Contracts with Framing*

Yuval Salant and Ron Siegel†

Northwestern University

November 2013

Abstract

We propose a theory of contracts with frames. Frames are used by a contract designer to affect how an agent evaluates various options in the contract. The effect of the frame is not persistent, and the agent can renege on the contract after the effect wears off. We observe that framing does not increase the designer's profit when the agent does not have private information or when framing decreases the agent's willingness to pay. Framing increases profit when it increases willingness to pay in a way that does not distort incentives too much. We characterize the profit-maximizing contract in specific environments, and study applications to price discrimination, insurance, and auctions.

1 Introduction

This paper begins with two observations. The first is that firms and other economic agents often influence individual behavior via framing. For example, when designing a brochure that describes membership tiers to potential members, institutions often highlight one tier in order to influence how different tiers are evaluated.[1] Similarly, by positioning big-ticket items in the

*We thank V. Bhaskar, Daniel Diermeier, Simone Galperti, Botond Koszegi, Margaret Meyer, Alessandro Pavan, Ran Spiegler, and seminar participants at Penn-State, Northwestern, the European Summer Symposium in Economic Theory on Behavioral Mechanism Design, and the Bounded Rationality in Choice conference at St. Andrews for helpful comments.

†Email: y-salant@northwestern.edu and r-siegel@northwestern.edu

[1] The Brookfield zoo membership brochure at http://www.brookfieldzoo.org/CZS/Membership highlights the "family plus" membership in at least three different ways.

entrance to every warehouse, Costco may increase consumers' willingness to pay for other items in the warehouse.

The second observation is that the extent to which individual behavior can be influenced by framing is limited. One reason is that individuals are often able to costlessly renege on their purchase decisions once the framing effect wears off. In some countries, e.g. in Israel, a return policy is mandated by law, while in other countries, e.g. in the United States, such a policy is an important part of firms' business strategies. Another reason is that consumers may learn to anticipate their frame-dependent behavior, and thus avoid the interaction with a seller altogether. Such anticipation may arise, for example, when consumers engage in similar interactions frequently.

The goal of this paper is to study the optimal design of product menus with frames by a monopolistic seller when consumers either anticipate their frame-dependent behavior or can renege on their purchase decisions.

We study a contracting environment in which a profit-maximizing principal introduces to an agent a menu of products along with a frame. The frame includes details that cause the agent to behave in a way that may be inconsistent with his preferences. We denote by U a function that summarizes the agent's preferences, or how the agent evaluates outcomes absent any framing, and by U^f a function that summarizes how the agent evaluates outcomes in a frame f. Given a product menu Z with a frame f, the agent chooses a U^f-maximal product if it is U^f- and U-superior to not making a purchase, and otherwise does not purchase anything. This specification of the agent's choice procedure aims to capture situations in which framing affects the agent's behavior at the point of sale (as described by U^f), but not the intrinsic value the agent assigns to each product (as described by U). That is, the framing effect is temporary in the sense that it does not change the agent's preferences over outcomes. The agent either realizes this ex-post and returns the product if he overpaid according to his preferences, or he anticipates the framing effect before interacting with the principal, and may therefore avoid the interaction.

We first investigate the impact of framing on the principal's profit. We observe that when the agent does not have private information, an optimal contract with framing does not generate more profit than an optimal frameless contract. This is because the agent's participation decision is consistent with his preferences. We then show that when the agent has private information, frames that reduce the agent's willingness to pay decrease the principal's profit and will therefore not be used. We also show, by way of an example, that even frames that increase the agent's willingness to pay may decrease the principal's profit. Finally, we observe that frames that increase the agent's willingness to pay in a way that does not incentivize lower types to mimic higher types "too much" increase the principal's profit.

We then characterize the optimal contract with framing in environments in which the agent's private information has two possible values, low and high, and frames increase the agent's willingness to pay (but not "too much"). In contrast to the standard theory of contracts, we establish that the low-type agent always participates in the contract, that the high type may not participate, and that when the high type participates, he over-consumes relative to the efficient outcome and may not obtain any information rents even though the low type participates.

To illustrate our results, we consider an insurance setting a-la Stiglitz (1977), in which a monopolistic insurance provider offers a menu of insurance policies to a population of risk-averse individuals. Each individual's privately-known risk of having an accident is either low or high. In addition to offering the menu, the provider can also highlight one policy in the menu, which causes individuals to anticipate regret in case of an accident if they obtain less coverage than in the highlighted policy. We show that the optimal menu of insurance policies differs from the optimal menu in Stiglitz's setting in two ways. First, the menu always includes a policy with partial coverage, which low-risk individuals choose. This is in contrast to Stiglitz's setting, in which low-risk individuals are not insured if their proportion in the population is small. Second, the menu includes either an over-insuring policy that high-risk individuals choose or an over-priced policy that is never chosen. This is again in contrast to Stiglitz's setting, in which high-risk individuals are always efficiently insured.

The relevance of frames as a design parameter may extend beyond contracting environments. For example, in Delgado et al. (2008) experimental auction, frames that highlight the possibility of losing lead to aggressive bidding and higher revenue. We conclude by analyzing a simple example of an efficient auction, in which the designer can increase via framing the degree of bidders' anticipated disappointment from losing and thus their bidding behavior. As in our contracting environment, the framing effect is temporary, and bidders anticipate it. This implies that inducing participation in a framed auction is harder than in a frameless one. We observe that despite this fact, the increase in bidders' willingness to pay for the item conditional on participation may be so large that the revenue in an efficient auction with an appropriately chosen frame is larger than in a revenue-maximizing (inefficient) frameless auction.

Our paper is related to several growing literatures. The specification of the agent builds on the framework of individual choice with frames developed by Salant and Rubinstein (2008) and Rubinstein and Salant (2012). The primitives that describe the agent in our model correspond to their framework, but our specification of the agent's choice procedure is different. Another paper that studies individual choice with frames is Ahn and Ergin (2010), who axiomatize frame-dependent preferences over acts, where a frame is a partition over the state space and acts are measurable with respect to the frame. In contrast to these papers, we study how frame-dependent behavior influences strategic interactions.

In the context of strategic interactions with frame-dependent behavior, Piccione and Spiegler (2012) and Spiegler (2013) study competition between two firms in a complete-information setting in which frames influence consumers' ability to compare the firms' actions, such as prices. Firms choose "marketing messages," in addition to their actions, and these messages jointly determine the frame. The frame and the actions determine how the market is split between the firms. We study a different question, namely the optimal design of product menus with frames in an incomplete-information monopolistic setting. Our model of consumer behavior is also different: frames distort behavior at the point of sale, and consumers either anticipate this distortion or can ex-post renege on their purchase.

Another related literature is the literature on behavioral contract theory (see Kőszegi (2013) for an excellent survey), and in particular the literature on screening agents with non-standard preferences. In this literature, the agent has at the outset some private information, either on his degree of inconsistency (see Eliaz and Spiegler (2006), Esteban and Miyagawa (2006), Esteban, Miyagawa, and Shum (2007), and Galperti (2013)), or on some payoff-relevant parameter, such as his willingness to pay (see Esteban and Miyagawa (2006) and Carbajal and Ely (2012)). The focus is on the design of an optimal product menu or menus from which the agent makes choices. In our framework the principal has an additional tool, frames, which he uses to temporarily influence how consumers evaluate different outcomes at the point of sale. Our focus is on the optimal use of profit-enhancing frames, and product menus that complement them, to screen agents with payoff-relevant private information.

There are also papers that study implementation with boundedly-rational agents. De Clippel (2013) studies implementation with general choice functions. Glazer and Rubinstein (2013) study a persuasion model in which agents are limited in their ability to find arguments that satisfy a set of rules specified by a principal in order to screen agents. We focus on framing as the cause for boundedly-rational behavior, and study the effect of frame-dependent behavior on implementation and on the design of profit-maximizing contracts.

The paper proceeds as follows. Section 2 presents a price discrimination example that illustrates how the predictions of the theory of contracts with frames differ from those of the standard theory. Section 3 introduces the framework. Section 4 discusses how frames that change the agent's willingness to pay influence the principal's profit. Section 5 analyzes the optimal contract in environments in which the agent's private information has two possible values. Section 6 studies an application to monopolistic insurance. Section 7 concludes with an application to auction design. The Appendix contains proofs that do not appear in the main text.

2 Example: Second-degree price discrimination with framing

We begin with a simple example of contracting with frames. A risk-neutral profit-maximizing seller wishes to sell a good to a population of buyers, each with unit demand. Producing the good is costless (costly production in discussed in Example 3 in Section 4). Each buyer's preferences over bundles (x, t), where $x \in [0, \rho]$ for some $\rho > 0$ is the quality of the good and t is the price of the good, are summarized by the utility function $U(x,t,\theta) = x\theta - t$, where $\theta \in \{L, H\}$ with $0 < L < H$ is the buyer's privately-known valuation. The proportion of Low-valuation buyers is $\pi_L > 0$, and the proportion of High-valuation buyers is $1 - \pi_L > 0$. Absent any framing, each buyer maximizes the function U when choosing among bundles.

In reality, sellers can increase buyers' willingness to pay at the point of sale via framing. For example, by positioning big ticket items such as LCD televisions at the entrance to its warehouses, Costco may increase buyers' willingness to pay for other products in the warehouse. We model this by allowing the seller to choose a frame $f \in F = \{0 < f_1 < \ldots < f_N\} \subset R_+$, which causes a buyer with valuation θ to behave at the point of sale as if his valuation were $\theta + f$. That is, in the frame f the buyer maximizes the function $U^f(x,t,\theta) = x(\theta + f) - t$.

The interaction between the seller and the buyers is as follows. The seller chooses a contract to offer buyers. A contract includes a frame f and a menu of product qualities and associated prices. We refer to the contract in which $f = 0$ and thus $U^f = U$ as the frameless contract. At the point of sale, each buyer chooses a U^f-maximal bundle from the menu if this bundle is weakly U^f-superior to not purchasing anything, i.e., obtaining the outside option $(0, 0)$. After the effect of the frame wears off, the buyer reevaluates his purchase decision according to U, and returns the purchased product to the seller for a full refund if it is strictly U-inferior to not purchasing anything. Returns are costless to the seller and the buyer.

We now compare the optimal contracts with and without framing. As we will see, the optimal contract with framing differs from the optimal frameless contract in two respects. First, the contract offers low-valuation buyers a positive-quality good that they purchase (and do not return) independently of their proportion in the population. Second, if the proportion of low-valuation buyers is not "too large," high-valuation buyers do not obtain information rents, even though low-valuation buyers purchase a positive-quality good.

2.1 Optimal contract without framing

In any frameless contract, if the buyer decides to purchase some bundle, he will not return it. This is because his interim (i.e., at the point of sale) and ex-post (i.e., after making the purchase) decisions are made according to the same function U. We can thus ignore the return possibility. Moreover, because the seller can replicate the outcome of any contract in which some type of buyers does not participate by offering a contract in which this type participates and obtains the outside option $(0, 0)$, the seller's profit maximization program can be written as follows:

Choose $\{(x_L, t_L), (x_H, t_H)\}$ to maximize $\pi_L t_L + (1 - \pi_L) t_H$ subject to:

$$\mathrm{IR}^U_\theta \; : \; x_\theta \theta - t_\theta \geq 0 \text{ for } \theta \in \{L, H\},$$

$$\mathrm{IC}^U_\theta \; : \; x_\theta \theta - t_\theta \geq x_{\theta'} \theta - t_{\theta'} \text{ for } \theta, \theta' \in \{L, H\} \text{ and } \theta' \neq \theta.$$

The Individual Rationality constraint IR^U_θ means that a type θ buyer wishes to participate in the interaction with the seller. The Incentive Compatibility constraint IC^U_θ means that conditional on participating, a type θ buyer chooses his intended bundle (x_θ, t_θ) over the other type's bundle. It is well-known that the optimal contract has the following properties (see, for example, Fudenberg and Tirole (1992, Chapter 7)):

- The low type obtains no information rents (IR^U_L binds), i.e., $t_L = x_L L$,
- The high type purchases the first-best quality, i.e., $x_H = \rho$,
- The high type is U-indifferent between his bundle and the low type's bundle (IC^U_H binds), i.e., $\rho H - t_H = x_L H - t_L$, so $t_H = \rho H - x_L(H - L)$,
- IC^U_L holds (because IC^U_H binds), and
- IR^U_L and IC^U_H imply IR^U_H.

The seller's problem therefore reduces to choosing x_L to maximize the linear function $\pi_L x_L L + (1 - \pi_L)(\rho H - x_L(H - L))$. Consequently, the profit-maximizing frameless contact is:

- The single-bundle contract $(\rho, \rho H)$ that excludes the low type if $\pi_L < 1 - L/H$, and
- The single-bundle pooling contract $(\rho, \rho L)$ otherwise.[2]

To summarize, in the optimal frameless contract the seller excludes low-valuation buyers if their proportion in the population is small enough, and when low-valuation buyers purchase positive quality, high-valuation buyers obtain information rents.

2.2 Optimal contract with framing

Consider now the seller's problem with framing. We first solve for the optimal contract with a given frame $f > 0$, and then optimize over frames.

Fix a frame $f > 0$. Proposition 2 in Section 5 shows that both types obtain positive qualities in the optimal contract with the frame f. We can thus write the seller's problem given the frame f as follows:

Choose $\{(x_L, t_L), (x_H, t_H)\}$ to maximize $\pi_L t_L + (1 - \pi_L) t_H$ subject to:

$$\mathrm{IR}^f_\theta \;:\; x_\theta(\theta + f) - t_\theta \geq 0 \text{ for } \theta \in \{L, H\},$$

$$\mathrm{IC}^f_\theta \;:\; x_\theta(\theta + f) - t_\theta \geq x_{\theta'}(\theta + f) - t_{\theta'} \text{ for } \theta, \theta' \in \{L, H\} \text{ and } \theta' \neq \theta,$$

$$\mathrm{IR}^U_\theta \;:\; x_\theta \theta - t_\theta \geq 0 \text{ for } \theta \in \{L, H\}.$$

[2] Both contracts are optimal when $\pi_L = 1 - L/H$.

The constraints IR^f_θ and IC^f_θ correspond to the behavior of type θ buyers at the point of sale. The constraint IR^U_θ means that type θ buyers do not return the product. Note that IR^U_θ implies IR^f_θ, so we can ignore IR^f_θ.

If the optimal contract is a pooling contract, then it is the contract $(\rho, \rho L)$. This is because $(\rho, \rho L)$ satisfies all the constraints and maximizes the seller's profit subject to IR^U_L. Otherwise, the optimal contract is a separating contract. Proposition 3 in section 5 characterizes the optimal separating contract. In this contract:

- The low type obtains no information rents (IR^U_L binds), i.e., $t_L = x_L L$,
- The high type over-consumes relative to the first-best quality. Because quality is bounded above by ρ, which is also the first-best quality, we have that $x_H = \rho$,
- The high type is U^f-indifferent between his bundle and the low type's bundle (IC^f_H binds), i.e., $\rho(H + f) - t_H = x_L(H + f) - t_L$, so $t_H = \rho(H + f) - x_L(H + f - L)$, and
- IC^f_L holds (because IC^f_H binds).

A key difference from the optimal frameless contract is that IR^U_H is not implied by the other constraints.

The seller's problem therefore reduces to choosing x_L to maximize the linear function $\pi_L x_L L + (1 - \pi_L)(\rho(H + f) - x_L(H + f - L))$ subject to IR^U_H, i.e., $x_L \geq \rho f/(H + f - L)$.

Consequently, the profit-maximizing contract with the frame f is:

- the two-bundle separating contract $\{(\rho f/(H+f-L), \rho L f/(H+f-L)), (\rho, \rho H)\}$ if $\pi_L < 1 - L/(H + f)$, and
- the single-bundle pooling contract $(\rho, \rho L)$ otherwise.

Note the two differences from the optimal frameless contract. First, both types obtain positive-quality goods independently of the distribution of types. Second, even though the low type obtains a positive-quality good, the high type may not obtain information rents.

Consider now optimizing over the frame f. If $\pi_L \geq 1 - L/(H + f_N)$, then the optimal contract with any frame f is the pooling contract with the bundle $(\rho, \rho L)$, so framing does not increase the principal's profit. If $\pi_L < 1 - L/(H + f_N)$, then the optimal contract with the frame f_N is the above separating contract, and this contract generates more profit than any other contract. In particular, the optimal contract involves framing. This, however, relies on production being costless: Example 3 in Section 4 shows that when production is costly and frames are large enough, the optimal contract is frameless.

3 Framework

This section describe the framework of contracting with frames. Subsection 3.1 defines the environment, the agent's choice procedure, the contract, implementation, and the principal's objective. Subsection 3.2 discusses a modified revelation principle that holds in the framework. Subsection 3.3 studies the full-information benchmark.

3.1 Definitions

Environment. The environment is a pair (Y, F). The set of outcomes $Y = X \times \mathbb{R} \cup \{stayout\}$ includes bundles (x, t), where $x \in X$ is a policy (e.g., quantity or quality) and $t \in \mathbb{R}$ is a transfer from the agent to the principal. The outcome *stayout* obtains if the agent and principal do not agree on a different outcome via the contract. We refer to a set Z of outcomes that includes *stayout* as a menu. The set F is the set of possible frames.

Agent. We first describe the primitives of the agent's choice correspondence, which are summarized by the vector $(\Theta, U, \{U^f\}_{f \in F})$. The set Θ is the set of possible agent types. The function $U : Y \times \Theta \to R$ reflects the agent's preferences, or how he evaluates different outcomes absent any framing. Each function $U^f : Y \times \Theta \to R$ describes how the agent evaluates outcomes in the frame f.

We now describe the agent's choice correspondence C^θ for every $\theta \in \Theta$. A choice problem is a pair (Z, f), where Z is a menu, i.e., $stayout \in Z \subset Y$, and f is a frame. For every choice problem (Z, f), the set of possible choices of a type θ agent is the set $C^\theta(Z, f) \subseteq Z$ that consists of (1) all the U^f-maximal outcomes in Z that are weakly U-superior to *stayout*, and (2) *stayout* if it is weakly U-superior to some U^f-maximal outcome in Z.

The choice correspondence C^θ describes the agent's behavior in at least two types of situations. The first is situations in which reneging on the contract ex-post is possible, e.g., because the principal willingly adopts a return policy or because the law mandates such a policy. In this case, the correspondence C^θ describes an agent who is affected at the point of sale by framing (and thus chooses according to U^f) but reevaluates his purchase according to U after the effect of the frame wears off, and returns the product if he overpaid relative to his outside option.

The second type of situations is situations in which the agent anticipates his frame-dependent behavior. Such anticipation may arise because the agent interacts with the principal frequently, because the agent was involved in similar interactions in the past, or because the agent communicates with other agents who are aware of the effect of the frame. The correspondence C^θ then describes the behavior of an agent who cannot avoid being affected by framing at the point of sale, but anticipates this effect and may therefore choose not to interact with the principal.

Contract. The principal offers a contract to the agent. A contract is a triple (A, M, f), where A is a set of actions, $M : A \to Y$ is a mapping from actions to outcomes, and $f \in F(A, M) \subseteq F$

is a frame. The restriction $f \in F(A, M)$ allows for the possibility that not all frames are available for every pair (A, M). Given a contract (A, M, f), the set of possible choices of a type θ agent is the set $C^{\theta}(M(A) \cup \{stayout\}, f)$, where $M(A)$ is the image of the mapping M. The set $M(A) \cup \{stayout\}$ is the contract's menu.

"Null" frame. We assume that the principal can always offer a contract without framing to the agent. Formally, we let $\phi \in F$ denote the "null" frame, and assume that $\phi \in F(A, M)$ for every (A, M). In the null frame, the agent evaluates outcomes according to U, i.e., $U^{\phi} = U$, so for any choice problem (Z, ϕ), we have that $C^{\theta}(Z, \phi)$ is the set of U-maximal outcomes in Z. We refer to (A, M, ϕ) as a frameless contract.

Implementation. An allocation rule g assigns to each $\theta \in \Theta$ an outcome $g(\theta) \in Y$. A contract (A, M, f) (partially) implements g if $g(\theta) \in C^{\theta}(M(A) \cup \{stayout\}, f)$ for every $\theta \in \Theta$.

Principal. The principal has a prior distribution over Θ, and knows the functions U and $\{U^f\}_{f \in F}$ and the correspondences $\{C^{\theta}\}_{\theta \in \Theta}$. The principal's cost of providing the policy x to a type θ agent is $c(x, \theta)$. The principal wishes to maximize his profit. i.e., he offers the agent a contract that implements a profit-maximizing allocation rule.

3.2 Modified revelation principle

In the standard framework of contracts without frames, the revelation principle states an allocation rule is implementable if and only if it is implementable by a direct revelation contract. This principle may not hold in the framework of contracts with frames. That is, it may be that a contract (A, M, f) implements an allocation rule g but the contract (Θ, g, f) does not. This happens when $g(\theta) = stayout$ and type θ chooses *stayout* in (A, M, f) only because it is weakly U-superior to some U^f-maximal outcome that is in the image of M but not in the image of g. This is illustrated by the following example.

Example 1 *Consider a two-type setting, with* $\Theta = \{L, H\}$ *and* $F = \{\phi, f\}$*. Let* L *and* H *also denote two outcomes in* Y*. Suppose that both types' preferences over outcomes satisfy* $L \succ stayout \succ H$*. Suppose also that the frame* f *does not affect the low type but makes the outcome* H *more appealing to the high type. Specifically, in the frame* f *the high type evaluates outcomes in a way that is consistent with the ranking* $H \succ L \succ stayout$*.*

Suppose that the principal wishes to implement the allocation rule $g(H) = stayout$ *and* $g(L) = L$*. While it is possible to implement* g *with the contract* (Θ, M^g, f) *in which* $M^g(\theta) = \theta$ *for every* $\theta \in \Theta$*, it is impossible to implement* g *with any contract* $(\Theta, g, \cdot)$*.*$\diamondsuit$

To deal with this issue, we define direct revelation contracts as follows.

Direct Revelation Contract (DRC). A Direct Revelation Contract (DRC) for an allocation rule g is a contract (Θ, M^g, f) in which $M^g(\theta) = g(\theta)$ if $g(\theta) \neq stayout$.

Another issue that arises in our framework is that not all frames may be part of a DRC for an allocation rule g. This happens, for example, if the image of g does not include *stayout*, and the frame highlights a bundle that is not in the image of g.

To state a modified revelation principle that accounts for this issue as well, we require the following definition. The triple (Θ, M^g, f) is an *infeasible DRC* for the allocation rule g if (1) $M^g(\theta) = g(\theta)$ for $g(\theta) \neq stayout$ and (2) $f \notin F(\Theta, M^g)$. We then have the following version of the revelation principle.

Observation 1 *The set of implementable allocation rules is bounded below by the set of allocation rules implementable by DRCs, and bounded above by the set of allocation rules implementable by DRCs and infeasible DRCs.*

The first part is trivial. The second part follows from the same reasoning that underlies the standard revelation principle. That is, if some contract (A, M, f) implements g and $g(\theta) \neq$ *stayout*, then in the associated (perhaps infeasible) DRC (Θ, M^g, f) we have $M^g(\theta) = g(\theta)$. And if $g(\theta) = stayout$ and type θ weakly U^f-prefers *stayout* to all outcomes in the image of M, we also have $M^g(\theta) = g(\theta)$. But if $g(\theta) = stayout$ and type θ strictly U^f-prefers some outcome in the image of M to *stayout*, then $M^g(\theta)$ is (any) U^f-maximal outcome in the image of M such that type θ weakly U-prefers *stayout* to this outcome, so $M^g(\theta) \neq g(\theta)$.

The following example illustrates cases in which the set of implementable allocation rules differs from the bounds specified in Observation 1.

Example 2 *Consider a two-type setting, with $\Theta = \{L, H\}$ and $F = \{\phi, f\}$. Let L, H, and J denote three outcomes in Y. Suppose that both types' preferences over outcomes satisfy $L \succ H \succ J \succ stayout$. Suppose also that the frame f highlights the outcome J. In particular, for f to be part of a contract the agent must be able to choose J in the contract. The frame affects the behavior of type H (but not L) as described below. Suppose that the principal wishes to implement the allocation rule $g(\theta) = \theta$.*

To see that the set of implementable allocation rules is strictly larger than the set of allocation rules implementable by DRCs, suppose that in the frame f type H evaluates outcomes in a way that is consistent with the ranking $H \succ L \succ J \succ stayout$. Then, a contract with the frame f whose menu includes $\{L, H, J\}$ implements g. However, since only an infeasible DRC for g can include f, the rule g is not implementable by a DRC.

To see that the set of implementable allocation rules is strictly smaller than the set of allocation rules implementable by DRCs and infeasible DRCs, suppose that in the frame f type H evaluates outcomes in a way that is consistent with the ranking $J \succ H \succ L \succ stayout$. Then, g is implementable by the infeasible DRC (Θ, g, f), but is not implementable by any contract. This is because g is not implementable by a frameless contract, and for f to be part of a contract the agent must be able to choose J in the contract. But then type H will choose J instead of H.$\diamondsuit$

3.3 Full-information benchmark

As a benchmark, we consider the case in which the agent has no private information, i.e., the case in which the set of types Θ consists of the single type θ. The following simple observation shows that framing does not increase the principal's profit in this case.

Observation 2 *Every implementable allocation rule is implementable with a frameless contract.*

To prove the observation, fix an implementable allocation rule $g(\theta) \equiv (x, t)$, and let (A, M, f) be a contract that implements g, i.e., $(x, t) \in C^{\theta}(M(A) \cup \{stayout\}, f)$. By the definition of C^{θ}, the outcome (x, t) is weakly U^f-superior and weakly U-superior to *stayout*. This implies that $(x, t) \in C^{\theta}(\{(x, t), stayout\}, \phi)$, because (x, t) is weakly U-superior to *stayout*. Thus, g is implementable with a frameless contract.

While Observation 2 is mathematically trivial, it suggests that a seller cannot benefit from framing in full-information environments in which buyers can renege costlessly on their decisions or in which buyers anticipate their biases. The observation also suggests that if a seller benefits from framing in a full-information environment, then some of the buyers would prefer not to interact with him.

4 Frames that change willingness to pay

An immediate corollary of Observation 2 is that a necessary condition for framing to increase the principal's profit is that the agent has private information. There may then be an allocation rule that is not implementable by a frameless contract, is implementable by a contract with framing, and generates higher profit than any allocation rule implementable by a frameless contract. One example is the optimal allocation rule in the price discrimination setting of Section 2 when the proportion of low-valuation buyers is small. This allocation rule is not implementable without framing because high-valuation buyers strictly U-prefer the bundle aimed at low-valuation buyers to the bundle aimed at high-valuation buyers. But the rule is implementable with the largest frame f_N because, in the frame, high-valuation buyers are willing to pay more for each additional unit of quality, and thus weakly U^{f_N}-prefer the bundle aimed at them to the bundle aimed at low-valuation buyers. More generally, however, increasing buyers' willingness to pay may change the set of implementable allocation rules in a way that decreases the principal's profit, as Example 3 below demonstrates.

This section studies how frames that change willingness to pay influence the principal's profit when the agent has private information. We say that a frame f *reduces the agent's willingness to pay* if for any pair of bundles $(x, t) > (x', t')$ and any type θ, $U^f(x, t, \theta) \geq U^f(x', t', \theta)$ implies that $U(x, t, \theta) > U(x', t', \theta)$. That is, the amount that the agent is willing to pay for $x - x'$ additional units of the policy in the frame is smaller than without the frame. A frame f

increases the agent's willingness to pay if for any pair of bundles $(x,t) > (x',t')$ and any type θ, $U(x,t,\theta) \geq U(x',t',\theta)$ implies that $U^f(x,t,\theta) > U^f(x',t',\theta)$. For example, in Section 2 frames increase the agent's willingness to pay. Of course, there are natural manipulations that do not increase (or decrease) the agent's willingness to pay. For example, the effect of highlighting a bundle on the agent's willingness to pay for other bundles may depend on whether the bundles are smaller or larger than the highlighted bundle. We analyze such an example in Section 6.

To discuss how changing the agent's willingness to pay via framing affects profit, we begin by making several assumptions. We then show that if frames reduce the agent's willingness to pay, every profit-maximizing contract is frameless. We also show that frames that slightly increase the agent's willingness to pay increase the principal's profit, but those that increase willingness to pay substantially may decrease the principal's profit.

4.1 Assumptions

Let $\Theta = \{\theta_1, \ldots, \theta_N\}$ be an ordered set of N types, with $\theta_i < \theta_j$ for $i < j$. Let F be a set of frames that includes the null frame ϕ. Let $X = [0,d]$ for some $d > 0$, and let $stayout = (0,0)$, i.e., we identify the agent's outside option with the lowest policy and a transfer of 0. Fix a cardinal representation $\{U^f\}_{f \in F}$, and assume that every U^f is strictly increasing in x, strictly decreasing in t, and continuous in x and t. Assume that the principal's cost c is strictly increasing and continuous in x, and satisfies $c(0,\theta) = 0$. We denote by (x^*_θ, t^*_θ) the bundle that solves the principal's full-information profit maximization program subject to the agent of type θ obtaining a U-utility of $U(stayout, \theta)$, and we assume that the associated profit is strictly positive.

We now present two assumptions on frames. The first assumption states that all frames are available in any DRC. This implies that there are no infeasible DRCs, so by Observation 1 it suffices to analyze DRCs.

Assumption A1. $F(\Theta, M) = F$ for every mapping $M : \Theta \to Y$.

In Example 2, (A1) fails because the frame f can only be part of a contract if the outcome J belongs to the contract's menu.

The second assumption postulates a standard single crossing property. In the context of contracting with frames, it states that the effect of framing is limited in the sense that frames cannot "reverse" the relative willingness to pay across types.

Assumption A2. For any frame $f \in F$, any two bundles $(x,t) > (x',t')$, and any two types $\theta > \theta'$, if $U^f(x,t,\theta') \geq U^f(x',t',\theta')$ then $U^f(x,t,\theta) > U^f(x',t',\theta)$.

4.2 Frames that reduce willingness to pay

Let f be a frame that reduces the agent's willingness to pay, i.e., for any pair of bundles $(x,t) > (x',t')$ and any type θ, $U^f(x,t,\theta) \geq U^f(x',t',\theta)$ implies that $U(x,t,\theta) > U(x',t',\theta)$. Consider the agent's behavior when facing a menu with the frame f. First, if the agent U^f-prefers some positive bundle to *stayout*, then the agent strictly U-prefers this bundle to *stayout*. This implies that if the agent makes a purchase from the menu with the frame f, then he will also make a purchase from a frameless menu with the same policies and slightly higher transfers. Second, conditional on making a purchase from a given menu, the agent chooses weakly larger bundles without the frame than with the frame, even if the transfers are slightly increased. This may reduce the principal's profit because the principal's cost may be type-dependent. The following result shows that the transfers can be increased in a way that incentivizes each type to purchase the same policy with and without the frame, and thus the principal's profit is strictly larger without the frame.

Proposition 1 *If frames reduce willingness to pay, then every optimal contract is frameless.*

4.3 Frames that increase willingness to pay

Proposition 1 shows that frames that reduce willingness to pay will not be used by the principal. We now turn to frames that increase willingness to pay.

Assumption A3. For any frame $f \in F \setminus \{\phi\}$, any two bundles $(x,t) > (x',t')$, and any type θ, if $U(x,t,\theta) \geq U(x',t',\theta)$ then $U^f(x,t,\theta) > U^f(x',t',\theta)$.

To discuss how increased willingness to pay affects the principal's profit, consider the set of allocation rules implementable by frameless contracts, and suppose that one of the profit-maximizing rules in this set is not fully pooling. Consider a frameless DRC that implements this allocation rule. Changing the null frame to a frame $f \neq \phi$ has three effects on the agent's behavior. First, if the agent U-prefers some bundle to *stayout*, then he also U^f-prefers this bundle to *stayout*. Second, the incentives of type θ_i to choose smaller bundles, which are aimed at lower types, are reduced. Third, the incentives of type θ_i to choose larger bundles, which are aimed at higher types, are increased.

The third effect does not play a role when each type θ_i weakly U^f-prefers his intended bundle to the bundles aimed at higher types. In this case, the principal's profit in an optimal contract with the frame f is strictly higher than in the optimal frameless DRC. This is because in the optimal frameless DRC each type θ_i is U-indifferent between his intended bundle and the bundle aimed at type θ_{i-1} (the downward incentive constraints bind), so the second effect and the fact the contract is not fully pooling imply that the principal can slightly increase the transfer in the bundle aimed at the highest type, θ_N, without affecting the behavior of any of the types.

On the other hand, when some types strictly U^f-prefer the bundles aimed at some higher types, an optimal contract with the frame f may actually lead to strictly lower profit than the optimal frameless DRC. To see this, consider a two-type setting in which the low type's U-willingness to pay is much lower than that of the high type, in every optimal frameless contract the low type buys a positive policy and a significant portion of the principal's profit comes from selling a high policy to the high type, and framing increases the willingness to pay of the low type so much that his behavior in the frame becomes very similar to that of the high type. Then, in order to generate high profit in the frame, the principal has to provide similar high policies to both types. But because the U-willingness to pay of the low type is small, selling a high policy to this type may be very costly due to production costs. This may decrease the principal's profit despite the frame's positive effect of increasing the agent's willingness to pay. The following example illustrates this in the price discrimination setting of Section 2 with added production costs.

Example 3 *Consider the price discrimination setting of Section 2 with marginal production cost $MC(x) = x$ for $x \leq 1$ and $MC(x) = 1 + (x-1)/B$ for $x > 1$, where B is large.*[3] *Note that the production cost is type-independent. Suppose that the seller can only increase the buyer's willingness to pay substantially. Specifically, suppose that $F = \{\phi, f\}$, where $f = 9$. Suppose also that the high type's U-willingness to pay for quality is much higher than that of the low type. Specifically, $L = 1$ and $H = 2$. Finally, suppose that $\pi_L > 1/2$.*

We now specify two frameless contracts D and E by describing the menus D and E that they induce, and show that the profit that any contract with the frame f generates is strictly lower than the maximum of the profit that these two contracts generate. Let $D = \{(x_L^*, t_L^*), (x_H^*, t_H), stayout\}$, where $x_L^* = 1$, $x_H^* = 1 + B$, $t_L^* = 1$, and $t_H = 2B + 1$. Then, $(x_L^*, t_L^*) \in C^L(D, \phi)$ and $(x_H^*, t_H) \in C^H(D, \phi)$. When buyers choose these bundles, D generates profit $\pi_H (B+1)/2$ from the high type, which is only π_H less than the first-best profit from selling to the high type, and generates the first-best profit from the low type. Let $E = \{(\varepsilon, \varepsilon), (x_H^*, t_H^* - \varepsilon), stayout\}$ for some small $\varepsilon > 0$. Then, $(\varepsilon, \varepsilon) \in C^L(E, \phi)$ and $(x_H^*, t_H^* - \varepsilon) \in C^H(E, \phi)$. When buyers choose these bundles and ε is sufficiently small, the profit that E generates is strictly higher than the first-best profit from selling to the high type, because $\pi_L > \pi_H$.

Consider a contract with the frame f that excludes one of the types, i.e., one of the types chooses stayout. If the contract excludes the high type, then the profit it generates is bounded above by the first-best profit from selling to the low type, which is strictly lower than the profit generated by D. If it excludes the low type, then the profit it generates is bounded above by the first-best profit from selling to the high type, which is strictly lower than the profit generated by E.

Now consider a non-excluding contract G with the frame f, denote by $(x'_\theta, t'_\theta) \neq stayout$ the bundle that type θ chooses, and suppose that G generates more profit than any excluding contract. To generate more profit than D, the contract G must generate a profit of at least $\pi_H (B+1)/2$ from the high type, because D already generates the first-best profit from the low type. This implies that $x'_H > B/4$, because the high type's marginal willingness to pay under

[3] The cost of producing x units is therefore $c(x) = x^2/2$ for $x \leq 1$ and $c(x) = \left(1 - B + 2(B-1)x + x^2\right)/2B$ for $x > 1$.

U is 2. Because the low type weakly U^f-prefers (x'_L, t'_L) to (x'_H, t'_H), we must also have that $x'_L(1+f) - t'_L \geq x'_H(1+f) - t'_H$. Because $t'_L \geq 0$ (otherwise, excluding the low type and selling the first-best to the high type is profit enhancing), $t'_H \leq 2x'_H$ (otherwise, the high type would strictly U-prefer stayout to (x'_H, t'_H)), and $f = 9$, we obtain that $x'_L \geq 4x'_H/5 > B/5$. But for B large enough, every unit above $B/8$ sold to the low type leads to a loss of at least $1/16$, even if the low type is charged his willingness to pay under U. This implies that for a large enough B the loss in G on the low type is larger than the possible gain on the high type.$\diamondsuit$

The following observation summarizes our discussion of frames that increase the agent's willingness to pay.

Observation 3 *Consider the set of allocation rules implementable by frameless contracts, and suppose that one of the profit-maximizing rules in this set is not fully pooling. If there exists a frame f such that every type weakly U^f-prefers his bundle in this allocation rule to higher types' bundles, then every optimal contract involves framing. If there does not exist such a frame, then it may be that every optimal contract is frameless.*

5 Optimal contract with two types

This section characterizes the optimal contract when the agent's private information has two possible values. In order to rule out situations like the one in Example 3, in which increasing the agent's willingness to pay decreases the principal's profit, we first introduce Assumption (A4) on frames that limits the distortion that frames create. We then analyze the optimal contracts under Assumptions (A1)-(A4), and discuss the qualitative differences between optimal contracts with and without frames.

5.1 Assumptions

Consider the setting of Section 4 with $\Theta = \{L, H\}$, where $L < H$. Denote by $\pi_\theta > 0$ the probability of type $\theta \in \Theta$. In addition to the assumptions in Subsection 4.1, assume that every function U^f is differentiable in x and t and that the principal's cost c is type-independent (Section 6 studies an insurance setting with type-dependent cost). Assume also that the principal's full-information profit maximization program subject to a type θ agent obtaining a given level of U-utility is strictly concave in the policy and has a unique solution, in which the policy is $x^*_\theta \in (0, d]$. In particular, x^*_θ is independent of the U-utility level of the agent.[4]

We also strengthen Assumptions (A2) and (A3) as follows. First, we assume that for the function U the marginal rate of substitution (MRS) of x to t at $(0,0)$ exists and is strictly higher

[4] This is always the case if U is quasi-linear in t, and is also the case in the insurance setting of Section 6.

for the high type than for the low type (Assumption (A2) implies a weak inequality).[5] This assumption implies that when the low type's probability is low enough an optimal frameless contract excludes him, i.e., the low type chooses *stayout*.[6] Second, we assume that the MRS of x to t of the high type at (x_H^*, t) for $t \le t_H^*$ is strictly higher for every function U^f than for U (Assumption (A3) implies a weak inequality). This assumption is used only in the proof of the second half of part (4) of Proposition 3 below.

The new non-technical assumption on frames, Assumption (A4), states that the distortion created by framing is limited in the sense that in the first-best solution to the principal's profit maximization problem the high-type agent wants to mimic the low-type agent, just like in the standard model.

Assumption A4. For every frame f, $U^f(x_L^*, t_L^*, H) > U^f(x_H^*, t_H^*, H)$.

5.2 Analysis

We now characterize the set of profit-maximizing contracts under Assumptions (A1)-(A4). By (A1), it suffices to analyze allocation rules implementable by DRCs. Given an allocation rule, we denote by (x_θ, t_θ) the bundle that the allocation rule assigns to type θ. By (A2), any implementable allocation rule has to satisfy $x_H \ge x_L$ and $t_H \ge t_L$. Moreover, the single-crossing property of U and the properties of the principal's cost function and profit maximization program imply that $x_H^* \ge x_L^*$.

5.2.1 Participation

In the standard model without framing, when the probability of the low type is small enough, the profit-maximizing contract excludes this type, i.e., $(x_L, t_L) = (0, 0)$, and assigns the first-best bundle to the high type, i.e., $(x_H, t_H) = (x_H^*, t_H^*)$.[7] In the model of contracts with frames, this is

[5] That is, $\frac{\partial U(x,t,H)/\partial x}{\partial U(x,t,H)/\partial t}|_{(0,0)} > \frac{\partial U(x,t,L)/\partial x}{\partial U(x,t,L)/\partial t}|_{(0,0)}$.

[6] Without this assumption, the low type is "essentially excluded" in the sense that as π_L converges to 0, so does the policy of the low type.

[7] To see that excluding the low type is optimal, consider the standard model and a contract in which the incentive compatibility constraint of the high type binds, the participation constraint of the low type binds, and in which $x_H = x_H^*$. For any $x_L \in (0, x_H^*]$, the revenue from the low type is finite, and the reduction in revenue from the high type (relative to making him indifferent to *stayout*) is strictly positive. The ratio $p(x_L)$ of the two numbers is therefore finite on $[\varepsilon, x_H^*]$ for any $\varepsilon > 0$, and because the MRS of the high type at $x = 0$ is strictly higher than that of the low type, by setting $p(0) = \lim_{x \to 0} p(x)$ we have that p is finite and continuous on $[0, x_H^*]$. Therefore, for sufficiently small $\pi_L > 0$ we have that $p(x_L) < (1 - \pi_L)/\pi_L$ for all $x_L \in (0, x_H^*]$, so it is optimal to set $x_L = 0$.

no longer true. Under (A1)-(A4), the low type agent is never excluded in an optimal contract.[8]

Proposition 2 *Both types' policies in a profit-maximizing contract are bounded away from* 0 *regardless of the distribution of types.*

Because both types purchase positive policies in an optimal contract, their bundles in the corresponding allocation rule have to be U- and U^f-superior to *stayout*. Thus, the following two participation constraints have to hold:

IR_θ^f : $U^f(x_\theta, t_\theta, \theta) \geq U^f(0,0,\theta)$ for $\theta \in \{L, H\}$, and

IR_θ^U : $U(x_\theta, t_\theta, \theta) \geq U(0,0,\theta)$ for $\theta \in \{L, H\}$.

Because frames increase willingness to pay, IR_θ^U implies IR_θ^f, so IR_θ^f is redundant. In addition, an optimal contract has to satisfy the Incentive Compatibility constraint for each type. The principal's program can therefore be written as:

Choose $((x_L, t_L), (x_H, t_H), f)$ to maximize $\pi_L(t_L - c(x_L)) + \pi_H(t_H - c(x_H))$ subject to:

IR_θ^U : $U(x_\theta, t_\theta, \theta) \geq U(0,0,\theta)$ for $\theta \in \{L, H\}$,

IC_θ^f : $U^f(x_\theta, t_\theta, \theta) \geq U^f(x_{\theta'}, t_{\theta'}, \theta)$ for $\theta, \theta' \in \{L, H\}$ and $\theta' \neq \theta$.

5.2.2 Optimal contracts

The set of optimal contracts may include pooling and separating contracts. If some optimal contract is pooling, then the set of optimal pooling contracts is the set of contracts that include the single bundle (x_L^*, t_L^*) and some frame $f \in F$. This is because (x_L^*, t_L^*) is the profit-maximizing bundle subject to IR_L^U, and it satisfies all the other constraints. In particular, the frameless contract with the single bundle (x_L^*, t_L^*) is optimal. The following proposition characterizes optimal contracts that are separating.

Proposition 3 *Any optimal contract that is separating involves framing and has the following properties:*

1. *The low type is U-indifferent between his bundle and stayout (IR_L^U binds);*
2. *The high type is U^f-indifferent between his bundle and that of the low type (IC_H^f binds);*
3. *Downward distortion at the bottom: The low type's bundle has a strictly positive policy that is weakly lower than the first-best policy;*

[8] Non-exclusion results due to other reasons appear in Eliaz and Spiegler (2006), Esteban and Miyagawa (2006), Esteban, Miyagawa, and Shum (2007), and Galperti (2013). Carbajal and Ely (2012) show that loss aversion may decrease the set of agent types that are optimally excluded.

4. *Upward distortion at the top: The high type's bundle has a policy that is weakly higher than the first-best policy. The policy is strictly higher if the first-best policy is an interior one.*

Note that unlike in the standard model, the high type may not obtain information rents in an optimal contract in which the low type purchases a positive policy, i.e., (IR_H^U) may bind. This is illustrated in the price discrimination example of Section 2.

An immediate corollary of Proposition 3 is that when every optimal frameless contract is separating, framing strictly increases the principal's profit. Indeed, when every optimal frameless contract is separating, then every optimal contract is separating (because an optimal contract that is pooling has to include the single bundle (x_L^*, t_L^*), as discussed above, so a frameless pooling contract with the same bundle is optimal as well); by Proposition 3, every optimal contract that is separating involves framing. There are many settings in which every optimal frameless contract is separating. This happens for example when $x_L^* < x_H^*$, or when $x_H^* = x_L^*$ is an interior policy and for the function U, the MRS of x to t of the high type is strictly larger than the MRS of the low type when both are evaluated at (x_L^*, t_L^*).

5.2.3 Optimal frame

We now characterize the optimal frame. For two frames f and k, we say that $f \succeq k$ if for every two bundles $(x, t) > (x', t')$ such that $U^k(x, t, H) \geq U^k(x', t', H)$, we have that $U^f(x, t, H) \geq U^f(x', t', H)$. We say that $f \succ k$ if for every two bundles $(x, t) > (x', t')$ such that $U^k(x, t, H) \geq U^k(x', t', H)$, we have that $U^f(x, t, H) > U^f(x', t', H)$.

Proposition 4 *If $f \succeq k$, then the profit in an optimal contract with f is weakly higher than in an optimal contract with k. If $f \succ k$ and some optimal contract with k is separating, then the profit in an optimal contract with f is strictly higher than in an optimal contract with k.*

In particular, if the relation $\succeq$ has a maximal element f, then there is an optimal contract with the frame f. And if every optimal contract is separating and the relation $\succ$ has a maximal element f, then the frame f is the only optimal frame.

5.3 Discussion

In two-type environments, the model of contracting with frames has several predictions that differ qualitatively from the standard model. The first relates to the participation of the low type. Proposition 2 shows that it is optimal for the principal to contract with the low type independently of the low type's probability. This is in contrast to the standard model, in which if the low type's probability is low enough it is optimal for the principal to exclude the low type in order to eliminate the high type's information rents.

The second prediction relates to the participation of the high type. While Proposition 2 shows that when the principal's cost is type-independent it is always optimal to contract with the high type, this is no longer true when the cost is type-dependent. In such cases, the cost of contracting with the high type may be so high that the principal may wish to exclude him. This cannot be done in the standard model with the single-crossing property, because if the low type participates then the high type prefers the low type's bundle to *stayout*. But excluding the high type may be possible in the model with frames. To exclude the high type, the principal has to offer him a bundle that is weakly U^f-superior to the low type's bundle and to *stayout*, but is weakly U-inferior to *stayout*.[9] Section 6 illustrates this possibility in an insurance setting.

A third departure from the standard model relates to cases in which both types participate and the optimal contract is a separating one. Proposition 3 shows that in such cases the high type over-consumes, as long as this is technologically feasible. This is in contrast to the standard model, in which the high type's policy is always efficient.[10]

6 Application to insurance with a reference bundle

This section studies a monopolistic insurance setting a-la Stiglitz (1977), and illustrates how the predictions of the model with frames differ from those of Stiglitz's original model.

A risk-neutral profit-maximizing insurance provider offers a menu of insurance bundles to a population of risk-averse individuals. Each individual has initial wealth w, and may suffer an accident of size $A > 0$. An individual's privately-known probability of an accident is $\theta \in \{L, H\}$, with $0 < L < H < 1$. The proportion of Low-risk individuals in the population is $\pi_L > 0$ and of High-risk individuals is $\pi_H = 1 - \pi_L > 0$. Each individual's preferences over wealth are summarized by a strictly increasing, strictly concave, and continuously differentiable function u.

An insurance bundle is a pair (x, t), where t is the premium paid by the individual to the insurance provider upfront and $x \geq 0$ is the amount paid by the provider to the individual if the accident occurs. The expected utility of an individual with risk level θ from the bundle (x, t) is

$$U(x, t, \theta) = \theta u(w - t - A + x) + (1 - \theta)u(w - t).$$

In addition to offering a menu of insurance bundles, the provider can also highlight one bundle in the menu. We denote the highlighted bundle by $f = (x_f, t_f)$, and identify no highlighting with $f = (0, 0)$. A contract is thus a menu of bundles and a highlighted bundle that belongs to the menu. The set F of frames is the set of all possible bundles.

[9] The offered bundle serves a similar role to the "unused options" that appear in the models of Esteban and Miyagawa (2006) and Galperti (2013), in which the principal offers the agent a menu of menus.

[10] Over-consumption arises for other reasons in Carbajal and Ely (2012) and Galperti (2013).

In a frame f, an individual treats the highlighted bundle as a reference point. He anticipates that if he purchases an insurance bundle (x,t) with $x \leq x_f$, he will experience regret of $r(x_f - x)$ if the accident occurs, in addition to the effect the accident has on his wealth. That is, in the frame f an individual chooses from the menu a bundle (x,t) that maximizes

$$U^f(x,t,\theta) = \theta \left(u\left(w - t - A + x\right) - 1_{x \leq x_f} r(x_f - x) \right) + (1-\theta)\, u\left(w - t\right).$$

We assume that the regret function r satisfies the following properties:

- $r'(\Delta) > 0$ for $\Delta \geq 0$: Regret is increasing in the difference in coverage $\Delta = x_f - x$ between the reference coverage and the chosen coverage,
- $r''(\Delta) < 0$ for $\Delta > 0$: Marginal regret is decreasing,
- $r(0) = 0$: There is no regret if the chosen coverage is higher than the reference coverage.[11]

Individuals anticipate their frame-dependent behavior, and if an individual decides not to interact with the insurance provider, he obtains no insurance.

Before discussing properties of the optimal insurance contract, we consider Assumptions (A1)-(A4). Assumption (A1) does not hold, because the reference bundle must be included in the contract. Nevertheless, we will proceed as if (A1) holds, and show that the optimal reference bundle can be chosen to coincide with the high-risk individuals' bundle. Assumption (A2) holds: a high-risk individual is willing to pay more than a low-risk individual to buy an additional unit of insurance, regardless of the reference bundle, because a high-risk individual is more likely to have an accident. Assumption (A3) does not hold, but Proposition 2 and Proposition 3 (except for the strict upward distortion in part (4) that we will prove directly below) rely only on a weaker property that does hold.[12] Assumption (A4) holds, because $x_H^* = x_L^* = A$ and $t_H^* > t_L^*$. An important departure from the previous section is that the principal's cost depends on the individual's type. The only change this introduces to the analysis is that it may be optimal to exclude high-risk individuals. This can in fact happen, as we show below.

We now characterize the optimal contract. The first property is similar to Stiglitz's (1977) model.

Property 1 *In an optimal contract, high-risk and low-risk individuals purchase different insurance bundles.*

[11] Note that r does not depend on the premium in the reference bundle. Our characterization of the optimal contract extends to cases in which r decreases in the reference premium, as long as r's dependency on the premium satisfies conditions that parallel those in the first two bullet points.

[12] This weaker property is that for any $(x,t) > (x',t')$ and every frame $f \neq \phi$, (i) if $U(x,t,\theta) > U(x',t',\theta)$ then $U^f(x,t,\theta) > U^f(x',t',\theta)$, (ii) if $(x',t') \neq$ *stayout* and $U(x,t,\theta) = U(x',t',\theta)$ then $U^f(x,t,\theta) \geq U^f(x',t',\theta)$, and (iii) if $(x',t') =$ *stayout* and $U(x,t,\theta) = U(x',t',\theta)$ then $U^f(x,t,\theta) > U^f(x',t',\theta)$.

To see why, suppose to the contrary that there is an optimal contract that is pooling with the bundle (x, t). Clearly, $x > 0$ and $t > 0$, because an optimal contract generates positive profit. In addition, $x \leq A$, because selling $x > A$ units of insurance to everybody at the low-risk individuals' willingness to pay is dominated by selling A units to everybody at the low-risk individuals' willingness to pay.

If $x = A$, then decrease x by a small $\varepsilon > 0$ and decrease t slightly so that $U(x, t, L)$ remains unchanged. The profit in the new pooling contract is higher because the provider makes essentially the same profit on low-risk individuals but gains approximately $\varepsilon(H - L)$ on high-risk individuals.

Now suppose that $0 < x < A$. Since $x > 0$ and IR_L^U holds, IR_H^U holds strictly. Then, add another insurance bundle (x_H, t_H) aimed at the high-risk individual, with $x_H < A$ slightly larger than x, such that IR_H^U continues to hold and IC_H^f holds with equality. By (A2), IC_L^f continues to hold. Profit strictly increases, because high-risk individuals are risk-averse (as are low-risk individuals) and are therefore U-willing to pay more for the additional unit of insurance than the cost to the risk-neutral provider, and their U^f-willingness to pay is weakly higher.

The next property is qualitatively different from Stiglitz's (1977) model.

Property 2 *In an optimal contract, high-risk individuals may not purchase insurance. Low-risk individuals purchase insurance regardless of the distribution of types.*

For the second part of Property 2, let $f = (x_H^*, t_H^*)$ and apply the arguments for non-exclusion of the low type in the proof of Proposition 2. For the first part of the property, consider a case in which π_L and π_H are close to $1/2$ and H is close to 1, so the provider's maximal profit from selling insurance to high-risk individuals in a full-information setting is some small ε. Suppose also that the maximal profit from selling to low-risk individuals in a full-information setting is substantial (so L is close to $1/2$). Denote this profit by $P >> \varepsilon$. If the anticipated regret is sufficiently high, then the provider can obtain profit P by excluding the high-risk individuals and fully insuring low-risk individuals. To do this, the provider offers and highlights an over-insuring bundle $f = (x_f, t_f)$ that high-risk individuals weakly U^f-prefer to the low-risk individuals' full insurance bundle, but which violates IR_H^U. Suppose that the regret function is such that x_f must be substantially greater than A. Now consider any non-pooling contract in which high-risk individuals are not excluded. Because the profit from insuring high-risk individuals is at most ε, to generate profit P low-risk individuals must be almost fully insured and charged close to their willingness to pay. But then, in order to offer high-risk individuals a different bundle that they weakly U^f-prefer to the low-risk individuals' bundle and weakly U-prefer to *stayout*, this additional bundle has to include substantial over-insurance. Because the cost of providing each additional unit of insurance is constant, while individuals' willingness to pay for an additional unit of over-insurance is smaller than this cost and decreases in the amount of insurance, we obtain that the provider loses on high-risk individuals in this contract, so his profit is lower than P.

The next property characterizes the optimal frame.

Property 3 *Every optimal contract involves a non-trivial reference bundle $(x_f, t_f) \neq (0,0)$. Moreover, in an optimal contract in which high-risk individuals purchase insurance, the reference coverage x_f is identical to the high-risk individuals' coverage level x_H.*

Consider the first part of Property 3. By Properties 1 and 2, in an optimal contract either high-risk individuals do not buy insurance, or individuals with different risk levels purchase different insurance bundles. To induce high-risk individuals not to buy insurance, the reference bundle must be non-trivial. And if individuals with different risk levels buy different bundles, then Proposition 3 implies that the optimal contract involves framing.

For the second part of the property, assume to the contrary that there exists an optimal contract in which high-risk individuals buy insurance and in which the reference coverage x_f differs from the high type's coverage x_H. By part (2) of Proposition 3, IC_H^f holds with equality in this contract, and because $x_H > x_L > 0$, by (A2) IC_L^f holds strictly. We now modify this contract by modifying the reference coverage to derive a contradiction to Proposition 3. If $x_f < x_H$, then increase x_f slightly to $x_{\tilde{f}}$ (or slightly above the low-risk individual's coverage x_L if $x_f < x_L$) so $\mathrm{IC}_L^{\tilde{f}}$ still holds. This increases the regret associated with purchasing the low-risk individuals' bundle (but not with purchasing the high-risk individuals' bundle), so $\mathrm{IC}_H^{\tilde{f}}$ holds strictly and all other constraints hold.[13] If $x_f > x_H$, then decrease x_f slightly to $x_{\tilde{f}}$ so $\mathrm{IC}_L^{\tilde{f}}$ still holds. This makes the low-risk individuals' bundle less attractive relative to that of the high-risk individuals, because r is concave and $x_H > x_L$. Again, this implies that $\mathrm{IC}_H^{\tilde{f}}$ holds strictly and all other constraints hold. In both cases, the new separating contract generates the same profit as the original one, and is therefore optimal, but in contradiction to part (2) of Proposition 3, the constraint $\mathrm{IC}_H^{\tilde{f}}$ holds strictly.

The last property is also qualitatively different from Stiglitz's (1977) model, in which high-risk individuals purchase full insurance.

Property 4 *In an optimal contract in which high-risk individuals purchase insurance, they are strictly over-insured.*

By Proposition 3 (except for the strict upward distortion in part (4)) and Properties 1 and 3, it suffices to verify that any contract in which low-risk individuals are partially insured, high-risk individuals are fully insured, and full coverage is highlighted, is not optimal. Consider such a contract, and increase the high-risk individuals' coverage and premium slightly along their U-indifference curve. This does not change the provider's profit to a first order, because when

[13] By the weak version of (A3) IR_θ^U implies $\mathrm{IR}_\theta^{\tilde{f}}$, just like with (A3).

high-risk individuals are fully insured their willingness to pay for an additional unit of insurance is identical to the provider's cost of providing this unit. Because the new coverage is larger than the reference coverage, the U-indifference implies that a high-risk individual is also U^f-indifferent between his original bundle and the new bundle, so IC_H^f continues to hold; and IC_L^f continues to hold because it held strictly before the change. Now increase x_f to equal the new coverage $x_{\tilde{f}}$ for the high-risk individual. Then $\mathrm{IC}_H^{\tilde{f}}$ holds strictly, and $\mathrm{IC}_L^{\tilde{f}}$ continues to hold if the change in coverage is small enough. Finally, increase the low-risk individuals' coverage and premium slightly along their U-indifference curve, which strictly increases profit to a first order and does not violate any of the constraints.

7 Conclusion: Framing in auctions

This paper incorporates framing into contract theory. By using frames, a contract designer affects how an agent evaluates various options in the contract, and the agent either anticipates this effect or can renege on his choice after the framing effect wears off.

We observed that framing does not increase the designer's profit when the agent does not have private information or when framing decreases the agent's willingness to pay. We also showed that framing increases the designer's profit when it increases the agent's willingness to pay but does not distort incentives too much. When the agent's private information has only two possible values, we established that — in contrast to the standard theory of contracts — the low-type agent always participates in the contract, the high type may not participate, and when the high type participates, he over-consumes and may not obtain information rents even though the low type participates. This section concludes with an example that illustrates the potential relevance of frames as a design parameter in multi-agent environments and, more specifically, in auctions.

In an auction, bidders' behavior may be influenced by the framing of the auction environment. For example, in Filiz-Ozbay and Ozbay's (2007) experimental auction, announcing before the auction starts that the winning bid will be revealed after the auction ends causes bidders to bid more aggressively. In Delgado's et al. (2008) experimental auction, subjects bid more aggressively in an auction with a frame that highlights the possibility of losing than in a baseline frameless auction. It seems that in both experiments, framing triggers bidders to anticipate a larger disappointment from losing than in a frameless setting, which leads to more aggressive bidding behavior conditional on participation. We now consider a reduced-form example of a single-unit efficient auction in which the auction designer can increase bidders' anticipated disappointment from losing.

An auction designer wishes to maximize his profit from selling a non-divisible good to one of N potential bidders, subject to allocating the good to the bidder with the highest value.

Bidders' private values for the good are independently and uniformly distributed on $[0,1]$. The preferences of a bidder with private value θ over lotteries (x,t), where x is the probability of winning the item and t is the monetary cost of obtaining this probability, are summarized by the function $U(x,t,\theta) = x\theta - t$.

To maximize profit, the designer chooses a frame $f \in F$ that affects bidders' anticipated disappointment from losing. In a frame f, with $f : [0,1] \to \mathbb{R}_+$, a bidder with value θ evaluates outcomes according to the function $U^f(x,t,\theta) = x\theta - (1-x)f(\theta) - t$. We would like the frame to affect the bidder only if he decides to compete with other bidders in the auction, so we assume that $U^f(stayout, \theta) = 0 = U(stayout, \theta)$. As in the contracting setting, bidders anticipate their frame-dependent behavior, so given U and $\{U^f\}$, the correspondence $C^\theta(Z,f)$ describes the behavior of a bidder with value θ.

Note that we allow the effect of a frame to vary with the bidder's type. When this is not the case, the revenue equivalence theorem implies that the revenue in an efficient auction with a frame is identical to the revenue in an efficient frameless auction. In addition, unlike in the earlier analysis, we do not identify *stayout* with the outcome $(0,0)$. This captures the idea that the bidder is affected by the frame only if he decides to actually submit a bid and thereby to compete with the other bidders in the auction. This is different from contracting environments with a single agent, in which the competitive aspect does not exist. Finally, because the auction designer has to implement an efficient allocation, he is restricted to choosing f such that IR_θ^U, IR_θ^f, and IC_θ^f hold for every θ. Unlike in the previous analysis, where IR_θ^U implied IR_θ^f, in this setting IR_θ^f implies IR_θ^U for every type θ, so we can ignore IR_θ^U.

The last point implies that inducing participation in a framed efficient auction is harder than in a frameless one. But conditional on participating, bidders' willingness to pay for the item is higher in the framed auction. The following observation shows that the increase in revenue that results from the increased willingness to pay (conditional on participation) may be much larger than the possible decrease in revenue due to the more demanding participation constraint.

Observation 4 *Consider an auction with a frame f that implements an efficient allocation such that for every type θ, $f(\theta)$ is strictly larger than the expected surplus of type θ in a frameless efficient auction. Then, the revenue in the auction with the frame f is strictly larger than the revenue in a revenue-maximizing (inefficient) frameless auction.*

8 Appendix

Proof of Proposition 1. Consider an allocation rule g and a DRC (Θ, M^g, f) that implements g, with a frame $f \neq \phi$ that reduces the agent's willingness to pay. Then, the DRC (Θ, g, f) also implements g. Otherwise, there exists a bundle $(x,t) \neq stayout$ in the image of M^g and a type θ with $g(\theta) = stayout$ such that type θ weakly U^f-prefers (x,t) to *stayout* and weakly U-prefers *stayout* to (x,t). This contradicts reduced willingness to pay if $(x,t) > (0,0)$, and contradicts the monotonicity of U and U^f if $x = 0$ or $t \leq 0$.

We thus consider the DRC (Θ, g, f), and let $y_\theta = g(\theta) = (x_\theta, t_\theta)$. We have that $x_\theta \geq x_{\theta'}$ and $t_\theta \geq t_{\theta'}$ for $\theta > \theta'$, by (A2) and reduced willingness to pay.

We now show that there exists a frameless DRC that achieves a strictly higher profit. If $y_{\theta_N} = stayout$, then the profit in the DRC (Θ, g, f) is 0, and a frameless contract that includes only the bundle $\left(x^*_{\theta_N}, t^*_{\theta_N}\right)$ generates a positive profit (by (A2), all types other than θ_N choose *stayout*).

Otherwise, $y_{\theta_N} \neq stayout$. Replace the frame f by the null frame, leaving the rest of the DRC unchanged. Reduced willingness to pay implies that every type θ for which $g(\theta) = y_{\theta_N}$ strictly U-prefers y_{θ_N} to all other bundles in the contract's menu. Therefore, if the frameless contract still implements g, the profit can be increased by slightly increasing t_{θ_N}.

If the frameless contract does not implement g, then reduced willingness to pay implies that this is because some type strictly U-prefers the bundle of some higher type. Denote by θ_i the lowest type that strictly U-prefers the bundle of some higher type to his own bundle y_{θ_i}. Denote by $\theta_j > \theta_i$ the lowest type whose bundle type θ_i strictly U-prefers to y_{θ_i}. Modify y_{θ_j} by increasing t_{θ_j} until type θ_i is U-indifferent between the resulting bundle and y_{θ_i}, and denote the modified bundle by y_{θ_j}. By (A2), type θ_j weakly U-prefers y_{θ_j} to *stayout* and to $y_{\theta_1}, \ldots, y_{\theta_{j-1}}$, because type θ_i is U-indifferent between y_{θ_j} and y_{θ_i} and weakly U-prefers y_{θ_i} to *stayout* and to $y_{\theta_1}, \ldots, y_{\theta_{j-1}}$ (by the definition of j). Thus, it is still the case that if some type θ strictly U-prefers the bundle of another type θ' to his own bundle, then $\theta' > \theta$.

The modification process can therefore be iterated, with i or j increasing in every iteration. The process ends with a contract in which every type U-prefers his bundle to the other bundles in the contract's menu. This contract generates a higher profit than the original framed DRC, because each type chooses the policy he did in the original DRC but the transfers are higher. ■

Proof of Proposition 2. Because the cost is type-independent, a contract that excludes the high type is strictly dominated by a pooling contract with the single bundle (x^*_L, t^*_L). And because the profit in any contract that the excludes the low type is bounded above by $\pi_H(t^*_H - c(x^*_H))$, it suffices to find a bundle (x_L, t_L) and an implementable allocation rule g that assigns the bundle (x_L, t_L) to the low type and the bundle (x^*_H, t^*_H) to the high type such that $t_L - c(x_L) > 0$.

Fix a frame $f \neq \phi$. Denote by (x_L, t_L) the bundle such that the high type is U^f-indifferent between (x^*_H, t^*_H) and (x_L, t_L) and the low type is U-indifferent between *stayout* and (x_L, t_L). That is, the bundle (x_L, t_L) is the point of intersection of the high type's U^f-indifference curve through (x^*_H, t^*_H) and the low type's U-indifference curve though *stayout* $= (0,0)$. By construction, $(x^*_H, t^*_H) \in C^H\left(\{(x_L, t_L), (x^*_H, t^*_H), stayout\}, f\right)$ and $(x_L, t_L) \in C^L\left(\{(x_L, t_L), (x^*_H, t^*_H), stayout\}, f\right)$, so g is implementable.

Assumption (A3) implies that $x_L > 0$, because the high type is U-indifferent between

(x_H^*, t_H^*) and *stayout*, and therefore strictly U^f-prefers (x_H^*, t_H^*) to *stayout*. Assumption (A4) implies that $x_L < x_L^*$. Otherwise, because (x_L, t_L) and (x_L^*, t_L^*) are on the low type's U-indifference curve through *stayout*, we have that $(x_L, t_L) \geq (x_L^*, t_L^*)$. Assumption (A2) then implies that $U(x_L, t_L, H) \geq U(x_L^*, t_L^*, H)$, so (A3) implies that $U^f(x_L, t_L, H) \geq U^f(x_L^*, t_L^*, H)$. But $U^f(x_H^*, t_H^*, H) = U^f(x_L, t_L, H)$ by construction, so $U^f(x_H^*, t_H^*, H) \geq U^f(x_L^*, t_L^*, H)$, which contradicts (A4).

Because $0 < x_L < x_L^*$, our assumption on the principal's profit function implies that the profit from the bundle (x_L, t_L) is strictly positive, so the profit from the allocation rule g is higher than only selling (x_H^*, t_H^*) to the high type.

Finally, note that (x_L, t_L) is independent of the distribution of types. This implies that in a profit-maximizing contract the policy for the low type (and therefore for the high type) is bounded away from 0, independently of the type distribution. This is because as the low-type's policy approaches 0, the principal's profit from the low type's bundle approaches 0, which is smaller than his profit from the bundle (x_L, t_L), whereas his profit from the high-type's bundle cannot be higher than his profit from (x_H^*, t_H^*), so the profit from g is higher than the profit from any contract in which the low type's policy is sufficiently small. ■

Proof of Proposition 3. Fix a profit-maximizing implementable allocation rule g that is separating, and denote the bundle of type θ by (x_θ, t_θ). By Proposition 2 and because g is separating, $0 < x_L < x_H$.

To verify that any optimal separating contract involves framing, suppose to the contrary that g is implementable by a frameless DRC. The standard theory then tells us that IR_L^U and IC_H^U bind, $x_L \leq x_L^*$, and $x_H = x_H^*$. Consider some frame $f \neq \phi$. By (A3), IC_H^f holds strictly but IC_L^f may fail. We now modify g in a way that increases profit until all the constraints are satisfied. First, increase t_H until IC_H^f or IR_H^U binds. If IR_H^U binds but IC_H^f does not, increase x_L and t_L along the U-indifference curve of the low type through *stayout*. By (A4), IC_H^f will bind before (x_L, t_L) reaches (x_L^*, t_L^*). The modified allocation rule is implementable by a DRC with the frame f, and generates a strictly higher profit than g, a contradiction.

Now suppose that g is implementable by a DRC with the frame f. We first note that if IC_θ^f holds strictly, then IR_θ^U binds, otherwise t_θ can be increased slightly without violating any of the constraints. This implies that either IC_H^f or IC_L^f bind, otherwise, because by (A4) $\{(x_L, t_L), (x_H, t_H)\} \neq \{(x_L^*, t_L^*), (x_H^*, t_H^*)\}$, some x_θ can be increased or decreased slightly along the U-indifference curve of agent θ to decrease $|x_\theta - x_\theta^*|$, which increases the principal's profit, without violating any of the constraints.

In fact, IC_H^f must bind. Indeed, suppose that IC_L^f binds. By (A2), because $x_L < x_H$, IC_H^f holds strictly, so IR_H^U binds. We now modify g in a series of steps in a way that increases profit, such that either at some point along the sequence all the constraints are satisfied, so the modified rule is implementable and generates more profit than g, a contradiction, or the modified rule assigns (x^*_θ, t^*_θ) to every type θ and IC^f_H holds, which contradicts (A4). The first step applies if $x_H > x^*_H$. In this case, decrease (x_H, t_H) continuously along the high type's U-indifference curve until either IC^f_H binds or $x_H = x^*_H$. In the former case, (A2) implies that IC^f_L holds,[14] so all the constraints are satisfied and the principal's profit increases, a contradiction. We therefore have that $x_H \le x^*_H$ and IC^f_H holds strictly. Now increase t_L until IR^U_L binds. This further relaxes IC^f_H. Finally, if $x_L < x^*_L$, increase (x_L, t_L) continuously along the low type's U-indifference curve until either IC^f_H binds or $x_L = x^*_L$. In the former case, we obtain a contradiction as in the first step. We have therefore reached a situation in which (i) $x_L \ge x^*_L$ and IR^U_L binds, (ii) $x_H \le x^*_H$ and IR^U_H binds, and (iii) IC^f_H holds strictly. Now, (i), (A2), and (A3) imply that

$$U(x_L, t_L, L) = U(x^*_L, t^*_L, L) \Rightarrow U(x_L, t_L, H) \ge U(x^*_L, t^*_L, H) \Rightarrow U^f(x_L, t_L, H) \ge U^f(x^*_L, t^*_L, H),$$

and (ii) and (A3) imply that

$$U(x^*_H, t^*_H, H) = U(x_H, t_H, H) \Rightarrow U^f(x^*_H, t^*_H, H) \ge U^f(x_H, t_H, H),$$

so by (iii) we have $U^f(x^*_H, t^*_H, H) > U^f(x^*_L, t^*_L, H)$, which contradicts (A4).

Because IC^f_H binds, by (A2) we have that IC^f_L holds strictly, so IR^U_L binds. This proves parts 1 and 2 of the proposition. It remains to show that $x_L \le x^*_L$ and $x_H \ge x^*_H$. If $x_L > x^*_L$, then decrease (x_L, t_L) slightly along the low type's U-indifference so that IC^f_L continues to hold. By (A2) and (A3) this relaxes IC^f_H, so all constraints hold and the profit increases, a contradiction. If $x_H < x^*_H$, then increase (x_H, t_H) slightly along the high type's U-indifference curve so that IC^f_L continues to hold. By (A3) this relaxes IC^f_H, so all constraints hold and the profit increases, a contradiction.[15]

Finally, suppose that $x^*_H < d$ and $x_H = x^*_H$. If IR^U_H holds strictly, then $x_L < x^*_L$, similarly to the standard setting.[16] And if IR^U_H binds, then $x_L < x^*_L$, because (A4) implies that

[14] It must be that $x_H \ge x_L$, because by IR^U_L and (A2) (x_L, t_L) lies below the high type's U-indifference curve through *stayout*, so IC^f_H binds before x_H reaches x_L.

[15] Recall that moving x_H up to x^*_H along the high type's U-indifference curve increases profit, regardless of the U-utility level.

[16] If $x^*_L = x_L$, then $x^*_L = x_L < x_H = x^*_H < d$, because the contract is separating. That $x^*_L < d$ implies that the principal's marginal cost at x^*_L is equal to the low type's U-MRS, whereas $x^*_L < x^*_H$ implies that the high type's U-MRS at x^*_L is strictly higher (because the profit function is concave along each type's U-indifference curve). Therefore, by (A3), decreasing x_L by some small ε along the low type's U-indifference curve decreases the high type's U^f-utility from the bundle (x_L, t_L) by at least $\delta\varepsilon$ for some $\delta > 0$ that is independent of ε. On the other hand, the high type's U^f-marginal utility with respect to t at (x_H, t_H) is finite, so decreasing his U^f- utility by $\delta\varepsilon$ without changing x_H requires increasing t_H by at least $\gamma\delta\varepsilon$ for some $\gamma > 0$ that is independent of $\varepsilon\delta$. Thus, for sufficiently small ε this leads to an increase in the principal's profit, because to a first order the change in profit from changing the the low type's bundle is 0, and this change allows an increase in profit from the high type that is positive to a first order.

$\{(x_L, t_L), (x_H, t_H)\} \neq \{(x_L^*, t_L^*), (x_H^*, t_H^*)\}$. But $x_L < x_L^*$ implies that the principal's marginal profit at x_L along the low type's U-indifference curve is positive, while $x_H = x_H^*$ implies that the principal's marginal profit at x_H along the high type's U-indifference curve is 0. Therefore, the profit can be increased by increasing x_H slightly along the high type's U-indifference curve, which relaxes IC_H^f and makes it possible to increase x_L along the low type's U-indifference curve.[17] ■

Proof of Proposition 4. Suppose that $f \succeq k$, and consider an optimal DRC with the frame k, in which type θ chooses the bundle $(x_\theta, t_\theta) \neq$ *stayout* (Proposition 2 shows that no type is excluded). If the contract is a pooling one, then the same bundle with the frame f will generate the same profit. Otherwise, replace the frame k by the frame f, leaving the rest of the DRC unchanged.

Suppose that no constraints are violated. Then, we are done with the first part of the proposition. For the second part, suppose that $f \succ k$. Then, IC_H^f holds strictly. Also, $x_H \geq x_H^*$ by Proposition 3. If IR_H^U does not bind, increase the profit by increasing t_H slightly. If IR_H^U binds and $x_H > x_H^*$, increase the profit by decreasing (x_H, t_H) slightly along the high type's U-indifference curve. Otherwise $(x_H, t_H) = (x_H^*, t_H^*)$, which implies that $x_L < x_L^*$ (otherwise by Proposition 3 we have $\{(x_L, t_L), (x_H, t_H)\} = \{(x_L^*, t_L^*), (x_H^*, t_H^*)\}$, which contradicts (A4) for the frame k). In this case, increase (x_L, t_L) slightly along the low type's U-indifference curve, which increases the profit.

If some constraints are violated, then $f \succeq k$ implies that only IC_L^f is violated, which implies by (A2) that IC_H^f holds strictly. By Proposition 3, we have that $x_L \leq x_L^*$; increase (x_L, t_L) along the low type's U-indifference curve until either IC_H^f binds (in which case we are done, because then IC_L^f holds so all constraints are satisfied and the new contract generates a higher profit than the original one) or $x_L = x_L^*$. Once $x_L = x_L^*$, increase t_H until either IC_H^f binds (in which case we are done) or IR_H^U binds. By Proposition 3, we have that $x_H \geq x_H^*$; once IR_H^U binds, decrease (x_H, t_H) along the high type's U-indifference curve - it must be that IC_H^f binds before (x_H^*, t_H^*) is reached, otherwise (A4) is violated. And once IC_H^f binds, all the constraints are satisfied, and the principal's profit is higher than in the DRC with the frame k. ■

Proof of Observation 4. It suffices to show that the revenue in an efficient auction with the frame f that assigns to every type θ an anticipated disappointment that equals his expected surplus in an efficient frameless auction is larger than in the optimal frameless auction.

---

[17] More precisely, increasing x_H by some small ε along the high type's U-indifference curve increases the high type's U^f-utility from the bundle (x_H, t_H) by at least $\delta\varepsilon$ for some $\delta > 0$ that is independent of ε. And increasing x_L by some small γ along the low type's U-indifference curve increases the high type's U^f-utility from the bundle (x_L, t_L) by no more than $\alpha\gamma$ for some $\alpha > 0$. Thus, the increase of x_H by ε allows to increase x_L by at least $\delta\varepsilon/\alpha$. And because the marginal effect on the profit of such an increase in x_H is 0, whereas the marginal effect on the profit of the increase in x_L is positive, for small ε the profit increases.

Let N denote the number of bidders in the auction. By Myerson (1981), one optimal frameless auction is a second-price auction with a reserve price of $1/2$. The revenue in this auction is:

$$\left(\frac{1}{2}\right)^N \cdot 0 + N\left(\frac{1}{2}\right)^N \cdot \frac{1}{2} + \sum_{i=2}^{N} \binom{N}{i} \left(\frac{1}{2}\right)^N \left(\frac{1}{2} + \frac{i-1}{i+1}\frac{1}{2}\right) = \frac{N-1}{N+1} + \frac{1}{2^N (N+1)}.$$

By the revenue equivalence theorem, the revenue in an efficient auction with the frame f is identical to the revenue in a second price auction with the frame f, where our assumption on f implies that $f(\theta) = \theta^N/N$. Given that a type θ bidder places a bid in this second price auction, it is weakly dominant for him to bid $\theta + f(\theta)$. And if all bidders bid in this way, then a simple calculation shows that bidding in the auction is U^f-superior to not bidding. Therefore, the revenue in this auction is the second-order statistic of the valuations, $(N-1)/(N+1)$, plus the second-order statistic of $f(\theta)$, which a straightforward calculation shows is $(N-1)/(2N(2N-1))$. Thus, the revenue is $(N-1)/(N+1) + (N-1)/(2N(2N-1))$.

It remains to verify that

$$\frac{N-1}{2N(2N-1)} \geq \frac{1}{2^N (N+1)}.$$

This holds for $N = 2$. For $N \geq 3$, it suffices to show that $2^N \geq 2\left(2 - \frac{1}{N}\right)/(1 - 1/N^2)$. This inequality holds, because for $N \geq 3$ we have that

$$2^N \geq 8 > \frac{4}{1-\frac{1}{9}} > \frac{2\left(2-\frac{1}{N}\right)}{1-\frac{1}{N^2}}. \blacksquare$$

References

[1] **Ahn, David S. and Haluk Ergin.** 2010. "Framing Contingencies." *Econometrica*, 78(2), 655–695.

[2] **Carbajal, Juan Carlos and Jeffrey C. Ely.** 2012. "Optimal contracts for loss-averse consumers." Mimeo.

[3] **De-Clippel, Geoffroy.** 2013. "Behavioral Implementation." Mimeo.

[4] **Delgado, Mauricio R., Andrew Schotter, Erkut Y. Ozbay, and Elizabeth A. Phelps**. 2008. "Understanding Overbidding: Using the Neural Circuitry of Reward to Design Economic Auctions." *Science*, 321(5897), 1849–1852.

[5] **Eliaz, Kfir and Ran Spiegler.** 2006. "Contracting with Diversely Naive Agents." *Review of Economic Studies*, 73(3), 689–714.

[6] **Esteban, Susanna and Eiichi Miyagawa.** 2006. "Optimal menu of menus with self-control preferences." Mimeo.

[7] **Esteban, Susanna, Eiichi Miyagawa, and Matthew Shum.** 2007. "Nonlinear pricing with self-control preferences." *Journal of Economic Theory*, 135(1), 306–338.

[8] **Filiz-Ozbay, Emel and Erkut Y. Ozbay.** 2007. "Auctions with Anticipated Regret: Theory and Experiment." *American Economic Review*, 97(4), 1407–1418.

[9] **Fudenberg, Drew and Jean Tirole.** 1992. "Game Theory." MIT Press, Cambridge, MA.

[10] **Galperti, Simone.** 2013. "Optimal Screening of Time Inconsistency." Mimeo.

[11] **Glazer, Jacob and Ariel Rubinstein.** 2013. "A Model of Persuasion with a Boundedly Rational Agent." *Journal of Political Economy*, forthcoming.

[12] **Kőszegi, Botond.** 2013. "Behavioral Contract Theory." Mimeo.

[13] **Myerson, Roger B.** 1981. "Optimal auction design." *Mathematics of Operations Research*, 6(1), 58–73.

[14] **Piccione, Michele and Ran Spiegler.** 2012. "Price Competition under Limited Comparability." *Quarterly Journal of Economics*, 124(1), 51–104.

[15] **Rubinstein, Ariel and Yuval Salant.** 2008. "Some Thoughts on the Principle of Revealed Preference." *In Handbooks of Economic Methodologies (Andrew Caplin and Andrew Schotter, eds.)*, New York: Oxford University Press, 115–124.

[16] **Rubinstein, Ariel and Yuval Salant.** 2012. "Eliciting Welfare Preferences from Behavioral Datasets." *Review of Economic Studies*, 79(1), 375–387.

[17] **Salant, Yuval and Ariel Rubinstein.** 2008. "(A, f): Choice with Frames." *Review of Economic Studies*, 75(4), 1287–1296.

[18] **Spiegler, Ran.** 2013. "Competitive Framing." *American Economic Journal: Microeconomics*, forthcoming.

[19] **Stiglitz, Joseph E.** 1977. "Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market." *Review of Economic Studies*, 44(3), 407–430.