly price discrimination. Higher quality levels are more attractive to a θ-consumer with low willingness to pay, but also more attractive to consumers with more favorable state signals, to whom the monopolist was hoping to sell an even higher quality product. The increase in revenues from the θ-consumer will be offset by information rents ceded to these consumers, to discourage them from choosing the quality product designed for the θ-consumer. Reference dependence enters, and modifies this standard tradeoff because the higher taste for quality implies that for any given quality and reference levels, the higher type consumer experiences a larger loss (or smaller gain)

[3]Monotonicity on the reference plan restricts how wrong buyers are allowed to be: a consumer with a low state signal, and consequently low intrinsic valuation, may expect to buy as much quality as, but not more than, a consumer with a high state signal.

[4]Yet it is hard to justify, purely in terms of welfare considerations, products like the mechanical stop watch Carrera Mikrograph Chronograph offered by Tag Heuer, which measures time at the 1/100th second precision.

[5]These predictions are consistent with the findings reported by Heyman, Orhun, and Ariely (2004), who tested the pseudo-endownment effect in experimental auctions to explain multiple bidding in online auctions. One explanation they advanced is that the anticipation of ownership increases the valuation of the object from its initial level.

than the θ-consumer. Thus, it is possible for the firm to increase profits by expanding its product line to *both high and low* ends of the market, in response to (or in anticipation of) consumers' high expectations.[6] This loss aversion effect can also account for three part tariffs and other complex contract schemes that have become increasingly popular among mobile phone operators, Internet providers, and other subscription services,[7] when for instance, low type consumers overestimate usage prior to choosing a contract.

The second effect is a novel distortion due entirely to loss aversion and has no counterpart in loss-neutral screening model. Consider a monopolist contemplating increasing the quality q offered to a given θ-consumer from just below his state-contingent reference level to just above it. Such a change has a discrete effect on the attractiveness of q to consumers who received higher signals, thus have higher willingness to pay, but whose reference levels are relatively similar. As a result, the monopolist would incur a discrete drop in profits due to information rents, were it to boost quality levels above the reference point of the θ-consumer. We quantify this *lump-sum* incentive cost and show its effects on the optimal product line and comparative statics. It implies an additional downward distortion in quality levels to consumers who would otherwise be offered products that surpass their reference points. It also implies that increased reference qualities may actually decrease offered qualities, so the interaction between reference quality levels and offered qualities under loss aversion and incomplete information is quite complex. Finally, it impacts the profitability of shaping reference points through, for example, advertising campaigns. Parallel shifts upward in reference plans always increase profits, but changes in the shape of the reference plan can have ambiguous effects. We illustrate these issues in a simple example with uniform signals, linear valuations, and quadratic cost.

The fact that the reference plan determines to a great extent qualitative features of the optimal product line leads to the question of belief manipulation, both on the part of the firm and on the part of the consumers. This issue is explored in Section 5, where we impose a consistency requirement on admissible reference plans. Specifically, we focus on *self-confirming* reference consumption plans; i.e., subjective beliefs about future consumption outcomes that coincide, in equilibrium, with the actual outcomes. Thus, our model allows us to analyze the consequences of reference plans as correct endogenous beliefs in optimal product line design. We find that, while the set of optimal self-confirming contracts is strictly smaller than the set of optimal contracts, it nonetheless contains quality schedules with most of the features described in the previous paragraphs. On the other hand, it is surprisingly simple to show that there exists a unique self-confirming contract menu that is preferred by the firm. Indeed, in any given self-confirming product line, each consumer buys his state-contingent reference quality level. Since a higher reference plan weakly increases the net total willingness to pay, as it makes the external, minimal quality option less desirable (due, again, to the pseudo-endowment effect), it follows that profits are increasing in the reference plan. Thus, the preferred self-confirming product line for the firm usually excludes fewer consumers from the market and has quality levels distorted upward from second best levels under loss neutrality to the levels that maximize net virtual surplus for loss-averse consumers. While these upward quality distortions improve allocative efficiency for low and intermediate type consumers, buyers with high state parameters end up purchasing overly sophisticated goods.

In practice, firms can manipulate reference points through advertising and other marketing practices prior to the introduction of the product line into the market.[8] Marketing

[6] So, in addition to overly sophisticated and exclusive items, luxury brand product lines include minor accessories: keychains, bookmarks, and phone straps.

[7] See for example Lambrecht and Skiera (2006) and Lambrecht, Seim, and Skiera (2007).

[8] Product shows in anticipation of market entrance are standard practice in, among others, the luxury goods, cars, and consumer electronics industries.

efforts will be credible as long as they promote optimal self-confirming product lines. A higher self-confirming reference plan never decreases the firm's expected profits or offered qualities, but it may or may not hurt consumer's surplus —note we are comparing how different self-confirming reference plans affect loss averse consumers. This is because a higher reference quality plan implies higher quality offers for (potentially more) active consumers, which translates into more information rents assigned to buyers with positive consumption levels. Yet, higher reference quality levels may be detrimental because, by diminishing the value of the outside option, they increase the net willingness to pay of active consumers.[9] We specify conditions under which the positive information rents effect associated with a higher self-confirming reference plan dominates the negative participation effect. In this case, the firm's preferred self-confirming contract menu is also the consumer's preferred contract menu. This result relies on two assumptions. First, a higher state parameter implies higher marginal intrinsic consumption valuation for quality due to single-crossing, but these effects are diminishing in the quality dimension. Second, consumers have quasi-linear utilities for the good offered by the firm and face no budgetary restrictions.

Section 6 offers some concluding remarks. Omitted proofs from the text are gathered in Section 7.

Related literature

Our work contributes to the line of inquiry pioneered by DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006), and Heidhues and Kőszegi (2008), among others, that investigates how profit maximizing firms operate in a market context where consumers have systematic deviations from traditional preferences. Galperti (2011) and Grubb (2009) are additional recent contributions.

Following Kőszegi and Rabin (2006), we model the consumer's gain-loss valuation in terms of differences in the consumption valuation. Importantly, we differ in how comparisons take place. In some economic contexts it is reasonable to assume, as Kőszegi and Rabin (2006) and Heidhues and Kőszegi (2012), that all buyers share an ex ante stochastic reference point and evaluate each realization of stochastic consumption with each potential realization of the reference point. However, in other situations —for instance, when heterogeneous consumers face a discriminating monopolist and make purchasing decisions at the interim stage— it may be more appropriate to let each buyer assess his quality consumption relative to his state-contingent reference quality level, and this is the approach we follow here. Our work in this respect is closer to Sugden (2003); see also De Giorgi and Post (2011). Recent papers that follow Kőszegi and Rabin's (2006) approach include Rosato (2013), who studies how bait-and-switch tactics manipulate reference points and raise profits even when consumers rationally expect the bait and switch; Hahn, Kim, Kim, and Lee (2012), who study non-linear pricing when consumers form reference points at an ex ante stage before learning their valuation but anticipating the eventual type-dependent consumption (thus consumers have a type-independent stochastic reference point); and Eisenhuth (2012), who looks at the optimal auction for bidders with expectations-based reference points —see also Lange and Ratan (2010).

Our work is related to Orhun (2009), who considers a two-type model in which the reference point of the high-type consumer is influenced by the quality offered to the low-type consumer and vice versa. We consider optimal product line design in response to an arbitrary, and fixed, reference plan. This enables us to study the incentives of the monopolist to manipulate reference points, perhaps via advertising. Karle (2013) studies advertising to loss-averse consumers using a model of expectations-based reference point formation with stochastic consumption a la Kőszegi and Rabin (2006). In his model,

[9]We thank an anonymous referee for pointing out this effect, which was previously omitted from our analysis.

advertising creates uncertainty about future consumption and this impacts reference point formation, whereas the logic of our model indicates that the firm will try to drive up each consumer's reference quality level unambiguously.

Throughout this paper, we model reference points in terms of quality levels, departing from recent work in the area such as Herweg and Mierendorff (2013) and Spiegler (2011), that specifies reference points in terms of prices. Despite the evidence supporting the existence of reference price effects on consumer behavior, empirical work from the marketing literature suggests that loss aversion on product quality is at least as important as, if not more important than, loss aversion in prices.[10] This point is also suggested by experimental data reported by Fogel, Lovallo, and Caringal (2004), who confirm the existence of loss aversion for quality in a laboratory setting, and Novemsky and Kahneman (2005), who stress that there is no loss aversion for monetary transactions that are expected to occur and thus accounted for (in the sense of Thaler (1985), for instance). In our setting —as in the motivating example— consumers are not subject to meaningful budgetary restrictions, so we treat potential expenses on the good in question as part of a budget established for transaction purposes. Thus, price differences among goods of various qualities are evaluated solely in terms of willingness to pay and do not register neither as gains nor as losses.

2. Product Line Design under Reference-Dependent Preferences

In this section we lay out the notation and main assumptions of our model, which builds on work by Mussa and Rosen (1978) and Maskin and Riley (1984), among others. In our framework, a consumer derives utility from consumption and, in addition, from comparisons between actual consumption and a state-contingent reference point. Following Tversky and Kahneman (1991), we consider loss-averse consumers.

2.1. **The firm.** A revenue-maximizing monopolist produces a good of different characteristics captured by the product attribute parameter $q \geq 0$. This parameter can be interpreted as either a one-dimensional measure of quality (exclusive features in a luxury product line) or quantity (amount of data offered by a mobile operator). We follow Mussa and Rosen (1978) and maintain the first interpretation, thus referring to q as the *quality* attribute.

The cost of producing one unit of the good with quality q is represented by $c(q)$, where the *cost function* $q \mapsto c(q)$ is non-decreasing, twice continuously differentiable, and satisfies $c(0) = 0$. In addition, we assume that there exists a positive number ϵ such that $c_{qq}(q) \geq \epsilon > 0$ for all quality levels.[11] The firm's problem is to design a revenue maximizing product line (i.e., a contract menu of posted quality–price pairs) to offer to potential buyers with differentiated demands.

2.2. **Consumers.** There is a population of consumers with quasi-linear preferences and unit demands for the good offered by the firm, but heterogeneous tastes for the product attribute. Preference heterogeneity depends on a state parameter (type) $\theta \in \Theta = [\theta_L, \theta_H]$, where $0 < \theta_L < \theta_H < +\infty$. The realization of the state signal is private information, so that the firm only knows the distribution F, with full support on Θ and density $f > 0$. We assume that the *inverse hazard rate* $\theta \mapsto h(\theta) \equiv (1 - F(\theta))/f(\theta)$ is non-increasing and twice continuously differentiable.

---

[10]See Hardie, Johnson, and Fader (1993) for instance. Bell and Lattin (2000) suggest that accounting for heterogeneity in consumers' price response significantly lowers previous estimates of the loss aversion price-coefficient. Their work does not consider estimates for loss aversion in product attributes.

[11]Throughout the paper we use subindices to denote (partial) derivatives. Therefore c_{qq} positive and bounded away from zero means that the cost function is strongly convex.

A θ-consumer has a *consumption valuation* for quality $q \geq 0$ given by $m(q,\theta)$. To avoid any complication arising from the classical screening framework, we impose the following regularity assumptions: (a) $(q,\theta) \mapsto m(q,\theta)$ is twice continuously differentiable; (b) for every $\theta \in \Theta$, the function $q \mapsto m(q,\theta)$ is strictly increasing and concave, with $m(0,\theta) = 0$; (c) for every quality level $q \geq 0$, the function $\theta \mapsto m(q,\theta)$ is non-decreasing, with $m_\theta(0,\theta) = 0$; (d) for all pairs (q,θ), $m_{q\theta}(q,\theta) > 0$, so that the single-crossing condition holds. In addition, we assume that for all types θ the following holds:

$$\lim_{q\to 0} \big(m_q(q,\theta) - c_q(q)\big) > 0 \quad \text{and} \quad \lim_{q\to+\infty} \big(m_q(q,\theta) - c_q(q)\big) < 0.$$

This specification includes, among others, Mussa and Rosen's (1978) monopoly model with linear consumption valuation and quadratic costs. It ensures that the quality schedule that maximizes consumption surplus is continuously differentiable on Θ and non-decreasing in types. An alternative to accommodate Maskin and Riley's (1984) model is to impose strong concavity of m and convexity of c.

We step aside from standard monopoly pricing theory and consider buyers who exhibit reference-dependent preferences for the product attribute (but not its price). Specifically, we assume that in addition to his consumption valuation, a θ-consumer derives utility from comparing quality q to a type-contingent reference quality level $r(\theta)$. A reference quality level may encompass different concepts: it can reflect (in)correct subjective expectations of future consumption; it may be determined by past experiences or by current aspirational considerations, etc. At this stage, it is convenient to study a general reference formation process, which we capture by the *reference consumption plan* $\theta \mapsto r(\theta) \geq 0$. We assume that this reference plan is non-decreasing, continuous and piecewise continuously differentiable, with bounded left and right derivatives for all state signals $\theta \in \Theta$.

Following Kőszegi and Rabin (2006), we let comparisons between consumption outcomes and reference points be evaluated in terms of changes in the consumption valuation. On the other hand, we assume that each buyer assesses quality consumption relative to his own state-contingent reference quality level. Thus, after observing state parameter θ, a consumer has a *gain-loss valuation* for q given by $\mu \times (m(q,\theta) - m(r(\theta),\theta))$, where $\mu = \eta$ if $q > r(\theta)$ and $\mu = \eta\lambda$ if $q \leq r(\theta)$. The parameter $\eta > 0$ is the weight attached to the gain-loss valuation and the parameter $\lambda \geq 1$ is the loss aversion coefficient. We treat the loss neutrality case (i.e., $\lambda = 1$) as a baseline scenario. The total valuation for the θ-consumer derived from purchasing one unit of the good with quality $q \geq 0$ is then:

$$m(q,\theta) + \mu \times \big(m(q,\theta) - m(r(\theta),\theta)\big).$$

Since preferences are quasi-linear, the utility that the θ-consumer obtains from buying a good with quality q at price p is his total valuation minus the price. The value of the outside option for a buyer, if he chooses not to trade with the firm, is taken to be the payoff derived from purchasing a minimal quality substitute good in a secondary market. For simplicity, we let both quality and price of the substitute good be equal to zero. This means that the θ-consumer's utility from the outside option is $-\eta\lambda m(r(\theta),\theta)$, and he buys from the firm only if his utility is no less than this value. The *net total valuation* of the θ-consumer from purchasing a good of quality q is then:

$$v(q,\theta) \equiv (1+\mu)\, m(q,\theta) + (\eta\lambda - \mu)\, m(r(\theta),\theta), \tag{1}$$

where, as before, $\mu = \eta$ if $q > r(\theta)$ and $\mu = \eta\lambda$ if $q \leq r(\theta)$. Presented with a contract to purchase quality q at price p, a θ-consumer with reference quality level $r(\theta)$ chooses to do so as long as his net total utility $v(q,\theta) - p$ is non-negative. This constitutes the (endogenous) participation constraint.

2.3. **Comment.** We provide the following interpretation of our framework. There is a mass of ex ante identical consumers interacting with the firm on a given time period. Prior to entering the market, consumers have a common reference consumption plan, based on correct or incorrect subjective beliefs or other aspirational considerations about future consumption outcomes. Later, each consumer receives a state parameter that affects his willingness to pay for the product attribute and, in addition, fixes his reference quality level according to the reference plan. The firm knows the distribution of state signals but does not observe their realizations. Thus, the firm designs a menu of individually rational and incentive compatible posted contracts to maximize expected revenue. Once contracts are posted, each θ-consumer evaluates a quality level in any given contract relative to his state-contingent reference quality level $r(\theta)$. The consumer then purchases his most preferred contract from the firm as long as the net total utility derived from this selection is non-negative.

Throughout the paper, we assume that the consumption valuation, the gain-loss valuation and the reference plan are common knowledge. That is, the firm is fully aware of the consumers' behavioral bias. We take a partial equilibrium approach and ignore any budgetary restriction on consumer's behavior. Insofar as the total willingness to pay of loss-averse consumers for a given level of product quality is influenced by the reference plan, we focus on allocative efficiency alone when discussing welfare implications of loss aversion, taking loss neutrality as a baseline scenario from which to compare effects of reference-dependent preferences, so that we can highlight the effects of loss aversion.

3. Price Discrimination under Complete Information

We begin by analyzing optimal product line design under complete information. Fixing a reference plan $\theta \mapsto r(\theta)$, when types are observable the firm cannot do better than charging the θ-consumer the whole of his net total valuation and earn (per customer) profits equal to (per customer) *total surplus*

$$TS(q,\theta) \;\equiv\; (1+\mu)m(q,\theta) \;+\; (\eta\lambda-\mu)m(r(\theta),\theta) \;-\; c(q). \tag{2}$$

The firm's profits are therefore maximized by offering the θ-consumer a quality level that maximizes total surplus:

$$q^{fb}(\theta) \;=\; \arg\max_{q\geq 0} TS(q,\theta).$$

Observe that total surplus depends directly and indirectly on the quality level, as the position of q relative to the reference level $r(\theta)$ determines the value that μ adopts in Equation 2. To find the solution to the firm's problem, define

$$S(q,\theta;\mu) \;\equiv\; (1+\mu)\, m(q,\theta) \;-\; c(q)$$

as the part of total surplus from the θ-consumer directly affected by the choice of quality, for each $\mu \in \{\eta, \eta\lambda\}$, and the quality schedule $\theta \mapsto \overline{q}(\theta;\mu)$ as the mapping

$$\overline{q}(\theta;\mu) \;=\; \arg\max_{q\geq 0} S(q,\theta;\mu), \qquad \text{for } \mu = \eta, \eta\lambda.$$

By our assumptions, $\overline{q}(\cdot;\mu)$ is continuously differentiable and non-decreasing.[12] In particular, since the consumption valuation is strictly increasing in q, we have $0 < \overline{q}(\theta;\eta) < \overline{q}(\theta;\eta\lambda)$ for all state signals $\theta \in \Theta$.

Note that under loss neutrality ($\lambda = 1$), the net total valuation is independent of the reference quality level. From Equation 2, total surplus in this case is given by

$$TS(q,\theta) \;=\; S(q,\theta;\eta),$$

[12]Recall that, for each θ, the consumption valuation is twice continuously differentiable and concave in q, and the cost function is strongly convex.

and thus the first-best monopoly qualities are $q^{fb}(\theta) = \overline{q}(\theta;\eta)$, independently of $r(\theta)$. We therefore interpret $\overline{q}(\theta;\eta)$ as the classic efficient (i.e., surplus maximizing) quality level under loss neutrality.

The solution of the monopolist problem under loss aversion ($\lambda > 1$) is now easily obtained by noticing that $TS(\cdot,\theta)$ coincides with $S(\cdot,\theta;\eta\lambda)$ when q is less than or equal to $r(\theta)$, and with a constant-shifted loss-neutral surplus $S(\cdot,\theta;\eta)$ whenever q is greater than $r(\theta)$. Since $S(\cdot,\theta;\eta)$ has a strictly smaller slope than $S(\cdot,\theta;\eta\lambda)$, the total surplus function exhibits a kink at $q = r(\theta)$. The quality level that maximizes profits is determined by the location of the kink relative to the two maximizers $\overline{q}(\theta;\eta)$ and $\overline{q}(\theta;\eta\lambda)$. Refer to Figure 1 for an illustration. If the reference quality level $r(\theta)$ is below $\overline{q}(\theta;\eta)$, then profits are increasing at the kink point and the firm chooses the efficient quality level $\overline{q}(\theta;\eta)$ —Figure 1(A). Similarly, if $r(\theta)$ lies above $\overline{q}(\theta;\eta\lambda)$, then profits are decreasing at the kink point and the firm sets quality at the latter level —Figure 1(B). If the reference point $r(\theta)$ lies in intermediate ranges, any deviation from the reference quality level will hurt profits and the optimal quality level is therefore $r(\theta)$ —Figure 1(C).

This leads to the following observation.

Proposition 1. *Given a reference plan $\theta \mapsto r(\theta)$, the complete information contract menu designed by the monopolist for loss-averse consumers consists of quality–price schedules $\theta \mapsto (q^{fb}(\theta), p^{fb}(\theta))$ such that:*

$$q^{fb}(\theta) = \begin{cases} \overline{q}(\theta;\eta\lambda) & : \ r(\theta) \geq \overline{q}(\theta;\eta\lambda), \\ r(\theta) & : \ \overline{q}(\theta;\eta\lambda) > r(\theta) > \overline{q}(\theta;\eta), \\ \overline{q}(\theta;\eta) & : \ \overline{q}(\theta;\eta) \geq r(\theta); \end{cases}$$

and

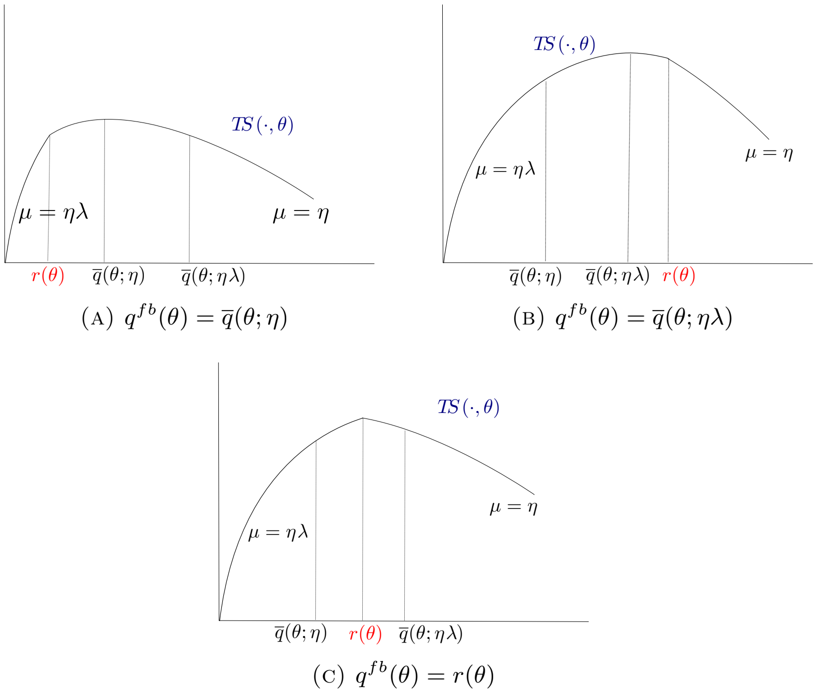
$$p^{fb}(\theta) = (1+\mu)\, m(q^{fb}(\theta),\theta) + (\eta\lambda - \mu)\, m(r(\theta),\theta),$$

where $\mu = \eta$ whenever $q^{fb}(\theta) > r(\theta)$ and $\mu = \eta\lambda$ otherwise.

Proposition 1 shows the effects of reference-dependent preferences and loss aversion on price discrimination in the absence of screening issues. For a low reference level, the firm's optimal quality will be in the domain of gains and therefore coincides with the loss-neutral case. When the reference quality at state θ exceeds the $\overline{q}(\theta;\eta\lambda)$ threshold, the optimal quality must be in the domain of losses and the firm exploits consumer's loss aversion by increasing its offer from the classic efficient level to $\overline{q}(\theta;\eta\lambda)$. Note that the reference plan entirely determines the shape of first-best quality offers for consumers with intermediate reference levels, that is when $\overline{q}(\theta;\eta) \leq r(\theta) \leq \overline{q}(\theta;\eta\lambda)$. Since the reference consumption plan may in principle be very general, first-best contracts can take various shapes. In particular, there may be pooling among certain consumer segments due to a common reference quality level.

We can understand these results better if we consider the comparative statics effect on firm profits and offered qualities of an increase in the reference level. These comparative statics are also of independent interest as they inform extensions of the model that would enable the firm to manipulate the reference level. The key observation is that a change in the reference level affects how the consumer evaluates not only the contracted quality but also the outside option —this is similar to the pseudo-endowment effect noticed by Ariely and Simonson (2003). In particular, if marketing increases the consumer's anticipated quality, then it also reduces the attractiveness of the outside option, further binding the consumer to the contract.

Suppose that $r(\theta) < \overline{q}(\theta;\eta)$, so that the reference quality lies strictly between the outside option (i.e. the zero quality good sold in the secondary market) and the optimal quality offered by the firm. The consumer is comparing his outside option in the domain of losses with the firm's offer in the domain of gains. An increase in $r(\theta)$ has countervailing



(A) $q^{fb}(\theta) = \overline{q}(\theta;\eta)$ (B) $q^{fb}(\theta) = \overline{q}(\theta;\eta\lambda)$

(C) $q^{fb}(\theta) = r(\theta)$

FIGURE 1. Total surplus function $TS(\cdot,\theta)$: the optimal quality is determined by the position of the kink.

effects: it increases the loss associated with the outside option but reduces the gain associated with the contract. Because marginal losses weigh more heavily than gains under loss aversion, the net effect increases the relative attractiveness of the firm's contract. The firm's quality offer is unchanged, but the consumer's willingness to pay, hence the firm's price, increases by

$$(\eta\lambda - \eta)m_q(r(\theta),\theta).$$

As soon as reference qualities exceed the loss-neutral efficiency level $\overline{q}(\cdot;\eta)$, both the quality and the outside option will be in the domain of losses and any further increase in the reference quality reduces the value of both equally, leaving the consumer's net valuation unchanged. However, the firm may now profitably increase quality above loss-neutral efficient levels. In particular, since offered qualities will be in the domain of losses where total surplus rises more steeply, there are larger surplus gains from quality. The firm will capture these gains by increasing quality up to the reference level where the net valuation and total surplus exhibit kinks. This increases profits by

$$(1 + \eta\lambda)m_q(r(\theta),\theta) \; - \; c_q(q).$$

Finally, once the reference quality $r(\theta)$ exceeds $\overline{q}(\theta;\eta\lambda)$, all gains from loss aversion have been exhausted and the firm's offered quality and price are unaffected by further increases in reference levels.

We stress that the ability to exploit a higher reference plan and distort qualities above and beyond efficiency levels depends on loss aversion: when consumers are loss-neutral, the optimal product line is independent of the reference consumption plan. These observations are gathered in the next result.

Proposition 2. *The following holds under complete information. For loss-neutral consumers:*

1. *Optimal qualities offered by the firm coincide with classic efficient qualities,* $q^{fb}(\theta) = \overline{q}(\theta;\eta)$*, regardless of the reference consumption plan.*

For loss-averse consumers:

2. *Optimal qualities offered by the firm are weakly greater than the loss-neutral efficient levels, and strictly greater when* $r(\theta) > \overline{q}(\theta;\eta)$.
3. *An increase in the reference quality level weakly increases the firm's profits, and strictly increases profits whenever* $r(\theta) \leq \overline{q}(\theta;\eta\lambda)$.
4. *An increase in the reference quality level weakly increases the offered quality by the firm, and the increase is strict whenever* $\overline{q}(\theta;\eta) \leq r(\theta) \leq \overline{q}(\theta;\eta\lambda)$.

4. Price Discrimination under Incomplete Information

In this section we study optimal product line design for a monopolist facing loss-averse consumers, when the realization of the state parameter is private information and the monopolist only knows the distribution F and the support Θ.

4.1. **The design problem.** Fixing a reference plan $\theta \mapsto r(\theta)$, the problem of the firm can be formally stated as follows. Choose a menu of posted quality–price contracts $\theta \mapsto (q(\theta), p(\theta))$ that maximizes expected profits

$$\Pi^{sb} = \int_{\theta_L}^{\theta_H} \Big\{ p(\theta) - c(q(\theta)) \Big\} f(\theta)\, d\theta$$

subject to the incentive compatibility constraints:

$$v(q(\theta),\theta) - p(\theta) \geq v(q(\theta'),\theta) - p(\theta'), \qquad \text{for all } \theta,\theta' \in \Theta; \tag{3}$$

and the individual rationality constraints:

$$v(q(\theta),\theta) - p(\theta) \geq 0, \qquad \text{for all } \theta \in \Theta. \tag{4}$$

A menu of contracts that satisfies both informational constraints is said to be *incentive feasible.* When there is no risk of confusion, we let

$$U(\theta) = v(q(\theta),\theta) - p(\theta)$$

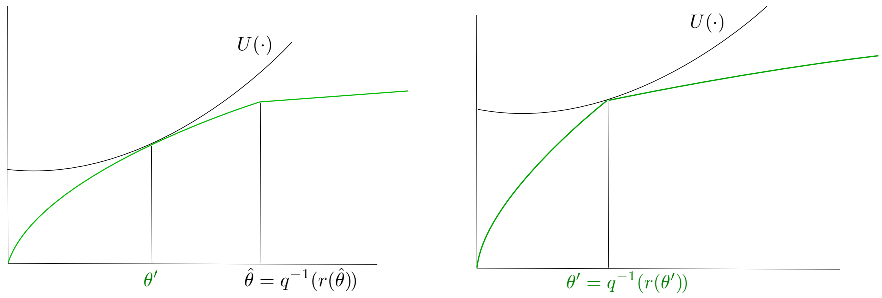
denote the indirect utility function associated with an incentive feasible product line.

Notice that the value the gain-loss coefficient μ implicitly takes in each side of the incentive compatibility inequality Equation 3 may differ, as it depends on comparison of $q(\theta)$ with $r(\theta)$ on the left-hand side, and comparison of the alternative offer $q(\theta')$ with $r(\theta)$ on the right-hand side. Suppose for a moment that the reference plan is strictly increasing. If $r(\theta)$ coincides with the quality offered to the θ-consumer, then the gain-loss coefficient will change abruptly depending on whether the bundle for the θ-consumer is selected by a lower type consumer, who experiences a gain with respect to his personal reference level, or a higher type consumer, who experiences a loss relative to his also higher reference quality level. This sudden change in the net total valuation due to the presence of loss aversion complicates the application of standard contract theoretic techniques, based on the usual integral representation of incentive compatibility, to characterize incentive feasible contracts when the monopolist faces a continuum of consumers.[13] Figure 2 illustrates the source of the problem.

Given an incentive feasible menu of contracts, $U(\theta)$ represents the maximum utility the θ-consumer can obtain among all of the available options. Therefore, when we consider any particular contract $(q(\theta'), p(\theta'))$ and plot the utility

$$\theta \mapsto v(q(\theta'),\theta) - p(\theta'),$$

[13]As proposed by Mussa and Rosen (1978) and Myerson (1981), and more recently by Williams (1999), Krishna and Maenner (2001), Milgrom and Segal (2002), among others.



(A) Determinacy (B) Indeterminacy

FIGURE 2. Kinks cause standard techniques based on the envelope theorem to fail.

the indirect utility function $\theta \mapsto U(\theta)$ must lie everywhere above it, and coincide with it at $\theta = \theta'$. When, as in Figure 2(A), $v(q(\theta'), \cdot)$ is differentiable at θ', this pins down the derivative of the indirect utility, and if this is true for almost every θ-consumer, then these derivatives can be integrated to recover $U(\theta)$.[14] However, when the quality $q(\theta')$ allocated to the θ'-consumer coincides with the reference quality $r(\theta')$, then the mapping $v(q(\theta'), \cdot)$ exhibits a kink at the point $\theta = \theta'$ and this can lead to an indeterminacy, as illustrated in Figure 2(B).[15]

On the other hand, for all $q \geq 0$, the total valuation has bounded left and right partial derivatives for every type $\theta \in \Theta$, which we denote respectively by $v_\theta^-(q, \theta)$ and $v_\theta^+(q, \theta)$. One conjecture is that it is possible to use the left or the right derivative of the total valuation in place of its derivative, when this last does not exist, to express the indirect utility under an incentive feasible contract menu, and thus to express expected profits in terms of virtual surplus. To show a more general result, let us define the correspondence $\theta \rightrightarrows \varphi(q, \theta)$ associated with quality level q as

$$\varphi(q, \theta) \;\equiv\; \left\{\delta \in \mathbb{R} \mid v_\theta^+(q, \theta) \;\leq\; \delta \;\leq\; v_\theta^-(q, \theta)\right\}.$$

When the reference quality level for the θ-consumer does not coincide with q, the net total valuation has a partial derivative with respect to types, so that $\varphi(q, \theta) = v_\theta(q, \theta)$. When $r(\theta) = q$ and the reference plan is strictly increasing at θ, because of loss aversion, the right derivative of the total valuation is strictly smaller than its left counterpart, hence $\varphi(q, \theta)$ will be a closed, bounded interval. One readily obtains the following expression for each θ-consumer (see Section 7 for details):

$$\varphi(q, \theta) \;=\; \begin{cases} (1 + \eta\lambda)\, m_\theta(q, \theta) & : \; r(\theta) > q, \\ (1 + \eta)\, m_\theta(q, \theta) \;+\; (\eta\lambda - \eta)\frac{d}{d\theta}\big(m(r(\theta), \theta)\big) & : \; r(\theta) < q, \\ \big[(1 + \eta\lambda) m_\theta(q, \theta)\,,\, (1 + \eta) m_\theta(q, \theta) + (\eta\lambda - \eta)\frac{d}{d\theta}\big(m(r(\theta), \theta)\big)\big] & : \; q = r(\theta). \end{cases} \tag{5}$$

We stress that since product quality is a choice variable of the firm, it could be optimal to offer contracts such that $q(\theta) = r(\theta)$ for a subset of consumers of positive measure. If for these buyers the reference plan is strictly increasing, it follows that in equilibrium the total valuation may fail to be differentiable in types, and therefore our correspondence in Equation 5 will be multi-valued, on a non-negligible subset of consumers in Θ. In this case,

[14] This statement is simply the envelope theorem.

[15] The reference plan is piecewise continuously differentiable, hence we omit discussion of kinks in the total valuation due to kinks in the reference plan. This is inconsequential for the derivation of the optimal product line.

we characterize incentive feasible contracts based on an integral monotonicity condition and a generalization of the Mirrlees representation of the indirect utility.[16] Given a (measurable) quality schedule $\theta \mapsto q(\theta)$, its associated correspondence $\theta \rightrightarrows \varphi(q(\theta),\theta)$ derived using Equation 5 is non empty-valued, closed-valued, bounded and measurable. Thus, it admits integrable selections. We use the notation $\theta \mapsto \delta(q(\theta),\theta) \in \varphi(q(\theta),\theta)$ to indicate an integrable selection. The following proposition provides a characterization of the incentive feasible product line offered by the monopolist.

Proposition 3. *Under incomplete information, the product line $\theta \mapsto (q(\theta), p(\theta))$ designed by the firm for loss-averse consumers is incentive feasible, with associated indirect utility $U(\theta) = v(q(\theta),\theta) - p(\theta)$, if and only if there exists an integrable selection $\theta \mapsto \delta(q(\theta),\theta)$ of the correspondence $\theta \rightrightarrows \varphi(q(\theta),\theta)$ for which the following conditions are satisfied.*

(a) *Integral monotonicity: for all $\theta', \theta'' \in \Theta$,*

$$v(q(\theta''),\theta'') - v(q(\theta''),\theta') \geq \int_{\theta'}^{\theta''} \delta(q(\tilde{\theta}),\tilde{\theta})\, d\tilde{\theta} \geq v(q(\theta'),\theta'') - v(q(\theta'),\theta').$$

(b) *Generalized Mirrlees representation: for all $\theta \in \Theta$,*

$$U(\theta) = U(\theta_L) + \int_{\theta_L}^{\theta} \delta(q(\tilde{\theta}),\tilde{\theta})\, d\tilde{\theta}.$$

(c) *Participation of the θ_L-consumer: $U(\theta_L) \geq 0$.*

We can now reformulate the firm's objective function in terms of a generalized virtual surplus. Ignoring momentarily the restrictions imposed by the integral monotonicity condition, first note that the generalized Mirrlees equation yields immediately to an expression for the incentive payments in terms of the yet to be determined quality offers and selection. Since it is optimal to leave the lowest type consumer without rents, we have $U(\theta_L) = 0$. Thus, incentive prices are given by:

$$p(\theta) = v(q(\theta),\theta) - \int_{\theta_L}^{\theta} \delta(q(\tilde{\theta}),\tilde{\theta})\, d\tilde{\theta}, \qquad \text{for all } \theta \in \Theta. \tag{6}$$

Denote by $\mu(\theta)$ the value of the gain-loss coefficient when the θ-consumer buys his designated quality level. From expressions (5) and (6), it is clear that for any quality schedule, the firm uses the smallest possible selection, namely

$$\delta(q(\theta),\theta) = (1+\mu(\theta))\, m_\theta(q(\theta),\theta) + (\eta\lambda - \mu(\theta)) \tfrac{d}{d\theta}\big(m(r(\theta),\theta)\big), \tag{7}$$

where $\mu(\theta) = \eta$ for all states θ such that $q(\theta) > r(\theta)$, and $\mu(\theta) = \eta\lambda$ for all θ such that $q(\theta) \leq r(\theta)$ instead. Using (7) in Equation 6, replacing the resulting equation in the expression for expected profits, and integrating by parts, we obtain the following expression for the firm's expected profits in terms of the *virtual consumption valuation* $m^*(q,\theta) = m(q,\theta) - h(\theta) m_\theta(q,\theta)$:

$$\Pi^{sb} = \int_{\theta_L}^{\theta_H} \Big\{ (1+\mu(\theta))\, m^*(q(\theta),\theta) + (\eta\lambda - \mu(\theta))\, m^*(r(\theta),\theta) - c(q(\theta)) \\ - (\eta\lambda - \mu(\theta))\, h(\theta)\, m_q(r(\theta),\theta)\, r_\theta(\theta) \Big\} f(\theta)\, d\theta.$$

The first line in the integrand of the above equation is the loss aversion version of the *virtual total surplus* associated with the θ-consumer, and accordingly denoted by $TS^*(q(\theta),\theta)$. It expresses the tradeoff between marginal and infra-marginal revenues that the monopolist faces when increasing the quality allocated to this particular buyer. The

[16]See Carbajal and Ely (2013) for a general characterization of incentive compatible mechanisms when, as in this model, the valuation function fails to be convex or differentiable in types.

second line, which we denote by $LS(q(\theta),\theta)$, captures a novel effect in optimal product line design due to loss aversion. Accordingly, we write the firm's objective function as:

$$\Pi^{sb} \;=\; \int_{\theta_L}^{\theta_H} \Big\{ TS^*(q(\theta),\theta) \;-\; LS(q(\theta),\theta) \Big\} f(\theta)\, d\theta. \tag{8}$$

The next step of the analysis is to understand the tradeoffs that stem from the interaction between the two components of the firm's profits.

4.2. **The optimal contract menu.** The monopolist's problem is to find a quality schedule $\theta \mapsto q^{sb}(\theta)$ that maximizes expected profits in Equation 8, subject to the integral monotonicity condition.[17] It can be solved in a way that parallels the complete information case, and this route illuminates new aspects arising from loss aversion. As before,

$$S^*(q,\theta;\mu) \;\equiv\; (1+\mu)\, m^*(q,\theta) \;-\; c(q)$$

denotes the part of the virtual total surplus corresponding to the θ-consumer that is directly affected by his choice of quality, for each $\mu = \eta, \eta\lambda$, and $\theta \mapsto q^*(\theta;\mu)$ as the mapping

$$q^*(\theta;\mu) \;\equiv\; \arg\max_{q\geq 0} \; S^*(q,\theta;\mu), \qquad \text{for } \mu = \eta, \eta\lambda.$$

Analogously to the complete information setting (c.f. Figure 1), the virtual total surplus $TS^*(q,\theta)$ coincides with $S^*(q,\theta;\eta\lambda)$ when the quality offer is weakly below $r(\theta)$ and with an appropriate shift of $S^*(q,\theta;\eta)$ when instead the quality offer is strictly above $r(\theta)$. In particular, for a fixed $\theta \in \Theta$, $TS^*(q,\theta)$ is continuous in q but kinked at the point $q = r(\theta)$, and achieves its maximum at one of three points, $q^*(\theta;\eta\lambda)$, $q^*(\theta;\eta)$, or $r(\theta)$, depending on the position of $r(\theta)$ relative to $q^*(\theta;\eta)$ and $q^*(\theta;\eta\lambda)$. Therefore, a maximization based only on the surplus component of the firm's objective function would develop in a manner similar to the complete information case, with the understanding that $TS^*(q,\theta)$ represents *virtual total surplus* and therefore accounts for screening-based incentive effects. In particular, $m^*(q,\theta) = m(q,\theta) - h(\theta)m_\theta(q,\theta)$ discounts the welfare of the θ-consumer, and therefore adds the usual downward distortions in the optimal quality offers to the upward distortions attributable to loss aversion and enumerated in Proposition 2.

A novel effect that stems from the combined presence of incomplete information and loss aversion enters the analysis through the $LS(q,\theta)$ component of the firms's objective. To gain some insight, consider the equivalent formulation

$$LS(q,\theta) \;=\; \begin{cases} (\eta\lambda - \eta)h(\theta)m_q(r(\theta),\theta)r_\theta(\theta) & : \; q > r(\theta), \\ 0 & : \; q \leq r(\theta). \end{cases} \tag{9}$$

As our terminology reflects, $LS(q,\theta)$ represents a *lump-sum cost* incurred by the monopolist whenever it contracts to sell a product whose quality exceeds the consumer's state-contingent reference level. Increasing $q(\theta)$ above $r(\theta)$ moves the valuation of the θ-consumer from the domain of losses to the domain of gains. $TS^*(q(\theta),\theta)$ accounts for the effect this has on the price extracted from the θ-consumer. $LS(q(\theta),\theta)$ on the other hand captures an additional cost that arises because the θ'-consumer, who has a reference quality level $r(\theta')$ above $r(\theta)$ but below $q(\theta)$, now views the $q(\theta)$ offer as a gain, compared to the previous offer $r(\theta)$, which was considered a loss. This causes a discrete change in the value the θ'-consumer attaches to $q(\theta)$, measured by $(\eta\lambda - \eta)m(r(\theta'),\theta')$. Since there is a continuum of types, changes of this nature are captured by $LS(q(\theta),\theta)$ —the amount of surplus the monopolist passes to higher consumers in order to discourage them from choosing $q(\theta)$, a quality level that now appears in the gain domain for these consumers.

[17] Showing that a contract menu satisfies condition (a) in Proposition 3 is complicated by the fact that the optimal selection changes value depending on whether the quality offer by the firm is greater or less than the reference quality level. We defer this step entirely to Section 7.

The combined effect of $TS^*(q,\theta)$ and $LS(q,\theta)$ in the objective function implies that there is now, in addition to the kink at $r(\theta)$, a discontinuous jump downward (see Figure 3 below). The full solution to the firm's design problem is presented below.

Proposition 4. *Given a reference plan $\theta \mapsto r(\theta)$, the optimal incentive feasible contract menu offered by the firm to loss-averse consumers consists of quality–price schedules $\theta \mapsto (q^{sb}(\theta), p^{sb}(\theta))$ such that:*

$$q^{sb}(\theta) = \begin{cases} q^*(\theta;\eta\lambda) & : \ r(\theta) \geq q^*(\theta;\eta\lambda), \\ r(\theta) & : \ q^*(\theta;\eta\lambda) > r(\theta) > q^*(\theta;\eta), \\ r(\theta) & : \ q^*(\theta;\eta) \geq r(\theta), \ \theta \leq \theta_{c_k}, \\ q^*(\theta;\eta) & : \ q^*(\theta;\eta) \geq r(\theta), \ \theta > \theta_{c_k}, \end{cases} \tag{10}$$

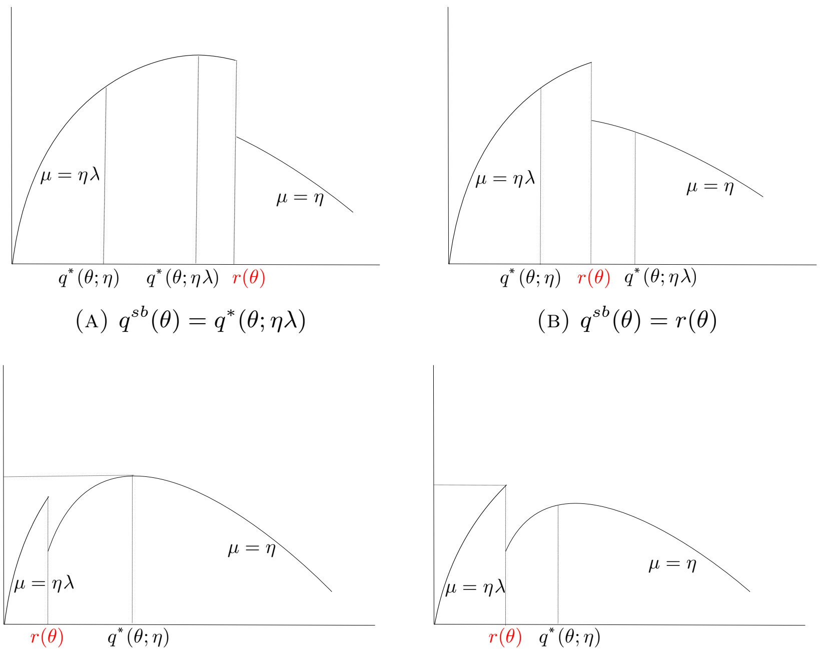
where the θ_{c_k}-consumer lies in the k-th subinterval of consumers with $q^*(\theta;\eta) \geq r(\theta)$, and

$$p^{sb}(\theta) = v(q^{sb}(\theta),\theta) - \int_{\theta_L}^{\theta} \delta(q^{sb}(\tilde{\theta}),\tilde{\theta})\, d\tilde{\theta}, \tag{11}$$

where the selection in the price schedule is given by

$$\delta(q^{sb}(\theta),\theta) = (1+\mu(\theta))\, m_\theta(q^{sb}(\theta),\theta) + (\eta\lambda - \mu(\theta))\tfrac{d}{d\theta}\big(m(r(\theta),\theta)\big)$$

with $\mu(\theta) = \eta\lambda$ when $q^{sb}(\theta) \leq r(\theta)$ and $\mu(\theta) = \eta$ otherwise.



(A) $q^{sb}(\theta) = q^*(\theta;\eta\lambda)$ (B) $q^{sb}(\theta) = r(\theta)$

(C) $q^{sb}(\theta) = q^*(\theta;\eta)$ (D) $q^{sb}(\theta) = r(\theta)$

FIGURE 3. The profit-maximizing quality is determined by the position and magnitude of the jump

Here we sketch the proof of Proposition 4, leaving the formal arguments to Section 7.

Step 1. If the θ-consumer has a reference quality above $q^*(\theta;\eta\lambda)$, then this last constitutes the optimal offer from the firm; see Figure 3(A). Indeed, this is the unique maximizer of $TS^*(q,\theta)$, and since $q^*(\theta;\eta\lambda) \leq r(\theta)$, the lump-sum cost $LS(q^*(\theta;\eta\lambda),\theta)$ is zero.

Step 2. If the θ-consumer has a reference level that lies between $q^*(\theta;\eta)$ and $q^*(\theta;\eta\lambda)$, then the optimal offered quality is $r(\theta)$. This follows because the virtual total surplus

$TS^*(q,\theta)$ is strictly decreasing for quality levels above the reference point, as $\mu(\theta)=\eta$, and strictly increasing for quality levels below the reference point, as $\mu(\theta)=\eta\lambda$, and here again the lump-sum cost is zero; see Figure 3(B).

Step 3. If the θ-consumer reference level lies below $q^*(\theta;\eta)$, the optimal offer by the firm is either $q^*(\theta;\eta)$ or $r(\theta)$. To see why, note that the unique maximizer of $TS^*(q,\theta)$ is $q^*(\theta;\eta)$ which is above the reference quality $r(\theta)$, so that the lump-sum incentive cost is active and amounts to $LS(q^*(\theta;\eta),\theta)$. There is therefore a tradeoff between the two components of the objective function: choosing $q^*(\theta;\eta)$ to capture efficiency gains and associated marginal revenues, or eschewing these and relaxing incentive constraints by offering $r(\theta)$ to ensure that this quality offer is viewed as a loss by higher types, thus avoiding the lump-sum cost. The sign of the difference between profits at $r(\theta)$ with gain-loss coefficient $\eta\lambda$, and profits at $q^*(\theta;\eta)$ with gain-loss coefficient η and an active lump-sum cost, depends on this trade-off.

Step 4. When the reference quality level is below but near $q^*(\theta;\eta)$, efficiency gains are small, hence the firm is more likely to offer $r(\theta)$. This is illustrated in Figure 3(D). For lower reference qualities, the tradeoff may go the other way, see Figure 3(C). Suppose there is an subinterval of types in Θ such that $r(\theta)=q^*(\theta;\eta)$ at its left and right endpoints, and $r(\theta)<q^*(\theta;\eta)$ everywhere else inside the subinterval. Then, it is possible that the monopolist may try to sell their reference levels to consumers with types at both the low and high ends of this subinterval, and $q^*(\theta;\eta)$ for θ-consumers in the middle range. As this pattern violates monotonicity of the quality schedule, it is not implementable.

Step 5. We observe that the reference plan $\theta\mapsto r(\theta)$ and the quality schedule $\theta\mapsto q^*(\theta;\mu)$ cross at most finitely many times, for $\mu=\eta,\eta\lambda$.[18] Thus, for any subinterval of types for whom reference qualities lie below $q^*(\theta;\eta)$, the optimal quality schedule corresponds to one of the following three cases: either it assigns $q^*(\theta;\eta)$ to each θ-consumer; or alternatively it assigns $r(\theta)$ for each state parameter θ; or there exists a cutoff type θ_c among them such that the firm offers $r(\theta)$ to each θ-consumer below θ_c, and $q^*(\theta;\eta)$ to each θ-consumer in the subinterval above θ_c. Which possibility is chosen by the firm depends of course on the details of the model (i.e., consumption valuation, distribution of types, reference plan, and the value of the gain-loss and loss aversion coefficients).

4.3. **Comparative statics.** The specifics of price discrimination under loss aversion exhibit novel elements, compared to the loss-neutral case. Relatively high reference plans for low type consumers can generate allocative efficiency gains under loss aversion, as quality offers get closer to the efficient qualities. In particular, there may be an increase in market coverage. High reference plans for high type consumers, on the other hand, can generate quality distortions above and beyond the efficient levels, so that there is an excess supply of the product attribute. Moreover, it is possible that for a non-negligible subset of buyers, the optimal quality schedule is determined entirely by the reference plan. This implies that optimal contract menu may exhibit a degree of complexity —pooling for some mid-range consumers, preceded and followed by separating contracts— that responds entirely to consumers expectations or aspirational considerations, as captured by the reference plan, and not to especial features of the cost function or the distribution of types. We spell out some of these properties below.

Corollary 1. *The following holds under incomplete information. For loss-neutral consumers:*

1. *Optimal qualities offered by the firm coincide with $q^*(\cdot;\eta)$, independently of the reference plan.*

[18] We are not stating that r and $q^*(\cdot;\mu)$ coincide only finitely many times, as it could be that $r(\theta)=q^*(\theta;\mu)$ for a subinterval of the type space, but only that the difference function $f^\mu=r-q^*(\cdot;\mu)$ changes from positive to negative a finite number of times.

For loss-averse consumers:

2. Downward distortions. *If the reference level $r(\theta)$ is weakly below $q^*(\theta;\eta)$, then so is the optimal quality offered by the firm.*
3. Efficiency gains. *If the reference level $r(\theta)$ lies between $q^*(\theta;\eta)$ and the loss-neutral efficient level $\overline{q}(\theta;\eta)$, then so does the optimal quality offered by the firm.*
4. Upward distortions. *For loss-averse buyers for whom $\overline{q}(\theta;\eta) \leq q^*(\theta;\eta\lambda)$, if the reference level $r(\theta)$ lies above the loss-neutral classic efficiency level, then so does the optimal quality offered by the firm.*

Recall that Proposition 2 analyses how profits and output of the firm changes with movements of the reference quality level. Under complete information, an increase in the reference level of the θ-consumer never decreases profits or quality offers. Under incomplete information, when $r(\theta)$ lies above $q^*(\theta;\eta\lambda)$, then —as in the complete information case— an increase in the reference level has no effect on profits or revenue maximizing offers. When $r(\theta)$ lies between $q^*(\theta;\eta)$ and $q^*(\theta;\eta\lambda)$, an increase in the reference level raises the consumer's virtual willingness to pay for quality, thus increasing output and profits in a way parallel to the complete information case.

Things are more subtle when $r(\theta)$ lies below $q^*(\theta;\eta)$. In this case, a higher reference level still raises the consumer's virtual willingness to pay for quality. However, because now the discontinuous jump in profits associated to $LS(q,\theta)$ becomes active, comparative statics are not straightforward. The lump-sum cost depends negatively on the magnitude of change in the reference quality of the θ-consumer through its effect on the marginal willingness to pay for quality, but positively on the rate of change of the reference plan itself through its effect on the reference level of higher types (cf. Equation 9). Hence there are countervailing forces at play. Despite this, we provide a condition that ensures monotonicity of profits.

Proposition 5. *Under incomplete information and loss aversion, the following holds.*

1. *If $r(\theta) \geq q^*(\theta;\eta)$, then an increase in the reference level weakly increases the firm's profits from the θ-consumer, and the effect is strict when $r(\theta) < q^*(\theta;\eta\lambda)$.*
2. *If $r(\theta) \geq q^*(\theta;\eta)$, then an increase in the reference level weakly increases the firm's optimal offer to the θ-consumer, and the effect is strict when $r(\theta) < q^*(\theta;\eta\lambda)$.*
3. *If $r(\theta) < q^*(\theta;\eta)$, then an increase in the reference level to $\hat{r}(\theta)$ increases the firm's profits when the following condition is satisfied:*

$$m_q(\hat{r}(\theta),\theta)\hat{r}_\theta(\theta) \;\leq\; m_q(r(\theta),\theta)r_\theta(\theta). \tag{12}$$

When Eq. (12) holds, the lump-sum cost associated to a higher reference level $\hat{r}(\theta)$ is smaller than the original lump-sum cost. This reinforces the effect on the willingness to pay associated to a worsening of the outside option, which is due to the pseudo-endowment effect. Note that the concavity of the consumption valuation $m(\cdot,\theta)$ immediately implies that a constant upward shift of the reference plan $\theta \mapsto r(\theta)$ never decreases profits. Finally, observe that as the lump-sum cost is associated to preventing potential deviations from higher consumers (through the inverse hazard rate, cf. Equation 9), any positive or negative effect will diminish as θ approaches θ_H.

We are not able to say precisely how optimal offers respond to higher reference levels when those levels lie below $q^*(\cdot;\eta)$. Even when the effect on profits is unambiguously positive, there may be some non monotonicities in the way optimal qualities react to movements in the reference plan.

Some of the new features of optimal product line design under loss aversion are illustrated in the following application.

4.4. **Price discrimination with uniform signals, linear valuations and quadratic cost.** We impose additional assumptions to obtain an explicit solution to the monopolist's problem: states are uniformly distributed on $\Theta = [1,2]$; consumers have a linear consumption valuation $m(q,\theta) = \theta q$; and the firm's cost function is quadratic, so $c(q) = q^2/2 + q$. Thus, the (pointwise) objective function of the firm is composed of:

$$\begin{aligned} TS^*(q,\theta) &= (1+\mu)(2\theta-2)q - q^2/2 - q + (\eta\lambda-\mu)(2\theta-2)r(\theta), \quad \text{and} \\ LS(q,\theta) &= (\eta\lambda-\mu)(2-\theta)\theta r_\theta(\theta). \end{aligned}$$

Let $\theta_\mu = (3+2\mu)/(2+2\mu)$, for $\mu = \eta, \eta\lambda$. Observe $\theta_{\eta\lambda} < \theta_\eta$ (cf. Figure 4). Readily, one obtains:

$$q^*(\theta;\mu) = \begin{cases} 0 & : 1 \le \theta \le \theta_\mu, \\ (1+\mu)(2\theta-2)-1 & : \theta_\mu \le \theta \le 2, \end{cases}$$

as the quality level that maximizes $S^*(q,\theta;\mu)$.

Following the interpretation provided in Section 2.3, we assume that all ex ante identical consumers have a common reference plan before state signals are received. To highlight the effects of the reference consumption plan in terms of optimal design, here we consider three different plans. Under the first plan $\theta \mapsto r_1(\theta)$, consumers naively believe the firm will offer the (ex ante) expected first best quality level under loss neutrality, so $r_1(\theta) = (1+\eta)3/2 - 1$ for all θ. Under the second reference plan, consumers anticipate first best offers, so $r_2(\theta) = \overline{q}(\theta;\eta) = (1+\eta)\theta - 1$ for all θ. Immediately from Proposition 4, the optimal quality schedule q_i^{sb} associated with the reference plan r_i is given by

$$q_1^{sb}(\theta) = \begin{cases} q^*(\theta;\eta\lambda) & : 1 \le \theta \le \underline{\theta}_1, \\ r_1(\theta) & : \underline{\theta}_1 \le \theta \le \overline{\theta}_1, \\ q^*(\theta;\eta) & : \overline{\theta}_1 \le \theta \le 2, \end{cases} \quad \text{and} \quad q_2^{sb}(\theta) = \begin{cases} q^*(\theta;\eta\lambda) & : 1 \le \theta \le \theta_2, \\ r_2(\theta) & : \theta_2 \le \theta \le 2; \end{cases}$$

where in the first case, $\underline{\theta}_1 = (7+4\eta\lambda+3\eta)/(4+4\eta\lambda)$ and $\overline{\theta}_1 = 7/4$, and in the second case $\theta_2 = (2+2\eta\lambda)/(1+2\eta\lambda-\eta)$.

Consider a third reference plan $\theta \mapsto r_3(\theta)$ defined by $r_3(\theta) = r_1(\theta)/2 + r_2(\theta)/2$. An interpretation is that each θ-consumer puts equal weight into his reference point being the loss-neutral efficient quality level, given his type, and the average efficient quality. The optimal offers to consumers with reference qualities below $q^*(\cdot;\eta)$ depends on the trade-off between efficiency gains and the lump-sum cost triggered by loss aversion. For such consumers, the difference between profits at $r_3(\theta)$ and $\mu = \eta\lambda$, and profits at $q^*(\theta;\eta)$ and $\mu = \eta$ (cf. Equation 18 in Section 7) is given by:

$$\triangle(r_3(\theta), q^*(\theta;\eta)) = \tfrac{1}{2}(1+\eta)(\eta\lambda-\eta)(2-\theta)\theta - \tfrac{1}{8}(1+\eta)^2\big(3\theta - \tfrac{11}{2}\big)^2.$$

The reference plan r_3 and the quality schedule $q^*(\cdot;\eta)$ intersect at $\theta = 11/6$. One has that the difference in profits at $\theta = 11/6$ is strictly positive and the difference in profits at $\theta_H = 2$ is strictly negative. Since this profit difference, as a function of types, is continuous and strictly decreasing for all types greater than $11/6$, it follows that there exists a unique cutoff $\overline{\theta}_3$ such that $\triangle(r_3(\overline{\theta}_3), q^*(\overline{\theta}_3)) = 0$. For consumers to the left of $\overline{\theta}_3$, the optimal qualities coincide with reference levels and is below quality schedule $q^*(\cdot;\eta)$. The optimal quality schedule under r_3 is:

$$q_3^{sb}(\theta) = \begin{cases} q^*(\theta;\eta\lambda) & : 1 \le \theta \le \underline{\theta}_3, \\ r_3(\theta) & : \underline{\theta}_1 \le \theta \le \overline{\theta}_3, \\ q^*(\theta;\eta) & : \overline{\theta}_3 \le \theta \le 2, \end{cases}$$

where $\underline{\theta}_3 = (11+8\eta\lambda-3\eta)/(6+8\eta\lambda-2\eta)$.

None of the contract menus studied in this application can be implemented using a single family of linear prices. We explicitly show this claim for the optimal menu $\theta \mapsto (q_2^{sb}(\theta), p_2^{sb}(\theta))$. Using Equation 11, one sees that

$$p_2^{sb}(\theta) \;=\; \begin{cases} 0 & :\; 1 \,\leq\, \theta \,\leq\, \theta_{\eta\lambda}, \\ (1+\eta\lambda)^2\theta^2 \,+\, K_{21} & :\; \theta_{\eta\lambda} \,\leq\, \theta \,\leq\, \theta_2, \\ \frac{1}{2}(1+\eta\lambda)(1+\eta)\theta^2 \,+\, K_{22} & :\; \theta_2 \,\leq\, \theta \,\leq\, 2, \end{cases}$$

where K_{21} and K_{22} are fixed fees associated to each segment of consumer types, which we ignore to simplify notation. We first transform the direct optimal contracts into (indirect) non-linear prices: $P_2^{sb}(q) = p_2^{sb}(\theta_2^{sb}(q))$, where $\theta_2^{sb}(q)$ is the inverse of $q_2^{sb}(\theta)$. Immediately, one has that

$$P_2^{sb}(q) \;=\; \begin{cases} \left(\frac{3}{2} + \eta\lambda\right)q \,+\, \frac{q^2}{4} & :\; q \,\leq\, \bar{q}_2, \\ \frac{1+\eta\lambda}{1+\eta}\left(q + \frac{q^2}{2}\right) & :\; q \,\geq\, \bar{q}_2; \end{cases}$$

where $\bar{q}_2 \,=\, q_2^{sb}(\theta_2)$. This non-linear pricing schedule is implemented using not one but two families of linear contracts:

$$\begin{aligned} L_{21} \;&=\; \left\{P_2^{sb}(\hat{q}) \,+\, \left(\tfrac{3}{2} \,+\, \eta\lambda \,+\, \tfrac{1}{2}\hat{q}\right)(q-\hat{q}) \;\middle|\; \hat{q} \leq \bar{q}_2\right\} \\ L_{22} \;&=\; \left\{P_2^{sb}(\hat{q}) \,+\, \left(\tfrac{1+\eta\lambda}{1+\eta}(1+\hat{q})\right)(q-\hat{q}) \;\middle|\; \hat{q} \geq \bar{q}_2\right\}. \end{aligned}$$

Let us now provide a simple illustration of the effects on profits and output when the reference plan increases, but all changes occur below $q^*(\cdot,\eta)$. We parameterize reference plans by $\alpha \in [0,1]$ as follows: $r^{\alpha}(\theta) = \alpha q^*(\theta;\eta)$, which generates optimal quality offer $q^{sb,\alpha}(\theta)$ to the θ-consumer. Thus, when $\alpha = 0$, the reference plan is identically zero and the monopolist offers qualities $q^{sb,0}(\theta) = q^*(\theta;\eta)$; and when $\alpha = 1$, both the reference plan and the optimal quality schedule are equal to $\theta \mapsto q^*(\theta;\eta)$. It is not difficult to see that for every $\alpha \in (0,1)$ there exists a cutoff type $\theta(\alpha)$, with $\theta_\eta < \theta(\alpha) < \theta_H = 2$, such that

$$q^{sb,\alpha}(\theta) \;=\; \begin{cases} \alpha q^*(\theta;\eta) & :\; 1 \,\leq\, \theta \,\leq\, \theta(\alpha), \\ q^*(\theta;\eta) & :\; \theta(\alpha) \,\leq\, \theta \,\leq\, 2. \end{cases}$$

Moreover, with some additional work one can show that $\theta(\alpha)$ increases with α. Immediately, this implies that optimal quality offers from the firm are not everywhere monotonically increasing in the reference quality level. It is also possible to show that there exists a threshold $\overline{\alpha} \in (0,1)$ such that if $\alpha > \overline{\alpha}$, then the effects on profits of an increase in the reference plan are unambiguously positive.

5. Self-Confirming Reference Plans

The analysis of Section 4 allows for differences between optimal quality offers from the firm and reference quality levels expected by consumers. Regardless of how subjective beliefs about qualities were formed, a θ-consumer buys his designated quality–price pair, even if the quality offer differs from his expectations, because the optimal product line is incentive compatible and individually rational. In this section we focus on correct belief formation, ruling out inconsistencies between expectation-based reference qualities and consumed qualities.

A reference plan $\theta \mapsto r(\theta)$ is said to be *self-confirming* if the optimal quality schedule generated by it is such that

$$q^{sb}(\theta) \;=\; r(\theta)$$

holds for every state parameter $\theta \in \Theta$. An optimal contract menu $\theta \mapsto (q^{sb}(\theta), p^{sb}(\theta))$ is called self-confirming if it is generated by a self-confirming reference plan. In a self-confirming product line, all potential buyers correctly anticipate their future consumption

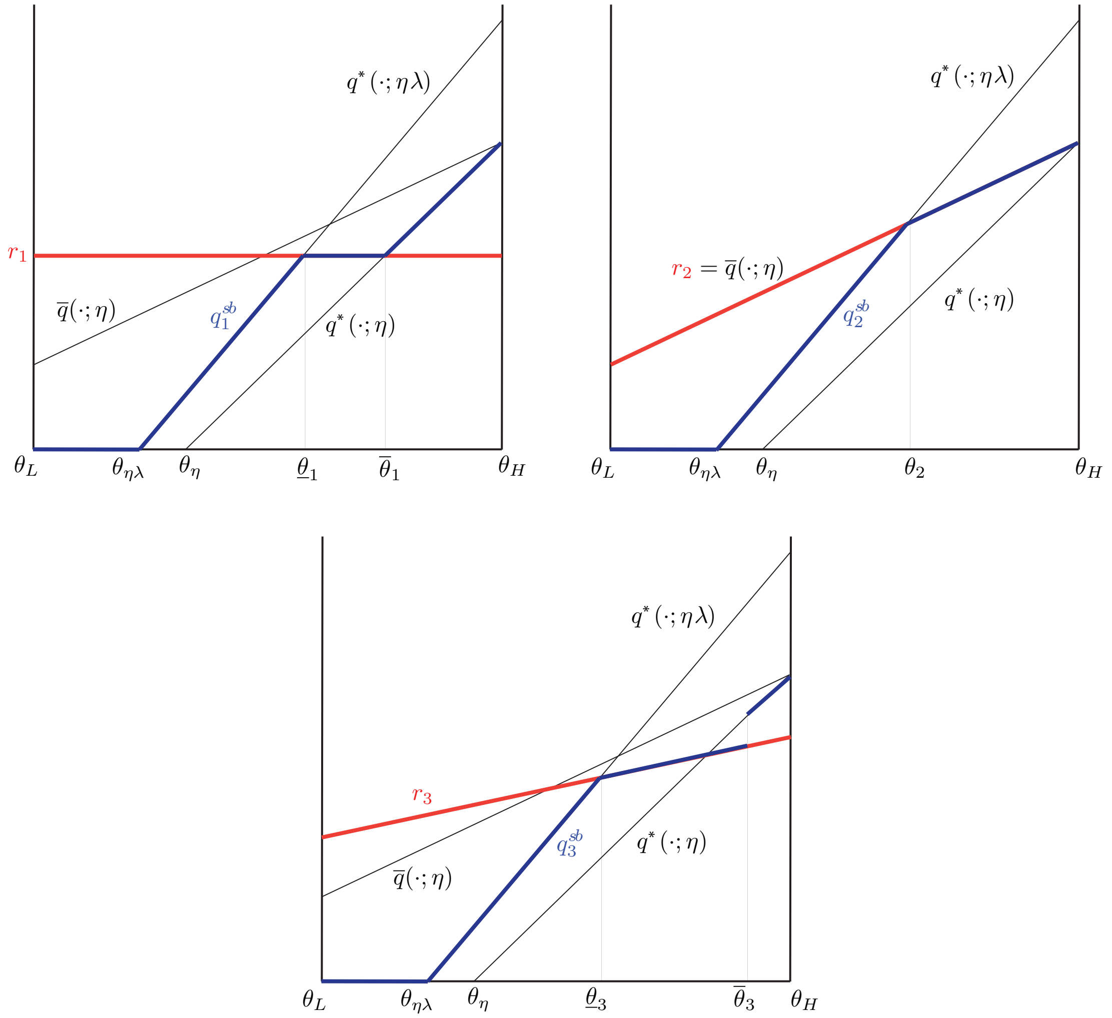


FIGURE 4. The optimal quality schedule q_i^{sb} for reference plan r_i.

outcomes and take those expectations as their reference quality levels. The set of self-confirming reference plans is clearly non-empty —for example, it contains $r = q^*(\cdot, \eta)$. While is difficult to provide a full characterization of the set of self-confirming reference plans, a sufficient condition is readily available (in the following results we maintain the assumptions made in Section 2.2 on reference plans).

Proposition 6. *A reference plan $\theta \mapsto r(\theta)$ is self-confirming if one of the following conditions is satisfied for every θ-consumer:*

(a) $q^*(\theta;\eta\lambda) \geq r(\theta) \geq q^*(\theta;\eta)$;
(b) $q^*(\theta;\eta) > r(\theta)$, *and the difference between profits at $r(\theta)$ with $\mu(\theta) = \eta\lambda$ and profits at $q^*(\theta;\eta)$ with $\mu(\theta) = \lambda$ is non-negative.*

The above result implies that there exists a multiplicity of self-confirming reference consumption plans. On the other hand, the existence of a self-confirming reference plan (partially) below $q^*(\cdot;\eta)$ depends on the existence of reference plan for which the lump-sum cost associated with loss aversion must be greater than efficiency gains, so that the firm maintains a non-negligible portion of consumers with $r(\theta)$ below $q^*(\theta;\eta)$ at their reference levels (cf. Equation 18). Note however that any such reference plan will need to satisfy $r(\theta_H) \geq q^*(\theta_H;\eta)$, as it is impossible for the monopolist to recover informational rents from higher type buyers by maintaining the θ_H-consumer at his reference point.

These observations lead to the following question: among all self-confirming optimal contract menus, which (if any) is the monopolist's preferred one? The answer, it turns out, is remarkably simple. Indeed, from Equation 8, the lump-sum cost $L(q(\theta),\theta)$ is never incurred when the firm's offers coincide with consumers' reference levels, as there is nowhere a change from the loss domain to the gain domain in the consumers' valuation. Thus, per customer profits in any self-confirming contract menu are

$$TS^*(r(\theta),\theta) \;=\; S^*(r(\theta),\theta;\eta\lambda) \;=\; (1+\eta\lambda)\, m^*(r(\theta),\theta) \;-\; c(r(\theta)).$$

Clearly, this expression is strictly increasing in the reference quality level, for all $r(\theta) \leq q^*(\theta;\eta\lambda)$. Since we have already established optimal prices for any incentive feasible quality schedule, it follows that the firm has a unique self-confirming product line, where the quality offer for each θ-consumer is $q^*(\theta;\eta\lambda)$ and its corresponding price is given by Equation 11.

What about consumers? A higher self-confirming reference consumption plan generates two opposite effects on consumer's welfare. On the one hand, a higher reference level increases the informational rents that the monopolist has to give to buyers that are active in the market. On the other, a higher reference level implies a lower value of the outside option, due to the pseudo-endowment effect, and this in particular means that active buyers are worse off (in a self-confirming product line, non-active buyers expect to be excluded from the market). To analyze these countervailing forces, first notice that, because $\mu(\theta) = \eta\lambda$ holds at every state in a self-confirming reference plan, the indirect utility for every θ-consumer, after discounting the value of the outside option, is

$$U(\theta) - \eta\lambda m(r(\theta),\theta) \;=\; \int_{\theta_L}^{\theta} m_\theta(r(\tilde\theta),\tilde\theta)\, d\tilde\theta \,+\, \eta\lambda \int_{\theta_L}^{\theta} \big\{ m_\theta(r(\tilde\theta),\tilde\theta) - m_\theta(r(\theta),\tilde\theta) \big\} d\tilde\theta \qquad (13)$$

—see Equations (1), (7) and condition (b) of Proposition 3. The first integral in the right-hand side of this expression captures the standard informational rents resulting from the screening process. It is positive for consumers buying a positive quality offer from the firm, and because of single crossing (i.e., $m_{q\theta} > 0$), its value increases with a higher reference plan. The second integral captures the value of the informational rents vis-à-vis the participation rents that the consumer concedes to the firm to avoid the outside option. Overall, the impact of a higher reference plan depends on the interaction of these two terms, which is also affected by the value of the gain-loss coefficient and the loss aversion coefficient. However, when single crossing has diminishing second order effects, the detrimental effects are relatively small, so that active consumers are also better off with a higher reference plan.

Proposition 7. *The following holds under incomplete information when the firm designs an optimal product line for loss-averse consumers:*

1. *A higher self-confirming reference plan strictly increases the firm's profits, whenever $r(\theta) \leq q^*(\theta;\eta\lambda)$.*
2. *If the function $q \mapsto m_\theta(q,\theta)$ is concave for every $\theta \in \Theta$, then a higher self-confirming reference plan strictly increases consumers' indirect utility, whenever $0 < r(\theta) \leq q^*(\theta;\eta\lambda)$.*
3. *Under the last hypothesis, the unique preferred self-confirming menu of contracts for both the firm and consumers is generated by the reference plan $\theta \mapsto r^*(\theta) = q^*(\theta;\eta\lambda)$.*

Our last result merits some comment, despite its simplicity. First, notice that under its preferred self-confirming reference plan, the firm exploits consumer loss aversion in two different, albeit related ways. On the one hand, a higher reference plan reduces the value of the outside option, thus driving up overall net (virtual) consumer surplus. On the other, by offering a quality level equal to the consumer's reference point, the firm takes advantage of the higher marginal willingness to pay for each additional unit of quality, which is captured in the choice of the selection used in the Mirrlees representation of the indirect utility to construct the optimal price schedule.

Second, that in our model consumers also prefer $r^* = q^*(\cdot; \eta\lambda)$ is somewhat counter-intuitive. A higher reference point diminishes the attractiveness of the outside option in the secondary market, which increases the willingness to pay for quality in the primary market served by the firm (recall we are ignoring budgetary restrictions from the part of the consumers). Under incomplete information, however, the firm has to pass some of the extra surplus to consumers in the form of information rents, which are increasing in quality. When the effects of the single crossing conditions are diminishing, it follows that the volume of information rents ceded by the firm to active consumers on the primary market exceeds the extra participation rents extracted from those consumers when the value of the outside option worsens. Thus, active consumers benefit with a higher self-confirming reference plan.

Third, there are various ways in which the firm may try to induce consumers to adopt its preferred reference plan (this may be the case even if the consumers' preferred reference plan were different than the firm's). For instance, the firm can announce a product line prior to actual market introduction —we leave aside any cost associated with advertising campaigns as they are irrelevant for our argument, as long as marginal advertising costs are small. These announcements can be made in terms of product specification and salient characteristics, may omit any mention of prices, and will be credible because they comply with a self-confirming reference consumption plan. This seems consistent with marketing practices spread across some industries, where both product announcements and advertising campaigns tend to precede actual market introduction and stress quality attributes over prices.

Fourth, there are allocative efficiency gains for all θ-consumers for whom $q^*(\theta; \eta\lambda)$ lies strictly above $q^*(\theta; \eta)$ —the optimal quality offered to loss-neutral consumers— but below $\overline{q}(\theta; \eta)$ —the efficient quality offered to loss-neutral consumers (see Figure 4). It is thus possible for some buyers with low intrinsic consumption valuation, who under loss neutrality would be excluded from the primary market, to have positive consumption outcomes precisely because of higher reference points. For θ-consumers with $q^*(\theta; \eta\lambda) > \overline{q}(\theta; \eta)$ the opposite is true, as these buyers end up with excessive quality levels —i.e., quality levels above the efficient quality offers to loss-neutral consumers. The expanded range of the preferred optimal menu of contracts seems to be consistent with stylized observations in certain industries (e.g., consumer electronics, luxury goods, etc.) where product lines include goods of increasingly high sophistication.

6. Concluding Remarks

In this paper, we study optimal contract design by a revenue maximizing monopolist who faces consumers with heterogeneous tastes, reference-dependent preferences and loss aversion. Our paper follows the line of work pioneered by DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006), Kőszegi and Rabin (2006), Heidhues and Kőszegi (2008), and Orhun (2009) among others, in studying the optimal responses of profit maximizing firms in a market context with consumers who have systematic behavioral biases.

We find that, while some general insights and intuition of standard price discrimination models are present, the reference consumption plan exerts considerable influence in specifics of the optimal product line. This is due to the appearance of new effects generated by loss aversion under incomplete information. Thus, depending on how potential buyers form their expectations of quality consumption, optimal contract menus may exhibit various distinct features —pooling for intermediate consumers, some discontinuities, efficiency gains, upward distortions from efficiency levels, etc.— and thus may not implemented by simple two-part tariffs.

Our research stresses the importance of understanding how the reference quality levels are formed or influenced. Most of the older empirical literature testing reference-dependent price and quality effects consider memory-based models of reference point formation process (e.g., Hardie, Johnson, and Fader (1993), Briesch, Krishnamurthi, Mazumdar, and Raj (1997), etc.). There is however recent evidence of expectation-based reference points in effort provision both in the field (e.g., Crawford and Meng (2011) and Pope and Schweitzer (2011)) and in the laboratory (e.g., Abeler, Falk, Goette, and Huffman (2011) and Gill and Prowse (2012)). In our monopoly pricing model with state-contingent reference qualities, there is a multiplicity of expectations-based, consistent reference consumption plans, many of which do not rule out marked complexities in optimal contracts. On the other hand, the firm's preferred self-confirming menu of contracts exhibits (allocative) efficiency gains and an increased coverage at the low end of the market, and excess supply of quality compared to the efficient quality levels for the high end of the market.

That the firm's preferred self-confirming menu is also the one preferred by consumers rests on higher informational gains being associated to higher quality offers, on the fact that there is no budgetary restrictions and the strict single-crossing condition holds everywhere and has diminishing effects (so that informational rents trump participation costs of active consumers). This conclusion may change in a more general model that takes into account other goods and services demanded by consumers as well as initial endowments. Thus, it is important to understand how, in practice, consumers' (correct) expectations of future consumption may be influenced by fashion and mode cycles, by social and peer pressure, by directed marketing campaigns, etc. This seems specially important in dynamic settings where a firm interacts with consumers over multiple time periods via long term contracts, or where there is short product cycles due to innovation, or in environments of oligopolistic competition where there may be more than one product attribute dimension that can be used as a tool to enter the market. We leave these questions for future research.

7. Proofs

Proof of Proposition 1.
Fix a type $\theta \in \Theta$ and suppose that $r(\theta) \geq \overline{q}(\theta;\eta\lambda)$. The unique maximum of the profit function in Equation 2 with $\mu = \eta\lambda$ is of course $\overline{q}(\theta;\eta\lambda)$, which generates net revenue to the firm equal to

$$TS(\overline{q}(\theta;\eta\lambda),\theta) \;=\; (1+\eta\lambda)m(\overline{q}(\theta;\eta\lambda),\theta) \;-\; c(\overline{q}(\theta;\eta\lambda)). \tag{14}$$

Choosing $q < r(\theta)$ does not change the objective function of the monopolist and strictly reduces per customer profits $TS(q,\theta)$. Choosing an alternative quality level $\hat{q} \geq r(\theta)$ shifts the objective function in Eq. (2) to incorporate $\mu = \eta$ instead of $\mu = \eta\lambda$. Since $r(\theta) > \overline{q}(\theta;\eta)$, the monopolist would choose a deviation to $\hat{q} = r(\theta)$ with associated profits equal to $(1+\eta\lambda)m(r(\theta),\theta) - c(r(\theta))$, which are less than or equal to profits derived from $\overline{q}(\theta;\eta\lambda)$. Hence, the firm has no profitable deviation.

Showing that the revenue maximizing quality level is $\overline{q}(\theta;\eta)$ when $r(\theta) \leq \overline{q}(\theta;\eta)$ is similar and therefore omitted. In this case, profits are equal to

$$TS(\overline{q}(\theta;\eta),\theta) \;=\; (1+\eta)m(\overline{q}(\theta;\eta),\theta) \;+\; (\eta\lambda-\eta)m(r(\theta),\theta) \;-\; c(\overline{q}(\theta;\eta)). \tag{15}$$

Now suppose that for the θ-consumer, $\overline{q}(\theta;\eta) < r(\theta) < \overline{q}(\theta;\eta\lambda)$. Choosing a quality $\hat{q} \geq r(\theta) > \overline{q}(\theta;\eta)$ leaves us with $\mu = \eta$ in Eq. (2), and thus we are in the strictly decreasing part of the total surplus. Similarly, choosing $\hat{q} \leq r(\theta) < \overline{q}(\theta;\eta\lambda)$ yields $\mu = \eta\lambda$

in Eq. (2), so that the monopolist is now in the strictly increasing section of the total surplus. It follows that the revenue maximizing quality is $r(\theta)$, which generates profits equal to

$$TS(r(\theta),\theta) \;=\; (1+\eta\lambda)m(r(\theta),\theta) \;-\; c(r(\theta)). \tag{16}$$

This completes the proof. □

Proof of Proposition 2.
1. Under loss neutrality, the total valuation is $v(q,\theta) = (1+\eta)m(q,\theta)$, independently of the consumers' reference plan. Readily from Equation 2, one has $q^{fb}(\theta) = \overline{q}(\theta;\eta)$ for all types $\theta \in \Theta$.

2. Immediately from Proposition 1.

3–4. Assume that $r(\theta) < \overline{q}(\theta;\eta) = q^{fb}(\theta)$. Then optimal revenue for the firm from the θ-consumer is given by Eq. (15). Increasing the reference level to $0 < \hat{r}(\theta) \leq \overline{q}(\theta;\eta)$ does not change the offered quality but strictly increases profits, as $(\eta\lambda-\eta)m_q(r(\theta),\theta) > 0$. When $\overline{q}(\theta;\eta) < r(\theta) = q^{fb}(\theta) < \overline{q}(\theta;\eta\lambda)$, optimal profits for the firm are given by Eq. (16). Thus, an increase in the reference level to $r(\theta) < \hat{r}(\theta) \leq \overline{q}(\theta;\eta\lambda)$ strictly increases the offer from the firm to this level and strictly raises profits, since $(1+\eta\lambda)m_q(r(\theta),\theta) - c_q(r(\theta)) > 0$. Finally, when the reference level is $\overline{q}(\theta;\eta\lambda) < r(\theta)$, optimal profits are given by Eq. (14). Thus, an increase in the reference level alters neither the optimal offer from the firm nor net revenues. □

Derivation of Equation 5.
Fix a quality level $\hat{q} > 0$ and assume that the reference plan r is strictly increasing around $\hat{\theta}$, for an arbitrary type $\hat{\theta}$ in the interior of Θ (all remaining cases are similarly obtained). For any $\theta \in \Theta$, let $\mu(\hat{q},\theta)$ denote the value that μ attains when comparing $\hat{q}$ to the reference level $r(\theta)$. The net total valuation as a function of types is given by:

$$\theta \;\mapsto\; v(\hat{q},\theta) \;=\; (1+\mu(\hat{q},\theta))\, m(\hat{q},\theta) \;+\; (\eta\lambda - \mu(\hat{q},\theta))\, m(r(\theta),\theta), \tag{17}$$

where $\mu(\hat{q},\theta) = \eta$ if $\hat{q} > r(\theta)$ and $\mu(\hat{q},\theta) = \eta\lambda$ if $\hat{q} \leq r(\theta)$. The reference plan is a piecewise continuously differentiable function, hence we omit discussion of kinks in the valuation function due to kinks in the reference plan. This will not have any consequence on the derivation of optimal contract menu. The function $v(\hat{q},\cdot)$ in (17) has bounded left and right partial derivatives at $\hat{\theta}$ defined, respectively, by:

$$v_\theta^+(\hat{q},\hat{\theta}) \;\equiv\; \lim_{\theta\downarrow\hat{\theta}} \frac{v(\hat{q},\theta) \;-\; v(\hat{q},\hat{\theta})}{\theta-\hat{\theta}} \qquad \text{and} \qquad v_\theta^-(\hat{q},\hat{\theta}) \;\equiv\; \lim_{\theta\uparrow\hat{\theta}} \frac{v(\hat{q},\theta) \;-\; v(\hat{q},\hat{\theta})}{\theta-\hat{\theta}}.$$

Suppose first that $r(\hat{\theta}) > \hat{q}$. Then for all types θ sufficiently close to $\hat{\theta}$, one has $\mu(\hat{q},\theta) = \mu(\hat{q},\hat{\theta}) = \eta\lambda$. It follows that $v_\theta^+(\hat{q},\hat{\theta}) = v_\theta^-(\hat{q},\hat{\theta}) = v_\theta(\hat{q},\hat{\theta}) = (1+\eta\lambda)m_\theta(\hat{q},\hat{\theta})$. Suppose next that $\hat{q} > r(\hat{\theta})$, so that for all types θ sufficiently close to $\hat{\theta}$, one has $\mu(\hat{q},\theta) = \mu(\hat{q},\hat{\theta}) = \eta$. It follows that $v_\theta^+(\hat{q},\hat{\theta}) = v_\theta^-(\hat{q},\hat{\theta}) = v_\theta(\hat{q},\hat{\theta}) = (1+\eta)m_\theta(\hat{q},\hat{\theta}) + (\eta\lambda-\eta)\frac{d}{d\theta}\big(m(r(\hat{\theta}),\hat{\theta})\big)$.

Finally, suppose that for the $\hat{\theta}$-consumer, $r(\hat{\theta}) = \hat{q}$. Note that for $\theta' < \hat{\theta} < \theta''$, one has $\mu(\hat{q},\theta') = \eta$ and $\mu(\hat{q},\hat{\theta}) = \mu(\hat{q},\theta'') = \eta\lambda$. By definition, we obtain

$$\begin{aligned} v_\theta^+(\hat{q},\hat{\theta}) &\equiv \lim_{\theta\downarrow\hat{\theta}} \frac{v(\hat{q},\theta) - v(\hat{q},\hat{\theta})}{\theta - \hat{\theta}} \\ &= \lim_{\theta\downarrow\hat{\theta}} \frac{1}{\theta - \hat{\theta}} \Big\{ (1+\mu(\hat{q},\theta))m(\hat{q},\theta) \;+\; (\eta\lambda - \mu(\hat{q},\theta))m(r(\theta),\theta) \\ &\qquad\qquad -\; (1+\mu(\hat{q},\hat{\theta}))m(\hat{q},\hat{\theta}) \;-\; (\eta\lambda - \mu(\hat{q},\hat{\theta}))m(r(\hat{\theta}),\hat{\theta}) \Big\} \\ &= \lim_{\theta\downarrow\hat{\theta}} (1+\eta\lambda) \frac{m(\hat{q},\theta) - m(\hat{q},\hat{\theta})}{\theta - \hat{\theta}} \;=\; (1+\eta\lambda)\, m_\theta(\hat{q},\hat{\theta}). \end{aligned}$$

Similarly, using the fact that $\hat{q} = r(\hat{\theta})$, we have

$$\begin{aligned} v_\theta^-(\hat{q},\hat{\theta}) &\equiv \lim_{\theta\uparrow\hat{\theta}} \frac{v(\hat{q},\theta) - v(\hat{q},\hat{\theta})}{\theta - \hat{\theta}} \\ &= \lim_{\theta\uparrow\hat{\theta}} \frac{1}{\theta - \hat{\theta}} \Big\{ (1+\eta)m(\hat{q},\theta) \;+\; (\eta\lambda - \eta)m(r(\theta),\theta) \\ &\qquad\qquad -\; (1+\eta\lambda)m(\hat{q},\hat{\theta}) \;\pm\; \eta\, m(\hat{q},\hat{\theta}) \Big\} \\ &= \lim_{\theta\uparrow\hat{\theta}} (1+\eta) \frac{m(\hat{q},\theta) - m(\hat{q},\hat{\theta})}{\theta - \hat{\theta}} \;+\; \lim_{\theta\uparrow\hat{\theta}} (\eta\lambda - \eta) \frac{m(r(\theta),\theta) - m(r(\hat{\theta}),\hat{\theta})}{\theta - \hat{\theta}} \\ &= (1+\eta)\, m_\theta(\hat{q},\hat{\theta}) \;+\; (\eta\lambda - \eta) \tfrac{d}{d\theta}\big(m(r(\hat{\theta}),\hat{\theta})\big). \end{aligned}$$

Note also that $v_\theta^+(\hat{q},\hat{\theta}) - v_\theta^-(\hat{q},\hat{\theta}) \;=\; -(\eta\lambda - \eta)m_q(r(\hat{\theta}),\hat{\theta})r_\theta(\hat{\theta}) \;\leq\; 0$. Thus, for the $\hat{\theta}$-consumer, one has that $\varphi(\hat{q},\hat{\theta})$ is equal to the non-empty closed interval

$$\big[(1+\eta\lambda)\, m_\theta(\hat{q},\hat{\theta})\,,\; (1+\eta)\, m_\theta(\hat{q},\hat{\theta}) + (\eta\lambda - \eta)\tfrac{d}{d\theta}\big(m(r(\hat{\theta}),\hat{\theta})\big)\big],$$

which collapses to a single point when $r_\theta(\hat{\theta}) = 0$. □

Proof of Proposition 3.
The equivalence between incentive compatibility of contracts $\theta \mapsto (q(\theta), p(\theta))$ and parts (a) and (b) follow from Theorem 1 in Carbajal and Ely (2013). Condition (c) clearly holds when contracts are individually rational. Suppose now that (c) is also in place. Using (b) we express $U(\theta) = U(\theta_L) + \int_{\theta_L}^{\theta} \delta(q(\tilde{\theta}),\tilde{\theta})d\tilde{\theta}$ for any θ-consumer. From (5), any integrable selection is such that $\delta(q(\theta),\theta) \geq 0$, so that $U(\theta) \geq 0$ follows readily. □

Proof of Proposition 4.
Step 1. Suppose that for the θ-consumer one has $r(\theta) \geq q^*(\theta;\eta\lambda)$. The unique maximizer of the integrand in the profit function of Equation 8 with $\mu(\theta) = \eta\lambda$ is precisely $q^*(\theta;\eta\lambda)$. Any deviation to an alternative quality $q \leq r(\theta)$ will only hurt profits as it decreases virtual total surplus in the objective function without changing the value of the lump-sum cost, which remains at zero. Now consider a deviation to $\hat{q} \leq r(\theta)$, which switches $\mu(\theta)$ in the objective function from $\eta\lambda$ to η. Since $r(\theta) \geq q^*(\theta;\eta)$, it follows by the (strong) concavity of $S^*(\cdot,\theta;\eta)$ that the optimal deviation in this case is $\hat{q} = r(\theta)$. The difference between profits at $r(\theta)$ and $\mu(\theta) = \eta\lambda$, and profits at $r(\theta)$ and $\mu(\theta) = \eta$ is given by:

$$(\eta\lambda - \eta)h(\theta)m_q(r(\theta),\theta)r_\theta(\theta) \;\geq\; 0.$$

It follows that profits at $q^{sb}(\theta) = q^*(\theta;\eta\lambda)$ and $\mu(\theta) = \lambda$ are strictly greater than profits at $r(\theta)$ and $\mu(\theta) = \eta\lambda$, which in turn are greater than profits at $r(\theta)$ and $\mu(\theta) = \eta$.

Step 2. Suppose that $q^*(\theta;\eta) < r(\theta) < q^*(\theta;\eta\lambda)$. Consider an alternative quality level $\hat{q}$, such that $\hat{q} > r(\theta) > q^*(\theta;\eta)$. The integrand of Eq. (8) has $\mu(\theta) = \eta$ for any such $\hat{q}$, so that the lump-sump cost is active. Clearly, profits are strictly decreasing in quality as long as $\hat{q} > r(\theta)$, so that there is no upward profitable deviation. One can use a similar argument to show that there is no downward profitable deviation from $q^{sb}(\theta) = r(\theta)$, as any deviation to $\hat{q} < r(\theta)$ has $\mu(\theta) = \eta\lambda$. Therefore $q^{sb}(\theta) = r(\theta)$ is optimal the optimal quality offered by the firm in this case.

Step 3. Suppose that for the θ-consumer, one has $r(\theta) \leq q^*(\theta;\eta)$. As before, the unique maximizer of the integrand of the profit function in Eq. (8) with $\mu(\theta) = \eta$ is $q^*(\theta;\eta)$. Any deviation to a quality level $\hat{q} > r(\theta)$ will not change the value of $\mu(\theta)$ to $\eta\lambda$ and thus will only decrease profits. Among deviations from $q^*(\theta;\eta)$ to quality levels $\hat{q} \leq r(\theta)$ that change the parameter $\mu(\theta)$ to $\eta\lambda$ in Eq. (8), thus avoiding the lump-sum cost, the one generating the highest profits is $\hat{q} = r(\theta)$. The difference between profits at $r(\theta)$ with associated $\mu(\theta) = \eta\lambda$ and profits at $q^*(\theta;\eta)$ with associated $\mu(\theta) = \eta$ is given by:

$$\begin{aligned}\triangle(r(\theta), q^*(\theta;\eta)) &= (\eta\lambda - \eta)h(\theta)m_q(r(\theta),\theta)\, r_\theta(\theta) \\ &\quad - \left\{S^*(q^*(\theta;\eta),\theta;\eta) - S^*(r(\theta),\theta;\eta)\right\}. \qquad (18)\end{aligned}$$

The sign of the above expression depends on the difference between gains associated with offering a quality level $r(\theta)$, which changes the value of $\mu(\theta)$ from η to $\eta\lambda$ in the profit function for the θ-consumer, cancelling lump-sum transfer to higher type consumers, and efficiency gains in virtual total surplus at $\mu(\theta) = \eta$ derived from shifting quality from $r(\theta)$ to $q^*(\theta;\eta)$.

Step 4. We formally show the following, from which our statement in the main text will be derived. Let $\theta' < \theta''$ be two types for whom $q^*(\theta'';\eta) \geq q^*(\theta';\eta) > r(\theta'') > r(\theta') \geq 0$. Then the monopolist either offers to them their respective reference quality levels; or $q^*(\theta';\eta)$ and $q^*(\theta'';\eta)$ to each of them, respectively; or it offers to the θ'-consumer his reference quality level and the quality level $q^*(\theta'';\eta)$ to the θ''-consumer.

That one of this possibilities must hold follows from Step 3. Suppose now that for θ' and θ'' satisfying the premise of this step, one has instead that $q^{sb}(\theta') = q^*(\theta';\eta)$ and $q^{sb}(\theta'') = r(\theta'')$. From Proposition 3, it suffices to show a violation of monotonicity: $v(r(\theta''),\theta'') - v(r(\theta''),\theta') < v(q^*_\eta(\theta'),\theta'') - v(q^*_\eta(\theta'),\theta')$. It is immediate to see that we can write the previous inequality as $m(r(\theta''),\theta'') - m(r(\theta''),\theta') < m(q^*(\theta';\eta),\theta'') - m(q^*(\theta';\eta),\theta')$. Since $\theta'' > \theta'$ and $q^*(\theta';\eta) > r(\theta'')$, the single-crossing condition on the valuation function implies that this inequality is indeed satisfied.

Step 5. From the assumptions in Section 2 it follows that $q^*(\cdot;\mu)$ is everywhere continuous and continuously differentiable except possibly at a type where $q^*(\cdot;\mu)$ turns from zero to positive. Therefore $\theta \mapsto f^\mu(\theta) = r(\theta) - q^*(\theta;\mu)$ is continuous and piecewise continuously differentiable, with bounded left and right derivatives everywhere on Θ. Let $A \subset \Theta$ be the set of types for which $f^\mu(\theta) = 0$ and $f^\mu_\theta(\theta) \neq 0$. Since $f^\mu_\theta(\theta)$ is continuous, it follows that $\theta \in A$ is an isolated point and thus A is a discrete subset of a compact set, hence it is finite. The construction of the optimal quality schedule $\theta \mapsto q^{sb}(\theta)$ in Eq. (10) follows from the previous steps.

It remains to show that the informational constraints, expressed as conditions (a) to (c) of Proposition 3, are in place. One immediately sees from the expression for incentive prices in Eq. (11) that both (b) and (c) are in place. To show that condition (a) —integral monotonicity— is satisfied, let $\theta', \theta'' \in \Theta$ be two consumer types such that $\theta' < \theta''$ and suppose $q^{sb}(\theta') < r(\theta') \leq r(\theta'') < q^{sb}(\theta'')$ holds —all remaining cases are similarly proven. By construction of the optimal quality offers, there exists a type θ_c, with $\theta' \leq \theta_c < \theta''$, for which one has $r(\theta') \leq r(\theta_c) = q^{sb}(\theta_c) \leq r(\theta'')$. Moreover, we can choose θ_c so that $q^{sb}(\theta) \leq r(\theta)$ for all $\theta' \leq \theta \leq \theta_c$, and $q^{sb}(\theta) > r(\theta)$ for all $\theta_c < \theta \leq \theta''$.

We first write the valuation differences in a suitable form:

$$\begin{aligned} &v(q^{sb}(\theta''),\theta'') \;-\; v(q^{sb}(\theta''),\theta') \\ &\qquad = \; v(q^{sb}(\theta''),\theta'') \;-\; v(q^{sb}(\theta''),\theta') \;\pm\; v(q^{sb}(\theta''),\theta_c) \qquad (19)\\ &\qquad \geq \; v(q^{sb}(\theta''),\theta'') \;-\; v(q^{sb}(\theta''),\theta_c) \;+\; v(q^{sb}(\theta_c),\theta_c) \;-\; v(q^{sb}(\theta_c),\theta'). \end{aligned}$$

where the inequality follows from the single crossing property assumed on the consumption valuation $m(q,\theta)$. A similar argument yields to:

$$\begin{aligned} &v(q^{sb}(\theta'),\theta') \;-\; v(q^{sb}(\theta'),\theta'') \qquad (20)\\ &\qquad \geq \; v(q^{sb}(\theta'),\theta') \;-\; v(q^{sb}(\theta'),\theta_c) \;+\; v(q^{sb}(\theta_c),\theta_c) \;-\; v(q^{sb}(\theta_c),\theta''). \end{aligned}$$

Notice now that

$$\begin{aligned} &v(q^{sb}(\theta''),\theta'') \;-\; v(q^{sb}(\theta''),\theta_c) \\ &\qquad = \int_{\theta_c}^{\theta''} \Big\{ (1+\eta) m_\theta(q^{sb}(\theta''),\tilde\theta) \;+\; (\eta\lambda-\eta)\tfrac{d}{d\theta}\big(m(r(\tilde\theta),\tilde\theta)\big) \Big\} d\tilde\theta \qquad (21)\\ &\qquad \geq \int_{\theta^*}^{\theta''} \delta(q^{sb}(\tilde\theta),\tilde\theta)\, d\tilde\theta, \end{aligned}$$

where the inequality follows from the monotonicity of q^{sb} and the fact that $\mu(\tilde\theta)=\eta$ for all $\theta_c<\tilde\theta\leq\theta''$. Furthermore, we can write

$$\begin{aligned} &v(q^{sb}(\theta_c),\theta_c) \;-\; v(q^{sb}(\theta_c),\theta') \\ &\qquad \geq \; (1+\eta\lambda) m(q^{sb}(\theta_c),\theta_c) \;-\; (1+\eta\lambda) m(q^{sb}(\theta_c),\theta') \qquad (22)\\ &\qquad = \int_{\theta'}^{\theta_c} (1+\eta\lambda) m_\theta(q^{sb}(\theta_c),\tilde\theta)\, d\tilde\theta \;\geq\; \int_{\theta'}^{\theta_c} \delta(q^{sb}(\tilde\theta),\tilde\theta)\, d\tilde\theta, \end{aligned}$$

where as before second inequality follow from the monotonicity of the optimal quality schedule and the fact that $\mu(\tilde\theta)=\eta\lambda$ for all $\theta'\leq\tilde\theta\leq\theta_c$. Combining expressions (21) and (22) with Eq. (19), we obtain:

$$v(q^{sb}(\theta''),\theta'') - v(q^{sb}(\theta''),\theta') \;\geq\; \int_{\theta'}^{\theta''} \delta(q^{sb}(\tilde\theta),\tilde\theta)\, d\tilde\theta,$$

which is the first inequality of the integral monotonicity condition of Proposition 3.

To obtain the second inequality, we write:

$$\begin{aligned} &v(q^{sb}(\theta'),\theta_c) \;-\; v(q^{sb}(\theta'),\theta') \qquad (23)\\ &\qquad = \int_{\theta'}^{\theta_c} (1+\eta\lambda) m_\theta(q^{sb}(\theta'),\tilde\theta)\, d\tilde\theta \;\geq\; \int_{\theta'}^{\theta_c} \delta(q^{sb}(\tilde\theta),\tilde\theta)\, d\tilde\theta, \end{aligned}$$

and similarly

$$\begin{aligned} &v(q^{sb}(\theta_c),\theta'') \;-\; v(q^{sb}(\theta_c),\theta_c) \\ &\qquad \leq \int_{\theta_c}^{\theta''} \Big\{ (1+\eta) m_\theta(q^{sb}(\theta_c),\tilde\theta) \;+\; (\eta\lambda-\eta)\tfrac{d}{d\theta}\big(m(r(\tilde\theta),\tilde\theta)\big) \Big\} d\tilde\theta \qquad (24)\\ &\qquad \leq \int_{\theta_c}^{\theta''} \delta(q^{sb}(\tilde\theta),\tilde\theta)\, d\tilde\theta. \end{aligned}$$

Combining expressions (23) and (24) with Eq. (20) above, we obtain the desired inequality. Thus, integral monotonicity is satisfied. □

Proof of Corollary 1.
The properties of the optimal offers $\theta \mapsto q^{sb}(\theta)$ follow readily from Equation 10. □

Proof of Proposition 5.
1–2. The results are deduced from Proposition 4, and expressions (8) and (9). Indeed, for all $r(\theta) \geq q^*(\theta;\eta\lambda)$, the optimal offer is $q^*(\theta;\eta\lambda)$ and profits from the θ-consumer are $TS^*(q^*(\theta;\eta\lambda),\theta) = S^*(q^*(\theta;\eta\lambda),\theta;\eta\lambda)$, since the lump-sum cost is inactive. For $q^*(\theta;\eta) \leq r(\theta) < q^*(\theta;\eta\lambda)$, on the other hand, the optimal offer is $r(\theta)$ and profits from the θ-consumer are $TS^*(r(\theta),\theta) = S^*(r(\theta),\theta;\eta\lambda)$. As $r(\theta) < q^*(\theta;\eta\lambda)$, this last expression is increasing in the reference level.

3. Suppose now that $r(\theta) < q^*(\theta;\eta)$ for the θ-consumer. Consider a change in the reference level to $\hat{r}(\theta)$ such that $r(\theta) < \hat{r}(\theta) < q^*(\theta;\eta)$ —the remaining case, when the reference level increases above $q^*(\theta;\eta)$, will follow from the analysis below. From Proposition 4, four possibilities emerge.

Case 1. The optimal quality for $r(\theta)$ is $q^{sb}(\theta) = r(\theta)$ and the optimal quality for $\hat{r}(\theta)$ is $\hat{q}^{sb}(\theta) = \hat{r}(\theta)$. Here the lump-sum cost is zero for both optimal offers, thus profits from the θ-consumer are $TS^*(r(\theta),\theta) = S^*(r(\theta),\theta;\eta\lambda) < S^*(\hat{r}(\theta),\theta;\eta\lambda) = TS^*(\hat{r}(\theta),\theta)$, as desired.

Case 2. The optimal quality for $r(\theta)$ is $q^{sb}(\theta) = r(\theta)$ and the optimal quality for $\hat{r}(\theta)$ is $\hat{q}^{sb}(\theta) = q^*(\theta;\eta) > \hat{r}(\theta)$. In this case the lump-sum cost is active at $\hat{q}^{sb}(\theta)$, so that profits from the θ-consumer with reference level $\hat{r}(\theta)$ equal $TS^*(q^*(\theta;\eta),\theta) - LS(q^*(\theta;\eta),\theta)$. However, since $\hat{r}(\theta)$ is an option available to the firm, it must be the case that

$$TS^*(q^*(\theta;\eta),\theta) - LS(q^*(\theta;\eta),\theta) \;\geq\; TS(\hat{r}(\theta),\theta) \;=\; S^*(\hat{r}(\theta),\theta;\eta\lambda) \;>\; S^*(r(\theta),\theta;\eta\lambda),$$

where the last term stands for profits from the θ-consumer when his reference level is $r(\theta)$. Thus, per customer profits are strictly greater at $\hat{r}(\theta)$ than at $r(\theta)$.

Case 3. The optimal quality for $r(\theta)$ is $q^{sb}(\theta) = q^*(\theta;\eta)$ and the optimal quality for $\hat{r}(\theta)$ is $\hat{q}^{sb}(\theta) = \hat{r}(\theta)$. In this case, the lump-sum cost is active at $q^{sb}(\theta)$ but becomes inactive at $\hat{q}^{sb}(\theta)$. Consequently, profits from the θ-consumer with $r(\theta)$ equal

$$\begin{aligned} TS^*(q^*(\theta;\eta),\theta) - LS(q^*(\theta;\eta),\theta) \;&=\; S^*(q^*(\theta;\eta),\theta;\eta) \;+\; (\eta\lambda-\eta)m^*(r(\theta),\theta) \\ &\quad-\; (\eta\lambda-\eta)h(\theta)m_q(r(\theta),\theta)r_\theta(\theta). \end{aligned} \tag{25}$$

Whereas profits from the θ-consumer with $\hat{r}(\theta)$ equal

$$\begin{aligned} TS^*(\hat{r}(\theta),\theta) \;&\geq\; S^*(q^*(\theta;\eta),\theta;\eta) \;+\; (\eta\lambda-\eta)m^*(\hat{r}(\theta),\theta) \\ &\quad-\; (\eta\lambda-\eta)h(\theta)m_q(\hat{r}(\theta),\theta)\hat{r}_\theta(\theta), \end{aligned} \tag{26}$$

where the inequality follows from the fact that $q^*(\theta;\eta)$ was an available alternative for the monopolist when the reference level is $\hat{r}(\theta)$.

Using Eq. (25) and Eq. (26), we see that the difference in profits for $\hat{r}(\theta)$ and $r(\theta)$, which we denote by $\Delta\pi(\hat{r}(\theta),r(\theta))$, is such that

$$\begin{aligned} \Delta\pi(\hat{r}(\theta),r(\theta)) \;&\geq\; (\eta\lambda-\eta)\big(m^*(\hat{r}(\theta),\theta) - m^*(r(\theta),\theta)\big) \\ &\quad-\; (\eta\lambda-\eta)h(\theta)\big(m_q(\hat{r}(\theta),\theta)\hat{r}_\theta(\theta) - m_q(r(\theta),\theta)r_\theta(\theta)\big). \end{aligned} \tag{27}$$

The first term in the right-hand side of the above expression is strictly positive, as the virtual consumption valuation $m^*(\cdot,\theta)$ is strictly increasing in q for $q < q^*(\theta;\eta)$. Thus, a sufficient condition for $\Delta\pi(\hat{r}(\theta),r(\theta)) > 0$ is that

$$m_q(\hat{r}(\theta),\theta)\hat{r}_\theta(\theta) - m_q(r(\theta),\theta)r_\theta(\theta) \;\leq\; 0,$$

which is precisely Eq. (12).

Case 4. The optimal qualities are $q^{sb}(\theta) = \hat{q}^{sb}(\theta) = q^*(\theta;\eta)$, so that in both cases the lump-sum cost is active. Here the difference in profits $\Delta\pi(\hat{r}(\theta),r(\theta))$ is equal to the right-hand side of Eq. (27). Thus the previous condition is also sufficient. □

Proof of Proposition 6.
When for any θ-consumer one has $q^*(\theta;\eta\lambda) \geq r(\theta) \geq q^*(\theta;\eta)$, it follows immediately from Eq. (10) that $q^{sb}(\theta) = r(\theta)$. Now suppose that $q^*(\theta;\eta)$ is strictly greater than $r(\theta)$ for some θ-consumer. Then, from Proposition 4, the optimal quality level is either $q^{sb}(\theta) = q^*(\theta;\eta)$ or $q^{sb}(\theta) = r(\theta)$. By (b), one has that the monopolist chooses $r(\theta)$ in this case. Thus the reference plan is self-confirming. □

Proof of Proposition 7.
1. Consider expected profits for the firm generated by any self-confirming reference plan $\theta \mapsto r(\theta)$. From Eq. (8), when $q^{sb}(\theta) = r(\theta)$ for all buyers, one has

$$\Pi^{sb} \;=\; \int_{\underline{\theta}}^{\theta_H} S^*(r(\theta),\theta;\eta\lambda)\, f(\theta)\, d(\theta).$$

For each state $\theta \in \Theta$, per customer profits equal $S^*(q,\theta;\eta\lambda)$ and are strictly increasing in q, for all $q < q^*(\theta;\eta\lambda)$, strictly decreasing in q for all $q > q^*(\theta;\eta\lambda)$ and attain a unique maximum at $q = q^*(\theta;\eta\lambda)$. Thus, per customer profits are strictly increasing in the self-confirming reference level, for $r(\theta) \leq q^*(\theta;\eta\lambda)$, with this last being the firm's preferred reference level.

2. For any given quality level q, from single crossing we obtain that $m_{q\theta}(q,\theta) > 0$. Thus, when the self-confirming reference quality plan specifies a positive quality level for the θ-consumer, an increase in the reference level also increases value of the first integral in the right-hand side of Eq. (13). On the other hand, when the function $q \mapsto m_\theta(q,\tilde{\theta})$ is concave for all $\tilde{\theta} \in \Theta$, we obtain that $m_{qq\theta}(q,\tilde{\theta})$ is non-negative for all quality levels q. Since $r(\tilde{\theta}) \leq r(\theta)$ holds for all $\theta_L \leq \tilde{\theta} \leq \theta$, we have that the value of the second integral of Eq. (13) is non-negative as well. The result is now established.

3. This follows immediately from the previous two points of the proposition. □

References

Abeler, J., A. Falk, L. Goette, and D. Huffman (2011): "Reference points and effort provision," *American Economic Review*, 101(2), 470–492.

Ariely, D., and I. Simonson (2003): "Buying, bidding, playing, or competing? Value assessment and decision dynamics in online auctions," *Journal of Consumer Psychology*, 13(1-2), 113–123.

Bell, D. R., and J. M. Lattin (2000): "Looking for loss aversion in scanner panel data: the confounding effect of price response heterogeneity," *Marketing Science*, 19(2), 185–200.

Briesch, R. A., L. Krishnamurthi, T. Mazumdar, and S. P. Raj (1997): "A comparative analysis of reference price models," *Journal of Consumer Research*, 24(2), 202–214.

Carbajal, J. C., and J. C. Ely (2013): "Mechanism design without revenue equivalence," *Journal of Economic Theory*, 148(1), 104–133.

Crawford, V. P., and J. Meng (2011): "New York City cab drivers' labor supply revisited: reference-dependent preferences with rational-expectations targets for hours and income," *American Economic Review*, 101(5), 1912–1932.

De Giorgi, E. G., and T. Post (2011): "Loss aversion with a state-dependent reference point," *Management Science*, 57(6), 1094–1110.

DellaVigna, S., and U. Malmendier (2004): "Contract design and self-control: theory and evidence," *Quarterly Journal of Economics*, 119(2), 353–402.

Eisenhuth, R. (2012): "Reference dependent mechanism design," Discussion paper, Northwestern University.

ELIAZ, K., AND R. SPIEGLER (2006): "Contracting with diversely naive agents," *Review of Economic Studies*, 73(3), 689–714.

FOGEL, S., D. LOVALLO, AND C. CARINGAL (2004): "Loss aversion for quality in consumer choice," *Australian Journal of Management*, 29(1), 45–63.

GALPERTI, S. (2011): "Commitment, Flexibility, and Optimal Screening of Time Inconsistency," Discussion paper, Northwestern University.

GILL, D., AND V. PROWSE (2012): "A structural analysis of disappointment aversion in a real effort competition," *American Economic Review*, 102(1), 469–503.

GRUBB, M. D. (2009): "Selling to overconfident consumers," *American Economic Review*, 99(5), 1770–1807.

HAHN, J.-H., J. KIM, S.-H. KIM, AND J. LEE (2012): "Screening loss averse consumers," Discussion paper, Seoul National University.

HARDIE, B. G. S., E. J. JOHNSON, AND P. S. FADER (1993): "Modeling loss aversion and reference dependence effects on brand choice," *Marketing Science*, 12(4), 378–394.

HEIDHUES, P., AND B. KŐSZEGI (2008): "Competition and price variation when consumers are loss averse," *American Economic Review*, 98(4), 1245–1268.

——— (2012): "Regular prices and sales," Forthcoming: *Theoretical Economics*.

HERWEG, F., AND K. MIERENDORFF (2013): "Uncertain demand, consumer loss aversion, and flat-rate tariffs," *Journal of the European Economic Association*, 11(2), 399–432.

HEYMAN, J. E., Y. ORHUN, AND D. ARIELY (2004): "Auction fever: the effect of opponents and quasi-endowment on product valuations," *Journal of Interactive Marketing*, 18(4), 7–21.

KAMENICA, E. (2008): "Contextual inference in markets: on the informational content of product lines," *American Economic Review*, 98(5), 2127–2149.

KARLE, H. (2013): "Creating attachment through advertising: loss aversion and prepurchase information," Discussion paper, ETH Zurich.

KŐSZEGI, B., AND M. RABIN (2006): "A model of reference-dependent preferences," *Quarterly Journal of Economics*, 121(4), 1133–1165.

KRISHNA, V., AND E. MAENNER (2001): "Convex potentials with an application to mechanism design," *Econometrica*, 69(4), 1113–1119.

LAMBRECHT, A., K. SEIM, AND B. SKIERA (2007): "Does uncertainty matter? Consumer behavior under three-part tariffs," *Marketing Science*, 26(5), 698–710.

LAMBRECHT, A., AND B. SKIERA (2006): "Paying too much and being happy about it: existence, causes, and consequences of tariff-choice biases," *Journal of Marketing Research*, 43(2), 212–223.

LANGE, A., AND A. RATAN (2010): "Multi-dimensional reference-dependent preferences in sealed-bid auctions – how (most) laboratory experiments differ from the field," *Games and Economic Behavior*, 68(2), 634–645.

MASKIN, E., AND J. RILEY (1984): "Monopoly with incomplete information," *RAND Journal of Economics*, 15(2), 174–196.

MILGROM, P. R., AND I. SEGAL (2002): "Envelope theorems for arbitrary choice sets," *Econometrica*, 70(2), 583–601.

MUSSA, M., AND S. ROSEN (1978): "Monopoly and product quality," *Journal of Economic Theory*, 18(2), 301–317.

MYERSON, R. B. (1981): "Optimal auction design," *Mathematics of Operations Research*, 6(1), 58–73.

NOVEMSKY, N., AND D. KAHNEMAN (2005): "The boundaries of loss aversion," *Journal of Marketing Research*, 42(2), 119–128.

ORHUN, A. Y. (2009): "Optimal product line design when consumers exhibit choice set-dependent preferences," *Marketing Science*, 28(5), 868–886.

Pope, D. G., and M. E. Schweitzer (2011): "Is Tiger Woods loss averse? Persistent bias in the face of experience, competition, and high stakes," *American Economic Review*, 101(1), 129–157.

Rosato, A. (2013): "Selling substitute goods to loss-averse consumers: limited availability, bargains, and rip-offs," Discussion paper, University of California Berkeley.

Simonson, I. (1989): "Choice based on reasons: the case of attraction and compromise effects," *Journal of Consumer Research*, 16(2), 158–174.

Spiegler, R. (2011): "Monopoly pricing when consumers are antagonized by unexpected price increases," Forthcoming: *Economic Theory*.

Sugden, R. (2003): "Reference-dependent subjective expected utility," *Journal of Economic Theory*, 111(2), 172–191.

Thaler, R. (1985): "Mental accounting and consumer choice," *Marketing Science*, 4(3), 199–214.

Tversky, A., and D. Kahneman (1991): "Loss aversion in riskless choice: a reference-dependent model," *Quarterly Journal of Economics*, 106(4), 1039–1061.

Williams, S. R. (1999): "A characterization of efficient, Bayesian incentive compatible mechanisms," *Economic Theory*, 14(1), 155–180.

School of Economics, The University of New South Wales
E-mail address: jc.carbajal@unsw.edu.au

Department of Economics, Northwestern University
E-mail address: jeffely@northwestern.edu