Optimal taxation in a habit formation economy*

Sebastian Koehne† Moritz Kuhn‡

September 25, 2014

Abstract

This paper studies habit formation in consumption preferences in a dynamic Mirrlees economy. We derive optimal labor and savings wedges based on a recursive approach. We show that habit formation creates a motive for subsidizing labor supply and savings. In particular, habit formation invalidates the well-known "no distortion at the top" result. We demonstrate that the theoretical findings are quantitatively important: in a parametrized life-cycle model, average labor and savings wedges fall by more than one-third compared with the case of time-separable preferences.

Keywords: optimal taxation; habit formation; recursive contracts

JEL Classification: D82, E21, H21

*This paper is a revised and extended version of an earlier manuscript titled "Optimal capital taxation for time-nonseparable preferences." The authors thank Erzo F. P. Luttmer, three anonymous referees, Arpad Abraham, Carlos da Costa, John Hassler, Per Krusell, Etienne Lehmann, Jean-Marie Lozachmeur, Nicola Pavoni, Pierre Pestieau, Rick van der Ploeg, Hakki Yazici, and participants at various seminars and conferences for many helpful comments. Sebastian Koehne gratefully acknowledges financial support from Torsten Söderbergs Stiftelse and Knut och Alice Wallenbergs Stiftelse.

†Corresponding author at: Institute for International Economic Studies (IIES), Stockholm University, SE-10691 Stockholm, Sweden, Phone: +46 8 16 35 64, sebastian.koehne@iies.su.se, and CESifo, Munich, Germany.

‡University of Bonn, Department of Economics, D-53113 Bonn, Germany, Phone: +49 228 73 62096, mokuhn@uni-bonn.de, and Institute for the Study of Labor (IZA), Bonn, Germany.

1 Introduction

What determines the optimal taxes on labor income and capital? Fundamental to this classic public finance question is a description of intertemporal decision making. Existing studies, following Diamond and Mirrlees (1978), have explored optimal taxation when decision makers aggregate across time in a separable way. The present paper proposes a model of decision making motivated by evidence from macroeconomics, psychology, and micro data—the *habit formation* model.[1] This model contains time-separable preferences as a special case but allows for intertemporal complementarities in consumption.

We introduce habit formation preferences into an otherwise standard dynamic Mirrlees economy. Agents face shocks to their abilities to generate labor income. Labor income is publicly observed, but abilities and labor supply are private information. In this environment, we characterize the solution of the social planning problem in terms of labor and savings wedges. As is common in this literature, positive wedges represent *implicit* taxes and indicate that decentralizations of the social planning allocation must correct individual labor or savings returns downward in one way or another.[2] To make the multiperiod social planning problem tractable for theoretical and numerical analysis, we transform it into a dynamic programming problem by generalizing insights from the recursive contract theory literature. This approach is common in dynamic private information problems with time-separable preferences (Spear and Srivastava, 1987; Phelan and Townsend, 1991). Our recursive formulation extends beyond optimal taxation and applies to a large class of private information problems.

We first study optimal labor taxation. For habit formation preferences, labor wedges are shaped by two countervailing forces. First, as in any self-selection problem with time-separable preferences, there is a motive for downward distortions to labor supply of all but the most productive type. This motive calls for positive labor wedges. Second, habit formation connects present and future self-selection problems. Because of complementarity between habits and consumption, self-selection becomes easier in the future if the worker consumes a lot in the present. This *habit effect* calls for subsidies to labor supply for all types and counteracts the

[1]See Messinis (1999) for a summary of habit formation in macroeconomics and Frederick and Loewenstein (1999) for a review of habit formation in the empirical and behavioral economics literature.

[2]The decentralization of optimal allocations is not unique; compare Golosov, Kocherlakota, and Tsyvinski (2003), Kocherlakota (2005), Albanesi and Sleet (2006), Golosov and Tsyvinski (2006), Werning (2011), Gottardi and Pavoni (2011), and Abraham, Koehne, and Pavoni (2014).

conventional self-selection distortion. As a consequence, the "no distortion at the top" result breaks down, and the most productive type obtains a negative labor wedge. For less productive types, labor wedges can be positive or negative, depending on the importance of the habit effect compared with the conventional self-selection distortion.

We next turn to optimal savings taxation. Our decomposition of savings wedges reveals three taxation motives. First, savings should be taxed because the agent has a better incentive to supply labor in the next period if he starts the next period with lower wealth *(wealth effect)*. This force is well known from models with time-separable preferences. Second, savings should be taxed, because stimulating present consumption increases the habit level in the next period. This effect makes high consumption in the next period more attractive and thereby reinforces the incentive to supply labor *(immediate habit effect)*. Third, savings should be subsidized, because stimulating next period's consumption increases the habit level in the remaining periods and thereby improves labor supply incentives in those periods *(subsequent habit effect)*. Habit formation thus affects savings taxation in opposing ways, and its impact will depend on the relative magnitude of immediate versus subsequent habit effects.

Our theoretical results identify forces that counteract the conventional Mirrleesian distortions to labor supply and savings. To demonstrate the quantitative importance of these results, we evaluate habit formation in a stylized life-cycle model. We parametrize the model according to empirical findings for the U.S. economy. We find the impact of habit formation on optimal savings and labor wedges to be negative and sizable. Averaged over the life cycle, optimal savings wedges of a typical worker fall by 40 percent, and optimal labor wedges by 35 percent, compared with the case of time-separable preferences. The negative impact on labor wedges was already suggested by our theoretical results. The negative impact on savings wedges is due to subsequent habit effects that prevail over immediate habit effects. Intuitively, incentive provision becomes more costly when rewards can be smoothed over fewer periods. Therefore, relaxing incentive problems later in life through subsequent habit effects is more important than relaxing incentive problems in the direct future through immediate habit effects.

Related literature. With few exceptions, most existing studies of dynamic taxation problems work with time-separable preferences. The contribution closest to ours is by Grochulski and Kocherlakota (2010) and explores a Mirrlees framework with time-nonseparable preferences similar to the present paper. Their focus is decentralization, and they show that social security systems (with history-dependent taxes and transfers upon retirement) can be used to implement optimal allocations when preferences are time-nonseparable. Apart from a three-period example with a negative savings wedge, they do not investigate savings or labor wedges any further.[3]

Several papers study Mirrleesian models with alternative forms of preference nonseparabilities. While habit formation differs from other nonseparabilities and requires an independent treatment, a general finding is that preference nonseparabilities affect Mirrleesian wedges in magnitude and sign. This finding applies to recursive preferences (Farhi and Werning, 2008), human capital effects (Bohacek and Kapicka, 2008; Grochulski and Piskorski, 2010; Stantcheva, 2014), and nonseparabilities between consumption and labor supply (Farhi and Werning, 2013), for example.

Another related paper is by Cremer, De Donder, Maldonado, and Pestieau (2010) and explores optimal commodity taxation in a framework with myopic habit formation. This framework gives rise to paternalistic taxation motives, because individuals do not foresee the habit formation relation when making consumption and savings decisions. Similar effects arise when myopic habit formation is introduced into a model of retirement; see Cremer and Pestieau (2011). The present paper is different in several key aspects, because we focus on labor and savings taxation and study time-consistent decision makers that anticipate their future preferences.

Finally, the paper builds on the extensive literature on habit formation preferences. Habit formation goes back to the theory of adaptation formalized in the psychological literature by Helson (1964). Habit formation postulates that individuals compare their current consumption with a historical reference level and derive utility both from consumption per se and from consumption growth.[4] Heien and Durham (1991) find support for habit formation based on micro-level consumption data. Frederick and Loewenstein (1999) review the substantial body of empirical research supporting the habit formation hypothesis. Moreover, habit formation has reconciled theory and evidence for several important questions in the macroeconomic literature,

[3] Our decomposition of savings wedges shows that the subsequent habit effect is responsible for their finding. However, we also reveal that incentive problems in the immediate future create countervailing forces because of wealth and immediate habit effects. Our quantitative analysis therefore finds that, even though it is possible to construct theoretical cases in which savings wedges are negative, those cases are not representative of typical taxation environments.

[4] In addition, there is the concept of *external* habit formation, where the reference point depends on the consumption levels of a peer group; see the discussion of "Catching up with the Joneses" in Abel (1990).

such as the equity premium puzzle (Abel, 1990; Constantinides, 1990; Campbell and Cochrane, 1999), the relationship between savings and growth (Ryder and Heal, 1973; Carroll, Overland, and Weil, 2000), and reactions to monetary policy shocks (Fuhrer, 2000).

2 Model

This section sets up a dynamic Mirrlees model of optimal taxation with habit formation preferences. The economy consists of a risk-neutral principal/planner and a unit measure of risk-averse agents facing a binary stochastic skill process. Time is discrete and indexed by $t = 1, 2, \ldots, T$, with $T < \infty$.

2.1 Preferences

Agents have identical von Neumann-Morgenstern preferences and maximize the expected value of

$$\sum_{t=1}^{T} \beta^{t-1} \left(u(c_t, h_t) - v(l_t) \right),$$

where c_t, h_t, l_t represent the agent's consumption, habit, and labor supply in period t, and $\beta \in (0, 1)$ is the agent's discount factor.[5] Labor disutility $v : \mathbb{R}_+ \to \mathbb{R}$ is continuous, strictly increasing, and weakly convex. Consumption utility $u : \mathbb{R}^2_+ \to \mathbb{R}$ is twice continuously differentiable, strictly concave, strictly increasing in its first argument, and strictly decreasing in its second argument. Consumption and habit are complements: $u''_{ch} > 0$. As usual, we use subscripts to denote partial derivatives.

The complementarity assumption $u''_{ch} > 0$ is standard in the habit formation literature. It holds for the widely used case of linear habit formation: $u(c_t, h_t) = \tilde{u}\left(c_t - \gamma h_t\right)$, with $\gamma \in (0, 1]$ and $\tilde{u} : \mathbb{R}_+ \to \mathbb{R}$ strictly increasing and strictly concave; compare Constantinides (1990) and Campbell and Cochrane (1999) among others. Another common specification of habit formation is the Cobb-Douglas case: $u(c_t, h_t) = \tilde{u}\left(c_t h_t^{-\gamma}\right)$; compare Abel (1990), Carroll, Overland, and Weil (2000), Fuhrer (2000), and Diaz, Pijoan-Mas, and Rios-Rull (2003). Here, $u''_{ch} > 0$ holds if the coefficient of relative risk aversion of $\tilde{u}$ is bounded below by one.[6]

[5] The preferences we use are time-consistent; see Johnsen and Donaldson (1985), for example.

[6] Write $\tilde{c} = ch^{-\gamma}$. Then $u''_{ch}(c, h) = \gamma h^{-\gamma-1} \tilde{u}'(\tilde{c}) \left[-\tilde{c}\tilde{u}''(\tilde{c})/\tilde{u}'(\tilde{c}) - 1\right]$.

2.2 Habits

We assume from now on that habits are short-lived: $h_t = c_{t-1}$, with c_0 being exogenous. This assumption simplifies the exposition and is empirically supported by results in Fuhrer (2000). Our results generalize easily to the case in which habits are a function of lagged consumption and lagged habit levels, $h_t = H(c_{t-1}, h_{t-1})$. See Section 3.1 for further discussion.

2.3 Skills

Agents differ with respect to their skills. An agent with hours l_t and skill realization θ_t produces $y_t = \theta_t l_t$ units of output in period t. Output is publicly observable, but hours and skills are private information.

For every t, let $\Theta_t = \left\{\theta_t^L, \theta_t^H\right\}$ be the set of possible skill realizations, with $0 < \theta_t^L \leq \theta_t^H$. Define $\Theta^t := \Theta_1 \times \cdots \times \Theta_t$. At the beginning of each period, a skill level $\theta_t \in \Theta_t$ is drawn for each agent. Draws are independent across agents. For now, we assume that draws are also independent across time. (In online Appendix C, we allow for skill processes with persistence.) Hence, there exist probability weights $\pi_t(\theta_t)$, with $\sum_{\theta_t \in \Theta_t} \pi_t(\theta_t) = 1$, such that the probability of a partial skill history $\theta^t = (\theta_1, \ldots, \theta_t) \in \Theta^t$ is given by $\Pi^t\left(\theta^t\right) = \pi_1\left(\theta_1\right) \cdots \pi_t\left(\theta_t\right)$. Without loss of generality, we assume $\pi_t(\theta_t) > 0$ for all $\theta_t \in \Theta_t$. We denote the expectation operator with respect to the unconditional distribution of skill histories θ^T by $\mathbb{E}[\,\cdot\,]$. As usual, the notation $\mathbb{E}_t[\,\cdot\,] := \mathbb{E}\left[\,\cdot\,\middle|\theta^t\right]$ represents expectations conditional on the time-t history θ^t.

The case of binary skills is a common simplification for discrete income taxation problems; compare Feldstein (1973), Stern (1982), and Stiglitz (1982), for example. Binary skills facilitate the exposition but are not essential to our results. Section 3.1 provides further discussion.

2.4 Social planner

We set up the social planning problem in its dual form: the social planner minimizes the costs of delivering a given level of ex ante welfare to the agents. The planner discounts future costs by a factor $q < 1$. Equivalently, the planner has access to a linear savings technology that transforms q units of date-t output into 1 unit of output at date $t + 1$.[7]

[7] It would not be difficult to endogenize the return of the savings technology by introducing an explicit production function that depends on capital and labor. Yet, this exercise would merely complicate the notation and generate no additional insights for the questions addressed in this paper.

2.5 Allocations

An *allocation* is a sequence $(\mathbf{c}, \mathbf{y}) = (c_t, y_t)_{t=1,\ldots,T}$ of consumption plans $c_t : \Theta^t \to \mathbb{R}_+$ and output plans $y_t : \Theta^t \to \mathbb{R}_+$. A *reporting strategy* is a sequence $\sigma = (\sigma_t)_{t=1,\ldots,T}$ of mappings $\sigma_t : \Theta^t \to \Theta_t$. Denote the set of all reporting strategies by Σ and set $\sigma^t(\theta^t) := \left(\sigma_1(\theta^1), \ldots, \sigma_t(\theta^t)\right)$. At the beginning of every period, the planner allocates consumption and output according to the history of reported skills. Because of short-lived habits, we have $h_t = c_{t-1}$. Hence, a reporting strategy $\sigma \in \Sigma$ yields ex ante expected utility according to

$$
\begin{aligned}
& w_1\left(\mathbf{c} \circ \sigma, \mathbf{y} \circ \sigma; c_0\right) \\
&:= \sum_{t=1}^{T} \sum_{\theta^t \in \Theta^t} \beta^{t-1} \left[u\left(c_t\left(\sigma^t\left(\theta^t\right)\right), c_{t-1}\left(\sigma^{t-1}\left(\theta^{t-1}\right)\right)\right) - v\left(\frac{y_t\left(\sigma^t\left(\theta^t\right)\right)}{\theta_t}\right)\right] \Pi^t\left(\theta^t\right).
\end{aligned}
$$

Since skills are privately observed, the planner needs to ensure that all agents reveal their information truthfully. An allocation that satisfies the truth-telling constraint

$$
w_1\left(\mathbf{c}, \mathbf{y}; c_0\right) \geq w_1\left(\mathbf{c} \circ \sigma, \mathbf{y} \circ \sigma; c_0\right) \quad \forall \sigma \in \Sigma
$$

is called *incentive compatible.*

2.6 Optimal allocations

The social planner seeks to provide a given level W_1 of ex ante welfare at minimal costs. Hence, an allocation $(\mathbf{c}, \mathbf{y})$ is called *optimal* if it solves the following problem:

$$
C_1(W_1, c_0) := \min_{\mathbf{c}, \mathbf{y}} \sum_{t=1}^{T} \sum_{\theta^t \in \Theta^t} q^{t-1} \left[c_t\left(\theta^t\right) - y_t\left(\theta^t\right)\right] \Pi^t\left(\theta^t\right) \tag{1}
$$

s.t.

$$
w_1\left(\mathbf{c}, \mathbf{y}; c_0\right) \geq w_1\left(\mathbf{c} \circ \sigma, \mathbf{y} \circ \sigma; c_0\right) \quad \forall \sigma \in \Sigma \tag{2}
$$

$$
w_1\left(\mathbf{c}, \mathbf{y}; c_0\right) = W_1. \tag{3}
$$

2.7 Recursive formulation

We use a recursive approach to derive labor and savings wedges and to study the quantitative importance of habit formation in a parametrized model. This subsection sets up the required notation and states the recursive formulation of the problem. We show that optimal allocations have a recursive formulation with two state variables: promised utility and the agent's habit level. Details and proofs are relegated to online Appendix B.

Given an allocation $(\mathbf{c}, \mathbf{y})$ and a history θ^t, $1 \leq t < T$, the *continuation allocation* $\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right)\right)$ is defined as the restriction of plans $(c_s, y_s)_{s=t+1,\ldots,T}$ to those histories $\theta^{t+1}, \ldots, \theta^T$ that succeed θ^t. The *continuation utility* associated with the continuation allocation is defined as

$$\begin{aligned}
& w_{t+1}\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right); c_t\left(\theta^t\right)\right) \\
:= & \sum_{s=t+1}^{T} \sum_{\theta^s \in \Theta^s} \beta^{s-t-1}\left[u\left(c_s\left(\theta^s\right), c_{s-1}\left(\theta^{s-1}\right)\right) - v\left(\frac{y_s\left(\theta^s\right)}{\theta_s}\right)\right] \Pi^s\left(\theta^s | \theta^t\right).
\end{aligned}$$

Note that the continuation utility w_{t+1} depends not only on the continuation allocation but also on lagged consumption $c_t\left(\theta^t\right)$ in order to capture the habit level at the beginning of period $t+1$. For any $c_- \in \mathbb{R}_+$ we define $\mathrm{dom}_{t+1}(c_-)$ to be the set of continuation utilities W with the property that, given habit level c_- in period $t+1$, there exists an incentive compatible allocation $\left(c_{t+1}^T, y_{t+1}^T\right)$ that generates utility

$$\mathbb{E}_t\left[\sum_{s=t+1}^{T} \beta^{s-t-1}\left(u(c_s, c_{s-1}) - v(y_s/\theta_s)\right)\right] = W, \quad \text{where } c_t = c_-.$$

Similar to the findings for time-separable preferences by Spear and Srivastava (1987) and Phelan and Townsend (1991), the constraint set and the objective of the social planner problem (1) can be given a sequential form.[8] This gives rise to the following reformulation of the problem.

Proposition 1 (Recursive formulation)**.** *Let $W_1 \in \mathrm{dom}_1(c_0)$. The value $C_1(W_1, c_0)$ of the social planner problem* (1) *can be computed by backward induction using the following equation for all*

[8] Following the approach by Fernandes and Phelan (2000), we can obtain a similar formulation when skill shocks are persistent.

t (with the convention $C_{T+1} = W_{T+1} = 0$):

$$C_t\left(W_t, c_{t-1}\right) = \min_{c_t^i, y_t^i, W_{t+1}^i} \sum_{i=L,H} \left[c_t^i - y_t^i + qC_{t+1}\left(W_{t+1}^i, c_t^i\right)\right] \pi_t\left(\theta_t^i\right) \tag{4}$$

s.t.

$$u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i \geq u\left(c_t^j, c_{t-1}\right) - v\left(y_t^j/\theta_t^i\right) + \beta W_{t+1}^j, \quad i, j = L, H \tag{5}$$

$$\sum_{i=L,H} \left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right] \pi_t\left(\theta_t^i\right) = W_t \tag{6}$$

$$W_{t+1}^i \in \mathrm{dom}_{t+1}\left(c_t^i\right), \quad i = L, H. \tag{7}$$

Moreover, plans $(c_t, y_t)_{t=1,\dots,T}$ that solve the sequence of problems (4) *constitute an optimal allocation. Conversely, any optimal allocation solves the sequence of problems* (4)*.*

Proposition 1 separates the social planner problem (1) into a sequence of simpler problems in which the planner determines current consumption, current output, and continuation utility at every point in time as a function of the current skill. Choices are constrained by the temporary incentive compatibility constraint (5), the promise-keeping constraint (6), and the domain restriction (7). The only difference relative to the familiar recursive formulation for incentive problems with time-separable preferences is that the agent's habit level becomes an additional state variable.[9]

In what follows, we assume that continuation utilities are interior elements of the domain. This assumption can be justified by imposing appropriate boundary conditions on preferences.[10]

3 Labor and savings wedges

This section derives the *wedges* (tax distortions) imposed by optimal allocations. As is well known in the dynamic public finance literature, the decentralization of optimal allocations is not unique. Hence, the robust insights from the present analysis are not about explicit tax instruments but about wedges.

In order to define labor and savings wedges, we first examine the agent's marginal utility of consumption. With habit formation, current consumption influences future habit levels. Given

[9]The recursive formulation can be easily extended to the case of persistent habits. See online Appendix B.

[10]For instance, $\mathrm{dom}_t\left(c_-\right) = \mathbb{R}$ for any t and c_- if consumption utility is unbounded below and above, or if consumption utility and labor disutility are unbounded above.

a consumption history $(c_1, \ldots, c_T)$, the marginal utility of consuming at date t is given by

$$\tilde{U}_t := \begin{cases} u_c'\left(c_t, c_{t-1}\right) + \beta u_h'\left(c_{t+1}, c_t\right) & \text{if } t < T, \\ u_c'\left(c_T, c_{T-1}\right) & \text{if } t = T. \end{cases}$$

If consumption in period $t+1$ is uncertain from the point of view of period t, marginal consumption utility becomes a random variable. We write $U_t := \mathbb{E}_t\left[\tilde{U}_t\right]$ for the expectation of this random variable conditional on date-t information.

Given an allocation $(\mathbf{c}, \mathbf{y})$, define the *labor wedge* in period t as

$$\tau_{y,t} := 1 - \frac{v'\left(y_t/\theta_t\right)}{\theta_t U_t}$$

and the *savings wedge* in period t as

$$\tau_{s,t} := 1 - \frac{qU_t}{\beta \mathbb{E}_t[U_{t+1}]}.$$

Note that $\tau_{y,t}$ and $\tau_{s,t}$ are random variables that depend on the date-t history θ^t, even though we have omitted this argument for notational convenience. Apart from the fact that habit formation changes the formula for marginal consumption utility U_t, the above definitions are standard. The labor wedge is the implicit tax rate that equates the agent's marginal rate of substitution between consumption and leisure to the after-tax income of an additional unit of labor supply. Similarly, the savings wedge is the implicit tax rate that aligns the agent's marginal rate of intertemporal substitution with the relative price of future consumption.

We solve a relaxed problem in which only downward incentive compatibility constraints are imposed.[11] Lemma 1 justifies this approach. The proof of Lemma 1 and all further proofs are relegated to online Appendix A.

Lemma 1. *The solution to the social planner problem* (4) *coincides with the solution to the*

[11] In addition, we assume that consumption and output are nonzero. This assumption can be justified by boundary conditions of the form $v'(0) = 0$ and $\lim_{c \to 0} u_c'(c, h) = \infty$ for all $h > 0$, for instance.

following relaxed problem:

$$C_t(W_t, c_{t-1}) = \min_{c_t^i, y_t^i, W_{t+1}^i} \sum_{i=H,L} \left[c_t^i - y_t^i + qC_{t+1}\left(W_{t+1}^i, c_t^i\right)\right] \pi_t\left(\theta_t^i\right) \tag{8}$$

s.t.

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H \geq u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta W_{t+1}^L \tag{9}$$

$$\sum_{i=H,L} \left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right] \pi_t\left(\theta_t^i\right) = W_t. \tag{10}$$

In what follows, we fix the period-t state vector (W_t, c_{t-1}). Equivalently, we fix the associated skill history θ^{t-1}. We denote the Lagrange multiplier for the incentive compatibility constraint (9) by μ_t and the multiplier for the promise-keeping constraint (10) by λ_t. We begin our analysis with the following preliminary insight.

Remark 1 (Homogeneous skills)**.** *Let* $(\mathbf{c}, \mathbf{y})$ *be an optimal allocation and suppose* $\theta_t^L = \theta_t^H$ *for* $t \geq t_0$*. Then the labor and savings wedges are zero:* $\tau_{y,t} = \tau_{s,t} = 0$ *for* $t \geq t_0$*.*

Remark 1 implies that tax distortions in our model are entirely due to skill heterogeneity, exactly as in the case of time-separable preferences. Thus, habit formation does not create a direct taxation motive. However, habit formation does create an important indirect taxation motive because it changes the structure of the incentive problem to report skills truthfully.

Proposition 2 (Labor wedges)**.** *Let* $(\mathbf{c}, \mathbf{y})$ *be an optimal allocation. For each history* θ^{t-1}*,* $t < T$*, there exist numbers* $A_t^L, B_t^L, B_t^H \geq 0$ *and Lagrange multipliers* $\mu_t, \mu_{t+1}^L, \mu_{t+1}^H \geq 0$ *associated with the incentive compatibility constraints in periods* t *and* $t+1$ *such that*

$$\tau_{y,t}\left(\theta^{t-1}, \theta_t^H\right) = -\mu_{t+1}^H B_t^H \leq 0, \tag{11}$$

$$\tau_{y,t}\left(\theta^{t-1}, \theta_t^L\right) = \mu_t A_t^L - \mu_{t+1}^L B_t^L \gtrless 0. \tag{12}$$

For $t = T$*, equations* (11) *and* (12) *hold with* μ_{t+1}^L, μ_{t+1}^H *replaced by zero. Finally, in the limit case of time-separable preferences* ($u_h' = 0$)*, we have* $B_t^L = B_t^H = 0$*.*

For time-separable preferences, Proposition 2 states that the labor wedge of the high-skilled worker is zero ("no distortion at the top"). The low-skilled worker faces the positive labor wedge $\mu_t A_t^L$. As usual in self-selection problems, this downward distortion is efficient because it reduces the incentive of the high-skilled worker to pretend being low-skilled.

With habit formation, the same self-selection distortion continues to apply. In addition, there is a motive for subsidizing the labor supply of high-skilled as well as low-skilled workers, captured by the terms $\mu_{t+1}^i B_t^i$ for $i = L, H$. As the Lagrange multiplier μ_{t+1}^i indicates, this motive is due to the incentive problem in period $t+1$. The proof of Proposition 2 reveals that B_t^i can be expressed as

$$B_t^i = b_t \left[u_h' \left(c_{t+1}^H, c_t \right) - u_h' \left(c_{t+1}^L, c_t \right) \right] = b_t u_{ch}'' \left(\xi, c_t \right) \left[c_{t+1}^H - c_{t+1}^L \right], \tag{13}$$

where $b_t = b_t\left(\theta^t\right)$ is a strictly positive number, $\theta^t = \left(\theta^{t-1}, \theta_t^i\right)$, while $c_t = c_t\left(\theta^t\right)$ and $c_{t+1}^H = c_{t+1}\left(\theta^t, \theta_{t+1}^H\right)$, $c_{t+1}^L = c_{t+1}\left(\theta^t, \theta_{t+1}^L\right)$ are the consumption levels in periods t and $t+1$, and $\xi = \xi\left(\theta^t\right)$ is some number between c_{t+1}^L and c_{t+1}^H. Since habit and consumption are by assumption complements, B_t^i is positive and enters negatively into the labor wedge. The intuition for this finding is as follows. A low labor wedge encourages work at date t. This increases date-t consumption and results in a higher habit level c_t at date $t+1$. Because of complementarity, the difference between the utility of a high-skilled worker $u\left(c_{t+1}^H, c_t\right)$ and a low-skilled worker $u\left(c_{t+1}^L, c_t\right)$ increases. This effect is socially desirable because it facilitates self-selection at $t+1$.

At a more general level, Proposition 2 shows that optimal intraperiod distortions take into account intertemporal preference dependencies. Since high habit levels are helpful for future incentive problems, this generates a motive for subsidizing labor across all skill types. We label this the *habit effect* and denote it by B_t^i. As a consequence, the labor wedge for high-skilled agents is negative ("subsidies at the top"), while the labor wedge for low-skilled agents consists of the standard taxation motive for current incentive provision A_t^L minus the habit effect B_t^L.

We now turn to the analysis of savings wedges. For time-separable preferences, savings wedges can be analyzed by variational arguments that perturb optimal allocations in two adjacent time periods. The result is the seminal Inverse Euler equation.[12] Unfortunately, this approach does not extend to the class of habit formation preferences. The key problem is that consumption at any given point in time affects future habit levels. Therefore, the contribution of consumption in periods t and $t+1$ to the worker's lifetime utility depends on subsequent consumption levels and hence on subsequent skill realizations. It is thus impossible to find a

[12] See Rogerson (1985) and Golosov, Kocherlakota, and Tsyvinski (2003), for instance.

consumption perturbation that is incentive-neutral and uses only information from periods t and $t+1$ (unless $t = T-1$); see Grochulski and Kocherlakota (2010).

The Lagrangian techniques adopted in this paper deliver insights on savings wedges for the habit formation case.

Proposition 3 (Savings wedges)**.** *Let* $(\mathbf{c}, \mathbf{y})$ *be an optimal allocation. For each history* θ^t, $t < T-1$, *there exist numbers* $D_t, E_t, F_t^j \geq 0$, $j \in \{L, H\}$, *and Lagrange multipliers* $\mu_{t+1}, \mu_{t+2}^j \geq 0$, $j \in \{L, H\}$, *associated with the incentive compatibility constraints in periods* $t+1$ *and* $t+2$ *such that*

$$\tau_{s,t}\left(\theta^t\right) = \mu_{t+1} D_t + \mu_{t+1} E_t - \sum_{j=L,H} \pi_{t+1}\left(\theta_{t+1}^j\right) \mu_{t+2}^j F_t^j. \tag{14}$$

For $t = T-1$, *equation* (14) *holds with* μ_{t+2}^L, μ_{t+2}^H *replaced by zero. Finally, in the limit case of time-separable preferences* ($u_h' = 0$), *we have* $E_t = F_t^L = F_t^H = 0$.

Proposition 3 shows that savings wedges for habit formation preferences have three components denoted by D_t, E_t, and F_t^j. Intuitively, the three components can be demonstrated by considering the following hypothetical situation. The agent, after working in period t and receiving the transfer $c_t\left(\theta^t\right)$, saves one unit of consumption for the following period. Three effects then change the agent's preferences over future states, and thereby the incentive to supply labor (or, put differently, the incentive to report truthfully) in the future.

First, there is the familiar *wealth effect* D_t. Saving one consumption unit at time t yields a fixed number of extra consumption units in all states at time $t+1$. Since preferences are concave in consumption, the value of extra consumption is higher in states with low c_{t+1}. Low-consumption states thus become relatively more attractive, and the agent's incentive to supply labor in period $t+1$ is reduced. This concavity/wealth effect is captured by the term

$$D_t = d_t\left(\mathbb{E}\left[\tilde{U}_{t+1}\middle|\theta^t, \theta_{t+1}^L\right] - \mathbb{E}\left[\tilde{U}_{t+1}\middle|\theta^t, \theta_{t+1}^H\right]\right), \tag{15}$$

where $d_t = d_t\left(\theta^t\right)$ is a strictly positive number, and $\tilde{U}_{t+1}$ is the marginal utility of consumption in period $t+1$. Since the marginal utility of consumption is higher in low-consumption (low-skill) states, D_t is positive and calls for a positive tax on savings. For time-separable preferences, Proposition 3 shows that D_t is in fact the only component of the savings wedge.

The second component of the savings wedge is the *immediate habit effect* E_t. Saving in period t reduces the agent's consumption and thereby diminishes the habit level at time $t+1$. Because of complementarity between habit and consumption, low-consumption states at time $t+1$ become relatively more attractive. This result reduces the incentive to supply labor. Formally, the immediate habit effect can be expressed as

$$E_t = e_t \left[u_h' \left(c_{t+1}^H, c_t \right) - u_h' \left(c_{t+1}^L, c_t \right) \right], \tag{16}$$

where $e_t = e_t \left(\theta^t\right)$ is strictly positive, while $c_t = c_t \left(\theta^t\right)$ and $c_{t+1}^H = c_{t+1} \left(\theta^t, \theta_{t+1}^H\right)$, $c_{t+1}^L = c_{t+1} \left(\theta^t, \theta_{t+1}^L\right)$ are the consumption levels in periods t and $t+1$. Since the cross derivative u_{ch}'' is positive by assumption, E_t is positive. Hence, the immediate habit effect goes in the same direction as the wealth effect and generates an additional motive for taxing savings.

Finally, the savings wedge has components F_t^j that capture a *subsequent habit effect*. As the Lagrange multiplier μ_{t+2}^j in equation (14) suggests, these components relate to the incentive problem in period $t+2$ and can be written as

$$F_t^j = f_t \left[u_h' \left(c_{t+2}^H, c_{t+1} \right) - u_h' \left(c_{t+2}^L, c_{t+1} \right) \right], \tag{17}$$

where $f_t = f_t \left(\theta^{t+1}\right)$ is strictly positive, $\theta^{t+1} = \left(\theta^t, \theta_{t+1}^j\right)$, while $c_{t+2}^H = c_{t+2} \left(\theta^{t+1}, \theta_{t+2}^H\right)$, $c_{t+2}^L = c_{t+2} \left(\theta^{t+1}, \theta_{t+2}^L\right)$ represents consumption in period $t+2$. Complementarity between habit and consumption implies that F_t^j is positive. Since the subsequent habit effect enters with a negative sign in equation (14), this effect calls for savings subsidies. The intuition is as follows. Saving at time t increases consumption at $t+1$, and thereby the habit at $t+2$. Because of complementarity between habit and consumption, this helps with the incentive problem at $t+2$ by making consumption relatively more attractive. Therefore, saving at t should be encouraged in order to relax the incentive problem in period $t+2$.

In summary, Propositions 2 and 3 identify forces that counteract the conventional distortions from time-separable Mirrlees models. Time-separable reasoning generates downward distortions on labor supply arising from present self-selection problems, whereas habit formation adds a motive to subsidize labor supply in order to facilitate self-selection in the future. Similarly, time-separable reasoning generates savings distortions arising from wealth effects, whereas habit formation calls for savings subsidies as a means of changing the valuation of consumption in the future.

Note that the implications of habit formation for savings wedges are somewhat less clear-cut than those for labor wedges, because immediate effects on preferences have to be traded off against subsequent effects. Yet, as long as incentive problems exacerbate over time, the forces pushing for savings subsidies will dominate. Finite-horizon models are a prime example of this effect, because the planner can spread rewards over fewer and fewer periods as time progresses. This makes incentive provision more costly over time and causes the (conditional) consumption variance and the shadow cost of the incentive constraint to grow over time, other things being equal. As equations (14), (16), and (17) indicate, both of these forces increase the subsequent habit effect relative to the immediate habit effect. We demonstrate the quantitative importance of this channel in Section 4.

3.1 Generalizations of the basic model

We made a number of simplifying assumptions that deserve a brief discussion. First, *nonbinary skill types* would make the model mathematically more tedious but do not change the arguments underlying our results. The effect of habit formation on labor and savings wedges is precisely due to the fact that the downward incentive compatibility constraint (9) is relaxed if habits increase. Nonbinary skill types generate a multitude of (local) downward incentive compatibility constraints. Each of these constraints is relaxed if habits increase, and so we find the same habit effects on labor and savings wedges that we found above.

Our results also generalize to the case of *persistent habits*. Yet, in this case, the model quickly becomes intractable. For instance, if habits follow the weighted average specification

$$h_t = (1-\eta)c_{t-1} + \eta h_{t-1} = (1-\eta)\sum_{k=1}^{t-1}\eta^{k-1}c_{t-k} + \eta^{t-1}c_0,$$

then raising the persistence parameter η from zero to a positive number entails that the habit at any given point in time affects the habits for the remainder of the agent's life. In that case, increasing the habit level relaxes the incentive compatibility constraints in all remaining periods, and the exposition of our results becomes more involved because we have to account for a large number of constraints and Lagrange multipliers. Apart from this complication, habit formation modifies labor and savings wedges in qualitatively the same way as above. In particular, the impact on savings wedges still involves a trade-off between immediate and subsequent effects: habit h_{t+1} is a function of c_t, while habit levels $h_{t+2}, h_{t+3}, \ldots, h_T$ react more strongly to c_{t+1} than to c_t.

Moreover, our results extend to the case of *persistent skills (Markov skills).* This case may seem somewhat less obvious than the previous two, since skill persistence requires a novel recursive formulation: it becomes necessary to add promised utility for deviators as well as the past skill level to the vector of state variables. Moreover, we obtain an additional promise-keeping constraint for agents who deviated in the past period. Yet, Propositions 2 and 3 hold true if the wedge components are suitably generalized. Further details can be found in online Appendix C.

4 A parametrized life-cycle model

By means of a parametrized life-cycle model, this section addresses the quantitative importance of our theoretical findings on labor and savings taxation. The model captures several key features of the U.S. economy. In particular, the skill process matches the empirical life-cycle profile and the cross-sectional variance of wages. For computational reasons, the skill process is transitory as in the theoretical model. All of our results are qualitatively robust to persistent shocks, as the theoretical analysis in online Appendix C shows. However, the quantitative findings may depend on that assumption.[13]

The recursive formulation from Section 2.7 gives rise to a straightforward computational approach. We first solve for the sequence of domain restrictions $(\mathrm{dom}_t)_{t=1,\ldots,T}$. We then exploit the Bellman equation (4) to obtain the sequence of cost functions $(C_t)_{t=1,\ldots,T}$ of the planner's problem using standard numerical optimization procedures. The associated policy functions are then iterated forward to generate the optimal allocation.[14]

[13] The computational difficulties arising from persistent shocks are beyond the scope of this paper. See the concluding remarks for further discussion.

[14] For computational reasons, we restrict the spaces for consumption and output to compact intervals. We verify ex post that the quantitative results do not depend on the choice of the interval bounds.

4.1 Parameters

There are $T = 11$ periods with a duration of five years each. Agents enter the model at age 25, retire at age 65, and die at age 80. In each period before retirement, skill level θ_t is randomly drawn from a set $\{\underline{\theta}_t, \overline{\theta}_t\}$, where both realizations have equal probability and $\underline{\theta}_t < \overline{\theta}_t$. Draws are independent across agents and time. We choose the life-cycle profile of expected skills in line with Hansen (1993, Table II), who estimates relative efficiency profiles of workers in the United States over the years 1955 to 1988. Expected skills are hump-shaped over the life cycle and peak in period 5 (ages 45–49). The variance of log-skills is 0.351 and matches the cross-sectional variance of log-wages in the United States in the period 1967–2006 (Heathcote, Storesletten, and Violante, 2012, Table 3). Skills are deterministic after retirement and amount to one-half of the average skill prior to retirement. We interpret the skills after retirement as skills for home production activities.

We set up habit formation in a Cobb-Douglas form: $u(c_t, h_t) = \tilde{u}\big(c_t h_t^{-\gamma}\big)$, where γ is a number between zero and one that controls the importance of habits.[15] In line with Diaz, Pijoan-Mas, and Rios-Rull (2003), we choose $\gamma = 0.75$. This value corresponds to the case of "strong habits" explored by Carroll, Overland, and Weil (2000) and is reasonably close to empirical results by Fuhrer (2000), who estimates a value of 0.80 based on aggregate consumption data. In line with our theoretical model and estimations by Fuhrer (2000), habits are short-lived: $h_t = c_{t-1}$ for $t > 1$. Period utility is of the CRRA type: $\tilde{u}(x) = x^{1-\sigma}/(1-\sigma)$, with $\sigma = 3$. The discount factor for agent and planner equals $q = \beta = 0.98^5$. The labor disutility function is $v(l) = \alpha l^{1+\frac{1}{\psi}}/(1+\frac{1}{\psi})$, with a Frisch elasticity of labor supply of $\psi = 0.5$ and $\alpha = 1$.

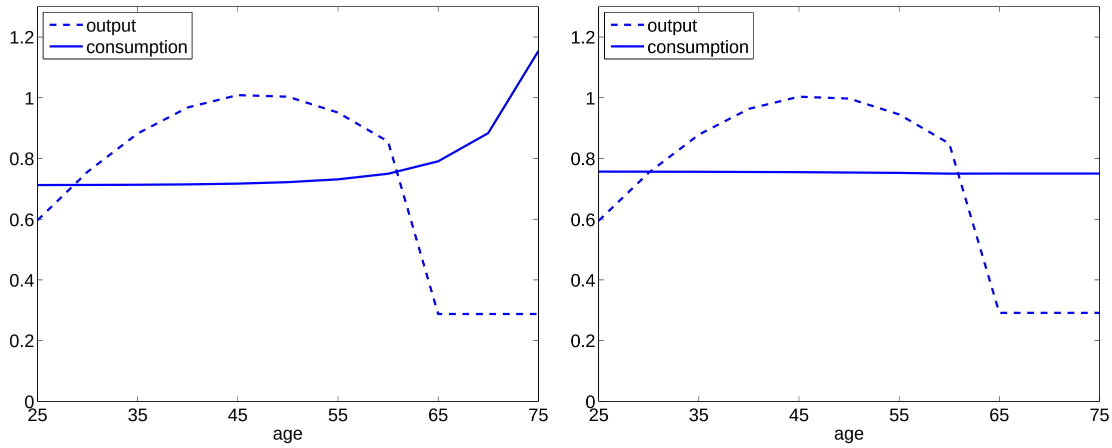
We set the initial utility promise W_1 such that the planner's budget is balanced, that is, $C_1(W_1, c_0) = 0$. We choose the initial habit level c_0 so that it coincides with the agent's expected consumption in the first period.

4.2 Results

Figure 1(a) presents the paths of expected output and consumption for the habit formation case ($\gamma = 0.75$). Expected output follows the hump-shaped pattern of the skill process, comple-

[15] Another common specification of habit formation is the linear one: $u(c_t, h_t) = \tilde{u}(c_t - \gamma h_t)$. For our present purposes, the Cobb-Douglas formulation is more convenient, since period utilities are well defined whenever c_t and h_t are positive. The linear formulation has the drawback of ruling out all pairs (c_t, h_t) with $c_t < \gamma h_t$, which makes the computation of the domain restriction and the optimal allocation somewhat more cumbersome.

Figure 1: Expected consumption and output over the life cycle



(a) Habit formation (b) Time-separable preferences

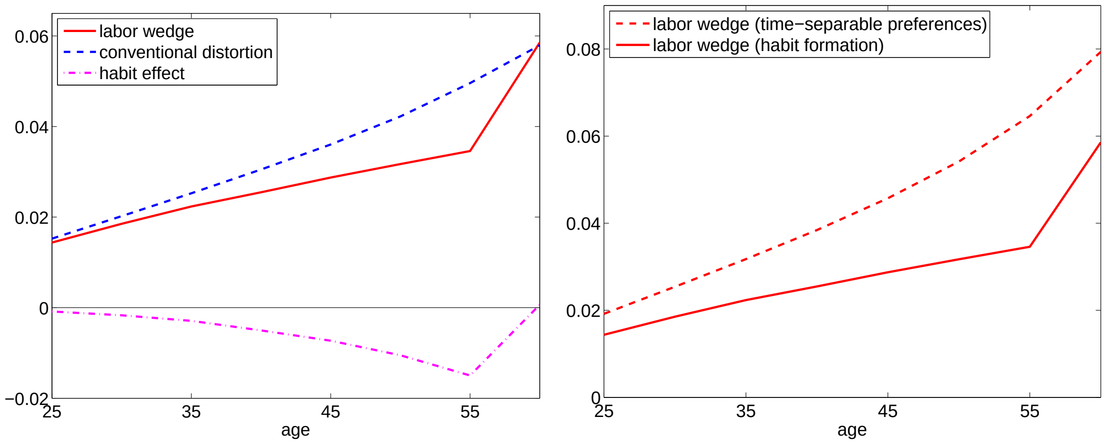
mented by a moderate level of home production output after retirement. Expected consumption increases over the life cycle and grows by about 10 percent from ages 25 to 65. Toward the end of the life cycle, consumption growth accelerates as effects on future habits become less of a concern.[16] Figure 1(b) shows the corresponding paths for the case of time-separable preferences ($\gamma = 0$).[17] The expected output path is very similar to the habit formation case. Expected consumption, however, is virtually flat (but slightly monotonically decreasing) for time-separable preferences. This shows that habit formation has a positive impact on the optimal growth rate of consumption.

Figure 2(a) displays the components of expected labor wedges for the habit formation case. The habit effect B_t calls for labor subsidies as outlined in our theoretical analysis. This effect is smaller in magnitude than the conventional Mirrleesian motive for labor taxation A_t. Thus, expected labor wedges are positive throughout the life cycle but significantly smaller than in the case of time-separable preferences (Figure 2(b)). Averaged over the life cycle, labor wedges in the habit formation case drop by approximately 35 percent compared with the time-separable case.

[16] We acknowledge that the consumption path during retirement is not well in line with empirical findings. A more sophisticated model of retirement would allow for stochastic mortality and potentially for a structural change in the habit formation relation at the time of retirement. Stochastic mortality alone already mitigates consumption growth during retirement to a large extent, because the effects of consumption on future preferences can never be fully ignored.

[17] To make the allocations comparable, we choose a scaling parameter of $\alpha = 4.3$ for the time-separable case, such that the discounted value of lifetime output (and consumption) coincides with the habit formation case. This adjustment has a negligible effect on labor and savings wedges: averaged over the life cycle, labor wedges are 0.046 with $\alpha = 1$ and 0.045 with $\alpha = 4.3$, while average savings wedges amount to 0.011 in both cases.

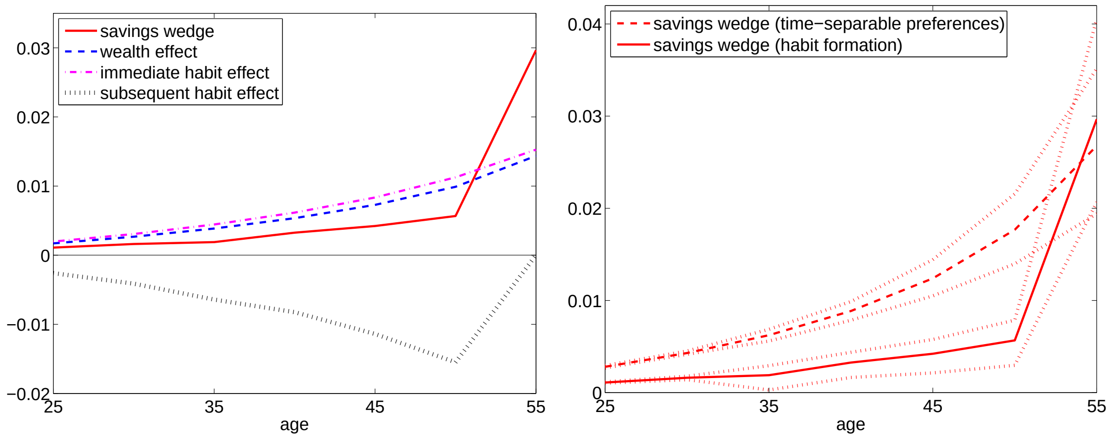
Figure 2: Expected labor wedges



(a) Decomposition

(b) Comparison with time-separable case

Figure 3: Expected savings wedges



(a) Decomposition

(b) Comparison with time-separable case

Notes: The dotted lines in panel (b) display the 10th and 90th percentiles of the savings wedges.

Figure 3(a) decomposes expected savings wedges for the habit formation case into the wealth effect, immediate habit effect, and subsequent habit effect. Both habit effects are sizable and in fact are larger in magnitude than the conventional taxation motive caused by wealth effects. As argued in the theoretical section above, the subsequent habit effect calls for savings subsidies. This effect dominates the immediate habit effect (calling for savings taxes), and thus the total impact of habit formation on savings wedges is negative. The life-cycle average of the savings wedge with habit formation is 0.0068 (corresponding to a 7.1 percent tax on net interest). In the time-separable case, it is 0.0113 (corresponding to a 11.8 percent tax on net interest); see Figure

3(b).[18] The variance of savings wedges is relatively small, as the plots of the 10th and 90th percentiles of the savings wedges (dotted lines) in Figure 3(b) indicate. Hence, savings wedges are lower in the habit formation case than in the time-separable case for the vast majority of possible realizations.

Recall that the subsequent habit effect encourages saving (and thus next period's consumption) in order to relax incentive problems in the subsequent future. The immediate habit effect, by contrast, discourages saving in order to relax the incentive problem in the period immediately following. Over time, incentive provisions must rely less on future promises and more on costly consumption rewards. Therefore, relaxing incentive problems later in life is relatively more important, which explains why the subsequent habit effect exceeds the immediate habit effect. The only exception to this rule appears at the very end of the working life, when the subsequent habit effect by definition falls to zero.

4.3 Sensitivity analysis

First, we note that the problem of incentive provision becomes more intricate if there are more skill types. To explore the role of additional skill types, we extend the quantitative model to three types. We set the life-cycle profile of expected skills, the variance of log-skills, and all other parameters as in our baseline model in Section 4.2. Table 1 reports the life-cycle averages of expected labor and savings wedges for habit formation preferences and time-separable preferences. As in the case with two skill types, the impact of habit formation on labor and savings wedges is negative. In the 2-type model, habit formation causes labor wedges to fall by 35 percent, and savings wedges by 40 percent. In the 3-type model, habit formation causes labor wedges to fall by 33 percent, and savings wedges by 39 percent. In the 3-type model, labor distortions below the top apply to a larger fraction of agents and therefore labor wedges are higher than in the 2-type model. Savings wedges are also higher in the 3-type model but the difference is less pronounced.

Second, we note that the problem of incentive provision exacerbates as the time horizon shrinks. To examine the role of the time horizon for the quantitative results, we explore models with different time horizons and compare wedges in the first period of those models. We set

[18] The difference becomes even more pronounced if we focus on workers between ages 25 and 50. For those workers, the average savings wedge with habit formation is roughly one-third of the average savings wedge with time-separable preferences.

Table 1: Expected wedges (life-cycle averages) for skill processes with two and three types

	habit formation		time-separable	
skill types	labor wedge	savings wedge	labor wedge	savings wedge
2	0.0293	0.0068	0.0448	0.0113
3	0.0489	0.0097	0.0727	0.0160

$T \in \{5, 10, 20, 30\}$ and parametrize the models as before, except that we replace the hump-shaped profile of expected skills by a flat profile for the sake of comparability across models. Table 2 shows the expected labor and savings wedges for habit formation preferences and time-separable preferences.

Table 2: Expected wedges (in the first period) for different time horizons

	habit formation		time-separable	
T	labor wedge	savings wedge	labor wedge	savings wedge
5	0.0288	0.0028	0.0438	0.0125
10	0.0185	0.0020	0.0262	0.0033
20	0.0140	0.0010	0.0188	0.0014
30	0.0128	0.0008	0.0170	0.0009

For both preference specifications, labor and savings wedges fall as the time horizon increases. This result mirrors the observation that labor and savings wedges rise over the life cycle in our baseline model in Section 4.2. The dependence of wedges on the life cycle is a typical finding for dynamic Mirrlees models (Golosov, Troshkin, and Tsyvinski, 2011). Qualitatively, we find that the impact of habit formation on labor and savings wedges is negative at all time horizons. Quantitatively, the impact diminishes as the time horizon increases, but it remains sizable for all specifications. In the case with the longest time horizon ($T = 30$), labor wedges with habit formation are approximately 25 percent lower, and savings wedges 13 percent lower, than in the case of time-separable preferences.

5 Concluding remarks

Findings from macroeconomics, psychology, and micro data provide evidence for habit formation in consumption preferences. This paper studies the effect of habit formation on optimal taxation in a model with private information. We characterize optimal allocations in terms of labor and savings wedges and identify several novel taxation motives.

Habit formation generates a motive to subsidize labor supply in order to encourage work (and indirectly consumption), because this motive makes agents hungrier for consumption in the future and thereby relaxes future incentive problems. Hence, optimal labor wedges tend to be smaller in the presence of habit formation preferences. Habit formation also generates a motive for savings subsidies. If the worker consumes less in the present and more in the following period, because of habit formation the agent will be hungrier for consumption in subsequent periods. Thus, incentive problems in subsequent periods are relaxed if consumption in the present period becomes relatively more expensive (i.e., if savings are subsidized). Optimal savings wedges trade off this effect against the motive to tax savings to make the agent hungrier in the period immediately following (because of wealth and immediate habit effects).

We demonstrate the quantitative importance of habit formation in a parametrized life-cycle model. Averaged over the life cycle, optimal labor wedges for habit formation preferences are 35 percent lower, and optimal savings wedges 40 percent lower, than for time-separable preferences. Our parametrization captures several key aspects of the U.S. economy. For computational reasons, we assume that skill shocks are transitory. It is beyond the scope of this paper to deal with the computational challenges that arise when habit formation is combined with persistent shocks. The recursive formulation will then involve three continuous state variables (habits, promised utility, threat utility). The main difficulty, however, is that the domain of feasible utilities becomes a two-dimensional nonrectangular set that depends on time, the past shock, and the habit level. To the best of our knowledge, the recursive contracting literature has not yet found numerical approaches to dealing with such problems.

Kapicka (2013) and Farhi and Werning (2013) compute models with time-separable preferences and persistent shocks that are continuous. Relying on the first-order approach and balanced-growth preferences, they are able to reduce the number of state variables to two. In principle, the first-order approach can also be applied in the habit formation case. Since the balanced-growth property breaks down, the number of state variables increases to four and the curse of dimensionality persists.

References

Abel, A. B. (1990): “Asset Prices under Habit Formation and Catching up with the Joneses,” *American Economic Review*, 80(2), 38–42.

Abraham, A., S. Koehne, and N. Pavoni (2014): "Optimal Income Taxation with Asset Accumulation," Institute for International Economic Studies. Mimeo.

Albanesi, S., and C. Sleet (2006): "Dynamic Optimal Taxation with Private Information," *Review of Economic Studies*, 73(1), 1–30.

Bohacek, R., and M. Kapicka (2008): "Optimal human capital policies," *Journal of Monetary Economics*, 55(1), 1–16.

Campbell, J. Y., and J. H. Cochrane (1999): "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107(2), 205–251.

Carroll, C. D., J. Overland, and D. N. Weil (2000): "Saving and Growth with Habit Formation," *American Economic Review*, 90(3), 341–355.

Constantinides, G. M. (1990): "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98(3), 519–43.

Cremer, H., P. De Donder, D. Maldonado, and P. Pestieau (2010): "Commodity Taxation under Habit Formation and Myopia," *BE Journal of Economic Analysis & Policy*, 10(1).

Cremer, H., and P. Pestieau (2011): "Myopia, redistribution and pensions," *European Economic Review*, 55(2), 165 – 175.

Diamond, P. A., and J. A. Mirrlees (1978): "A model of social insurance with variable retirement," *Journal of Public Economics*, 10(3), 295 – 336.

Diaz, A., J. Pijoan-Mas, and J. Rios-Rull (2003): "Precautionary savings and wealth distribution under habit formation preferences," *Journal of Monetary Economics*, 50(6), 1257–1291.

Farhi, E., and I. Werning (2008): "Optimal savings distortions with recursive preferences," *Journal of Monetary Economics*, 55(1), 21–42.

Farhi, E., and I. Werning (2013): "Insurance and taxation over the life cycle," *Review of Economic Studies*, 80(2), 596–635.

Feldstein, M. (1973): "On the optimal progressivity of the income tax," *Journal of Public Economics*, 2(4), 357 – 376.

Fernandes, A., and C. Phelan (2000): "A recursive formulation for repeated agency with history dependence," *Journal of Economic Theory*, 91(2), 223–247.

Frederick, S., and G. Loewenstein (1999): "Hedonic Adaptation," in *Well-being: The foundations of hedonic psychology*, ed. by D. Kahneman, E. Diener, and N. Schwarz, pp. 302–329. Russell Sage Foundation Press.

Fuhrer, J. C. (2000): "Habit Formation in Consumption and Its Implications for Monetary-Policy Models," *American Economic Review*, 90(3), pp. 367–390.

Golosov, M., N. Kocherlakota, and A. Tsyvinski (2003): "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, 70(3), 569–587.

Golosov, M., M. Troshkin, and A. Tsyvinski (2011): "Optimal dynamic taxes," Discussion paper, National Bureau of Economic Research.

Golosov, M., and A. Tsyvinski (2006): "Designing Optimal Disability Insurance: A Case for Asset Testing," *Journal of Political Economy*, 114(2), 257–279.

Gottardi, P., and N. Pavoni (2011): "Ramsey Asset Taxation under Asymmetric Information," European University Institute. Mimeo.

Grochulski, B., and N. Kocherlakota (2010): "Nonseparable preferences and optimal social security systems," *Journal of Economic Theory*, 145(6), 2055 – 2077.

Grochulski, B., and T. Piskorski (2010): "Risky human capital and deferred capital income taxation," *Journal of Economic Theory*, 145(3), 908 – 943.

Hansen, G. (1993): "The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, 8(1), 71–80.

Heathcote, J., K. Storesletten, and G. Violante (2012): "Consumption and labor supply with partial insurance: An analytical framework," Federal Reserve Bank of Minneapolis and New York University, Mimeo.

Heien, D., and C. Durham (1991): "A test of the habit formation hypothesis using household data," *Review of Economics and Statistics*, pp. 189–199.

Helson, H. (1964): *Adaptation-Level Theory.* Harper & Row New York.

Johnsen, T. H., and J. B. Donaldson (1985): "The Structure of Intertemporal Preferences under Uncertainty and Time Consistent Plans," *Econometrica*, 53(6), pp. 1451–1458.

Kapicka, M. (2013): "Efficient Allocations in Dynamic Private Information Economies with Persistent Shocks: A First-Order Approach," *Review of Economic Studies*, 80(3), 1027–1054.

Kocherlakota, N. R. (2005): "Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation," *Econometrica*, 73(5), 1587–1621.

Messinis, G. (1999): "Habit Formation and the Theory of Addiction," *Journal of Economic Surveys*, 13(4), 417–42.

Phelan, C., and R. M. Townsend (1991): "Computing Multi-Period, Information-Constrained Optima," *Review of Economic Studies*, 58(5), 853–881.

Rogerson, W. P. (1985): "Repeated Moral Hazard," *Econometrica*, 53(1), 69–76.

Ryder, Jr., H. E., and G. M. Heal (1973): "Optimum Growth with Intertemporally Dependent Preferences," *Review of Economic Studies*, 40(1), 1–33.

Spear, S., and S. Srivastava (1987): "On repeated moral hazard with discounting," *Review of Economic Studies*, 54(4), 599–617.

Stantcheva, S. (2014): "Optimal Taxation and Human Capital Policies Over the Lifecycle," MIT. Mimeo.

Stern, N. (1982): "Optimum taxation with errors in administration," *Journal of Public Economics*, 17(2), 181 – 211.

Stiglitz, J. E. (1982): "Self-selection and Pareto efficient taxation," *Journal of Public Economics*, 17(2), 213 – 240.

Werning, I. (2011): "Nonlinear capital taxation," MIT. Mimeo.

Appendices for online publication

Optimal taxation in a habit formation economy

Sebastian Koehne* Moritz Kuhn†

This document is organized as follows. Appendix A collects the proofs that are omitted from the main text. Appendix B derives a recursive formulation of the social planning problem with habit formation. The setup allows for general recursive habit processes and contains the case of one-period habits discussed in the main text as a special case. Appendix C derives labor and savings wedges when the skill process is persistent.

Appendix A: Proofs

Proof of Lemma 1. Since the constraint set of the unrelaxed problem is a subset of the constraint set of the relaxed problem, it suffices to show that the solution of the relaxed problem is feasible for the unrelaxed problem. In other words, it suffices to show that the solution of the relaxed problem satisfies the upward incentive compatibility constraint.

Without loss of generality, we assume $\theta_t^H > \theta_t^L$. We first show that the downward incentive constraint is binding for the relaxed problem. Suppose to the contrary that the solution of the relaxed problem has a slack downward incentive constraint. By inspection of the Kuhn-Tucker conditions, the solution then takes the form $c_t^H = c_t^L$, $W_t^H = W_t^L$, and $y_t^H > y_t^L$. However, allocations of such form violate the downward incentive constraint. Hence the assumption that the solution of the relaxed problem has a slack downward incentive constraint must be false.

We now show that a binding downward incentive constraint implies that the upward incentive constraint is satisfied. Formally, a binding downward incentive constraint implies

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H = u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta W_{t+1}^L. \tag{1}$$

*Institute for International Economic Studies (IIES), Stockholm University, SE-10691 Stockholm, Sweden, Phone: +46 8 16 35 64, sebastian.koehne@iies.su.se, and CESifo, Munich, Germany.

†University of Bonn, Department of Economics, D-53113 Bonn, Germany, Phone: +49 228 73 62096, mokuhn@uni-bonn.de, and Institute for the Study of Labor (IZA), Bonn, Germany.

Recall that labor disutility v is a convex function. Since $1/\theta_t^L \geq 1/\theta_t^H$, convexity of v implies that the difference $v\left(y/\theta_t^L\right) - v\left(y/\theta_t^H\right)$ increases in y. Moreover, it is easy to see that a binding downward incentive constraint implies $y_t^H \geq y_t^L$. Combining the last two insights, we obtain

$$v\left(y_t^L/\theta_t^L\right) - v\left(y_t^L/\theta_t^H\right) \leq v\left(y_t^H/\theta_t^L\right) - v\left(y_t^H/\theta_t^H\right). \tag{2}$$

We rewrite this inequality as

$$v\left(y_t^L/\theta_t^L\right) - v\left(y_t^H/\theta_t^L\right) \leq v\left(y_t^L/\theta_t^H\right) - v\left(y_t^H/\theta_t^H\right). \tag{3}$$

We combine the binding downward incentive constraint with the above inequality and obtain

$$v\left(y_t^L/\theta_t^L\right) - v\left(y_t^H/\theta_t^L\right) \leq u\left(c_t^L, c_{t-1}\right) - u\left(c_t^H, c_{t-1}\right) + \beta W_{t+1}^L - \beta W_{t+1}^H. \tag{4}$$

Hence the upward incentive constraint is satisfied. □

Proof of Remark 1. Since the incentive compatibility constraint has a Lagrange multiplier of zero in all periods $t \geq t_0$, we have $\mu_t = 0$ for $t \geq t_0$. Now the result follows from Propositions 2 and 3. □

Proof of Proposition 1. See online Appendix B. □

Proof of Proposition 2. The (finite horizon) Bellman equation of the social planner problem is

$$C_t\left(W_t, c_{t-1}\right) = \min_{c_t^i, y_t^i, W_{t+1}^i} \sum_{i=H,L} \left[c_t^i - y_t^i + qC_{t+1}\left(W_{t+1}^i, c_t^i\right)\right] \pi_t\left(\theta_t^i\right) \tag{5}$$

s.t.

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H \geq u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta W_{t+1}^L \tag{6}$$

$$\sum_{i=H,L} \left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right] \pi_t\left(\theta_t^i\right) = W_t. \tag{7}$$

Problem (5) has the following first-order conditions for consumption

$$0 = \pi_t\left(\theta_t^H\right)\left[1 + qC_{t+1,h}\left(W_{t+1}^H, c_t^H\right)\right] - \lambda_t u_c\left(c_t^H, c_{t-1}\right)\pi_t\left(\theta_t^H\right) - \mu_t u_c\left(c_t^H, c_{t-1}\right), \tag{8}$$

$$0 = \pi_t\left(\theta_t^L\right)\left[1 + qC_{t+1,h}\left(W_{t+1}^L, c_t^L\right)\right] - \lambda_t u_c\left(c_t^L, c_{t-1}\right)\pi_t\left(\theta_t^L\right) + \mu_t u_c\left(c_t^L, c_{t-1}\right), \tag{9}$$

for output

$$0 = -\pi_t\left(\theta_t^H\right) + \lambda_t \frac{v'\left(y_t^H/\theta_t^H\right)}{\theta_t^H}\pi_t\left(\theta_t^H\right) + \mu_t \frac{v'\left(y_t^H/\theta_t^H\right)}{\theta_t^H}, \tag{10}$$

$$0 = -\pi_t\left(\theta_t^L\right) + \lambda_t \frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L}\pi_t\left(\theta_t^L\right) - \mu_t \frac{v'\left(y_t^L/\theta_t^H\right)}{\theta_t^H}, \tag{11}$$

and for continuation utilities

$$0 = \pi_t\left(\theta_t^H\right) qC_{t+1,W}\left(W_{t+1}^H, c_t^H\right) - \lambda_t\beta\pi_t\left(\theta_t^H\right) - \mu_t\beta, \tag{12}$$

$$0 = \pi_t\left(\theta_t^L\right) qC_{t+1,W}\left(W_{t+1}^L, c_t^L\right) - \lambda_t\beta\pi_t\left(\theta_t^L\right) + \mu_t\beta. \tag{13}$$

We begin with the labor wedge of the high-skilled worker. Combine the first-order condition for y_t^H with that for c_t^H to obtain

$$\frac{1 + qC_{t+1,h}\left(W_{t+1}^H, c_t^H\right)}{u_c\left(c_t^H, c_{t-1}\right)} = \frac{\theta_t^H}{v'\left(y_t^H/\theta_t^H\right)}. \tag{14}$$

By the envelope theorem, applied to the Bellman equation (5) at date $t+1$, we have

$$C_{t+1,W}\left(W_{t+1}^H, c_t^H\right) = \lambda_{t+1}^H, \tag{15}$$

$$C_{t+1,h}\left(W_{t+1}^H, c_t^H\right) = -\lambda_{t+1}^H \sum_j u_h\left(c_{t+1}^{Hj}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^j\right) - \mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right] \tag{16}$$

Hence we can rewrite (14) as

$$\frac{\theta_t^H}{v'\left(y_t^H/\theta_t^H\right)}u_c\left(c_t^H, c_{t-1}\right) = 1 - qC_{t+1,W}\left(W_{t+1}^H, c_t^H\right)\sum_j u_h\left(c_{t+1}^{Hj}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^j\right) - q\mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]. \tag{17}$$

Combine the first-order condition for W_{t+1}^H with the first-order condition for y_t^H to obtain

$$qC_{t+1,W}\left(W_{t+1}^H, c_t^H\right) = \beta\frac{\theta_t^H}{v'\left(y_t^H/\theta_t^H\right)}. \tag{18}$$

Use this to rewrite (17) as follows:

$$\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1}, \theta_t^H\right.\right] = \frac{v'\left(y_t^H/\theta_t^H\right)}{\theta_t^H}\left(1 - q\mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]\right). \tag{19}$$

Therefore the labor wedge is

$$\tau_{y,t}^{H} = -\mu_{t+1}^{H} \frac{qv'\left(y_t^H/\theta_t^H\right)}{\theta_t^H \mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^H\right.\right]} \left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]. \tag{20}$$

Using the first-order condition for y_t^H and the identity $q\pi_t\left(\theta_t^H\right)\lambda_{t+1}^H = \beta\left(\lambda_t \pi_t\left(\theta_t^H\right) + \mu_t\right)$, and defining

$$B_t^H = \frac{\beta\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]}{\lambda_{t+1}^H \mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^H\right.\right]}, \tag{21}$$

the labor wedge is $\tau_{y,t}^H = -\mu_{t+1}^H B_t^H$.

We now turn to the labor wedge of the low-skilled worker. First we write the first-order condition for c_t^L as

$$\left[\lambda_t \pi_t\left(\theta_t^L\right) - \mu_t\right] u_c\left(c_t^L, c_{t-1}\right) - \pi_t\left(\theta_t^L\right) = q\pi_t\left(\theta_t^L\right) C_{t+1,h}\left(W_{t+1}^L, c_t^L\right). \tag{22}$$

The envelope theorem, applied to the Bellman equation (5) at date $t+1$, yields

$$C_{t+1,W}\left(W_{t+1}^L, c_t^L\right) = \lambda_{t+1}^L, \tag{23}$$

$$C_{t+1,h}\left(W_{t+1}^L, c_t^L\right) = -\lambda_{t+1}^L \sum_j u_h\left(c_{t+1}^{Lj}, c_t^L\right)\pi_{t+1}\left(\theta_{t+1}^j\right) - \mu_{t+1}^L\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right] \tag{24}$$

Combined with the first-order condition for W_{t+1}^L, we obtain

$$\begin{aligned} q\pi_t\left(\theta_t^L\right) C_{t+1,h}\left(W_{t+1}^L, c_t^L\right) &= -\lambda_t \pi_t\left(\theta_t^L\right)\beta \sum_j u_h\left(c_{t+1}^{Lj}, c_t^L\right)\pi_{t+1}\left(\theta_{t+1}^j\right) \\ &\quad + \mu_t \beta \sum_j u_h\left(c_{t+1}^{Lj}, c_t^L\right)\pi_{t+1}\left(\theta_{t+1}^j\right) \\ &\quad - \mu_{t+1}^L \pi_t\left(\theta_t^L\right) q\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]. \end{aligned} \tag{25}$$

We substitute this in the first-order condition for c_t^L to obtain

$$\lambda_t \pi_t\left(\theta_t^L\right)\mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^L\right.\right] - \pi_t\left(\theta_t^L\right) \tag{26}$$

$$= \mu_t \mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^L\right.\right] - \mu_{t+1}^L \pi_t\left(\theta_t^L\right) q\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]. \tag{27}$$

Now we use the first-order condition for y_t^L to replace $\pi_t\left(\theta_t^L\right)$:

$$\lambda_t \pi_t\left(\theta_t^L\right)\left\{\mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^L\right.\right] - \frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L}\right\} \tag{28}$$

$$= \mu_t \left\{\mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^L\right.\right] - \frac{v'\left(y_t^L/\theta_t^H\right)}{\theta_t^H}\right\} - \mu_{t+1}^L \pi_t\left(\theta_t^L\right) q\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]. \tag{29}$$

This can be rewritten as

$$\left(\lambda_t \pi_t\left(\theta_t^L\right)-\mu_t\right)\left\{\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1}, \theta_t^L\right.\right]-\frac{v^{\prime}\left(y_t^L / \theta_t^L\right)}{\theta_t^L}\right\} \tag{30}$$

$$= \mu_t\left\{\frac{v^{\prime}\left(y_t^L / \theta_t^L\right)}{\theta_t^L}-\frac{v^{\prime}\left(y_t^L / \theta_t^H\right)}{\theta_t^H}\right\}-\mu_{t+1}^L \pi_t\left(\theta_t^L\right) q\left[u_h\left(c_{t+1}^{LH}, c_t^L\right)-u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]. \tag{31}$$

Using the identity $\pi_t\left(\theta_t^L\right) q \lambda_{t+1}^L=\beta\left(\lambda_t \pi_t\left(\theta_t^L\right)-\mu_t\right)$, and defining

$$A_t^L = \frac{\beta}{q \pi_t\left(\theta_t^L\right) \lambda_{t+1}^L \mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1}, \theta_t^L\right.\right]}\left[\frac{v^{\prime}\left(y_t^L / \theta_t^L\right)}{\theta_t^L}-\frac{v^{\prime}\left(y_t^L / \theta_t^H\right)}{\theta_t^H}\right], \tag{32}$$

$$B_t^L = \frac{\beta\left[u_h\left(c_{t+1}^{LH}, c_t^L\right)-u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]}{\lambda_{t+1}^L \mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1}, \theta_t^L\right.\right]}, \tag{33}$$

the labor wedge is hence $\tau_{y,t}^L=\mu_t A_t^L-\mu_{t+1}^L B_t^L$. This completes the proof. □

Proof of Proposition 3. We begin with the savings wedge for the high-skilled worker. Combine the first-order condition for consumption (8) and the envelope condition (16) to obtain

$$\frac{\lambda_t \pi_t\left(\theta_t^H\right)+\mu_t}{\pi_t\left(\theta_t^H\right)} u_c\left(c_t^H, c_{t-1}\right)-1 \tag{34}$$

$$= -q \lambda_{t+1}^H \sum_j u_h\left(c_{t+1}^{Hj}, c_t^H\right) \pi_{t+1}\left(\theta_{t+1}^j\right)-q \mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right)-u_h\left(c_{t+1}^{HL}, c_t^H\right)\right].$$

Using the identity $q \pi_t\left(\theta_t^H\right) \lambda_{t+1}^H=\beta\left(\lambda_t \pi_t\left(\theta_t^H\right)+\mu_t\right)$, we can rewrite the previous equation as

$$\frac{q \lambda_{t+1}^H}{\beta} \mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1}, \theta_t^H\right.\right]=1-q \mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right)-u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]. \tag{35}$$

The first-order conditions for consumption in period $t+1$ are

$$0 = \pi_{t+1}\left(\theta_{t+1}^H\right)\left[1+q C_{t+2,h}\left(W_{t+2}^{HH}, c_{t+1}^{HH}\right)\right]-\lambda_{t+1}^H u_c\left(c_{t+1}^{HH}, c_t^H\right) \pi_{t+1}\left(\theta_{t+1}^H\right)-\mu_{t+1}^H u_c\left(c_{t+1}^{HH}, c_t^H\right) \tag{36}$$

$$0 = \pi_{t+1}\left(\theta_{t+1}^L\right)\left[1+q C_{t+2,h}\left(W_{t+2}^{HL}, c_{t+1}^{HL}\right)\right]-\lambda_{t+1}^H u_c\left(c_{t+1}^{HL}, c_t^H\right) \pi_{t+1}\left(\theta_{t+1}^L\right)+\mu_{t+1}^H u_c\left(c_{t+1}^{HL}, c_t^H\right). \tag{37}$$

Summing up these conditions and substituting the result into the previous equation yields

$$\begin{aligned}\frac{q \lambda_{t+1}^H}{\beta} \mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1}, \theta_t^H\right.\right] = & -\pi_{t+1}\left(\theta_{t+1}^L\right) q C_{t+2,h}\left(W_{t+2}^{HL}, c_{t+1}^{HL}\right)-\pi_{t+1}\left(\theta_{t+1}^H\right) q C_{t+2,h}\left(W_{t+2}^{HH}, c_{t+1}^{HH}\right) \\ & +\lambda_{t+1}^H\left[u_c\left(c_{t+1}^{HH}, c_t^H\right) \pi_{t+1}\left(\theta_{t+1}^H\right)+u_c\left(c_{t+1}^{HL}, c_t^H\right) \pi_{t+1}\left(\theta_{t+1}^L\right)\right] \\ & -\mu_{t+1}^H\left[u_c\left(c_{t+1}^{HL}, c_t^H\right)-u_c\left(c_{t+1}^{HH}, c_t^H\right)\right]-q \mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right)-u_h\left(c_{t+1}^{HL}, c_t^H\right)\right].\end{aligned} \tag{38}$$

We use the envelope conditions for period $t+2$ to replace $C_{t+2,h}$. This gives, after some algebra,

$$
\begin{aligned}
q\lambda_{t+1}^{H}\mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1},\theta_t^H\right.\right] \quad = \quad & \beta\lambda_{t+1}^{H}\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_t^H\right.\right] \\
& -\mu_{t+1}^{H}\beta\left(\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_t^H,\theta_{t+1}^L\right.\right]-\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_t^H,\theta_{t+1}^H\right.\right]\right) \\
& -q\mu_{t+1}^{H}\beta\left[u_h\left(c_{t+1}^{HH},c_t^H\right)-u_h\left(c_{t+1}^{HL},c_t^H\right)\right] \\
& +q\pi_{t+1}\left(\theta_{t+1}^{H}\right)\mu_{t+2}^{HH}\beta\left[u_h\left(c_{t+2}^{HHH},c_{t+1}^{HH}\right)-u_h\left(c_{t+2}^{HHL},c_{t+1}^{HH}\right)\right] \\
& +q\pi_{t+1}\left(\theta_{t+1}^{L}\right)\mu_{t+2}^{HL}\beta\left[u_h\left(c_{t+2}^{HLH},c_{t+1}^{HL}\right)-u_h\left(c_{t+2}^{HLL},c_{t+1}^{HL}\right)\right].
\end{aligned}
\tag{39}
$$

Setting $i=H$ and defining

$$
D_t^i = \frac{\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_t^i,\theta_{t+1}^L\right.\right]-\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_t^i,\theta_{t+1}^H\right.\right]}{\lambda_{t+1}^{i}\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_t^i\right.\right]}, \tag{40}
$$

$$
E_t^i = \frac{q\left[u_h\left(c_{t+1}^{iH},c_t^i\right)-u_h\left(c_{t+1}^{iL},c_t^i\right)\right]}{\lambda_{t+1}^{i}\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_t^i\right.\right]}, \tag{41}
$$

$$
F_t^{ij} = \frac{q\left[u_h\left(c_{t+2}^{ijH},c_{t+1}^{ij}\right)-u_h\left(c_{t+2}^{ijL},c_{t+1}^{ij}\right)\right]}{\lambda_{t+1}^{i}\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_t^i\right.\right]}, \quad j=L,H, \tag{42}
$$

the savings wedge is hence

$$
\tau_{s,t}^{i}=\mu_{t+1}^{i}D_t^i+\mu_{t+1}^{i}E_t^i-\pi_{t+1}\left(\theta_{t+1}^{H}\right)\mu_{t+2}^{iH}F_t^{iH}-\pi_{t+1}\left(\theta_{t+1}^{L}\right)\mu_{t+2}^{iL}F_t^{iL}. \tag{43}
$$

For the savings wedge of the low-skilled worker, we follow the same steps to show that formula (43) applies if we set $i=L$ in definitions (40), (41), (42). This completes the proof. □

Appendix B: Recursive formulation

We rewrite the multiperiod private information problem as a dynamic programming problem with two state variables: promised utility and the agent's habit level. We derive this property in a setting with recursive habit processes: $h_t = H(c_{t-1}, h_{t-1})$, with h_1 being exogenous. Our results extend findings from Spear and Srivastava (1987) and Phelan and Townsend (1991) to the class of habit formation preferences.

We consider the following optimization problem:

$$C_1(W_1, h_1) := \min_{\mathbf{c},\mathbf{y}} \sum_{t=1}^{T} \sum_{\theta^t \in \Theta^t} q^{t-1} \left[c_t\left(\theta^t\right) - y_t\left(\theta^t\right)\right] \Pi^t\left(\theta^t\right) \tag{44}$$

s.t.

$$w_1\left(\mathbf{c}, \mathbf{y}; h_1\right) \geq w_1\left(\mathbf{c} \circ \sigma, \mathbf{y} \circ \sigma; h_1\right) \quad \forall \sigma \in \Sigma \tag{45}$$

$$w_1\left(\mathbf{c}, \mathbf{y}; h_1\right) = W_1. \tag{46}$$

First we introduce some notation. A consumption allocation $\mathbf{c}$, combined with a fixed initial habit h_1, generates a unique sequence of habit levels $(h_t(\theta^{t-1}))_{t=1,\ldots,T}$ according to the sequence of equations $h_t = H(c_{t-1}, h_{t-1})$, $t = 2, \ldots, T$. Given an allocation $(\mathbf{c}, \mathbf{y})$ and a history θ^t, the *continuation allocation* $\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right)\right)$ is defined as the restriction of plans $(c_s, y_s)_{s=t+1,\ldots,T}$ to those histories $\theta^{t+1}, \ldots, \theta^T$ that succeed θ^t. The *continuation utility* associated with $\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right)\right)$ is defined as

$$\begin{aligned} & w_{t+1}\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right); h_{t+1}\left(\theta^t\right)\right) \\ & := \sum_{s=t+1}^{T} \sum_{\theta^s \in \Theta^s} \beta^{s-t-1} \left[u\left(c_s\left(\theta^s\right), h_s\left(\theta^{s-1}\right)\right) - v\left(\frac{y_s\left(\theta^s\right)}{\theta_s}\right)\right] \Pi^s\left(\theta^s | \theta^t\right). \end{aligned} \tag{47}$$

Note that, in contrast to the time-separable case, the continuation utility w_{t+1} depends not only on the continuation allocation but also on the consumption history $c^t(\theta^t)$ as summarized by the one-dimensional statistic $h_{t+1}(\theta^t)$.

For any $h \in \mathbb{R}_+$ we define $\mathrm{dom}_t(h)$ to be the set of time-t continuation utilities W with the property that, given time-t habit level $h_t = h$, there exists an incentive compatible allocation $\left(c_t^T, y_t^T\right)$ that generates utility

$$\mathbb{E}_{t-1}\left[\sum_{s=t}^{T} \beta^{s-t}\left(u(c_s, h_s) - v(y_s/\theta_s)\right)\right] = W, \quad \text{where } h_t = h,\ h_s = H(c_{s-1}, h_{s-1}) \text{ for } s > t. \tag{48}$$

The following result transforms the incentive compatibility constraint (45) into a sequence of temporary constraints.

Lemma (One-shot deviation principle). *The allocation* $(\mathbf{c}, \mathbf{y})$ *is incentive compatible if and only if it* *satisfies the following condition for all t and all $\theta^t \in \Theta^t$, $\hat{\theta} \in \Theta_t$:*

$$\begin{aligned} & u\left(c_t\left(\theta^t\right), h_t\left(\theta^{t-1}\right)\right) - v\left(\frac{y_t\left(\theta^t\right)}{\theta_t}\right) + \beta w_{t+1}\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right); H\left(c_t\left(\theta^t\right), h_t\left(\theta^{t-1}\right)\right)\right) \\ \geq\; & u\left(c_t\left(\theta^{t-1}, \hat{\theta}\right), h_t\left(\theta^{t-1}\right)\right) - v\left(\frac{y_t\left(\theta^{t-1}, \hat{\theta}\right)}{\theta_t}\right) \\ & \quad + \beta w_{t+1}\left(c_{t+1}^T\left(\theta^{t-1}, \hat{\theta}\right), y_{t+1}^T\left(\theta^{t-1}, \hat{\theta}\right); H\left(c_t\left(\theta^{t-1}, \hat{\theta}\right), h_t\left(\theta^{t-1}\right)\right)\right). \end{aligned} \tag{49}$$

Proof. Since one-shot deviations are special cases of reporting strategies, incentive compatibility clearly implies that the temporary incentive constraint (49) holds for all t and all $\theta^t \in \Theta^t$, $\hat{\theta} \in \Theta_t$.

For the reverse implication, we proceed by induction. Induction basis: Consider any function $\tilde{\sigma}_1 : \Theta_1 \to \Theta_1$. Define reporting strategy $\sigma^{(1)}$ by $\sigma_1^{(1)}\left(\theta_1\right) = \tilde{\sigma}_1\left(\theta_1\right)$ and $\sigma_t^{(1)}\left(\theta^t\right) = \theta_t$ for all $t > 1$. Since the temporary incentive constraint (49) holds for $t = 1$ we obtain the inequality

$$\begin{aligned} & w_1\left(\mathbf{c}, \mathbf{y}; h_1\right) \\ = & \sum_{\theta_1 \in \Theta_1} \left[u\left(c_1\left(\theta_1\right), h_1\right) - v\left(\frac{y_1\left(\theta_1\right)}{\theta_1}\right) + \beta w_2\left(c_2^T\left(\theta_1\right), y_2^T\left(\theta_1\right);\, H\left(c_1\left(\theta_1\right), h_1\right)\right)\right] \pi_1\left(\theta_1\right) \\ \geq & \sum_{\theta_1 \in \Theta_1} \left[u\left(c_1\left(\tilde{\sigma}_1\left(\theta_1\right)\right), h_1\right) - v\left(\frac{y_1\left(\tilde{\sigma}_1\left(\theta_1\right)\right)}{\theta_1}\right)\right] \pi_1\left(\theta_1\right) \\ & \quad + \beta \sum_{\theta_1 \in \Theta_1} w_2\left(c_2^T\left(\tilde{\sigma}_1\left(\theta_1\right)\right), y_2^T\left(\tilde{\sigma}_1\left(\theta_1\right)\right);\, H\left(c_1\left(\tilde{\sigma}_1\left(\theta_1\right)\right), h_1\right)\right) \pi_1\left(\theta_1\right) \\ = & \, w_1\left(\mathbf{c} \circ \sigma^{(1)}, \mathbf{y} \circ \sigma^{(1)}; h_1\right). \end{aligned}$$

Hence, truth-telling dominates any strategy $\sigma^{(1)}$ involving deviations only in period 1.

Induction step: Suppose that the inequality $w_1\left(\mathbf{c}, \mathbf{y}; h_1\right) \geq w_1\left(\mathbf{c} \circ \sigma^{(t-1)}, \mathbf{y} \circ \sigma^{(t-1)}; h_1\right)$ holds for all strategies $\sigma^{(t-1)}$ involving deviations only in periods $1, \ldots, t-1$. Let $\sigma^{(t)}$ be a reporting strategy that involves deviations only in periods $1, \ldots, t$. Given a history $\theta^{t-1} \in \Theta^{t-1}$, let $\hat{\theta}^{t-1} = \sigma^{(t)}\left(\theta^{t-1}\right) = \left(\sigma_1^{(t)}\left(\theta^1\right), \ldots, \sigma_{t-1}^{(t)}\left(\theta^{t-1}\right)\right)$ be the corresponding history of reports. Let $\sigma^{(t-1)}$ be the strategy that coincides with $\sigma^{(t)}$ in periods $1, \ldots, t-1$ and corresponds to truth-telling in periods $t, \ldots, T$. Since by assumption the temporary incentive constraint (49) holds for all histories $\left(\hat{\theta}^{t-1}, \theta_t\right)$, $\theta_t \in \Theta_t$, we obtain

$$
\begin{aligned}
& w_t\left(\left(\mathbf{c} \circ \sigma^{(t-1)}\right)_t^T\left(\theta^{t-1}\right), \left(\mathbf{y} \circ \sigma^{(t-1)}\right)_t^T\left(\theta^{t-1}\right); h_t\left(\hat{\theta}^{t-1}\right)\right) \\
= & \sum_{\theta_t}\left[u\left(c_t\left(\hat{\theta}^{t-1}, \theta_t\right), h_t\left(\hat{\theta}^{t-1}\right)\right) - v\left(\frac{y_t\left(\hat{\theta}^{t-1}, \theta_t\right)}{\theta_t}\right)\right] \pi_t\left(\theta_t\right) \\
& + \beta \sum_{\theta_t} w_{t+1}\left(c_{t+1}^T\left(\hat{\theta}^{t-1}, \theta_t\right), y_{t+1}^T\left(\hat{\theta}^{t-1}, \theta_t\right); H\left(c_t\left(\hat{\theta}^{t-1}, \theta_t\right), h_t\left(\hat{\theta}^{t-1}\right)\right)\right) \pi_t\left(\theta_t\right) \\
\geq & \sum_{\theta_t}\left[u\left(c_t\left(\hat{\theta}^{t-1}, \sigma_t^{(t)}\left(\theta^t\right)\right), h_t\left(\hat{\theta}^{t-1}\right)\right) - v\left(\frac{y_t\left(\hat{\theta}^{t-1}, \sigma_t^{(t)}\left(\theta^t\right)\right)}{\theta_t}\right)\right] \pi_t\left(\theta_t\right) \\
& + \beta \sum_{\theta_t} w_{t+1}\left(c_{t+1}^T\left(\hat{\theta}^{t-1}, \sigma_t^{(t)}\left(\theta^t\right)\right), y_{t+1}^T\left(\hat{\theta}^{t-1}, \sigma_t^{(t)}\left(\theta^t\right)\right); H\left(c_t\left(\hat{\theta}^{t-1}, \sigma_t^{(t)}\left(\theta^t\right)\right), h_t\left(\hat{\theta}^{t-1}\right)\right)\right) \pi_t\left(\theta_t\right) \\
= & w_t\left(\left(\mathbf{c} \circ \sigma^{(t)}\right)_t^T\left(\theta^{t-1}\right), \left(\mathbf{y} \circ \sigma^{(t)}\right)_t^T\left(\theta^{t-1}\right); h_t\left(\hat{\theta}^{t-1}\right)\right).
\end{aligned}
$$

This implies

$$
\begin{aligned}
& w_1\left(\mathbf{c} \circ \sigma^{(t-1)}, \mathbf{y} \circ \sigma^{(t-1)}; h_1\right) \\
= & \sum_{s=1}^{t-1} \beta^{s-1} \sum_{\theta^s \in \Theta^s}\left[u\left(c_s\left(\sigma^{(t-1)}\left(\theta^s\right)\right), h_s\left(\sigma^{(t-1)}\left(\theta^{s-1}\right)\right)\right) - v\left(\frac{y_s\left(\sigma^{(t-1)}\left(\theta^s\right)\right)}{\theta_s}\right)\right] \Pi^s\left(\theta^s\right) \\
& + \beta^{t-1} \sum_{\theta^{t-1} \in \Theta^{t-1}} w_t\left(\left(\mathbf{c} \circ \sigma^{(t-1)}\right)_t^T\left(\theta^{t-1}\right), \left(\mathbf{y} \circ \sigma^{(t-1)}\right)_t^T\left(\theta^{t-1}\right); h_t\left(\hat{\theta}^{t-1}\right)\right) \Pi^{t-1}\left(\theta^{t-1}\right) \\
\geq & \sum_{s=1}^{t-1} \beta^{s-1} \sum_{\theta^s \in \Theta^s}\left[u\left(c_s\left(\sigma^{(t)}\left(\theta^s\right)\right), h_s\left(\sigma^{(t)}\left(\theta^{s-1}\right)\right)\right) - v\left(\frac{y_s\left(\sigma^{(t)}\left(\theta^s\right)\right)}{\theta_s}\right)\right] \Pi^s\left(\theta^s\right) \\
& + \beta^{t-1} \sum_{\theta^{t-1} \in \Theta^{t-1}} w_t\left(\left(\mathbf{c} \circ \sigma^{(t)}\right)_t^T\left(\theta^{t-1}\right), \left(\mathbf{y} \circ \sigma^{(t)}\right)_t^T\left(\theta^{t-1}\right); h_t\left(\hat{\theta}^{t-1}\right)\right) \Pi^{t-1}\left(\theta^{t-1}\right) \\
= & w_1\left(\mathbf{c} \circ \sigma^{(t)}, \mathbf{y} \circ \sigma^{(t)}; h_1\right),
\end{aligned}
$$

and hence, using the induction hypothesis, we have $w_1\left(\mathbf{c}, \mathbf{y}; h_1\right) \geq w_1\left(\mathbf{c} \circ \sigma^{(t)}, \mathbf{y} \circ \sigma^{(t)}; h_1\right)$. Since $\sigma^{(t)}$ was an arbitrary strategy involving deviations only in periods $1, \ldots, t$, the induction step is complete. This completes the proof. □

Equation (49) states that it is not profitable to misreport one's skill in period t and report the truth in all periods thereafter. If this condition holds for all periods and all possible histories, the lemma shows that no reporting strategy (potentially involving deviations in multiple time periods) yields more utility than truth-telling.

Based on definition (47), the promise-keeping constraint (46) can be written as

$$W_1 = \sum_{\theta_1 \in \Theta_1} \left[u\left(c_1\left(\theta_1\right), h_1\right) - v\left(\frac{y_1\left(\theta_1\right)}{\theta_1}\right) + \beta w_2\left(c_2^T\left(\theta_1\right), y_2^T\left(\theta_1\right); H\left(c_1\left(\theta_1\right), h_1\right)\right) \right] \pi_1\left(\theta_1\right). \tag{50}$$

Similarly, for periods $t > 1$ definition (47) is equivalent to

$$\begin{aligned} & w_t\left(c_t^T\left(\theta^{t-1}\right), y_t^T\left(\theta^{t-1}\right); h_t\left(\theta^{t-1}\right)\right) \\ & = \sum_{\theta_t \in \Theta_t} \left[u\left(c_t\left(\theta^{t-1}, \theta_t\right), h_t(\theta^{t-1})\right) - v\left(\frac{y_t\left(\theta^{t-1}, \theta_t\right)}{\theta_t}\right) \right] \pi_t\left(\theta_t\right) \\ & \quad + \beta \sum_{\theta_t \in \Theta_t} w_{t+1}\left(c_{t+1}^T\left(\theta^{t-1}, \theta_t\right), y_{t+1}^T\left(\theta^{t-1}, \theta_t\right); H\left(c_t\left(\theta^{t-1}, \theta_t\right), h_t\left(\theta^{t-1}\right)\right)\right) \pi_t\left(\theta_t\right). \end{aligned} \tag{51}$$

In summary, the incentive compatibility constraint (45) of the social planner problem is equivalent to the sequence of temporary constraints (49), whereas the promise-keeping constraint (46) is equivalent to condition (50) in combination with the sequence (51) of constraints for continuation utilities w_t, $t > 1$.

Since the constraint set can be given the sequential form (49), (50), (51), and since the objective function is a sum of period payoffs, the social planner problem is a standard dynamic programming problem. In particular, the Bellman Principle of Optimality holds. This establishes the following result.[1]

Proposition (Recursive formulation)**.** *Let $W_1 \in \mathrm{dom}_1(h_1)$. The value $C_1(W_1, h_1)$ of the social planner problem* (44) *can be computed by backward induction using the following equation for all t (with the convention $C_{T+1} = W_{T+1} = 0$):*

$$C_t(W_t, h_t) = \min_{c_t, y_t, W_{t+1}} \sum_{\theta \in \Theta_t} \left[c_t(\theta) - y_t(\theta) + q C_{t+1}\left(W_{t+1}(\theta), H(c_t(\theta), h_t)\right)\right] \pi_t(\theta) \tag{52}$$

s.t.

$$u\left(c_t(\theta), h_t\right) - v\left(y_t(\theta)/\theta\right) + \beta W_{t+1}(\theta) \geq u\left(c_t(\theta'), h_t\right) - v\left(y_t(\theta')/\theta\right) + \beta W_{t+1}(\theta') \quad \forall \theta, \theta' \in \Theta_t \tag{53}$$

$$\sum_{\theta \in \Theta_t} \left[u(c_t(\theta), h_t) - v(y_t(\theta)/\theta) + \beta W_{t+1}(\theta)\right] \pi_t(\theta) = W_t \tag{54}$$

$$W_{t+1}(\theta) \in \mathrm{dom}_{t+1}\left(H(c_t(\theta), h_t)\right) \quad \forall \theta \in \Theta_t. \tag{55}$$

Moreover, plans $(c_t, y_t)_{t=1,\ldots,T}$ that solve the sequence of problems (52) *constitute an optimal allocation. Conversely, any optimal allocation solves the sequence of problems* (52)*.*

In the numerical section of the paper, it is inevitable to work with compact spaces for consumption and output. For the numerical section we therefore pick bounds $\underline{c}, \overline{c}, \underline{y}, \overline{y} \in \mathbb{R}_{++}$ with $\underline{c} < \overline{c}$, $\underline{y} < \overline{y}$, and add the boundary constraints $\overline{c} \geq c_t \geq \underline{c}$ and $\overline{y} \geq y_t \geq \underline{y}$ for all t to the social planner problem. The

[1] The recursive formulation generalizes without difficulty to infinite time horizons if utilities are bounded.

bounds allow us to find a straightforward expression for the domain restriction $\mathrm{dom}_t(h)$. Based on the monotonicity properties of our preference specification, we obtain the upper bound of $\mathrm{dom}_t(h)$ by simply setting consumption to $\bar{c}$ and output to $\underline{y}$ for all realizations and all remaining periods. Similarly, the lower bound of $\mathrm{dom}_t(h)$ is obtained by setting consumption to $\underline{c}$ and output to $\bar{y}$ for all realizations and all remaining periods. By continuity, all points in the interval between the upper and lower bound of $\mathrm{dom}_t(h)$ are feasible promises.

Appendix C: Persistent skills

We assume that skills form a Markov chain with transition probabilities $\pi_t\left(\theta_t|\theta_{t-1}\right)$, where $\pi_t(\theta_t^H|\theta_{t-1}^H) > \pi_t(\theta_t^H|\theta_{t-1}^L)$. Following the insights from Fernandes and Phelan (2000), the Markov property imposes two additional state variables (past skill type θ_{t-1}, threat utility $\hat{W}_t$) and one additional constraint (threat-keeping constraint). As usual, we study a relaxed problem in which only the downward incentive compatibility constraints are imposed. With this approach, a high skill report may only come from a high-skilled worker and there is common knowledge of preferences in that case. A low skill report may come from both types of workers. Since those workers face different probability distributions over future uncertainty, we need to impose a threat-keeping constraint in that case.

If the past skill is low, the Bellman equation of the social planning problem is therefore

$$C_t\left(W_t, \hat{W}_t, c_{t-1}, \theta_{t-1}^L\right) = \min_{c_t^i, y_t^i, W_{t+1}^i, \hat{W}_{t+1}^L} \sum_{i=H,L} \left[c_t^i - y_t^i + qC_{t+1}\left(W_{t+1}^i, \hat{W}_{t+1}^i, c_t^i, \theta_t^i\right)\right] \pi_t\left(\theta_t^i|\theta_{t-1}^L\right) \quad (56)$$

s.t.

$$W_t = \sum_{i=H,L} \left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right] \pi_t\left(\theta_t^i|\theta_{t-1}^L\right) \quad (57)$$

$$\hat{W}_t = \sum_{i=H,L} \left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right] \pi_t\left(\theta_t^i|\theta_{t-1}^H\right) \quad (58)$$

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H \geq u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta \hat{W}_{t+1}^L. \quad (59)$$

If the past skill is high, the Bellman equation is

$$C_t\left(W_t, c_{t-1}, \theta_{t-1}^H\right) = \min_{c_t^i, y_t^i, W_{t+1}^i, \hat{W}_{t+1}^L} \sum_{i=H,L} \left[c_t^i - y_t^i + qC_{t+1}\left(W_{t+1}^i, \hat{W}_{t+1}^i, c_t^i, \theta_t^i\right)\right] \pi_t\left(\theta_t^i|\theta_{t-1}^H\right) \quad (60)$$

s.t.

$$W_t = \sum_{i=H,L} \left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right] \pi_t\left(\theta_t^i|\theta_{t-1}^H\right) \quad (61)$$

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H \geq u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta \hat{W}_{t+1}^L. \quad (62)$$

Introduce symbol $\hat{\lambda}$ for the Lagrange multiplier of the threat-keeping constraint (58) and define

$$
\begin{aligned}
B_t^H &= \frac{\beta\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]}{\lambda_{t+1}^H \mathbb{E}\left[\tilde{U}_t|\theta^{t-1}, \theta_t^H\right]} \geq 0, && (63)\\
B_t^L &= \frac{\beta\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]}{\left(\lambda_{t+1}^L + \hat{\lambda}_{t+1}^L\right)\mathbb{E}\left[\tilde{U}_t|\theta^{t-1}, \theta_t^L\right]} \geq 0, && (64)\\
A_t^L &= \beta\frac{\frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L} - \frac{v'\left(y_t^L/\theta_t^H\right)}{\theta_t^H} + \hat{U}_t^L - \mathbb{E}\left[\tilde{U}_t|\theta^{t-1}, \theta_t^L\right]}{q\pi_t\left(\theta_t^L|\theta_{t-1}\right)\left(\lambda_{t+1}^L + \hat{\lambda}_{t+1}^L\right)\mathbb{E}\left[\tilde{U}_t|\theta^{t-1}, \theta_t^L\right]} \geq 0. && (65)
\end{aligned}
$$

Proceeding as in the proof of Proposition 2, the labor wedges can be represented as $\tau_{y,t}^H = -\mu_{t+1}^H B_t^H$ and $\tau_{y,t}^L = \mu_t A_t^L - \mu_{t+1}^L B_t^L$. Note that the habit effects B_t^H, B_t^L are exact analogies to the case with transitory shocks. The instantaneous labor distortion A_t^L includes one additional term:

$$
\begin{aligned}
& \hat{U}_t^L - \mathbb{E}\left[\tilde{U}_t|\theta^{t-1}, \theta_t^L\right] && (66)\\
=\ & \beta\sum_j u_h\left(c_{t+1}^{Lj}, c_t^L\right)\pi_{t+1}\left(\theta_{t+1}^j|\theta_t^H\right) - \beta\sum_j u_h\left(c_{t+1}^{Lj}, c_t^L\right)\pi_{t+1}\left(\theta_{t+1}^j|\theta_t^L\right) && (67)\\
=\ & \beta\left[\pi_{t+1}\left(\theta_{t+1}^H|\theta_t^H\right) - \pi_{t+1}\left(\theta_{t+1}^H|\theta_t^L\right)\right]\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right] \geq 0. && (68)
\end{aligned}
$$

Savings wedges can be derived by following the proof of Proposition 3. For the high-skilled worker ($i = H$) we define

$$
\begin{aligned}
D_t^i &= \frac{\mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1}, \theta_t^i, \theta_{t+1}^L\right] - \mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1}, \theta_t^i, \theta_{t+1}^H\right]}{\lambda_{t+1}^i \mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1}, \theta_t^i\right]}, && (69)\\
E_t^i &= \frac{q\left[u_h\left(c_{t+1}^{iH}, c_t^H\right) - u_h\left(c_{t+1}^{iL}, c_t^i\right)\right]}{\lambda_{t+1}^i \mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1}, \theta_t^i\right]}, && (70)\\
F_t^{ij} &= \frac{q\left[u_h\left(c_{t+2}^{ijH}, c_{t+1}^{ij}\right) - u_h\left(c_{t+2}^{ijL}, c_{t+1}^{ij}\right)\right]}{\lambda_{t+1}^i \mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1}, \theta_t^i\right]}, \quad j = L, H, && (71)
\end{aligned}
$$

and obtain the savings wedge

$$
\tau_{s,t}^i = \mu_{t+1}^i D_t^i + \mu_{t+1}^i E_t^i + \sum_j \pi_{t+1}\left(\theta_{t+1}^j|\theta_t^i\right)\mu_{t+2}^{ij} F_t^{ij}. \tag{72}
$$

This is again an exact analogy to the case with transitory shocks. For the low-skilled worker ($i = L$) we replace λ^i_{t+1} by the sum $\lambda^L_{t+1} + \hat{\lambda}^L_{t+1}$ in the definitions of D^i_t, E^i_t, F^{ij}_t and we define

$$\hat{D}^L_t = \frac{\sum_j \left[\pi_{t+1}\left(\theta^j_{t+1}|\theta^L_t\right) - \pi_{t+1}\left(\theta^j_{t+1}|\theta^H_t\right)\right] \mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1}, \theta^L_t, \theta^j_{t+1}\right]}{\left(\lambda^L_{t+1} + \hat{\lambda}^L_{t+1}\right) \mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1}, \theta^L_t\right]}, \tag{73}$$

$$\hat{E}^L_t = \frac{q\sum_j u_h\left(c^{Lj}_{t+1}, c^L_t\right)\left[\pi_{t+1}\left(\theta^j_{t+1}|\theta^H_t\right) - \pi_{t+1}\left(\theta^j_{t+1}|\theta^L_t\right)\right]}{\left(\lambda^L_{t+1} + \hat{\lambda}^L_{t+1}\right) \mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1}, \theta^L_t\right]}. \tag{74}$$

The savings wedge is then

$$\tau^L_{s,t} = \mu^L_{t+1} D^L_t + \hat{\lambda}^L_{t+1}\hat{D}^L_t + \mu^L_{t+1} E^L_t + \hat{\lambda}^L_{t+1}\hat{E}^L_t + \sum_j \pi_{t+1}\left(\theta^j_{t+1}|\theta^L_t\right)\mu^{Lj}_{t+2} F^{Lj}_t. \tag{75}$$

The concavity/wealth effect is captured by the sum $\mu^L_{t+1} D^L_t + \hat{\lambda}^L_{t+1}\hat{D}^L_t$. Note that $\hat{D}^L_t$ is zero if $c^{LH}_{t+1} = c^{LL}_{t+1}$. Hence, even though the Lagrange multiplier μ^L_{t+1} does not show up directly, the part $\hat{\lambda}^L_{t+1}\hat{D}^L_t$ vanishes if $\mu^L_{t+1} = 0$. If $\mu^L_{t+1} > 0$, then due to concavity and $\pi_{t+1}\left(\theta^L_{t+1}|\theta^L_t\right) > \pi_{t+1}\left(\theta^L_{t+1}|\theta^H_t\right)$, the term $\hat{\lambda}^L_{t+1}\hat{D}^L_t$ is positive, just like $\mu^L_{t+1} D^L_t$. The immediate habit effect consists of the terms $\mu^L_{t+1}E^L_t + \hat{\lambda}^L_{t+1}\hat{E}^L_t$. The term $\mu^L_{t+1}E^L_t$ is familiar and looks just like in the case of the high-skilled worker. The term $\hat{\lambda}^L_{t+1}\hat{E}^L_t$ goes in the same direction, since $\pi_{t+1}\left(\theta^H_{t+1}|\theta^H_t\right) > \pi_{t+1}\left(\theta^H_{t+1}|\theta^L_t\right)$ and $u_h\left(c^{LH}_{t+1}, c^L_t\right) > u_h\left(c^{LL}_{t+1}, c^L_t\right)$ due to complementarity. Hence $\hat{\lambda}^L_{t+1}\hat{E}^L_t$ is also an immediate habit effect. Even though μ^L_{t+1} does not show up directly, we note that this term will be zero if $\mu^L_{t+1} = 0$, or equivalently if $c^{LH}_{t+1} = c^{LL}_{t+1}$. Finally we have the subsequent habit effect, consisting of the terms $\mu^{Lj}_{t+2}F^{Lj}_t$ just like in the case of the high-skilled worker.

References

Fernandes, A., and C. Phelan (2000): "A recursive formulation for repeated agency with history dependence," *Journal of Economic Theory*, 91(2), 223–247.

Phelan, C., and R. M. Townsend (1991): "Computing Multi-Period, Information-Constrained Optima," *Review of Economic Studies*, 58(5), 853–881.

Spear, S., and S. Srivastava (1987): "On repeated moral hazard with discounting," *Review of Economic Studies*, 54(4), 599–617.