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On the Empirical Evidence of Microeconomic Demand Theory

by

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"Facts do not cease to exist because they are ignored."
Aldous Huxley

1 Introduction

The concept of *'market demand'* plays a fundamental role in economic analysis, for example in price theory under perfect or oligopolistic competition. In each particular theory one defines an equilibrium concept, for example, a competitive price equilibrium (Walras) or a Nash-equilibrium in quantities (Cournot) or prices (Bertrand) as strategic variables. If one wants to use such a model for a comparative static analysis then the equilibrium concept should be well determined, which means that the equilibrium should be unique and stable with respect to an appropriate price adjustment process. In order to obtain a well determined equilibrium it is essential that the market demand has certain properties, for example, a qualitative prediction of how market demand reacts to a change in prices provided the other determinants of demand remain unchanged (the ceteris paribus clause). The most traditional property of market demand is the so-called 'Law of Demand' or to use a more neutral term, the monotonicity of market demand. This means that the *vector* Δp *of price changes* and the *vector* ΔF *of demand changes* point in opposite directions, that is to say

$$\Delta p \cdot \Delta F < 0 \; .$$

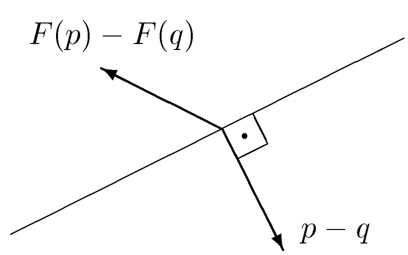

Figure 1

To model market demand as a monotone function requires a justification. The market demand function is a *hypothetical concept*, because it relates demand to all hypothetical price systems, thus the *function* cannot be observed. The relevant question is whether the hypothesis of a monotone market demand function or certain consequences of this hypothesis are, at least in principal, falsifiable by empirical data. This "inductive validation" of modelling market demand by a monotone function has to be carefully distinguished from a "deductive validation", which consists in deriving deductively the property of monotonicity from traditional hypotheses of microeconomic theory.

There is a general agreement in the profession on what microeconomics is. A concise definition is given by Malinvaud (1991): "Est dite microéconomique toute théorie qui prétend respecter dans ses formulations abstraites l'individualité de chaque bien et de chaque agent." In order to respect the individuality of commodities and agents microeconomics has to deal with a *high-dimensional* commodity space $\mathbb{R}^l$ and a *large* number of agents.

The behavior of each agent, in our context of each household, is modelled by an *individual demand function* f^i; this function (which typically is different for different households) relates to every price vector $p \in \mathbb{R}^l_+$ and every disposable income level x^i the demand vector $f^i(p, x^i) \in \mathbb{R}^l$. A population I of households (the consumption sector of an economy) is then described by the collection $\{f^i, x^i\}_{i \in I}$. The *joint distribution* of the households' characteristics (f, x) is denoted by μ. To respect the full individuality of each agent no restriction on the distribution μ, other than on its support, is made. Market demand is then defined by

$$F(p, \mu) = \frac{1}{\# I} \sum_{i \in I} f^i(p, x^i) \ .$$

Neo-classical microeconomics is more specific about the individual demand functions. The demand vector is viewed as a result of a maximization problem:

$$f^{\precsim^i}(p, x) = \quad \arg \quad \max_{p \cdot z \leq x} u^i(z)$$

$$z \in I\!R^l_+$$

where u^i denotes a utility representation of the preference relation $\precsim^i$ of household i.

The implications of the hypothesis of utility maximization on the individual demand function are well understood. Let $Sf(p,x)$ denote the matrix of the Slutsky substitution effects, i.e.,

$$Sf(p,x) = \partial_q f(q, q \cdot f(p,x)|_{q=p}$$

then (up to some regularity assumptions) a demand function f can be derived from same utility function if and only if the matrix $Sf(p,x)$ is negative semidefinite and symmetric.

I shall not discuss whether the hypothesis of utility maximization is acceptable. Much, probably too much, has been written on this point. In any case, most economists have a vested interest in defending their view on this point, which I am afraid, can not be changed by good arguments or empirical evidence.

I am more interested in the pragmatic question of whether this hypothesis is "useful" for a theory of market demand because I am interested in a theory of market demand and not in a theory of individual demand.[1] That is to say, I want to discuss whether the hypothesis of rational individual behavior implies some desirable properties of market demand, like the 'Law of Demand'.

A sufficient condition for monotonicity of the market demand function $F(p)$ is that the Jacobian matrix $\partial F(p)$ is negative definite.

$$\partial F(p) = \frac{1}{\#I} \partial p f^i(p, x^i) \ ,$$

and since by the Slutsky decomposition

$$\partial_p f^i(p,x) = Sf^i(p,x^i) - Af^i(p,x^i) \ ,$$

[1] *"In all our discussions so far, we have been concerned with the behaviour of a single individual. But economics is not, in the end, much interested in the behaviour of single individuals. Its concern is with the behaviour of groups. A study of individual demand is only a means to the study of market demand."* (Hicks), p. 34

where $Sf(p,x)$ denotes the matrix of the Slutsky substitution effects and $Af(p,x)$ denotes the matrix of income effects, we obtain for the Jacobian matrix of market demand

$$\begin{aligned} \partial F(p) &= \frac{1}{\#I}\sum_{i\in I} Sf^i(p,x^i) \quad - \quad \frac{1}{\#I}\sum_{i\in I} Af^i(p,x^i) \\ &= \qquad \bar{S}(p) \qquad - \qquad \bar{A}(p)\ . \end{aligned}$$

It is well-known that the only implication of the hypothesis of utility maximization which is useful for the theory of market demand is that the average Slutsky substitution effect matrix $\bar{S}(p)$ is negative semi-definite. One might reject the hypothesis of utility maximization yet still accept, as a behvioral assumption of the population, the negative semi-definiteness of the *average* Slutsky substitution effect matrix. Nothing can be concluded on the structure of the average income effect matrix $\bar{A}(p)$ and hence, on the Jacobian matrix $\partial F(p)$ without making strong specific assumptions on the utility functions. In this sense the neo-classical theory of consumer behavior is *incomplete*, at least, as a foundation for a theory of market demand.[2]

There are two possibilities to overcome this lack of structure of the microeconomic model of a consumption sector:

1. To complement the neo-classical model by a hypothesis on the *form of the individual utility function u^i* still respecting the individuality of each agent (i.e., no restriction is made on the form of the distribution μ, only the support of μ is restricted).

2. To complement the microeconomic model by a hypothesis on the *form of the joint distribution μ of agents' characteristics.* By doing so one gives up the strict individuality of each agent.

A simple example for the first approach is to assume that the preference relations of all households are homothetic which implies that the vector of

[2]For the *excess demand* of an exchange economy the lack of structure is made explicite by the well-known contributions of Sonnenschein (1973) Mantel (1974) and Debreu (1974) and Kirman-Koch (1986).

marginal propensity to consume $\partial_x f(p,x)$ and the vector of demand $f(p,x)$ are colinear. This implies that the matrix $Af(p,x)$ of income effects, and hence, the matrix $\bar{A}$ are positive semi-definite.

A difficulty with this approach[3] is that it is hard to justify the additional assumptions on the utility functions even if one accepts the general hypothesis of utility maximization. Are such additional assumptions falsifiable? Surely not without auxiliary hypotheses on individual behavior, for example the time invariance of utility functions.

A simple example for the second approach is to assume that all households have the same, yet arbitrary, demand function and the density of the income distribution is a decreasing function on an interval $[0,\xi]$. One can show that these assumptions imply that the *average* income effect matrix $\bar{A}$ is positive semi-definite.

From a methodological point of view the second approach has an advantage. Hypotheses on the form of the distribution of households' characteristics are easier to falsify. For the above example this is obvious by Figure 2 and 3. The assumption that all households have the same demand function is clearly in contradiction with the data as shown in Figure 2, and the estimates of the densities of income distributions in Figure 3 are certainly not decreasing for low income.

[3] A different and very stimulating model has recently been developed by Grandmont (1992).

Figure 2

Figure 3

In the next section I shall formulate an hypothesis on the distribution of households' characteristics which is less restrictive than the one in the above example, yet which is not falsified by empirical data! Then I shall show in Section 3 that this hypothesis has an important consequence for the theory of market demand.

"... le recours à l'observation systématique tient une place importante, pour montrer lesquelles des spécifications alternatives concevables ont une réalité. L'approche purement microéconomique entretient trop souvent l'illusion qu'on peut se passer de cette référence aux données, alors même que ses résultats négatifs sont autant de preuves qu'on ne peut pas conclure sans faire appel à l'observation."

Edmond Malinvaud (1991), p. 147.

2 The Hypothesis of Increasing Spread of Conditional Demand

This section deals with data analysis and not with speculative economic modelling.

Consider the "cloud" of demand vectors $\{y^i\}_{i \in I}$ in the commodity space of a population I of households of an economy. This cross-section demand data defines a distribution ν on the commodity space. Traditional microeconomics does not imply any specific pattern of this distribution.

Consider all households in the population with income x – I call this subpopulation the x-households. Let $\nu(x)$ denote the distribution on the commodity space of x-households' demand. As I shall show there is a tendency that the distribution $\nu(x + \Delta)$ is more "heterogeneous" than the distribution $\nu(x)$. This claim can be made precise in different ways, for details I refer to Hildenbrand (1993). For example, if one measures the degree of heterogeneity by the degree of positive-definiteness of the matrix $m^2\nu(x)$ of second moments of the distribution $\nu(x)$, then the above claim means that the matrix

$$m^2\nu(x + \Delta) - m^2\nu(x)$$

has a tendency to be positive semi-definite for $\Delta > 0$. This motivates to consider the following matrix

$$M := \int \partial_x m^2 \nu(x) \rho(x) dx$$

where ρ denotes the density of the income distribution.

The hypothesis of increasing spread of conditional demand means that the matrix M is positive definite.

The matrix M can be estimated[4] from Family Expenditure Surveys which are available for many countries. I should mention that these data refer to *commodity-aggregates* and not to commodities in the sense of microeconomics. The link is made by the Composite-Commodity Theorem of Hicks-Leontief. Details can be found in Hildenbrand (1993). In order to show that an estimate of the matrix M is positive definite, one computes an estimate of the smallest eigenvalue with confidence bounds. If the upper confidence bound is positive then the hypothesis of a positive definite matrix M is not rejected. Furthermore, if the lower confidence bound is positive then the opposite hypothesis, that is to say, the matrix M is not positive definite, is rejected. Tables 1 and 2 show the estimates and confidence bounds for the smallest eigenvalue of the matrix M for two data sets; Table 1 for the United Kingdom Family Expenditure Survey for the years 1968-1984, with nine commodity aggregates, and Table 2 for the French Enquête Budget de Famille for the years 1979, 1984 and 1989 with fourteen commodity aggregates. The results clearly show that the hypothesis of increasing spread of conditional demand is very well supported by empirical evidence.

[4] For details, I refer to Härdle-Hildenbrand-Jerison (1991) and Hildenbrand-Kneip (1993).

Year	lower confidence bound	smallest eigenvalue	upper confidence bound	largest eigenvalue	sample size
1968	7.9	14.0	20.0	2316	7098
1969	11.0	22.8	32.7	2210	6954
1970	10.6	17.1	24.5	2301	6331
1971	4.5	12.8	21.5	2225	7171
1972	8.5	20.9	30.6	2294	7059
1973	10.3	19.1	27.8	2193	7059
1974	8.6	18.0	26.5	2206	6626
1975	13.2	23.4	32.7	2203	7139
1976	5.8	15.4	24.5	2181	7133
1977	3.1	9.8	16.8	2176	7124
1978	7.7	11.8	15.1	2143	6950
1979	3.5	7.3	11.6	2243	6712
1980	7.1	11.7	15.9	2045	6889
1981	5.2	9.0	12.9	2050	7415
1982	5.0	8.1	11.8	2131	7358
1983	1.8	4.9	8.7	2106	6915
1984	7.9	13.6	19.0	2193	7009

Table 1: Estimates (times $10^4/\bar{x}_t$) of smallest eigenvalue with confidence bounds and largest eigenvalue of the matrix M for U.K.-FES.

Year	lower confidence bound	smallest eigenvalue	upper confidence bound	largest eigenvalue	sample size
1979	0.99	1.23	1.62	2107	9052
1984	0.69	0.78	0.90	2265	11023
1989	0.53	0.64	0.74	2418	8458

Table 2: Estimates (times $10^4/\bar{x}_t$) of smallest eigenvalue with confidence bounds and largest eigenvalue of the matrix M for the French-EBF.

3 Implications of the Hypothesis of Increasing Spread

As I said before, the hypothesis of utility maximization does not imply the observed property of increasing spread. Does this matter? The answer depends on whether the property of increasing spread is relevant for a theory of market demand. Indeed, if increasing spread were not relevant there would be no point of taking into account in modelling the consumption sector. However one can show (for details I refer again to Hildenbrand (1993)) that there is a link between the average income effect matrix $\bar{A}$ and the matrix M of the last section. Under certain assumptions (metonymy) one can show that positive definiteness of the matrix M implies the positive definiteness of the matrix $\bar{A}$! Thus, a microeconomic model of a consumption sector which is useful for the theory of market demand should be such that it possesses the property of increasing spread of conditional demand. Yet, this property cannot be obtained by respecting the *individuality of commodities and agents* which is part of the traditional definition of microeconomics. Obviously, there is an exception to this claim: if the respect of individuality goes so far that one is willing to swallow homothetic preferences for all households.

In conclusion: In order to obtain a useful foundation for the theory of market demand one has to complement the hypothesis of a negative semi-definite average substitution effect matrix $\bar{S}$ – a consequence of the hypothesis of utility maximization or the weak axiom of revealed preference – by the hypothesis of increasing spread. This last hypothesis is not in the spirit of traditional microeconomics. However, it has good empirical support.

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