

# Endogenous Inequality in Integrated Labor Markets with Two-Sided Search

By GEORGE J. MAILATH, LARRY SAMUELSON, AND AVNER SHAKED\*

*We consider a market with “red” and “green” workers, where labels are payoff irrelevant. Workers may acquire skills. Skilled workers search for vacancies, while firms search for workers. A unique symmetric equilibrium exists in which color is irrelevant. There are also asymmetric equilibria in which firms search only for green workers, more green than red workers acquire skills, skilled green workers receive higher wages, and the unemployment rate is higher among skilled red workers. Discrimination between ex ante identical individuals arises in equilibrium, and yet firms have perfect information about their workers, and strictly prefer to hire minority workers. (JEL C70, D40, J30)*

Bad things happen to good people—two people can appear to be similar in all economically relevant aspects, including skills and opportunities, and yet one of them can fare much better than the other. In its most visible form, this heterogeneity appears as discrimination, in which people hindered by no disadvantage, other than a characteristic that is seemingly irrelevant for any economic purpose, consistently achieve substandard economic outcomes.

Theories of statistical discrimination are commonly invoked to explain group-based inequality (see Glen G. Cain [1986] for a survey). Suppose workers’ skills are not observable, but

that there is a noisy signal of skill. Workers also have a payoff-irrelevant characteristic, such as being colored red or green, that is observable. If firms can condition their evaluation of the signal on this characteristic, then there is an equilibrium in which green workers acquire skills and are hired at a high wage, while red workers do not acquire skills and are hired at a low wage. Because red workers choose to be less skilled than green workers, firms pay them a lower equilibrium wage, providing less incentives to be skilled. Firms and red workers thus coordinate on a low-skill, low-wage outcome, while firms and green workers coordinate on a high-skill, high-wage outcome.<sup>1</sup> Paradoxically, theories of statistical discrimination yield no economic discrimination (Cain, 1986): all workers, including reds, are paid their marginal product. Given skills, color plays no role in explaining wages.

While statistical discrimination may explain part of what we see, it cannot be the whole story. Theories of statistical discrimination predict that the red labor market clears, while discrimination typically leads to persistent unemployment. Moreover, theories of statistical discrimination focus on an informational friction, arising because firms cannot identify the

\* Mailath: Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104 (e-mail: gmailath@econ.sas.upenn.edu); Samuelson: Department of Economics, University of Wisconsin, 1180 Observatory Drive, Madison, WI 53706 (e-mail: larrysam@ssc.wisc.edu); Shaked: Department of Economics, University of Bonn, Adenauerallee 24-26, D 53113 Bonn, Germany (e-mail: shaked@glider.econ3.uni-bonn.de). We thank the National Science Foundation and the Deutsche Forschungsgemeinschaft, SFB 303 at the University of Bonn for financial support, and thank Michael Morris for research assistance. This work began while Mailath and Samuelson were visiting the University of Bonn. An earlier version was written while Mailath was visiting the University of Wisconsin. This version was partly written while the three authors attended the 1997 Warwick Summer Research Workshop on Dynamic Games and Their Economic Applications at the University of Warwick, and while Samuelson visited the University of Pennsylvania. We gratefully acknowledge the hospitality of these institutions. We also thank Drew Fudenberg, John Kennan, and the referees for helpful comments.

<sup>1</sup> Bradford Cornell and Ivo Welch (1996) show that statistical discrimination can arise, even if workers make no skill-level choice and skill distributions are identical across groups, if employers are better able to screen one group than another.

skill level of workers, that is not always plausible. Finally, many of these theories have a “separability” feature, common to models of multiple equilibria, the implausibility of which is discussed below.

In this paper we focus on search, rather than informational, frictions as an explanation of inequality in outcomes. The popular explanation for people with identical skills and education achieving quite different outcomes is often given in the maxim: “It’s not what you know, it’s who you know.” Success can depend critically not only on the choices we make in our interactions with others, but also on the identity of those with whom we interact. As a result, people throw tremendous amounts of energy into searching for the right people. Schools, clubs, neighborhoods, professional organizations, and academic conferences are all chosen because they allow association with a desired group.

We examine a dynamic labor model in which newborn workers acquire skills at an idiosyncratic cost. Unemployed skilled workers search for firms with vacancies, while also being a potential target of search on the part of firms with vacancies. Once an unemployed skilled worker meets a firm with a vacancy, they bargain over the division of the surplus.<sup>2</sup> Workers come in two varieties, red and green. A worker’s color has no direct effect on payoffs, but firms can observe workers’ colors and can focus their search on one color, potentially giving rise to endogenously generated, search-based inequality.

Because color is payoff irrelevant, there is always a symmetric equilibrium in which colors are ignored. However, there are also asymmetric equilibria in which fewer red workers acquire skills than green workers, firms choose to search only among skilled green workers, and skilled red workers suffer from lower wages and higher unemployment rates than skilled green workers. The payoff-irrelevant characteristic of redness identifies its bearer as a target for discrimination, and reds fare systematically worse than greens.

In our model, red skilled workers have no difficulty in convincing firms of their skills. Moreover, reds have precisely the same oppor-

tunity to search firms as greens. In an asymmetric equilibrium, a firm with a vacancy contacted by a searching red worker happily hires that worker, obtaining a skilled worker at a low wage. However, since firms seek only green workers, reds have a higher unemployment rate and hence a lower expected payoff from market participation. This lower outside option causes worker-firm bargaining to yield lower wages for red workers than for identical skilled green workers. This is unambiguous economic discrimination: workers of identical skills receive different wages based on color.

Since reds do not acquire skills because employers do not seek reds because reds do not acquire skills, the asymmetric equilibria in our model superficially exhibit a coordination failure similar to that in theories of statistical discrimination. Moreover, while worker skills are observable (and so firms are not forming posteriors about worker skills conditional on their color), the asymmetric firm search behavior is due to the different average—or statistical—properties of the red- and green-worker pools (there are more skilled workers in the green pool). In this sense, our model simply pushes the source of the statistical discrimination back one step.

In other crucial respects, however, our model and the typical statistical discrimination models part company. Statistical discrimination models, like most other models of coordination failure, yield separable outcomes: there is no interaction between the different groups.<sup>3</sup> In addition to the asymmetric equilibrium in which reds receive a lower wage, these models also exhibit a *symmetric* equilibrium in which *all* workers receive the same low payoff as do reds in the asymmetric equilibrium (in this sense, the greens are not imposing an externality on the reds). There is also a symmetric equilibrium in which all workers receive the same high payoff as do greens in the asymmetric equilibrium. In our model, the asymmetric equilibrium is *not* a

<sup>2</sup> See Martin J. Osborne and Ariel Rubinstein (1990) for a survey of the large literature on search and bargaining.

<sup>3</sup> An instructive exception is the statistical discrimination model of Andrea Moro and Peter Norman (1998). Workers in their model must be assigned to either a simple or a complementary complex task within the firm, with only skilled workers capable of performing the complex task. In an asymmetric equilibrium, the marginal product of each group depends upon the characteristics of the other, introducing an externality between groups. This interaction between skilled and unskilled workers also implies that (as here), there is a unique symmetric equilibrium.

coordination failure. There is a unique symmetric equilibrium.<sup>4</sup> Payoffs to workers in an asymmetric equilibrium cannot be replicated in the symmetric equilibrium. Green workers can obtain high payoffs in the asymmetric equilibrium only because red workers obtain low payoffs.

The workers in our model thus impose an important externality on one another. It is only the existence of the green workers that allows firms to direct their search to the detriment of red workers. This externality leads to markedly different policy considerations than coordination-failure models. The coordination-failure aspect of statistical discrimination ensures that any policy intervention in the red market has no effect on green workers. One can subsidize red skill acquisition, impose restrictions on red wages, or impose red hiring quotas without the slightest impact on green workers. In such a world, discrimination policy could not have given rise to the concept of “reverse discrimination.” In contrast, our red and green workers participate in a single labor market. Any policy designed to affect the outcome of red workers inevitably has consequences for greens, and greens may have a great deal to lose or gain from discrimination policy. Our model is also suggestive concerning the appropriate discrimination policy, a topic to which we return in Section VI.

This externality between workers implies that relative group sizes are relevant in determining the possible directions of discrimination. While it is natural to think of minorities as being the targets of discrimination, this is not necessarily the case. It can be the case that an asymmetric equilibrium is more likely to exist in our model if firms search the minority color worker, though we also find circumstances under which discrimination must target the minority.<sup>5</sup>

There are three key features of our model. The pattern of firm and worker matches, including the rates at which matches occur, is endogenous, depending upon firms’ choices of which workers to

seek and the size of the unemployed skilled worker and vacant firm pools. The workers’ choices of whether to acquire skills are also endogenous, depending upon the market values of these skills. Together, these choices allow asymmetries between different groups of *ex ante* identical workers to be generated endogenously. Finally, the terms of trade between matched agents are determined in a bargaining process that reflects market conditions.<sup>6</sup> This builds a natural antidote to discrimination into the model by causing the disadvantaged group of workers to be more attractive to firms because they command a lower wage for a given, observable skill level. We identify conditions under which discrimination persists despite this ameliorating effect.

In practice, firms search for workers through formal means, such as advertisements, and through informal networks of contacts and referrals. We interpret a strategy of searching only for greens as the cultivation of a contact network that involves primarily greens. The importance of firms pursuing potential workers through networks of contacts should not be underestimated. Formal studies emphasizing such networks are reinforced by the more popular job-search literature.<sup>7</sup> Job-seeking guides routinely emphasize the exploitation of informal contacts. Anecdotes concerning the importance of the “old boy network” are reinforced by popular claims that most jobs are obtained with employers where the new worker already has a personal acquaintance.

Our results show that groups of people can be trapped in outcomes featuring low skill levels, low wages, and high unemployment, not because there are barriers to their seeking employers or securing a job once an employer is found, but because employers are optimally choosing not to seek them. Given that workers always have the option of seeking firms, can firm search decisions really be that important? We suspect that firm search is

<sup>4</sup> Kenneth Burdett and Eric Smith (1995) study a search-based model in which firms make nontrivial decisions about vacancies. As a result, coordination failures in the form of multiple symmetric equilibria can arise, some in which firms have many vacancies, workers acquire high skills and earn high wages, and others in which firms have few vacancies, workers acquire low skills and earn low wages.

<sup>5</sup> For simplicity, the bulk of our formal analysis is restricted to the case of equal-sized groups. Examples 1 and 2 in Section IV explore the implications of unequal group sizes.

<sup>6</sup> These features distinguish our analysis from that of Michael Sattinger (1998), who examines a model in which firms can choose which groups of workers to seek but in which worker groups have exogenously fixed asymmetries, contact rates between firms and workers are insensitive to the sizes of the unemployed worker and vacant firm pools, and the terms of trade are fixed.

<sup>7</sup> The importance of personal contacts and referrals in seeking jobs is stressed by Mary Corcoran et al. (1980), Harry J. Holzer (1988), Douglas Staiger (1990), Howard Wial (1991), and Jacqueline Berger (1995).

vitaly important in real labor markets, especially markets for skilled labor. Jobs are frequently filled not through formal procedures by which potential employees apply to firms, but through formal and informal efforts on the part of firms to identify candidates for the job.<sup>8</sup> Academic labor markets are a superb example, where the hiring process for new Ph.D.s typically begins with a department search phase. More generally, “head-hunting” firms exist because firm search is important.

We also suspect that the importance of employer search is growing in our economy. Employers face increasingly stringent legal restrictions on the information they can seek from job applicants. In many settings, it is illegal to ask about an applicant’s religion, marital and family status, nationality, health, criminal record, and a variety of personal habits, even though many of these may be important in ascertaining the value of the employee to the firm. As a substitute for seeking this information, a firm can offer incentives for existing employees or other contacts to recommend new employees, with the recommendation of the existing employee signaling information that the firm cannot legally seek. As a result, we expect firm search to become an increasingly important force in shaping economic outcomes.

The following section introduces the model. Section II examines the symmetric equilibrium of the model. Sections III–V examine asymmetric equilibria. Section VI discusses the results and their potential policy implications. Proofs whose arguments are potentially distracting are collected in the Appendix.

### I. The Model

We consider an economy with a continuum of firms and workers. Firms’ and workers’ lifetimes are independently distributed according to a Poisson process with death rate  $\delta$ . Births of new firms and workers also occur at rate  $\delta$ , so that the sizes of the populations of firms and workers are constant.<sup>9</sup> The total populations of both workers and firms are normalized to be of measure one. Time is continuous, with interest rate  $r$ .

<sup>8</sup> David Scoones (1995) discusses firm search.

<sup>9</sup> We assume that the continua of independent random variables describing firm and worker outcomes yield market outcomes with no aggregate randomness.

All firms are identical. Each worker has a label, red or green, that has no direct payoff implications. For convenience, we assume that half of the population of workers has a red label and half has a green label. Upon entering the market, or being “born,” each worker makes an irrevocable decision either to acquire skills or to eschew skills and enter the unskilled sector of the economy. Workers differ in the opportunity cost of acquiring skill, denoted by  $\alpha \geq 0$ , where this opportunity cost includes both the direct cost of skill acquisition as well as the forgone value to entering the unskilled labor market. Each worker’s opportunity cost  $\alpha$  is the realization of a random variable, independent of the worker’s color, with continuous cumulative distribution function  $C$ . A worker makes the skill-acquisition decision knowing his opportunity cost of skill.

Each firm can hire at most one worker. If a firm employs a skilled worker, whether red or green, a flow surplus of  $x$  is generated, while a firm hiring an unskilled worker generates a flow surplus of zero. Notice that our model is partial equilibrium in that the flow surplus,  $x$ , is independent of the total number of skilled workers hired.

A firm currently without an employee is “vacant,” while a firm with an employee is “occupied.” All meetings between vacant firms and unemployed workers arise from either firm or worker search.<sup>10</sup> Consider a firm searching for both colors of worker when the skilled-worker population is of size  $H_W$  and the unemployment rate of skilled workers is  $\rho_W$ . The process describing meetings between unemployed skilled workers and the searching firm follows a Poisson process with meeting rate  $\lambda_F \rho_W H_W$ . The parameter  $\lambda_F$  captures the intensity of firm search, while  $\rho_W H_W$  captures the idea that the firm can more quickly find a member of a large group than of a small one. In addition, note that this meeting rate is independent of the size of the searching firm population (i.e., of the vacancy rate). This yields an aggregate matching process with increasing returns to scale. This is not an innocuous specification; we discuss our

<sup>10</sup> Since search is costless and the firm always has the option of declining to hire any worker found, not searching is weakly dominated by searching. We accordingly assume vacant firms always search. A similar comment applies to unemployed workers.

modelling of the matching process and alternatives in Section VI.

Alternatively, the firm can decide to search only for unemployed green workers or only for unemployed red workers.<sup>11</sup> If the firm searches only for unemployed green workers and there are  $\rho_G H_G$  unemployed skilled green workers,<sup>12</sup> the process describing meetings of the firm and unemployed green workers as a result of firm search follows a Poisson process with rate  $2\lambda_F \rho_G H_G$ . In this case, there are no meetings between the firm and red workers generated by firm search, though worker search may still lead to meetings with reds. Restricting search to one color has the effect of doubling the search intensity on that color, since the firm is then concentrating its search on half as many potentially skilled workers (recall that there are equal numbers of red and green workers), while reducing the search intensity to zero on the other color. In particular, we can think of a firm who searches *both* colors of worker as facing two Poisson processes, one generating meetings of the firm and green workers at rate  $\lambda_F \rho_G H_G$ , and one generating meetings of the firm and red workers at rate  $\lambda_F \rho_R H_R$ , so that meetings of the firm and *all* workers as a result of firm search follow a Poisson process with rate  $\lambda_F \rho_G H_G + \lambda_F \rho_R H_R = \lambda_F \rho_W H_W$  if  $\rho_G = \rho_R = \rho_W$ . Restricting search to only green workers allows the search efforts formerly dedicated to red workers to be transferred to greens, giving a meeting rate of  $\lambda_F \rho_G H_G + \lambda_F \rho_G H_G = 2\lambda_F \rho_G H_G$ .<sup>13</sup>

Unemployed skilled workers simultaneously search for vacant firms with intensity  $\lambda_W$ . Since we have assumed that the population of firms is fixed at one, meetings generated by worker search follow a Poisson process with rate  $\lambda_W \rho_F$ , where  $\rho_F$  is the vacancy rate of firms.

Two types of tie can arise in the matching process. The first is a firm (or worker) being contacted by more than one worker (firm) at the

same time. The second is a worker contacting a firm at the same time as that firm contacts a worker. We ignore these ties when calculating an agent's value function, because the agent assigns them zero probability. Moreover, we assume that the measure of agents involved in such ties is zero.

After an unemployed skilled worker and a vacant firm make contact, they bargain. We postulate a simple bargaining game: a fair coin determines a proposer, who makes a take-it-or-leave-it wage offer to the responder. In any sequentially rational equilibrium, the proposer will make an offer that leaves the responder indifferent between accepting and rejecting, and the offer will be accepted. A variety of more complicated bargaining conventions suffice for the result and might be more realistic. The essential features of the bargaining process are that each agent captures a share of the surplus that is increasing in the agent's expected value of returning to the search process and decreasing in his rival's expected value of returning to the search process.<sup>14</sup>

We assume that an occupied firm cannot abandon its current worker to bargain with a new worker, nor can a worker abandon a firm to seek a new one. This is a strong assumption in the context of our simple bargaining model, which implicitly requires high transactions costs for dissolving an employment relationship. For example, the worker may have won the initial coin toss and proposed a wage that extracts all the surplus from the firm, leaving the firm anxious for a chance to dismiss the current employee, if it can be done cheaply, and bargain again with a new employee. This assumption would be unnecessary in a more realistic bargaining process that allowed both the firm and worker to capture sufficient surplus *ex post* as well as *ex ante*.

<sup>11</sup> Searching firms cannot distinguish previously employed workers from workers who have never been employed. Notice, however, that once the firm and worker meet, the worker has no difficulty convincing the firm of her skill, making employment history irrelevant for the equilibrium we construct.

<sup>12</sup> We denote the size of the green (red) skilled-worker population by  $H_G$  ( $H_R$ ) and the unemployment rate of green (red) skilled workers by  $\rho_G$  ( $\rho_R$ ).

<sup>13</sup> Note that if  $\rho_R H_R = \rho_G H_G$ , then searching all workers gives the same meetings rate as searching green workers.

<sup>14</sup> The impact of outside options in infinite-horizon bargaining depends upon whether the outside option is implemented as result of exogenous breakdown, and or as a result of a choice by a player (see Osborne and Rubinstein, 1990). Our modelling is consistent with exogenous breakdown, perhaps as a result of the bargainers being contacted by other searchers. The rate of exogenous breakdown would then occur at a rate that depended upon the number of people searching in the market, introducing a new complication that we ignore.

## II. The Symmetric Steady-State Equilibrium

In this section we examine a symmetric, steady-state equilibrium in which firms pay no attention to workers' colors. The equilibrium unemployment rates of red and green workers are identical. Any meeting between an unemployed worker and a firm with a vacancy results in the vacancy being filled.

Let  $V_w$  denote the value of skills to a worker, or equivalently, the value of entering the market for skilled labor. Since a worker acquires skills if  $\alpha < V_w$ , the fraction of new workers who become skilled is  $C(V_w)$ . Let  $H_w$  denote the size of the skilled workforce, or equivalently, the proportion of workers who are skilled. In a steady state, the size of the inflow of newly skilled workers [given by the product of the rate at which new workers appear,  $\delta$ , and the proportion,  $C(V_w)$ , of new workers acquiring skills] equals the size of the outflow of existing skilled workers (given by  $\delta H_w$ , the product of the death rate of workers and the size of the skilled labor force), and hence we have the *skilled-worker steady-state* condition:

$$(1) \quad H_w = C(V_w).$$

In a steady state, the rate at which new vacancies are created equals the rate at which they are filled. New vacancies appear at the rate  $2\delta(1 - \rho_F)$ , since  $1 - \rho_F$  of the firms are currently occupied, and at rate  $2\delta$  either a worker dies, creating a vacancy at a previously occupied firm, or an occupied firm dies and is replaced by a new, vacant firm. Vacancies are filled as a result of both firm and worker search. There are  $\rho_F$  firms vacant, and hence searching, generating meetings at the rate  $\rho_F \lambda_F \rho_w H_w$ . At the same time, there are  $\rho_w H_w$  workers searching, generating additional meetings at the rate  $\rho_w H_w \lambda_w \rho_F$ . Adding these two sources of filled vacancies, vacancies are filled at the rate  $\rho_F \rho_w H_w (\lambda_F + \lambda_w)$ . Thus, the *vacancies steady-state* condition is

$$(2) \quad 2\delta(1 - \rho_F) = \rho_F \rho_w H_w (\lambda_F + \lambda_w).$$

Flows into and out of unemployment also balance in steady state. Newly unemployed workers arrive at the rate  $2\delta H_w (1 - \rho_w)$  [because there are  $H_w (1 - \rho_w)$  employed skilled

workers, with worker and firm deaths adding unemployed workers at the rate  $2\delta$ ]. Since unemployed workers find jobs at the same rate as vacancies are filled, or  $\rho_F \rho_w H_w (\lambda_F + \lambda_w)$ , we have (dividing by  $H_w$ ) the *unemployment steady-state* condition:

$$(3) \quad 2\delta(1 - \rho_w) = \rho_F \rho_w (\lambda_F + \lambda_w).$$

Let  $w$  be the expected flow payoff of an employed worker and  $Z_w$  the steady-state value of an employed, skilled worker. To see how the values of an employed and unemployed worker are related, consider temporarily a discrete-time model.<sup>15</sup> Time intervals are of length  $\tau$ , with the death probability, discount rate, and search intensities given by  $\delta\tau$ ,  $r\tau$ , and  $\lambda_F\tau$  and  $\lambda_w\tau$ . Consider first the recursive equation determining  $Z_w$ . An employed worker earns a flow payoff of  $w\tau$  in the current period, survives until the next period with probability  $(1 - \delta\tau)$ , and then loses employment due to firm death with probability  $\delta\tau$ , for an outcome whose present value is  $V_w/(1 + r\tau)$ , and retains employment with complementary probability, for a present value of  $Z_w/(1 + r\tau)$ . Thus,

$$Z_w = w\tau + (1 - \delta\tau) \left( \delta\tau \frac{V_w}{(1 + r\tau)} + (1 - \delta\tau) \frac{Z_w}{(1 + r\tau)} \right).$$

Turning to the equation for  $V_w$ , an unemployed worker survives to the next period with probability  $(1 - \delta\tau)$ . If she survives, then with probability  $\rho_F(\lambda_F + \lambda_w)\tau$  she is matched with a vacant firm and begins employment, for a present value of  $Z_w/(1 + r\tau)$ . With complementary probability, she is again unemployed, for a present value of  $V_w/(1 + r\tau)$ . Hence,

$$V_w = (1 - \delta\tau) \left( \rho_F(\lambda_F + \lambda_w)\tau \frac{Z_w}{(1 + r\tau)} + (1 - \rho_F(\lambda_F + \lambda_w)\tau) \frac{V_w}{(1 + r\tau)} \right).$$

<sup>15</sup> An alternative derivation of the value functions that avoids the discrete approximation can be found in Mailath et al. (1999).

Solving the first equation for  $Z_W$  and then taking the limit as  $\tau$  approaches zero gives an expression that relates the continuous-time values of  $Z_W$  and  $V_W$ :

$$Z_W = \lim_{\tau \rightarrow 0} \frac{w\tau(1+r\tau) + (1-\delta\tau)\delta\tau V_W}{1+r\tau - (1-\delta\tau)^2}$$

$$= \frac{w + \delta V_W}{r + 2\delta}.$$

To interpret this equation, we note that the employed worker's value from her current job can be calculated by discounting the flow of  $w$  at rate  $r + 2\delta$ , reflecting the discount rate  $r$  and the death rates of both partners. The value  $Z_W$  consists of this value plus the term  $\delta V_W/(r + 2\delta)$ , capturing the expected present value of being returned to the unemployed pool by surviving a firm death.

Similarly,

$$V_W = \lim_{\tau \rightarrow 0} \frac{(1-\delta\tau)\rho_F(\lambda_F + \lambda_W)\tau Z_W}{(1+r\tau) - (1-\delta\tau)(1-\rho_F(\lambda_F + \lambda_W)\tau)}$$

$$= \frac{\rho_F(\lambda_F + \lambda_W)Z_W}{\rho_F(\lambda_F + \lambda_W) + r + \delta}.$$

Analogous calculations for the firms yield

$$Z_F = \frac{f + \delta V_F}{r + 2\delta}$$

and

$$V_F = \frac{\rho_W H_W (\lambda_F + \lambda_W) Z_F}{\rho_W H_W (\lambda_F + \lambda_W) + r + \delta},$$

where  $f$  is the expected flow payoff to an occupied firm,  $V_F$  is the steady-state value of a vacant firm, and  $Z_F$  is the steady-state value of a firm currently employing a worker.

Rather than calculating the equilibrium values of  $w$  and  $f$ , we use the observation that  $w + f = x$ , the total flow surplus, to calculate the equilibrium values of  $V_W$  and  $V_F$  directly. Letting  $S$  denote the surplus a matched firm and worker divide, we have

$$S = Z_W + Z_F = \frac{x + \delta(V_W + V_F)}{r + 2\delta}.$$

Firms and workers bargain over the surplus  $S$  by

making take-it-or-leave-it wage proposals with equal probability. In equilibrium, any such proposal makes the responding agent indifferent between accepting the proposal and rejecting, and so

$$(4) \quad Z_W = \frac{1}{2} V_W + \frac{1}{2} (S - V_F)$$

and

$$(5) \quad Z_F = \frac{1}{2} V_F + \frac{1}{2} (S - V_W).$$

Solving for  $V_W$  and  $V_F$ , we have the *value equations*:

$$(6) \quad V_W =$$

$$\frac{\rho_F(\lambda_F + \lambda_W)x}{(r + \delta)[(\rho_F + \rho_W H_W)(\lambda_F + \lambda_W) + 2(r + 2\delta)]},$$

and

$$(7) \quad V_F =$$

$$\frac{\rho_W H_W (\lambda_F + \lambda_W)x}{(r + \delta)[(\rho_F + \rho_W H_W)(\lambda_F + \lambda_W) + 2(r + 2\delta)]}.$$

Finally, suppose that no worker acquires skills, giving  $H_W = 0$ . Then substituting  $H_W = 0$  and  $\rho_F = 1$  in (6) gives the value to a worker who decides to become skilled when no other worker acquires skills,

$$V_W^0 \equiv \frac{(\lambda_F + \lambda_W)x}{(r + \delta)(\lambda_F + \lambda_W + 2(r + 2\delta))}.$$

Note that  $V_W^0$  is an upper bound for  $V_W$ . We are now in position to define a symmetric steady state.

*Definition 1:* A *symmetric steady state* is a 5-tuple of values  $(H_W, \rho_F, \rho_W, V_W, V_F)$  satisfying the skilled-worker steady-state condition (1), the vacancies steady-state condition (2), the unemployment steady-state condition (3), and the value equations (6) and (7).

*Definition 2:* A *symmetric equilibrium* is a symmetric steady state in which every firm finds it optimal to search both colors of worker.

If the opportunity cost of acquiring skills is

too large, then the only symmetric steady state is trivial, in that no worker becomes skilled. The following proposition focuses attention on *non-trivial* equilibria—some workers acquire skills ( $H_W > 0$ )—by assuming that there are some workers with sufficiently low opportunity costs of skill acquisition ( $C(V_W^0) > 0$ ). Search frictions ensure that some (but not all) of these workers are unemployed ( $\rho_W \in (0, 1)$ ).

At the other extreme, low opportunity costs may lead all workers to become skilled. A sufficient condition for at least some workers not to acquire skills ( $H_W < 1$ ) is  $C(x/(r + \delta)) \leq 1$ . This ensures that there are some workers whose opportunity cost of acquiring skills exceeds the maximum possible payoff of  $x/(r + \delta)$ , the value of receiving the entire flow surplus  $x$  and being immediately rematched with a new firm upon firm death.<sup>16</sup> An equilibrium is *interior* if  $0 < H_W < 1$ . The proof of the following proposition is in the Appendix, as are other omitted proofs.

**PROPOSITION 1:** *Every symmetric steady state is a symmetric equilibrium. A symmetric equilibrium exists and is unique. If  $C(V_W^0) > 0$ , then  $H_W > 0$  and  $\rho_W \in (0, 1)$ . If  $C(x/(r + \delta)) \leq 1$ , then  $H_W < 1$ .*

It is immediate that every symmetric steady state is a symmetric equilibrium, since the two colors of worker act, and so can be treated, identically.

A basic asymmetry between firms and workers is built into the model, because only workers have the opportunity of opting out of the skilled sector. This asymmetry is reflected in the equilibrium (when it is interior), since skilled workers are in short supply. Workers use this imbalance to extract a larger share of the surplus when bargaining with the firm, which in turn gives workers a larger value of participating in the market:

**PROPOSITION 2:** *If the symmetric equilibrium is interior,*

$$\rho_F > \rho_W, \quad Z_F < Z_W, \quad \text{and} \quad V_F < V_W.$$

<sup>16</sup> While sufficient, this condition is not necessary for interior symmetric equilibria. In later sections, we construct interior symmetric equilibria for parameter values that violate this condition.

If firms must also make an investment to participate in the skilled labor market, the asymmetry between firms and workers reflected in Proposition 2 would disappear.<sup>17</sup> However, the economic forces in which we are interested are most conveniently displayed in a model which ignores this complication.

Asymmetries between firms and workers in the ability to search have no effect on equilibrium. The search intensities  $\lambda_F$  and  $\lambda_W$  only affect the equilibrium through their sum  $\lambda \equiv \lambda_F + \lambda_W$ , while the distribution of search opportunities between firms and workers is irrelevant. As long as firms search both reds and greens, it is thus immaterial whether contacts are initiated by firms or by workers. Increasing the sum  $\lambda$  of the search intensities has the expected effects of increasing the rate at which an unemployed worker encounters vacant firms ( $\rho_F \lambda$ ) and the rate at which a vacant firm encounters unemployed workers ( $\rho_W H_W \lambda$ ). This reduces the proportion of vacant firms and the unemployment rate of workers, increasing the value of acquiring skill and the proportion of workers acquiring skill:

**PROPOSITION 3:** *The symmetric equilibrium is unchanged by variations in  $\lambda_F$  and  $\lambda_W$  that leave the sum  $\lambda = \lambda_F + \lambda_W$  unchanged. Moreover, if the equilibrium is interior,*

$$\frac{d(\rho_F \lambda)}{d\lambda} > 0, \quad \frac{d(\rho_W H_W \lambda)}{d\lambda} > 0$$

and hence

$$\frac{d\rho_F}{d\lambda} < 0, \quad \frac{d\rho_W}{d\lambda} < 0,$$

$$\frac{dV_W}{d\lambda} > 0, \quad \text{and} \quad \frac{dH_W}{d\lambda} > 0.$$

Since firms are in fixed supply, it is intuitive that an increase in the search intensity should reduce the vacancy rate of firms ( $d\rho_F/d\lambda < 0$ ). In contrast, it initially appears as if the increased search intensity, and the corresponding increase in the value of a worker, could prompt a suffi-

<sup>17</sup> Adrian M. Masters (1998) studies such a model, examining the inefficiencies due to bargaining and search.



ciently large increase in the number of skilled workers to raise the unemployment rate of skilled workers. However, an increase in the unemployment rate is inconsistent with the unemployment steady-state condition (3).

The value of vacant firms is not monotonically increasing in aggregate search intensity  $\lambda$ . If the symmetric equilibrium is interior, so  $H_W < 1$ , there is an excess supply of firms. If an increase in search intensity produces only a modest increase in the supply of skilled workers, the attendant increase in the workers' value may suffice to move the balance of bargaining power sufficiently in favor of workers that firms are worse off. In particular,  $V_F = 0$  when  $\lambda = 0$  and, when  $H_W < 1$ ,  $V_F \rightarrow 0$  as  $\lambda \rightarrow \infty$  (since workers capture all the surplus when matching frictions disappear).

We now describe how the symmetric equilibrium responds to supply-and-demand pressures. Parameterize the distribution of opportunity costs by fixing a distribution function  $K$  with support  $\mathfrak{R}_+$  and setting  $C(V_W) = K(V_W - Y)$ . An increase in  $Y$  can be interpreted as an increase in the attractiveness of the unskilled labor sector and hence a decrease in the supply of skilled labor. The symmetric equilibrium is trivial for sufficiently large  $Y$  and is interior otherwise. The following proposition follows from a straightforward application of the implicit function theorem, and its proof is omitted.

**PROPOSITION 4:** *Let the distribution of opportunity costs be given by  $K(V_W - Y)$ . If the symmetric equilibrium is interior ( $0 < H_W < 1$ ), then  $d\tilde{p}/dY = d(\rho_W H_W)/dY < 0$ , and hence,*

$$\frac{d\rho_F}{dY} > 0, \quad \frac{d\rho_W}{dY} < 0, \quad \frac{dV_F}{dY} < 0,$$

$$0 < \frac{dV_W}{dY} < 1, \quad \text{and} \quad \frac{dH_W}{dY} < 0.$$

An increase in  $Y$ , by increasing the opportunity cost of acquiring skills, reduces the quantity of skilled labor supplied ( $dH_W/dY < 0$ ). As a result, the vacancy rate of firms increases, while the unemployment rate of workers decreases. This shift in unemployment rates is accompanied by a shift of bargaining power in favor of

workers, and the wage increases. As a result, the equilibrium value of a firm falls, while the equilibrium value of a skilled worker must increase to offset the higher opportunity cost of acquiring skills.

In a frictionless world ( $\lambda_W, \lambda_F = \infty$ ) in which  $H_W < 1$ , the value of a match between a worker and a firm is  $x/(r + \delta)$  and workers capture all of the surplus. There is no unemployment, and hence no excess expenditure on acquiring skills, and all workers for whom  $\alpha \leq x/(r + \delta)$  enter the skilled sector. In our model, time-consuming search and bargaining lead to two distortions. First, there is unemployment. The total expenditure on acquiring skills, including the opportunity cost of sacrificing participation in the unskilled market, is thus inefficiently higher than would be needed in a frictionless world to achieve the realized volume of employment. Second, too few workers acquire skill, both because unemployment reduces the value of acquiring skills and because costly bargaining prevents workers from capturing the entire surplus from a match.<sup>18</sup>

Using the symmetric equilibrium as a foundation, we can construct a host of asymmetric equilibria. For example, we need only let half the firms search exclusively for green workers and half for reds to achieve an equilibrium in which firms are completely segregated, but in which aggregate outcomes and payoffs duplicate those of the symmetric equilibrium. In this case, the various colors of worker receive separate but equal treatment. The following section considers asymmetric equilibria in which separate treatment is inherently unequal.

<sup>18</sup> Because an increase in the search intensity can decrease  $V_F$ , reducing search frictions need not lead to Pareto-superior outcomes. However, an increase in search intensity must increase total economic surplus, net of opportunity costs, generated by the market. In particular, an increased search intensity reduces the expected length of an unemployment spell for existing workers, increasing the surplus generated by each such worker (with no new opportunity costs incurred). New workers are attracted into the market only if the value of entry exceeds their opportunity cost, which in turn implies that the value of the additional surplus they create must exceed their opportunity cost, since they do not capture all of this surplus.

### III. Asymmetric Steady States

We turn now to the study of asymmetric equilibria in which firms search only green workers. There is clearly nothing special about greens here, and an analogous equilibrium exists in which firms search only for reds.

Consider first a market in which all workers automatically enter the skilled sector and in which every meeting between a firm and worker results in employment of the worker at an exogenously fixed wage. Suppose moreover that firms search only for green workers. Then there will be more unemployed reds than greens, because reds acquire employment only as a result of their own search, while greens acquire employment either as a result of their own search or firm search. As a result, firms earn a higher payoff from seeking reds than greens, and firm behavior cannot be sustained in an equilibrium.

In our model, two additional forces appear. First, wages are determined endogenously. If firms search only greens, then red workers are in a relatively weak bargaining position and so earn a lower wage than greens. This makes it even less likely that firms will find it optimal to search only for greens. However, fewer reds may acquire skills, making reds less attractive because it is harder to find a skilled red, and reinforcing the decision to search only for greens. The question is then whether the market wage for reds can fall enough to disrupt an asymmetric equilibrium by making it optimal for firms to search reds, or whether the lower wages will be overwhelmed by a sufficiently large adjustment in the number of workers acquiring skills as to make searching reds suboptimal.

We break our study of the asymmetric equilibrium into three parts. This section defines and characterizes the steady state induced by asymmetric firm behavior. Section IV investigates the optimality of asymmetric search and hence the existence of an asymmetric equilibrium. Section V explores some implications of asymmetric equilibria. The notation follows that of the symmetric case, but with the single subscript “ $W$ ” for workers now replaced by “ $R$ ” and “ $G$ ,” for red and green workers.

The equilibrium conditions are analogous to those of the symmetric case, with two exceptions. First, in equilibrium, firms must find it optimal to only search greens. Second, if there

are unemployed skilled red workers, then some vacant firms will meet them as a result of worker search, and we must specify a firm’s reaction to such a meeting. The steady-state conditions are derived under the presumption that a vacant firm reaches an agreement with any unemployed skilled red worker it happens to meet. We then confirm that in the steady state, vacant firms strictly prefer to reach such an agreement rather than remaining vacant (Proposition 6).

The inflows of new skilled workers are given by  $\delta C(V_G)/2$  for greens and  $\delta C(V_R)/2$  for reds, since half of the workers are green and half are red. In a steady-state equilibrium, these inflows match the outflows of skilled workers caused by death (given by  $\delta H_G$  and  $\delta H_R$ ), so the *green- and red-skilled-worker steady-state* conditions are

$$(8) \quad H_G = C(V_G)/2$$

and

$$(9) \quad H_R = C(V_R)/2.$$

The *vacancies steady-state* condition is:

$$(10) \quad 2\delta(1 - \rho_F) = 2\rho_F\lambda_F H_G \rho_G + (\rho_G H_G + \rho_R H_R)\lambda_W \rho_F.$$

This again reflects a balancing of the rate at which deaths create firm vacancies, given by  $(1 - \rho_F)2\delta$ , and the rate at which vacancies are filled. The first term on the right side captures the rate at which vacancies are filled as a result of firm search, since there are  $\rho_F$  firms searching and each firm is generating meetings at rate  $2\lambda_F \rho_G H_G$  (given a green unemployed skilled-worker population of size  $\rho_G H_G$ ). The second term captures the effect of worker search. These terms are asymmetric because firms search only green workers while both types of workers search for firms.

The *green and red unemployed steady-state* conditions are given by:

$$(11) \quad 2\delta(1 - \rho_G) = \rho_G \rho_F (\lambda_W + 2\lambda_F)$$

and

$$(12) \quad 2\delta(1 - \rho_R) = \rho_R \rho_F \lambda_W.$$

These are analogous to the unemployment rate for the symmetric equilibrium, given by (3), with the exceptions that red workers find employment only as a result of their own search, and the entire unit measure of firms is searching the half-unit measure of green workers, giving a firm search intensity of  $2\lambda_F$ .

Analogously to the symmetric case, the various value functions are (where, for example,  $w_R$  is the expected wage of a red skilled worker and  $Z_{F,R}$  is the value of the firm when currently matched with a red worker):

$$Z_R = \frac{w_R + \delta V_R}{r + 2\delta}, \quad Z_G = \frac{w_G + \delta V_G}{r + 2\delta},$$

$$Z_{F,R} = \frac{f_R + \delta V_F}{r + 2\delta}, \quad Z_{F,G} = \frac{f_G + \delta V_F}{r + 2\delta},$$

$$V_R = \frac{\rho_F \lambda_W Z_R}{\rho_F \lambda_W + r + \delta},$$

$$V_G = \frac{\rho_F (2\lambda_F + \lambda_W) Z_G}{\rho_F (2\lambda_F + \lambda_W) + r + \delta},$$

and

$$V_F = \frac{(2\lambda_F + \lambda_W) \rho_G H_G Z_{F,G} + \lambda_W \rho_R H_R Z_{F,R}}{(2\lambda_F + \lambda_W) \rho_G H_G + \lambda_W \rho_R H_R + r + \delta}.$$

The surpluses of the different matches are given by

$$S_R = Z_R + Z_{F,R} = \frac{x + \delta(V_R + V_F)}{r + 2\delta}$$

and

$$S_G = Z_G + Z_{F,G} = \frac{x + \delta(V_G + V_F)}{r + 2\delta}.$$

Finally, bargaining gives

$$\begin{aligned} Z_R &= \frac{V_R}{2} + \frac{(S_R - V_F)}{2} \\ &= \frac{x}{2(r + 2\delta)} + \frac{(r + 3\delta)V_R - (r + \delta)V_F}{2(r + 2\delta)}, \end{aligned}$$

$$Z_G = \frac{V_G}{2} + \frac{(S_G - V_F)}{2}$$

$$= \frac{x}{2(r + 2\delta)} + \frac{(r + 3\delta)V_G - (r + \delta)V_F}{2(r + 2\delta)},$$

$$Z_{F,R} = \frac{V_F}{2} + \frac{(S_R - V_R)}{2}$$

$$\begin{aligned} &= \frac{x}{2(r + 2\delta)} \\ &\quad + \frac{(r + 3\delta)V_F - (r + \delta)V_R}{2(r + 2\delta)}, \end{aligned}$$

and

$$Z_{F,G} = \frac{V_F}{2} + \frac{(S_G - V_G)}{2}$$

$$\begin{aligned} &= \frac{x}{2(r + 2\delta)} \\ &\quad + \frac{(r + 3\delta)V_F - (r + \delta)V_G}{2(r + 2\delta)}. \end{aligned}$$

Some tedious algebra allows us to solve for the value functions:<sup>19</sup>

$$\begin{aligned} (13) \quad V_F &= \frac{x}{(r + \delta)\Delta} \left[ (2\lambda_F + \lambda_W) \rho_F \lambda_W \right. \\ &\quad \times (\rho_G H_G + \rho_R H_R) \\ &\quad + 2(r + 2\delta) \{ (2\lambda_F + \lambda_W) \rho_G H_G \\ &\quad \left. + \lambda_W \rho_R H_R \} \right], \end{aligned}$$

$$(14) \quad V_R = \frac{\rho_F \lambda_W (\rho_F (2\lambda_F + \lambda_W) + 2(r + 2\delta)) x}{(r + \delta)\Delta},$$

<sup>19</sup> The details can be found in Mailath et al. (1999).

and

$$(15) \quad V_G = \frac{\rho_F(2\lambda_F + \lambda_W)(\rho_F\lambda_W + 2(r + 2\delta))x}{(r + \delta)\Delta},$$

where  $\Delta \equiv 2(r + 2\delta)\{(2\lambda_F + \lambda_W)(\rho_F + \rho_G H_G) + \lambda_W(\rho_F + \rho_R H_R) + 2(r + 2\delta)\} + \rho_F\lambda_W(2\lambda_F + \lambda_W)(\rho_F + \rho_R H_R + \rho_G H_G)$ .

*Definition 3:* A green asymmetric steady state is a 8-tuple  $(H_G, \rho_G, V_G, H_R, \rho_R, V_R, \rho_F, V_F)$  solving the balance equations (8)–(12) and the value functions (13)–(15).

As before, the green asymmetric steady state is *nontrivial* if some workers acquire skills, and nontriviality requires the existence of some workers with low opportunity costs of skill acquisition. The Appendix uses a fixed-point argument to establish the following proposition.

**PROPOSITION 5:** *There exists a green asymmetric steady state. If  $C(V_W^0) > 0$ , every green asymmetric steady state is nontrivial.*

We have not asserted the uniqueness of a green asymmetric steady state. Exploiting the recursive nature of the steady-state conditions, we can use (10)–(12) to show that for any fixed worker entry rates  $H_G$  and  $H_R$ , there is a unique set of vacancy and unemployment rates. However, there may be multiple green asymmetric steady states, characterized by different entry rates  $H_G$  and  $H_R$ .

The following proposition characterizes the different treatment received by red and green workers in an asymmetric steady state.

**PROPOSITION 6.** *In a nontrivial green asymmetric steady state, some green workers acquire skills ( $H_G > 0$ ), and*

$$V_R < V_G, \quad H_R < H_G, \quad Z_{F,R} > Z_{F,G},$$

$$Z_{F,R} > V_F, \quad w_R < w_G, \quad f_R > f_G,$$

$$\text{and} \quad \rho_R > \rho_G.$$

In a green asymmetric equilibrium, red workers face a less attractive value of entering the market than do green workers ( $V_R < V_G$ ), and so if any workers acquire skills, then some green workers do ( $H_G > 0$ ), while fewer red than green workers acquire skills ( $H_R < H_G$ ). Red workers are thus at a disadvantage when bargaining with firms. Firms exploit this weaker position to extract a larger share of the surplus from red workers. As a result, average red-worker wages fall short of green-worker wages ( $w_R < w_G$ ), while the firm receives a larger portion of the flow surplus from a red worker ( $f_R > f_G$ ). Given this wage difference, a vacant firm given a choice between a red worker and a green worker would strictly prefer a red worker ( $Z_{F,R} > Z_{F,G}$ ), and a vacant firm prefers to enter an employment relationship with a red worker rather than continue searching ( $Z_{F,R} > V_F$ ). Our assumption that the firm would hire any red workers it meets, used to calculate the steady state, is thus consistent with optimal behavior. At the same time, since reds are not being searched by firms, the unemployment rate is higher for reds than greens ( $\rho_R > \rho_G$ ). In conventional terms, reds thus suffer lower wages, lower labor-force participation rates, and higher unemployment rates. Finally, notice that Proposition 6 does not rule out  $H_R = 0$ , and we will construct nontrivial asymmetric steady states in which no reds acquire skills.

#### IV. Asymmetric Equilibria

In the symmetric case, red and green workers achieve identical outcomes, making the firm indifferent between the two colors of worker and ensuring that the firm's decision to treat red and green workers identically is automatically optimal. In contrast, (green) asymmetric equilibria involve an additional consideration beyond the steady-state conditions: it must be optimal for the firm to search only green workers.

Instead of searching only green workers, the firm has the option of either searching only red workers or searching both types of workers. Searching only greens is optimal if

$$(16) \quad V_F \geq \max\{V_F(R|G), V_F(W|G)\},$$

where  $V_F(R|G)$  ( $V_F(W|G)$ ) is the value of a firm searching red (all) workers in a steady state in which all other firms are searching green workers. Hence,

$$V_F(R|G) = \frac{\lambda_W \rho_G H_G Z_{F,G} + (2\lambda_F + \lambda_W) \rho_R H_R Z_{F,R}}{\lambda_W \rho_G H_G + (2\lambda_F + \lambda_W) \rho_R H_R + r + \delta}$$

and

$$V_F(W|G) = \frac{(\lambda_W + \lambda_F)(\rho_G H_G Z_{F,G} + \rho_R H_R Z_{F,R})}{(\lambda_W + \lambda_F)(\rho_G H_G + \rho_R H_R) + r + \delta}.$$

*Definition 4:* A green asymmetric equilibrium is a green asymmetric steady state satisfying (16).

Direct manipulation of the firm-search optimality condition  $V_F \geq V_F(R|G)$  allows us to identify an essential feature of asymmetric equilibria, yielding (proof omitted) the following proposition.

**PROPOSITION 7:** *In all nontrivial green asymmetric equilibria,*

$$\rho_R H_R < \rho_G H_G.$$

Since vacant firms in a green asymmetric equilibrium prefer to hire reds, conditional on meeting a skilled worker, a vacant firm would direct search at reds if red and green workers were equally easy to find. An equilibrium in which firms do not seek red workers can then only be supported if there are more unemployed skilled green workers than red workers, giving  $\rho_R H_R < \rho_G H_G$ .

We have already seen that skilled red workers in a green asymmetric steady state must have a higher unemployment rate than green workers. This is consistent with the paucity of skilled red workers, required for the optimality of asymmetric search, only if sufficiently few red workers acquire skills. The existence of a green asymmetric equilibrium thus requires a sufficiently vigorous supply response on the part of workers. In particular, an asymmetric steady

state in which  $H_G \leq H_R$  cannot be a nontrivial green asymmetric equilibrium, and a green asymmetric equilibrium is inconsistent with a degenerate opportunity cost distribution that fixes skilled acquisition rates so that there is no supply response ( $H_G = H_R$ ).

When does an asymmetric equilibrium exist? In addition to sufficiently responsive worker supply decisions, the relative sizes of the search intensities  $\lambda_W$  and  $\lambda_F$  are now important. This represents a departure from the symmetric equilibrium, where only the sum  $\lambda_W + \lambda_F$  mattered. A nontrivial green asymmetric equilibrium requires that there be at least some firm search, or  $\lambda_F > 0$ . The conditions for the existence of an asymmetric equilibrium involve the interplay between the relative sizes of  $\lambda_F$  and  $\lambda_W$  and the worker supply responses induced by the opportunity cost distribution  $C$ , with larger values of  $\lambda_F$  allowing asymmetric equilibria to exist under weaker assumptions on  $C$ . It is convenient to vary  $\lambda_F$  and  $\lambda_W$ , while preserving the sum  $\lambda = \lambda_F + \lambda_W$ .

We begin with the result that asymmetric equilibria exist when  $\lambda_F$  is large.

**PROPOSITION 8:** *Fix  $\lambda = \lambda_F + \lambda_W$  and suppose  $C(V_W^0) > 0$ . There exists  $\lambda_F^* < \lambda$  such that a nontrivial asymmetric equilibrium exists for all  $\lambda_F \in (\lambda_F^*, \lambda]$ .*

If virtually all contacts between firms and workers arise as a result of firm search, then a decision on the part of firms to search only green workers imposes an insurmountable obstacle to red workers. Red skilled workers face such limited employment prospects that red workers overwhelmingly opt into the unskilled sector of the economy. Since it is much more likely that a firm will be successful in finding an unemployed green skilled worker than an unemployed red one, firms optimally search only greens. The requirement  $C(V_W^0) > 0$  rules out the possibility that no green worker finds it optimal to acquire skills even when firms are searching only greens.

It is not surprising that asymmetric equilibria exist if firm search is sufficiently more important than worker search. The more important observation is that if the distribution of the cost of acquiring skills tends to concentrate its mass near the value of being a skilled worker, then worker supply responses will be large and

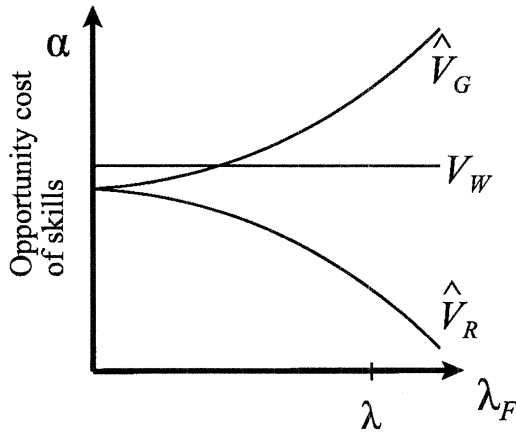


FIGURE 1. EXTREME ASYMMETRIC VALUE FUNCTIONS

Notes: The position of  $V_W$  is determined by the fraction of skilled workers in the symmetric equilibrium. It is possible that  $V_W < \hat{V}_G(0)$ .

asymmetric equilibria can exist even for small values of  $\lambda_F$ . Fix total search intensity  $\lambda = \lambda_W + \lambda_F$ . Denote by  $\hat{V}_G(\lambda_F)$  and  $\hat{V}_R(\lambda_F)$  the values of acquiring skills, to a green and a red worker (respectively), in a hypothetical asymmetric steady state in which the firms' search intensity is  $\lambda_F$ , all green workers acquire skills ( $H_G = 1/2$ ), and no red workers do ( $H_R = 0$ ). We refer to such a steady state as an *extreme (green) steady state*. Since  $\lambda_F \in [0, \lambda]$ ,  $\hat{V}_G, \hat{V}_R: [0, \lambda] \rightarrow \mathfrak{R}_+$ . A lengthy algebraic manipulation yields:<sup>20</sup>

$$(17) \quad \frac{d\hat{V}_G(\lambda_F)}{d\lambda_F} > 0 \quad \text{and} \quad \frac{d\hat{V}_R(\lambda_F)}{d\lambda_F} < 0.$$

These extreme asymmetric value functions are illustrated in Figure 1. Under the maintained assumption that all green workers enter and no red workers enter, the asymmetric steady state in which firms search only greens is equivalent to a symmetric equilibrium in which the total number of workers is  $1/2 (= H_G)$  and in which all of the workers necessarily enter. Increasing  $\lambda_F$  while holding the total of  $\lambda_F + \lambda_W$  constant at  $\lambda$  has the effect of increasing the effective search rate, given by  $2\lambda_F + \lambda_W$ . But increasing the search rate in a symmetric equilibrium increases the value of workers, ensuring that  $\hat{V}_G(\lambda_F)$  is increasing in  $\lambda_F$ . To ascertain the

behavior of  $\hat{V}_R(\lambda_F)$ , notice that  $\rho_F$  decreases as  $\lambda_F$  increases, meaning that a red worker who did acquire skills would find it more difficult to find a vacant firm, which tends to decrease the value  $\hat{V}_R(\lambda_F)$ . An increase in  $\lambda_F$  may also decrease  $V_F$ , decreasing the firm's bargaining power and tending to increase  $\hat{V}_R(\lambda_F)$ , but this force is overwhelmed by the decrease in  $\rho_F$ , ensuring that  $\hat{V}_R(\lambda_F)$  decreases.

Note that when  $\lambda_F = 0$ , all contacts between firms and workers come about as a result of worker search. It is then irrelevant whether firms search reds or greens, and red and green workers face identical environments and opportunities, giving  $\hat{V}_G(0) = \hat{V}_R(0)$ .

Figure 1 also displays the function  $V_W(\lambda_F)$ , showing the value of acquiring skills in a *symmetric* equilibrium. Since we have fixed  $\lambda = \lambda_F + \lambda_W$ , this function is constant in  $\lambda_F$  (Proposition 3). The relative positions of the curves  $V_W, \hat{V}_G$ , and  $\hat{V}_R$  depend upon the number of workers who acquire skills in the symmetric equilibrium. If  $H_W = 1/2$ , then  $V_W = \hat{V}_G(0) = \hat{V}_R(0)$  (note that when  $\lambda_F = 0$ , so that only workers search, an asymmetric steady state in which all greens but no reds acquire skills— $H_G = 1/2, H_R = 0$ —gives precisely the same outcome as a symmetric steady state in which half of the workers acquire skills— $H_W = 1/2$ ). From Proposition 4, we then know that if more than half of the workers acquire skills in the symmetric equilibrium, then the value of worker skills will be lower, giving  $V_W < \hat{V}_G(0) = \hat{V}_R(0)$ . If less than half of the workers acquire skill in the symmetric equilibrium, then  $V_W > \hat{V}_G(0) = \hat{V}_R(0)$ , as shown in Figure 1.

We now examine the interplay between the importance of firm search, indicated by  $\lambda_F$ , and the distribution of skill costs. Suppose  $V_W$  is the value of skills in the unique symmetric equilibrium when the cost distribution is  $C$ . Then, for any cost distribution  $C_1$  satisfying  $C_1(V_W) = C(V_W)$ ,  $V_W$  is also the value of skills in the unique symmetric equilibrium when the cost distribution is  $C_1$ .

**PROPOSITION 9:** Fix a cost distribution  $C$  and an aggregate search intensity  $\lambda = \lambda_F + \lambda_W$ . Suppose there exists  $\bar{\lambda}_F$  such that  $\hat{V}_G(\bar{\lambda}_F) > V_W > \hat{V}_R(\bar{\lambda}_F)$ , where  $V_W$  is the value of skills in the unique symmetric equilibrium given  $C$ . Then, for all  $\lambda_F \geq \bar{\lambda}_F$ , there

<sup>20</sup> Again, see Mailath et al. (1999).

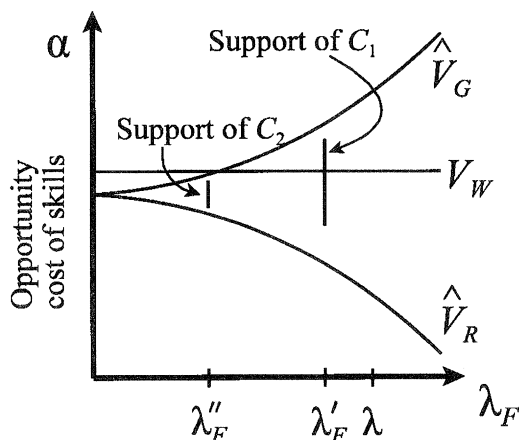


FIGURE 2. EXISTENCE OF EXTREME GREEN ASYMMETRIC EQUILIBRIA

Notes:  $V_W$  is the symmetric value function for the distribution  $C_1$  (but not for  $C_2$ ). There is an extreme equilibrium when the cost distribution is  $C_1$  and  $\lambda_F = \lambda'_F$ , and also when the distribution is  $C_2$  and  $\lambda_F = \lambda''_F$ .

exists  $C_1$  such that  $C_1(V_W) = C(V_W)$  (so that  $V_W$  is the value of skills in the symmetric equilibrium for  $C_1$ ) and such that a nontrivial asymmetric equilibrium exists for the pair  $(\lambda_F, C_1)$ .

Figure 2 illustrates the construction used in the proof: letting  $\lambda_F = \lambda'_F$ , we can find a cost distribution, such as  $C_1$ , with a sufficiently small support so that, under the hypothesized behavior in the extreme steady state, the value to a green worker acquiring skills satisfies  $C_1(\hat{V}_G(\lambda_F)) = 1$ , and the value to a red worker acquiring skills satisfies  $C_1(\hat{V}_R(\lambda_F)) = 0$ . This ensures that the hypothesized behavior of workers is optimal in the extreme steady state. It is then optimal for the firm to search only greens, giving an asymmetric equilibrium. More generally, the key to existence of an asymmetric equilibrium is that workers' decisions of whether to acquire skills are sufficiently sensitive to changes in the value of skills. If they are, then the increase in the value of skills to greens and the decrease in the value of skills to reds, caused by firms searching only greens rather than all workers, will prompt enough greens to acquire skills and enough reds to forgo skills as to make it optimal for the firm to search only greens. In the proof of Proposition 9, we have found it convenient to force such a dramatic

worker response as to cause all greens and no reds to acquire skills, but this extreme case is unnecessary. Reds need not be excluded from the skilled labor market, nor must all greens be included, as long as the supply response to firms' decisions to search only greens is sufficiently large.

While Proposition 9 potentially yields existence for smaller values of  $\lambda_F$  than does Proposition 8, it does not apply to very small values of  $\lambda_F$  if  $H_W \neq 1/2$ , since the cost function  $C_1$  must yield a symmetric equilibrium with worker value  $V_W$ . The same argument, however, immediately yields asymmetric equilibria for any value of  $\lambda_F$ .

**PROPOSITION 10:** For any  $\lambda_F \in (0, \lambda]$  and  $V \in (\hat{V}_R(\lambda_F), \hat{V}_G(\lambda_F))$ , there exists a nontrivial asymmetric equilibrium for any cost distribution sufficiently concentrated around  $V$ .

This is illustrated in Figure 2, where we take  $\lambda_F = \lambda''_F$  and the cost distribution  $C_2$ . Note that  $V_W$  is not the worker value in the symmetric equilibrium for  $C_2$ .

The key to the existence of an asymmetric equilibrium is thus not that firm search is very important, but rather that by deciding to search only green workers, firms can prompt sufficiently large adjustments in worker skill decisions to justify this decision. This can occur even for very small  $\lambda_F > 0$  if the cost of acquiring skills is likely to be near the benefits of being skilled.

Proposition 8 establishes conditions under which a nontrivial asymmetric equilibrium coexists with a nontrivial symmetric one. We could obtain alternative sufficient conditions for such coexistence by adding the condition  $C(V_W^0) > 0$  to Proposition 9. However, Propositions 9 and 10 leave open the possibility that a nontrivial asymmetric equilibrium may exist in an economy which allows only trivial symmetric equilibria. The next proposition gives parameter restrictions for which nontrivial green asymmetric equilibria exist while the symmetric equilibrium is trivial.

**PROPOSITION 11:** Fix  $\kappa > 0$ . For sufficiently low search intensities satisfying  $\lambda_W \leq \kappa\lambda_F$ , there are opportunity cost distributions for which the nontrivial green asymmetric equilibrium exists and the symmetric equilibrium is trivial.

The smaller is  $\kappa$ , and hence the greater is the relative importance of firm search, the larger is the range of total search intensities  $\lambda_W + \lambda_F$  consistent with this result.

An asymmetric equilibrium increases the value of acquiring skills for the advantaged group. Proposition 11 shows that if the opportunity cost of becoming skilled is relatively high, then the symmetric equilibrium may be trivial, but some workers may be willing to acquire skills in order to attain the higher returns provided by an asymmetric equilibrium. An asymmetric equilibrium is then necessary in order to induce any workers to acquire skills: the skilled labor market opens only if it is discriminatory.

We conclude this section with two examples that illustrate the role of group size in the existence of asymmetric equilibria. Let  $N_G$  and  $N_R$  be the sizes of the green and red worker pools, where  $N_G + N_R = 1$ . Section VI discusses the appropriate specification for firm search intensity when a particular group is targeted and the groups are of different sizes. One natural specification, followed here, is that targeting green workers yields a meeting rate of  $[(N_G + N_R)/N_G]\lambda_F = \lambda_F/N_G \equiv \lambda_F^G$ , while targeting red workers yields a meeting rate of  $[(N_G + N_R)/N_R]\lambda_F = \lambda_F/N_R \equiv \lambda_F^R$ . When the two groups are of equal size, targeting a group doubles the search intensity, as we have assumed.

We begin with an example in which an asymmetric equilibrium exists only if the target of discrimination is a sufficiently large *majority*.

*Example 1:* To simplify the calculations, we assume that  $\lambda_F = \lambda$ , so that all search is firm search, and that  $C(0) = 0$ . Then in any asymmetric green steady state, only green workers can be skilled. The value to a green worker in a green asymmetric steady state is given by [from (15)]

$$\hat{V}_G(\lambda_F, N_G) = \frac{\rho_F \lambda_F^G x}{(r + \delta)[(\rho_F + \rho_G N_G) \lambda_F^G + 2(r + 2\delta)]},$$

where  $\hat{V}_G(\lambda_F, N_G)$  contains the argument  $N_G$  to identify the composition of the labor force. Since  $\lambda_W = 0$ , it is straightforward to verify  $\hat{V}_G(\lambda_F, N_G) > V_W^0 > 0 = \hat{V}_R(\lambda_F, N_G)$ . Suppose now that  $C(V_W^0) = 0$ , so that the

symmetric equilibrium is trivial, and let the distribution  $C$  concentrate all its mass on a value  $c^* > C(V_W^0)$ . Then a green asymmetric equilibrium will exist only if  $\hat{V}_G(\lambda_F, N_G) \geq c^*$ . Now notice that  $\hat{V}_G(\lambda_F, N_G)$  is decreasing in  $N_G$ . As a result, a green asymmetric equilibrium will exist only if  $\hat{V}_G(\lambda_F, N_G)$  is sufficiently large, and hence only if  $N_G$  is sufficiently small.<sup>21</sup> In this market, workers have attractive outside options, and the market for skilled labor may be unable to provide a sufficiently large payoff to attract any workers. As a result, there is no nontrivial symmetric equilibrium. Asymmetry, by allowing firms to concentrate their search only on green workers, increases green-worker payoffs, potentially attracting green workers into the market. The value of the asymmetry to green workers is larger when there are fewer green workers, making firm search more concentrated and hence more effective. As a result, a green (or red) asymmetric equilibrium may fail to exist when the groups are of approximately equal size, but may exist when there is a sufficiently small group of greens who can be made the *beneficiaries* of discrimination.

There are two potential obstacles to the existence of an asymmetric equilibrium, one involving the generation of sufficiently high worker values to attract workers in the market, and the other involving the optimality of the firm's asymmetric search strategy. The latter can give rise to considerations that make an asymmetric equilibrium more likely to exist when the *minority* group bears the brunt of discrimination. We now illustrate this.

*Example 2:* Let  $V_W^1$  be the worker value from acquiring skills when all workers become skilled. For simplicity, suppose the support of the opportunity cost distribution consists of two points, 0 and  $\bar{c}$ , with  $C(0) = \gamma$ . Suppose moreover that  $\bar{c} < V_W^1$ . If  $\lambda_W > 0$ , then at least a fraction  $\gamma$  of each worker color group will acquire skills in any equilibrium. Moreover, the symmetric equilibrium is nontrivial with all

<sup>21</sup> This example shares with Proposition 11 the property that the symmetric equilibrium is trivial while skill acquisition occurs in the asymmetric equilibrium. Proposition 11 differs in not requiring all search to be conducted by firms, while examining only equal-sized worker groups.



workers becoming skilled. Finally, if the value of acquiring skills for a red worker in an asymmetric equilibrium is less than  $\bar{c}$ , then precisely a fraction  $\gamma$  of red workers will acquire skill.

We now argue that for  $\lambda_W$  and  $N_R$  small, so that firm search is relatively important and there are few red workers, there is a green asymmetric equilibrium, but no red asymmetric equilibrium exists. Consider first the existence of the green asymmetric equilibrium. For any  $\gamma$ , with vacant firms searching green workers, the green-worker value exceeds  $V_W^1$  and so all green workers become skilled. Since red workers are not being searched by firms, red-worker value can be made less than  $\bar{c}$  by setting  $\lambda_W$  sufficiently small. Then only a fraction  $\gamma$  of red workers acquire skills, and so the pool of unemployed red skilled workers  $\rho_R H_R$  is bounded above by  $\gamma N_R$ . It remains to verify that firms find it optimal to search greens only. The meeting rate for a firm searching green workers is  $\lambda_F^G \rho_G H_G$  while the meeting rate for a firm searching red workers is  $\lambda_F^R \rho_R H_R$ . Now, as  $N_R$  becomes small,  $\lambda_F^G$  approaches  $\lambda_F$  while  $\lambda_F^R$  is unbounded. However,  $\lambda_F^R \rho_R H_R$  is bounded above by  $2\lambda_F \gamma$ , and so by choosing  $\gamma$  small, the meeting rate from searching reds (relative to that from searching greens) can be made sufficiently small that it is not profitable for firms to search reds. Note that the bound on  $\gamma$  is independent of  $N_R$ .

We now argue that no red asymmetric equilibrium exists. Note first that in a red asymmetric steady state, a fraction  $\gamma$  of green workers are skilled, and since  $\lambda_W$  is small,  $\rho_G$  is close to one. Thus the unemployed green-skilled-worker population is approximately of size  $\gamma N_G$ . By choosing  $N_R$  sufficiently small,  $\lambda_F^R$  can be made arbitrarily large and so the unemployment rate of red skilled workers can be made arbitrarily small.<sup>22</sup> Thus, for sufficiently small  $N_R$ , a vacant firm has a sufficiently higher meeting rate from searching greens rather than reds and so there is no red asymmetric equilibrium.

Example 2 captures the sense in which small groups are likely to be the target of discrimination. The constraint on the existence of an asymmetric equilibrium is that the firm may find it optimal to search for red, disadvantaged workers. Because they are victims of discrimi-

nation, skilled red workers face higher unemployment rates and secure lower wages than do green workers. Both characteristics make red workers attractive targets for firms. The countervailing force is that fewer red workers acquire skills, making skilled red workers harder to find. The firm will neglect red workers only if they are scarce enough to be unprofitable search targets, and this will be the case if and only if the pool of red workers is sufficiently small. How small is “sufficiently small” depends upon the characteristics of the market. Propositions 8–10 identify cases in which asymmetric equilibria exist with red- and green-worker pools of equal size. However, it is clear from Example 2 that there are other cases in which asymmetric outcomes exist only if the target of discrimination is a sufficiently small group.

## V. The Welfare Implications of Discrimination

In the green asymmetric equilibria constructed in the previous section, green workers fare better than in the symmetric equilibrium, while red workers fare worse (i.e.,  $V_G > V_W > V_R$ ). This is the expected result that by discriminating in favor of greens when searching for workers, firms make green workers better off and red workers worse off. Is this a characteristic of all green asymmetric equilibria? We first present two examples showing that these equalities may hold only weakly. In both examples, we find that discrimination need not harm red workers, while the second example shows that discrimination may allow a weak Pareto improvement in worker payoffs. We assume throughout that red and green workers each comprise half of the population.

*Example 3:* Consider a symmetric equilibrium giving the value  $V_W$  illustrated in Figure 3. Notice that  $H_W < 1/2$ , since  $V_W > \hat{V}_G(0) = \hat{V}_R(0)$ . Now fix  $\lambda_F$  such that  $\hat{V}_G(\lambda_F) > V_W$ . Then if  $C(\hat{V}_G(\lambda_F)) = 1$  and  $C(\hat{V}_R(\lambda_F)) = 0$ , there exists an asymmetric equilibrium in which  $H_G = 1/2$  and  $H_R = 0$ . But as  $\lambda_F$  declines toward the value  $\lambda_F^*$  at which  $\hat{V}_G(\lambda_F^*) = V_W$ ,  $V_G$  declines to  $V_W$  as  $V_R$  remains well below  $V_W$ . In the limit, we have an equilibrium corresponding to  $\lambda_F^*$ , supported by a cost distribution that places all of its mass on the value  $\alpha = V_W$ , for which  $\hat{V}_G(\lambda_F^*) = V_W$  and  $\hat{V}_R(\lambda_F^*) < V_W$ . This demonstrates that being the targets of pref-

<sup>22</sup> The red unemployed steady-state condition is  $2\delta(1 - \rho_R)/\rho_R = \rho_F(\lambda_W + \lambda_F^R)$  [cf., (11)].

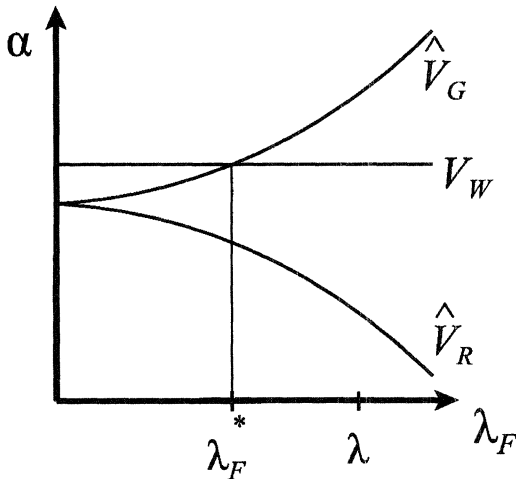


FIGURE 3. THE EQUILIBRIUM OF EXAMPLE 3

Note: If the cost distribution is sufficiently concentrated around  $V_W$  and  $\lambda_F$  is sufficiently close to (but larger than)  $\lambda_F^*$ , then all workers are almost indifferent between the extreme green asymmetric equilibrium and the symmetric equilibrium.

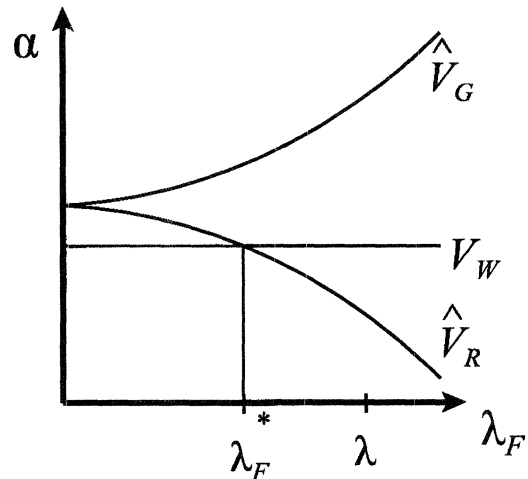


FIGURE 4. THE EQUILIBRIUM OF EXAMPLE 4

Note: If the cost distribution is sufficiently concentrated around  $V_W$  and  $\lambda_F$  is sufficiently close to (but larger than)  $\lambda_F^*$ , then red workers are almost indifferent between the extreme green asymmetric equilibrium and the symmetric equilibrium, while green workers strictly prefer the asymmetric equilibrium.

erable search activity need not make greens better off (or at least, for nontrivial cost distribution, need not make greens very much better off). At the same time, reds do not suffer any payoff losses from discrimination in the limiting equilibrium corresponding to  $\lambda_F^*$ . The value of skills to a skilled red worker falls precipitously when firms search only greens, and reds accordingly opt out of the skilled labor market, but the opportunity cost of  $\alpha = V_W$  ensures that they suffer no payoff loss in doing so.

*Example 4:* Consider a symmetric equilibrium giving the value  $V_W$  illustrated in Figure 4. Notice that  $H_W > 1/2$ , since  $V_W < \hat{V}_G(0) = \hat{V}_R(0)$ . Now fix  $\lambda_F$  such that  $\hat{V}_R(\lambda_F) < V_W$ . Then if  $C(\hat{V}_G(\lambda_F)) = 1$  and  $C(\hat{V}_R(\lambda_F)) = 0$ , there again exists an asymmetric equilibrium in which  $H_G = 1/2$  and  $H_R = 0$ . But as  $\lambda_F$  declines toward the value  $\lambda_F^*$  at which  $\hat{V}_G(\lambda_F^*) = V_W$ ,  $V_R$  increases to  $V_W$  as  $V_G$  remains well above  $V_W$ . In the limit, we have an equilibrium for  $\lambda_F^*$  featuring a cost distribution that places all of its mass on the value  $\alpha = V_W$  and featuring  $\hat{V}_R(\lambda_F^*) = V_W$  while  $\hat{V}_G(\lambda_F^*) > V_W$ . This demonstrates that having firms ignore

reds to search only greens need not make reds worse off (or at least, for nontrivial cost distribution, need not make reds very much worse off). Reds opt out of the skilled labor market when firms search only greens, but the value of skills to a red worker has not fallen and the opportunity cost of  $\alpha = V_W$  ensures that they suffer no payoff loss from declining to become skilled. In this case, however, green workers gain significantly from being the targets of firm search, so that the discriminatory search produces a weak Pareto improvement in worker payoffs.

In general, asymmetric equilibria can yield much lower utility levels for red workers than does the symmetric equilibrium. In the equilibrium illustrated in Figure 2, for example, all the red workers who choose to become skilled in the symmetric equilibrium, except for the marginal red worker (for whom  $\alpha = V_W$ ), strictly prefer the symmetric equilibrium to the green asymmetric equilibrium. However, the possibility remains that total welfare is always higher in the asymmetric rather than symmetric equilibrium, in the sense that the gains to green workers and (possibly)

firms outweigh the losses to red workers.<sup>23</sup> The next example shows that this need not be the case.

*Example 5:* Let  $\lambda_F = \lambda$ , so that all search is conducted by firms. Let the cost distribution  $C$  put a mass of  $1/2$  on  $\hat{V}_G(\lambda_F)$  and a mass of  $1/2$  on  $\hat{V}_R(\lambda_F) = 0$ . In the symmetric equilibrium, half of the workers, equally divided between reds and greens, enter and  $V_W = \hat{V}_G(0) = \hat{V}_R(0)$ . We consider an asymmetric equilibrium in which all of the green and none of the red workers enter, so that we again have half of the workers entering. We use a hat to denote values from the asymmetric equilibrium and suppress the argument  $\lambda$ . To compute the change in welfare in moving from the symmetric to asymmetric equilibrium, note that (i)  $1/4$  of the workers have gained  $\hat{V}_G - V_W$  (these are the greens who acquired skills in the symmetric equilibrium and now receive  $\hat{V}_G$  rather than  $V_W$ ), (ii)  $1/4$  of the workers have lost  $V_W$  (these are the red workers who acquired skills in the symmetric equilibrium, and must now take their outside option of zero), and (iii) firms have gained  $\hat{V}_F - V_F$ , which may be positive or negative. The remaining workers, whose opportunity cost of skills is given by  $\hat{V}_G$ , experience no change in payoff when moving to the asymmetric equilibrium, either because they still do not enter the market (red workers) or they enter to obtain a payoff of  $\hat{V}_G$  (green workers).

We are thus interested in the circumstances under which

$$\frac{1}{4}(\hat{V}_G - V_W) - \frac{1}{4}V_W + \hat{V}_F - V_F < 0,$$

and hence the asymmetric equilibrium entails a welfare loss. Rearranging, this is

$$4\hat{V}_F + \hat{V}_G < 4V_F + 2V_W.$$

Noting that half of the workers acquire skills in

<sup>23</sup> Norman (1998) identifies other cases in which discrimination can be efficient. He examines a model of statistical discrimination in which workers must be assigned to either a simple or a complex job, with only workers who have acquired skills being able to perform the complex task. Statistical discrimination distorts skill-acquisition decisions, but can improve the assignment of workers to tasks by allowing the firm to concentrate on the preferred, more skilled group in assignments to the complex task. The latter force may overwhelm the former, causing discrimination to be efficient.

both the symmetric and asymmetric equilibria, and that the value functions for an extreme asymmetric equilibrium match those of the symmetric equilibrium (replacing  $\lambda_F$  by  $2\lambda_F$ ), we can use (6)–(7) to obtain the equivalent inequality:

$$\frac{(2\hat{\rho}_G + \hat{\rho}_F)2\lambda_F x}{(r + \delta)[(\hat{\rho}_F + \hat{\rho}_G \frac{1}{2})2\lambda_F + 2(r + 2\delta)]} < \frac{(\rho_W + \rho_F)2\lambda_F x}{(r + \delta)[(\rho_F + \rho_W \frac{1}{2})\lambda_F + 2(r + 2\delta)]}.$$

That is,

$$(18) \quad \frac{[(\hat{\rho}_F + \frac{1}{2}\hat{\rho}_G)2\lambda_F + 2(r + 2\delta)](\rho_W + \rho_F)}{[(\rho_F + \frac{1}{2}\rho_W)\lambda_F + 2(r + 2\delta)](2\hat{\rho}_G + \hat{\rho}_F)} > 1.$$

Now consider large values of  $\lambda_F = \lambda$ . As  $\lambda_F$  (and hence  $\lambda$ ) increases, both  $\hat{\rho}_G$  and  $\rho_W$  approach zero while  $\hat{\rho}_F$  and  $\rho_F$  approach  $1/2$ . This is the statement that, if search opportunities arise very rapidly, then virtually all of the workers will be employed, and hence nearly half of the firms will be employed. But this in turn ensures that the limit of the left side of (18), as  $\lambda_F$  approaches infinity, is 2. The asymmetric equilibrium thus yields a strict efficiency loss, in the sense that increased payoffs to green workers and firms are insufficient to overcome red-worker losses.

It is intuitive that asymmetric equilibria should involve welfare losses when  $\lambda$  is large. The benefit of an asymmetric equilibrium is that search becomes more effective because the fixed pool of firms can concentrate its search efforts on a smaller pool of workers. The cost of asymmetry is that the value of acquiring skills decreases for a red worker, because red-worker opportunities to meet firms are restricted, and some red workers accordingly opt into the unskilled sector. When  $\lambda_F$  is close to  $\lambda$  and  $\lambda$  in turn is large, the costs of the asymmetric equilibrium are large (because red-worker meeting opportunities are severely restricted and hence many red workers are pushed out of the skilled sector) and the benefits are small (because

search is already quite efficient, leaving small gains to be captured by more efficient search).

All our examples have had the property that green workers prefer the green asymmetric equilibrium to the symmetric equilibrium, while red workers at least weakly prefer the symmetric equilibrium. Is this always the case? This is easily shown to be the case if  $\lambda_F = \lambda$ , but we are interested in a more general result. In a model in which it is simply *assumed* that each party receives a fixed share of any flow surplus, the answer is also easily shown to be yes. Our non-trivial bargaining procedure, however, complicates things dramatically. When firms search only for green workers, the unemployment rate of green workers falls while that of firms rises (as expected). Green workers thus find matches with vacant firms more quickly, increasing their values, and firms find it harder to find unemployed workers, decreasing their bargaining power and again increasing the value of a green worker. However, red workers find it much harder to find vacant firms, since no firms search for red workers. This decreases the value of a red worker, increasing the value of a firm and potentially making the firm a more aggressive bargainer when dealing with green workers. We must then show that this increase in firm bargaining power cannot decrease the value of a green worker. Alternatively, there is the possibility that the firms' bargaining position is sufficiently weak that even red workers do very well in bargaining, yielding high red-worker values.

While we have not been able to provide a complete analytic answer, we can provide a partial one supplemented by numerical calculations. We proceed in two stages. First, let  $V_W(H_G + H_R)$  be the value of an unemployed worker in a hypothetical symmetric steady state in which the proportion of skilled workers is arbitrarily fixed at  $H_G + H_R$  (with equal numbers of reds and greens acquiring skills).

**PROPOSITION 12:** *Suppose  $(H_G^*, V_G^*, H_R^*, V_R^*)$  are the skilled population sizes and values of green and red workers in a green asymmetric equilibrium. If*

$$(19) \quad V_G^* > V_W(H_G^* + H_R^*) > V_R^*,$$

*and if a nontrivial symmetric equilibrium exists, then*

$$V_G^* \geq V_W^* \geq V_R^*,$$

*where  $V_W^*$  is the value of an unemployed worker in the symmetric equilibrium with no restrictions on entry.*

Notice that Proposition 12 includes the case of  $H_R^* > 0$ , and hence is not limited to extreme equilibria.

**PROOF:**

Suppose  $V_G^* < V_W^*$ , and hence  $V_W^* > V_W(H_G^* + H_R^*)$ . An argument mimicking the proof of Proposition 4 establishes that a symmetric equilibrium can yield a higher worker value than  $V_W(H_G^* + H_R^*)$  only if the number of workers entering in the symmetric equilibrium is less than  $H_G^* + H_R^*$ . But since  $V_G^* < V_W^*$  and  $V_R^* < V_W^*$ , at least as many workers acquire skills in the symmetric equilibrium as in the asymmetric equilibrium, and hence at least  $H_G^* + H_R^*$  workers acquire skills, a contradiction. A similar argument precludes the possibility that  $V_W^* < V_R^*$ .

It remains to verify (19). As before, we fix  $\lambda = \lambda_W + \lambda_F$ , and let  $\lambda_F$  vary. Denote by  $\hat{V}_G(H_G, H_R, \lambda_F)$  and  $\hat{V}_R(H_G, H_R, \lambda_F)$  the values to a green and red worker in an asymmetric steady state with population sizes of green and red skilled workers arbitrarily fixed at  $H_G$  and  $H_R$ .<sup>24</sup> We again have  $\hat{V}_G(H_G, H_R, 0) = \hat{V}_R(H_G, H_R, 0) = V_W(H_G + H_R)$ , since firm search decisions and the composition of the skilled labor force are irrelevant when  $\lambda_F = 0$ . It is also immediate that, for all  $H_G$  and  $H_R$ ,

$$(20) \quad \begin{aligned} \hat{V}_G(H_G, H_R, \lambda) &> V_W(H_G + H_R) \\ &> \hat{V}_R(H_G, H_R, \lambda) = 0. \end{aligned}$$

*Remark:* A sufficient condition for (19) is that, for all  $H_G, H_R$ , and all  $\lambda_F \leq \lambda$ ,

$$(21) \quad \begin{aligned} \hat{V}_G(H_G, H_R, \lambda_F) &> V_W(H_G + H_R) \\ &> \hat{V}_R(H_G, H_R, \lambda_F). \end{aligned}$$

<sup>24</sup> So that  $\hat{V}_G(\lambda_F) = \hat{V}_G(1/2, 0, \lambda_F)$  and  $\hat{V}_R(\lambda_F) = \hat{V}_R(1/2, 0, \lambda_F)$ .

Extending (20) to (21) is a much easier task than working with the full model, since we have exogenously fixed worker entry decisions, allowing us to ignore any considerations arising out of the specification of the cost distribution  $C$ . However, the inequality is still sufficiently complicated that we have not found a direct analytical argument. Extensive numerical investigation verifies the inequality for combinations of parameters satisfying  $\lambda \in (1, 50]$ ,  $\lambda_F \in (0.05, \lambda]$ ,  $r \in (1, 10]$ ,  $\delta \in (1, 10]$ ,  $H_G \in (0.05, 0.4]$ , and  $H_R \in [0, H_G]$ .

## VI. Discussion

### A. The Matching Process

In our model, the probability that a searching firm meets an unemployed skilled worker is increasing in the number of unemployed skilled workers, rather than in the number of unemployed skilled workers *per searching firm*. This implies that the aggregate meetings technology exhibits increasing returns to scale. As the number of skilled workers and vacancies for skilled workers doubles, each agent experiences an increased matching frequency, and hence the total number of matches more than doubles.<sup>25</sup> This assumption contrasts with a large search-theoretic literature modeling aggregate unemployment, which uses a constant-returns-to-scale aggregate matching function.<sup>26</sup> Moreover, there is a significant body of empirical work with mixed results on whether the aggregate matching function exhibits constant returns to scale.<sup>27</sup> As Robert E. Hall (1989) and Blanchard and Diamond (1992) have argued, however, the observed matching function is a reduced-form object, and estimates of constant returns do not necessarily imply that the underlying meetings technology has constant returns to scale.<sup>28</sup> Indeed, as Blanchard and Diamond (1992,

p. 357) write, “[t]he mechanical process, which reflects the process through which workers and jobs find each other, surely has increasing returns over some range.”

Rather than being intended as a description of the aggregate labor market, our model is particularly relevant for markets in which firm search is important and workers make skill-acquisition decisions. In such markets, one might expect informational and idiosyncratic matching considerations to be particularly important. As a result, different unemployed skilled workers are not perfect substitutes. Having a larger pool of unemployed skilled workers then increases the probability of finding an appropriate match, leading to increasing returns.<sup>29</sup> Our increasing-returns matching process is meant to capture these considerations, though we do not explicitly model them.

In order for firms to be able to target workers of a particular color, there must already exist a pattern of differentiation. It may be that greens and reds attend different schools and colleges, join different social institutions, or live in different areas. By focussing their search on particular schools, social institutions, or locations, firms can then effectively target workers of a particular color. Alternatively, if firms rely on social networks, such as an “old boy” network of managers or the social networks of employees, then greens should be interpreted as those workers in the social network. Any group that can be identified with a search-relevant attribute is a candidate for discrimination. We are thus not attempting to show that discrimination is possible in the absence of differences between workers. Rather, we are interested in how discriminatory equilibria exploit existing payoff-irrelevant differences. Since the labels “red” and “green” admit a variety of interpretations, including race, sex, and location, we do not view the existence of such differentiation as unreasonable.

It may be useful to explicitly describe a model of search that illustrates the notion of targeting groups, in the context of locations.

<sup>25</sup> This follows from (2)–(3). See Peter A. Diamond (1982) for a similar matching technology.

<sup>26</sup> A leading example of this work is Dale T. Mortensen and Christopher A. Pissarides (1994).

<sup>27</sup> In addition to Olivier J. Blanchard and Diamond (1989, 1990), see Melvyn G. Coles and Eric Smith (1996) and Lourens Broersma and Jan C. van Ours (1999) and the references therein.

<sup>28</sup> In a more complete model, search intensities are endogenous (for example, there may be a discouragement effect in that workers search less intensely when unemployment is high). The aggregate matching function combines

the meeting technology with the determination of search intensities.

<sup>29</sup> See Coles and Smith (1998) for a model with this flavor. Interestingly, they find some empirical support for such a model over a standard constant-returns-to-scale matching model.

There are  $\ell_G(\ell_R)$  locations at which greens (reds, respectively) may be found. Unemployed workers are uniformly distributed over their respective locations. If a firm is at a green location, for example, for a length of time  $\tau'$ , the probability that the firm does *not* meet an unemployed green worker at that location is  $\exp\{-\tau'\lambda'_F(\rho_G H_G/\ell_G)\}$ . Now, suppose a firm spends a length of time  $\tau'$  at each location. The probability that the firm does not meet any unemployed workers is then  $(\exp\{-\tau'\lambda'_F\rho_G H_G/\ell_G\})^{\ell_G}(\exp\{-\tau'\lambda'_F\rho_R H_R/\ell_R\})^{\ell_R} = \exp\{-\tau'\lambda'_F(\rho_G H_G + \rho_R H_R)\}$ , yielding a meetings process with firm search intensity  $\lambda_F \equiv \lambda'_F(\ell_G + \ell_R)$  [since the total length of time spent without a meeting is  $(\ell_G + \ell_R)\tau' \equiv \tau$ ]. Now suppose the firm allocates the same quantity of its scarce resource, time, to visiting only green locations. Then the firm spends  $\tau' \equiv \tau/\ell_G$  at each green location, so that the probability of no meeting is  $(\exp\{-\tau'\lambda'_F\rho_G H_G/\ell_G\})^{\ell_G} = \exp\{-\tau'\lambda'_F\rho_G H_G\} = \exp\{-\tau\lambda_F^G \rho_G H_G\}$ , where  $\lambda_F^G \equiv (\ell_G + \ell_R)/\ell_G$  is the firm search intensity when only searching greens. If the number of locations for each color is proportional to the size of the population of that color, then we obtain the matching function used in Examples 1 and 2 of Section IV.

### B. The Unskilled Sector

We have said nothing about the unskilled sector of the economy. The firms and workers in this sector may be engaged in a search process identical to that of the skilled sector. But in this case, a green asymmetric equilibrium will induce more reds than greens to remain unskilled, suggesting that firms in the unskilled labor market should favor reds and discriminate against greens in their search for workers. This in turn produces a somewhat counterintuitive model of discrimination, since we ordinarily think of disadvantaged workers as being disadvantaged at all levels.

A more realistic model would not only explicitly model the unskilled sector, but would involve a hierarchy of skill levels, with firm search being especially important for higher skill levels and relatively inconsequential at lower levels. Higher skill sectors would be characterized by an abundance of green workers and green asymmetric equilibria, while the lowest skill sectors would be characterized by a virtual

dearth of green workers. Firms may search reds in the latter, but the unimportance of firm search, coupled with the scarcity of green workers, would ensure that one only very rarely observed a disadvantaged green worker, who would be only slightly disadvantaged. Alternatively, low-skill labor markets may lack the idiosyncracies, typical of skilled labor markets, that lead to increasing returns in the meeting technology, ensuring that discrimination of the type we study cannot occur in low-skill markets. In either case, reds are effectively disadvantaged at all levels, suffering discrimination in high-skill jobs and being confined to low-skill markets inhabited overwhelmingly by red workers.

### C. Policy Implications

Red and green workers are identical, but receive different wages in our equilibrium. This is discrimination in its starkest form. What can be done about such discrimination?

The first step in any program to eliminate discrimination is typically to prohibit the practice of paying different wages to workers who are identical except for their color, i.e., to impose the constraint

$$w_R = w_G.$$

For example, we might implement this constraint by forcing firms to participate in “color-blind” bargaining in which they make wage offers without knowing the color of their bargaining partners. Because red and green workers are equally productive, this will imply  $f_R = f_G$  and  $Z_{F,R} = Z_{F,G}$ , meaning that firms receive identical shares of the flow surplus and identical present values from red and green workers.

While an equal-wage requirement has great normative appeal, it can enhance the firm’s incentive to discriminate between workers. Without mandated wage equality, skilled red workers have the attraction that they are cheaper than green workers, though firms do not find this advantage sufficient to overwhelm the paucity of skilled red workers. With mandated wage equality, red workers are no longer less expensive, causing their scarcity to pose a more powerful deterrent to firm search. The result can be an even more concentrated focus on searching for green workers and higher unemployment

ment rates for red workers. If this combination of higher wage rates but higher unemployment rates does not induce significant red-worker entry, asymmetric equilibria will continue to exist in which firms search only greens. Employed red workers earn the same payoffs as employed green workers in such an equilibrium, but red workers still suffer from higher unemployment rates and lower expected values from acquiring skills, and so fewer red workers acquire skills. This suggests not that equal-pay provisions are misguided, but that they are most likely to be effective if coupled with measures to address firm search and hiring behavior.

Affirmative action programs designed to address search behavior fall into two categories, those based on procedures and those based on outcomes. Outcome-based schemes involve requirements that firms hire more reds, in turn prompting more reds to acquire skills, while leading employers to revise their assessments of red skill levels and voluntarily hire reds.<sup>30</sup> Procedure-based approaches involve requirements that firms devote sufficient energy to making vacancies known to minority candidates and seeking such candidates.

In our model, outcome-based affirmative action schemes are unnecessary for convincing firms of the merits of red workers. The value of skilled reds is evident once a match is made. However, search procedures play an important role in shaping the equilibrium and procedure-based programs have great potential to be effective.

One possibility is to eliminate the asymmetric equilibrium by prohibiting firms from seeking workers. However, it seems very unlikely that firms will ever be told that they cannot seek candidates for jobs, or that the efficiency costs of such a move would be worth the benefits. There are two potentially effective alternatives. First, steps can be taken to increase the rate and effectiveness of worker search (i.e., to increase  $\lambda_w$ ). The provision of labor-market information, training in job search techniques, and logistical support in the job search process may all enhance the search rate for disadvantaged workers. This in turn may increase the value of acquiring skills, and attract sufficiently many

workers into the skilled sector, as to make it optimal for firms to search such workers, breaking the asymmetric equilibrium.

Second, firms can be required to search both reds and greens. This will again eliminate asymmetric equilibria. Color-blind search is the goal behind a host of procedural affirmative action requirements, including requirements that firms advertise positions widely, include members of disadvantaged groups in their target search pools, and include them in the interviewing process. It remains an unfortunate characteristic of such programs that a token effort is difficult to distinguish from a sincere one, though it is important to note that once a symmetric equilibrium is established, then searching both colors is optimal.

#### D. Worker Heterogeneity

Motivated by our interest in cases in which identical agents are faced with different outcomes, we have examined only the case in which red and green workers are identical. There may well be differences between red and green workers. For example, they may have different search opportunities, leading to different search intensities for red and green workers. They may also have different cost distributions for acquiring skills. Unfortunately, it is easy to imagine circumstances in which asymmetric equilibria, in which firms search only greens, lead to reductions in the search opportunities and increases in the costs of acquiring skills facing reds, reinforcing the asymmetry in the equilibrium and creating a market-induced "poverty trap."

The mere fact that one color of worker faces higher costs of acquiring skills than the other does not doom the former to being disadvantaged in an asymmetric equilibrium. As long as the distributions of schooling costs are not too dissimilar, it is possible that the high-cost color of worker is the advantaged worker in an asymmetric equilibrium. For example, it could be that greens face higher costs of acquiring skills than reds, but the concentration of firms on searching greens confers a sufficient advantage that more greens acquire skills (especially if the greens are *ex ante* a larger group). If we interpret higher costs of acquiring skills as arising out of lower natural abilities, we then have a case in which the less able group fares better in

<sup>30</sup> See Stephen Coate and Glenn C. Loury (1993) for a discussion.

equilibrium.<sup>31</sup> Considerable attention has recently been devoted to the question of whether the inferior economic performance of various demographic and racial groups should be attributed to deficiencies in individuals' abilities or in some aspect of their environment. Our asymmetric equilibrium provides yet another reason for caution in linking seemingly inferior outcomes to differences in ability.

#### APPENDIX

##### PROOF OF PROPOSITION 1:

Because  $V_W \leq V_W^0$ , it is immediate that there is a unique, trivial symmetric steady state when  $C(V_W^0) = 0$ . Suppose then  $C(V_W^0) > 0$  and set  $\lambda \equiv \lambda_F + \lambda_W$ . The equations determining steady-state equilibria are recursive, with the unemployment rate entering most equations only in the form  $\rho_W H_W \equiv \tilde{\rho}$ . From (2), we have

$$(A1) \quad \rho_F = \frac{2\delta}{(2\delta + \tilde{\rho}\lambda)},$$

from (1), (2), and (3),

$$\tilde{\rho} = C(V_W) - 1 + \rho_F,$$

and finally, from (6),

$$V_W = \frac{\rho_F \lambda}{(r + \delta)[(\rho_F + \tilde{\rho})\lambda + 2(r + 2\delta)]} x.$$

Combining these three equations yields

$$(A2) \quad \tilde{\rho} = C\left(\frac{2\delta\lambda}{(r + \delta)[(2\delta(1 + \tilde{\rho}) + \tilde{\rho}^2\lambda)\lambda + 2(r + 2\delta)(2\delta + \tilde{\rho}\lambda)]} x\right) - 1 + \frac{2\delta}{2\delta + \tilde{\rho}\lambda}.$$

There is a unique value of  $\tilde{\rho} \in (0, 1)$  solving (A2), since the right side is continuous (because  $C$  is a continuous distribution function) and decreasing in  $\tilde{\rho}$ , is strictly positive at  $\tilde{\rho} = 0$  (the

argument of  $C$  at  $\tilde{\rho} = 0$  is just  $V_W^0$ ) and is less than one at  $\tilde{\rho} = 1$ .

Given  $\tilde{\rho}$ , the values of  $\rho_F$  and  $V_W$  are uniquely determined by (A1) and (6), with  $H_W$  then determined by (1), and  $V_F$  by (7).

Because the unique equilibrium satisfies  $\tilde{\rho} = \rho_W H_W \in (0, 1)$ , we have  $\rho_W \in (0, 1)$  and  $H_W > 0$ . If  $C(x/(r + \delta)) < 1$ , then  $H_W < 1$ , since  $V_W < x/(r + \delta)$ .

##### PROOF OF PROPOSITION 2:

The number of occupied firms and employed skilled workers must be equal. However, some workers face opportunity costs arbitrarily close to  $x/(r + \delta)$ , which is strictly larger than the payoff to acquiring skills (because it ignores the possibilities of having to wait to find a vacant firm and of firm deaths), and hence opt into the unskilled sector. As a result,  $H_W < 1$ , and so, from (2) and (3),

$$(A3) \quad 1 - \rho_F = H_W(1 - \rho_W) < 1 - \rho_W,$$

so that  $\rho_W < \rho_F$ . The inequality  $V_F < V_W$  then follows from (6)–(7), while  $Z_F < Z_W$  follows from (4)–(5).

##### PROOF OF PROPOSITION 3:

Letting  $m = \tilde{\rho}\lambda$ , (A2) can be rewritten as

$$m = \lambda C - \lambda + \frac{2\delta\lambda}{2\delta + m}.$$

Differentiating this expression with respect to  $\lambda$  and then substituting from (A2) shows that  $dm/d\lambda > 0$ , that is,

$$\frac{d(\rho_W H_W \lambda)}{d\lambda} > 0.$$

Equation (A1) then shows that  $d\rho_F/d\lambda < 0$ .

We now construct an argument by contradiction. Suppose  $d(\rho_F \lambda)/d\lambda \leq 0$ . Then,  $d(\rho_W H_W)/d\lambda > 0$  [from (2)], and hence  $dV_W/d\lambda < 0$  [(6)], which in turn implies  $dH_W/d\lambda < 0$  [(1)], and so  $d\tilde{\rho}/d\lambda < 0$  [(A3)]. But, from (A1),

$$\frac{d(\rho_F \lambda)}{d\lambda} = \frac{2\delta}{(2\delta + \tilde{\rho}\lambda)^2} \left[ 2\delta + \tilde{\rho}\lambda - \lambda^2 \frac{d\tilde{\rho}}{d\lambda} \right],$$

<sup>31</sup> Robert Shimer (1995) examines a model in which information frictions may induce firms to prefer less qualified workers to more qualified workers.



so that  $d(\rho_F\lambda)/d\lambda > 0$ , a contradiction. Hence,  $d\rho_F\lambda/d\lambda > 0$  and (3) then gives  $d\rho_W/d\lambda < 0$ .

Suppose now that  $dV_W/d\lambda \leq 0$ . This is equivalent to

$$(A4) \quad \frac{d(\rho_F\lambda)}{d\lambda} (\tilde{\rho}\lambda + 2(r + 2\delta)) \\ \leq \rho_F\lambda \frac{d(\tilde{\rho}\lambda)}{d\lambda}.$$

But differentiating the equation  $\tilde{\rho}\lambda = \lambda H_W - \lambda + \lambda\rho_F$  yields

$$\frac{d(\tilde{\rho}\lambda)}{d\lambda} = H_W + \lambda \frac{dH_W}{d\lambda} - 1 + \frac{d(\rho_F\lambda)}{d\lambda} \\ = \tilde{\rho} - \rho_F + \lambda \frac{dH_W}{d\lambda} + \frac{d(\rho_F\lambda)}{d\lambda},$$

and substituting into (A4) gives, after rearrangement,

$$\frac{d(\rho_F\lambda)}{d\lambda} ((\tilde{\rho} - \rho_F)\lambda + 2(r + 2\delta)) \\ \leq \rho_F\lambda \left( \tilde{\rho} - \rho_F + \lambda \frac{dH_W}{d\lambda} \right).$$

Since  $d(\rho_F\lambda)/d\lambda > 0$ , we have (recall that  $dV_W/d\lambda \leq 0$  implies  $dH_W/d\lambda \leq 0$ ),

$$\frac{d(\rho_F\lambda)}{d\lambda} ((\tilde{\rho} - \rho_F)\lambda) < \rho_F\lambda(\tilde{\rho} - \rho_F).$$

From Proposition 2,  $\tilde{\rho} < \rho_F$ , and so  $d(\rho_F\lambda)/d\lambda > \rho_F$ . That is,  $d\rho_F/d\lambda > 0$ , a contradiction. Hence, we have  $dV_W/d\lambda > 0$ , and, from (1),  $dH_W/d\lambda > 0$ .

#### PROOF OF PROPOSITION 5:

The green and red unemployed and vacancies steady-state conditions (10)–(12) have the form

$$2\delta(1 - \rho_X) = \rho_X A_X,$$

where  $A_X \geq 0$  is a function of the search intensities and the other unemployment and/or vacancy rates but not  $\rho_X$ . Solving this equation for  $\rho_X$  gives

$$\rho_X = 2\delta/(2\delta + A_X) \in [0, 1].$$

Since (8) and (9) imply  $H_G, H_R \in [0, 1]$ , and (13)–(15) imply  $V_F, V_R, V_G \in [0, x/(r + \delta)]$ , the 8-equation system described by (8)–(15) maps the compact set  $[0, 1]^5 \times [0, x/(r + \delta)]^3$  into itself in a continuous manner. Thus, by Brouwer's fixed-point theorem, the system has a fixed point, and this is a green asymmetric steady state.

If the steady state is trivial, then  $H_G = H_R = 0$  and  $\rho_F = 1$ . Evaluating (15) at these values gives

$$V_G = \frac{(2\lambda_F + \lambda_W)x}{(r + \delta)(2\lambda_F + \lambda_W + 2(r + 2\delta))} > V_W^0,$$

and  $C(V_W^0) > 0$  thus implies  $H_G > 0$ , so the steady state cannot be trivial.

#### PROOF OF PROPOSITION 6:

The inequality  $V_R < V_G$  follows immediately from (14)–(15), and in turn implies  $H_G > H_R$ , and so  $H_G > 0$  if  $\max\{H_G, H_R\} > 0$ . The inequality  $V_R < V_G$  also implies  $Z_{F,R} > Z_{F,G}$ , and so  $Z_{F,R} > V_F$ . The inequality  $Z_{F,R} > Z_{F,G}$  can hold only if the firm extracts a larger portion of the flow surplus from red workers, giving  $w_R < w_G$  and  $f_R > f_G$ . The inequality on unemployment rates,  $\rho_R > \rho_G$ , follows directly from (11)–(12).

#### PROOF OF PROPOSITION 8:

If there is no worker search ( $\lambda_W = 0$  and  $\lambda_F = \lambda$ ) and firms search only greens, then  $V_R = 0$  and hence  $H_R = 0$ . From Proposition 5, the asymmetric steady state is nontrivial, and so  $V_G > 0$  and  $H_G > 0$ . Moreover, (11) implies  $\rho_G > 0$ . It is then optimal for firms to search only greens, and we have a green asymmetric equilibrium.

Since steady states are upper-hemicontinuous in  $\lambda_F$ , both the number of green workers who acquire skills ( $H_G$ ) in a green asymmetric steady state and the number of unemployed skilled green workers ( $\rho_G H_G$ ) are bounded away from zero, while  $H_R$  and hence  $\rho_R H_R$  approach zero, as  $\lambda_F$  approaches  $\lambda$ . For sufficiently large  $\lambda_F$ , it is then again optimal for firms to search only greens, yielding an asymmetric equilibrium.

## PROOF OF PROPOSITION 9:

For sufficiently concentrated  $C$ , we have  $C(\hat{V}_G(\lambda_F)) = 1$  and  $C(\hat{V}_R(\lambda_F)) = 0$ , so that all workers' opportunity costs of acquiring skills  $\alpha$  satisfy  $\hat{V}_R(\lambda_F) < \alpha < \hat{V}_G(\lambda_F)$ , yielding an asymmetric steady state in which  $H_G = 1/2$  and  $H_R = 0$ . But then there are no skilled red workers, making it pointless for the firm to search red workers and ensuring  $V(R|G) < V_F$  and  $V(W|G) < V_F$ .

## PROOF OF PROPOSITION 11:

The value of a green worker in an extreme green asymmetric steady state is given by [from (15)]

$$\hat{V}_G(\lambda_F) = \frac{\rho_F(2\lambda_F + \lambda_W)x}{(r + \delta)\{(\rho_F + 1/2\rho_G)(2\lambda_F + \lambda_W) + 2(r + 2\delta)\}}$$

From (14) and (15),  $\hat{V}_R(\lambda_F) < \hat{V}_G(\lambda_F)$ . Since  $V_W^0$  is an upper bound for  $V_W$ , the worker value in a symmetric equilibrium, the symmetric equilibrium is trivial if  $C(V_W^0 + \varepsilon) = 0$  for small  $\varepsilon > 0$ . If  $\hat{V}_G(\lambda_F) > V_W^0$ , then for any opportunity cost distribution with support contained in the open interval  $(\max\{V_W^0, \hat{V}_R(\lambda_F)\}, \hat{V}_G(\lambda_F))$ , the symmetric equilibrium is trivial and the extreme green steady state is an equilibrium.

The inequality  $\hat{V}_G(\lambda_F) > V_W^0$  is equivalent to

$$2(r + 2\delta)(2\rho_F - 1) > 2(r + 2\delta)\left(\frac{\lambda_W}{\lambda_F}\right)(1 - \rho_F) + \frac{\rho_G}{2}\left(2 + \left(\frac{\lambda_W}{\lambda_F}\right)\right)(\lambda_F + \lambda_W).$$

The second term on the right side can be made arbitrarily close to zero by choosing  $\lambda_F$  and  $\lambda_W$  small with  $\lambda_W \leq \kappa\lambda_F$ . At the same time, for small  $\lambda_W$  and  $\lambda_F$ ,  $\rho_F$  is close to 1, giving the desired inequality.

## REFERENCES

- Berger, Jacqueline.** "Were You Referred by a Man or a Woman? Gender of Contacts and Labor Market Outcomes." Mimeo, Princeton University, 1995.
- Blanchard, Olivier J. and Diamond, Peter A.** "The Beveridge Curve." *Brookings Papers on Economic Activity*, 1989, (1), pp. 1–60.
- . "The Aggregate Matching Function," in Peter A. Diamond, ed., *Productivity/growth/unemployment*. Cambridge, MA: MIT Press, 1990, pp. 159–201.
- . "The Flow Approach to Labor Markets." *American Economic Review*, May 1992 (*Papers and Proceedings*), 82(2), pp. 354–59.
- Broersma, Lourens and van Ours, Jan C.** "Job Searchers, Job Matches, and the Elasticity of Matching." *Labour Economics*, March 1999, 6(1), pp. 77–93.
- Burdett, Kenneth and Smith, Eric.** "The Low Skill Trap." Department of Economics Discussion Paper No. 95-40, University of British Columbia, 1995.
- Cain, Glen G.** "The Economic Analysis of Labor Market Discrimination: A Survey," in Orley Ashenfelter and Richard Layard, eds., *Handbook of labor economics*, Vol. 1. New York: Elsevier Science, 1986, pp. 693–785.
- Coate, Stephen and Loury, Glenn C.** "Will Affirmative-Action Policies Eliminate Negative Stereotypes?" *American Economic Review*, December 1993, 83(5), pp. 1220–40.
- Coles, Melvyn G. and Smith, Eric.** "Cross-Section Estimation of the Matching Function: Evidence from England and Wales." *Economica*, November 1996, 63(252), pp. 589–97.
- . "Marketplaces and Matching." *International Economic Review*, February 1998, 39(1), pp. 239–54.
- Corcoran, Mary; Datcher, Linda and Duncan, Greg J.** "Most Workers Find Jobs Through Word of Mouth." *Monthly Labor Review*, August 1980, 103(8), pp. 33–35.
- Cornell, Bradford and Welch, Ivo.** "Culture, Information, and Screening Discrimination." *Journal of Political Economy*, June 1996, 104(3), pp. 542–71.
- Diamond, Peter A.** "Aggregate Demand Management in Search Equilibrium." *Journal of Political Economy*, October 1982, 90(5), pp. 881–94.
- Hall, Robert E.** "Comments and Discussions." *Brookings Papers on Economic Activity*, 1989, (1), pp. 61–64.
- Holzer, Harry J.** "Search Method Use by Unemployed Youth." *Journal of Labor Economics*, January 1988, 6(1), pp. 1–20.

- Mailath, George J.; Samuelson, Larry and Shaked, Avner.** "Endogenous Inequality in Integrated Labor Markets with Two-Sided Search—Omitted Calculations." Mimeo, University of Pennsylvania, 1999; <http://www.ssc.upenn.edu/gmailath/pubpaper.html>.
- Masters, Adrian M.** "Efficiency of Investment in Human and Physical Capital in a Model of Bilateral Search and Bargaining." *International Economic Review*, May 1998, 39(2), pp. 477–94.
- Moro, Andrea and Norman, Peter.** "Affirmative Action in a Competitive Economy." Mimeo, University of Minnesota and University of Wisconsin, 1998.
- Mortensen, Dale T. and Pissarides, Christopher A.** "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies*, July 1994, 61(3), pp. 397–415.
- Norman, Peter.** "Statistical Discrimination and Efficiency." Mimeo, University of Wisconsin, 1998.
- Osborne, Martin J. and Rubinstein, Ariel.** *Bargaining and markets*. San Diego, CA: Academic Press, 1990.
- Sattinger, Michael.** "Statistical Discrimination with Employment Criteria." *International Economic Review*, February 1998, 39(1), pp. 205–37.
- Scoones, David.** "When Do Firms Search for Workers?" Mimeo, University of Texas, 1995.
- Shimer, Robert.** "Do Good Guys Come in First? Wage Determination and the Ranking of Job Applicants." Mimeo, Massachusetts Institute of Technology, 1995.
- Staiger, Douglas.** "The Effect of Connections on the Wages and Mobility of Young Workers." Mimeo, Massachusetts Institute of Technology, 1990.
- Wial, Howard.** "Getting a Good Job: Mobility in a Segmented Labor Market." *Industrial Relations*, Fall 1991, 30(3), pp. 396–416.