Expropriation and Control Rights:  
A Dynamic Model of Foreign Direct Investment

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Abstract: This paper studies the strategic interaction between a foreign direct investor and a host country. We analyze how the investor can use his control rights to protect his investment if he faces the risk of “creeping expropriation” once his investment is sunk. It is shown that this hold-up problem may cause underinvestment if the bargaining position of the investor is too weak and overinvestment if it is too strong. We also analyze the impact of spillover effects, we give a rationale for “tax holidays” and we examine how stochastic returns affect the strategic interaction of investor and host country.

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1 Introduction

Direct investments in foreign countries are notoriously prone to the risk that once the investment is sunk the expected returns from the investment are diluted by, say, changes in the host government’s tax policy. This ”sovereign” risk reflects that a government cannot commit to a certain action through explicit contracts like individuals can because there is no legal recourse if it chooses to violate the terms of the agreement.

In this paper we analyze to what extend implicit agreements can protect the economic interests of a foreign investor if explicit guarantees cannot be relied upon. The problem of implicit agreements is that they have to be self-enforcing. This requires in particular that the investor has means to punish the host government if he is taxed excessively. In our analysis we focus on the control rights of the investor and on how they can be used for this purpose. The investor may, for example, exercise his control rights to shift production to some other countries, switch to different sources for inputs etc.

In Section 2 we develop a formal model in which host government and investor interact repeatedly. Initially, the investor has to make some investment and in all the following periods he decides whether to continue production in the host country or to switch to other production facilities abroad. The host government in turn decides in each period how much of the realized return to appropriate through taxation. We assume that the host country can impose taxes (or other charges) after returns in the host country have been realized. Thus, the host country can expropriate the entire return stream in any given period.

In Section 3 we describe the set of investment levels that can be sustained with an implicit self-enforcing contract. We show that the investor can recover his initial investment outlays only if his threat to shift production in case of excessive taxation is credible, i.e. if his outside option is not too inefficient. But at the same time the outside option should not be too attractive since otherwise the investor cannot credibly commit to produce in the host country and thus the host government is tempted to expropriate as much as it can right away. A second interesting observation is that some minimum level of investment may be needed for FDI to be viable at all. Otherwise the returns
from producing in the host country are not high enough as compared to switching to the outside option and thus again the investor cannot prove his commitment to the host country.

In Section 4 we consider several extensions of our basic model. In particular, we analyze the impact of spillover effects of the investment on the host country, we give a rationale for the frequently observed phenomenon of “tax holidays” and we analyze the effect of stochastic returns on the strategic interaction and the set of sustainable investments. If the returns from the investment are stochastic, we find that FDI is less likely to be viable the higher the risk involved. The problem is that the host government is more tempted to deviate from the implicit agreement when the realized returns are high and thus has to be offered high taxes. This may be compensated by offering lower taxes in case of low returns, but only limitedly so. Interestingly, just the opposite effect may obtain if instead the return from the outside option (shifting production abroad) is stochastic. Here, higher risk can imply that in some states of the world the outside option becomes so attractive that it should actually be chosen in equilibrium. This means of course that the investor is less dependent on the goodwill of the host government and this in turn tends to make FDI more likely to be viable.

Section 5 summarizes our results and concludes.

In contrast to the sovereign debt problem which is by now well understood, the sovereign risk of foreign direct investment has received much less attention in the literature. Eaton and Gersovitz (1983) discuss a reputation model of FDI with many potential investors. If the host country taxes excessively potential future investors are deterred and the host country loses access to foreign capital. They also note, however, that the prospects for an embargo of direct investments following expropriation are not as good as in case of financial investments. The problem is that investments differ very much and that potential investors may not consider the past record of a country as very relevant for their own investment. Thomas and Worrall (1994) consider a bilateral monopoly situation with a government and one investor. The investor decides in each period how much to invest and the host government decides how much to tax. Thomas and Worrall study the dynamic investment path and show that there will be underinvestment initially, but
that investment will rise over time. In Thomas and Worrall, the alternative to investing is not modelled explicitly and its return normalized to zero throughout their paper. In contrast, in our model, the investor has to choose between producing locally or abroad, which allows us to study the impact of changes in the outside option.

2 The Model

Consider the following relationship between a multinational enterprise (MNE) and a less developed host country (HC). MNE would like to exploit an investment opportunity in the host country. The investment cannot be carried out by HC itself, because HC has no funds available to finance the project and is so highly indebted already that it cannot obtain any additional credit on international capital markets. MNE, which is not credit constrained, discounts future payoffs with discount factor \( \delta_1 < 1 \) which reflects the riskless world interest rate, while HC’s discount factor is \( \delta_2 \leq \delta_1 \).\(^1\) Since we want to focus on the problems of financing this investment opportunity in the host country, the technology of the project is assumed to be very simple: If MNE invests an amount of \( K \geq 0 \) (which is then sunk) and takes action \( a_t \) in period \( t \), then the project will generate a return stream

\[
R_t = R(K, a_t) ,
\]

\( t = 1, \ldots, \infty \), which accrues in the host country. This return stream is net of all costs but before transfers to HC have been made. Note that the technology is stationary, deterministic, and that there is no depreciation of capital. Some of these assumptions will be relaxed in Section 4 below. Action \( a_t \) represents the control rights exercised by MNE as the owner of the investment project. In principle, the owner of an asset can decide unilaterally what to do with his asset. In practice, however, ownership rights may be severely restricted by all kinds of regulation or direct government intervention. But, as long as the owner has not been formally expropriated, he can always decide to leave his assets unproductive and to exclude others from using them. We model this in a highly

\(^1\)If HC’s credit constraint is binding, HC is willing to pay an interest rate equal to or higher than the riskless world interest rate in order to get additional funds. Hence, its discount rate must be (weakly) smaller than the discount rate of MNE.
There are only two possible actions in every period, \( a_t \in \{ \bar{a}, \pi \} \). The efficient action, denoted by \( \bar{a} \), means that MNE is actively involved in the country and uses the investment project in the most efficient way. But MNE can also choose action \( \pi \), e.g., reduce its commitment to HC, shift production to plants in other countries, and leave the investment project at least partially idle. Given this interpretation, it is natural to assume that

\[
R(K, \bar{a}) > R(K, a) \quad \forall \ K > 0 .
\]  

(2)

Note, however, that \( R(K, a_t) \) is just the return of the investment project accruing in the host country. The action \( a_t \) may also affect returns of MNE in other countries. For example, if some production is shifted to a plant in another country, then profits realized in this other country will be affected. These other returns are denoted by \( r(a_t) \), and, since action \( a_t \) is intended to shift profits out of HC, we assume that \( r(a) \geq r(\pi) \geq 0 \).

**Assumption 1** \( R(\cdot, a) \) is twice continuously differentiable, strictly increasing and concave in \( K \) with \( R(0, \cdot) = 0 \) and \( \lim_{K \to \infty} R(K, \cdot) < \bar{R} < \infty \). For all \( K > 0 \), \( R(K, \bar{a}) - R(K, a) \) is strictly positive, increasing, and concave in \( K \). Furthermore, there exists a \( K > 0 \) such that

\[
\frac{\delta_1}{1 - \delta_1} [R(K, \bar{a}) + r(\bar{a})] > K .
\]  

(3)

Given Assumption 1 there exist unique investment levels \( K^*(a) \) such that

\[
K^*(a) = \arg \max \left\{ \frac{\delta_1}{1 - \delta_1} [R(K, a) + r(a)] - K \right\} .
\]  

(4)

Note that \( K^*(\pi) \) is strictly positive (because of (3)) and is uniquely characterized by the first order condition

\[
\frac{\delta_1}{1 - \delta_1} \frac{\partial R(K^*(\pi), \pi)}{\partial K} = 1 .
\]  

(5)

**Assumption 2** It is efficient to invest \( K^*(\pi) \) and to choose action \( \pi \) in every period, i.e.,

\[
\frac{\delta_1}{1 - \delta_1} [R(K^*(\pi), \pi) + r(\pi)] - K^*(\pi) > \frac{\delta_1}{1 - \delta_1} [R(K^*(a), a) + r(a)] - K^*(a) .
\]  

(6)
Let $K^*(\pi) = K^{FB}$. The first best allocation generates a surplus
\[
\frac{\delta}{1-\delta} \left[ R(K^{FB}, \pi) + r(\pi) \right] - K^{FB} > 0,
\] (7)
which can be shared between MNE and HC. The problem is, however, that the parties are unable to commit on how to share the returns of the investment project. Once investment costs are sunk, HC is tempted to expropriate the “quasi-rents” of the investment by imposing additional taxes, by raising import or export duties, or by asking for outright bribes in return for allowing production to take place. This is the classical “hold-up problem”.\(^2\) We model this by assuming that at the end of each period $t$ HC chooses the level of taxes $T_t$ to be imposed on MNE subject to the constraint $T_t \leq R(K, a_t)$. Thus, HC can expropriate the entire return stream (which accrues in the host country) of the project. But, of course, if this is anticipated by MNE, no positive amount of investment can be sustained. Note, however, that the parties are engaged in an (infinitely) repeated relationship, so it may be possible to sustain some positive share of profits for MNE by an implicit contract. Such an implicit agreement has to be made before investment costs are sunk in period 0. We will analyze the scope for implicit contracts in detail in the next section.

The time structure of the model is summarized in Figure 1:

\[\begin{array}{cccc}
\text{initial} & \text{period 0} & \text{period } t, \\
\text{bargaining} & & t = 1, \ldots, \infty \\
\hline
\text{Implicit contract} & \text{Investment} & \text{MNE} & \text{HC} \\
\text{on investment } K, & \text{costs } K & \text{chooses} & \text{chooses} \\
\text{future actions } a_t, & \text{sunk} & \text{action } a_t & \text{tax } T_t \\
& & \text{realized} & \\
\end{array}\]

Figure 1: Time structure of the model

\(^2\)See e.g. Klein, Crawford and Alchian (1978), Grout (1984) and Williamson (1985). This problem is particularly prone in transition economies in Eastern Europe or in less developed countries where there is no well established independent judicial system yet which could be used to enforce a commercial contract between the government and a foreign direct investor.
The payoffs of MNE and HC from the repeated relationship are given by

\[ U^{MNE} = -K + \sum_{t=1}^{\infty} \delta_t^1 \left[ R(K, a_t) + r(a_t) - T_t \right] \]  

and

\[ U^{HC} = \sum_{t=1}^{\infty} \delta_t^2 T_t, \]

respectively.

3 Implicit Contracts and FDI

An implicit contract cannot be enforced by the courts, but has to be self-enforcing, i.e. the behavior described by the implicit agreement has to be a subgame perfect equilibrium of the repeated game between the involved parties. Since there is symmetric information at the initial bargaining stage the Coase theorem suggests that the parties will agree to an implicit contract which maximizes the joint surplus in the relationship subject to the constraint that this contract has to be self-enforcing. How the joint surplus will be split depends on the relative bargaining power of the two players, in particular on the outside options available to MNE and HC, and will not be modelled explicitly here. Instead, we want to characterize the set of investment levels that can be sustained through an implicit contract.

Without loss of generality we can restrict attention to the following class of implicit contracts: In period 0 MNE invests some amount \( K > 0 \). Thereafter, MNE chooses \( a_t = \bar{a} \) in every period and HC chooses the tax \( T_t = T \). If any deviation occurs, MNE chooses \( a_t = a \) in all subsequent periods, and HC chooses \( T_t = R(a_t) \). These implicit contracts use the worst punishment equilibrium which gives each of the players his minmax payoff (given that the investment \( K \) has been sunk already). It is well know that any allocation that can be sustained at all in a subgame perfect equilibrium can be sustained by deterring a deviation with the threat of the worst punishment equilibrium.\(^3\) Note further that no positive investment level can be sustained if MNE is supposed to choose \( a \) in every period

\(^3\)See Abreu (1988).
along the equilibrium path. In this case there is no possibility for MNE to punish HC, and HC will expropriate all of the returns in every period.

For an implicit contract to form a subgame perfect equilibrium, it has to satisfy the following constraints:

- HC must get at least its outside option utility of 0. Hence, HC’s participation constraint requires
  \[
  \frac{1}{1-\delta_2} T \geq 0 .
  \] (10)

- Given that HC sticks to the terms of the agreement, it must be profitable for MNE to invest \( K \), i.e.,
  \[
  \frac{\delta_1}{1-\delta_1} [R(K, \pi) + r(\pi) - T] \geq K ,
  \] (11)
  which will be referred to as MNE’s participation constraint.

- HC must be better off imposing the tax \( T \) in every period, rather than expropriating the entire return \( R(K, \pi) \) in one period and \( R(K, a) \) in all periods thereafter, i.e., HC’s incentive constraint
  \[
  \frac{1}{1-\delta_2} T \geq R(K, \pi) + \frac{\delta_2}{1-\delta_2} R(K, a)
  \] (12)
  has to be satisfied.

- MNE must prefer to take action \( \pi \) and pay tax \( T \) in every period, rather than choosing action \( a \) and paying \( T = R(K, a) \) in every period. Note that the tax is imposed after HC observes the action taken in any given period. Therefore, MNE’s incentive constraint reduces to
  \[
  R(K, \pi) + r(\pi) - T \geq r(a) .
  \] (13)

Condition (10) is implied by (12) and \( R(K, \pi), R(K, a) > 0 \). Conditions (11) - (13) are equivalent to:

\[
T \leq R(K, \pi) + r(\pi) - \frac{1-\delta_1}{\delta_1} K
\] (14)
\[
(1-\delta_2) R(K, \pi) + \delta_2 R(K, a) \leq T
\] (15)
\[
T \leq R(K, \pi) + r(\pi) - r(a)
\] (16)
Note that (16) and $r(\bar{a}) > r(\bar{\pi})$ imply $T < R(K, \bar{\pi})$, i.e., $T$ is indeed feasible.

**Proposition 1** There exists an implicit contract which induces MNE to invest $K$ and to take action $\bar{\pi}$ in every period if and only if

$$K \leq \frac{\delta_1}{1 - \delta_1} [r(\bar{\pi}) + \delta_2 (R(K, \bar{\pi}) - R(K, \bar{a}))] \quad (17)$$

and

$$r(\bar{a}) - r(\bar{\pi}) \leq \delta_2 [R(K, \bar{\pi}) - R(K, \bar{a})] \quad (18)$$

**Proof:** See Appendix.

To interpret this result consider first the case of an investment project with fixed size $K$. Condition (17) says that the project has to be sufficiently profitable for MNE to cover the initial capital outlay $K$. If $K$ has been sunk, the maximum per period return MNE can get in the host country is given by $\delta_2(R(K, \bar{\pi}) - R(K, \bar{a}))$ (otherwise HC cannot be prevented from fully expropriating MNE). In addition, MNE receives $r(\bar{\pi})$ which accrues outside HC. Since $K$ has to be paid in period 0 while the returns start only in period 1, the RHS is an upper bound on the net present value of the return stream accruing to MNE.

Consider now condition (18) which is equivalent to

$$R(K, \bar{\pi}) - R(K, \bar{a}) \leq \frac{1}{1 - \delta_2} [R(K, \bar{\pi}) + r(\bar{\pi}) - R(K, \bar{a}) - r(\bar{a})] \quad (19)$$

Note that HC will always expropriate at least $R(K, \bar{a})$ in every period given that investment costs $K$ have been sunk already. On the LHS of (19) we have $R(K, \bar{\pi}) - R(K, \bar{a})$, i.e., the additional amount that could be expropriated if MNE takes action $\bar{\pi}$. This amount has to be smaller than the net present value (evaluated using HC’s discount factor $\delta_2$) of the total surplus $[R(K, \bar{\pi}) + r(\bar{\pi}) - R(K, \bar{a}) - r(\bar{a})]$ that can be generated in this and in all future periods by taking action $\bar{\pi}$ instead of action $\bar{a}$. Note that this is also the maximum amount that can be offered to HC for not expropriating all of $R(K, \bar{\pi})$ in any given period. Otherwise, MNE would prefer to take action $\bar{a}$. Hence, if condition (19) is violated, it is impossible to deter HC from fully expropriating MNE.

Condition (19) has an interesting implication:
Corollary 1 Consider an investment project with fixed size $K$. The project cannot be financed through FDI if MNE has too much flexibility in shifting production abroad, i.e., if the cost of withdrawal

$$R(K, \bar{a}) + r(\bar{a}) - R(K, a) - r(a)$$

(20)

is sufficiently small.

One might have suspected that if MNE can easily shift production between countries, it has a strong bargaining position vis à vis HC and will not be taxed too much. While this is true, another constraint may become binding in this case. If it is possible to shift production at low costs, then the surplus generated from taking $\bar{a}$ (i.e. producing in the host country) instead of taking $a$ (i.e. producing in some other country) is small, while the difference in profits which accrue in the host country, $R(K, \bar{a}) - R(K, a)$, may still be significant. Thus, HC’s profit from fully expropriating this amount may be bigger than the total amount of future taxes that can credibly be offered by MNE.

This observation is also important when it comes to the choice of investment projects. MNE may prefer to choose a type of investment which implies a rather strong commitment to HC instead of an investment which leaves a lot of flexibility ex post. The idea is to increase the ex post bargaining power of HC. If HC’s ex post bargaining position is very weak, it will not receive enough of the future returns of the project, so it might as well fully expropriate all of the returns from the very beginning.

Let us now consider an investment project with variable size $K$. The question is under what conditions the efficient amount of investment $K^{FB}$ can be sustained.

Proposition 2 There exists a unique $\bar{K}$ such that

$$\bar{K} = \frac{\delta_1}{1 - \delta_1} \left[ r(\bar{a}) + \delta_2 \left( R(\bar{K}, \bar{a}) - R(\bar{K}, a) \right) \right] .$$

(21)

$\bar{K}$ is an upper bound on the level of investment that can be sustained through an implicit contract. If there exist $K$ satisfying (18), then there exists a unique $K > 0$ such that

$$r(a) - r(\bar{a}) = \delta_2 \left[ R(K, \bar{a}) - R(K, a) \right] .$$

(22)
$K$ is a lower bound on the set of sustainable investment levels.

Proof: See Appendix.

Propositions 1 and 2 are illustrated in Figure 2. Lines (14), (15) and (16) represent the constraints on $T$ given by the respective inequalities. Given Assumption 1 these lines are concave, and (14) and (15) have to intersect eventually. Condition (18) requires $(K, T)$ to lie between (16) and (15), while condition (17) says that only $(K, T)$ combinations above line (15) and below line (14) are feasible. If these lines intersect at $K$ and $\bar{K}$ as shown in Figure 2, then any investment level between $K$ and $\bar{K}$ can be sustained with an implicit contract. Note that if $K^{FB}$ happens to be larger than $\bar{K}$ or smaller than $K$, then the first best investment level cannot be implemented. If (15) and (16) intersect to the right of $\bar{K}$
or do not intersect at all, then the dotted set is empty and no positive investment level can be sustained.

$\overline{K}$ and $\underline{K}$ are characterized in more detail by the following corollaries.

**Corollary 2** Suppose that the set of investment levels that can be financed through FDI is non-empty. The maximal amount of investment, $\overline{K}$, is

(a) increasing with $\delta_1$ and $\delta_2$,
(b) increasing with $r(a)$, and
(c) increasing if the difference $R(K, \pi) - R(K, a)$ increases for all $K$.

The interpretation of Corollary 2 is as follows: The maximal amount of investment is given by the constraint that the amount of taxes withhold by HC does not render the project unprofitable. The larger $\delta_1$, the smaller is the return after taxes required per period to pay off the investment. The larger $\delta_2$, the less inclined is HC to fully expropriate in one period at the expense of future tax revenues, so the amount of taxes per period can be reduced. The larger $r(a)$, the higher is MNE’s return from taking action $\pi$ outside the host country which makes it easier to sustain this action in equilibrium. Finally, the larger $R(K, \pi) - R(K, a)$ (for any given $K$), the larger is the maximum per period return that can be given to MNE.

**Corollary 3** Suppose that the set of investment levels that can be financed through FDI is non-empty. The minimal amount of investment, $\underline{K}$.

(a) decreases as $\delta_2$ increases,
(b) increases as $r(a) - r(\overline{\pi})$ increases, and
(c) decreases if the difference $R(K, \pi) - R(K, a)$ increases for all $K$.

At first glance it may be surprising that there is a minimum level of investment at all. This is due to the fact that HC has to be given a sufficiently large share of the return stream in order to prevent it from fully expropriating MNE. Increasing $K$
increases the quasi-rents of the project and thus the future payoff going to HC if it does not expropriate. The interpretation of Corollary 3 is straightforward. The larger $\delta_2$, the less are future payoffs discounted by HC and the smaller is the amount of future taxes that has to go to HC in order to prevent full expropriation. The smaller $r'(g) - r'(\pi)$, the less attractive it is for MNE to withdraw from HC and the weaker is MNE’s bargaining position. Hence, HC can expect a larger share of the future surplus. Finally, the larger $R(K, \pi) - R(K, g)$ (for any given $K$), the larger is the maximum per period return that can be given to HC. In all of these cases it becomes less difficult to sustain any given level of investment and therefore the minimum level of investment becomes smaller.

Let us briefly summarize the main insights from the basic model. First, in a repeated relationship the hold-up problem associated with FDI can be mitigated by an implicit contract which is sustained by MNE’s threat to withdraw from HC if taxes become too high. This threat gives some bargaining power to MNE which can be used to get some share of the quasi-rents of the project in order to recover the initial capital outlay. However, MNE’s bargaining position may not be too strong. If MNE has too much flexibility in shifting production abroad, it cannot commit to give a sufficient share of future revenues to HC which in turn induces HC to fully expropriate all of the returns right away.

Second, even if it is possible to sustain some level of investment, it may be that the efficient investment level $K^{FB}$ cannot be sustained. There may be underinvestment ($K < K^{FB}$) if HC cannot be prevented from expropriating too much of the return stream. But there may also overinvestment ($K > K^{FB}$) if investing too much is the only possibility to commit to giving a sufficiently large share of the return stream to HC in order to prevent it from fully expropriating MNE.

4 Some Extensions of the Model

4.1 Spillover Effects in the Host Country

The preceding sections assumed that the only effect of the investment project on HC’s welfare is the amount of taxes that can be collected. However, such a project may have
significant spillover effects on HC. For example, MNE may bring technological or managerial know-how to the country, support local suppliers, or pay wages in excess of market wages to local employees. What is the effect of such positive external effects on the set of sustainable investment levels and taxes?\textsuperscript{4}

Let $B(K, a_t) \geq 0$ denote all positive external effects on HC accruing in period $t$, $t \in \{1, \ldots, \infty\}$, with $B(K, \bar{a}) > B(K, \bar{a})$ for all $K > 0$. There may be some additional spillover $B_0(K) \geq 0$ accruing in period 0 when investment costs are sunk, e.g. the rents going to local firms and workers for building up a plant in the host country. These spillovers effects affect HC’s participation constraint

$$B_0(K) + \sum_{t=1}^{\infty} \delta_t^t [T + B(K, \bar{a})] \geq 0, \quad (23)$$

and HC’s incentive constraint

$$\frac{1}{1-\delta_2} [T + B(K, \bar{a})] \geq R(K, \bar{a}) + B(K, \bar{a}) + \frac{\delta_2}{1-\delta_2} [R(K, \bar{a}) + B(K, \bar{a})]. \quad (24)$$

Solving for $T$ these constraints are equivalent to:

$$T \geq -\left[B(K, \bar{a}) + \frac{1-\delta_2}{\delta_2} B_0(K)\right], \quad (25)$$

$$T \geq (1-\delta_2) R(K, \bar{a}) + \delta_2 R(K, \bar{a}) - \delta_2 [B(K, \bar{a}) - B(K, \bar{a})]. \quad (26)$$

Obviously, positive spillover effects relax the constraints on the set of sustainable investment levels, since the RHS of (26) is strictly smaller than the RHS of (15). In particular, if the spillover effects are significantly affected by $a_t$ ($B(K, \bar{a}) - B(K, \bar{a})$ large), if HC is sufficiently patient ($\delta_2$ close to 1), and if MNE can shift almost all profits out of the host country ($R(K, \bar{a})$ small), then the RHS of (26) may become negative.

An interesting case is depicted in Figure 3. The dotted area bounded by (14), (16) and (26) is the set of all $(K, T)$ combinations satisfying the incentive and participation constraints. However, all sustainable investment levels require $T$ to be negative. Hence, if

\textsuperscript{4}Negative external effects are also possible. For example, the project may deplete natural resources which could have been exploited by the country itself at some later point in time, or it may pollute the environment. We will focus on the case of positive spillovers in this section. The extension to negative external effects is straightforward.
HC has no funds available to subsidize the investment project, it cannot go ahead. Note that this is the case even though there are some $K$ which are privately profitable, namely all $K$ for which (14) is positive.

Figure 3: Spillover effects and the necessity of subsidies

Suppose now that HC can use some export revenues in period $t, t = 1, \ldots, \infty$, to subsidize MNE, but that it has no funds available at date 0, when investment costs are sunk. In this case the highest investment level that can be sustained is $\mathcal{K}$. Note, however, that $\mathcal{K}$ still falls short of the efficient investment level $K^{FB}$, which obtains at the point where the difference between lines (14) and (25) is maximized. Even though HC would like to commit to a higher level of subsidies, this commitment is not credible. In particular, after investment costs are sunk HC cannot be induced to pay for $B_0(K)$ and $B(K, a)$ which accrue independently of whether MNE chooses $\bar{a}$ or $a$. 

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4.2 Short-Term Commitment and Tax Holidays

So far we only considered stationary implicit contracts in which the tax paid by MNE is the same in every period. In reality, however, the tax rate paid by a foreign direct investor is typically non-stationary. In particular, many less developed countries offer "tax holidays" to foreign direct investors, i.e., investors do not have to pay any taxes in the beginning of the project for a given number of years.\(^5\) This practice is puzzling. If MNE and HC have similar discount rates, why does the reduction in tax rates invariably take this form rather than a uniform reduction over time. In Section 2 we argued that HC’s discount factor is typically smaller than MNE’s because HC is often credit constrained or has to pay a higher risk premium on international capital markets. In this case we would expect exactly the opposite time pattern: High taxes in the beginning of the project (as a form of credit from MNE to HC) and low taxes thereafter.

Our model can be used to give a rationale for tax holidays.\(^6\) So far we assumed that HC cannot make any commitment not to tax MNE excessively once investment costs are sunk. This assumption is useful to explore the set of investment levels that can be sustained by an implicit contract alone. In practice, however, some short-term commitment of the government may be credible. For example, a foreign direct investor may be confident that the present government in a host country will honour its promises (for reputational or other reasons), but he may be afraid that future governments do not feel obliged in the same way.

The simplest way to model this is to assume that the government can fully commit for a maximum of \(\tau\) periods.\(^7\) We continue to assume that HC is cash constrained and cannot pay any subsidies to MNE. Thus, the best the government can do is to offer a tax \(T = 0\) for \(\tau\) periods to MNE (a tax-holiday). This relaxes MNE’s participation constraint

\(^5\)Caves (1982) and Gersovitz (1987) survey the literature on tax holidays. Caves reports that the average length of tax holidays in less developed countries is five years.

\(^6\)See also Bond and Samuelson (1986) who offer a signaling model to explain tax holidays. In their model HC has private information about local conditions which affect the profitability of the investment project. Offering a tax holiday is a signal that the country offers a high productivity environment.

\(^7\)The qualitative results are unchanged if \(\tau\) is a random variable the realization of which is unknown at the time investment costs have to be sunk.
which is now given by
\[
\frac{\delta_1 - \delta_1^{\tau+1}}{1 - \delta_1} [R(K, \bar{\pi}) + r(\bar{\pi})] + \frac{\delta_1^{\tau+1}}{1 - \delta_1} [R(K, \pi) + r(\pi) - T] \geq K
\]  
(27)

Solving for the maximum amount of taxes \( T \) that may be imposed after period \( \tau \), we get
\[
T \leq \frac{1}{\delta_1} [R(K, \bar{\pi}) + r(\bar{\pi})] - \frac{1 - \delta_1}{\delta_1^{\tau+1}} K .
\]  
(28)

Note that the incentive constraints (12) and (13) for HC and MNE, respectively, are unaffected by the possibility of a tax holiday.

**Proposition 3** Suppose that HC can commit to a tax holiday of \( \tau \) periods.
Then there exists an implicit contract which induces MNE to invest \( K \) and to take action \( \bar{\pi} \) in every period if and only if
\[
K \leq \frac{\delta_1}{1 - \delta_1} [r(\bar{\pi}) + (1 - \delta_1^\tau) R(K, \bar{\pi}) + \delta_2 \delta_1^\tau [R(K, \bar{\pi}) - R(K, \bar{a})]]
\]  
(29)

and
\[
r(a) - r(\bar{\pi}) \leq \delta_2 [R(K, \bar{\pi}) - R(K, a)] .
\]  
(30)

**Proof:** See Appendix.

Proposition 3 shows that the possibility of a tax holiday increases the upper bound on the level of investment that can be financed through FDI. Comparing (29) with (17) it is easy to see that
\[
\delta_2 (R(K, \bar{\pi}) - R(K, \bar{a})) < (1 - \delta_1^\tau) R(K, \bar{\pi}) + \delta_2 \delta_1^\tau (R(K, \bar{\pi}) - R(K, \bar{a}))
\]  
(31)

which is equivalent to
\[
0 \leq (1 - \delta_1^\tau) [(1 - \delta_2) R(K, \bar{\pi}) + \delta_2 R(K, \bar{a})] .
\]  
(32)

It can be shown that the RHS of (29) exceeds the RHS of (17) by an amount which equals the discounted sum over the first \( \tau \) periods of the minimum tax that would have to be paid without a tax holiday. This is exactly the additional return that can be guaranteed to MNE if a short-term commitment by HC government is feasible.
4.3 Stochastic Returns

Finally, suppose that there is some uncertainty about the return stream of the investment project. In this section it will be shown that stochastic returns may affect the strategic behavior of both parties even if they are risk neutral. Two types of uncertainty have to be distinguished, however. If the MNE’s outside option is a random variable, it may become easier to sustain any given level of investment with an implicit contract. On the other hand, if MNE’s return in the host country is affected by noise, then the more risky the project the more difficult it is to sustain the desired investment level.

Stochastic Outside Option

Consider the case of a stochastic outside option first. Suppose that \( \tilde{r}_t(a) \), the return generated by MNE abroad in period \( t \) if he shifts production out of the host country, is an i.i.d. random variable with two possible realizations. In every period \( \tilde{r}(a) \) equals \( \bar{r} \) with probability \( \mu \) and \( r \) with probability \( 1 - \mu \). MNE observes the realization of \( \tilde{r}(a) \) before deciding on \( a_t \). For notational simplicity we set \( R(K, a) = 0 \). To make the results in this section comparable to our results in the basic model, assume that \( \mu \bar{r} + (1 - \mu)r = r(a) \).

Furthermore, we focus on the case where \( r \geq R(K, \bar{a}) + r(\bar{a}) \), i.e., where efficiency requires to shift production out of the host country if the realization of the outside option is favorable.\(^8\)

Clearly, a stochastic \( \tilde{r}(a) \) improves the overall efficiency of the project, because in every period there is some positive probability \( 1 - \mu \) that a return \( \bar{r} \geq R(K, \bar{a}) + r(\bar{a}) \) can be realized, while \( \bar{r} \) does not matter for efficiency. Furthermore, HC can collect taxes only in those periods where MNE does not shift production abroad. Indeed, it is easy to see that this relaxes MNE’s participation constraint, which is now given by

\[
\frac{\delta_1}{1 - \delta_1} [\mu (R(K, \bar{a}) + r(\bar{a}) - T) + (1 - \mu)\bar{r}] \geq K.
\]

Furthermore, MNE’s incentive constraint is relaxed. Only if the realization of his outside option is unfavorable, MNE has to be prevented from withdrawing from the host country.

\(^8\)The case \( r < R(K, \bar{a}) + r(\bar{a}) \) is analytically similar to the case of stochastic returns in the host country (see below). The qualitative results of this section carry over if there are more than two possible realizations of \( \tilde{r}(a) \), as long as some realizations are larger than \( R(K, \bar{a}) + r(\bar{a}) \).
forever:

\[ R(K, \bar{\pi}) + r(\bar{\pi}) - T + \frac{\delta_1}{1 - \delta_1} \left[ \mu (R(K, \bar{\pi}) + r(\bar{\pi}) - T) + (1 - \mu)\bar{\pi} \right] \geq r + \frac{\delta_1}{1 - \delta_1} [(1 - \mu)\bar{\pi} + \mu r] . \]  

(34)

On the other hand, HC’s incentive constraint becomes tighter. Since HC can tax profits in each future period only with probability \( \mu \), the temptation increases to fully expropriate all of the returns in a period where MNE did not shift production abroad:

\[ T + \frac{\delta_2}{1 - \delta_2} \mu T \geq R(K, \bar{\pi}) . \]  

(35)

Nevertheless, the following proposition shows that the positive effects always dominate, i.e., the set of investment levels that can be sustained with an implicit contract increases as compared to the case of a deterministic outside option.

**Proposition 4** Suppose that \( \mu r + (1 - \mu)\bar{\pi} = r(\bar{a}) \) and \( \bar{\pi} \geq R(K, \bar{\pi}) + r(\bar{\pi}) \).

There exists an implicit contract which induces MNE to invest \( K \) and to take action \( \bar{a} \) whenever \( \tilde{r}(\bar{a}) = r \) if and only if

\[ K \leq \frac{\delta_1}{1 - \delta_1} \left[ \frac{\delta_2 \mu ^2}{1 - \delta_2 + \delta_2 \mu} R(K, \bar{\pi}) + \mu r(\bar{\pi}) + (1 - \mu)\bar{\pi} \right] \]  

(36)

and

\[ r \leq \frac{\delta_2 \mu}{1 - \delta_2 + \delta_2 \mu} R(K, \bar{\pi}) + r(\bar{\pi}) . \]  

(37)

The set of investment levels that can be sustained with a stochastic outside option is strictly larger as compared to the case of a deterministic outside option.

**Proof:** See Appendix.

Note that in the limit, as \( \mu \to 1 \), conditions (36) and (37) converge to conditions (17) and (18) in Proposition 1.
Stochastic Returns in the Host Country

Consider now the case where $\tilde{R}_t(K, \bar{a})$ is an i.i.d. random variable. To see the basic effect, it is sufficient to consider the following simple example. Suppose that $\tilde{R}(K, \bar{a})$ equals $R(K)$ with probability $\mu$, $0 < \mu < 1$, and 0 with probability $1 - \mu$. Let $R^e(K, \bar{a}) = \mu \bar{R}(K)$ denote the expected value of $\tilde{R}(K, \bar{a})$, and suppose that $R^e(K, \bar{a})$ satisfies all the assumptions of Section 2. Since HC can expropriate only the realized return, the taxes imposed in each period will also be a random variable. Consider a stationary implicit contract which says that MNE invests $K$ in period 0 and chooses $a_t = \bar{a}$ in each period $t$, while HC imposes tax $T = \bar{T}$ if $\tilde{R} = \bar{R}(K)$ and $T = 0$ if $\tilde{R} = 0$. Let $T^e = \mu \bar{T}$ denote the expected tax payment.

The participation constraint and the incentive constraint of MNE (inequalities (11) and (13), respectively), are unaffected by the introduction of uncertainty. We just have to replace $R(K, \bar{a})$ and $T$ by their expected values. The incentive constraint of HC (inequality (12)) is affected however. Suppose that the realized return in some period $t$ is 0. In this case it is impossible for HC to expropriate. If $\bar{R}(K)$ has been realized, it does not pay for HC to fully expropriate this return if and only if

$$T + \frac{\delta_2}{1 - \delta_2} T^e \geq \bar{R}(K) + \frac{\delta_2}{1 - \delta_2} R(K, \bar{a})$$

(38)

Note that HC compares a lottery which it receives on the equilibrium path to a deterministic payment if it deviates and fully expropriates MNE. Therefore, if HC is risk-averse, $T$ and thus the expected value of taxes have to be higher in order to induce HC to bear this risk. This effect reinforces the conclusion of Corollary 4 below.

Substituting $\bar{T} = \frac{T^e}{\mu}$ and $\bar{R}(K) = \frac{R^e(K, \bar{a})}{\mu}$, and solving for $T^e$, we get:

$$T^e \geq \frac{1 - \delta_2}{1 - \delta_2 + \delta_2 \mu} R^e(K, \bar{a}) + \frac{\delta_2 \mu}{1 - \delta_2 + \delta_2 \mu} R(K, \bar{a})$$

(39)

Comparing (39) with (15) in Section 3, it is easy to see that $\frac{1 - \delta_2}{1 - \delta_2 + \delta_2 \mu} < 1 - \delta_2$ for all $\mu < 1$. Thus, if we compare a deterministic project with a return $R(K, \bar{a})$ to a stochastic

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9It is easy to see that if $R(K, \bar{a})$ is random this has no effect on strategic behavior of either party because it matters only off the equilibrium path. All relevant decisions have to be taken before $R(K, \bar{a})$ has been realized. Therefore, only its expected value matters.
project with an expected return \( R^e(K, \pi) = R(K, \pi) \), then the stochastic project requires a higher expected tax to be paid to HC in every period. The problem is that HC has to be prevented from expropriating MNE in each state of the world. In order to minimize the expected tax that has to be paid to HC one would like to equalize the gains from expropriating over all states. However, if \( \tilde{R} = 0 \), then the constraint \( T \geq 0 \) becomes binding. Hence it is impossible to fully compensate the tax increase in the good state by a tax reduction in the bad state.\(^{10}\)

**Proposition 5** In the stochastic case there exists an implicit contract which induces MNE to invest \( K \) and to take action \( \pi \) in every period if and only if

\[
K \leq \frac{\delta_1}{1 - \delta_1}\left[r(\bar{\pi}) + \frac{\delta_2\mu}{1 - \delta_2 + \delta_2\mu} (R^e(K, \bar{\pi}) - R(K, \bar{\pi}))\right]
\]

(40)

and

\[
r(\bar{a}) - r(\bar{\pi}) \leq \frac{\delta_2\mu}{1 - \delta_2 + \delta_2\mu} [R^e(K, \bar{\pi}) - R(K, \bar{\pi})].
\]

(41)

Proof: See Appendix.

**Corollary 4** Compare a deterministic project which generates \( R(K, \pi) \) to a stochastic project with \( \mu R(K) = R^e(K, \pi) = R(K, \pi) \). The set of investment levels that can be implemented in the stochastic case is strictly smaller than in the deterministic case. Keeping \( R^e(K, \pi) \) constant, this set is strictly decreasing with the riskiness of the project, i.e. with the parameter \( \mu \). For all \( K > 0 \) there exists a \( \mu(K) > 0 \) such that for all \( \mu < \mu(K) \) the investment level \( K \) cannot be sustained.

Proof: See Appendix.

If HC could subsidize MNE in the bad state of the world, then the expected amount of taxes to be paid could be reduced, possibly up to the amount in the deterministic case.

\(^{10}\)This argument carries over immediately to the case with more than two possible outcomes as long as there is at least one state of the world with a return sufficiently small for the constraint \( T \geq 0 \) to become binding.
This would be beneficial to both parties if it allows to sustain a more efficient investment level. Note also that the constraint $T \geq 0$ is due to the fact that HC has no funds available and is credit constraint on international capital markets, but has nothing to do with HC’s participation constraint.

5 Conclusions

The preceding sections analyzed the scope of implicit contracts to sustain foreign direct investment in the presence of sovereign risk in a dynamic framework. The implicit contract has to prevent the host country from engaging in “creeping expropriation”, i.e., from imposing specific taxes, tariffs or other charges on the foreign investor in order to capture the quasi-rents of his project. The results are consistent with our results in Schnitzer (1995), where we analyzed the choice between FDI and a combination of debt finance and a licensing agreement in a static model. However, the dynamic model considered here allows for a deeper understanding of the strategic interaction between HC and MNE, and it offers several additional predictions. Let us briefly summarize the main insights from this model and compare them to our companion paper.

In a repeated relationship the hold-up problem associated with foreign direct investment may be mitigated by an implicit or self-enforcing contract which is sustained by MNE’s threat to withdraw from HC if taxes become too high. This threat gives some bargaining power to MNE which can be used to get a positive share of the quasi-rents of the project in order to recover the initial investment cost. Similarly, in the static model of Schnitzer (1995) we argued that MNE will get some share of the quasi-rents of the project because it has private information about the value of its outside option. Both models suggest that FDI is viable only if the threat to withdraw from the host country is credible, which is more likely to be the case if production is rather high tech and destined for export markets.

The dynamic model considered here offers several additional insights. First, it shows that MNE’s bargaining position must not be too strong. If MNE has too much flexibility in shifting production abroad, it cannot commit to give a sufficient share of future
revenues to HC which in turn induces HC to fully expropriate all of the returns right away.\footnote{In Schnitzer (1995) there is a similar effect. If HC does not expect to receive sufficiently high taxes, then it will choose to nationalize the project. Note that in the model considered here there is no possibility to nationalize the project. Instead, HC can tax away the entire return stream which would have never been optimal in our previous paper.} Therefore, MNE may want to choose a technology which implies a rather strong commitment to produce in the host country.

Second, even if it is possible to sustain some level of investment, it may be that the efficient investment level $K_{FB}$ cannot be sustained. There may be underinvestment ($K < K_{FB}$) if HC cannot be prevented from expropriating too much of the return stream. This will be the case if HC’s bargaining position is very strong, for example, because shifting production out of HC is very costly, or if HC’s discount factor is rather small. Hence, this problem will be particularly severe if production is destined for local markets, or if HC has to pay a very high risk premium on international capital markets.

The underinvestment problem becomes more severe if investment and production in the host country have significant positive spillover effects. Since the foreign investor will not take these external effects into account when he makes his investment decision the government would like to subsidize a higher level of investment. However, if HC is cash and credit constrained at the time of the investment, it may be impossible to commit to tax reductions or subsidies in the future in order to sustain the socially efficient level of production.

Perhaps more surprisingly, there may also be overinvestment ($K > K_{FB}$). This is going to happen if investing too much is the only possibility to commit to giving a sufficiently large share of the return stream to HC in order to prevent it from fully expropriating MNE. Thus, this problem is likely to occur if MNE has a lot of flexibility in shifting production abroad and thus a too strong bargaining position vis à vis HC.

Third, if HC can credibly commit to a tax policy for a (possibly random) number of periods, it may want to commit to no taxes in the beginning of the project in order to make a more efficient investment level sustainable. This offers an explanation for the widely observed phenomenon of tax holidays.
Finally, stochastic returns affect the strategic interaction of both parties. If MNE’s outside option is a random variable, and if this outside option may be so attractive in some periods that efficiency requires to shift production abroad, then HC is more tempted to fully expropriate the returns in a period where MNE produced in the host country. However, the efficiency gain which can be realized from choosing the outside option in some periods is always sufficient to compensate HC and to prevent it from expropriating. Hence, a stochastic outside option unambiguously improves the set of implementable allocations. This result suggests that FDI in less developed countries is particularly interesting for multinationals which produce the same component of a final product in several less developed countries and which can allocate production according to the comparative cost advantages and exchange rate developments in the different countries.

On the other hand, if the revenues generated in the host country are stochastic, the set of implementable allocations shrinks because HC has to be given more taxes on average in order to prevent expropriation. This is more likely to be a problem if production is for domestic markets and if the local environment is very unstable.

These predictions seem to be consistent with the little empirical evidence available on FDI in less developed countries. However, it would be very desirable to have additional and more disaggregated data in order to put the hypotheses derived in this paper to a more specific test.
6 Appendix

Proof of Proposition 1: We know already that (14) - (16) are necessary conditions for the implicit contract to form a subgame perfect equilibrium. Given that the prescribed behavior off the equilibrium path forms a subgame perfect equilibrium these conditions are also sufficient. Hence, if a transfer $T$ can be found which satisfies (14) - (16), then there exists an implicit contract which implements the desired behaviour. Such a $T$ exists, if the LHS of (15) is smaller or equal to the RHS of (14), i.e., if

$$(1 - \delta_2)R(K, \bar{\pi}) + \delta_2R(K, a) \leq R(K, \bar{\pi}) + r(\bar{\pi}) - \frac{1 - \delta_1}{\delta_1} K,$$

which is equivalent to (17), and if the LHS of (15) is smaller or equal to the RHS of (16), i.e., if

$$(1 - \delta_2)R(K, \bar{\pi}) + \delta_2R(K, a) \leq R(K, \bar{\pi}) + r(\bar{\pi}) - r(a),$$

which is equivalent to (18). Q.E.D.

Proof of Proposition 2: Condition (17) of Proposition 1 requires that

$$F(K) = \frac{\delta_1}{1 - \delta_1} [r(\bar{\pi}) + \delta_2 (R(K, \bar{\pi}) - R(K, a))] - K \geq 0.$$  

Clearly, $F(0) > 0$. Furthermore, given Assumption 1, $F(K)$ is continuous, concave and tends to $-\infty$ as $K \to \infty$. Hence, by the intermediate value theorem there exists a unique upper bound $\bar{K}$ on the sustainable investment levels satisfying $F(\bar{K}) = 0$.

Condition (18) of Proposition 1 requires that

$$G(K) = \delta_2 [R(\bar{K}, \bar{\pi}) - R(K, a)] - r(a) + r(\bar{\pi}) \geq 0.$$  

Note that $G(0) < 0$ and that $G(K)$ is continuous and monotonically increasing by Assumption 1. However, $R(K, \bar{\pi}) - R(K, a)$ is bounded above as $K \to \infty$. Hence, it is possible that there does not exist a $K$ satisfying this condition. But if the set of $K$ satisfying (45) is non-empty, then there exists a unique lower bound $\underline{K} > 0$ such that $G(\underline{K}) = 0$. Q.E.D.

Proof of Proposition 3: The proof is analogous to the proof of Proposition 1 with (14) replaced by (28). Q.E.D.
Proof of Proposition 4: Solving inequalities (33) - (35) for $T$, we get:

$$T \leq R(K, \overline{a}) + r(\overline{a}) + \frac{1 - \mu}{\mu} \bar{\tau} - \frac{1 - \delta_1}{\delta_1 \mu} K$$  
(46)

$$T \leq R(K, \overline{a}) + r(\overline{a}) - \frac{1 - \delta_2}{1 - \delta_2 + \delta_2 \mu} R(K, \overline{a})$$  
(48)

It is straightforward to show that a tax $T$ satisfying these constraints can be found if and only if (36) and (37) hold.

It remains to be shown that the constraints on $K$ are relaxed as compared to the case of Proposition 1. First, we want to show that the RHS of (36) is strictly larger than the RHS of (17), i.e.,

$$\frac{\delta_1}{1 - \delta_1} [r(\overline{a}) + \delta_2 R(K, \overline{a})] < \frac{\delta_1}{1 - \delta_1} \left[ \frac{\delta_2 \mu^2}{1 - \delta_2 + \delta_2 \mu} R(K, \overline{a}) + \mu r(\overline{a}) + (1 - \mu) \bar{\tau} \right]$$  
(49)

By assumption $\bar{\tau} \geq R(K, \overline{a}) + r(\overline{a})$. Therefore, inequality (49) holds if

$$r(\overline{a}) + \delta_2 R(K, \overline{a}) < \frac{\delta_2 \mu^2}{1 - \delta_2 + \delta_2 \mu} R(K, \overline{a}) + \mu r(\overline{a}) + (1 - \mu) [R(K, \overline{a}) + r(\overline{a})] ,$$  
(50)

which is equivalent to $0 < (1 - \delta_2)^2$, which is clearly true.

Finally, we want to compare (18) to (37). Given that $r(a) = \mu \bar{\tau} + (1 - \mu) \bar{\tau}$ (18) can be rewritten as

$$\bar{\tau} \leq \frac{\delta_2}{\mu} R(K, \overline{a}) + \frac{r(\overline{a})}{\mu} - \frac{1 - \mu}{\mu} \bar{\tau} .$$  
(51)

It has to be shown that the RHS of (51) is strictly smaller than the RHS of (37), i.e.,

$$\frac{\delta_2}{\mu} R(K, \overline{a}) + \frac{r(\overline{a})}{\mu} - \frac{1 - \mu}{\mu} \bar{\tau} < \frac{\delta_2 \mu}{1 - \delta_2 + \delta_2 \mu} R(K, \overline{a}) + r(\overline{a}) .$$  
(52)

Since by assumption $\bar{\tau} \geq R(K, \overline{a}) + r(\overline{a})$, inequality (52) is satisfied if

$$\frac{\delta_2}{\mu} R(K, \overline{a}) + \frac{r(\overline{a})}{\mu} - \frac{1 - \mu}{\mu} [R(K, \overline{a}) + r(\overline{a})] < \frac{\delta_2 \mu}{1 - \delta_2 + \delta_2 \mu} R(K, \overline{a}) + r(\overline{a}) .$$  
(53)

Again, it can be shown that this inequality is equivalent to $(1 - \delta_2)^2 > 0$. Q.E.D.

Proof of Proposition 5: The proof follows the lines of the proof of Proposition 1, replacing (15) by (39). Q.E.D.
Proof of Corollary 4: Proposition 2 differs from Proposition 1 only by the term $\frac{\mu}{1-\delta_2+\delta_2\mu} < 1$ on the RHS of inequalities (40) and (41). Hence, the set of $K$ satisfying these conditions is reduced. Furthermore, $\frac{\mu}{1-\delta_2+\delta_2\mu}$ is strictly decreasing with $\mu$ and so is the set of investment levels satisfying (40) and (41). Finally, $\lim_{\mu \to 0} \frac{\mu}{1-\delta_2+\delta_2\mu} = 0$. Since $r(\underline{q}) - r(\pi) > 0$, condition (41) must be violated for any $K$ if $\mu$ is sufficiently close to 0. \[ Q.E.D. \]
References


