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HETEROGENEOUS CONSUMERS AND PRODUCT DIFFERENTIATION IN A MARKET FOR PROFESSIONAL SERVICES*

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1. Introduction

Some rather formidable difficulties surround the formal analysis of markets with differentiated products. These concern both the specification of consumer preferences over the 'similar but distinct' products of rival producers, and the implications of this for the analysis of the firm's choice of product.

Such difficulties have prompted, or enhanced, interest in certain special cases of the problem, which involve choice over a very particular space of product characteristics. The most familiar example of this is in location theory [Hotelling (1929)] where preferences can be defined as a simple function of the distance to a producer. More recently, some progress has been made by analysing the case where goods differ only in 'quality', so that all consumers agree on the ordering of alternative products [Jaskold Gabszewicz and Thisse (1980)].

The present paper is concerned with an example of the latter type of problem. Elsewhere, we have analysed a market for a particular kind of professional service, initially provided by a monopolistic profession to a population of consumers *identical* in incomes and tastes. We have examined the effects of allowing the entry of a rival group of 'paraprofessionals', supplying a service of inferior quality at a lower price.[1] Considerable interest surrounds the issue of whether such a 'paraprofession' will be viable, in affording its members an income in excess of their transfer earnings as 'non-professionals' [Shaked and Sutton (1980)].

In the present paper we extend this study to the case where consumers differ in income; the main focus of our study concerns the analysis of the manner in which the heterogeneity of consumers may enhance the viability of the paraprofessional group.

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[1]The immediate empirical motivation for the study lay in the ongoing debate as to whether a rival group should be allowed to compete with the British Legal profession in providing conveyancing (escrow) services in house purchase.

2. The model

We briefly summarise here the basic model developed in Shaked and Sutton (1980), together with some extensions relevant to our present concerns.

We consider a group of 'workers' each of whom supplies exactly one unit of labour services to some quite separate group of 'consumers'.

We divide the N workers into three groups, L 'professionals', P 'paraprofessionals', and $l=(N-P-L)$ 'labourers'. Workers are assumed to be alike when employed as labourers but to differ in their ability to provide professional services. We order the workers on $[0,N]$ in increasing order of (some appropriate measure of) 'ability'.

We take as given, throughout, the existence of a self-regulating profession which sets a lower bound, in terms of ability, for entry. It will follow from our analysis that, over the relevant range, all workers satisfying this entry requirement choose to be members of the profession, and so this entry requirement determines the size L of the profession.

In a similar manner, the paraprofession chooses a minimum ability requirement and so determines its size P. We assume that a consumer is unable to distinguish the differing abilities of particular individuals, but, through 'hearsay' or 'general knowledge', is aware of what degree of confidence he may typically place in the reliability of services offered by a 'professional', and also by a 'paraprofessional'. We describe his preferences over potential professions consisting of individuals whose ability lies in some subinterval (a,b) of $[0,N]$. His preference ordering $\gtrsim$ over professions (a,b) is assumed to satisfy the axioms:

Q1: The preference order $\gtrsim$ is complete, reflexive, transitive and continuous.

Q2: If $(a_1,b_1)\leqq(a_2,b_2)$ then $(a_1,b_1)\lesssim(a_2,b_2)$.

The second axiom states that if we remove the least able members, and/or introduce more able newcomers, the consumer prefers the new profession thus formed to the old.

Let $Q(a,b)$ be a function representing this ordinal preference ordering. We shall refer to $Q(a,b)$ as the quality of the profession (a,b).

Consumers are assumed to purchase either one unit of the services of the 'profession', whose quality we denote β, or one unit of those of the 'paraprofession', of quality γ, or neither. His remaining income is spent on the services of 'labourers', his consumption of which is represented by a continuous variable x.

The consumers are assumed to be identical in their tastes, which are fully described by the pair of functions, $V(x)$, giving the utility of the individual purchasing x unit of labourers' services only, and none of professionals', and

$U(x,\alpha)$ giving the level achieved where, in addition to x units of labourers' services, the consumer enjoys one unit of professional services from a profession (either our original profession or the paraprofession) of quality α.

We assume that $V(x)$ and $U(x,\alpha)$ are increasing in all arguments, that $U(x,\alpha)>V(x)$, and that $U(0,\alpha)=V(0)=0$. (To achieve positive utility a consumer must enjoy some 'labourers' services'.)

Given x, α we define $M(x,\alpha)$ as follows. Let x denote a consumer's income in terms of the quantity of labourers' services he can purchase. Then he is willing to forego $M(x,\alpha)$ units of labourers' services in return for 1 unit of professional services of quality α. M is defined by the relationship

$$U(x-M(x,\alpha),\alpha)=V(x).$$

Differentiating with respect to x, and representing the partial derivative of $U(x,\alpha)$ w.r.t. x as U_x^α, and that of $M(x,\alpha)$ as M_x^α,

$$U_x^\alpha(1-M_x^\alpha)=V_x \quad \text{or} \quad M_x^\alpha=1-V_x/U_x^\alpha.$$

We assume that, for all x, α:

Assumption 1. $U_x^\alpha>V_x>0$, or equivalently

$$1>M_x^\alpha>0. \tag{1}$$

This states that $U_x^\alpha>V_x$, i.e. for two individuals achieving the same level of utility, one of whom enjoys the services of x labourers only while the other enjoys the services of the smaller number $x-M$ of labourers, along with one unit of professional services, then the marginal utility of additional services from labourers is greater for the latter individual (reflecting his lesser consumption of such services).

It is worth noting that this assumption is ordinal, being preserved when any increasing transformation is applied to U, V.

We now proceed to develop a related function.

Consider a consumer whose income in terms of labourers' services is x, and let M^γ denote the price of paraprofessional services relative to the services of labourers. Then we define the function $\mathscr{M}(M^\gamma,x,\beta)$, being the number of units of labourers' services the consumer is willing to forego for a unit of professional services of quality β, via the relationship

$$U(x-\mathscr{M},\beta)=U(x-M^\gamma,\gamma).$$

We now make an assumption, analogous to that introduced above, that the marginal utility of labourers' services to the individual consuming β

exceeds that of the individual consuming γ, reflecting his lesser consumption of such services:

Assumption 2. $U^{\beta}_{x} > U^{\gamma}_{x} > 0.$

Using our definition of $\mathscr{M}(M^{\gamma}, x, \beta)$ in conjunction with Assumption 2, we can obtain a number of elementary relationships which will be found useful below. From our definition

$$U(x - \mathscr{M}(M^{\gamma}, x, \beta), \beta) = U(x - M^{\gamma}, \gamma),$$

we have on differentiating w.r.t. x, and denoting the partial derivative of $\mathscr{M}(M^{\gamma}, x, \beta)$ w.r.t. x as $\mathscr{M}^{\beta}_{x}$,

$$U^{\beta}_{x}(1 - \mathscr{M}^{\beta}_{x}) = U^{\gamma}_{x},$$

whence

$$\mathscr{M}^{\beta}_{x} = 1 - U^{\gamma}_{x}/U^{x}_{\beta}, \tag{2}$$

so that, from Assumption 2,

$$0 < \mathscr{M}^{\beta}_{x} < 1. \tag{3}$$

Moreover, differentiating w.r.t. M^{γ},

$$-U^{\beta}_{x}\mathscr{M}^{\beta}_{M^{\gamma}} = -U^{\gamma}_{x},$$

whence

$$\mathscr{M}^{\beta}_{M^{\gamma}} = U^{\gamma}_{x}/U^{\beta}_{x}. \tag{4}$$

Furthermore, combining (2) and (4),

$$\mathscr{M}^{\beta}_{x} + \mathscr{M}^{\beta}_{M^{\gamma}} = 1, \tag{5}$$

whence, using (3),

$$0 < \mathscr{M}^{\beta}_{M^{\gamma}} < 1. \tag{6}$$

Finally, it may be helpful to illustrate the definition of our two functions $M(x, \alpha)$ and $\mathscr{M}(M^{\gamma}, x, \beta)$ which play a central role in our subsequent analysis (fig. 1).

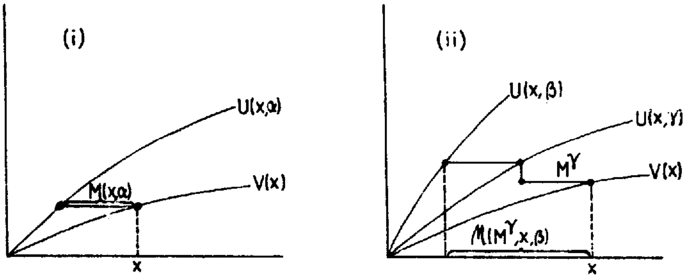

Fig. 1. (i) The function $M(x,\alpha)$. (ii) The function $\mathcal{M}(M^{\gamma},x,\beta)$.

3. Market equilibrium with heterogeneous consumers

We consider a population of n consumers, divided into two income groups. A fraction δ, $0 \leqq \delta \leqq 1$, are 'rich' enjoying income Y_1, while the remainder are 'poor' with income $Y_2 < Y_1$. We define the income relative $\mu = Y_1/Y_2 > 1$. We shall be concerned below with the effects of varying the distribution of income through changes in δ, and in μ. As δ increases from zero, the minority of 'rich' consumers expands, up to $\delta = \frac{1}{2}$, the point at which incomes may be thought of as 'most unequal'. Thereafter, increases in δ involve a shrinkage in the number of poor consumers. At $\delta = 1$, incomes again coincide. Moreover, all 'real' variables in the model coincide at $\delta = 0$ and at $\delta = 1$, all prices having risen by a factor μ. (This simply reflects the fact that our consumers, as a whole, spend all their money incomes in purchasing services from a fixed body of workers — the 'resources' of the economy.)

For fixed δ, on the other hand, an increase in $\mu \geqq 1$ makes the rich relatively richer and the poor relatively poorer, $\mu = 1$ corresponding to perfect equality.

For given parameters δ and μ, then, we investigate market equilibrium, being a vector of prices $(q, p^{\gamma}, p^{\beta})$, denoting the price of the labourers, paraprofessionals and professionals, respectively. Since we assume that each worker supplies exactly one unit of services these prices also represent workers' incomes.

We say that the paraprofession is *viable* if $p^{\gamma} \geqq q$, i.e. if paraprofessionals earn at least their transfer earnings as labourers. In general, as we shall see, the viability of the paraprofession depends *inter alia* on its quality γ, and hence on its size. As the paraprofession expands, so that its quality deteriorates, we will eventually reach a maximum size at which the profession remains viable. We shall speak of the *range* of viable paraprofessions as being from size zero up to this maximum level.

We shall be interested primarily in whether this range of viable paraprofessions expands, or otherwise, as the distribution of incomes varies.

We investigate the equilibrium vector of prices (q, p^γ, p^β) for given parameters δ, μ, and for a given size L, P and quality β, γ, of profession and paraprofession.

We distinguish the following cases:

(i) $\delta n < L$,

(i′) $\delta n = L$,

(ii) $L < \delta n < L + P$,

(ii′) $\delta n = L + P$,

(iii) $L + P < \delta n$.

Leaving aside for the moment the borderline cases (i′), (ii′), throughout the three remaining zones, a single equation represents the condition for market equilibrium, viz.

$$\delta n t^1 + (1-\delta) n t^2 = L M^\beta + P M^\gamma + l, \tag{7}$$

where

$$t^i = Y^i/q \qquad \text{(so that } t^1 = \mu t^2\text{)},$$

$$M^\beta = p^\beta/q, \qquad M^\gamma = p^\gamma/q,$$

$$l = N - L - P.$$

The interpretation of eq. (7) is as follows. The LHS represents consumers' total income measured in terms of the number of units of labourers' services which it can purchase. On the RHS, $(p^\beta/q) = M^\beta$ represents the number of units of labourers' services which each of the L individuals purchasing one unit of professional services foregoes, in return for those professional services. Thus the RHS represents total consumer expenditure on labour services.

While eq. (7) holds for all three of our cases, the determination of M^γ, M^β must be dealt with separately for each.

Case (i): $\delta n < L$

Here the number of rich consumers falls short of the number of professionals. Hence some poor consumers purchase professionals' services, some those of paraprofessionals', and some neither. Thus at equilibrium a poor consumer is indifferent between these three alternatives, so that

$$M^\beta = M(t^2, \beta), \qquad M^\gamma = M(t^2, \gamma).$$

Now since $t^1 = \mu t^2$, eq. (7) can be solved for t^2. We write $M(t^2, \gamma)$, where t^2 is the solution for (7), as $M^{(1)}(\delta, \mu, P)$. (We here write $M^{(1)}$ as a function of δ, μ, P only, since we shall hold L constant throughout.)

Case (ii): $L < \delta n < L + P$

The number of rich consumers exceeds the number of professionals, so that some rich individuals patronise paraprofessionals, whence

$$M^{\beta} = \mathscr{M}(M^{\gamma}, t^1, \beta).$$

Some poor consumers buy paraprofessionals' services, while the remainder purchase only labourers' services. Thus

$$M^{\gamma} = M(t^2, \gamma).$$

Eq. (7) can again be solved for t^2; we denote the value of $M(t^2, \gamma)$ corresponding to the solution as $M^{(2)}(\delta, \mu, P)$.

Case (iii): $L + P < \delta n$

The number of rich consumers exceeds the number of professionals and paraprofessionals combined. Hence some rich consumers patronise professionals, some paraprofessionals, and the rest neither. Hence

$$M^{\beta} = M(t^1, \beta), \qquad M^{\gamma} = M(t^1, \gamma).$$

Eq. (7) can now be solved for t^1. We denote the value of $M(t^1, \gamma)$ corresponding to the solution as $M^{(3)}(\delta, \mu, P)$.

Our functions $M^{(i)}(\delta, \mu, P)$ thus defined then give over their respective ranges, the value of M^{γ} at equilibrium. The criterion for the viability of the paraprofession is simply that $M^{\gamma} = p^{\gamma}/q \geqq 1$.

Our development rests, then, on the characterisation of the functions $M^{(i)}(\delta, \mu, P)$. We state their properties in a number of lemmas, whose proofs are given in the penultimate section.

We aim to examine, as remarked earlier, the maximum size of paraprofession P consistent with viability; in order to ensure that this is well defined we establish:

Lemma 1. (i) *For all δ there exists at most one P such that $M^{(1)}(\delta, \mu, P) = 1$.* (ii) *For all δ satisfying $\delta n \geqq L$ there exists at most one P such that $M^{(2)}(\delta, \mu, P) = 1$.* (iii) *For all δ satisfying $\delta n \geqq L$ there exists at most one P such that $M^{(3)}(\delta, \mu, P) = 1$.*

We further establish:

Lemma 2. *For all δ such that $\delta n > L$, and all P, $M^{(3)}(\delta, \mu, P) > M^{(2)}(\delta, \mu, P) > M^{(1)}(\delta, \mu, P)$.*

The effect of changes in the income distribution parameters is summarised in:

Lemma 3. *For all δ, $M_\delta^{(1)}(\delta, \mu, P) < 0$, and for all δ such that $\delta n \geqq L$, $M_\delta^{(2)}(\delta, \mu, P) < 0$ and $M_\delta^{(3)}(\delta, \mu, P) < 0$.*

Lemma 4. *As μ increases the intersection of $M^{(1)}$, and of $M^{(2)}$, respectively, with $M = 1$ occurs at a lower value of P. As μ increases the intersection of $M^{(3)}$ with $M = 1$ occurs at a higher value of P.*

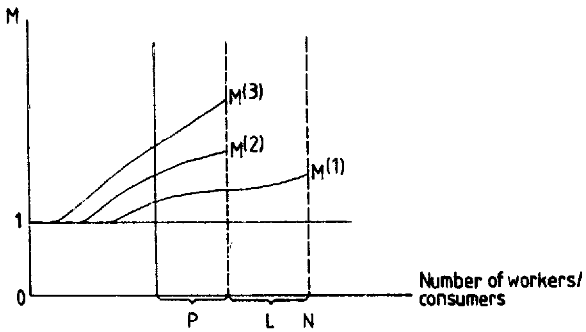

Fig. 2. The determination of $M^\gamma = p^\gamma/q$, being the income of paraprofessionals relative to non-professionals, as a function of the size P of the paraprofession. M^γ is given by the appropriate member of the family of function $M^{(i)}$, as described in the text.

We may now apply Lemmas 1–4 to characterise the largest viable paraprofession. The functions $M^{(i)}$ are represented in fig. 2.

When δn lies within one of the three ranges specified earlier, the equilibrium value of M^γ is given by the appropriate $M^{(i)}$.

For $\delta n = L + P$ there is a continuum of equilibria represented by the range of values lying between $M^{(3)}$ and $M^{(2)}$. For our purposes it suffices to note that by continuity of the $M^{(i)}$ the 'end points' corresponding to the case where M^γ coincides with $M^{(3)}$ and with $M^{(2)}$, respectively, are equilibrium points.

The case $\delta_n = L$ is analysed in an identical manner.

A paraprofession is said to be *viable* if there exists an equilibrium such that $M^\gamma \geqq 1$.

In order to ensure that a paraprofession of *some* size is viable, we assume[2] that the $M^{(i)}$ all intersect the vertical $N-L$ above $M=1$.

We now consider how the range of viable paraprofessions is determined as δ increases from 0 to 1.

For $\delta<L/n$, i.e. $\delta n<L$, the viable paraprofessions are those for which $M^{(1)}\geqq 1$. For $\delta=L/n$ the viable paraprofessions are those for which $M^{(2)}\geqq 1$ [fig. 3 (i), (i′)].

As δ increases further, we pass successively through three further zones, as indicated in fig. 3 (ii), (ii′), (iii).

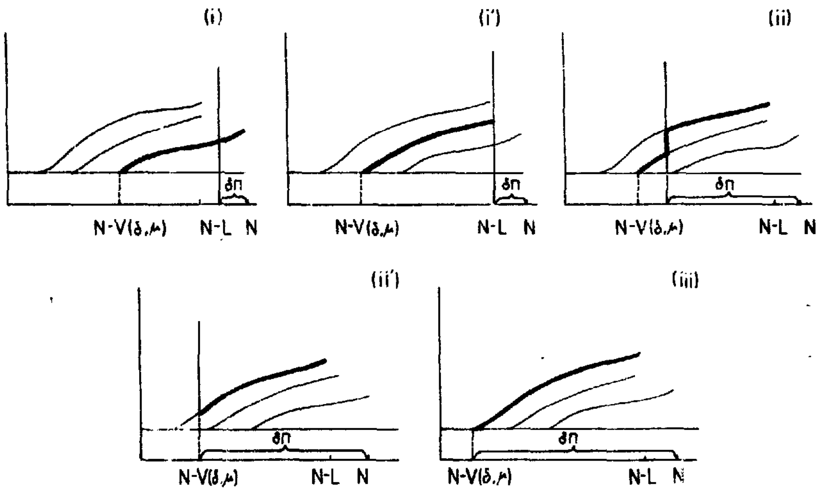

Fig. 3. The determination of M^{γ} for successively larger values of δ. [The heavy line shows the appropriate $M^{(i)}$ function which gives M^{γ} as a function of P, and $V(\delta,\mu)$ is the size of the largest viable paraprofession.]

We define the critical values $\delta^{(i)}$ such that $n\delta^{(i)}=L+P$ for that value of P which satisfies $M^{(i)}(\delta,\mu,P)=1$. Then for δ below $\delta^{(2)}$ the largest viable paraprofession is that satisfying $M^{(2)}=1$ [fig. 3 (ii)]. For $\delta^{(2)}<\delta<\delta^{(3)}$, it is that satisfying $\delta n=L+P$ [fig. 3 (ii′)]. Finally, for $\delta>\delta^{(3)}$ the largest viable paraprofession is that satisfying $M^{(3)}=1$ [fig. 3 (iii)].

We have, as an immediate consequence of the properties of $M^{(i)}$ adduced in Lemmas 1 and 2 above, the preliminary proposition:

Proposition 1. For all δ, L, there exists a range of viable paraprofessions, i.e. if a paraprofession of size P is viable, then so also is any paraprofession of size $P'<P$.

[2]In fact it is enough to assume that for $\delta_n=L$, $M^{(1)}$ lies above 1 at $N-L$ and that for $\delta=1$, $M^{(3)}$ lies above 1 at $N-L$ [i.e. to borrow the terminology of Shaked and Sutton (1980) the profession can be threatened by its last member at $\delta=L/n$ and at $\delta=1$].

Proof. The result follows immediately from our preceding discussion, on appealing to the fact that $M^{(3)} > M^{(2)}$ (Lemma 2).

Corr. There exists some maximum size of paraprofession consistent with viability.

This follows immediately, using Lemma 1. We denote the size of this largest viable paraprofession as $V(\delta, \mu)$.

We may now proceed to investigate how the size of the maximum viable paraprofession changes as the distribution of income is varied by raising δ.

From Lemma 3, we have that, as δ increases, the curves $M^{(i)}$ shift downwards. Using fig. 3, we may deduce the behaviour of $V(\delta, \mu)$ by considering a leftward shift in the vertical at $N - \delta n$ and a downward shift in the $M^{(i)}$. The results are illustrated in fig. 4, in which we have indicated the zones corresponding to the five cases shown in fig. 2.

For $\delta < L/n$ [fig. 3 (i)], an increase in δ causes $M^{(1)}$ to shift downwards so that $N - V(\delta, \mu)$ moves to the right, i.e. $V(\delta, \mu)$ decreases. At $\delta = L/n$ [fig. 3 (i')], there is a discontinuity, as $V(\delta)$ jumps to a higher value. Thereafter, on $L/n < \delta < \delta^{(2)}$, the zone represented in fig. 3 (ii), increases in δ are associated with a fall in $V(\delta)$ as $M^{(2)}$ declines.

Referring to fig. 3 (ii'), $V(\delta, \mu)$ *increases* with δ over phase (ii'), since here we have $N - V(\delta, \mu) = \delta n$. On the last phase, from fig. 3 (iii), however, $V(\delta)$ again falls as δ increases, since $M^{(3)}$ shifts downwards.

The above remarks establish, inter alia, the result which we wish to emphasise, and which we state as:

Proposition 2. The maximum viable size of paraprofession $V(\delta)$ first declines as the fraction of rich consumers δ increases, but at the critical value $\delta = L/n$, $V(\delta)$ shifts discontinuously upwards, after which it again decreases over a neighbourhood to the right of the critical point.

We now turn to the interpretation of the results just summarised.

The central feature of the analysis is illustrated by phases (i)–(ii) in fig. 4. The behaviour of the maximum viable size of paraprofession $V(\delta, \mu)$ as δ increases involves two distinct effects.

The first effect is intuitively obvious: throughout phases (i)–(ii), the marginal (poorest) consumer purchasing paraprofessional services is 'poor'. Raising δ involves a reduction in the real income of such consumers, and this tends to reduce the relative price of paraprofessional services. It is this effect which operates in phases (i) and (ii).

The second effect is more subtle, and leads to the discontinuity in $V(\delta)$ at the point where $\delta n = L$. To the right of this, only rich individuals consume the services of the original profession; to the left, the marginal (poorest) consumer purchasing such services is 'poor'.

The effect of an increase in δ through this critical value, then, is to change the identity of the marginal (poorest) consumer purchasing the high quality services offered by the original profession from a 'poor' consumer to a 'rich' one. The determination of the relative price of the higher quality services is now through the equation

$$M^{\beta}=\mathcal{M}(M^{\gamma},t^{1},\beta),$$

reflecting the real income t^1 of our marginal 'rich' consumer, rather than through

$$M^{\beta}=M(t^{2},\beta).$$

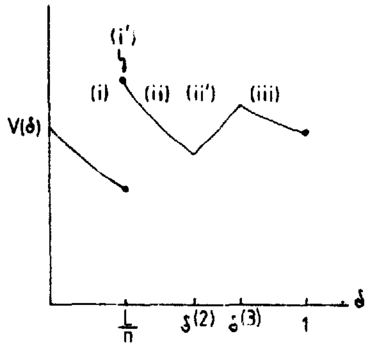

Fig. 4. The maximum viable size of paraprofession $V(\delta,\mu)$ as a function of the fraction of rich consumers δ. [*Note:* $V(L/n)$ may be above or below $V(0)=V(1)$.]

This implies a 'bidding' up of the relative price of these superior services through competition among our rich consumers. One result of this is a loss of welfare by our hitherto intramarginal rich consumers. However, this shift in expenditure towards members of the original profession is accompanied by a concomitant shift away from expenditure on 'other things' — our residual category of labourers' services, and so to a fall in their price q. Since some of our 'poor' consumers consume only these services, their welfare is thereby enhanced. Thus, paradoxically, the increase in the money income of some poor consumers here *improves* the welfare of the remaining 'poor' consumers. It is this which in turn leads to a bidding up of the relative price of the paraprofessional series which they, among others, consume, and so to the enhanced viability of the paraprofessional group.

It is perhaps the main virtue of the rather special 'discrete' model we have used here, that it allows these two distinct effects of income redistribution to be unravelled.

The latter phases, (iv) and (v), may be described more cursorily.

In stage (iv), the number of rich consumers coincides with the size of the profession and paraprofession combined. The marginal consumer is a 'rich' consumer, and the size of the paraprofession accordingly expands pari passu with an increase in the number of rich consumers.

But eventually, as δ increases towards unity, the real purchasing power of these 'rich' individuals is (eventually) falling, while the quality of the expanding paraprofession is deteriorating, so that eventually the 'rich' consumer is no longer willing to support a paraprofession of size $P=\delta n-L$; this is the point $\delta^{(3)}$.

The final phase is the obverse of phase (i); here, the introduction of a small number of poor consumers into an initially egalitarian society ($\delta=1$) raises the real incomes of the rich individuals who support the paraprofession and enhances its viability.

Lastly, we remark on the effects of changes in income distribution associated with changes in our income relative μ. Here we obtain as an immediate consequence of Lemma 3 above:

Proposition 3. The maximum viable size of paraprofession $V(\delta,\mu)$ decreases with μ for small δ $(0\leq\delta\leq\delta^{(2)})$ and increases with μ for large δ $(\delta>\delta^{(3)})$.

The interpretation of this is straightforward. Here the relative numbers of rich and poor consumers remains fixed, so that only the first of the two effects identified above operates. In a phase where the marginal (poorest) consumer supporting the paraprofession is poor [phases (i)–(ii)], raising μ reduces the relative income of such consumers and the viability of the paraprofession is reduced. In the opposite case [phase (iii)], raising μ enhances its viability. As μ tends to unity, moreover, so that the incomes of our two groups coincide, the curve $V(\delta,\mu)$ converges to a horizontal, at a height $V(0)=V(1)$.

4. Proofs

4.1. Proof of Lemma 1

If P is such that $M^{\gamma}=1$, then by (7)

$$\delta nt^1+(1-\delta)nt^2=LM^{\beta}+P+l=LM^{\beta}+(N-L),$$

whence

$$\delta nt^1+(1-\delta)nt^2-LM^{\beta}=(N-L)=\text{constant}.$$

We demonstrate that for each of our cases (i), (ii) and (iii) there is a unique t^1 (and equivalently t^2) which satisfies this equation. It follows immediately then that there is a unique γ such that $M(t^i,\gamma)=1$, and this γ uniquely identifies P.

To establish the uniqueness of the solution in t^i we show that

$$\delta n t^1+(1-\delta)nt^2-LM^\beta$$

is an increasing function of t^2.

For case (i),

$$M^\beta=M(t^2,\beta),$$

and writing $t^1=\mu t^2$,

$$\begin{aligned}&\delta n t^1+(1-\delta)nt^2-LM(t^2,\beta)\\&=nt^2(\delta\mu+1-\delta)-LM(t^2,\beta)\\&=L(t^2-M(t^2,\beta))+[n(\delta\mu+1-\delta)-L]t^2.\end{aligned}$$

But $n(\delta\mu+1-\delta)>n>L$ so that the second term is an increasing function of t^2, while by virtue of Assumption 1, $M_x^\beta<1$, so that the first term is also an increasing function of t^2, whence our result follows.

For case (ii),

$$M^\beta=\mathcal{M}(M^\gamma,t^1,\beta),$$

and writing $t^2=t^1/\mu$,

$$\delta n t^1+(1-\delta)nt^2-LM^\beta=L(t^1-M^\beta)+(\delta n-L)t^1+(1+\delta)t^1/\mu.$$

Assume $\delta n>L$. Then the last two terms here are increasing in t^1. We noted above [eq. (3)] that $\mathcal{M}_x^\beta<1$, whence the first term is also an increasing function of t^1 as required.

The proof for case (iii) parallels that of case (ii) exactly.

4.2. *Proof of Lemma 2*

We first establish that $M^{(3)}>M^{(2)}$.

The equation for t^1 which defines $M^{(2)}$ is, from (7),

$$\delta n t^1+(1-\delta)nt^2=L\mathcal{M}(M(t^2,\gamma),t^1,\beta)+PM(t^2,\gamma)+l. \tag{8}$$

We compare this with the corresponding equation for $M^{(3)}$, viz.

$$\delta n t^1 + (1-\delta) n t^2 = LM(t^1, \beta) + PM(t^1, \gamma) + l. \tag{9}$$

Now for the value of t^1 which solves the latter equation we have, since $t^1 = \mu t^2 > t^2$, that

$$M(t^1, \gamma) > M(t^2, \gamma). \tag{10}$$

We can further show that

$$M(t^1, \beta) > \mathcal{M}(M(t^2, \gamma), t^1, \beta).$$

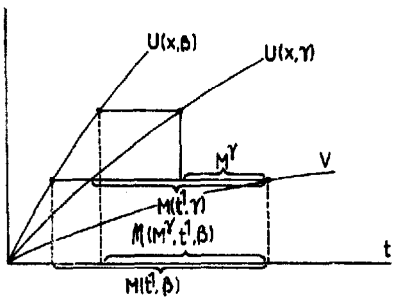

Fig. 5. The function $\mathcal{M}(M^\gamma, t^1, \beta)$.

This second relationship follows from the fact that $M(t^2, \gamma) < M(t^1, \gamma)$; this is illustrated in fig. 5. For, in this phase some rich consumers patronise professionals while others patronise paraprofessionals. Hence

$$\begin{aligned} U(t^1 - \mathcal{M}(M(t^2, \gamma), t^1, \beta), \beta) &= U(t^1 - M(t^2, \gamma), \gamma) \\ &> U(t^1 - M(t^1, \gamma), \gamma) \\ &= U(t^1 - M(t^1, \beta), \beta), \end{aligned}$$

whence

$$M(t^1, \beta) > \mathcal{M}(M^\gamma, t^1, \beta). \tag{11}$$

It follows from eqs. (10) and (11) that, for the t^1 which solves eq. (9) above, we have

$$\delta n t^1 + (1-\delta) n t^2 - L\mathcal{M}(M(t^2, \gamma), t^1, \beta) - PM(t^2, \gamma) > l. \tag{12}$$

We now show that the LHS of this inequality is an increasing function of t^1. Hence, by comparison with eq. (8), the value of t^1 which solves (8) is less than the corresponding value defined by (9). Thus $M^{(2)} < M^{(3)}$ as required.

It remains to establish that the expression on the LHS of (12) is increasing in t^1. We rewrite the expression in the form

$$L[t^1 - \mathcal{M}(M(t^2,\gamma),t^1,\beta)] + P(t^2 - M(t^2,\gamma))$$
$$+ [(\delta n - L)\mu + (1-\delta)n - P]t^2.$$

We denote the partial derivative of $M(x,\gamma)$ w.r.t. x as M_x, and that of $\mathcal{M}(M^\gamma,x,\beta)$ w.r.t. x as $\mathcal{M}^\beta_x$.

Then the derivative of $\mathcal{M}(M(t^2,\gamma),t^1,\beta)$ w.r.t. t^1 becomes

$$[\mathcal{M}(M(t^2,\gamma),t^1,\beta)]'_{t^1} = \mathcal{M}^\beta_{M^\gamma} - M^\gamma_x \cdot \frac{1}{\mu} + \mathcal{M}^\beta_x$$
$$= 1 - \mathcal{M}^\beta_{M^\gamma}\left(1 - M^\gamma_x \cdot \frac{1}{\mu}\right),$$

where the last equality follows from (5) above.

But $M^\gamma_x < 1 < \mu$ (Assumption 1), hence $[\mathcal{M}(M(t^2,\gamma),t^1,\beta)]'_{t^1} < 1$ so that our first term,

$$L[t^1 - \mathcal{M}(M(t^2,\gamma),t^1,\beta)],$$

is an increasing function of t^1.

The second term also increases in t^2, and so in t^1, since $M^\gamma_x < 1$.

Moreover, on the range $\delta n \geqq L$, we have

$$(\delta n - L)\mu + (1-\delta)n - P \geqq (\delta n - L) + (1-\delta)n - P = n - L - P > 0,$$

so that the third term is also an increasing function of t^1, whence our result follows.

We now show that $M^{(2)} > M^{(1)}$, which completes our proof of Lemma 2.

For case (i), we have as our defining equation for $M^{(1)}$, from (7) above,

$$\delta n t^1 + (1-\delta) n t^2 = LM(t^2,\beta) + PM(t^2,\gamma) + l. \tag{13}$$

Now for the value of t^2 which solves this equation we have

$$M(t^2,\beta) < \mathcal{M}(M(t^2,\gamma),t^1,\beta), \tag{14}$$

since

$$\mathcal{M}(M(t^2,\gamma),t^1,\beta)=M(t^2,\gamma)+X,$$

where $X>M(t^2,\beta)-M(t^2,\gamma)$ as illustrated in fig. 6.

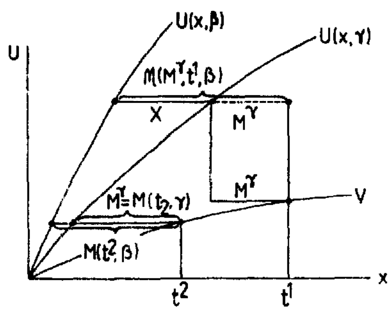

Fig. 6

The last step here follows from the fact that the horizontal distance between $U(x,\beta)$ and $U(x,\gamma)$ increases with U, i.e. $M(x,\beta)-M(x,\gamma)$ is increasing in x. This is a direct consequence of Assumption 2, for on differentiating,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\{M(x,\beta)-M(x,\gamma)\}&=M_x^\beta-M_x^\gamma\\&=\left(1-\frac{V_x}{U_x^\beta}\right)-\left(1-\frac{V_x}{U_x^\gamma}\right)\\&=V_x\left(\frac{1}{U_x^\gamma}-\frac{1}{U_x^\beta}\right)>0.\end{aligned}$$

Now it follows from (14) that, for the value of t^2 which solves eq. (13) that

$$\delta nt^1+(1-\delta)nt^2-L\mathcal{M}(M(t^2,\gamma),t^1,\beta)-PM(t_2,\gamma)<l.$$

But we have established above that the LHS of this inequality is an increasing function of t^2. Hence the value of t^2 which solves eq. (8) above — and so defines $M^{(2)}$ — is greater than that which solves eq. (13); whence our result follows immediately.

4.3. Proof of Lemma 3

From eq. (7),

$$\delta n t^1 + (1-\delta) n t^2 = LM^\beta + PM^\gamma + l.$$

Consider an increase in δ to δ'. The RHS here is independent of δ, and the LHS increases with δ, so for the original t^1, t^2 and the new value δ',

$$\delta' n t^1 + (1-\delta') n t^2 > LM^\beta + PM^\gamma + l.$$

But the function

$$\delta' n t^1 + (1-\delta') n t^2 - LM^\beta + PM^\gamma$$

has been shown above to be an increasing function of t^1 (t^2).

Hence the new value $t^{1'}$ ($t^{2'}$) which solves (1) lies below t^1 (t^2).

It follows from our definition of $M^{(1)}$ as the value of $M(t^2, \gamma)$ corresponding to the t^2 which solves (7) that $M^{(1)}$ is decreasing in δ.

A similar argument applies to $M^{(2)}$, $M^{(3)}$.

4.4. Proof of Lemma 4

When P is such that $M^\gamma = 1$, then eq. (7) becomes

$$\delta n t^1 + (1-\delta) n t^2 - LM^\beta = N - L. \tag{7'}$$

In the proof of Lemma 1 above, we showed that the LHS here is an increasing function of t^1 (t^2).

We now proceed to show that, for cases (i) and (ii), the left hand expression is also an increasing function of μ. From this it follows immediately that as μ increases, the t^1 (t^2) value which satisfies (7′), i.e. foı which $M^\gamma = 1$, decreases. Thus the corresponding level of quality γ satisfying $M^\gamma = 1$ rises, since $M(t, \alpha)$ is increasing in t and α. Hence the corresponding size P of the paraprofession falls as required.

It remains to establish that the LHS of (7′) is indeed an increasing function of μ.

For case (i), the expression becomes

$$n t^2 (\delta\mu + 1 - \delta) - LM(t^2, \beta),$$

which is clearly increasing in μ.

For case (ii) the expression becomes

$$nt^2(\delta\mu+1-\delta)-L\mathscr{M}(M(t^2,\gamma),\mu t^2,\beta), \qquad (7'')$$

whose derivative with respect to μ is

$$\delta nt^2 - L\mathscr{M}_x^\beta t^2 = (\delta n - LM_x^\beta)t^2 > 0,$$

since

$$\delta n > L > L\mathscr{M}_x^\beta,$$

where the last inequality follows from Assumption 2.

For case (iii), the expression becomes

$$nt^1\left(\delta+\frac{1-\delta}{\mu}\right)-LM(t^1,\beta),$$

which is clearly *decreasing* in μ. Hence, reversing the above line of argument, the size P of the paraprofession at which $M^{(3)}=1$ increases with μ.

5. Summary and conclusions

We have considered a model in which consumers choose between the services of a professional group, and those of a paraprofessional group offering a lower quality service at a lower price.

Our consumers, whose preferences are identical, are divided into two income classes, 'rich' and 'poor'. Each demands at most one unit of professional services; and spends the remainder of his income on a residual category of 'other things', here represented by the services of non-professionals, or 'labourers'. Given a fixed size of profession, we examine to what maximum size the paraprofession can expand — where such expansion occurs through a lowering of entry standards and a reduction in the quality of its services — while still remaining viable, in the sense that its members earn at least their transfer earnings as non-professionals.

It might be expected that, in some sense, greater heterogeneity among consumers might enhance the 'viability' (measured by the maximum size to which it can expand) of the 'low quality' service offered by the paraprofessional group.

In fact, the relationship between the distribution of income and the viability of the paraprofession is a complex one. The central feature which we have emphasised is that two effects are involved, whose role is illustrated by considering a change in income distribution brought about by increasing the fraction of consumers who are 'rich'. When this fraction is small, some of the remaining 'poor' consumers purchase the high quality services offered by the original profession. The services of the paraprofession are purchased only by such 'poor' consumers. An increase in the fraction of 'rich' consumers at first reduces the relative purchasing power of the 'poor' consumers supporting the paraprofession so that its viability is *reduced*. Thus introducing heterogeneity among consumers can render the paraprofession *less viable*.

However, a discontinuity occurs at the point where the number of 'rich' consumers expands to the point where only they consume the high quality services offered by the original profession. As the number of 'rich' consumers exceeds this level, the associated bidding up of the price of high quality services by rich consumers now leads to a shift in total expenditure away from our residual category — the services provided by 'non-professionals' or 'labourers'. The fall in the price of such services leads to a rise in the welfare of our 'poor' consumers, and in turn to an increase in the viability of the para-profession, whose services they purchase.

Thus, over this range, an increase in heterogeneity *enhances* the viability of the paraprofessional group. Further small increases in the proportion of rich consumers again, however, reduce its viability; our first effect again operates.

Eventually, we reach a point where the fraction of rich consumers is so large, that only they consume professional services of either type; here, the maximum viable size of paraprofession simply rises pari-passu with the number of 'rich' consumers, until a point is reached at which the implied deterioration in its quality causes its viability to once again decline.

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