



---

Product Differentiation and Industrial Structure

Author(s): Avner Shaked and John Sutton

Source: *The Journal of Industrial Economics*, Vol. 36, No. 2 (Dec., 1987), pp. 131-146

Published by: Blackwell Publishing

Stable URL: <http://www.jstor.org/stable/2098408>

Accessed: 15/09/2008 09:24

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=black>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit organization founded in 1995 to build trusted digital archives for scholarship. We work with the scholarly community to preserve their work and the materials they rely upon, and to build a common research platform that promotes the discovery and use of these resources. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).



Blackwell Publishing is collaborating with JSTOR to digitize, preserve and extend access to *The Journal of Industrial Economics*.

<http://www.jstor.org>

## PRODUCT DIFFERENTIATION AND INDUSTRIAL STRUCTURE

AVNER SHAKED AND JOHN SUTTON\*

Some recent literature on “vertical product differentiation” has developed the idea that if the nature of technology and tastes in some industry take a certain form, then the industry must necessarily be “concentrated”; and must remain so, no matter how large the economy becomes. The present paper develops this idea further, and looks at some of its implications. This approach offers a simple unified framework within which to re-explore many issues which arise in considering the relationship between advertising, R & D, and market structure.

A NUMBER of recent papers have been concerned with models of vertical product differentiation (i.e. differentiation by quality). Such models permit a unified treatment of certain situations in which firms incur increased fixed costs with a view to enhancing consumers’ willingness to pay for their respective products. These include, in particular, the case of R & D expenditures devoted to product development or improvement, and the case of advertising outlays aimed at increasing perceived quality. What such situations have in common is the fact that the level of fixed cost incurred by firms is endogenously determined, and its equilibrium level reflects the interplay of demand conditions and the underlying technology—the latter being expressed as a relationship between costs and product attributes. What the present paper is concerned with is the relationship between these exogenous parameters, and the equilibrium form of industry structure.

A useful point of departure is given by considering a different and more familiar setting, in which the level of fixed costs is treated as a constant. In this setting, the size of fixed costs, relative to the size of the economy, determines the number of firms, and therefore the degree of concentration. As the size of the economy increases, the level of concentration falls. Economies of scale thus become “less important” as a barrier to entry in “large economies”.

This idea can be captured formally by reference to a number of familiar models, such as Hotelling’s model of “horizontal” product differentiation. In such models, as the size of the economy increases, the market share of every firm present will become arbitrarily small.

Our aim in the present paper is to delineate some conditions which are sufficient to exclude this tendency towards “fragmentation”. Now the recent

\* The authors would like to acknowledge the financial support of the International Centre for Economics and Related Disciplines (LSE). John Sutton would also like to thank the ESRC for financial support.

literature on vertical product differentiation provides some models in which the property fails in a very extreme manner: the number of firms and the pattern of market shares may remain completely unaffected as the size of the economy increases (the “finiteness” property, Shaked and Sutton [1982]). This very extreme failure of the “fragmentation” property is particular to a quite special type of “pure vertical differentiation” model, however. In the present paper, we set out to develop conditions on technology and tastes which will lead to the *minimal* failure of the “fragmentation” property, i.e. at least one firm should continue to retain some particular level of market share no matter how large the economy becomes (though the total number of firms present may become arbitrarily large).

It is worth emphasizing at the outset that our aim is to set out very weak and general requirements, and so it is appropriate to begin from this extremely weak notion of “concentration”, i.e. a failure of the fragmentation property.

In doing this, we seek to set out suitable conditions which will hold over a wide range of underlying models. One of the main complaints about the recent increase in the use of oligopoly models in industrial organization is that “with oligopoly, anything may happen”. It is therefore of particular interest to draw out properties which are relatively robust in respect of alternative specifications of the model (for example: Cournot versus Bertrand equilibria; simultaneous or sequential patterns of entry and product choice; whether firms are single product or multiproduct). It is especially important in the present context to ensure that our results hold for cases where products are differentiated both in respect of vertical (quality) attributes, and in terms of “horizontal” attributes—since a striking feature of the existing literature is the fact that the results obtaining in the cases of the “pure vertical” and “pure horizontal” differentiation literatures are sharply different, but the—empirically relevant—case in which both types of attribute are present, has not been widely studied.<sup>1</sup>

The price which we pay, in trying to come to terms with the need to provide such a robust characterization result, is that we cannot provide a correspondingly wide treatment of existence problems. It is notorious in models of product differentiation that equilibria may fail to exist for many reasonable specifications of the standard models. *Here, we aim merely to point out some characteristics which equilibria must have, if they do exist.* We do not prove existence, though we offer some remarks as to particular specifications where existence problems may arise, and to some cases where existence is assured. The models discussed here are based on a two-stage process, where firms

<sup>1</sup> It is easily shown, for example that the “finiteness” property which is central to the “pure vertical differentiation” case, fails to hold once both horizontal and vertical attributes are present. Nonetheless, the conditions which we develop for the following general case are motivated by, and are precisely in the same spirit as, the conditions leading to the “finiteness” result in the “pure vertical differentiation” case.

incur sunk costs in choosing (“developing”) their products at the first stage. Then, at the second stage, taking product specifications as fixed, they compete in prices. Existence problems may arise at both stages.

In the case of “pure horizontal” differentiation, a (pure strategy) price equilibrium may fail to exist at the second stage (d’Aspremont, Gabszewicz and Thisse [1979]). To ensure existence, suitable convexity assumptions need to be imposed on consumer preferences (Neven [1985]). The case of “pure vertical” differentiation is relatively “well behaved” in this respect (Gabszewicz, Shaked, Sutton and Thisse [1981]). Assuming that a price equilibrium exists for any set of products, existence problems may still arise at the first (product choice) stage, if firms choose their products simultaneously. If, on the other hand, (a finite number of) firms choose products sequentially, existence of a pure strategy equilibrium is assured (Börger [1985], Harris [1985]).

The structure of the present paper is as follows. We first make clear precisely what we mean by the “fragmentation” property, by illustrating it in the context of the familiar Hotelling model (section I). In section II we set out our model, and we develop our main proposition. Section III sets out our conclusions.

#### I. PRELIMINARIES: A “FRAGMENTATION” RESULT

The simplest representation of “product differentiation” is provided by Hotelling’s location model, in which sellers are positioned at various points along some line segment. Consumers are distributed along the line segment, and each purchases one unit from the lowest cost supplier; cost equals price plus transport cost.

It is standard in this context to employ a 2-stage equilibrium concept. In the first stage each firm either enters, by paying sunk cost  $\sigma > 0$ , and chooses its location, or else it decides not to enter. In the second stage, each firm may produce any volume of output at zero cost. Here, we seek a Nash equilibrium in prices (which will depend upon the entry and location choices made by all firms at the first stage). We then seek a Perfect Equilibrium in this two-stage game.<sup>2</sup>

What we mean by the size of the economy in what follows is the total number of consumers present. We increase the size of the economy by replication, i.e. the distribution of consumer tastes remains unchanged.

<sup>2</sup> i.e. Having solved the “price” game played at the second stage, to find the Nash equilibrium, we express the profits (payoffs) earned at this equilibrium as a function of the locations of the firms. (These locations are *fixed*, once the second stage is reached, and they appear as parameters in our solution). We then define a new game, in which we use the functions thus calculated (profit as a function of locations) as our payoff functions; and we seek a Nash equilibrium in that game. This is a “Perfect Equilibrium” in the 2-stage game.

For this model, the following proposition holds (the assumptions are stated, and the proof is set out, in the Appendix):

*Proposition I.* For any  $\varepsilon > 0$ , there exists an economy size  $S^*$  such that for any size  $S > S^*$ , every firm has an equilibrium market share less than  $\varepsilon$ .

What Proposition I ensures is that all firms have a “small” market share at equilibrium. Traditionally, such a market was described as being “atomistic”. Currently the term appears to cause confusion, since in the technical literature the terms “atom” and “atomic” are used with precisely the opposite connotation—a usage borrowed from measure theory. To avoid ambiguity, we will refer to this form of market structure as “fragmented”, a usage which is not uncommon.

The type of product differentiation which is represented by Hotelling’s “location” framework, is labelled “horizontal differentiation”. In this framework, given any number of sellers offering distinct products at equal prices each seller has a strictly positive market share.

We may contrast this with the case of “vertical differentiation”, defined as follows: given any two distinct products, *if they were sold at the same price*, then *all* consumers would choose the same one (the “higher quality” product).<sup>3</sup> At the equilibrium, of course, higher quality products will sell at higher prices: we assume that consumers differ in their willingness to pay for quality improvement. To motivate this, it is assumed that consumers differ in income, or in tastes (i.e. sensitivity to quality), or both.

Clearly, products will in practice be differentiated both in respect of attributes which correspond to the “horizontal” case, and also in respect of “vertical” attributes. “Horizontal” attributes might for instance include colour, or various “aesthetic” features (design). An example of a “vertical” attribute would be, say, the operating speed of a computer. Only in the latter case can we speak unambiguously of “product improvement”; and we shall simply refer to such attributes as corresponding to some level of “quality” henceforward.

A general theory of product differentiation, then, requires an analysis of situations in which products may be differentiated both “horizontally” and “vertically”; and it is to this case that we turn in the next section.

## II. THE MODEL

The model is as follows: a *product* is described by two characteristics,  $u$  and  $h$ , where we identify the first as vertical, and the second as horizontal. A consumer is specified by two parameters; his income  $Y$ , and his “most

<sup>3</sup> On a point of terminology: it might seem natural to reserve the term “differentiation by quality” for this case. However, many authors use the word “quality” quite loosely in this literature, treating it as a synonym for “variety”, within the context of “location” type models. Our present terminology, due to Lancaster, avoids ambiguity.

preferred" value of the horizontal attribute,  $\alpha$ . (For simplicity, we confine ourselves to a set-up in which a consumer's willingness to pay for a higher quality is determined solely by his income; but the results can be extended trivially to allow consumers to differ in their tastes over the vertical attribute also.)

The utility score of the consumer, when he buys one unit of product  $(u, h)$  at price  $p$ , is given by

$$U(u, d, y)$$

where  $d = |h - \alpha|$  and  $y = Y - p$  denotes income spent on "outside goods". We assume that  $Y$  is distributed according to a density  $f(Y, \alpha)$  on some compact set, where  $\bar{f} \geq f(Y, \alpha) \geq \underline{f} > 0$ .

We assume that  $U_u > 0$ ,  $U_d < 0$ . We also assume  $U_{uy} > 0$ , which ensures that richer consumers are willing to pay more for a given quality increment.

To complete our description of the model, we need to specify the firms' choice of products, and the nature of price competition. Here, a number of alternative specifications appear reasonable. We note the following points in regard to which our results turn out to be robust:

(i) In respect of the "final" stage in which firms set prices, we use the standard "Bertrand" assumption (Nash equilibrium in prices). Our result remains true, however, for the alternative "Cournot" scheme (Nash equilibrium in quantities). Clearly, other possibilities might be considered, but this should suffice to reassure the reader that the result is fairly robust in this respect.

(ii) In respect of the preceding stage in which firms choose their product specifications, two standard formulations are used in the literature. The proof which follows refers to the "simultaneous entry" case, but we note below that it extends to the case of "sequential entry" also (i.e. where a finite number of potential entrants choose their products in a predetermined order, and thereafter compete in price. (For a full description of this approach see Lane [1980].)

(iii) Our characterization holds good both for single product firms and also for the more realistic—though relatively neglected—case of competition between multiproduct firms (with an appropriate definition of the cost structure).

Now, since we aim to formulate a result which is independent of these several choices, it should be clear that a sharp dividing line cannot be drawn between those combinations of technology and tastes which must lead to a fragmented structure, and those which must lead to a concentrated one. Clearly there will be some middle ground, over which the choices made in respect of (i)–(iii) will influence the result. We confine ourselves to the delineation of conditions *sufficient* to ensure a concentrated structure.

For the sake of concreteness, we will set out the analysis which follows in the context of the simplest possible model, and then we will note how the

proof can be extended to cover the several additional possibilities just noted. To this end, we define the equilibrium concept as follows: We consider a 2-stage game. In stage 1, each firm either chooses a single product, of specification  $(u, h)$ , at cost  $F(u)$ ; or else it chooses not to enter. In the second stage, we seek a Nash equilibrium in prices (Bertrand equilibrium). We characterize a Perfect Equilibrium in the 2-stage game.

We now introduce two assumptions on technology which turn out to be sufficient to imply a concentrated structure. Let  $F(u)$  denote the fixed cost incurred in developing a product with attributes  $(u, h)$ , and let  $c(u)$  denote the unit variable cost incurred in its production. Our first assumption relates to the form of  $c(u)$ , and it *embodies the basic idea which we carry over from the "pure vertical differentiation" literature* (Shaked and Sutton [1983]).

Assumption 1 may be described loosely as requiring that the costs of quality improvement involve at most only a modest rate of increase in unit variable cost. Specifically, what it says is that for any given improvement in quality, *the increase in unit variable cost is strictly less than the marginal valuation of the richest consumer*. Formally, we require:

*Assumption 1:* There exists some strictly positive triple  $(\mu, p, \Delta)$ , such that, if each other firm offers a good with  $u \leq \bar{u}$ , and any  $h$ , at unit variable cost, then a firm offering a good of quality  $\bar{u} + \Delta$ , and any  $h$ , at a price  $p + c(\bar{u} + \Delta)$  will capture at least fraction  $\mu$  of consumers.

It may seem more attractive to restate directly in terms of utility and cost functions. Assumption 1 may, for example, be replaced by the (stronger) requirement that:  $U_u$  is bounded away from zero;  $U_y, |U_d|$  are bounded above, and  $c(u)$  is bounded above by some income level which is less than the maximum value of consumer income, which we will denote as  $\bar{U}$ .

In what sort of context, then, is Assumption 1 likely to be satisfied? The burden of quality improvement might fall primarily on fixed costs in some cases (R & D effort to improve computer speed, say), but on unit variable cost in others (the production of higher quality furniture, by the way of using more expensive raw materials, say). No *a priori* restriction can be placed on the relationship between product quality—measured in terms of consumers' willingness to pay—and the level of *unit variable costs*. Assumption 1 is more likely to be valid in those industries in which the main burden of product improvement falls on fixed cost, rather than variable costs.

Our second Assumption relates to the relationship between product quality and fixed cost  $F(u)$ . We assume that *quality improvement always involves an increase in fixed costs; and that the proportionate increase in fixed cost required to achieve a given quality increment is bounded*. Formally, we require:

*Assumption 2:* On  $u \in (0, \infty)$ ,  $F(u) > 0$ ,  $F'(u) > 0$  and  $F'(u)/F(u)$  is bounded above. (Denote an upper bound to  $F'/F$  as  $\beta$ .)

The role of Assumption 2 is as follows. As the size of the economy increases, we expect that, given our assumption on  $c(u)$ , there will be a tendency for the maximum quality offered to increase. This leads to the possibility that, as  $S$  increases, the fractional increase in cost required to achieve a given increment to quality might become arbitrarily large; Assumption 2 excludes this possibility.

We have chosen, for ease of exposition, to write our Assumptions in terms of the “quality index”  $u$ . The requirement of Assumption 2 is well defined in this representation. It is very important to emphasize, however, that all that matters here is the relationship between costs and consumers’ willingness to pay. In fact, the combination of Assumptions 1 and 2 can be restated as follows: there is some factor  $k$  such that, by incurring  $k$  times more fixed cost than any other market participant, a firm can guarantee sales of  $\mu S$  at a mark-up  $p$  over unit variable cost, irrespective of the prices set by rival firms. (In fact,  $k$  is equal to  $e^{\beta\Delta}$  in the terminology of Assumptions 1 and 2).<sup>4</sup> The role of Assumption 2 is illustrated by an example, below.

We now show how Assumptions 1 and 2 imply that in any equilibrium, industrial structure must remain “concentrated”, in the sense that no “limit theorem” of the kind described in Proposition 1 above can hold good.

We assume throughout the presence of a “large” number of potential entrants.

*Proposition II:* Under Assumptions 1, 2, it follows that there exists some  $\varepsilon > 0$ , such that at equilibrium at least one firm has a market share greater than  $\varepsilon$ , irrespective of the size of the economy.

*Remark:* We first establish this for the case of a Nash equilibrium in prices, single product firms, and simultaneous entry. For the several extensions, see the following Remarks.

*Proof:* The proof follows immediately from A1 and A2.

Choose

$$\varepsilon < \frac{\mu p}{\bar{Y}(1 + e^{\beta\Delta})}$$

where  $\mu$ ,  $p$  and  $\Delta$  are given by Assumption 1,  $\bar{Y}$  is the maximum level of consumer income, and  $\beta$  is an upper bound to  $F'/F$ . We now show that any configuration of products in which each firm has market share  $\varepsilon$  or less is not an equilibrium. We do this by showing that any firm can increase its profit by choosing a different, higher, quality. Note that if each firm has a market share  $\leq \varepsilon$ , then *all* firms earn revenue, and so profit, less than  $\varepsilon \bar{Y} S$ .

Let  $\bar{u}$  denote the maximum quality on offer. Note that the firm producing  $\bar{u}$  incurs fixed cost  $F(\bar{u})$ , and this cannot exceed its revenue, at equilibrium. But

<sup>4</sup> By writing  $F'(u)/F(u) = \beta$  and integrating from  $u$  to  $u + \Delta$ . This observation makes it clear that the present analysis does not rest on the use of some unobservable “quality index”. For the empirical use of “willingness to pay” measures, see Beggs, Cardell and Hausman (1981).



its revenue is at most  $\varepsilon \bar{Y}S$ , whence  $F(\bar{u}) \leq \varepsilon \bar{Y}S$ . Suppose some firm offers a product  $(\bar{u} + \Delta, h)$ , for some  $h$  (the product choices of other firms being held fixed). Now, this will affect the equilibrium vector of prices (in the "second stage"). We know, however, that no firm with positive sales can have price below unit variable cost, at a Nash equilibrium in prices. From Assumption 1, the entrant can capture a share  $\mu$  at a price  $p + c(\bar{u} + \Delta)$  independently of the prices set by rivals. Hence its revenue, net of variable cost, at equilibrium, exceeds  $\mu pS$ . But from Assumption 2, (by integrating),

$$F(\bar{u} + \Delta) \leq F(\bar{u}) \cdot e^{\beta \Delta}$$

Hence its profit  $\pi \geq \mu pS - F(\bar{u})e^{\beta \Delta} = (\mu p - \varepsilon \bar{Y}e^{\beta \Delta})S > \varepsilon \bar{Y}S$  (by our choice of  $\varepsilon$ ). Thus this change in product specification raises profits, and contradicts our assumption that the original configuration was an equilibrium.

This completes our proof.

*Remark 1 (Cournot Equilibrium).* The above proof holds good, with a trivial modification, if we replace the Bertrand equilibrium in the final stage, by Cournot equilibrium. To see this, note that all we require, concerning the final stage equilibrium, is the following: that the firm offering quality  $\bar{u} + \Delta$  will enjoy profits at least equal to  $\mu pS - F(\bar{u} + \Delta)$ . In the proof, we used the fact that, under Bertrand competition, it can certainly achieve this level of profit by setting price  $c(\bar{u} + \Delta) + p$ , whatever the prices set by rival firms. To extend the result to Cournot competition, we note that it can certainly achieve this level of profit by setting quantity  $\mu$ , whatever the quantities chosen by other firms. For, as we show in the Appendix, any vector of quantities  $X$  is mapped into a non-negative vector of prices  $\bar{p}$ , where either  $p_i > 0$ ,  $D_i(\bar{p}) = X_i$ ; or  $p_i = 0$ ,  $D_i(\bar{p}) \leq X_i$  where  $D_i(p)$  denotes demand. But Assumption 1 ensures that, for price at or below  $c(\bar{u} + \Delta) + p$ , the firm offering quality  $\bar{u} + \Delta$  has demand  $\mu$  or greater. Hence the desired property follows.

*Remark 2 (Sequential Entry).* In the case in which, for any  $S$ , a finite number of firms choose their products in a predetermined sequence, the above argument may be applied directly to the last firm in the sequence. As noted above, the existence of an equilibrium configuration of products in this setting is ensured, so long as price equilibrium exists for any set of products in the ensuing subgame. (see p. 133). To permit the possibility of entry by an arbitrarily large number of firms, the number of potential entrants may be increased *pari passu* with the size of the economy. (We have not been able to extend the proof of Proposition II to the case where we have an infinite sequence of potential entrants; here, we cannot exclude the appearance of certain pathological equilibria).

*Remark 3 (Multiproduct Firms).* Suppose each firm can produce a number of products  $(u_1, h_1), (u_2, h_2), \dots, (u_n, h_n)$ . Suppose that it incurs cost  $F(\bar{u}) + (n-1)\alpha$  where  $\alpha > 0$  and  $\bar{u} = \max(u_1, u_2, \dots, u_n)$ . This seems a natural

way of extending our cost structure to the multiproduct firm. Then the preceding proof goes through as before (where the relevant market share is that of a *firm*, not a product).

It is of interest to note, incidentally, that the “fragmentation” result for horizontal product differentiation, which we stated as Proposition 1 above, *does* rely on the notion that each firm offers a single product. With multiproduct firms, and sequential entry, for example, it may be the case that the first mover would enter “many” products, and so the fragmentation property may fail. (On such “product proliferation” strategies, see Schmalensee [1978], Bonanno [1987]. For a full analysis, see Shaked and Sutton [1987].)

*Remark 4.* It is worth noting that in models of the present kind, free entry does not exclude the possibility that intramarginal firms earn positive profits (see Shaked and Sutton [1982]). All that can be said is that, given the strategy of entrants, any firm which at equilibrium chooses not to enter, would make negative profits by entering.

What Proposition II says, loosely, is that where the burden of quality improvement falls primarily on fixed cost (in the sense of Assumption 1), then we cannot obtain a fragmented market structure, even in the limit.

What, then, determines the *degree* of concentration in the present setting? This will depend upon the preferences of consumers, and the shape of the technology ( $c(u)$ ,  $F(u)$ ). It is worth emphasizing that it is the *interplay* of these two factors which matters. As noted above, all that matters is the relationship between costs and consumers’ willingness to pay. In order to specify the *degree* of concentration, then, we need to impose more structure on the model. In the present paper, we have confined our attention to the minimal departure from a “fragmented” structure—the failure of the “convergence” property.

If market shares do not converge to a fragmented structure, then, what is the impact of an increase in the size of the economy? This question, it is worth noting, is of particular interest in the context of trade theory, where the standard starting point for analyzing trade in differentiated products, produced under increasing returns, is to consider the impact of joining together a number of identical economies.

We are unable to give a general characterization of this. In what follows, we confine ourselves to the case of “simultaneous entry”. Here we can readily establish the following, intuitively appealing, corollary to Proposition II.

*Corollary.* Under “simultaneous entry”, the top quality on offer,  $\bar{u}$ , and the associated level of fixed costs  $F(\bar{u})$ , increase to infinity with the size of the economy  $S$ .

*Proof.* Note first that, since  $F(0) > 0$ , it follows that, for any  $S$ , only a finite number of firms enter. Hence we may define the maximum level of  $u$  over all firms; call this  $\bar{u}$ . Now consider any firm which chooses not to enter at

equilibrium. Such a firm can earn revenue of at least  $\mu pS$ , by entering with a quality level  $\bar{u} + \Delta$ , whence its costs would be at most  $e^{\beta\Delta}F(\bar{u})$ . So at any equilibrium,

$$\mu pS < F(\bar{u})e^{\beta\Delta}$$

(since equilibrium requires that nonentrants, should they enter, make non-positive profits).

Hence:  $\mu pS e^{-\beta\Delta} < F(\bar{u})$

from which it follows that  $F(\bar{u})$ , and so  $\bar{u}$ , tends to infinity with  $S$ .

This result implies that increases in the size of the economy (or the linking of similar economies via free trade) will lead not to a fragmented industry, but to a concentrated industry, operating at higher levels of fixed costs, and producing “better” products. In other words, the equilibrium level of expenditure on fixed costs (R & D and advertising, say) will increase with the size of the economy. Thus, in contrast to Proposition I, there is no tendency for fixed costs to become less important as a “barrier to entry”, as the size of the economy increases. This is a theme which we have explored elsewhere, within the “pure vertical differentiation” model (Shaked and Sutton [1984]).

The role of Assumption 2 deserves further comment. This is seen most clearly by considering an extreme example, as follows: Consider the family of functions parameterized by  $n$ :

$$F_n(u) = \max(1, \exp n(u - \bar{u})^3)$$

each of which violates Assumption 2. Take the limiting function as  $n \rightarrow \infty$ , viz. the fixed cost schedule  $F(u)$  is constant on  $[0, \bar{u}]$  but is  $+\infty$  for  $u > \bar{u}$  (Figure 1). Suppose, for example, that unit variable cost  $c(u) = 0$  for all  $u$ , so

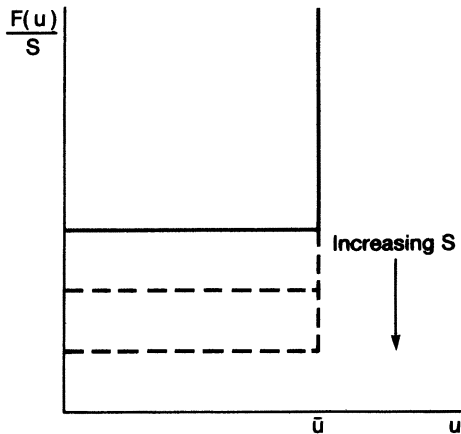


Figure 1

that Assumption 1 is satisfied. Let firms enter simultaneously, and then compete in prices *à la* Bertrand.

In this case, as  $S \rightarrow \infty$ , we converge to a fragmented structure in the manner of Proposition I. As  $S$  becomes large, a large number of products of quality  $\bar{u}$ , and various  $h$ , are offered; and each product has an arbitrarily small market share.

To see this, consider the horizontal differentiation case, following the proof of Proposition I (Appendix): in order to ensure that each firm has an arbitrarily small market share at equilibrium, it suffices to show that a  $\delta$ -net of products of the form  $(u, h)$ , for various  $h$ , will be offered in the limit  $S \rightarrow \infty$ .

Notice that this argument relies crucially on the fact that products are differentiated horizontally as well as vertically. In the special case of *pure vertical differentiation* the preceding assumption in regard to the cost function  $F(u)$  leads to an equilibrium in which only a *finite* number of firms enter (see Shaked and Sutton [1982]).

We remark, finally, on one further assumption implicit in our analysis. This is the notion that the cost  $F(u)$  incurred in product development is independent of the qualities offered by other firms. In the R & D context, we can think of this as embodying the notion of patent protection; clearly, any relaxation of this will alter our conclusions. This raises a number of familiar issues concerning R & D and patent protection, which lie beyond our present scope.

### III. CONCLUDING REMARKS

The theory of product differentiation offers a useful framework within which to explore the relationship between the level of fixed costs (notably R & D and advertising expenditures) and industrial concentration.

*Our present analysis focusses attention on the idea, that what matters in regard to the influence which any such fixed costs will have, is not the size of fixed costs per se (the usual focus of attention), but rather the extent to which these fixed expenditures (can) substitute for higher variable cost, in product improvement (whether real or "perceived").*

In the present model, given the size of the market, it is the interplay between consumers' tastes and the underlying technology (specified by a series of feasible cost-quality combination) which simultaneously determine the degree of concentration and the level of fixed costs.

Thus, for example, we are led to question the familiar argument that certain industries are highly concentrated because the level of fixed costs involved is high. Instead, we would argue that the manufacture of aircraft, or mainframe computers, say, is limited to a small number of firms, not *because* the fixed costs of product development are so high, relative to the size of the market—but rather because the possibility exists, primarily through incurring additional *fixed* costs, of shifting the technological frontier constantly forward

towards more sophisticated products.<sup>5</sup> Indeed, we argue that, given such a technology as we posit here, fixed costs in these industries are high *because* the market is large. This is the point made by the Corollary to Proposition II above, and in our earlier work.<sup>6</sup>

Finally, some clarificatory remarks are in order, regarding the relationship between the present analysis and the traditional Bain paradigm, which states that the presence of various “Barriers to Entry” to an industry determines its level of concentration (structure); that high levels of concentration in turn facilitate cooperative price behaviour (conduct); and this collusive behaviour then leads to high profits (performance). This idea of a chain of causation running from Structure to Conduct to Performance has, of course, been increasingly questioned in the more recent literature. In the present paper, taking technology and tastes as our exogenous variables, we have chosen to proceed by exploring the implications of *non-cooperative* behaviour, in the context of a model in which structure and performance are simultaneously determined.

Our theme has been, that *even under competitive pricing*, a concentrated structure necessarily obtains if technology and tastes take a certain form. We wish to emphasize that we do not question the view that a concentrated structure may in turn facilitate collusion, as Bain’s view emphasized. A richer dynamic structure might allow such collusive outcomes to be supported as noncooperative equilibria; but this lies outside our present scope.<sup>7</sup>

AVNER SHAKED AND JOHN SUTTON,  
*The London School of Economics and  
 Political Science,  
 Houghton Street,  
 London WC2A 2AE.*

ACCEPTED OCTOBER 1987

#### APPENDIX

##### I. STATEMENT AND PROOF OF PROPOSITION I

###### *Assumptions*

1. Consumers are distributed along  $[0, 1]$  according to some atomless density function  $S \cdot f(\alpha)$ . We assume that  $f$  is bounded, and strictly positive, viz.

<sup>5</sup> It is helpful in some contexts to think of the curve  $F(u)$  as the envelope of a series of curves pertaining to successive “technologies” in the manner of the familiar “overlapping technologies” diagram. What we then determine, within the model, is which of these technologies will be exploited.

<sup>6</sup> Insofar as the level of fixed costs and the degree of concentration are determined simultaneously within the model, we also question arguments which rely on the reverse direction of causation here: as in the idea that “high concentration induces innovation”.

<sup>7</sup> What if behaviour is in fact collusive? Here, a familiar, if paradoxical, story suggests itself: more collusion, insofar as it raises profits, will tend to induce further entry, and so *reduce* measured concentration levels. This of course implies a *reversal* of the direction of causation between concentration and conduct suggested by Bain.

$0 < \underline{f} < f(\alpha) < \bar{f}$  on  $0 \leq \alpha \leq 1$  and that  $\int_0^1 f(\alpha) d\alpha = 1$ . Thus  $S$  denotes the population of consumers in the market.

2. A consumer located at  $\alpha$  patronizing a seller at location  $h$ , incurs a premium  $H(|h - \alpha|)$ , where  $H(\cdot)$  is a strictly increasing convex function.

3. Firms are free to enter at any particular location by paying a sunk cost of entry  $\sigma$ , and may thereafter produce at zero marginal cost.

*Remark 1.* It is well known that a number of questions concerning existence of equilibrium in this model remain open. As noted earlier, we do not address these questions here.<sup>8</sup>

*Remark 2.* For any set of locations, and prices, the assumptions that  $H(\cdot)$  is convex implies that the market share of each firm has the form of an *interval* of consumers. Note, however, that this interval need not include the consumers located in the neighbourhood of the firm itself.

*Proposition I.* For every  $\epsilon > 0$ , there exists an economy size  $S^*$  such that in every equilibrium in a market of size  $S > S^*$ , each firm has a market share less than  $\epsilon$ .

*Proof.* Our proof proceeds in two steps. We first show that:

(i) as  $S \rightarrow \infty$ , the spacing between firms becomes arbitrarily close, i.e. for any  $\delta > 0$ , there exists  $\bar{S}$  such that in any equilibrium set of products in an economy of any size  $S > \bar{S}$ , the distance between any two neighbouring firms is less than  $\delta$  (as is the distance from 0 to the left-most firm, and from the right-most firm to 1).

To show this, note that if two firms are a distance  $\delta$  apart, a firm entering midway between them can certainly capture a market share corresponding to an interval of length  $(\delta/2)$  by setting price zero, i.e. a share of at least  $f \cdot \delta/2$ . Hence it can certainly capture a market share of  $f \cdot \delta/4$ , say, at a strictly positive price *independently of the prices set by other firms, and the size of economy  $S$ .* (We here rely on the fact that  $H(\cdot)$  is strictly increasing.)

Now if assertion (i) is false, we can find a sequence of market sizes  $S^k \rightarrow \infty$ , and an equilibrium for each  $k$ , such that in each of these equilibria, some pair of firms is more than distance  $\delta$  apart (or, that one "end" firm is more than distance  $\delta$  from the endpoint). But as  $S^k \rightarrow \infty$ , it follows that the profits of such a firm (which are proportional to  $S^k$ ) must exceed entry cost  $\sigma$ , for  $k$  sufficiently large. *Thus entry is profitable, and so this is not an equilibrium.* This establishes (i).

(ii) We now establish Proposition I. Suppose it were false. Then for some market share  $\epsilon > 0$ , there exists a sequence of market sizes  $S^k \rightarrow \infty$ , together with an equilibrium for every  $k$  such that some firm in each equilibrium has a share of at least  $\epsilon$ . But from (i), it follows that by choosing  $k$  sufficiently large, we can ensure that the minimum distance between neighbouring firms is less than any  $\delta > 0$ .

Let

$$(1) \quad \delta = \frac{\epsilon^2}{36(1-\epsilon)\bar{f}} \frac{H(\epsilon/6\bar{f})}{H(1)}$$

and choose a corresponding value of  $k$  (as prescribed in (i) above).

We now show that, at a Nash equilibrium in prices, it cannot be the case that one firm has a market share of  $\epsilon$  or greater; thus obtaining a contradiction.

<sup>8</sup> See for instance d'Aspremont, Jaskold Gąbszewicz and Thisse (1979), Neven (1985). It may be worth noting that a formulation in which firms choose price and location simultaneously is much more problematic. It is easily shown that no Nash equilibrium in (price, location) pairs exist (Novshek [1980]).

Take a firm whose market share is  $\varepsilon$  or greater. Label it firm 1. We can find an interval containing fraction  $\varepsilon/2$  of all consumers, all served by firm 1, and lying wholly on one or other side of firm 1's location. (See Figure A1.) Divide that interval into three subintervals, each containing a fraction  $\varepsilon/6$  of all consumers. Each subinterval is at least  $\frac{\varepsilon}{6\bar{f}}$  in length. Take the middle subinterval. Note that any firm in this subinterval is a distance  $\varepsilon/6\bar{f}$  or more from firm 1, and so it can capture the fraction  $\frac{\varepsilon}{6}$  of consumers in the outer subinterval by setting any price below  $p_1 + H(\varepsilon/6\bar{f})$ .

From (i), we know that this middle subinterval contains at least  $\varepsilon/6\bar{f}\delta$  firms. At least one of these firms must therefore have a market share of at most  $r$ , where  $r$  satisfies

$$\varepsilon + r \cdot \varepsilon/6\bar{f}\delta = 1$$

i.e. we have  $r = \frac{1-\varepsilon}{\varepsilon} \cdot 6\bar{f}\delta$ . Label this firm as "2". Now if firm 2 sets a price at or above  $p_1 + H(1)$ , it certainly has sales zero. Hence, since its revenue is necessarily positive at equilibrium,

$$p_2 < p_1 + H(1)$$

and so its revenue is at most

$$(2) \quad S \cdot [p_1 + H(1)] \cdot \frac{1-\varepsilon}{\varepsilon} 6\bar{f}\delta$$

But by setting price

$$p'_2 = p_1 + H(\varepsilon/6\bar{f})$$

it can capture a market share of  $\varepsilon/6$ , and so will earn revenue

$$(3) \quad S \cdot [p_1 + H(\varepsilon/6\bar{f})] \cdot \frac{\varepsilon}{6}$$

Our definition (1) of  $\delta$  ensures that expression (3) exceeds expression (2) and therefore that  $p_2$  is not an optimal choice of price for firm 2. (It is optimal for firm 2 to lower its

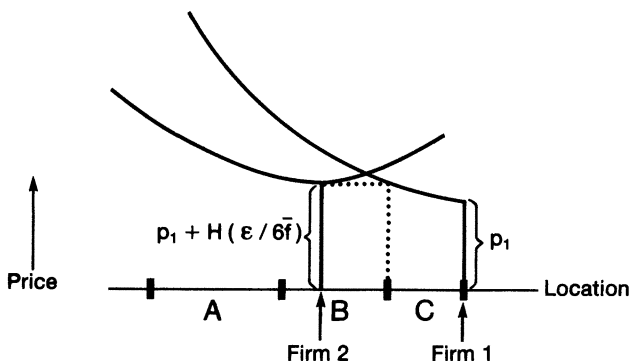


Figure A1: To illustrate proof of Proposition I, part (ii). Firm 1 has a market share exceeding  $\varepsilon$ . Each interval  $A, B, C$  corresponds to a market share of at least  $\varepsilon/6$ , and is of length  $\varepsilon/6\bar{f}$ , or greater. Any firm in interval  $B$  can capture all consumers in interval  $A$  by setting price  $p_1 + H(\varepsilon/6\bar{f})$

price, and so capture a larger market share.) This contradicts the supposed optimality of the prices.

This completes our proof.

II. COURNOT EQUILIBRIUM WITH DIFFERENTIATED PRODUCTS

Since it is standard in the product differentiation literature to employ a Bertrand equilibrium, it may be worth setting out the method of analysis appropriate to the Cournot case.

Given  $n$  goods, then for each vector of quantities  $\bar{X} = (X_1, \dots, X_n)$  we map this vector into a vector of prices  $\bar{p}(\bar{X}) = (p_1, \dots, p_n)$  and demand levels  $(D_1, \dots, D_n)$  satisfying

$$D_i(\bar{p}) = X_i, \quad \text{if } p_i > 0$$

$$D_i(\bar{p}) \leq X_i, \quad \text{if } p_i = 0$$

To establish the existence of such a price vector, we proceed as follows: define the function

$$M_i(\bar{p}) = \max(-p_i; D_i(\bar{p}) - X_i), \quad i = 1, \dots, n$$

This function clearly satisfies the condition:

- (i) if  $p_i > 0$  and  $X_i < D_i(\bar{p})$ , then  $M_i(\bar{p}) > 0$
- (ii) if  $p_i = 0$  and  $X_i \geq D_i(\bar{p})$ , then  $M_i(\bar{p}) = 0$

Within our setting, where each consumer buys 1 unit or none,  $D_i(\bar{p})$ , and so  $M_i(\bar{p})$ , is bounded by the total population of the economy, which we denote  $S$ .

Define the function  $T(\bar{p}) = \bar{p} + M(\bar{p})$  on  $\bar{p} \geq 0$ . Clearly,  $T(\bar{p}) \geq 0$ . Define the domain,  $0 \leq p_i \leq \bar{Y} + S$ , where  $\bar{Y}$  is the upper bound to the income of any individual consumer. The mapping  $p \rightarrow T(p)$  is continuous, and maps the domain into itself, as is easily checked. Hence there exists a fixed point, and it satisfies conditions (i), (ii) above. It is necessary, within any specific model, to check that this price vector is uniquely defined. The vectors  $\bar{X}$  and  $\bar{p}(\bar{X})$  determine the payoff of each firm, and so we can solve for the Nash equilibrium, taking the quantities as strategies.

REFERENCES

BEGGS, S., CARDELL, S. and HAUSMAN, J., 1981, 'Assessing the Potential Demand for Electric Cars', *Journal of Econometrics*, 17, pp. 1-20.

BONANNO, G., 1987, 'Product Proliferation and Entry Deterrence Revisited', *Review of Economic Studies*, 54, pp. 37-46.

BORGERS, TILMAN, 1985, 'Existence of Sequential Entry Equilibrium in Product Differentiation Models', mimeo.

D'ASPROMONT, C., GABSZEWICZ, J. JASKOLD and THISSE, J. F., 1979, 'On Hotelling's "Stability in Competition"', *Econometrica*, 47, pp. 1145-1150.

GABSZEWICZ, J. JASKOLD, SHAKED, A., SUTTON, J. and THISSE, J. F., 1981, 'Price Competition among Differentiated Products: A Detailed Study of a Nash Equilibrium', ICERD working paper no. 37, LSE.

HARRIS, CHRISTOPHER, 1985, 'Existence and Characterization of Perfect Equilibrium in Games of Perfect Information', *Econometrica*, 53, pp. 613-616.

LANE, W., 1980, 'Product Differentiation in a Market with Endogenous Sequential Entry', *Bell Journal of Economics*, 11, pp. 237-260.



- NEVEN, D., 1985, 'Two Stage (Perfect) Equilibrium in Hotelling's Model'; *Journal of Industrial Economics*, 33, pp. 317-326.
- NOVSHEK, W., 1980, 'Equilibrium in Simple Spatial (or Differentiated Product) Models', *Journal of Economic Theory*, 22, pp. 313-326.
- SCHMALENSEE, R., 1978, 'Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry', *Bell Journal of Economics*, 9, pp. 305-327.
- SHAKED, AVNER and SUTTON, JOHN, 1982, 'Relaxing Price Competition Through Product Differentiation', *Review of Economic Studies*, 49, pp. 3-14.
- SHAKED, AVNER and SUTTON, JOHN, 1983, 'Natural Oligopolies', *Econometrica*, 51, pp. 1469-1484.
- SHAKED, AVNER and SUTTON, JOHN, 1984, 'Natural Oligopolies and International Trade' in H. KIERZKOWSKI (ed.), *Monopolistic Competition and International Trade* (Oxford: Oxford University Press).
- SHAKED, AVNER and SUTTON, JOHN, 1987, 'Multiproduct Firms and Market Structure', ICERD Working Paper no. 154, LSE.