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How to deal with unobservable variables in economics

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Abstract

The paper discusses different methods to deal with unobservable variables: Kalman-Filtering, principal components, factor analysis, LISREL, MIMIC, DYMIMIC, PLS with respect to parameter estimation and forecasting. We got very good results by an extension of Kalman-Filtering called AS (general stationary parameter model). LISREL proved to be superior to PLS in parameter estimation. Explicit introduction of the latent variables "mood" of the economic agents, the "political trend" and "social stability" improved the forecasting performance of an econometric model of the FRG.

How to deal with unobservable variables in economics

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1 Introduction

Unobservable variables are quite common in economies: expectations, beliefs, spirits, degrees of risk aversion, information, entrepreneurship and others directly non-measurable concepts play an important role in determining the decisions of economic agents. There are different methods to deal with this problem:

1. One could disregard it, consider only measurables variables (like prices, production, income) and leave the non measurable ones behind the scene. They are implicitly taken into account by their effects on the measurable variables. This may lead to systematic errors and higher variances of forecasts, because changes in the "unobservable" determinants on the observable ones are disregarded. In the case of expectations this is often rationalized by assuming rational expectations which reduce the expectations to the measurable aims of the actions. But there is now a new school in economics which questions the rationality of human actions. This destroys the base for rational expectations.

2. A second approach would be to take these influences into account by letting the parameters of the system of observable variables also become variables and estimating their dependence on other variables and on time simultaneously. This can be done by variants of Kalman-Filtering.

3. The third approach is to try to estimate the latent variables by some indicators. The methods of principal components or factor analysis are well known in statistics and often used in psychology or in sociology, but seldom in economics. Here two new methods have been developed recently: the PLS-method of Herman Wold and the LISREL-procedure of Jöreskog and others and the MIMIC- and DYMIMIC-procedure of Aigner and others.

At the Special Research Unit (Sonderforschungsbereich) 303 at Bonn University there was a research unit (Teilprojekt) which tried to compare these approaches from the practical point of view. Hans Schneeweiß was interested in this projects. He looked at it from the statistical point of view. He worked out sufficient conditions that the two estimation methods approach each other (in general they lead to different parameter values): Hans Schneewiß suggested two different interpretations of the PLS-method: it may be thought of as *defining* consistently parameters and latent variables or as *estimating* parameters and latent variables given from outside.

Hans Schneeweiß accepts the first interpretation, see Schneeweiß 1990b, p. 38. We look into the problem more from the second point of view. Hans Schneeweiß' papers (and letters to me) have deeply influenced the research presented here. The results have been partly published at different places, partly they are only available as research reports. Thus it may be a suitable present to my dear friend and colleague Hans Schneeweiß at the occasion of his 65^{th} anniversary to report on this research which is terminated now. I take this also as an opportunity to thank my former collaborators Alfons Kirchen, Gábor Körösi and Kalmán Féhér, who did the practical work from which I quote those parts which are relevant in this context.

2 Kalman-Filtering

We start with the second approach, the Kalman-Filtering. The Kalman-Filtering is a recursive procedure which allows to estimate time dependent structural parameters of linear systems. We explain it for the simple case of a single regression equation:

$$y_t = x_t' \cdot \beta_t + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma^2) \tag{1}$$

where y_t is the dependent variable to be explained, $x_t' = (x_{1t}, \dots, x_{nt})$ the vector of the explaining variables and $\beta_t' = (\beta_{1t}, \dots, \beta_{nt})$ the vector of the time dependent structural parameters. ϵ_t is a random term, normally distributed with expectation 0 and variance σ^2. There are observations on y_t and $x_{1t}, \dots, x_{nt}$. In this form the model (1) cannot be estimated. We need informations on the "hyperstructure" of the model, i. e. informations on the chance process which determines β_t. There are different possibilities; the following three approaches may be found in the literature:

1. the "random walk" model: $\beta_t = \beta_{t-1} + \eta_t$ where η_t is a chance vector with covariance matrix Q
2. the "random coefficient" model: $\beta_t = \bar{\beta} + \eta_t$
3. the "return to normal" model: $\beta_t = \bar{\beta} + A(\beta_t - \bar{\beta}) + \eta_t$, where A is a matrix and η_t a chance vector.

We used a model which comprised these models and others. Details may be found in this dissertation of the scholar in charge of his work, *Alfons Kirchen*. The approach and the numerical results are due to him.

The “hyperstructure” we used was:

$$y_t = x_t'\beta_t + \epsilon_t$$

$$C\alpha_t = \beta_t - \bar{\beta}$$

$$\alpha_t = A\alpha_{t-1} + D\eta_t,$$

where C is a selection matrix (consisting of zeros and ones), A a transition matrix with a spectral radius < 1, D an innovation matrix, ϵ_t also “white noise”. α_t may be interpreted as “state of the environment”. This approach is called “general stationary parameter model” (Allgemeines stationäres Parametermodell, AS). It seems to be intuitively acceptable, but at first sight, it seems to complicate the whole thing: instead of β_t alone one has to estimate $\beta_t, \bar{\beta}, \sigma^2, Q, A, C, D$. All elements of these vectors and matrices have to be estimated. This is made possible by a two-stage procedure due to *Akaike*. He showed that a k-dimensional stochastic process

$$z_t = \sum_{m=0}^{\infty} W_m \cdot \eta_{t-m}$$

where η_t is a chance vector, is equivalent to a state variable model

$$z_t = C\alpha_t, \quad \alpha_t = A_{t-1} + D\eta_t$$

if the relation $W_m = CA^mD$ holds. The algorithm of Akaike allows to infer from observations $z_t, z_{t-1}, \ldots$ the elements of Q, C, A, D. The two- stage procedure runs like this. First the “hyperstructure” σ^2, Q, A, C, D is predetermined. Then it is possible to estimate β_t and $\bar{\beta}$ by Kalman-Filtering. With given β_t and $\bar{\beta}$ the Akaike-algorithm allows to estimate Q, C, A, D. By iteration one gets consistent solutions for all parameters. Details may be seen in *Kirchen*.

This algorithm has been examined at several constructed examples and gave very good results. Afterwards a medium sized quarterly ecomometric model of the german economy (79 equations) has been used and the parameters estimated by the usual least square method, by Kalman-Filtering under the assumption of a random walk hyperstructure, then on the assumption of a return to normal hyperstructure and finally under the assumption of a general stationary parameter model (AS) as explained above. In order to compare the performance of the different models, all parameters have been estimated using the reference period 1962.1 to 1981.4 whereas the information of the following three years (1982.1 to 1984.4) was already available. The performance has been judged by the precision of the forecast compared to the precision of the forecast by using the usual OLS procedure of parameter estimation. It turned out that no improvement could be made by using the normal Kalman-Filtering with random walk and return to normal hyperstructure. This may explain why Kalman filtering is scarcely used in econometrics. But with parameters estimated by the AS hyperstructure a substantial improvement of the forecasting performance of the model could be reached.

Table 1: The root mean square error of the ex ante forecasts for 12 quarters by the same econometric model for the Federal Republic of Germany where the parameters have been estimated by OLS or by AS

Variable	$C'P$	$I'P$	M	X	P	UE	$Y'L$	$Y'PP$
Model OLS	7.802	3.554	5.178	7.521	0.023	0.346	19.337	13.427
Model AS	5.512	2.927	3.456	4.552	0.015	0.209	18.412	12.053

Notations:
$C'P$ = real private consumption
$I'P$ = real investments in installments
M = real imports
X = real exports
P = price index of GDP
UE = unemployed persons
$Y'L$ = available wage and transfer income
$Y'PP$ = gross entrepreneureal and property income (Brutto-einkommen aus Unternehmertätigkeit und Vermögen)

Source: Krelle, 1987b, p. 7

Table 1 shows the results in terms of the root mean square error (RMSE). Figures 1-4 illustrate some of the results. As one can see, substantial improvements can be reached by estimating the parameters of an econometric forecasting model by Kalman-Filtering with the AS-approach, but not with other Kalman-Filtering approaches which are familiar from the literature. But, of course, the latent variables stay behind the scene. The forces which induce the changes of the parameters cannot be identified that way. Thus it seems advisable to use this Kalman-Filtering approach only for short term forecasts where one could assume that the unknown forces which are responsible for the changes of the parameters stay more or less the same.

3 The measurement of one latent variable by the method of principal components

Assume that there are a number of measurable variables $y_1, \ldots, y_n$ (called: factors), but for one reason or another we do not want or do not find it possible to work with so many variables but want to represent these variables by one variables η which represents the time shape of all the variables as good as possible. We also could say: we want to identify a latent variable η which exerts a common influence on all the factors $y_1, \ldots, y_n$, but there are many other influences on $y_1, \ldots, y_n$, which could be treated as white noise. After transforming the variables $y_1, \ldots, y_n$ to zero mean and unit variance we would have to estimate parameters $a_1, \ldots, a_n$ (called: factor loadings) by

$$y_i = a_i \eta + \nu_i, \qquad i = 1, \ldots, n \tag{1a}$$

or to estimate $\alpha_1, \dots, \alpha_n$ (called factor weights) such that

$$\eta = \alpha_1 x_1 + \cdots + \alpha_n x_n + \eta \tag{1b}$$

where ν_i and η are white noise stochastic termes with zero mean. The estimations methods are well known (see e. g. Dhrymes, p. 53ff.).

We used this method to identify a latent variable "entrepreneurship" (u) by three indicators: the rate of technical progress (w_τ), the saving ratio (s) and the time discount rate approximated by the negative of the yield on government bonds ($-\rho$): the lower this yield, the shorter the time horizon. *Table 2* shows the results of these estimations for different countries.

Table 2: Factor Weights and Factor Loadings

variable		rate of technical progress w_t factor		savings rate s factor		bond yield ($\approx$ time discount rate) ϱ factor		relative variance
		loading	weight	loading	weight	loading	weight	
country		a_1	α_1	a_2	α_2	a_3	α_3	
1	USA	.474	.883	.322	.600	−.457	−.851	.621
2	FRG	.441	.893	.348	.705	−.422	−.855	.675
3	Japan	.030	.048	.554	.886	−.564	−.902	.533
4	France	.555	.901	–	–	−.555	−.901	.811
5	UK	.592	.845	.592	.845	–	–	.714
6	Italy	.351	.809	.413	.951	−.375	−.863	.768
7	NL	.532	.961	.502	.905	−.138	−.250	.601
8	B / L	.638	.905	.469	.665	−.279	−.396	.473
9	Canada	.536	.934	–	–	−.536	−.934	.872

Source: Krelle, 1987a, p. 392

The factor weights and loadings are very similar for the USA and Germany, but rather different to other countries. The latent variable u showed long-term fluctuations which may explain the long-term Kondratieff cycles. Unfortunately, only one wave may be observed so that inferences on existence and frequency of this waves are rather heroic. Details may be seen in the article mentioned at *Table 2*.

4 Systems of latent and observable variables: an overview

It is reasonable to start with the most general linear model of this kind, the so called LISREL-model (Linear Structural Relations Model) of Jöreskog and Sörbom (1978). This model distinguishes between endogenous latent variables η with indicators y and exogenous latent variables ξ with indicators x. All manifest variables are calibrated to zero mean so that no absolute term is necessary in the following *LISREL-model*:

$$\text{The set of structural relations} \quad \eta = A\eta + B\xi + \epsilon \tag{2a}$$

$$\text{endogenous measurement relations} \quad y = C\eta + \nu \tag{2b}$$

$$\text{exogenous measurement relations} \quad x = D\xi + \omega \tag{2c}$$

where small letters indicate vectors, capital letters matrices, ϵ, ν, ω are IID disturbance terms, and $[I - A] \neq 0$.

In this general form the parameters of the matrices A, B, C, D cannot be identified. Some restrictions must be imposed to guarantee identifiability. They may be derived from the covariance matrix of the manifest variables (x, y).[1].

Special cases of the LISREL-model are

1. The MIMIC–Model (= Multiple Indicator Multiple Cause–Model):

$$\eta = b'\xi + \epsilon, \qquad \eta \text{ a scalar}, \quad \xi \text{ a vector}$$

$$y_i = c_i\eta + \nu_i, \qquad \eta \text{ a scalar}, \quad i = 1, \dots, n$$

$$x = \xi$$

2. The error in the variables–model. Here the matrices C and D in $(2b)$ and $(2c)$ are unity matrices.

3. The principal component model: Here only the part $y_i = c_i\eta + \nu_i$ is retained.

But there are also extensions of the LISREL model in order to include dynamic features in the model. I only mention the so called DYMIMIC model of Watson and Engle (1983) and Engle and Watson (1985):

$$\eta = E\eta_{-1} + Ax + \epsilon$$

$$y = B\eta + Fx + \nu$$

For other extensions: see Bánkövi and others (1979), (1986) or Geweke and others (1977), (1981).

Herman Wold takes a different approach with his *PLS-Method* (=partical least squares). Whereas the LISREL system is estimated by the maximum likelihood method so that the statistical meaning of the estimation process and the statistical properties of the parameters are known, the PLS-Method starts with a different system and uses a two stage estimation process. The model is supposed to be strictly recursive. There is no

[1]For details, see Schneeweiß, 1984, p. 6ff

formal difference between endogenous and exogenous variables, but the model may contain "mode A"- (or: outwards directed) latent variables, or "mode B"- (inwards directed) latent variables or both. "Outwards" directed means that the latent variable influences the indicator variables, and a "inwards" directed means the inverse. Formally the PLS-System may be written:

$$\eta = A\eta + \epsilon \tag{3a}$$

$$y = C\eta + \nu \tag{3b}$$

$$\eta = Gy + \mu \tag{3c}$$

A is a triangular matrix, C and G are block diagonal matrices. (3a) and (3b) are conform to the LISREL model, but (3c) is new: the latent variables are explained as a weighted average of their indicators.

Given this situation we tried to find out whether LISREL or PLS is preferably to be used given a model where both methods are applicable. Afterwards we introduced three latent variables (each of which are estimated by three indicators) into the quarterly econometric model for Germany which was also used for the Kalman-Filtering estimation of the parameters, see section 2 above. All these estimations have been carried out within the Special Research Unit (Sonderforschungsbereich) 303 in one of the special projects (Teilprojekt) under my responsibility by *Gábor Körösi* and *Kálmán Féhér*. The results are documented in research reports from which I present the main results in the following two sections.

5 Comparison of PLS and LISREL

Hans Schneeweiß compared the two methods in his paper "Modelle mit latenten Variablen" of 1984. He found that in Herman Wolds approach parameters are not identifiable and nevertheless Herman Wolds solution algorithm PLS yields – if it converges – definite figures for all parameters. How is that possible? Hans Schneeweiß suggests that the estimation procedure itself defines the values of the parameters, and that seems to be right. But which properties have the estimated parameters? Do they keep any relations to the "true" parameters? This question seems to be difficult to answer on the theoretical base. Therefore we decided to use the Monte-Carlo method to find out the relation empirically. We used different models, but the results are very similar in all cases. Thus I present here only some results for model 5b[2]:

$$y_i = c_{i1}\eta_1 + \nu_i, \qquad i = 1, \dots, 4 \qquad y_j = c_{j2}\eta_2 + \nu_j, \qquad j = 5, \dots, 8$$

[2]The results for all other models are to be found in the research reports of Körösi and Féhér, from which these and the following figures are taken.

and the correlation coefficient $\rho(\eta_1, \eta_2) = a_{12} = 0.3$, which means $\eta_1 = 0.3\,\eta_2 + \epsilon$. The error terms ν_i, ν_j and ϵ are taken from a normal distribution. *Tables 3 and 4* show some of the results.

The LISREL figures for this model are available from the literature, see Boomsma (1982) and reproduced here. It is apperent that LISREL reproduces the exact figures if the sample size is large enough. In Féhér (1989) also the figures for a sample size of 1000 are available, but the estimated average of the PLS-parameters does not come nearer to the true values. We do not reproduce these results here. The PLS-process converges rather soon also if the sample size is small. There are no computational problems. The

Table 3: Comparison of PLS and LISREL Estimates, model 5b

Sample size: 25, iteration: average = 7.3, maximum = 18

loading	value	average	st.dev.	RMSE	skewness	kurtosis	LISREL[a]
$c_{1,1}$	0.6	0.371	0.566	0.610	−1.330	3.891	0.58
$c_{2,1}$	0.6	0.571	0.434	0.435	−1.390	3.341	0.59
$c_{3,1}$	0.8	0.539	0.502	0.566	−1.783	4.714	0.80
$c_{4,1}$	0.8	0.610	0.459	0.497	−1.976	5.329	0.80
$c_{5,2}$	0.6	0.531	0.189	0.201	0.520	2.820	0.57
$c_{6,2}$	0.6	0.609	0.317	0.318	−1.589	4.462	0.60
$c_{7,2}$	0.8	0.656	0.280	0.315	−1.062	3.233	0.80
$c_{8,2}$	0.8	0.788	0.294	0.294	−2.154	5.893	0.80
$a_{1,2}$	0.3	0.277	0.370	0.371	−0.928	2.190	0.28

Percentage of LISREL iterations not converging in 250 steps: 2%

Sample size: 50, iteration: average = 6.1, maximum = 10

loading	value	average	st.dev.	RMSE	skewness	kurtosis	LISREL[a]
$c_{1,1}$	0.6	0.674	0.170	0.185	−0.988	2.883	0.60
$c_{2,1}$	0.6	0.704	0.127	0.164	−2.012	16.750	0.59
$c_{3,1}$	0.8	0.873	0.052	0.089	−3.650	39.648	0.79
$c_{4,1}$	0.8	0.819	0.066	0.069	−0.362	3.315	0.80
$c_{5,2}$	0.6	0.743	0.126	0.191	−0.062	2.040	0.58
$c_{6,2}$	0.6	0.663	0.029	0.069	0.640	3.725	0.58
$c_{7,2}$	0.8	0.854	0.038	0.066	0.428	1.794	0.79
$c_{8,2}$	0.8	0.807	0.095	0.095	−0.622	2.248	0.78
$a_{1,2}$	0.3	0.288	0.055	0.056	−7.685	100.209	0.29

Sample size: 100, iteration: average = 4.0, maximum = 5

loading	value	average	st.dev.	RMSE	skewness	kurtosis	χ^2	P-val(%)	LISREL[a]
$c_{1,1}$	0.6	0.708	0.060	0.124	−0.603	2.018	546.0	100.0	0.59
$c_{2,1}$	0.6	0.725	0.077	0.147	−0.684	2.125	630.3	100.0	0.59
$c_{3,1}$	0.8	0.829	0.020	0.035	−0.010	2.487	437.0	100.0	0.80
$c_{4,1}$	0.8	0.829	0.024	0.038	1.035	2.468	843.0	100.0	0.81
$c_{5,2}$	0.6	0.662	0.082	0.103	−0.997	2.326	1307.0	100.0	0.60
$c_{6,2}$	0.6	0.697	0.064	0.116	−0.341	2.712	651.7	100.0	0.60
$c_{7,2}$	0.8	0.866	0.031	0.073	0.338	2.598	731.2	100.0	0.80
$c_{8,2}$	0.8	0.851	0.028	0.058	−0.332	1.453	800.1	100.0	0.80
$a_{1,2}$	0.3	0.336	0.039	0.053	0.181	2.014	798.4	100.0	0.30

[a] Taken from *Boomsma* [1982], Model 44

Source: Körösi (1989), p. A11-12

Table 4: Comparison of PLS and LISREL Estimates, model 5b

Sample size: 200, iteration: average = 3.3, maximum = 4

loading	value	average	st.dev.	RMSE	skewness	kurtosis	χ^2	P-val(%)	LISREL[a]
$c_{1,1}$	0.6	0.710	0.019	0.111	−0.319	1.416	548.9	100.0	0.60
$c_{2,1}$	0.6	0.697	0.038	0.104	0.035	1.419	557.9	100.0	0.60
$c_{3,1}$	0.8	0.851	0.017	0.053	−0.373	1.464	586.7	100.0	0.80
$c_{4,1}$	0.8	0.869	0.007	0.070	−0.331	1.460	373.3	99.8	0.80
$c_{5,2}$	0.6	0.731	0.029	0.134	−0.214	1.374	594.0	100.0	0.60
$c_{6,2}$	0.6	0.708	0.034	0.113	0.283	2.244	554.1	100.0	0.59
$c_{7,2}$	0.8	0.854	0.010	0.055	−0.350	2.126	424.8	100.0	0.80
$c_{8,2}$	0.8	0.871	0.013	0.072	−0.071	1.342	396.1	100.0	0.80
$a_{1,2}$	0.3	0.340	0.034	0.052	0.168	1.612	672.3	100.0	0.31

Sample size: 400, iteration: average = 3.0, maximum = 3

loading	value	average	st.dev.	RMSE	skewness	kurtosis	χ^2	P-val(%)	LISREL[a]
$c_{1,1}$	0.6	0.706	0.016	0.107	−0.112	1.839	344.4	97.0	0.59
$c_{2,1}$	0.6	0.735	0.015	0.136	−0.429	1.357	492.3	100.0	0.60
$c_{3,1}$	0.8	0.859	0.008	0.060	0.209	1.343	317.0	79.7	0.80
$c_{4,1}$	0.8	0.860	0.007	0.060	0.216	1.207	473.7	100.0	0.80
$c_{5,2}$	0.6	0.716	0.009	0.116	−0.795	2.853	581.3	100.0	0.60
$c_{6,2}$	0.6	0.761	0.006	0.161	−0.092	2.371	444.2	100.0	0.60
$c_{7,2}$	0.8	0.860	0.005	0.060	−0.215	3.402	533.8	100.0	0.80
$c_{8,2}$	0.8	0.849	0.006	0.050	−0.213	1.341	438.8	100.0	0.80
$a_{1,2}$	0.3	0.320	0.017	0.026	0.136	1.497	480.2	100.0	0.30

[a] Taken from *Boomsma* [1982], Model 44

Source: Körösi (1989), p. A11-12

LISREL procedure may not converge if the sample size is too small (that means: under 100), but it performs well for larger sample sizes.

The distributions of the PLS-estimates are often bi- or multi-modal. Details may be found in the paper of Fëhér, 1989, Appendix C1–C14.

I think these results show clearly that one should use the LISREL approach whenever the model conforms to the basic assumptions of the LISREL procedure. PLS could possibly be used as a first approximation and for forecasts. We did not check the forecasting behavior of a system with PLS parameters. If this is acceptable the PLS procedure may nevertheless have its place since it is simple, independent of the characters of the error terms and of the structure of the covariance matrix and of identifiability restrictions. Thus it may be worthwhile to look into the forecasting performance of systems with PLS parameters.

6 Latent variables in an econometric forecasting system

The climax of our research on latent variables should be the introduction of these variables into an econometric forecasting system. There is a permanent interaction between "the economy" represented by an econometric forecasting system and its socio-political environment. If we could model it by introducing latent variables we could hopefully improve the performance of the model. We introduced three latent variables:

ECON a variable which should represent the "mood" of the economic agents

POLIT a variable which should represent the general political trend (more right wing or more left wing)

INSEC a variable to show the degree of social stability or social tensions

INSEC is influenced by per capita GNP, by the GDP deflator and by the consumer price index. POLIT is explained by the growth rate of GNP, by the unemployment rate and by the ratio of government consumption to GNP. ECON is caused by the consumer price index, the per capita GNP and the rate of private consumption to GNP. These equations (called predictor equations) are necessary to predict the latent variables in LISREL.

The manifest indicators for ECON are: the stock exchange index, the percentage of persons having pessimistic resp. optimistic expectations for the future (according to the polls of the Allensbach Institute) and the reciprocal of the average yield on securities.

The three indicators for POLIT are: the popularity of the three main political parties in the past (CDU, SPD, FDP) measured by the monthly opinion polls of the Allensbach Institute.

For INSEC we used as indicators: the number of criminal acts per 1000

persons, murder attempts and murder and the number of insolvent firms. INSEC influences the two income equations, POLIT the two consumption equations, ECON the two investment equations in our model.

The underlying econometric forecasting system was the same Bonn quarterly model as used for Kalman-Filtering (see section 2). The model has 34 stochastic equations and 45 identities. Six equations were modified in order to introduce the latent variables: those which explain private and government consumption, the wage rate, transfer payments to wage earnes, corporate investments in equipment and in building. Since LISREL is based on the covariance matrix, no constant term can be estimated. Thus some equation had to be changed. The parameters of this system are estimated by OLS, by SUR (Seemingly Unrelated Regressions), LISREL 1 and LISREL 2 (restricted or unrestricted estimates) and PLS.

From the results only the estimation for the two investment functions are reproduced, see *Table 5*. The f_{ij} are the parameters of the explaining variables in the investment functions which are not reproduced here. The results for the other equations are similar.

The parameter values estimated by different methods are rather similar with the exception of the PLS parameters. These are often very different.

The improvement by the introduction of the latent variables was rather small with respect to the fit of the equations as well as with respect to the result of forecasts. Perhaps

Table 5: Parameter estimates of investment functions

Eq.	par.	OLS	SUR	LISREL1	LISREL2	PLS
I'PDE	$f_{5,14}$	0.8064	0.7926	0.790	0.841	0.781
Corporate		(15.063)	(15.645)	(15.5)	(16.0)	—
investment	$f_{5,15}$	0.09878	0.09999	0.100	0.081	0.115
in		(6.4694)	(6.8979)	(6.9)	(5.4)	—
equipment	$f_{5,16}$	−0.3670	−0.3669	−0.367	−0.358	−0.082
		(−4.5131)	(−4.7650)	(−4.7)	(−4.8)	—
	$f_{5,17}$	−1.5793	−1.4854	−1.464	−1.259	−0.058
		(−3.7175)	(−3.7053)	(−3.6)	(−2.9)	—
	$f_{5,18}$	0.0293	0.0328	0.033	0.023	0.067
		(1.9551)	(2.3096)	(2.3)	(1.6)	—
	$f_{5,24}$	−4.2628	−4.2612	−4.261	−4.312	−0.380
		(−16.810)	(−17.713)	(−17.6)	(−18.5)	—
	$f_{5,25}$	−2.2333	−2.2318	−2.231	−2.281	−0.200
		(−8.8269)	(−9.2985)	(−9.2)	(−9.8)	—
	$f_{5,26}$	−3.4291	−3.4285	−3.428	−3.457	−0.304
		(−13.586)	(−14.319)	(−14.2)	(−14.9)	—
	$\gamma_{5,3}$	—	—	—	0.061	0.043
					(1.3)	—
	SEE	0.797	0.757	0.762	0.748	—
	R^2	0.976	0.976	0.976	0.977	0.977
	DW	1.960	1.952	—	—	—
I'PDF	$f_{6,19}$	0.8374	0.8410	0.842	0.865	0.656
Corporate		(15.083)	(15.997)	(15.9)	(14.3)	—
investment	$f_{6,20}$	0.0581	0.0584	0.058	0.052	0.216
in		(7.5899)	(8.0392)	(8.0)	(6.9)	—
buildings	$f_{6,21}$	−0.0575	−0.0581	−0.058	−0.069	−0.068
		(−2.1249)	(−2.2671)	(−2.3)	(−2.7)	—
	$f_{6,22}$	0.1965	0.1895	0.188	0.154	0.109
		(2.8519)	(2.9047)	(2.9)	(2.0)	—
	$f_{6,23}$	0.4309	0.4555	0.461	0.489	0.103
		(3.4112)	(3.8245)	(3.9)	(4.1)	—
	$f_{6,24}$	−1.9972	−1.9987	−1.999	−2.012	−0.542
		(−15.526)	(−16.379)	(−16.3)	(−16.7)	—
	$f_{6,25}$	0.1167	0.1166	0.117	0.103	0.031
		(0.9092)	(0.9575)	(1.0)	(0.9)	—
	$f_{6,26}$	0.0287	0.0289	0.029	0.018	0.007
		(0.2237)	(0.2370)	(0.2)	(0.1)	—
	$\gamma_{6,3}$	—	—	—	0.015	0.015
					(0.6)	—
	SEE	0.405	0.385	0.387	0.386	—
	R^2	0.942	0.942	0.942	0.943	0.942
	DW	1.372	1.357	—	—	—

Source: Körösi (1989), p. B-8

the equations have already been "too good" so that the introduction of latent variables could not bring much improvement. It seems that latent variables of the kind considered here exert a longterm influence: small changes maintained over long time may change the economy and the society substantially. Thus, as usual in science, much rests unexplained and there remains space for future research.

Figure 1: Real GDP of the FRG (price level of 1976)

——— observed

- - - - - forecast with OLS–parameters

— — — — forecast with AS–parameters

Quelle: Krelle, [17]

Figure 2: Price level of GDP, FRG)

——— observed

- - - - - forecast with OLS–parameters

— — — — forecast with AS–parameters

Quelle: Krelle, [17]

Figure 3: Real private consumption (price level of 1976)

observed

forecast with OLS–parameters

forecast with AS–parameters

Quelle: Krelle, [17]

Figure 4: Employment in billion of working hours

observed

forecast with OLS–parameters

forecast with AS–parameters

Quelle: Krelle, [17]

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