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# Multiproduct firms and market structure

Avner Shaked\*

and

John Sutton\*

*Models of (horizontal) product differentiation generally admit many equilibria. These include concentrated equilibria, in which few firms each offer many products, and fragmented equilibria, in which many firms each offer one product. Since these outcomes depend in a delicate way on features of the models that are hard to identify or proxy empirically, these models may seem empirically empty in regard to predictions about industrial structure. This article proposes a simple and natural reparameterization of such models in terms of empirically observable market characteristics, thereby generating testable predictions about the relationship between market size and market structure (concentration).*

## 1. Introduction

■ In reviewing a quarter century of development in the theory of imperfect competition, Joan Robinson (1953) remarked that the most glaring gap in the literature lay in the absence of any serious treatment of market equilibrium with multiproduct firms. It is only in the past decade that a series of studies tackling particular aspects of this problem has appeared. The early work, following Schmalensee (1978), focused on the question of whether a first mover could establish a monopoly outcome by means of a strategy of product proliferation under conditions of sequential entry. (See, in particular, Hay (1976), Bonanno (1987), and Judd (1985).) Later work emphasized the role of cost factors (economies of scope) in generating outcomes in which some firms produce several products (Panzar and Willig, 1981).

More recently, some authors have begun to examine the role of demand factors (Wolinsky, 1986). It is this last strand in the literature that provides the point of departure for the present article. We abstract from any cost-based influences in that we assume that the cost of production of any particular product is the same for each firm, regardless of what other products it may produce (no scope economies). We also assume that the sunk costs that must be incurred in setting up production are exogenously given; that is, we abstract from the role of endogenous sunk costs explored in the vertical product differentiation literature (Shaked and Sutton, 1987).

The basic question addressed in this literature may be phrased as follows. In the presence of multiproduct firms, we might expect to find fragmented equilibria, in which a large number of firms each offer a small range of products, and/or concentrated equilibria in

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which a small number of firms each offer many products. What, then, are the basic features of consumer preferences (i.e., the structure of demand) that lead to the appearance of the various types of equilibria?

The answer that we give to this question is couched in terms of two basic features of market demand, which have an intuitively natural interpretation. The central object of this article is to set out this way of looking at the problem. The two basic features of market demand, in terms of which we present our analysis, are best motivated by comparing the incentive to introduce a new variety faced by a monopolist with that faced by a new entrant.

For the monopolist, the incentive to introduce a new variety is determined by the demand for the new product *net of any loss of sales* incurred on his existing products. This *expansion effect*, which can for our purposes be measured by the increase in monopoly profit, is the first of the two basic features on which our analysis rests. For the new entrant, however, what matters is the level of demand for the new variety; he escapes the negative externality suffered by the monopolist on his existing products. On the other hand, the entrant's incentive to introduce the new product is weakened, *vis-à-vis* the monopolist, insofar as competition reduces the equilibrium price of the new offering. Thus, the equilibrium outcome reflects the balance of two effects: the expansion effect just mentioned and the *competition effect*, which can be measured by the gap between prices under a competitive outcome and those under a monopolistic one.

The main novelty of the present treatment lies, then, in presenting this way of parameterizing the problem. This approach may be contrasted with the more popular parameterizations of such problems in the recent literature, which have emphasized the degree of substitutability between the products as a key determinant of structure. Useful though such a representation is, we argue in what follows that the degree of substitutability between the products affects the outcome in two distinct ways (represented by the expansion and the competition effects, respectively) and that unravelling these two contributions in the way proposed here can lead to a better understanding of what is happening in these models.

Our central aim, then, is to show how the equilibrium pattern of outcomes can be characterized in terms of these two effects. The effects are most easily illustrated in the context of a model in which there are only two goods and in which no good is produced by more than one firm. Here, there is just one relevant measure of market expansion (from one to two goods) and competition (a two-good monopolist versus two single-product firms). This permits a particularly simple graphical representation of the relevant relationships. When more goods are introduced, the same underlying effects operate to determine the pattern of equilibrium outcomes. But, in this setting, a graphical representation is less useful, as we must distinguish several expansion effects (from one to two goods, from two to three goods, *etc.*) and several competition effects.

We begin our analysis with a rather full treatment of the two-good case. In Section 2, we set out an elementary analytical framework for the two-good setup. In Section 3, we analyze a specific model (the basic linear model) within this framework. In Section 4, we informally describe a number of extensions of the basic linear model before going on to develop one of these in detail (the extended linear model). This last model is then extended to the three-good case, which is analyzed in Section 5. Finally, in Section 6 we look at the case in which our assumption of strategic symmetry (simultaneous entry) is replaced by one of sequential entry.

Running through the article is a central thread regarding the relationship between market size and equilibrium outcomes. The reason for emphasizing this relationship is that it appears to offer a particularly helpful route towards the empirical testing of this type of model.<sup>1</sup>

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<sup>1</sup> See, for example, Bresnahan and Reiss (1987) and the discussion in Sutton (1989).

## 2. The two-good case: an elementary framework

■ We begin by setting out an elementary two-good framework, within which we can discuss the key parameters that induce the appearance of various kinds of equilibrium configurations across a broad class of models. Then, in the next section, we set out some specific models and show how they can be analyzed using this framework. In later sections, we extend the framework in various ways.

The basic two-good framework of the present section applies to a class of models with the following characteristics:

- (i) Only two distinct varieties of goods can be produced.
- (ii) The market is characterized in terms of a two-stage game, in which each of a number of firms (potential entrants) chooses at stage 1 which product(s) it will produce. It incurs a “sunk cost” of  $\epsilon > 0$  per product entered.
- (iii) Each variety can be produced by at most one firm. We simply *assume* this for the moment.<sup>2</sup>
- (iv) In the second stage of the game, the firms choose their respective prices. At this point, we do not impose any restrictions on the form of price competition involved. The examples developed below include both the case of a Nash equilibrium in prices (Bertrand equilibrium) and the case of joint profit maximization.
- (v) Products enter symmetrically into firms’ profit functions, so we can denote the profit functions corresponding to each of the possible product configurations entered at stage 1 by  $\pi(1, 0)$ ,  $\pi(2, 0)$ , and  $\pi(1, 1)$ , where  $\pi(k, l)$  denotes the profit of a firm with  $k$  products in the presence of a rival with  $l$  products.

We focus on characterizing the equilibria of this game.<sup>3</sup> The menu of configurations which form equilibria in this setting depends *only* on the values of the stage 2 profits,  $\pi(1, 0)$ ,  $\pi(2, 0)$ , and  $\pi(1, 1)$ . This leads us to begin by offering a general characterization of these equilibria in terms of two functions of the  $\pi(k, l)$  to which we can attach a simple economic interpretation. We assume that  $\pi(2, 0) \geq \pi(1, 0)$ , i.e., that the introduction of a second product will not reduce a monopolist’s total profit, and that  $\pi(2, 0) > 2\pi(1, 1)$ , i.e., that monopolization will not reduce total profit. We also assume that firms enter simultaneously at stage 1. (The case of sequential entry is considered in Section 4.)

As we noted in the introduction, the incentives for an incumbent or entrant to introduce a new product depend on two effects. The first is the *expansion effect*, which we measure by

$$E = \frac{\pi(2, 0) - \pi(1, 0)}{\pi(1, 0)}.$$

$E$  denotes the fractional increase in profit enjoyed by an incumbent monopolist who introduces a second product. This measures the degree to which total demand is increased when a new product is introduced. The second effect is the *competition effect*, which we measure by

$$C = \frac{\pi(2, 0) - 2\pi(1, 1)}{2\pi(1, 1)}.$$

$C$  denotes the fractional difference between the industry’s profits under monopoly and its

<sup>2</sup> Some of the cases examined later involve the use of a Nash equilibrium in prices (Bertrand equilibrium); in this setting, the present assumption turns out to be superfluous, as it can be shown that no firm would wish to produce a product identical to its rivals’ at equilibrium.

<sup>3</sup> If we characterize the second-stage price competition game as a Nash equilibrium in prices, then our present procedure amounts to the conventional one of seeking a (subgame) perfect equilibrium in the two-stage game.

profits under competition. Thus,  $C$  measures the incentive to monopolize; the stronger the degree of price competition in the market, the larger is  $C$ .

Given the restrictions on the  $\pi(k, l)$  noted above, we have  $E \geq 0$  and  $C \geq 0$ . Now, the conditions for a configuration of each possible kind to be supported as a (subgame perfect) equilibrium are readily stated in terms of the optimal reply functions as follows. Let  $(i \rightarrow j)$  denote “ $j$  is the optimal reply to  $i$ .” Then, the necessary and sufficient conditions for each possible optimal reply are given in Table 1.

We assume throughout that  $\pi(1, 0) > \epsilon$ , i.e., the market is viable in that it is certainly profitable to enter one product. (This assumption is used in writing down the conditions for  $(0 \rightarrow 2)$  in Table 1.)<sup>4</sup> To simplify the notation, we normalize our model by choosing the units such that  $\pi(1, 0) = 1$ ; we assume henceforward that  $0 < \epsilon < 1$ .

These optimal replies suffice to determine the equilibria of the model. The (simultaneous entry) Nash equilibria correspond to those pairs of strategies, each of which is an optimal reply to the other. (For a discussion of sequential entry, see Section 6.)

Table 1 can be used to partition  $(E, C)$  space into a number of zones that correspond to different patterns of optimal replies. Any particular (two-good) model can be represented by a point in  $(E, C)$  space, and this serves to specify the equilibria of the model. It remains, therefore, to demonstrate that a range of reasonable models that span various  $(E, C)$  combinations can be adduced.

To illustrate the way in which different market environments correspond to different  $(E, C)$  combinations, consider the following examples.

*Example 1: Perfect substitutes.* Consider a situation in which the two distinct products converge in their characteristics to the point where they are virtually identical. The introduction of the second product now causes no market expansion, but competition is likely to be very strong; so,  $E = 0$ , while  $C$  becomes large.

*Example 2: Islands.* Assume that the two goods are sold to different groups of consumers and that neither group values the product favored by the other. Here, the two products are essentially independent of each other; we get a maximal expansion effect but no competition effect ( $E = 1, C = 0$ ).

*Example 3: Joint profit maximization.* Consider a market in which close substitute goods are offered, but in which the prices of rival firms are determined by joint profit maximization. Then, we have  $E = 0$  and  $C = 0$ .

We return to these and other examples in later sections.

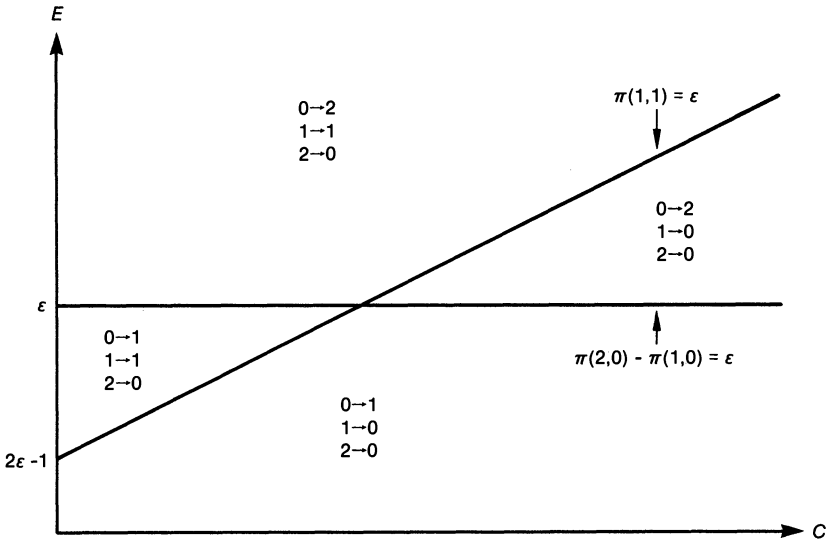
Using the definitions of  $E$  and  $C$  and the fact that  $\pi(1, 0) = 1$ , we can illustrate the zones in  $(E, C)$  space which are associated with the various forms of optimal reply. (See Figure 1.) To construct Figure 1, recall that  $\pi(1, 0) = 1$  and note that, in terms of  $E$  and  $C$ , the condition  $\pi(1, 1) = \epsilon$  becomes  $E = 2\epsilon C + 2\epsilon - 1$  and  $\pi(2, 0) - \pi(1, 0) = \epsilon$  becomes

TABLE 1 Conditions for Optimal Replies

Optimal Reply	Conditions
$(0 \rightarrow 1)$	$\pi(1, 0) > \epsilon, \quad \pi(2, 0) - \pi(1, 0) < \epsilon$
$(0 \rightarrow 2)$	$\pi(1, 0) > \epsilon, \quad \pi(2, 0) - \pi(1, 0) > \epsilon$
$(1 \rightarrow 0)$	$\pi(1, 1) < \epsilon$
$(1 \rightarrow 1)$	$\pi(1, 1) > \epsilon$
$(2 \rightarrow 0)$	(always true)

<sup>4</sup> The conditions for  $(0 \rightarrow 2)$  are that  $\pi(2, 0) > 2\epsilon$  and that  $\pi(2, 0) - \pi(1, 0) > \epsilon$ . Since we assume that  $\pi(1, 0) > \epsilon$ , this reduces to the requirement specified in Table 1.

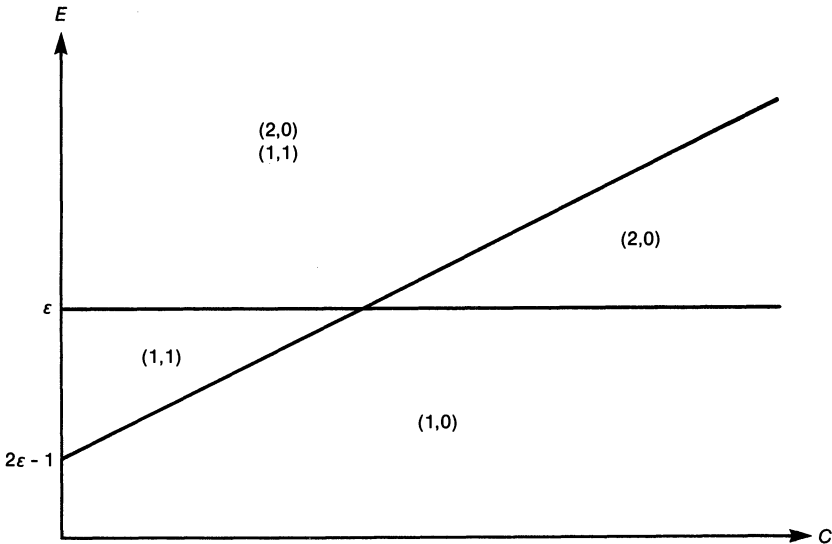
FIGURE 1  
OPTIMAL REPLIES IN THE TWO-GOOD CASE



$E = \epsilon$ . Now, from Figure 1, we may immediately deduce which configurations appear as equilibria in this (simultaneous entry) model. (See Figure 2.)

The interpretation of Figure 2 is as follows. Fix the competition effect, i.e., take a point on the  $C$  axis, and consider an increase in the expansion effect, (i.e., move vertically upward in the diagram). Eventually, both  $(1, 1)$  and  $(2, 0)$  appear as equilibria. Depending on the value of  $C$  chosen, however, either  $(1, 1)$  or  $(2, 0)$  may appear first. Note that it is at *higher* values of  $C$  that the monopoly equilibrium,  $(2, 0)$ , appears first, while at *lower* values of  $C$ , it is the competitive equilibrium that appears first. Now, reversing this process, hold the

FIGURE 2  
EQUILIBRIA IN THE TWO-GOOD MODEL (SIMULTANEOUS ENTRY)



(The case  $\frac{1}{2} < \epsilon < 1$  is illustrated.)

expansion effect,  $E$ , constant (choose a point on the vertical axis) and increase the degree of competition (move horizontally to the right in the figure). Notice that, for sufficiently large  $C$ , only the **monopoly** solutions appear.

These features of the pattern of equilibrium configurations are a reflection of a basic idea that plays a central role in these models: *the stronger (weaker) the degree of postentry competition, the stronger the tendency for monopoly (competitive) solutions to appear*. This, however, is only part of the story. The nature of the equilibria in this framework depends in general on the **interplay** of the two effects we have identified.

To see this, consider how an increase in the size of the market (in the sense of successive replications of the population of consumers) changes the set of equilibria in this framework. In doing this, we shall assume that all of the profit functions,  $\pi(k, l)$ , increase in direct proportion to the population of consumers; this will be true of all the models we consider. Our equilibrium conditions involve a comparison of linear combinations of these  $\pi(k, l)$  with the sunk cost of entry,  $\epsilon$ . Hence, the equilibria obtained by increasing the population by replication can equivalently be obtained by reducing the value of  $\epsilon$  from one to zero.

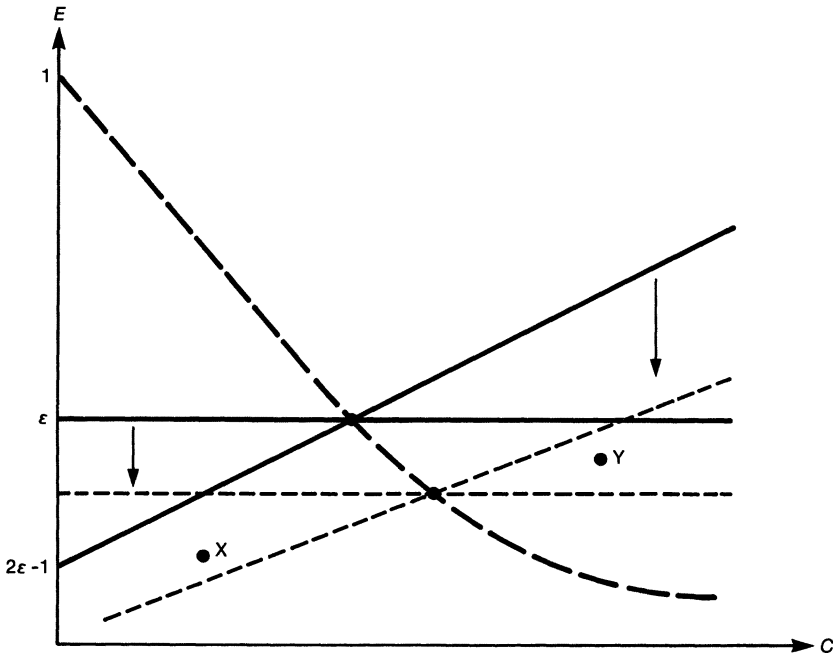
Now, as shown in Figure 3 below, reducing  $\epsilon$  shifts the two lines which divide our zones in the direction indicated by the arrows, while the point of intersection of the two lines follows the locus shown by the hatched line in the figure,

$$E = 1/(2C + 1).$$

This critical locus plays an important part in what follows, and it has a direct economic interpretation: along this locus,

$$\pi(2, 0) - \pi(1, 0) = \pi(1, 1),$$

FIGURE 3  
THE EFFECT OF INCREASING THE SIZE OF THE MARKET IN THE TWO-GOOD MODEL



[Note the role played by the critical locus (hatched line) traced out by the intersection of the two (solid) lines as  $\epsilon$  varies.]

so the profit derived from introducing the second product is the same for a monopolist as it is for a new entrant.

Now, by taking a given point in  $(E, C)$  space to represent the characteristics of the market in question and shifting the two lines shown in Figure 3 in the directions indicated, we can immediately see how the equilibria vary as the size of the market increases. For a point such as  $X$ , which lies below the hatched line, as the size of the market increases, the  $(1, 1)$  equilibrium enters *before* the  $(2, 0)$  equilibrium; that is, there exists some range of market sizes such that the  $(1, 1)$  equilibrium is viable, but the  $(2, 0)$  equilibrium is not. For points such as  $Y$ , lying above the hatched line, the  $(2, 0)$  equilibrium enters first. Thus, it is in markets in which both the competition and expansion effects are weak that the fragmented equilibrium,  $(1, 1)$ , appears first. Where both effects are strong, it is the  $(2, 0)$  equilibrium which appears first.

The preceding discussion has been directed towards establishing some basic features of two-good models in terms of two easily interpretable market parameters,  $E$  and  $C$ . It is now time to look at a specific model that we can place within this framework.

### 3. The two-good case: the basic linear model

■ In this section, we use the above framework to analyze the simplest of the models of (horizontal) product differentiation which have been explored in the recent literature, namely, the linear demand schedule model. (See, for example, Shubik and Levitan (1980) and Deneckere and Davidson (1985).) Let each consumer have the same quadratic utility function defined over  $n$  varieties,

$$U(x_1, x_2, \dots, x_n) = \sum_i (\alpha x_i - \beta x_i^2) - \beta \sigma \sum_{i < j} x_i x_j + M, \quad (1)$$

where

$$M = Y - \sum_i p_i x_i$$

denotes income spent on outside goods. We assume that  $\alpha > 0$ ,  $\beta > 0$ , and  $0 \leq \sigma \leq 2$ . The above formula then defines utility over the restricted domain of  $\{x_i\}$  for which all the marginal utilities,  $U_{x_i}$ , are nonnegative. The consumer's optimal purchases will be interior to this domain for all positive price vectors,  $\{p_i\}$ . We assume that each consumer has an income,  $Y$ , that is sufficiently large to ensure that his optimal purchases are defined as the interior solution defined by setting  $U_{x_j} = p_j$  for  $j = 1, \dots, n$ ; whence, the demand schedule for good  $i$  takes the form

$$x_i = \frac{\alpha(2 - \sigma) + \sigma \sum_{j \neq i} p_j - p_i(2 + \sigma(n - 2))}{\beta(2 - \sigma)(2 + \sigma(n - 1))}. \quad (2)$$

For the one-good case, this reduces to the monopoly demand schedule,

$$x_i = \frac{\alpha - p_i}{2\beta}. \quad (3)$$

For the two-good case, the demand schedule becomes

$$x_i = \frac{\alpha(2 - \sigma) + \sigma p_j - 2p_i}{\beta(2 - \sigma)(2 + \sigma)}. \quad (4)$$

The parameter  $\sigma$  measures the degree of substitution between the products. If  $\sigma = 0$ , then the cross-product term in the utility function (1) vanishes, and (given our assumption



of “large  $Y$ ”) the demand schedule (4) collapses to the monopoly schedule (3). At  $\sigma = 2$ , on the other hand, all varieties become perfect substitutes, and the utility function collapses to

$$U = \alpha \sum_i x_i - \beta \left( \sum_i x_i \right)^2.$$

In this limit, the demand schedule for good  $i$  becomes infinitely elastic, as indicated by (4).

We now characterize a Nash equilibrium in prices (Bertrand equilibrium) for each of the subgames which follow the entry of one or two products. We assume that marginal costs are zero. (The extension to constant marginal cost is trivial.)

When only one product is offered, it follows immediately from inspection of the monopoly demand schedule (3) that

$$\pi(1, 0) = \alpha^2/8\beta.$$

We normalize by setting  $\alpha^2/8\beta = 1$  (which is equivalent to measuring all profit functions in units of  $\alpha^2/8\beta$ ). Routine calculations show that the profit functions then take the form

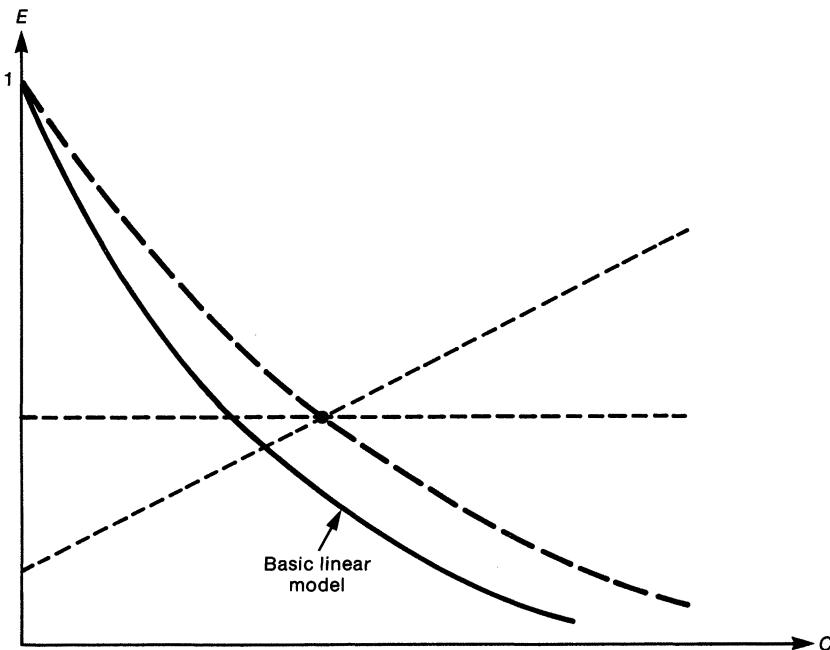
$$\pi(2, 0) = \frac{4}{2 + \sigma} \tag{5a}$$

and

$$\pi(1, 1) = \frac{16(2 - \sigma)}{(2 + \sigma)(4 - \sigma)^2}. \tag{5b}$$

We now consider the effect of varying the degree of substitutability between the two

FIGURE 4  
BASIC LINEAR MODEL



[This model traces out a negatively sloped locus in  $(E, C)$  space, as the substitution parameter  $\sigma$  varies (solid line). This locus lies wholly below the critical locus of Figure 3 (hatched line).]

products, as measured by  $\sigma$ , on the competition effect,  $C$ , and on the expansion effect,  $E$ . In terms of the parameter  $\sigma$ , the two effects may be written as

$$E(\sigma) = \frac{2 - \sigma}{2 + \sigma} \quad (6a)$$

and

$$C(\sigma) = \frac{\sigma^2}{8(2 - \sigma)}. \quad (6b)$$

When  $\sigma = 0$ , we obtain  $E = 1$  and  $C = 0$ , corresponding to Example 2 outlined above; as  $\sigma \rightarrow 2$ , we obtain  $E \rightarrow 0$  and  $C \rightarrow \infty$ , corresponding to Example 1. As  $\sigma$  varies, we trace out a negative relationship in  $(E, C)$  space, as indicated in Figure 4, *viz.*,

$$C = \frac{(1 - E)^2}{8E(1 + E)}.$$

For all  $C > 0$ , this curve lies below the critical locus,  $E = 1/(2C + 1)$ , defined above (see Figure 4), and so we can immediately deduce the results appropriate to this case, as developed in the preceding section. For example, as the size of the economy increases, the fragmented equilibrium enters first, and the monopoly equilibrium later.

#### 4. Some variations on the basic linear model

■ Viewed in terms of the elementary two-good framework developed above, the basic linear model is, in effect, a one-parameter model; its properties are determined by the value of  $\sigma$  alone, which determines  $E$  and  $C$  via equation (6). Moreover, as we have just seen, the model corresponds to one particular case arising within the present framework irrespective of the value of  $\sigma$ .

We now look at some variations of this model that lead to new  $(E, C)$  combinations that span the various zones of interest within our present framework.

Our first extension is motivated by the fact that our earlier analysis suggests that there are two quite separate influences at work, and so it seems natural to extend the basic linear model to incorporate (at least) one further parameter. There are many ways in which—in principle—this can be done, although the algebraic tractability of the resulting models varies considerably. We note a few of these in passing before exploring one of them in detail. First, the assumption of symmetry might be abandoned, making some markets (varieties) smaller (less popular) than others. This can be done by varying the coefficient  $\beta$  in the above model, *i.e.*, for the two-good case, setting

$$U = \alpha x_1 - \beta_1 x_1^2 + \alpha x_2 - \beta_2 x_2^2 - \frac{1}{2}\sigma(\beta_1 + \beta_2)x_1 x_2.$$

This will have the effect of *reducing* the expansion effect enjoyed by a firm entering the smaller market, given that the larger market is occupied. This model does not retain symmetry between the two products, however. To reintroduce symmetry, suppose that there are two groups of consumers of equal size, one with the above utility function, and the other with the utility function

$$U = \alpha x_1 - \beta_2 x_1^2 + \alpha x_2 - \beta_1 x_2^2 - \frac{1}{2}\sigma(\beta_1 + \beta_2)x_1 x_2.$$

This restores symmetry to the model; it is easily shown, moreover, that an increase in  $|\beta_1 - \beta_2|$  has the effect of *increasing* the expansion effect,  $E$ , associated with any value of  $C$ .

Note that, in this way, we can obtain all  $(E, C)$  combinations lying above the locus traced out by the basic linear model. Our next example generates some points below the locus. If we replace the Nash equilibrium in prices by joint profit maximization, we have

$$\pi(1, 1) = \pi(2, 0) = \frac{4}{2 + \sigma},$$

and so,

$$E = \frac{2 - \sigma}{2 + \sigma} \quad \text{and} \quad C = 0.$$

In this case, as  $\sigma$  varies from zero to two,  $E$  falls from one to zero.

One final variant, which leads to  $(E, C)$  combinations lying above those traced out by the basic linear model, arises in an incomplete information setting and leads to a particularly simple formulation. It goes as follows. We divide consumers into some fraction,  $f$ , who are aware of the availability of the several varieties of the good in question, and the remaining fraction, who are not aware of it. The entry of a new variety (or equivalently, the selling of the good at a new location) makes a new group of consumers aware of the good in question. Once they become aware of its existence, they automatically learn of the availability and price of all competing varieties (as with the local advertising effects of Mattewson and Winter (1984), for example).

In this setting, we let  $f(n)$  denote the number of some underlying set of consumers who are aware of the existence of (all competing varieties of) the good when  $n$  varieties are available, and let each of these consumers have the quadratic utility function (1) defined over the  $n$  varieties. The profit functions then become

$$\begin{aligned} \pi(1, 0) &= f(1), \\ \pi(2, 0) &= \frac{4}{2 + \sigma} f(2), \end{aligned}$$

and

$$\pi(1, 1) = \frac{16(2 - \sigma)}{(2 + \sigma)(4 - \sigma)^2} f(2).$$

It follows that, for a given  $\sigma$ , the competition effect is unchanged, but the expansion effect is now multiplied by  $f(2)/f(1)$ , viz.

$$E(\sigma) = \frac{2 - \sigma}{2 + \sigma} \cdot \frac{f(2)}{f(1)} \quad \text{and} \quad C(\sigma) = \frac{\sigma^2}{8(2 - \sigma)}.$$

To simplify the notation, let  $f(1) = 1$ , so that  $\pi(1, 0) = 1$  as before, and write  $f(2)/f(1)$  as  $\varphi$ . If  $\varphi = 1$ , then the locus  $(E(\sigma), C(\sigma))$  coincides with the  $(E, C)$  locus in the basic linear model. As  $\varphi$  increases, this curve shifts upwards. We assume, in line with our above interpretation, that  $f(2) \geq f(1)$ , so that  $\varphi \geq 1$ .

This model, which we shall call the extended linear model, turns out to be a useful source of examples because of its extremely simple and tractable structure, and we shall develop it further in the next section. We can illustrate its equilibria in the two-product case in terms of  $E$  and  $C$  as before. Alternatively, we can represent them in terms of the parameters  $\varphi$  and  $\sigma$ .

## 5. A three-good example

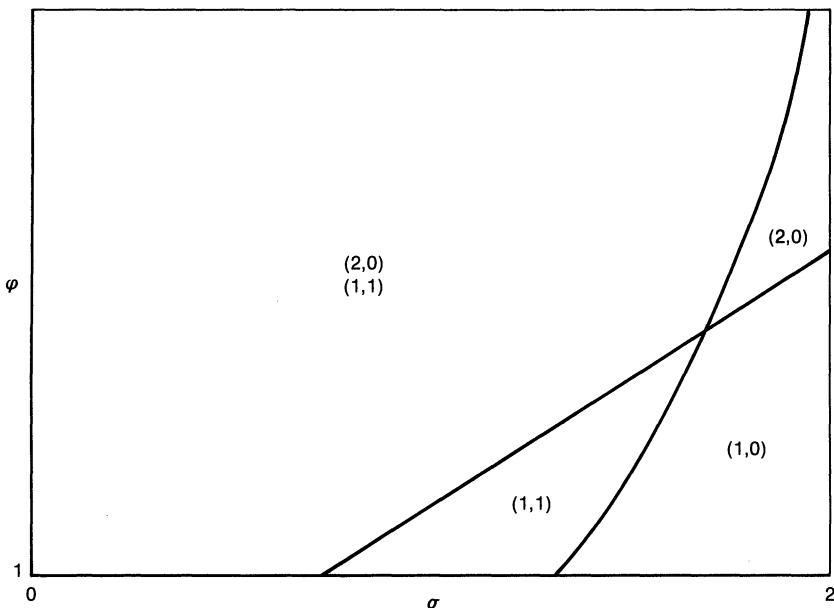
■ While the two-good version of the basic linear model and its various extensions serve to illustrate some basic principles in a simple way, they suffer from one obvious limitation: the fact that we simply constrain the permitted number of goods to be two may lead to the artificial appearance of equilibria involving two goods for certain parameter configurations,

$(\sigma, \varphi)$ , which do not constitute equilibria once a third good is allowed to enter (i.e., once the underlying utility function is defined over three, or  $n$ , goods). To illustrate this point, consider the following exercise, which relates to the effect of varying the degree of substitutability,  $\sigma$ , in the basic linear model (i.e., where we hold  $\varphi = 1$ ). In Figure 5, we can examine the effect of reducing  $\sigma$  by moving leftward along the horizontal axis, so that we move through a sequence of equilibria  $(1, 0)$ ,  $(1, 1)$ ,  $[(2, 0), (1, 1)]$ . In fact, as we show in what follows, the regime involving the equilibrium  $(2, 0)$  will not appear at all once a third good is permitted (for, at the values of  $\sigma$  in question, the optimal reply to a firm entering two goods is to enter a third good). The simple point which we wish to make here is that a full characterization of equilibria involving two goods should be based on an analysis that allows at least three goods to enter, and so on. (In principle, of course, one should allow any number,  $n$ , of goods to enter, but explicit computation of all profit functions does not appear to be feasible).

Once we introduce a third good, of course, a diagrammatic representation, whether in terms of the expansion and competition effects or in terms of the parameters of the utility function, becomes difficult. Some strong and rather arbitrary regularity conditions must be imposed in order to reduce the model to a two-parameter representation. Here, we confine ourselves to a simple example, which extends our representation of the extended linear model in terms of the utility function parameters,  $\varphi$  and  $\sigma$ , as illustrated in Figure 5 for the two-good case.

In specifying the extended linear model for the three-good case, we need to specify the numbers of consumers present in the market when one, two, or three goods are offered. To this end, we normalize by fixing  $f(1) = 1$ , so that  $\pi(1, 0) = 1$  as before, and we take  $f(2) = \varphi$  as a parameter. To keep the model as a two-parameter model (in  $\sigma$  and  $\varphi$ ), we further assume, arbitrarily, that  $f(3) = \varphi$ , i.e., that the entry of a third variety does not inform any additional consumers of the good's existence. (The qualitative features of the

FIGURE 5  
EXTENDED LINEAR MODEL



(This is an alternative representation of the equilibria of the extended linear model. The case  $\frac{1}{2} < \varepsilon < 1$  is illustrated)

results which follow continue to hold if  $f(3)$  exceeds  $\varphi$  by a sufficiently small amount. If  $f(3)$  is sufficiently large, certain zones involving one or two products will be displaced as it becomes more attractive to enter a third product). The relevant profit functions then become

$$\begin{aligned} \pi(1, 0) &= 1 & \pi(2, 0) &= \frac{4}{2 + \sigma} \varphi & \pi(3, 0) &= \frac{3}{1 + \sigma} \varphi \\ \pi(1, 1) &= \frac{16(2 - \sigma)}{(2 + \sigma)(4 - \sigma)^2} \cdot \varphi & \pi(1, 1, 1) &= \frac{(2 - \sigma)(2 + \sigma)}{4(1 + \sigma)} \cdot \varphi \\ \pi(1, 2) &= \frac{4(2 - \sigma)(2 + \sigma)^3}{(1 + \sigma)(8 + 4\sigma - \sigma^2)^2} \cdot \varphi & \pi(2, 1) &= \frac{4(2 - \sigma)(4 + 3\sigma)^2}{(1 + \sigma)(8 + 4\sigma - \sigma^2)^2} \cdot \varphi. \end{aligned}$$

As in the two-good case, we can express the conditions for the various optimal reply functions,  $(i \rightarrow j)$ , in terms of the  $\pi(\cdot)$  and  $\epsilon$ , i.e., in terms of  $\varphi$ ,  $\sigma$ , and  $\epsilon$ . These functions are shown in Figure 6. The equilibrium configurations for the model are shown in Figure 7, which may be compared to the corresponding diagram for the two-good case depicted in Figure 5. The details of these constructions are described in the Appendix. The qualitative features illustrated in Figure 7 (the relative ordering of the curves and the pattern of zones) depend *only* on some inequalities on the profit function (see (A1)–(A3) in the Appendix), which have simple economic interpretations and which hold good within the particular model studied here. These inequalities guarantee the existence of all zones shown in Figure 6, with the possible exception of the shaded zone which may or may not exist depending on the value of  $\epsilon$ .

FIGURE 6  
THE PATTERN OF OPTIMAL REPLIES IN THE EXTENDED LINEAR MODEL: THREE-GOOD CASE

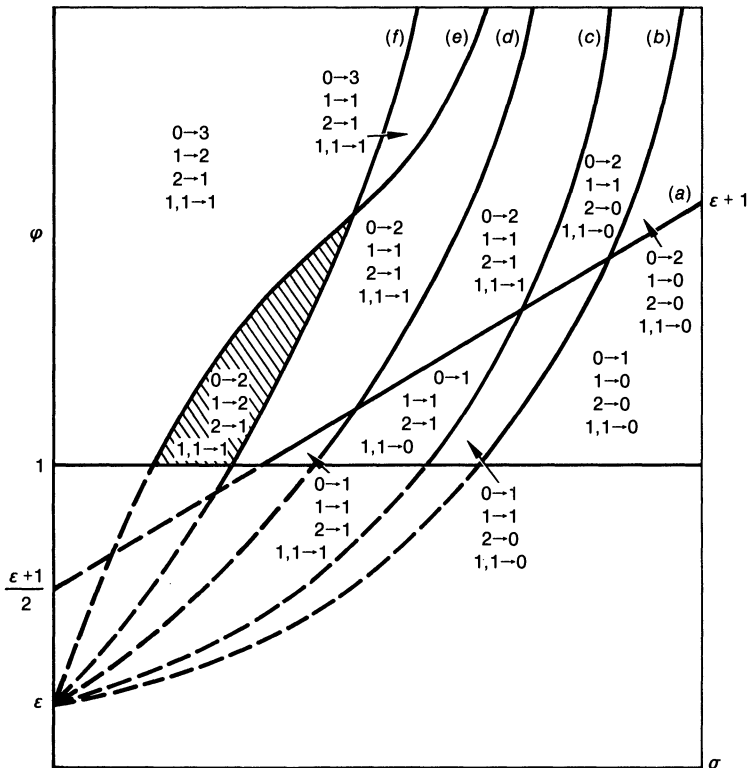
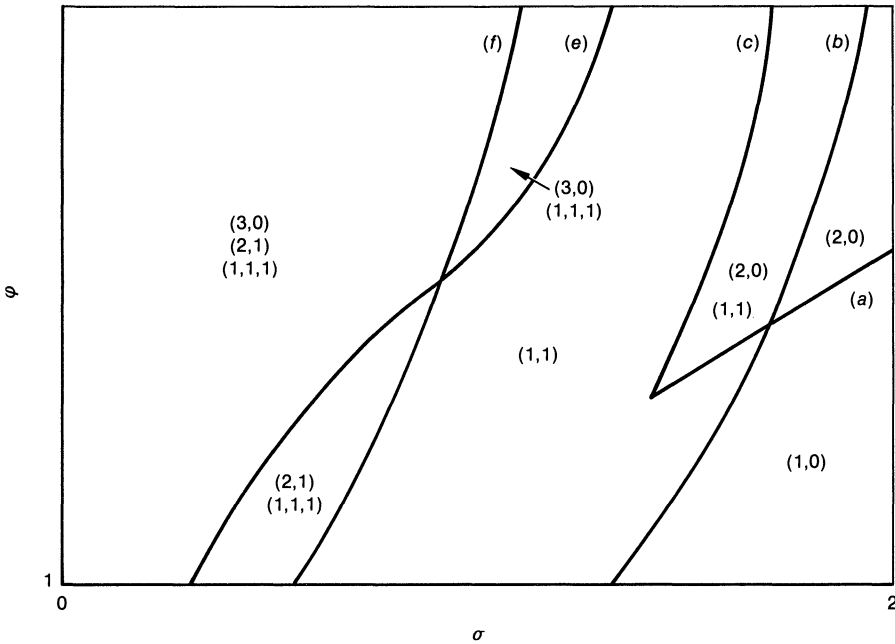


FIGURE 7

SIMULTANEOUS ENTRY EQUILIBRIA IN THE EXTENDED LINEAR MODEL: THREE-GOOD CASE



[Note that the diagram coincides with Figure 5 to the right of the line  $\pi(1,2) = \epsilon$ , labelled (c), where no optimal reply involves the introduction of a third good. The labels on the curves refer to those used in Figure 6.]

The basic linear model ( $\varphi = 1$ ) corresponds to the horizontal axis in Figure 7. As  $\sigma$  falls, the equilibria in the case of the three-good model enter in the order  $(1, 0)$ ,  $(1, 1)$ , followed by three-good equilibria. One feature of the equilibrium configurations shown in Figure 7 is worth noting: at the left-hand side of the figure, where  $\sigma = 0$ , we have the islands case identified earlier (Example 2). Here, the model reduces to the case of  $n$  independent markets, and so, once  $\epsilon$  is low enough to permit entry of even one good, we necessarily obtain *all* possible equilibrium configurations involving the full complement of  $n$  goods.

## 6. Sequential entry and market preemption

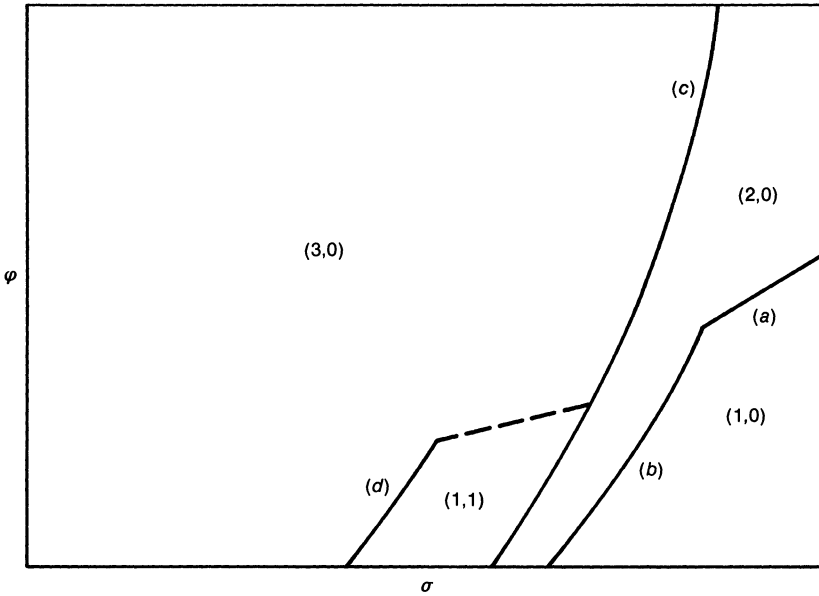
■ Up to this point we have confined our attention to the simultaneous entry case, in which neither firm enjoys a strategic advantage over its rival. Under sequential entry, one firm is assumed to enter first, and its rival(s) enters later, conditioning its product choice on the first mover's product choice. Several authors have suggested that, where this form of strategic asymmetry is present, the first mover will preempt entry by rivals, leading to a monopoly outcome. The vehicle in which this question has been examined, for the most part, is that of Hotelling's location paradigm. Here we show that, at least for the basic linear model and its extensions, this preemption result need not necessarily hold. The outcome depends on a delicate interplay between a number of factors.

We illustrate this point by means of a comparison between the simultaneous entry Nash equilibria in our three-good example of the extended linear model, as shown in Figure 7, and the corresponding sequential entry equilibria, which are shown in Figure 8. (For the analytical details, the reader is referred to the Appendix.)

As Figure 8 shows, there is, in general, a strong tendency towards preemption by the

FIGURE 8

SEQUENTIAL ENTRY EQUILIBRIA IN THE EXTENDED LINEAR MODEL: THREE-GOOD CASE



[Note that the fragmented equilibria (1,1) appear for intermediate values of  $\sigma$  (see text). The hatched line segment is not a boundary on the optimal reply map shown in Figure 6(see Appendix).]

first mover; the only fragmented equilibrium to appear is (1, 1). In order to develop the intuition underlying the appearance of (1, 1) as an equilibrium in the sequential entry case, we first note a basic property of these models. *If the first mover can preempt entry by offering k goods, (i.e.,  $k \rightarrow 0$ ), then a configuration of the form (1, 1, . . . 1) involving k single-product firms cannot be supported as an equilibrium under sequential entry.* For example, (1, 1) cannot be an equilibrium under sequential entry if the first firm can impose the outcome (2, 0) by introducing two products, since, if the rival's optimal reply to the entry of one product is to follow with a second product, the profits earned by that rival from this second product exceed the entry cost,  $\epsilon$ . But the first mover can achieve at least this (incremental) profit level by introducing the second product and then setting whatever prices hold under the fragmented configuration (1, 1); indeed, by setting optimal (monopoly) prices, it can achieve more. Hence, it will always pay the first mover to preempt.

A corollary of this argument is that in an  $n$ -good model, fragmented equilibria under sequential entry can involve at most  $(n - 1)$  goods.

The possibility that (1, 1) may appear as an equilibrium under sequential entry turns on the fact that for certain underlying parameter combinations,  $(\sigma, \phi, \epsilon)$ , the optimal reply function has the following structure:

$$(2 \rightarrow 1), \quad \text{but} \quad (1, 1 \rightarrow 0).$$

That is, if the first mover enters two products, this will not deter further entry. (It takes three products to preempt.) On the other hand, if the first and second movers each enter one product, then no further entry occurs.

Now the profitability of entering a third product will, of course, be less in the case in which prices are more depressed by competition. So the above pair of requirements may indeed be satisfied for some model parameters. It is clear, moreover, that the requirement that  $(1, 1 \rightarrow 0)$  will fail (that is, we have  $(1, 1 \rightarrow 1)$ ) insofar as the expansion effect is

strong or the competition effect is weak. It is also clear that the requirement that  $(2 \rightarrow 1)$  will fail (that is, we have  $(2 \rightarrow 0)$ ) insofar as the expansion effect is weak or the competition effect is strong. Thus, in terms of our substitution parameter,  $\sigma$ , it follows that these conditions will fail at both extremes and will be satisfied in some intermediate range of values. (See the areas in Figure 8.)

These conditions are, however, necessary but not sufficient for the appearance of  $(1, 1)$  as an equilibrium. Given these conditions, the first mover may choose either to enter one product, yielding net profits of  $\pi(1, 1) - \epsilon$ , to enter two products, yielding  $\pi(2, 1) - 2\epsilon$ , or to enter three, yielding  $\pi(3, 0) - 3\epsilon$ . Since the zone in which these conditions are satisfied lies below curve  $(f)$  in Figure 6, the second of these three possibilities is dominated by the first. What we need, then, is simply that  $\pi(3, 0) - \pi(1, 1) < 2\epsilon$ . Now, both  $\pi(3, 0)$  and  $\pi(1, 1)$  increase in direct proportion to  $\varphi$ , as can be seen from the explicit representation of the profit functions cited earlier. Hence, this condition is violated for sufficiently large  $\varphi$ . Explicit calculation shows that if  $\varphi = 1$  (the case of the basic linear model) or if  $\varphi$  is sufficiently close to one, there exists a range of  $\epsilon$  values for which this condition is satisfied. Our point, then, is that while there is indeed a strong tendency toward monopolization in this context, the preemptive outcome does not hold good in general, as our present counterexample indicates.<sup>5</sup>

Finally, it may be worth remarking on the status of simultaneous versus sequential entry models in this context. Both of these formulations are special cases and embody rather strong assumptions on the strategy space. It is possible, for example, to have (dynamic) models that preserve strategic symmetry while allowing (mixed-strategy equilibrium) outcomes in which firms actually enter sequentially. (See, for example, Fudenberg and Tirole (1985).) Thus, the fact that firms are observed to enter markets sequentially is not in itself evidence of the kind of strategic asymmetry represented by the first-mover advantage incorporated into sequential entry models.<sup>6</sup>

## 7. Summary and conclusions

■ The main aim of this article is to illustrate a particular way of parameterizing some simple models of equilibria with multiproduct firms in terms of the competition effect and expansion effect defined above. These two effects arise in various ways in the existing literature. Wolinsky (1986) focuses on what we have called the competition effect, for example, while the clearest statement of the role played by what we have labelled the expansion effect is found in Scherer (1979). (Other relevant papers include Brander and Eaton (1984) and Bhatt (1987).) Our aim here has been to point out that a useful way of interpreting and classifying results in this area lies in representing (parameterizing) the underlying model(s) directly in terms of these two effects.

It should be emphasized that this manner of representing the models is offered, not as an alternative to the more familiar approaches, but as a complement to these. Its role is to aid intuition in regard to the interpretation of the (often complex) patterns of equilibrium outcomes found in these models; indeed, once many goods are present, any representation which uses a comparatively small number of parameters will impose some correspondingly strong restrictions on the underlying demand structure. Often, this leads to difficulties of interpretation.

<sup>5</sup> It may be worth noting that in the present model, in contrast to Bernheim (1984), our first mover does not have any entry-detering strategies of the type posited in that article.

<sup>6</sup> Although this point appears simple, it seems worth emphasizing in view of the apparently widespread view that sequential entry models are "more reasonable" than simultaneous entry models. The fact that they generate predominantly monopolized outcomes is then seen as an unattractive feature of these models, given that casual empiricism suggests a preponderance of nonmonopoly outcomes.



In the present article, we have juxtaposed a familiar and useful parameterization in terms of the elasticity of substitution with the alternative competition-effect/expansion-effect parameterization. We explored the relationship between the elasticity of substitution and these two effects, using the relatively simple two-good case as a guide. We were then able to offer a fairly clear intuitive explanation for the rather complex pattern of results obtained when we set up the three-good case in terms of the elasticity of substitution parameterization. (See the discussion of sequential equilibria in Section 6).

In the many-good case, the best choice of parameterization will doubtless vary with the model and the application in question. Our argument is that the easiest way to interpret the results may still be by reference to a (formal or informal) appeal to the competition-effect/expansion-effect distinction.

In developing the arguments set out in the present article, we have chosen, for simplicity, to work in terms of a nonspatial approach to product differentiation. The reason for doing this lay in the relative tractability of these models. We feel, however, that the same arguments could usefully be developed in a spatial framework, with similar results.

We also chose to confine ourselves to pure-strategy equilibria. In the simultaneous-entry case, mixed-strategy equilibria will also be present. Indeed, these are symmetric games, in which the only pure-strategy equilibria are asymmetric and thus come in mirror-image sets; these games also have a symmetric equilibrium in mixed strategies. A characterization of these latter equilibria lies outside our present scope.

Our main emphasis has been on the analysis of the strategically symmetric setup, in which firms enter simultaneously. We have also looked at the strategically asymmetric situation, in which one firm has a first-mover advantage. Contrary to most of the literature, we have found that in the latter setting, preemption of the market by the first mover does *not* necessarily occur, although there is a strong tendency toward such outcomes. A fragmented structure can only arise in this context for very particular parameter combinations.

One reason why a number of articles in this area have found preemption to occur appears to lie in the fact that they use particular "Hotelling type" models, which incorporate assumptions ensuring that the market expansion effect, defined above, is zero. (The most precise statement along these lines is that of Bonanno (1987) which explicitly assumes that all consumers buy one unit of the good given any configuration of products offered.) The range of parameter values at which we found a fragmented outcome in our present analysis lay in the midrange of the substitution parameter and thus involves both a positive competition effect and a positive expansion effect. Models which assume a zero expansion effect will tend to exclude some cases of interest. This underlines our general argument that a full understanding of the way in which demand-side factors impinge on equilibrium outcomes requires that we look at (a range of) examples that span various possible combinations of competition and expansion effects.

## Appendix

■ We provide here the analytical details pertaining to the three-good case within the extended linear model, as described in the text. The results for the other cases considered in the text follow readily as special cases of our present analysis.

The relevant profit functions are gathered here for convenience:

$$\begin{aligned} \pi(1, 0) &= 1 & \pi(2, 0) &= \frac{4}{2 + \sigma} \varphi & \pi(3, 0) &= \frac{3}{1 + \sigma} \varphi \\ \pi(1, 1) &= \frac{16(2 - \sigma)}{(2 + \sigma)(4 - \sigma)^2} \cdot \varphi & \pi(1, 1, 1) &= \frac{(2 - \sigma)(2 + \sigma)}{4(1 + \sigma)} \cdot \varphi \\ \pi(1, 2) &= \frac{4(2 - \sigma)(2 + \sigma)^3}{(1 + \sigma)(8 + 4\sigma - \sigma^2)^2} \cdot \varphi & \pi(2, 1) &= \frac{4(2 - \sigma)(4 + 3\sigma)^2}{(1 + \sigma)(8 + 4\sigma - \sigma^2)^2} \cdot \varphi. \end{aligned}$$

In order to partition the  $(\varphi, \sigma)$  diagram into relevant zones, we introduce five curves in  $(\varphi, \sigma)$  space, defined as follows:

- (a)  $\pi(2, 0) - \pi(1, 0) = \epsilon$ ;
- (b)  $\pi(1, 1) = \epsilon$ ;
- (c)  $\pi(1, 2) = \epsilon$ ;
- (d)  $\pi(1, 1, 1) = \epsilon$ ;
- (e)  $\pi(3, 0) - \pi(2, 0) = \epsilon$ ;
- (f)  $\pi(2, 2) - \pi(1, 1) = \epsilon$ .

From the above expressions, a number of relations follow, which serve to fix the relative positions of these curves on the  $(\varphi, \sigma)$  diagram. First,

$$\pi(1, 1, 1) \leq \pi(1, 2) \leq \pi(1, 1). \quad (A1)$$

Here, the first inequality reflects the fact that the merger of rivals raises a firm's profit. The second reflects the fact that an expansion of a rival's product range reduces a firm's profit. (This latter relation holds good in the special case in which  $f(3) = f(2) = \varphi$ , which we have chosen to analyze here; as  $f(3)$  rises relative to  $f(2)$ , this inequality will be reversed.) Second,

$$\pi(3, 0) - \pi(2, 0) \leq \pi(1, 1, 1) \quad (A2)$$

and

$$\pi(2, 1) - \pi(1, 1) \leq \pi(1, 1, 1). \quad (A3)$$

This states that the return to an incumbent from having entered a third product is less than the return to an entrant. (Again, this happens to be true for our special case  $f(3) = f(2) = \varphi$  but will be reversed if  $f(3)$  rises relative to  $f(2)$ .)

We may now construct Figure 6, which shows the pattern of optimal replies. We assume  $\epsilon < 1$  to ensure the viability of at least one good. Note that all five functions (a)–(f) are increasing in  $\sigma$ . Function (a) starts from the intercept  $(\epsilon + 1)/2 < 1$  and increases linearly. The curves (b)–(f) start from the negative intercept  $\epsilon$  and increase to infinity as  $\sigma \rightarrow 2$ . The order of the curves is fixed by the inequalities cited above, in the manner shown in the diagram. Curve (d) intersects (a) once only, at a point above the  $\varphi = 1$  line. Curve (e) starts above (f) but ends below it. (There may be more than one crossing point.) The zones of interest are those below curve (d), i.e., those involving equilibria in which one or two goods are present. All of these zones are nonempty (for a suitable range of  $\epsilon$ ) under our assumptions.

Figure 7 shows the simultaneous entry Nash equilibria; these follow immediately from inspection of Figure 6.

The sequential entry equilibria are sketched in Figure 8. As indicated in the text, the characterization of these equilibria requires the introduction of one further curve, defined by

$$(g) \quad \pi(3, 0) - \pi(1, 1) = 2\epsilon,$$

which is shown by a hatched line in Figure 8. This curve begins from the intercept  $\varphi = \epsilon$  and ends at  $\varphi = 2\epsilon$  for  $\sigma = 2$ . It can be shown that for suitable  $\epsilon$  (in particular for all  $\epsilon > .555$ ), the (1, 1) zone is nonempty.

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