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INTERNATIONAL TRADE IN DIFFERENTIATED PRODUCTS*

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1. INTRODUCTION

The present paper is concerned with the problem of analyzing international trade in differentiated products. Since this involves the introduction of new goods in each market, we require some specification of consumer preferences over a range of — actual or potential — substitute goods. A general analysis of this problem involves familiar difficulties. Recent progress in the area has depended on various, particular, structures of preferences.[1]

We here consider a special case whose salient features are that a number of firms produce rival products which differ in 'quality,' the 'quality' of each firm's product being fixed exogenously; that each consumer purchases (at most) *one* of the alternative products; and that market equilibrium is characterized as a non-cooperative price equilibrium.[2]

Our central theme is that the number of products which can coexist at equilibrium in such a market is bounded (even though we assume a continuum of consumers, and no costs); and that, on joining two economies to form a 'common market,' *there is a tendency for the total number of products coexisting at equilibrium to fall under free trade*; *the lowest quality goods disappear from the market.*

No general analysis is offered; instead, we confine ourselves to providing a simple, but suggestive, example of a partial equilibrium type. This is outlined in Section 2, for the case of a single economy. The effect of international trade between two such economies is examined in Section 3. The implication of our central proposition, as well as some brief comments on how the analysis might be generalized, are given in the concluding Section 4.

2. INDUSTRY EQUILIBRIUM IN A SINGLE COUNTRY

Consider an industry consisting of a number of firms producing at zero cost distinct, substitute, goods. We label their respective products by an index $k=1,\dots, n$, where firm k sells product k at price p_k.

Assume a continuum of consumers identical in tastes but differing in income;

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[1] For example, the 'characteristics' model of Lancaster [1980], or the Dixit-Stiglitz model (Dixit and Stiglitz [1977], Krugman [1979]).

[2] Our analysis rests on the model of Gabszewicz and Thisse [1980]. An analogous preference structure is postulated in Shaked and Sutton [1981].

incomes are uniformly distributed over some range, $[a, b]$, $a>0$ so that the density function of income t is

$$g(t) = \begin{cases} K & a \le t \le b \\ 0 & \text{otherwise.} \end{cases}$$

Thus $K(b-a)$ is a measure of the size of the economy.

Consumers make indivisible and mutually exclusive purchases from among our n substitute goods, in the sense that any consumer either makes no purchase, or else buys exactly one unit from one of the n firms. We denote by $U(t, k)$ the utility achieved by consuming one unit of product k and t units of 'other things' (the latter may be thought of as a Hicksian 'composite commodity,' measured as a continuous variable); and by $U(t, 0)$ the utility derived from consuming t units of income only.

Assume that the utility function takes the form

$$U(t, k) = u_k \cdot t \qquad k = 1, \ldots, n$$

(1) and

$$U(t, 0) = u_0 \cdot t$$

with $0<u_0<u_1<\cdots<u_n$ (i.e., the products are labeled in increasing order of quality). Let

$$C_k = \frac{u_k}{u_k - u_{k-1}}$$

(whence $C_k>1$). Then we may define the income level t_k such that a consumer with this income will be indifferent between good k at price p_k and good $k-1$ at price p_{k-1}, viz.,

$$t_k = p_{k-1}(1 - C_k) + p_k C_k$$

(2) and

$$t_1 = p_1 C_1.$$

This is easily checked by reference to (1).

Assuming zero costs, the profit (revenue) of the k-th firm is, for *any*[3] density function $f(t)$,

$$R_n = p_n \int_{t_n}^{b} f(t)dt$$

$$R_k = p_k \int_{t_k}^{t_{k+1}} f(t)dt \qquad 1 < k < n.$$

[3] It is only when discussing a single economy, in Lemmas 2 and 3, that we appeal to the assumption that the density function $g(t)$ is uniform.

The expression for R_1 depends on whether or not t_1 as defined by (2) is greater than or less than a (on whether the market is covered, in the sense that all consumers buy one of the n products)

$$R_1 = p_1 \int_{t_1}^{t_2} f(t)dt \qquad t_1 \geq a$$

or

$$R_1 = p_1 \int_{a}^{t_2} f(t)dt \qquad t_1 \leq a.$$

We may now proceed to characterize a non-cooperative price equilibrium (Cournot-Nash equilibrium) in which n firms have a positive market share, being a vector of prices $(p_1^*, \ldots, p_n^*)$ such that no one firm k can improve its profit by deviating from p_k^*, the other firms' prices being fixed.

The following Lemma characterizes some properties of the market shares of the various firms in equilibrium.

LEMMA 1. *A necessary condition*[4] *for an equilibrium in which goods* 1 *to* n *have a positive market share is*

$$\int_{t_n}^{b} f(t)dt > f(t_n) \cdot t_n;$$

$$\int_{t_k}^{t_{k+1}} f(t)dt > f(t_k) \cdot t_k \qquad 1 < k < n.$$

PROOF. A necessary condition for profit (revenue) maximization by firm k *is*:

$$R_k \Big|'_{p_k^*} = p_k \int_{t_k}^{t_{k+1}} f(t)dt \Big|'_{p_k^*}$$

$$= \int_{t_k}^{t_{k+1}} f(t)dt + p_k^* f(t_{k+1})(1 - C_{k+1}) - p_k^* f(t_k) C_k$$

$$= \int_{t_k}^{t_{k+1}} f(t)dt - f(t_k)t_k + p_k^* f(t_{k+1})(1 - C_{k+1}) + p_{k-1}^* f(t_k)(1 - C_k) = 0.$$

But C_k, $C_{k+1} > 1$, hence $\int_{t_k}^{t_{k+1}} f(t)dt > f(t_k) \cdot t_k$. It is easily verified that this first order condition corresponds to a maximum. It is shown in an exactly similar fashion that for $k = n$,

$$\int_{t_n}^{b} f(t)dt > f(t_n) \cdot t_n \qquad \text{Q.E.D.}$$

We next demonstrate that if the range of incomes is restricted in an appropriate

[4] The revenue function defined here for firm k refers of course to the situation in which the price of k is set at a level such that products $k+1$ and $k-1$ remain available, i.e., retain positive market shares. We stress that while this is a *necessary* condition for equilibrium, a full analysis of Nash equilibrium in the model requires that p_k^* be shown to be globally optimal.

manner, the number of products which retain positive market shares, i.e., 'remain on the market,' at equilibrium, is at most two.

This is part of a more general theorem established in Gabszewicz and Thisse [1980]. The intuitive explanation is as follows: if more goods enter, price competition between rival firms forces the prices of competing 'higher quality' goods down to a level at which some 'low quality' products would not be preferred by consumers, even at a price of zero.

LEMMA 2. *Let $b<4a$. Then for any equilibrium involving a sequence of n products, labelled in order of decreasing quality $n, n-1, \ldots, 1$, at most two products $(n, n-1)$ have a positive market share and charge a positive price, whereas the remaining products have a zero market share.*

PROOF. Applying Lemma 1 to the density function $g(t)$, we obtain

$$K(b - t_n) > Kt_n, \quad K(t_n - t_{n-1}) > Kt_{n-1} \quad \text{or} \quad b > 2t_n,$$

$$t_n > 2t_{n-1} \quad \text{i.e., } b > 4t_{n-1}.$$

But since by assumption $a>b/4$, we have $a>t_{n-1}$ which means that goods $n-2$, $n-3, \ldots, 1$ have zero market share. (Of course, if $b<2a$, the same proof can be adduced to show that only one good has a positive market share). The equilibrium market shares will depend in general both on the income distribution and the relationship between the quality levels of the different goods. A useful measure of the difference in qualities for two goods 1, 2 is,

$$\alpha = \frac{C_2 - 1}{C_1} = \frac{u_1 - u_0}{u_2 - u_0}$$

Q.E.D.

LEMMA 3. *If $2a<b<4a$ and if u_1, u_2 are such that*

$$\alpha > \frac{b - 2a}{3a}$$

then both products 1 and 2 have positive market shares, and positive prices, at equilibrium; and $t_1<a$, so that the market is covered (each consumer buys one or other of the two goods).

PROOF. Since

$$t_1 = p_1C_1, \quad t_2 = p_1(1 - C_2) + p_2C_2$$

we have

$$p_2C_2 = t_2 + t_1\alpha.$$

Rewriting the first order conditions for equilibrium (Lemma 1), for products 1, 2, we have, in terms of α, for product 1:

$$t_2 - a = t_1\alpha \qquad \text{if} \quad t_1 \leq a$$

$$t_2 - t_1 = t_1(1 + \alpha) \qquad \text{if} \quad t_1 \geq a$$

while for product 2:

$$b - t_2 = t_2 + t_1\alpha.$$

Solving these equations for $t_1 \leq a$ yields

$$t_1 = \frac{b - 2a}{3\alpha}, \quad t_2 = \frac{b + a}{3}$$

(independently of the qualities of the two goods!)
and

$$p_1 = \frac{b - 2a}{3\alpha C_1}, \quad p_2 = \frac{b - a}{3C_2}.$$

Now if $2a < b$ and $\alpha > (b-2a)/(3a)$ then $t_1 < a < t_2$ and $p_1 > 0$, $p_2 > 0$, i.e., both products have positive market shares and prices and the market is covered. If on the other hand we attempt to solve the equations for $t_1 \geq a$ we obtain a contradiction, viz., $t_1 < a$. Q. E. D.

R*emark.* While we have chosen a restricted range of α, the first part of the Lemma (that both products enjoy a positive market share and a positive price) remains true for all α. The proof of this is lengthy, however, and is not needed for our present example.

3. THE EFFECTS OF FREE TRADE

Suppose we introduce free trade between two economies of the type characterized above. The outcome will clearly depend in general on the relationship between the qualities of the several goods initially available on each market, as well as on the relationship between the pattern of income distribution in each economy. Intuitively, it is clear that if, for example, the 'top' products in each of the two markets were very similar in quality, then competition between these two might drive out the remaining goods, in the manner of Lemma 2 above.

In the following Proposition, we consider two economies, each of which satisfies the conditions of Lemmas 2–3, so that it supports two goods initially. Regarding the qualities of the goods, we assume only that they are distinct. We then present a condition on the distributions of income in the two economies — requiring that they be 'sufficiently similar' — which ensures that at most three goods can survive in the combined economy.

PROPOSITION. *Given two separate economies in each of which incomes are distributed uniformly over some range* $[a_i, b_i]$, $2a_i < b_i < 4a_i$, *suppose each of these economies initially supports two goods, and that all four goods are distinct. Then if* $b_1 > b_2$, $a_1 > a_2$ *and* $b_1 < 2b_2$, $a_1 < 2a_2$ *the combined economy supports at most three goods.*

Remark. From the above conditions on $[a_i, b_i]$

$$\frac{a_1}{2} < a_2 < a_1 < \frac{b_1}{2} < b_2 < b_1.$$

The pattern of income distribution in the combined economy is shown in Figure 1.

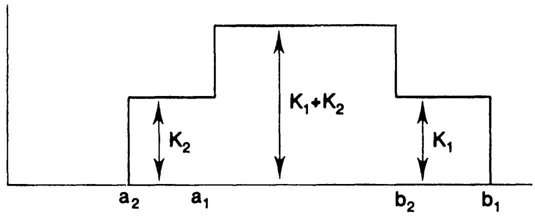

FIGURE 1: INCOME DISTRIBUTION IN THE COMBINED MARKET.

PROOF. We proceed in stages, showing that the market share of the highest quality good n extends below $b_1/2$ and that of good $(n-1)$ extends below a_1, while that of the third good $(n-2)$ extends below a_2.

(i) $t_n < b_2$ (i.e., the market share of the top quality product extends below b_2). If $t_n \geq b_2$, then from Lemma 1,

$$\int_{t_n}^{b_1} K_1 dt > K_1 t_n$$

i.e., $b_1 - t_n > t_n$ whence $(1/2)b_1 > t_n$ but $b_2 > (1/2)b_1$ by assumption, so that $b_2 > t_n$ which is a contradiction. Hence $t_n < b_2$.

(ii) $t_n < (b_1/2)$. Suppose first that $a_1 \leq t_n < b_2$. Then by Lemma 1

$$\int_{t_n}^{b_2} (K_1 + K_2) dt + \int_{b_2}^{b_1} K_1 dt > (K_1 + K_2) t_n.$$

Hence $(b_2 K_2 + b_1 K_1)/[2(K_1 + K_2)] > t_n$ but $b_1 > b_2$, whence $t_n < b_1/2$. If $t_n < a_1$, on the other hand, then by our assumption $a_1 < b_1/2$, we have $t_n < b_1/2$ as required.

(iii) $t_{n-1} < a_1$. This is obvious if $t_n < a_1$. Otherwise we have $a_1 < t_n < b_1/2$. Then, assuming that $t_{n-1} \geq a_1$, we obtain from Lemma 1 (for $k = n-1$)

$$t_{n-1} < \frac{t_n}{2}.$$

Moreover, from (ii),

$$\frac{t_n}{2} < \frac{b_1}{4} < a_1$$

so that $t_{n-1} < a_1$, which is a contradiction. Hence, $t_{n-1} < a_1$.

(iv) $t_{n-2} \leq a_2$. Suppose $t_{n-2} > a_2$. Then, by Lemma 1 for $k = n-2$,

$$t_{n-2} < \frac{1}{2} t_{n-1}$$

and from (iii)

$$\frac{1}{2} t_{n-1} < \frac{a_1}{2} < a_2$$

whence $t_{n-2} < a_2$ which is a contradiction. Hence $t_{n-2} \leq a_2$. Q.E.D.

4. SUMMARY AND CONCLUSIONS

We have here described a suggestive example of the manner in which international trade may impinge on a market for differentiated products. The central features of this example are that each consumer chooses (at most) one of a number of competing products of varying 'quality' supplied by rival firms, and that the outcome is characterized as a non-cooperative price equilibrium.

We have shown how competition between rival producers can reduce the prices of higher quality goods to the point where some 'lower quality' goods are driven out of the market. The idea here is that, given the price of rival higher quality products, no consumer would prefer the low quality alternatives even at price zero, were they to be on offer.

We have illustrated by means of an example how, when separate economies are combined in a common market, the outcome may involve the disappearance of certain goods hitherto produced. It is worth noting that the upper bound to the number of products which can survive in an economy, in the context of a model of this kind, is not associated with fixed costs, or increasing returns to scale, (as in Krugman [1979] for example). Indeed, we have here assumed costs to be zero. Instead, this upper bound follows intrinsically from the combination of our assumptions on consumer preferences, and our characterization of market equilibrium. In particular, this upper bound is independent of the *size* of the economy (all our equilibrium conditions are unchanged if the parameter K varies). Some special features of the present example which appear not to be fundamental to the result, and which might in principle be relaxed, are that

(i) a particular 'linear' form of the consumers utility function is used;

(ii) the distribution of incomes in each economy is uniform, and consumers are identical in tastes;

(iii) the quality of good produced by each firm is given exogenously, rather than chosen in an optimal manner.

It should finally be remarked that our analysis is a partial equilibrium one: no account is taken of the impact of consequent changes in output and employment, and so in relative unit costs, in each economy.

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