Experimental Evidence for Attractions to Chance

by

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Abstract

Divide the decisionmaker's future into: (i) a *pre-outcome period* (lasting from the decision until the outcome of that decision is known), and (ii) a sequel *post-outcome period* (beginning when the outcome becomes known). Anticipated emotions in both periods may influence the decision, in particular, with regard to an outcome that matters to the person, the enjoyable tension from not yet knowing what this outcome will be. In the experiments presented, lottery choice can be explained by this attraction to chance, and cannot be explained by either convex von Neumann-Morgenstern utility, or by rank dependent risk loving weights: attraction to chance is a separate motivator.

1. Introduction

This paper offers experimental evidence that for most decision makers, satisfaction or dissatisfaction with an option derives not only from the events the choice is about. Most also gain satisfaction or dissatisfaction directly from the riskiness of the situation.

The phenomenon of gaining satisfaction directly from the riskiness of the situation may be termed the attraction to chance, and of suffering dissatisfaction directly from the riskiness of the situation, repulsion from chance.

This paper investigates a much discussed instance of such satisfaction, the agreeable feelings of tension experienced before learning which of several events will happen, as in the case of gambling not for the sake of making material gains, but for the excitement involved. Interest in this satisfaction dates back to Blaise Pascal 1670:

'Some people live a perfectly pleasant existence in which they continually gamble small amounts. Give one of them every morning the amount he could have won during the day providing he does not gamble, and you will make him miserable. You may think this is because he is looking for the pleasure of the game rather than the winnings involved. If so, just try letting him play without stakes. He will betray no enthusiasm, only boredom.' (English translation by Allais 1979)

Such attractions to chance and dislike of boredom are often termed the utility of gambling. Von Neumann and Morgenstern recognised it as an important phenomenon. They reported however that they were unable to find axioms that

'practically defined numerical expectations [as] legitimate ... concepts like a "specific utility of gambling" cannot be formulated free of contradiction on this level. This may seem to be a paradoxical assertion. But anybody who has seriously tried to axiomatise that elusive concept, will probably concur with it. ... It is hoped that a way will be found to achieve this, but the mathematical difficulties seem to be considerable. [von Neumann and Morgenstern 1972, pp28, 632]

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However, the periodisation by the stages of knowledge of the outcome suggested by one of the authors avoids the contradiction on this level, Pope 1983, 1984, 1985, 1986, 1988, 1991, 1995, 1996/7, 1998 and 1999. This epistemic periodisation is described in the next section. It offers in place of the usual non-temporal analysis of the utility of gambling, a sound temporal basis for understanding this phenomenon.

2. Time Structure of Anticipated Decision Consequences

In this paper we shall restrict our attention to decision situations with a simple time structure of future consequences in which the only uncertainty concerns one outcome which will become known at a specific time in the future. One could look at more general situations involving a stream of outcomes but this will not be done here. We think of the future of the decision maker as divided into three periods:

1. the pre-decision period
2. the pre-outcome period
3. the post-outcome period

During the decision process of the pre-decision period, the decision maker seeks to find a balance between conflicting motivational forces which relate to emotions anticipated to be experienced in the pre- and post-outcome periods.

As soon as the decision maker has made her choice, the pre-outcome period begins. In this period she may experience attractions to chance, emotions like hope and excitement. She may additionally, or instead, experience in this pre-outcome period repulsions from chance, emotions like fear and worry. The emotions hope and excitement, fear and worry are due to the tension of not yet knowing the outcome.

The post-outcome period begins as soon as the outcome is known. The outcome involves a change in the decision maker's welfare, *eg* his asset position if we think of lotteries over money amounts. The valuation of the outcome independently of what might have been is what is captured by the usual interpretation of expected utility theory and rank dependent utility. Undoubtedly this valuation is an important motivational force, but frequently not the only one.

In the post-outcome period, the decision maker frequently experiences also emotions caused by the relationship of the actual outcome to previously potential outcomes which were not realised. She may feel attractions to chance like elation, relief if the outcome or her choice turned out to have been better than was previously possible or repulsions from chance, disappointment or regret if the outcome or her choice turned out to have been worse than was previously possible.

3. Subjects and Payment Procedure

All subjects were advanced students of business administration and economics at the University of Bielefeld. Group 1 comprised the entire 17 students taking a lecture course on auctions and markets. Group 2 comprised the entire 19 students taking a seminar on decision making.

Various lottery choice questions were posed to the two groups on different days. For every question with possible payoffs not greater than 100 DM, one participant was selected by chance and was paid in money after the lottery was played out. The outcomes of these lotteries were determined by turning a roulette wheel in front of the students. The pair of alternative outcomes were connected to red and black events. In the case of zeros, the wheel would be turned again.

Since some phenomena do not occur in the range of small money payoffs, it seemed to be important to also ask questions about lottery choices involving very high amounts of money. Of course for high amounts the manner of payment had to be different. In these cases lotto coupons were filled in, with the understanding that should they win (which in fact they did not), one decision after the other would be selected by chance and paid until no money was left. In all lotteries, including those with possible payoffs not greater than 100 DM, there was the condition that each student could be selected to be paid only once.

The students were fully informed about payment procedures beforehand. These procedures applied to all tasks involving questions in question set 1. Thus our Results 1 and 3 below are based on money pay-off incentives, even if there was only a small chance of being selected. New results, Cubitt, Starmer and Sugden 1998, suggest that this small selection chance may not matter as much as one might think.

The procedure used to elicit certainty equivalents of lotteries in question set 2 did not permit monetary incentives. But it avoids a problem with the frequently used procedure of G. M. Becker, M.H. de Groot and Jacob Marschak 1964 — hereafter called the BDM procedure. This is the problem that since its construction excludes the utility of chance, the BDM procedure can only correctly determine the certainty equivalent in the absence of any utility of chance, any tensions.

The number interpreted as a certainty equivalent in the BDM procedure is a parameter in a two stage lottery which on the one hand determines the probability with which the evaluated lottery is played out in second stage, and on the other hand determines the lower bound of the outcome space in the case that the compound lottery ends after the first stage. This creates a complex situation with respect to anticipations of reaping a utility of chance, anticipated tensions. If subjects are influenced in their choices by such anticipations, there is no reason for them to choose the parameter under this BDM procedure in such a way that it is also the certain amount equivalent to the evaluated lottery. The tensions involved in both parts of the compound lottery used in the BDM procedure may very well lead to a higher or lower choice of the parameter.

Albers forthcoming 2000a and 2000b compares the value yielded by the BDM procedure with subject's own statements on the upper and lower bound of the range of money amounts for which the subject feels indifferent between receiving the money and playing the lottery. The result was that the BDM value was in about equally many cases in each of the following five categories: below the range, at the lower bound, within the range, at the upper bound, and above the range. Thus in about 40 per cent of the cases it was outside this range of subjects' own perceptions of their upper and lower bound.

This discrepancy most probably stems from the problem that the BDM procedure involves a two-stage lottery which differs from the original lottery and thereby induces a different amount of tension from the original lottery. For a given binary lottery L with lowest payoff a and highest payoff b, the BDM procedure asks the subject to choose as if maximizing her utility in her choice of a value of x in a two stage lottery where in step 1 a number n is drawn from a uniform distribution on the interval [a, b], in stage 2 the subject receives the number (if $n > x$), or the lottery (if $n < x$). If $n = x$, a fair coin decides. Locating the original lottery in the second stage of a two-stage lottery alters the lottery being chosen. Moreover any negative tension induced by the discrepancy between original lottery L's two possible outcomes is likely to be reduced because of the fact that this lottery L has a reduced likelihood of occurring. Its likelihood of occurring has been reduced by [(x-a)/(b-a)] per cent. This reduction in tension makes the BDM lottery more desirable for subjects who perceive the tension induced by the original lottery L as negative, and less desirable for subjects who perceive the tension of the original lottery as positive.

The number of subjects who perceive the induced tension as positive or negative can be expected to depend on the particular lottery offered, with more finding the induced tension negative in the case of lotteries involving only possible losses. If the BDM discrepancies are biases introduced by tensions, we would therefore predict as follows. In the case of lotteries involving only possible losses, the BDM value on average lies *below* those yielded by subjects own statements on the upper and lower bound of the range of money amounts for which the subject feels indifferent between receiving the money and playing the lottery. Conversely, for lotteries yielding only gains. This set of predictions is borne out in the data presented in Albers forthcoming 2000a and 2000b. We therefore concluded that the BDM procedure results in biased estimates of certainty equivalent and should be avoided.

4. Question Set 1

The following question was asked for a sequence of values of S namely S = 50, 100, 1,000, 10,000, 100,000, 10 DM in this order.[1]

Which of the following alternatives (A) or (X) do you prefer:				
	(A)	S DM for sure		
or				
	(X)	S DM for sure	+	participation in a roulette lottery with if red Þ you receive X DM if black Þ you pay X DM if zero Þ repetition the size of X being your choice

The subjects knew that the chance of the ball falling on either a red or a black segment of the wheel was 50 per cent. Subjects could choose either (A) or (X). If a subject chose (X), she had to specify the size of X.

Choosing (A) is identical to choosing (X) and specifying X = 0. However we distinguished (A) from the cases (X) in order to present the choice of the sure amount as a separate alternative. The task of the subject was to select that alternative which she preferred.

Table 1 gives an overview over the choices of X for different values of X. The entries in the Table are frequencies of X as a percentage of S.

Table 1
Frequencies of X as a Percentage of S for Positive S

1 We asked last for S = 10 DM, in order to avoid starting the question sequence with an extremely small amount.

S	0%			10%			20%		30%	40%	50%	60%	70%	80%	90%		100%	>100%
S= 10	5										1		1				26 M	3
50	6			3						6	1	7 M		1			11	1
100	4		1	2			3		3	2	10 M	3	1				6	1
1000	7		2	7	1		4 M		3	2	6			1			2	1
10000	14	4 M	3	2		1	2	1	1		3				2		2	1
100000	20 M	5	1	1	1		3								1	1	2	1

Note: Percentages are entered rounded to the nearest positive multiple of 2.5%. Most values are exactly at such percentages.
M indicates the median.

In the table, the powers of ten in the value of S are equally spaced. This corresponds to a logarithmic scale. Accordingly 50 (whose logarithm is approximately 1.7 has been put between 10 and 100 but nearer to 100.

The answers of the two groups were very similar, with identical medians for each S.

Only one subject chose X = 0 for all S, the other 34 subjects chose positive amounts of X within a range $S_L \le S \le S_U$ which was not always the whole range between 0 and 100,000, but a smaller one. Only three subjects ever chose a zero X within their ranges $S_L \le S \le S_U$ but at most for two values of S.

The lower limit S_L and the upper limit S_U vary from person to person. For 31 subjects, the lower limit was 10. For the remaining 5 subjects the lower limit S_L was higher. For 16 subjects the upper limit S_U was 100,000 DM, but for the remaining 20 subjects lower.

Result 1

Subjects tend to choose positive X for amounts of S which are not too high. The amount X as a percentage of S tends to decrease with the size of S. This can be seen by the medians in Table 1.

The tendency toward a decrease in X as a percentage of S can also be seen in the individual data. There are only two subjects for which X is a constant percentage of S, namely 100 per cent. All the others exhibit a decreasing tendency, even if seven subjects show an increase of this percentage from one value of S to the next higher one, but each of them only once.

Interpretation

Since a subject who chose (X), a positive X, would opt for S + X with probability 0.5 and S - X with probability 0.5, a subject with a concave utility function for money, should always choose (A), S DM for sure, if she wants to maximise her expected utility. In terms of expected utility theory, one would have to conclude that within the range of positive X, the utility function for money is convex. However as we shall see later, we have other evidence which would imply that it is concave.

Our own interpretation relies on the idea that the subjects are attracted to the tension created by the uncertainty about whether X will be added or subtracted from S, or elation felt after the lottery result is known if the previously merely possible better outcome results. It is however of significance that this motivation seems to prevail only within a range of S.

Below the lower limit of S, S_L, the amounts may be too small to generate excitement if they do not exceed S. Amounts of X higher than S may well be unattractive through losses having a higher weight than gains.

Above the upper limit of S, S_U, a value of X sufficiently high in relation to S may be needed to generate excitement. However subjects may be reluctant to gamble for very high amounts because they are guided by a quasi-moral rule not to do this, and by the repulsion from chance. Subjects may also be reluctant to gamble for very high amounts because the fear of the bad lottery outcome may loom large during the pre-outcome period and the possibility of regret in the post-outcome period. These repulsions from chance may, when the bad possible outcome is absolutely so much below the sure thing of S, eclipse the positive attractions to chance of agreeable tension in the pre-outcome period followed by the possibility of elation in the post-outcome period if the previously merely possibly good outcome is in fact realised. These are plausible explanations of why most subjects have an upper limit S_U above which they do not opt for a positive X.

5. Question Set 2

The subjects were asked for the upper and lower bounds of their money equivalents of the following lotteries:

S DM with probability of 0.5 and 0 DM with probability of 0.5,
S = 10, 100, ... 100,000.

Subjects were asked for upper and lower bounds rather than for exact values of monetary equivalents since our experience indicates that there is a zone of indifference rather than a single exact money value of a lottery. Data about upper and lower bounds seem to be more reliable than answers to questions requiring a single exact money value response. In this context, see Albers forthcoming 2,000a and 2,000b.

Table 2 shows the frequencies of the average of these upper and lower bounds of the money equivalents of each subject as percentages of S.

Table 2

Frequencies of Money Equivalents as Percentages of S

for lotteries yielding 0 and S with probability one half each

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+------------------------------------------------------------------------------------------+
|        0%     10%     20%     30%     40%     50%     60%     70%     80%     90%    100%|
|        +-------+-------+-------+-------+-------+-------+-------+-------+-------+-------+ |
| S= 10  .       .       .   2   1   3   .   4 1 7 1 6   . 1 1   2   4   .       1       1 |
|        .       .       .       .       .       M       .       .       .       .       . |
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|   100  .       .   1   .   1   . 1 2 1 4 1 6 1 9   4   1   1   1   1   .       .       . |
|        .       .       .       .       .     M .       .       .       .       .       . |
|  1000  .       .       3       2 4 6 2 3 2 4 1 3   2   .   2   .   1   .       .       . |
|        .       .       .       .       M       .       .       .       .       .       . |
| 10000  .   1   .   1 2 . 2 4 3 2 3   5 1 1 1 2 3   2   .       1   1   .       .       . |
|        .       .       .       . M     .       .       .       .       .       .       . |
|100000  . 3   2 . 3 3 1 1 6 1 2 1 3 1 1 .   3   1   2   .       . 1     .       .       . |
|        .       .       . M     .       .       .       .       .       .       .       . |
+------------------------------------------------------------------------------------------+
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Note: One subject did not answer this set of questions.
Averages of each subject are entered rounded to the nearest 2.5%.
M indicates the median

Result 2

The money equivalent as a percentage of S tends to decrease as S increases. This is clearly shown by the medians in Table 2. This was the case for both groups, though the second group, the participants in the Decision Seminar, showed a slight tendency to select lower monetary equivalents: the median of this group was 5 percentage points lower in the case of S = 10 and S = 1,000.

The percentages shown in Table 2 also have the tendency to decrease or at least not increase for each individual. Only eight subjects show increases from one power of ten to the next. Six of them only in one case, while the other two show decreases in the majority of cases.

Interpretation

The data interpreted in terms of von Neumann Morgenstern expected utility theory suggest that utility functions tend to be concave, at least if they are assumed to be either concave everywhere or convex everywhere.

Moreover Table 2 suggests not only concavity, but actually increasing relative risk aversion in the Arrow-Pratt sense. Both in the case of expected utility theory, and its generalisation, rank dependent utility theory, for constant relative risk aversion in this sense, the certainty equivalent should be a constant proportion of S.

Results 1 and 2 together contradict von Neumann Morgenstern expected utility theory, at least for utility functions which are not composed of many concave and convex segments alternating in a queer way. But is an explanation possible when expected utility theory is generalised to allow rank dependent utilities as in *eg* John Quiggin 1982 and 1993, Maurice Allais 1988, or Daniel Kahneman and Amos Tversky 1992?

As shown in the appendix, under appropriate assumptions on the utility function, rank dependent utility theory can lead to conclusions which have some similarity to Results 1 and 2: A positive X is chosen and X/S is constant or decreasing with increasing S. The money equivalent of the lottery which yields zero and S with equal probabilities is a fraction of S which decreases with increasing S. However we show below that rank dependent utility is also in conflict with our results.

Consider a decision maker who conforms to rank dependent utility theory. Let *L* be a lottery which yields payoffs *A* and *B* with equal probabilities. Assume *A* ² *B*, then the decision maker attaches the utility

$$U(L) = (1-g)\, u(A) + g u(B)$$

to *L*. Here *u* is the utility function and g is the probability weight of 1/2. Since no other probabilities than a half appear in the context of our paper, only this probability weight *g* matters here. Suppose that u is concave. It can be seen without difficulty that in this case the decision maker cannot choose a positive X in any question in Set 1 unless g is greater than 1/2.

Tversky and Kahneman 1992 have estimated utility functions of the form

$$u(X) = X^{a}$$

together with a probability weighting function. They obtained a mean value of 0.88 for the exponent a which indicates that utility functions tend to be concave. Their weighting function depends on a parameter γ. For their mean value of 0.61 for γ, one obtains $g = 0.42$. The survey Camerer 1995 reports that results of other investigators also tend to yield concave utility functions and values of g smaller than 1/2. However, for concave utility functions, $g > 1/2$ is necessary for the selection of the typical response to questions in Set 1. Obviously rank dependent utility theory seems to be in conflict with our results.

Perhaps under some alternative (possibly very queer) specification of the utility function, a rank dependent utility theory could be found that is more compatible with results 1 and 2 and the data of other experimentalists yielding estimates of g. But even allowing for this possibility, we have reasons to believe that the idea of attractions to and repulsions from chance leads to a more adequate interpretation than rank dependent modelling. This will become clear from our third result.

6. Influence of Exogenous Sources of Tension

Coombs and Huang 1970 found that too little as well as too much variance in payoffs is disliked by subjects. The response to question set 1 may be related to this. We view the behaviour in question set 1 as motivated by a desire for agreeable tension in the pre-outcome period and the possibility of elation in the outcome period. According to Hebb 1949 such a desire results from the need of the central nervous system for stimulation. This suggests that under conditions already involving high tension followed by possibilities of elation, demand for more tension will be lower than otherwise. Furthermore, in such high tension times, it may be undesirable to be distracted from central tasks by either agreeable or disagreeable tension, or by either elation or regret.

Such times of high tension include oral credit point examinations. Our two groups of subjects faced oral credit point examinations conducted jointly by two of the authors. In these examinations, each student answers questions alone or in groups of two posed by one of the two examiners. At the end of the questioning, the student leaves the office while the two examiners confer, and then is called back a few minutes later to be informed of her grade in the course, partly or entirely determined by this oral examination.

The peak time of tension created by a subject's choices of positive Xs in question set 1 may be around the time they know they are about to learn their payoffs from these choices. This is the time when they watch the set of turnings of the roulette wheel determining the payoffs. We compared:

A the *original* answers to question set 1 in which these turnings of the roulette wheel occurs at a time of neutral background tension, namely immediately after the experimental session, with the answers

B if instead these turnings of the roulette wheel were to be immediately *before* the oral credit point examination

C if instead these turnings of the roulette wheel were to be immediately *after* the oral credit point examination.

The time right before the credit point examination and the time within the credit point examination already provides much pre-outcome period tension. If the motivation for choosing positive values of X in question set 1 is a desire for agreeable tension, we would expect that condition B would result in lower choices of X than in condition A.

Further in the period right before the credit point examination additional pre-outcome period tension can be anticipated until the end of the oral examination. By contrast in the period right after the credit point examination, no other special sources of tension are looming to substitute for agreeable tension from X. It might therefore be expected that for many subjects choices of X would be higher in condition C than in condition B.

Indeed in condition C, some subjects may be expected to have choices of X higher not only than condition B but also higher than in neutral condition A. These may be subjects who anticipate that after the exam there will be a drop in tension to an uncomfortably low level. Before the credit point examination, in an effort to be as calm as possible, they have been foregoing some of their normal activities that generate tension. Such anticipations of having an undesirably low level of tension and chance for elation after the examination are however likely to be absent in the case of students lacking information that they will pass reasonably well. For such anxious students, the negative side of tension in the pre-outcome period — fear of the bad possible outcome — may loom larger, as also repulsions from chance in the post-outcome period — the possibility of experiencing regret from the bad possible outcomes. Indeed in the case of very anxious students, those who have prior information that they are likely to receive overall a poor examination grade, the anticipations may even be reversed. They may anticipate being in an unusually glum mood after they learn their examination grade. They may anticipate that in this unusually glum mood they will experience none of the hope side of tension ending in elation and rejoicing, only fear of tension ending in disappointment and regret.

If therefore the motivation for choosing positive values of X in question set 1 is a desire for additional tension, in comparing condition C with neutral condition A, we would expect little change in the choice of X in the case of students quite uncertain of what grade they will receive in the credit point examination, a tendency to decrease in the case of students anticipating doing poorly, and a tendency to increase in the case of students anticipating a reasonable outcome in this credit point examination.
Group 1 had rather little information on their likely grade at the end of the credit point examination, especially relative to Group 2, and thus is the Group that would lack a tension motive to choose levels of X in condition C different from those they chose in neutral condition A. This lack of information may also have meant that the level of exogenous tension engendered by condition B may have been greater for subjects in Group 1, comprising the 17 students taking the lecture on auctions and markets. Their oral examination would cover the lecture material, constituted their entire grade and they had never before had an oral credit point examination. For subjects in Group 2, by contrast, comprising the 19 students taking the seminar on decision making, their oral examination would cover the contents of a paper previously submitted by the student on material dealt with in the seminar, a paper submitted jointly with another student, and they were examined in groups of two. For subjects in Group 2, their performance in the oral

credit point exam only partly determined their grade, which depended mainly on their written paper. In this regard, Group 2 could more easily predict what their overall grade would turn out to be at the time of the doing their oral credit point examination.

Result 3

Table 3 shows the directions of change in the chosen level of X — additional tension — as the level of past and future exogenous tension changes from A, a *neutral* level, to B, high *future* exogenous tension, to C, high *past* exogenous tension.

We show the results separately for the two groups of students given the possibility that the levels of exogenous future and past exogenous tension generated by the credit point examination may have been higher for Group 1 (denoted below as subjects 1 to 16[*]) than for Group 2 (denoted as subject A to S).

B versus A: The first row of each section of Table 3 depicts the direction of change in the chosen level of X from the neutral condition A to condition B when subjects learn the outcomes of their selected X's immediately *before* the credit point examination. Under condition B, there is already much tension *ahead* and it is undesirable to be distracted from the central examination task by feelings of gratuitously introduced extra tension, elation or regret arising out of their choice of a positive X, the majority of subjects choose a lower X than under condition A. Indeed fifty percent more subjects chose a lower than chose the same X, and none at all chose a higher X. This lowering is highly statistically significant. Under a binomial one-tailed test, there is a less than 0.1 per cent chance that so many would have lowered their choice of X purely by chance. **C versus A:** The second row of each section of Table 3 depicts the direction of change in the chosen level of X from the neutral condition A to condition C when subjects learn the outcomes of their selected X's immediately *after* the credit point examination when there will have *been* an extended period of tension, but no special tension to be anticipated as *still looming ahead*, the tendency toward a lower X is much less pronounced, and indeed significantly less pronounced. Compared to neutral condition A, under condition C, the tendency toward a lower X is so mild that about half of the 35 subjects who answered this question chose the same X, and 6 went so far as to choose a higher X than in the neutral condition A, perhaps because they felt reluctant to face such a sudden drop in tension after the examination. In condition C, under a binomial one-tailed test, the very slight tendency toward a lower X than in neutral condition A is not statistically significant for either group, or for the two groups taken together, not even regarding something as high as 10% as a reasonable significance level.

[*] One subject in this group did not answer the third set of questions.

Table 3

Change in selected X under conditions of Increased Exogenous Tension compared to A

B: outcomes to be learned immediately *before* the oral credit point examination

C: outcomes to be learned immediately *after* the oral credit point examination

*Change in X when oral credit point exam is that of the auctions and markets lecture course**																	*Sum 1 to 16*			
Subjects:	1	2	3	4	5	6	7	8	9	X	1	2	3	4	5	6	/	-	o	+
B versus A	o	/	o	/	o	-	/	o	o	/	/	/	/	o	/	o	8	1	7	-
C versus A	o	o	o	/	o	-	-	o	o	+	/	o	o	o	o	o	2	2	11	1
B versus C	o	/	o	(o)	o	-	/	o	o	/	/	/	/	o	/	o	7	1	8	-

Changes in X when oral credit point exam is that of the decision making seminar																				*Sum A to S*			
Subjects:	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	/	-	o	+
B versus A	o	/	/	o	o	/	o	o	/	/	/	/	/	o	/	-	o	/	/	11	1	7	-
C versus A	o	+	+	o	+	/	o	o	/	/	+	/	/	o	/	+	o	/	/	8	-	6	5
B versus C	o	/	/	o	/	/	o	o	/	o	/	/	o	o	(o)	/	o	o	/	9	-	10	-

All subjects	*Sum A-S + 1-16*			
	/	-	o	+
B versus A	19	2	14	-
C versus A	10	2	17	6
B versus C	16	1	18	-

o no changes in X

(o) subject always selected X=0 under A so that no reduction in X was possible under B

\+ majority of changes in X to higher values

\- majority of changes in X to lower values

/ X changed to X=0 for at least one S (and in all these cases the majority of changes in X was to a lower value)

* one of the subjects in this group did not answer this set of questions

In the direction of change in X from neutral condition A to condition C, there is greater diversity among subjects who had better prior knowledge of whether they were likely to perform well or poorly — those in Group 2 (A to S) — than among those with little such prior knowledge — those in Group 1 (1 to 16). The number of subjects who changed their value of X at least once was 13 in Group 2, but only 5 in Group 1, while those who selected identical values of X were 11 in Group 1, but only 6 in Group 2. Under Fisher's exact test, this difference is significant at the three per cent level

C versus A: The second row of each section of Table 3 depicts the direction of change in the chosen level of X from the neutral condition A to condition C when subjects learn the outcomes of their selected X's immediately *after* the credit point examination when there will have *been* an extended period of tension, but no special tension to be anticipated as *still looming ahead*, the tendency toward a lower X is much less pronounced, and indeed significantly less pronounced. Compared to neutral condition A, under condition C, the tendency toward a lower X is so mild that about half of the 35 subjects who answered this question chose the same X, and 6 went so far as to choose a higher X than in the neutral condition A, perhaps because they felt reluctant to face such a sudden drop in tension after the examination. In condition C, under a binomial one-tailed test, the very slight tendency toward a lower X than in neutral condition A is not statistically significant for either group, or for the two groups taken together, not even regarding something as high as 10% as a reasonable significance level.

In the direction of change in X from neutral condition A to condition C, there is greater diversity among subjects who had better prior knowledge of whether they were likely to perform well or poorly — those in Group 2 (A to S) — than among those with little such prior knowledge — those in Group 1 (1 to 16). The number of subjects who changed their value of X at least once was 13 in Group 2, but only 5 in Group 1, while those who selected identical values of X were 11 in Group 1, but only 6 in Group 2. Under Fisher's exact test, this difference is significant at the three per cent level

B versus C: The third row of each section of Table 3 depicts the direction of change in the chosen level of X from condition B when subjects already have much exogenous tension looming *ahead* to condition C when the period of heightened exogenous tension lies in the *past*. It can be seen that when heightened exogenous tension looms *ahead*, condition B, nearly fifty percent of subjects choose a lower X than when this period of heightened exogenous tension lies in the *past*, condition C. None at all chose any higher X's in condition B than in condition C. This difference in the direction of change in X is significant at the one per cent level. From Fisher's exact test, there is less than a one per cent probability that there would be so many more lower X's in condition B than in C purely by chance.

Interpretation

Neither expected utility theory nor rank dependent utility theory can explain our Result 3 because in these theories attractions to and repulsions from chance are not considered explicitly. In our view it would be desirable to develop theories of risk taking behaviour in which tension and maybe also other motivational factors are formally introduced as variables. In this way one could transcend the mere representation of choice behaviour as the result of the optimisation of a mathematical expression depending in some way on probabilities and money amounts. Explicit modelling of the motivational forces could result in a much richer theory which would not only explain lottery choices under fixed conditions, but also permit conclusions about the effect of exogenous influences.

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Appendix

To see how some features of results 1 and 2 can be captured by a rank dependent model, consider a subject with a utility function of the form

$$u(S) \quad = \quad \ln(1+[S/C]) \qquad \text{with } C>0,$$

and assume that the subject evaluates lotteries L with two monetary outcomes A and B obtained with equal probability, and A ² B by the formula

$$u(L) \quad = \quad (1-g)\, u(A) + g\, u(B) \qquad\qquad \text{with } 1/2<g<1.$$

This is in agreement with a form of rank dependent utility theory in which greater weight is given to the better outcome, at least in the case of equal probabilities. Note that this greater weight, *ie* $g>1/2$, is necessary to obtain result 1, the choice of positive Xs in question set 1

We first look at the determination of the optimal X for question set 1. Let L_{SX} be the lottery which yields the two outcomes S-X and S+X with equal probabilities 1/2. The optimal choice of X maximises:

$$u(L_{SX}) \quad = \quad (1-g)\ln(1+\{[S-X]/C\}) \; + \; g\ln(1+\{[S+X]/C\}).$$

This leads to

$$du(L_{SX})/dX \quad = \quad -(1-g)/(C+S-X) \; + \; g/(C+S+X) \quad = \quad 0.$$

We obtain

$$X \quad = \quad (2g-1)(C+S).$$

In view of $1/2<g<1$, this value of X is positive and smaller than C+S. We now show that not only the first order conditions, but also the second order conditions for a maximum are satisfied for X.

$$d^2u(L_{SX})/dX^2 \quad = \quad - \; (1-g)/(C+S-X)^2 \; - \; g/(C+S+X)^2 \, .$$

Since X is positive and smaller than C+S, this second derivative is negative. We can conclude that a local maximum is obtained at X. Since X is the only stationary value, this maximum is a global maximum, too.

As we have seen, X is positive. Moreover, in view of C^30, the fraction X/S is constant for C=0, and decreases with increasing S for C>0. This conclusion is somewhat similar to result 1. It differs in that we obtain no restriction on the range of S and that for most subjects X is not a linear function of S, but increases more slowly than this. This is also true for the medians shown in Table 2.

Let M_s be the money equivalent of the lottery which yields zero and S with equal probabilities. We have

$$\ln M_s \quad = \quad g\ln(1+\{S/C\})$$

and therefore

$$M_s \quad = \quad (1+\{S/C\})^g.$$

We have

$$\ln(M_s/S) \quad = \quad g\ln(1+(S/C) \; - \; \ln S.$$

This yields

$$d\ln(M_s/S)/dS \quad = \quad g/(C+S) \; - \; 1/S \quad < \quad 0.$$

Therefore M_s/S decreases with increasing S. This conclusion drawn from a rank dependent model with $g > 1/2$ bears some qualitative similarity to results 1 and 2. However as noted in section 5, its value of $g > 1/2$ conflicts with empirical estimates that $g< 1/2$.