**Natural Oligopolies**

Avner Shaked; John Sutton

*Econometrica*, Vol. 51, No. 5 (Sep., 1983), 1469-1484.

Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198309%2951%3A5%3C1469%3ANO%3E2.0.CO%3B2-3

*Econometrica* is currently published by The Econometric Society.

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at http://uk.jstor.org/about/terms.html. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at http://uk.jstor.org/journals/econosoc.html.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

---

JSTOR is an independent not-for-profit organization dedicated to creating and preserving a digital archive of scholarly journals. For more information regarding JSTOR, please contact support@jstor.org.

http://uk.jstor.org/
Sun Feb 6 18:30:32 2005

*Econometrica*, Vol. 51, No. 5 (September, 1983)

# NATURAL OLIGOPOLIES[1]

BY AVNER SHAKED AND JOHN SUTTON[2]

In a market where firms offer products which differ in quality, an upper bound may exist to the number of firms which can coexist at a noncooperative price equilibrium. We fully characterize the conditions under which this possibility arises.

## 1. BACKGROUND

THE PRESENT PAPER is concerned with the analysis of price competition in markets where consumers purchase a single unit of some good, the alternative brands of which differ in quality. The defining characteristic of this kind of product differentiation is that, were any two of the goods in question offered at the same price, then all consumers would agree in choosing the same one, i.e. that of "higher quality."

Little attention has been paid to the analysis of competition in this "vertical differentiation" case, in contrast to the widely studied case of "horizontal differentiation" where the defining characteristic is that consumers would differ as to their most preferred choice if all the goods in question were offered at the same price. The standard paradigm is that of the "locational" and associated models. In such models, the number of firms in the industry increases indefinitely as the fixed costs associated with entry decline, or, equivalently, as the size of the economy expands. That this can happen, depends in turn on the fact that the market can support an arbitrarily large number of firms, each with a positive market share and a price in excess of unit variable cost. This property is of fundamental importance: for, as firms become more closely spaced, price competition between them implies that prices approach the level of unit variable costs. It is this "Chamberlinian" configuration which forms the basis of the notion of "perfect monopolistic competition." (See [**5**].)

The central question posed in the present paper is whether this property will be available in the "vertical differentiation" case. In a large class of cases, it turns out not to hold; in such cases, no passage to an atomistic, competitive, structure will be possible. However low the level of fixed costs, and independently of any considerations as to firms' choices of product, the nature of price competition in itself ensures that only a limited number of firms can survive at equilibrium.

[1]This paper was presented at the "Agglomeration in Space" meeting (Habay-la-Neuve, Belgium, May 1982) organized with the financial support of the Fonds National de la Recherche Scientifique of Belgium.

Our thanks are due to the International Centre for Economics and Related Disciplines at LSE for financial support.

[2]We would like to acknowledge the help of two referees, whose comments prompted a considerable improvement in the exposition of these results.

## 2. INTRODUCTION

We will be concerned, in what follows, with elaborating a condition which is necessary and sufficient to allow an arbitrarily large number of firms to co-exist with positive market shares, and prices exceeding unit variable costs, at a Nash equilibrium in prices. It may be helpful to begin, in the present section, by setting out this condition in a quite informal manner.

Suppose a consumer with income $Y$ purchases *one unit* of a product of quality $u$, at price $p(u)$, thereby achieving a level of utility given by, say, the function

$$u \cdot (Y - p(u)).$$

Let each of a number of firms produce one product of some quality $u$ subject to constant unit variable cost, $c(u)$. We consider a hypothetical situation in which a number of products are offered at a price equal to their respective levels of unit variable cost. (The relevance of this case lies in the fact that some firms may not be able to achieve positive sales at a price which covers variable cost, and it is this which limits the number of firms surviving at equilibrium.)

Figure 1 shows the function $uc(u)$. Take a line of slope $Y_1$ through the point $(u, uc(u))$. Then the vertical intercept $AB = u \cdot (Y_1 - c(u))$ represents the utility attained by a consumer of income $Y_1$ in purchasing a product of quality $u$ at price $c(u)$. Again referring to the Figure, the consumer of income $Y_1$ is indifferent between $u$ at price $c(u)$ and $v$ at price $c(v)$. Finally, we illustrate the optimal quality choice for a consumer of income $Y_2$, who can purchase any quality at unit variable cost, as the point of tangency $q$ (chosen to maximize the associated intercept).

We are now in a position to identify a fundamental dichotomy which forms the basis of our subsequent analysis. Let consumer incomes lie in some range $[a, b]$ and suppose unit variable cost rises only slowly with quality. Then, if two products are made available at unit variable cost, all consumers will agree in

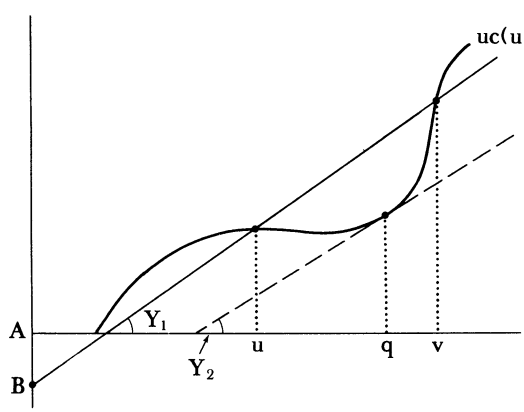


FIGURE 1.

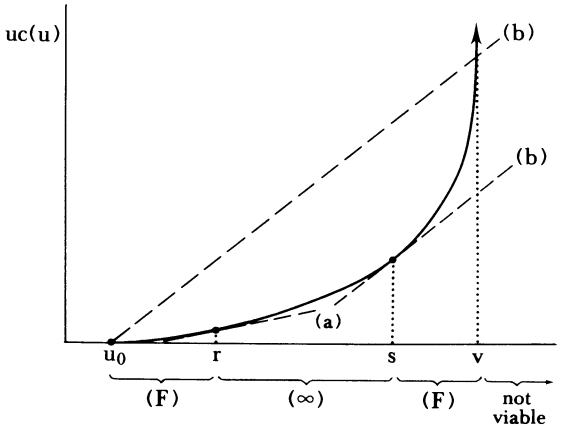


FIGURE 2(i).

preferring the higher quality product, i.e. all consumers rank the products in the same order. On the other hand, consider the cost function shown in Figure 2(i); here, we illustrate an example in which $uc(u)$ is convex, and we identify two points of tangency $r$ and $s$ where the slope of the curve coincides with our extreme income values $a$ and $b$. Here, if any set of products lying in the interval below $r$ is made available at unit variable cost, all consumers will agree in ranking them in increasing order of quality; and for a set of qualities drawn from the interval above $s$, and sold at unit variable cost, consumers will agree in ranking them in decreasing order of quality. In the intermediate quality range, however, consumers will differ in their ranking of products, at unit variable cost. Now this is reminiscent of the "location" paradigm noted above, and we shall show in the sequel that the basic property alluded to earlier continues to hold good here—an unbounded number of firms may coexist with positive market shares and prices exceeding unit variable cost, at equilibrium.

Now, in this "location-like" situation, the manner in which an arbitrarily large number of firms may be entered, is straightforward: for, within a certain interval, we can always insert an additional firm (product) *between* two existing firms, without precipitating the exit of any other firm.

A second, and quite distinct, kind of situation may arise, however, which is also consistent with the coexistence of an unbounded number of competing firms. While many subcases of this possibility arise, all are quite analogous, and a clear illustration of the mechanism involved is provided by the following example. Suppose costs were zero; and suppose further that the range of incomes extends downwards to zero. In this case, an unbounded number of products may be entered: for no product can have zero profits at equilibrium unless some higher quality product sells for price zero (remember the consumer of income zero is indifferent between all products at price zero, so any product can otherwise find some positive price at which it can earn positive profits). But it now suffices to notice that the highest quality product in the sequence will *not* be

sold at price zero; for clearly there exists some price at which it can earn positive profits.

Hence in this situation, an infinite number of products may again be entered—but now, the method by which they are entered is by introducing new products of successively lower quality at the end of the existing range.

What characterizes this situation, and all analogous subcases, is the presence of a consumer—here the consumer of income zero—who is (locally) indifferent between alternative products, at unit variable cost (i.e. the derivative of his utility score with respect to product quality is zero).

The condition which we develop below is designed to exclude these two types of situation. Where that condition is satisfied, all consumers will be agreed in ranking the products in the same strict order, at unit variable cost. When this is so, it follows that one firm could set a price which would drive the remaining firms out of the market. This will not in general occur at equilibrium, however. (For an elementary example, see [**4**].) What we show is that, in this case, *there will exist an upper bound independent of product qualities, to the number of firms which can coexist with positive market shares and prices exceeding unit variable costs, at a Nash Equilibrium in prices.*

It is worth stressing immediately that this property is extremely strong: the bound we define depends only on the pattern of tastes and income distribution and is independent of the qualities of the various products offered.

The mechanism through which the result comes about, is that whatever the set of products entered, competition between certain "surviving" products drives their prices down to a level where every consumer prefers either to make no purchase, or to buy one of these surviving goods *at its equilibrium price*, rather than switch to any of the excluded products, *at any price sufficient to cover unit variable cost.*

The implications of this "finiteness property" are far-reaching; for, if the technology is such that unit variable cost rises only slowly with quality, so that the "finiteness property" holds everywhere, then *irrespective of the manner in which product quality is chosen by firms*, the familiar "limiting process" by which we might arrive at a competitive outcome cannot occur. The number of firms which can coexist at equilibrium is no longer limited by the level of fixed costs, as in the familiar case, but is instead determined by the upper bound which we identify below. This means, in turn, that the effect of a further reduction in fixed costs, or an increase in the extent of the market, will, once that bound is attained, have no effect on the equilibrium number of firms in the industry.

It is this configuration, in which the finiteness property holds over the relevant quality range, which we label a *natural oligopoly*.

The "finiteness" property has already been demonstrated for a special case in which all costs are zero, by Jaskold Gabszewicz and Thisse [**2**]. The aim of the present paper is to provide a necessary and sufficient condition for such an outcome, where costs are present.

Finally, we emphasize that we shall not be concerned here with the question of optimal quality choice by firms; the "finiteness" property is independent of such

considerations. The range of qualities available on the market will, in general, of course depend *inter alia* on the relation between *fixed costs* (including R & D), and product quality. (We have elsewhere examined this problem of quality choice [**7**, **8**, **9**].) Here, however, we will take qualities as given and all such costs as sunk costs; and so we will be concerned only with variable cost.[3]

The structure of the paper is as follows. Section 3 presents the model, and in Section 4 we examine price equilibrium. In Section 5 we present a necessary and sufficient condition for "finiteness;" Section 6 is devoted to a discussion of the results.

## 3. THE MODEL

A number of firms produce distinct, substitute, goods. We label their respective products by an index $k = 1, \ldots, n$, where firm $k$ sells product $k$ at price $p_k$. (We take the goods to be distinct, here, since if two or more goods are identical, then all have price equal to unit variable cost, at a Nash Equilibrium in prices, by the usual Bertrand argument. The case where some firms produce an identical quality level is considered in the proof of Proposition 3 below.)

Assume a continuum of consumers identical in tastes but differing in income; incomes are uniformly distributed over some range, $0 < a \leq t \leq b$.

Consumers make indivisible and mutually exclusive purchases from among our $n$ substitute goods, in the sense that any consumer either makes no purchase, or else buys exactly one unit from one of the $n$ firms. We denote by $U(t,k)$ the utility achieved by consuming one unit of product $k$ and $t$ units of "other things" (the latter may be thought of as a Hicksian "composite commodity," measured as a continuous variable), and by $U(t,0)$ the utility derived from consuming $t$ units of income only.

Assume that the utility function takes the form

$$U(t,k) = u_k \cdot t \qquad (k = 1, \ldots, n) \tag{1}$$

and

$$U(t,0) = u_0 \cdot t$$

with $0 < u_0 < u_1 < \cdots < u_n$ (i.e. the products are labelled in increasing order of quality).

(The particular forms of the utility function, and income distribution, used here, play no crucial part in what follows. See Section 6 below.) Let

$$r_{k-1,k} = \frac{u_k}{u_k - u_{k-1}}$$

(whence $r_{k-1,k} > 1$). Then we may define the income level $t_k$ such that a consumer with this income will be indifferent between good $k$ at price $p_k$ and

[3] Labor, materials, and divisible capital equipment. It is of course *long run* unit variable costs which are relevant.

good $k-1$ at price $p_{k-1}$, by setting

$$u_{k-1} \cdot (t_k - p_{k-1}) = u_k \cdot (t_k - p_k)$$

to obtain

$$t_k = p_{k-1}(1 - r_{k-1,k}) + p_k r_{k-1,k} \tag{2}$$

$$= p_{k-1} + (p_k - p_{k-1}) \cdot r_{k-1,k}$$

and

$$t_1 = p_1 r_{0,1}.$$

It is immediate from inspection of our utility function that a consumer with income above $t_k$ will strictly prefer the higher quality good $k$, and conversely: the function (1) is designed to capture the property that richer consumers are willing to pay more for a higher quality product.

Given any set of prices, then, certain firms have positive market shares bounded by marginal consumers (income levels), firm $k$ selling to consumers of income $t_k$ to $t_{k+1}$ ($t_k$ to $b$ for firm $n$); the market shares of the higher quality firms corresponding to higher income bands. We shall find it convenient below to identify, sometimes, the set of firms with positive market shares; it is important to remember that a firm may be "just" excluded in the sense that $t_k = t_{k-1}$ so that it has market share zero: here an infinitesimal fall in its price, or an infinitesimal rise in the price set by either of this firm's neighbors, will cause its market share to become positive.

Let $c(u)$ represent the level of unit variable cost as a function of the quality of the product; it is assumed independent of the level of output. We will assume that $c(u)$ is continuously differentiable (but see Section 6 below). We will write $c(u_k)$ as $c_k$ in what follows.

The profit of any firm $k$ now becomes, for $k = 1, \ldots, n-1$,

$$\pi_k = (p_k - c_k)(t_{k+1} - t_k), \qquad p_k \geq c_k,$$

$$\pi_k = 0 \qquad \text{otherwise.}$$

From this we may deduce a necessary condition for profit maximization. For firm $k$ we require

$$(t_{k+1} - t_k) - (p_k - c_k)(r_{k,k+1} + r_{k-1,k} - 1) \leqq 0,$$

which is the requirement that an *increase* in $k$'s price reduces profit. The corresponding inequality required to ensure that a *reduction* in $k$'s price reduces profits splits into a number of cases according as $k$'s nearest neighbor from above, and/or from below, has market share zero.

## 4. PRICE EQUILIBRIUM

We seek a noncooperative price equilibrium (Nash Equilibrium), viz.: a vector of prices $p_n^*, p_{n-1}^*, \ldots, p_1^*$, such that, for all $k$, given the prices set by the remaining firms, $p_k^*$ is the profit maximizing price for firm $k$.

We begin by establishing the existence of such an equilibrium:

LEMMA 1: *For any given products $u_1, u_2, \ldots, u_n$ and corresponding prices $p_1, p_2, \ldots, p_{k-1}, p_{k+1}, \ldots, p_n$, for all k, the profit of the kth firm is a single peaked function of its price.*

PROOF: Note that for $p_k$ sufficiently high the sales of firm $k$ are zero; similarly, for $p_k = c_k$ revenue equals zero. We establish that for intermediate values of $p_k$, profit $\pi_k$ is a single peaked function of $p_k$.

We note that the market share of firm $k$ is sandwiched between that of two neighboring firms, $k-1$ and $k+1$. As its price falls, it will at some point squeeze out one or both of these neighboring firms, thus acquiring a new "neighbor."

Consider the function

$$\pi_k = (p_k - c_k)(t_{k+1} - t_k)$$

which is formally defined for all $p_k$, and which coincides with the profit of firm $k$ over that range of $p_k$ such that firm $k$ has a positive market share bounded by $(k-1)$ and $(k+1)$. We first show that any turning point of $\pi_k$ is a maximum, i.e. $\pi_k$ is single peaked. For, differentiating with respect to $p_k$ we have

$$\begin{aligned} \pi_k' &= (p_k - c_k)(1 - r_{k,k+1} - r_{k-1,k}) + t_{k+1} - t_k, \\ \pi_k'' &= 2(1 - r_{k,k+1} - r_{k-1,k}) < 0. \end{aligned} \tag{3}$$

Suppose now that $p_k$ falls so far as to drive one of its neighbors, $k-1$ say, out of the market. Then its new neighbors are $k-2$, $k+1$. Again, the profit function $\hat{\pi}_k$ for $k$ sandwiched between $k-2$, $k+1$, is a single peaked function of $p_k$. Moreover, at the price at which the market share of $k-1$ becomes zero, i.e. $t_k = t_{k-1}$, we shall show that

$$\hat{\pi}_k' > \pi_k'$$

so that if $\pi_k$ is increasing at this point then a fortiori $\hat{\pi}_k$ is increasing. From this it follows that the profit function is globally single peaked.

To show that $\hat{\pi}_k' > \pi_k'$ we compare $\pi_k'$ as defined by (3) with

$$\hat{\pi}_k' = (p_k - c_k)(1 - r_{k,k+1} - r_{k-2,k}) + t_{k+1} - t_{k-2}$$

and using $t_k = t_{k-1}$, and since (by inspection of the definitions of $r_{k-1,k}$) we have $r_{k-2,k} < r_{k-1,k}$, our result follows. *Q.E.D.*

From this we obtain the following proposition.

PROPOSITION 1: *For any set of products* $1, \ldots, n$ *a noncooperative price equilibrium* $p_1, \ldots, p_n$ *exists.*

PROOF: This follows immediately by appealing to the fact that each firm's profit function is quasi-concave by virtue of Lemma 1 [**1**, p. 152]. *Q.E.D.*

## 5. THE FINITENESS PROPERTY

We proceed by defining the following property:

DEFINITION 1: An interval $[\underline{u}, \overline{u}]$ of qualities possesses the *finiteness property* if there exists a number $K$ such that, at any Nash equilibrium involving a number of products drawn from this interval, at most $K$ enjoy positive market shares and prices exceeding unit variable cost.

REMARK: $K$ will depend on the range $[a, b]$ of consumer incomes.

We note that if this property does not hold, then it follows that for all $N$, there exists a sequence of at least $N$ products coexisting with positive market shares, and prices exceeding unit variable cost, at a Nash Equilibrium in prices.

We now turn to the condition required to ensure this "finiteness" property.

We begin by defining a function $t(u, v)$, which is the income level at which a consumer is indifferent between goods $u$ and $v$, where both are available at unit variable cost. Setting

$$u(t - c(u)) = v(t - c(v))$$

we have

$$t(u, v) = \frac{vc(v) - uc(u)}{v - u} = c(v)r_{uv} + c(u)(1 - r_{uv})$$

where $r_{uv} = v/(v - u)$.

Consumers of income above $t(u, v)$ strictly prefer the higher quality good; and conversely.

We begin by deleting from our interval $[\underline{u}, \overline{u}]$ any products for which $t(u_0, u) > b$; such products will not be viable in that even the richest consumer will prefer to make no purchase, rather than buy such a good, even at cost. In general this deletion will leave a number of closed subintervals of quality.

We now state a condition which will be shown, in Propositions 2, 3 below, to be necessary and sufficient for the finiteness property to hold on any such subinterval, whence it follows immediately that it is necessary and sufficient for finiteness on $[\underline{u}, \overline{u}]$.

We define the function $t(u, u)$,

$$t(u, u) = \lim_{v \to u} t(u, v).$$

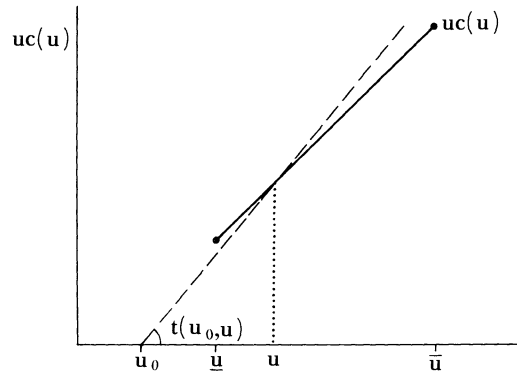


FIGURE 2(ii).

Since $c(u)$ is differentiable, and

$$t(u,u) = c(u) + uc'(u),$$

it follows that $t(u,u)$ is well defined.

Now $t(u,u)$ may be interpreted directly, as follows. If all goods are made available at unit variable cost, then a consumer of income $t(u,u)$ attains *either* a maximum, *or* a minimum, of utility, by choosing $u$. To see this, consider the problem

$$\max_u u(Y - c(u))$$

which leads to the first order condition (Figure 1)

$$Y = (uc(u))' = uc'(u) + c(u) = t(u,u).$$

Our condition for finiteness is that no such consumer is present; to exclude such cases we require that *either* (a) $t(u,u) \notin [a,b]$, *so that all such consumers lie outside our range of incomes*, *or* (b) $t(u,u) < t(u_0,u)$, *so that any such consumer strictly prefers to make no purchase, rather than buy* $u$. (The latter case is illustrated in Figure 2(ii).) Combining these two cases we obtain the following condition.

CONDITION (F): $t(u,u) \notin [\max(a, t(u_0,u)); b]$.

REMARK 1: Condition (F) implies that for all $(u,v)$, $t(u,v) \notin [\max(a, t(u_0, u)); b]$.

This latter condition excludes the appearance of any consumer indifferent between two goods $u$ and $v$, at unit variable cost. That it follows from (F) is immediate from inspection of Figure 1. (We might of course replace $t(u_0,u)$ here by $t(u_0,v)$, for if a consumer prefers $u_0$ to $u$, and is indifferent between $u$ and $v$, he prefers $u_0$ to $v$ also.)

REMARK 2: Since $t(u,v)$ is continuous, it follows that $t(u,v)$ is uniformly (in $u,v$) bounded away from its corresponding interval $[\max(a, t(u_0 u)); b]$.

REMARK 3: We here note two cases which will be of interest below.

For any two goods $u$ and $v$, with $v > u$: (i) If $t(u,v) < \max(a, t(u_0, u))$, then any consumer willing to buy $u$ at cost (in the sense of preferring this to making no purchase) will certainly prefer to buy $v$ rather than $u$, if both are made available at cost. (ii) If $t(u,v) > b$, then any consumer willing to buy $v$ at cost, will prefer to buy $u$ rather than $v$ if both goods are made available at cost.

In the Introduction, we noted that there are two ways in which the "finiteness" property may fail to hold; we can now interpret the restriction imposed by Condition (F) in terms of these two possibilities.

The first way in which Condition (F) may be violated is by the appearance of some $u \in (\underline{u}, \overline{u})$, such that a consumer of some income $t \in (a,b)$ attains a *maximum* of utility by consuming $u$, all products being available at unit variable cost. This case violates the requirement that all consumers rank products in the same order at cost; it is analogous to the familiar "location" models, in that consumers with income above (below) $t$ will prefer a quality above (below) $u$.

The remaining cases in which Condition (F) is violated are all analogous to the case noted in the Introduction, in which costs are zero, while the range of incomes extends to zero. These include the possibility that, for some $u \in (\underline{u}, \overline{u})$, a consumer of some income $t \in (a,b)$ attains a *minimum* of utility. (The analogy between this case to that in which the range of incomes extends to zero is developed in the proof of Proposition 3 below.) They also include a number of boundary cases; for example, where a consumer of income $t = a$ attains maximum utility at $u = \underline{u}$.

We now turn to our central results, showing that Condition (F) is necessary and sufficient for finiteness.

PROPOSITION 2 (Sufficiency): *Condition* (F) *implies the finiteness property.*

PROOF: First note, by virtue of the continuity of $t(u,v)$, and the differentiability of $c(u)$, that (F) implies that *either* $t(u,v) > b$ for all $(u,v)$ *or* $t(u,v) < \max(a, t(u_0, u))$. (Note Remark 1 above.) In the former case, unit variable cost rises so steeply with quality that all consumers rank products (at cost) in decreasing order of quality; while in the latter case $c(u)$ is "sufficiently flat," and all consumers rank products (at cost) in increasing order of quality.

We here establish the result for the latter case; the proof for the former case being similar.

We establish the result by showing that there exists some $\epsilon > 0$ such that the market share of any good, whose price exceeds unit variable cost, is greater than $\epsilon$.

We first note what happens if two or more goods have the same quality level. Then, by the familiar Bertrand argument, they have price equal to unit variable

cost at equilibrium. Moreover, it then follows immediately from Condition (F) that all products of lower quality have a zero market share. Now if the highest quality level is offered by two or more firms, our result therefore follows. Otherwise, denote the highest quality level produced by more than one firm as $u_s$, where we set $u_s = u_0$ in the case where all products are distinct.

Consider any good $k, k > 1$, which has a price exceeding unit variable cost, and for which $t_k > a$. Then the first order condition for profit maximization by firm $k$ implies

$$t_{k+1} - t_k \geq (p_k - c_k)(r_{k-1,k} + r_{k,k+1} - 1).$$

Clearly

$$t_k > \max(a, t(u_0, u_{k-1}), t(u_s, u_{k-1})) = m, \qquad \text{say.}$$

(Note $t_k > a$, and since the consumer of income $t_k$ prefers good $k-1$ at price $p_{k-1} \geq c_{k-1}$ to the zero good, we have $t_k > t(u_0, u_{k-1})$. Similarly, $t_k > t(u_0, u_s)$.) Hence

$$t_k = p_k r_{k-1,k} + p_{k-1}(1 - r_{k-1,k}) > m$$

or (remembering $r_{k-1,k} > 1$),

$$\begin{aligned}(p_k - c_k) r_{k-1,k} &\geq m + p_{k-1}(r_{k-1,k} - 1) - c_k r_{k-1,k} \\ &\geq m + c_{k-1}(r_{k-1,k} - 1) - c_k r_{k-1,k} = m - t(u_{k-1}, u_k).\end{aligned}$$

But bearing in mind the first order conditions above, we have that the market share

$$\begin{aligned}(t_{k+1} - t_k) > (p_k - c_k) r_{k-1,k} &\geq m - t(u_{k-1}, u_k) \\ &> \max(a, t(u_0, u_{k-1})) - t(u_{k-1}, u_k).\end{aligned}$$

By virtue of Remarks 1 and 2 above, this last expression is bounded away from zero, uniformly in $u$, from which our result follows. *Q.E.D.*

PROPOSITION 3 (Necessity): *If Condition* (F) *is violated, then the finiteness property does not hold.*

PROOF: As we noted above, we may divide instances in which (F) is violated into two cases according as: (i) $t(u,u)$ maximizes his utility by choosing $u$ (at cost) or (ii) $t(u,u)$ minimizes his utility by choosing $u$ (at cost).

Now in both of these cases the same construction can be used to show how any number of products can be entered in the neighborhood of $u$; the essential property used is that there exists a consumer who is locally indifferent between a certain range of goods.

Each of these cases includes a number of subcases; we deal with one subcase, viz. $t(u,u) \in (a,b)$ and $u \in (\underline{u}, \overline{u})$ and $t(u_0, u) < t(u,u)$. The remaining "boundary" cases can be dealt with in an obvious manner.

CASE (i): Here the proof is immediate. We have some interval of qualities for which $t(q,q) \in (\max(a, t(u_0, q)), b)$ at each point $q$ (from the differentiability of $uc(u)$ at $q$ (Fig. 1)).

Thus each quality in this interval is preferred, at cost, to any alternative, by consumers of some income level (as in "location" models).

Hence, given any sequence of products $u_1, \ldots, u_n$ in this neighborhood, each good $k$ certainly enjoys a positive market share at equilibrium. Thus any number of distinct products can coexist with positive market shares and prices exceeding variable cost, and the finiteness property does not hold.

CASE (ii): This case is more complicated; to show that there are $n$ qualities which can coexist we need to choose these qualities close to $u$. The construction of such a set of qualities is carried out in the Appendix.

We here illustrate the intuition underlying this construction by describing a limiting case, as follows. Suppose there exists a consumer who is indifferent between an interval of qualities (i.e. $uc(u)$ is linear over this interval). Then consider any finite sequence $u_1, \ldots, u_n$ of qualities in this interval.

A product within this sequence will not be driven out of the market unless the seller of some higher quality product in the sequence sets price equal to cost. It suffices then to show that the top quality $u_n$ is not sold at cost. But this is immediate, for this product can certainly earn positive profits by selling at a price exceeding cost.

This argument is of course analogous to that of the zero cost case, with $a = 0$, alluded to above. (As $t(u,u)$ is in the interior of the relevant interval, $u_n$ can be sold to a richer consumer at a price above cost.)

The proof in the Appendix shows how this kind of argument extends to the case of a turning point of $uc(u)$ and thus establishes that the finiteness property does not hold. *Q.E.D.*

## 6. DISCUSSION

The condition we have developed above is necessary and sufficient for the finiteness property. That condition refers to the relationship between consumers' willingness to pay for quality improvements, and the change in unit *variable* cost associated with those improvements; thus it involves the interplay of technology and tastes.

The "finiteness" condition is likely to hold in those industries where the main burden of quality improvement takes the form of R & D, or other fixed costs. Unit variable costs, on the other hand, being the sole costs relevant to our present concerns, may rise only slowly with increases in quality. Indeed, insofar as product innovation is often accompanied by concomitant process innovation, unit variable costs may even fall.

It is this situation, where the "finiteness" property holds along the relevant interval of qualities, which we have labelled a "natural oligopoly."

The finiteness condition does not *in itself* exclude an infinite number of firms; it is consistent with the presence of an arbitrarily large number of firms each selling an identical product at a price equal to unit variable cost, and a bounded number of firms offering a range of distinct, higher, qualities, at prices exceeding unit variable cost.

The implications of our present results are most clearly seen in the context of a

model in which firms first incur some *arbitrarily small* fixed cost in entering the industry; then choose the qualities of their respective products, and then compete in price. Here, the presence of any fixed cost, however small, excludes the viability of firms whose prices are not strictly greater than unit variable cost at equilibrium. We have, in [7], characterized a perfect equilibrium[4] in this three stage game (entry; choice of quality; choice of price). The outcome is that, given a large number of entrants, only a bounded number (there, two) will choose to enter; they will produce distinct products, and both will earn strictly positive profits at equilibrium. Further reductions in fixed costs, or an expansion in the size of the economy (once our bound is attained), have no effect on the equilibrium number of firms in the industry.

We remark, finally, on a number of directions in which certain assumptions of the present model may be relaxed:

(i) *Linearity of utility functions*; *uniform income distribution*: The special forms of the utility function and income distribution used here do not play a critical role, and our results may be extended to a wider class of function. (For a full treatment of existence, and finiteness, in the zero cost case, see [3].)

(ii) *Identical consumers*: The assumption that consumers be identical can be relaxed, once some ranking of consumers in order of their willingness to pay is available.

(iii) *Smoothness of the cost function*: The analysis extends readily to the case in which $c(u)$ is kinked. In fact a new possible case of "finiteness" arises here, in that $c(u)$ may be "flat" up to some point, and "steep" thereafter, so that the finiteness property holds (consider the first and third zones in Figure 2(i) joined at a kink). Consumers will now rank products (at cost) in increasing order of quality to the left of the kink, but in decreasing order to the right.

(iv) *Multiproduct firms*: The restriction that each firm produces a single product can be relaxed. If we allow firms to produce a number of products, the finiteness property still holds, in that a bound will exist to the number of *firms* which can enjoy positive market shares at a Nash Equilibrium in prices.

*London School of Economics*

*Manuscript received August, 1981; final revision received September, 1982.*

## APPENDIX

We here provide the construction referred to in the proof of Proposition 3 of the text. We show that where (F) is violated, then for any $n$, we can choose $n$ distinct qualities sufficiently close to the point $u$ at which (F) fails, such that all coexist with positive market share at a Nash Equilibrium in prices. This proof covers both the cases, (i) and (ii), referred to in Proposition 3; while a direct proof for case (i) is possible, the present construction is needed for case (ii). The fact that the present construction covers both cases demonstrates clearly that what enables an infinite number of products to coexist is the presence of a consumer who is locally (in quality) indifferent between products of differing qualities, where each is made available at cost.

The strategy of our construction is as follows: Choosing $n$ qualities in a particular manner we write down the system of first order conditions which define equilibrium, and we show that a choice of qualities which are "sufficiently close" ensures that all have a positive market share at equilibrium.

[4]See [6].

Let $u$ denote the quality at which (F) is violated. Choose $u_n = u$ and $u_k - u_{k-1} = \epsilon$. Note from the definition of $t_1, \ldots, t_k$, that we can express $p_k r_{k-1,k}$ as a function of $t_1, \ldots, t_k$, viz.

$$p_k r_{k-1,k} = t_k + \cdots + t_2 + t_1\alpha$$

where $\alpha = (u_1 - u_0)/\epsilon$. Similarly, we can express $c_k r_{k-1,k}$ as:

$$c_k r_{k-1,k} = t(u_{k-1}, u_k) + \cdots + t(u_1, u_2) + t(u_0, u_1)\alpha.$$

To simplify the equation, we define a new variable

$$s_k = t_k - t(u_{k-1}, u_k).$$

Then the first order conditions (3) take the form:

$$\begin{cases} s_2 + t(u_1, u_2) - a = s_1\alpha, & s_1 \leq a - t(u_0, u_1), \\ s_2 + [t(u_1, u_2) - t(u_0, u_1)] = s_1(2 + \alpha), & s_1 \geq a - t(u_0, u_1), \end{cases}$$

$$s_{k+1} - s_k + [t(u_k, u_{k+1}) - t(u_{k-1}, u_k)] = 2(s_k + s_{k-1} + \cdots + s_2 + s_1\alpha) \qquad (k = 2, \ldots, n-1),$$

$$b - s_n - t(u_{n-1}, u_n) = s_n + s_{n-1} + \cdots + s_2 + s_1\alpha.$$

(Note that the first order condition for firm 1 depends on whether 1's lower boundary is below $a$ or not, i.e., whether all consumers buy one of the available products, or otherwise. Hence we have two equations for firm 1.)

From the first order condition for firm $k$, we may deduce that $k$'s market share is

$$M^k = 2(s_k + \cdots + s_2 + s_1\alpha).$$

As $\epsilon \to 0$, the qualities chosen approach $u$ and $\alpha \to \infty$, $t(u_k, u_{k+1}) \to t(u, u)$ for $k = 1, \ldots, n-1$ and $t(u_0, u_1) \to t(u_0, u)$.

We wish to show that in the limit as $\epsilon \to 0$, the market shares of all products are positive.

Since in any solution the $s_k$ are bounded, it must be the case that $s_1$ approaches zero. Denote $s_1\alpha$ as $\bar{s}_1$, and write the equations for $\epsilon = 0$ (limit equations). Linearity and continuity guarantee that the solution of the limit system is the limit of the solutions. In the limit the relevant equation for firm 1 corresponds to the first of the pair cited above ($s_1 = 0$).

$$s_2 + [t(u, u) - a] = \bar{s}_1,$$

$$s_{k+1} - s_k = 2(s_k + \cdots + s_2 + \bar{s}_1) \qquad (k = 2, \ldots, n-1),$$

$$[b - t(u, u)] - s_n = s_n + \cdots + s_2 + \bar{s}_1.$$

Note that $b - t(u, u) > 0$ and $t(u, u) - a > 0$. To show that $k$'s market share $M^k = 2(s_k + \cdots + s_2 + \bar{s}_1)$ is positive, we split the system of equations into two subsystems, and introduce the new variable $M^k$:

$$\text{System (A)} \quad \begin{cases} s_2 + [t(u, u) - a] = \bar{s}_1, \\ s_3 - s_2 = 2(s_2 + \bar{s}_1), \\ s_4 - s_3 = 2(s_3 + s_2 + \bar{s}_1), \\ \quad \cdots \\ s_k - s_{k-1} = 2(s_{k-1} + \cdots + s_2 + \bar{s}_1), \\ M^k = 2(s_k + \cdots + s_2 + \bar{s}_1), \end{cases}$$

$$\text{System (B)} \quad \begin{cases} s_{k+1} - s_k = M^k, \\ s_{k+2} - s_{k+1} = 2s_{k+1} + M^k, \\ \quad \cdots \\ s_n - s_{n-1} = 2(s_{n-1} + \cdots + s_{k+1}) + M^k, \\ b - t(u, u) - s_n = s_n + \cdots + s_{k+1} + M^k/2. \end{cases}$$

We show that the first system (A) defines $s_k$ as an increasing linear function of $M^k$ with a negative value at $M^k = 0$ and that the second system (B) defines $s_k$ as a decreasing linear function with a positive value at $M^k = 0$. The two functions must therefore intersect at a positive $M^k$.

To verify these assertions, note that in (A) $s_2, s_3, \ldots, s_k, M^k$ are all strictly increasing linear functions of $\bar{s}_1$; hence $s^k$ can be written as an increasing linear function of $M^k$. To see that $s^k < 0$ when $M^k = 0$, set $M^k = 0$ in the last equation, and substitute this in the preceding equation, to obtain $s_{k-1} = 3s_k$. Continuing backwards, we represent each $s_j$ in turn, for $j \geq 2$, as $q^j s_j$, where $q^j$ is some positive constant, and $\bar{s}_1$ as $q^2 s_2 + [t(u, u) - t(u_0, u)]$. Substituting this in the last equation, we have

$$(1 + q^2 + q^3 + \cdots q^{k-1})s_k + [t(u, u) - t(u_0, u)] = 0.$$

Hence $s_k(0) < 0$.

From system (B), beginning from the first equation, we can write $s_{k+1}, \ldots, s_n$ as linear functions of $s_k, M^k$, increasing in both arguments. Substituting this in the last equation, we find $s^k$ as a decreasing function of $M^k$. This function is positive for $M^k = 0$ for (from the first equation) $s_{k+1} = s_k$, while from the second $s_{k+2} = 3s_k$, etc. All the $s_k$ can be written as the product of a positive constant and $s_k$, whence from the last equation we have $s_k > 0$.

Hence the solution $M^k$ is positive. This completes our construction.

## REFERENCES


[1] FRIEDMAN, JAMES W.: *Oligopoly and the Theory of Games*. Amsterdam: North-Holland, 1977.

[2] JASKOLD GABSZEWICZ, J., AND J.-F. THISSE: "Entry (and Exit) in a Differentiated Industry," *Journal of Economic Theory*, 22(1980), 327–338.

[3] JASKOLD GABSZEWICZ, J., AVNER SHAKED, JOHN SUTTON, AND J.-F. THISSE: "Price Competition Among Differentiated Products: A Detailed Study of a Nash Equilibrium," ICERD Discussion Paper No. 37, London School of Economics, 1981.

[4] ———: "International Trade in Differentiated Products," *International Economic Review*, 22(1981), 527–534.

[5] LANCASTER, K.: *Variety, Equity and Efficiency*. New York: Columbia University Press, 1979.

[6] SELTEN, R.: "Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 4(1975), 25–55.

[7] SHAKED, AVNER, AND JOHN SUTTON: "Relaxing Price Competition through Product Differentiation," *Review of Economic Studies*, 49(1982), 3–14.

[8] ———: "Natural Oligopolies and the Gains from Trade," ICERD Discussion Paper No. 48, London School of Economics, 1982.

[9] ———: "Natural Oligopolies and International Trade," in H. Kierzkowski (ed.), *Monopolistic Competition and International Trade*. Oxford: Oxford University Press, 1983.