BARGAINING WITH WEAK PRECOMMITMENTS

TYMON TATUR†

ABSTRACT. We study how weak precommitments affect bargaining outcomes. In the basic model, players can attempt to commit to a proposal before negotiations start, but these precommitments are weak in the sense that there is substantial uncertainty whether later a player will be actually committed or free to seek compromise.

We show that perfect Bayesian equilibria exist and that outcomes of all equilibria converge to the generalized Nash bargaining solution as the commitment power of both players goes to zero. We then consider a broader class of commitment technologies, to further understand what affects bargaining power when players can make weak precommitments.

Schelling (1956, 1963) popularized the idea that the ability to publicly commit to a course of action before a negotiation may matter and, in particular, can result in better bargaining outcomes for the player with this ability.

When we say that an economic agent can publicly commit to a certain course of action, we typically mean that the agent has access to a technology which allows her to publicly announce an action and later ensures sufficiently high costs of choosing different actions so that she will stick to the announced action no matter what others do. In practice, negotiating parties often only have access to commitment technologies of much lower quality. As an example, consider the negotiations between the newly elected Greek government, the

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† Department of Economics, University of Bonn, Adenauerralllee 24-42, D-53113 Bonn; Email: tatur@uni-bonn.de. The author thanks Paul Heidhues, Daniel Krähmer, George Mailath, Benny Moldovanu, Sven Rady, Ariel Rubinstein, Hamid Sabourian, Jakub Steiner, Alex Wolitzky, and others for helpful comments and discussions.

Euro zone countries, and the IMF that started in February 2015. On May 15 Alexis Tsipras, the Greek prime minister reconfirmed the most important “red lines” his government had previously drawn up:

> "The deal must close there is no doubt about it. However, some cannot have in the back of their minds the idea that, as time goes by, the Greek side’s resilience will be tested and its red lines will fade out. [...] From this podium I want to assure the Greek people that there is no possibility or chance that the Greek government will back down on pension and labor issues.“[1]

On July 13, the Greek prime minister signed a preliminary agreement in which in return for a 86 billion Euro rescue package he agreed to terms which violated almost all of the “red lines” he had announced, including pension cuts and labor market liberalization. Given that most observers believed that it is not very likely that Greece will actually stick to its “red lines”, what exactly is the role of announcements like the above and how does the fact that agents can make them affect bargaining outcomes? Note that while the negotiations between the Greek government and its creditors had many specific features, it is quite common in many negotiations for players to first make incompatible proposals and to later settle on a compromise. Examples where such behavior can frequently be observed include bargaining between labor unions and employer organizations, out of court settlements, bargaining over the price of an item in a bazaar, and so on.[2]

In this paper, we will consider a stylized model of distributional bargaining in which players can attempt to commit before an interaction but their ability to commit is poor in the sense that they only have access to very low-quality

[1]Translation by Reuters.

[2]Often attempts to commit take less stark forms than in the above example. In section 2 we will propose a possible explanation why that might be the case.

commitment devices. What do we mean by “low quality commitment device”? On an abstract level, a perfect commitment device is one which allows the agent to publicly announce an action and later ensures that, whenever the agent would choose a different action, she would face sufficiently high costs, so that the agent will stick to the announced action no matter what. (The exact nature of those costs, whether they are reputational, psychological, or related to some contractual arrangements the agent can enter, is not important.) In contrast, a low-quality commitment device is one which rarely works at all, in the sense that the agent is usually totally free to choose whatever she wants and does not face any additional costs if she does not follow the announced course of action and, moreover, even if the device works and the agent faces a cost of breaking her commitment that cost does not necessarily need to be large. Those are the kind of commitment devices considered in this paper. We will see that, despite their low-quality nature, the fact that agents have access to such devices has a profound effect on bargaining outcomes.

In the basic model analyzed first, we consider a situation where players have access to a particularly simple form of commitment device: one which results in full commitment with probability ε and no commitment with probability $1 - \varepsilon$. While the other player sees that her opponent attempted to commit, she does not observe whether the other player is actually committed, which corresponds to the idea that the the costs a player incurs if she breaks her commitment are private information to that player.[3] The main question of this paper is how the availability of such low-quality commitment devices affects bargaining outcomes. Our first result shows that if the commitment probability ε is sufficiently small, i.e. players' ability to commit is sufficiently poor, players will attempt to commit in any equilibrium, all equilibria have the same

[3]The idea of imperfect commitment devices, where each player is not entirely to what extent her opponent is actually committed appears already in Schelling (1956).

structure, and bargaining outcomes converge to the symmetric Nash bargaining solution as ε goes to zero. For the case where the commitment abilities of players are different or where players differ in terms of their haggling skills, the same result (Theorem 1) provides a foundation for the asymmetric Nash bargaining solution and shows how bargaining outcomes depend on commitment technology and haggling skills.

In the basic model we assumed that whenever an agent attempts to commit, she is later either fully committed or not committed at all. This corresponds to a situation where the costs of breaking a commitment are either prohibitive or zero. In the general model, we relax this assumption, assuming instead that when the commitment attempt is successful with probability ε, the player can break her commitment but faces a positive cost c_i of doing so, where the cost c_i is randomly drawn according to some distribution F_i. Theorem 2 characterizes equilibria in this case and shows that the distribution F_i does not affect the limiting outcomes as long as high enough costs occur with positive probability.[4]

The model presented here can be seen as a perturbation of a bargaining game without any commitment, in the sense that the case $\varepsilon = 0$ corresponds to a situation where players are not able to commit at all and that game has many equilibria. As such, our model differs from a literature starting with Nash (1953) that considers perturbations of the Nash Demand game, which is often seen as epitomizing the essence of what is involved when both sides can make binding commitments.[5] Thus, while Nash (1953) (see also Carlsson (1991), Muthoo (1996), Dutta (2012) and others) obtained the Nash Bargaining solution using a perturbation of a simple framework in which players can perfectly commit, we obtain the Nash bargaining solution using a perturbation of a simple framework in which players cannot commit at all.

[4]We also assume that, for each player i, the distribution F_i has finite support.

[5]See, for example, Binmore, Osborne, and Rubinstein (1992), p.197.

Our model is also related to a literature started by Crawford (1982) and later extended by Ellingsen and Miettinen (2008) and others. This literature formalizes Schelling's (1965) idea of imperfect commitment and investigates the role of imperfect commitments as a source of inefficiencies. Crawford's model is in many ways similar to ours. The major difference, however, is that Crawford (1982) assumes that a certain exogenously given division of surplus is implemented when both players do not attempt to commit or both players decide to back down from their commitments. As a result, in the class of equilibria which Crawford identifies for the case where players' commitment ability goes to zero (he does not have uniqueness), the utility of both players converges to that exogenously given division of surplus. This of course is not a problem if the goal is an investigation of inefficiencies, but it means that the model cannot be used to make interesting predictions about the way surplus is divided and what affects this division. The same is true for Ellingsen and Miettinen (2008).[6]

Finally, we should mention an interesting connection to a literature started by Abreu and Gul (2000) (see also Kambe (1999), Abreu and Pearce (2007), Wolitzky (2012)) which perturbs Rubinstein's (1982) model by introducing behavioral types, thereby allowing for reputational effects in the spirit of Kreps and Wilson (1982) and Milgrom and Roberts (1982). Despite the fact that these models are quite different mathematically, a small probability of being a certain behavioral type could be considered analogous to a small probability of being committed to some strategy. Of course, the crucial force in these models is different: in that literature, behavioral types can be mimicked over

[6]The work of Ellingsen and Miettinen (2008) is less related in the sense that both their model and the derived equilibria are substantially different. Oversimplifying somewhat, mathematically their model would be more similar to a modification of our model where players who were not successful in their attempt to commit can only compromise and, if both players compromise, an exogenously given division of surplus is implemented.

long periods, resulting in strategic interactions similar to a game of attrition. In contrast, in our model the possibility of long delays and opportunity costs of waiting play no role. In particular, our model could be applied to situations where the bargaining parties need to reach agreement with a short deadline and thus delays are not even viable. Also the predicted equilibria are quite different. Kambe (1999) shows that when players choose their potential behavioral types (corresponding to initial demands in Kambe's paper) endogenously[7], they will choose behavioral types that make compatible demands and therefore settle immediately without inefficiencies.[8] That is fundamentally different in our model, where, in equilibrium, players attempt to commit to somewhat incompatible proposals in the sense that the seller proposes a price that is slightly higher than the price proposed by the buyer. Thus, in terms of the actual economic predictions, our model predicts that, say, in a negotiation between a labor union leader and the management of a firm (or an industry representative) the two sides may initially vigorously defend somewhat incompatible proposals and then nevertheless settle on a compromise deal. In contrast, the equilibrium in Kambe's paper seems to describe a world where both sides propose the same deal – perhaps the equilibrium deal is seen as an established social convention – and vigorously insist that they will not agree to any deal that is worse for them.[9]

[7]In Kambe (1999) a player can choose an initial demand and then with some small probability becomes stubborn and insists on this initial demand. Chooosing an initial demand in Kambe (1999) is thefore somewhat similar to attempting to commit to some proposal in our model.

[8]See Propositions 1 and 2 in Kambe (1999).

[9]The two model also differ fundamentally in terms of predicted bargaining outcomes. Consider our model for the case where buyer and sellers have different commitment probabilities: $\varepsilon_b = k_b \cdot \varepsilon$ and $\varepsilon_s = k_s \cdot \varepsilon$. By Theorem 1, as $\varepsilon \to 0$, the division of surplus converges to the asymmetric Nash bargaining solution where the weights depend on the ratio k_b/k_s and the share of surplis of each player i is increasing in her coefficient k_i. In contrast, if in Kambe (1999) the buyer is stubborn with probability $\varepsilon_b = k_b \cdot \varepsilon$ and the seller is stubborn with probability $\varepsilon_s = k_s \cdot \varepsilon$, then the parameters k_b and k_s do not affect the bargaining outcome in the limit as ε goes to zero. Thus if, for instance, one player is three times more likely

The paper is organized as follows. Section 1 introduces the basic model and characterizes equilibria in Theorem 1. Section 2 presents the general model and provides an equilibrium characterization in Theorem 2. Section 3 sketches the ideas behind the proof of Theorem 1. Section 4 concludes.

1. Basic Model

For ease of exposition, we will present our results in a framework where a potential buyer and a potential seller bargain over a price p. However, the model can also capture more general situations where two players bargain over some parameter, have opposing preferences concerning this parameter, and are risk averse or risk neutral. For example, in the context of two players deciding how to split a dollar, p could correspond to the fraction received by the first player. Even more generally, the model can capture other situations where two players need to decide on a point on the Pareto frontier of some regular convex set of possible payoff profiles.

1.1. **Framework.** There are two players: a potential buyer (b) interested in buying some indivisible object and a potential seller (s) interested in selling that object. We will assume that each agent $i \in \{b, s\}$ has a twice differentiable utility function $v_i(p)$ which specifies that agent's Bernoulli utility if trade occurs at a price p. The fact that sellers prefer to trade at a higher price and buyers prefer to trade at a lower price is formally captured by the assumption that $v'_s > 0$ and $v'_b < 0$. We also assume that both players are risk averse or risk neutral, formally $v''_b, v''_s \leq 0$.

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to become stubborn than the other, then, if we consider the limit where the probability of becoming stubborn gets small, we will get the same division of surplus as if both become stubborn with the same probability.

Technically, the reason for this difference is that in the reputational models started by Abreu and Gul (2000), what matters is not the ratio $\frac{\varepsilon_b}{\varepsilon_s}$ but the ratio $\frac{\ln \varepsilon_b}{\ln \varepsilon_s}$.

The utility of both players when no trade happens (and no money changes hands) is normalized to zero. Since we are interested in voluntary trade, we will furthrmore assume that the interval of prices $(\underline{p}, \bar{p})$ for which both players prefer trade to no trade is non-empty.[10]

We want to study situations where players can attempt to commit to certain proposals before negotiations start, but their ability to commit is poor in the sense that their attempts are usually unsuccessful. Consider the following two stage game with observable actions:

Stage 1: Each player i proposes a price $p_i \in [\underline{p}, \bar{p}]$ and decides whether to attempt to commit to that price or not.

Stage 2: Players observe the actions from stage 1., i.e. they see the proposal of the opponent and learn whether the opponent attempted to commit.

If *both players make the same proposal*, i.e. $p_b = p_s$, then the game ends here, trade occurs under the price both proposed, and payoff of player i is $v_i(p)$ where $p = p_b = p_s$.

If *players make compatible proposals* in the sense that $p_b > p_s$, then trade is conducted at some price $p \in (p_s, p_b)$ and payoffs are $v_i(p)$, where the price $p = p(h)$ can depend on the stage 1 history h.

If *players make incompatible proposals*, i.e. $p_b < p_s$, then the game proceeds as follows. If a player $i \in \{b, s\}$ did not attempt to commit, then she is always uncommitted and free to decide between two actions: "insist" and "compromise". If she did attempt to commit in stage 1, then with an exogenously given probability $1 - \varepsilon_i$ she is again uncommitted and free to choose between "compromise" and "insist", but with probability ε_i she is committed and must play "insist".

[10]Note that the lowest price the seller is willing to accept is given by $v_s(\underline{p}) = 0$ and the highest price the buyer is willing to accept is given by $v_b(\bar{p}) = 0$. We assume that there exist prices $\underline{p}$ and $\bar{p}$ solving those two equalities and $\underline{p} < \bar{p}$.

Outcomes and payoffs in this case are now determined as follows. If both players insist on their proposal, then no agreement is reached and no trade conducted. If one player insists on her proposal and the other compromises then trade is conducted at the price proposed by the insisting party. If both players compromise then they "meet in the middle" and trade at the compromise price $p_{comp} = \alpha \cdot p_b + (1 - \alpha) \cdot p_s$, where α is a fixed parameter between zero and one that captures the relative haggling abilities of the two players.

In other words, if $p_b \neq p_s$ payoffs are given by the game matrix

	"insist"	"compromise"
"insist"	$0, 0$	$v_s(p_s), v_b(p_s)$
"compromise"	$v_s(p_b), v_b(p_b),$	$v_s(p_{comp}), v_b(p_{comp})$

where the seller's actions correspond to rows, the buyer's actions correspond to columns, and $p_{comp} = \alpha \cdot p_b + (1 - \alpha) \cdot p_s$.

The solution concept that we will consider is *pure-strategy perfect Bayesian Nash equilibrium.*[11] From now on, we will use the word *equilibrium* to denote pure-strategy perfect Bayesian Nash equilibria.

1.2. **Possible Interpretations.** The above stylized model tries to capture bargaining situations where perfect commitment is unavailable and agents only have access to low quality comitment devices. This is done by assuming that, whenever players attempt to commit, their attempt is only successful with some small probability.[12]

The model allows for a number of interpretations. In particular, the source of commitment could be:

[11]The restriction to pure strategies appears natural if we think about bargaining norms. From a technical point of view it allows us, among other things, to eliminate the unstable mixed equilibria of stage 2 that exist if both players propose prices $p_b > p_s$, where p_b is sufficiently larger than p_s.

[12]Later, in section 2, we will consider a more general model.

(i) reputational (following a different action than the announced may damage the reputation of the agent with a third party – a small ε could then capture the idea that the agent's commitment power is limited since she is unlikely to engage in an activity with that third party where her reputation will matter or, alternatively, a break of commitment in the current interaction is unlikely to be observed by that third party),

(ii) psychological (the agent may, for instance, dislike breaking her word because if this violates a moral norm – a small ε in this context could capture that the agent's commitment power is limited because she is unlikely to be in a mental state where such concerns will be relevant to her) or

(iii) contractual (the agent could attempt to commit to an action by signing a contract that results in costly contractual obligations if a different course of action is chosen – a small ε here could capture that the agent's commitment power is limited by the fact that most likely the contract will not be enforcable before court or the agent will be able to find a valid reason to renege on her contractual obligations).

In section 2 we will extend our results to a broader class of commitment devices.

Remark 1. We assumed that, whenever both players compromise after incompatible proposals, they trade at a price given by $\alpha \cdot p_b + (1 - \alpha) \cdot p_s$, where α captures the relative haggling abilities of both players. While this captures the notion of a compromise price quite nicely in the context of bargaining over prices, there might be applications where players bargain over some other parameter and where it is more natural to assume that whenever both players decide to compromise, a coin is tossed and with probability α the proposal of the buyer is implemented and with probability $1 - \alpha$ the proposal of the seller is implemented. Going through the proof of Theorem 1, it is straightforward to check that all the results of this section remain true if we modify our model in this way.

1.3. **Benchmark $\varepsilon = 0$.** Note that in the basic model from the last section the probabilities ε_i can be seen as measuring the quality of the commitment device available to player i. Indeed, if for some player i it is the case that $\varepsilon_i = 1$, then that player can perfectly commit to a price. If $\varepsilon_i = 0$, player i has no ability to commit whatsoever.

It is straightforward to check that for the case where ε_b and ε_s are both equal to zero, for any price $p^* \in [\underline{p}, \bar{p}]$, there exists an equilibrium where both players in stage 1 propose p^* and attempt to commit to that proposal.[13]

To see this, consider a strategy profile such that: both players propose the price p^*; in stage 2 strategies only depend on proposed prices but not on whether players attempted to commit or not; after a price pair (p_s, p^*) with $p_s > p^*$ the buyer insists and the seller compromises; after a price pair (p^*, p_b) with $p_b < p^*$ the seller insists and the buyer compromises; and after any other price pair (p_s, p_b) with $p_s < p_b$ one player compromises and the other insists.

In the following we will be interested how the set of equilibria of the above game looks like, if both players' ability to commit is poor.

1.4. **Equilibrium Characterization and Comparative Statics.** Consider the case where player i who attempts to commit is actually committed with commitment probability $\varepsilon_i = k_i \cdot \varepsilon$ where $k_i > 0$ are fixed constants.[14]

[13]The same is true for the case where ε_b and ε_s are both equal to one. However, in that case, the set of equilibrium outcomes remains large if we perturb the game slightly and consider ε_b and ε_s that are close to one.

[14]For the case where only one player has a positive commitment probability the structure of equilibria is different. In particular, if one introduces an appropriare refinement based on trembles in stage 2, one obtains that in all equilibria both players propose the price that maximizes the payoffs of the player whose commitment probability is positive.

Theorem 1. *There exists $\bar{\varepsilon} > 0$ such that for $\varepsilon \in (0, \bar{\varepsilon})$ an equilibrium exists and any equilibrium has the property that: players attempt to commit to prices $p_b < p_s$; are indifferent between compromising and insisting on the equilibrium path; and on the equilibrium path compromise whenever not committed.*

As $\varepsilon \to 0$, p_s and p_b both converge to the generalized Nash bargaining solution, formally, the unique price p^* maximizing $v_b(p)^{\frac{k_b}{\alpha}} \cdot v_s(p)^{\frac{k_s}{1-\alpha}}$.

Proof. See Appendix. □

Corollary. *For the special case where both players have access to the same commitment technology (i.e. $\varepsilon_b = \varepsilon_s$) and have the same haggling ability (i.e. $\alpha = \frac{1}{2}$), as ε becomes small, the proposed prices p_b and p_s both converge to the symmetric Nash bargaining solution, i.e. the price p^* maximizing $v_b(p) \cdot v_s(p)$.*

Theorem 1 tells us that players will in equilibrium always use their commitment devices and attempt to commit. Moreover, what commitment devices are at their disposal has an effect on bargaining outcomes. In particular, as the commitment power of both players goes to zero, trade will occur with probability converging to 1 at prices that converge to p^*, the price maximizing the Nash product

$$v_b(p)^{\frac{k_b}{\alpha}} \cdot v_s(p)^{\frac{k_s}{1-\alpha}}.$$

Thus, Theorem 1 gives us a new foundation for the Nash bargaining solution and a new interpretation of the weights corresponding to the bargaining power of both players.

Note that this has several implications in terms of comparative statics. Since $v_s(p)$ is increasing in p and $v_b(p)$ is decreasing in p, the price p^* maximizing the Nash Product will be increasing in k_s, decreasing in k_b, and increasing in α. This means that the prices are higher if the seller has more commitment power, lower if the buyer has more commitment power, and higher if the buyer has a relatively higher haggling ability. The first two of those comparative statics results may appear natural, after all they correspond to the general intuition that higher commitment ability corresponds to more bargaining power. The third comparative statics result may seem counter-intuitive at first glance. After all, for any given history where both players compromise, the buyer would prefer a higher α since it would mean that the final price will be closer to her own proposal p_b. Similarly, the seller would prefer a lower α since it would mean that the final price will be closer to her own proposal p_s. Thus, one could expect that a higher α also in equilibrium leads to prices that are better for the buyer and worse for the seller.

To understand those comparative statics results, consider the fundamental trade-off players face in the studied game. For the sake of concreteness, consider a seller who is thinking whether to propose a higher price p_s. For the seller the advantage of proposing a higher price is that if the buyer finds that price still acceptable and decides to compromise, trade will be conducted at a higher price. Potentially, however, there is a second, negative effect of a higher price. The buyer may be more likely to insist on her own price when facing a proposal by the seller that is less attractive for her. A buyer who insists on her own price is always bad news for the seller, since it means that either trade is more likely to be conducted at the lower price proposed by the buyer (if the seller was planning to compromise) or there is a higher probability of disagreement and no trade happening at all (if the seller was planning to insist).

Given that trade-off, it is easy to give a crude intuition for our comparative statics results. A higher commitment probability ε_s is good for the seller since it makes the other player less likely to insist if the seller attempts to commit to a higher price, reducing the negative effect of a high price in the above trade-off and therefore allowing the seller to propose higher prices. Similarly, a higher commitment probability ε_b for the buyer means that the seller will be less likely to insist if the buyer attempts to commit to a lower price and therefore allows the buyer to ask for lower prices more aggressively. In consequence, it is intuitive that a higher ε_s leads to higher prices and a lower ε_b leads to lower prices.

Why is a higher α good for the seller in the sense that it leads to higher equilibrium prices? Consider again the fundamental trade-off described above. A higher α means that if the seller asks for a higher price, the buyer is less likely to insist since the compromise outcome is more attractive for her. Thus the second, negative effect of a price increase will be smaller if α is large than if α is small. As a result, the seller should be able to choose prices more aggressively. Similarly, a buyer who considers proposing a lower price, will be more worried that the seller will insist on her own price if α is high and therefore the compromise outcome is less attractive for the seller. This intuition suggests that the seller will be more aggressive when considering a higher price and the buyer will be less agressive when considering asking for a lower price. As a result of both of those effects one would expect equilibrium prices to be higher if α is higher which is what our theorem tells us.

Remark 2. It is worth pointing out that the weights α and $1-\alpha$ in our model play a very different role than the weights in the partitioning rule of Carlsson (1991). Indeed, the weights in Carlsson determine how surplus is divided when prices are compatible. In our model, we allowed that whenever $p_b < p_s$ trade is conducted at an arbitrary price between p_b and p_s - what the price was had no effect on our results. So, in contrast to Carlsson (1991), in our model it is irrelevant how surplus is divided when prices are compatible. The weights α and $1-\alpha$ capture how surplus is divided when players make incompatible initial demands but then decide to compromise.

Remark 3. In this subsection we saw that each player's share of surplus is increasing in her commitment probability ε_i. This means that, if a player i could choose a commitment device before the entire game is played and her opponent would observe this choice, the player would prefer a commitment device with a higher commitment probability ε_i. However, this does not immediately imply that the agent would make the same choice when allowed to choose between devices at stage 1 since, in this case, her opponent would not observe her choice before choosing her own price.

Nevertheless, it is possible to show that, if we modify our model by (i) letting agents in stage 1 choose any commitment probability that is smaller equal to the numbers ε_i from our model and (ii) assume that this choice is observed in stage 2, then under the limit considered in Theorem 1, in equilibrium each player i chooses a commitment probability equal to ε_i and equilibria have the exact structure and convergence properties described in Theorem 1.

2. General Model

The commitment device of each player i in the basic model had the property that, whenever the player attempted commitmemt, with probability ε_i she was fully committed and with the remaining probability $1-\varepsilon_i$ she was not committed at all. This corresponds to a situation where a player's cost of breaking her commitment takes only two values: it is prohibitive with probability ε_i and zero with probability $1-\varepsilon_i$. In this section we will extend the analysis by allowing costs of breaking commitment to take more than two values.

2.1. **Extended Framework.**

Consider the following modification of the basic model from section 1. If an agent i attempts to commit in stage 1 and the proposed prices are incompatible in the sense that $p_b < p_s$ then, as before, with probability $1 - \varepsilon_i$ she is uncommitted and free to choose to insist or compromise without any penalties. Now, however, when her commitment attempt is successful with the complimentary probability ε_i, the agent is no longer forced to insist. Instead, in that case, a cost $c_i > 0$ is randomly drawn[15] according to a cumulative distribution function F_i with a finite support that contains high enough cost values.[16] Whenever the player breaks her commitment and agrees to compromise she has to incur the cost c_i. Formally, the payoffs of a player i are then given by the following table

	"insist"	"compromise"
"insist"	0	$v_i(p_i)$
"compromise"	$v_i(p_j) - c_i$	$v_i(p_{comp}) - c_i$

where rows correspond to actions of player i, columns correspond to actions of the other player j, and p_{comp} is again the compromise price given by $p_{comp} = \alpha \cdot p_b + (1-\alpha) \cdot p_s$.

2.2. **Equilibrium Characterization.** Assume again $\varepsilon_i = k_i \cdot \varepsilon$, where k_i for each player i is fixed.

Theorem 2. *There exists $\bar{\varepsilon} > 0$ such that for $\varepsilon \in (0, \bar{\varepsilon})$ the following is true. All equilibria have one of the following three structures:*

(i) Both players attempt to commit, $p_b < p_s$, on the equilibrium path both players are indifferent between compromising and insisting when uncommitted, and compromise on the equilibrium path if and only if uncommitted. As $\varepsilon \to 0$, the prices proposed in those equilibria converge to the unique price p^{NBS} maximizing $v_b(p)^{\frac{k_b}{\alpha}} \cdot v_s(p)^{\frac{k_s}{1-\alpha}}$.

(ii) Only the buyer attempts to commit, $p_b < p_s$, on the equilibrium path the buyer always insists and the seller always compromises. As $\varepsilon \to 0$, the prices proposed in those equilibria converge to $\bar{p}$.

[15] The draws for both players are independent.

[16] Formally, $F_i(\max_{p\in[\underline{p},\bar{p}]} v_i(p)) < 1$. The assumption that $F_i(\max_{p\in[\underline{p},\bar{p}]} v_i(p)) < 1$ simply means that there is a positive probability of costs that are prohibitive in the sense that the player will insist no matter what.

(iii) Only the seller attempts to commit, $p_b < p_s$, on the equilibrium path the seller always insists and the buyer always compromises. As $\varepsilon \to 0$, the prices proposed in those equilibria converge to $\underline{p}$.

Moreover, equilibria with the structure described in (i) exist.

Proof. See Appendix. □

Note that while the theorem tells us that equilibria of type (i) always exist, equilibria of type (ii) and (iii) need not, as we have seen in the special case covered in Theorem 1. Even in cases where equilibria of the type (ii) or (iii) do exist, there are a number of reasons why they could be perceived as not appealing. Imagine, for example, that there are some infinitesimal costs of going through the haggling process that occurs in stage 2 whenever players propose different prices. To capture this formally, assume that the agents have lexicographic preferences: they choose their strategy to maximize payoffs (as defined before) but, if they have several strategies yielding the same payoffs, they choose one that minimizes the probability of going through the haggling process.[17] Since players still maximize payoffs, assuming such preferences leads to a refinement of the equilibrium set. From the proof of Theorem 2 – see remark 4 in the appendix – it is clear that equilibria of type (i) constructed in that proof will survive the refinement. On the other hand, equilibria of type (ii) and (iii) will not survive since the player who does not attempt commitment would prefer to propose the price proposed by her opponent in equilibrium.

The theorem, therefore, suggests that if players have access to more general commitment devices, then what matters in terms of bargaining power is how often players achieve partial commitment (in the sense that they face some

[17]Such preferences could be due to a tiny delay whenever players go through the haggling process or a tiny chance of miscommunication during the haggling.

positive costs of breaking their commitment), while higher costs often yield no additional benefit.

This last observation has an interesting implication. If it is the case that it is harder to gain access to commitment devices with larger costs c_i (or there is a chance of an exogenous event that will force the player to break her commitment and suffer those costs), then we should expect players to use commitment devices where the costs of breaking the commitment are usually not that high.

3. Sketch of the Proof of Theorem 1

Logically, the statement of Theorem 1 can be divided into three parts: (i) equilibria of the described form exist, (ii) in those equilibria prices converge to the asymmetric Nash bargaining solution as ε goes to zero, and (iii) any equilibrium has the described form for small ε.

A formal and detailed proof of Theorem 1 is in the appendix. In the remainder of this section we will quickly and somewhat informally sketch the main ideas behind the proof. In the following we will assume that players are identical in terms of commitment and haggling power, i.e. $\varepsilon_b = \varepsilon_s$ (we will therefore drop the subscript) and $\alpha = \frac{1}{2}$.

Note first that if both players attempt to commit and are indifferent between compromising and insisting on the equilibrium path, then the proposed pair of prices p_b and p_s has to satisfy the system of equations

$$(1-\varepsilon) \cdot v_s(p_s) = \varepsilon \cdot v_s(p_b) + (1-\varepsilon) \cdot v_s(\frac{p_b + p_s}{2})$$

$$(1-\varepsilon) \cdot v_b(p_b) = \varepsilon \cdot v_b(p_s) + (1-\varepsilon) \cdot v_b(\frac{p_b + p_s}{2}).$$

To prove existence of the equilibria described in the theorem, we first use Brouwer's fixed point theorem to show that this system has a solution (Lemma

1) and then explicitly construct a strategy profile that has the structure described in the theorem and is an equilibrium. This strategy profile has the property that if the buyer deviates to a lower price or the seller deviates to a higher price than in stage 2, the player who deviated will compromise whenever uncommitted and the other player will always insist. (This can be done since, for the prices proposed in our equilibrium candidate, both players were exactly indifferent between insisting and compromising if the other player compromises whenever uncommitted.)

To see why the convergence statement in the theorem holds, note that

$$(1-\varepsilon)\cdot v_b(p_b) = \varepsilon \cdot v_b(p_s) + (1-\varepsilon)\cdot v_b(\frac{p_b+p_s}{2})$$

implies that $|p_b - p_s| \to 0$ as $\varepsilon \to 0$. Indeed, imagine there is a sub-sequence such that p_b converges to some p_b^* and p_s converges to some p_s^* as ε goes to zero where $p_b^* \neq p_s^*$. Then the above equality in the limit yields

$$v_b(p_b^*) = v_b(\frac{p_b^*+p_s^*}{2})$$

which, given the fact that v_b is strictly increasing implies $p_b^* = p_s^*$.

To show that both prices converge to p^{NBS} it is enough to show that whenever p_i, p_j converge to some price p^* it must be that $p^* = p^{NBS}$.[18] However, using the fact that

$$v_i(\frac{p_i+p_j}{2}) - v_i(p_i) \approx v'(p_i)\cdot\frac{p_j-p_i}{2}$$

for small ε since then $|p_b - p_s|$ is small, we get that

$$\frac{\varepsilon}{1-\varepsilon} \approx \frac{v_i'(p_i)}{v_i(p_i)}\cdot\frac{p_j-p_i}{2}.$$

[18]The fact that this is sufficient follows from the fact that $[\underline{p}, \bar{p}]$ is compact.

Since this holds for both players, we obtain that in the limit

$$-\frac{v_i'(p^*)}{v_i(p^*)} = \frac{v_j'(p^*)}{v_j(p^*)},$$

which is the first order condition for the maximization problem defining the symmetric NBS.

Before we sketch the arguments used to show that any equilibrium has the structure described in the theorem, let us make a simple preliminary observation.

Imagine the strategy of the buyer is to propose some price $p_b < \bar{p}$ (either attempting to commit or not). What happens if the seller attempts to commit to a price p_s' just slightly above p_b? Since the buyer knows that the seller will insist with probability ε, she will compromise even if she expects the seller to compromise whenever not committed. It is straightforward to check that compromising is strictly dominant as long as

$$(1-\varepsilon) \cdot v_b(p_b) < \varepsilon \cdot v_b(p_b) + (1-\varepsilon) \cdot v_b(\frac{p_b + p_s}{2}).$$

Note that this means that, as long as prices are bounded away from $\bar{p}$, the buyer will compromise if the seller attempts to commit to a price p_s' slightly higher than p_b, where $\Delta p = p_s' - p_b$ can be chosen to be of the same order of magnitude as the commitment probability ε - as long as all the proposed prices are bounded away from $\underline{p}$ and $\bar{p}$.

The argument that any equilibrium has the structure described in the theorem consists of a number of steps, which we will sketch under the simplifying assumption that all the proposed prices are bounded away from $\underline{p}$ and $\bar{p}$:

(i) For sufficiently small ε, it cannot be that $p_b = p_s$. This is formally proven in Lemma 6. The idea is to consider a deviation where the seller slightly increases the price so that it is still dominant for the buyer to compromise if uncommitted and then have the seller compromise herself in stage 2. The advantage of doing so is that trade will often be conducted at a better price for the seller, an advantage which is of order of magnitude Δp, i.e. the order of magnitude of ε. The disadvantage of doing this is that both players will be committed at the same time with probability ε^2 in which case they will both insist which will result in low payoffs. Since the disadvantages are of order of magnitude ε^2 and the advantages are of orderr of magnitude ε the advantages will dominate for small ε.

(ii) There are no equilibria in which $p_b < p_s$ and neither players attempts commitment. This is formally proven in Lemma 7. If neither player attempts commitment, there are two potential stage 2 equilibria: buyer insists and seller compromises or buyer compromises and seller insists. So in the end, trade is conducted with probability 1 at price p_b or with probability 1 at price p_s. If, for example, trade is conducted at p_b, the seller would benefit from attempting to commit to a price slightly below p_b and then insisting.

(iii) For sufficiently small ε, there are no equilibria in which $p_b < p_s$, one player attempts commitment, and the other does not. This is formally proven in Lemma 8. Imagine only the buyer attempts commitment. Note that on the equilibrium path the seller compromises. (If seller would insist, it must be that the buyer compromises. This means trade occurs at p_s. But then the buyer would prefer to attempt to a price slightly below p_s.) Since the seller compromises (and did not attempt commitment) the buyer will insist given that $p_b > p_s$. This means trade always occurs at p_b. Consider a deviation as in the second step.

(iv) In any equilibrium $p_b < p_s$, both players attempt commitment, and they both compromise on the equilibrium path. Given the previous steps we only need to show that both players compromise on the equilibrium path.

To see that this is the case, assume, for example, that the seller insists on the equilibrium path. Then the buyer would be better off just offering the price p_s.

(v) Since both players compromise on the equilibrium path, in stage 2 the payoff from compromising must be at least as high as the payoff from insisting. If one player would not be indifferent between compromising and insisting on the equilibrium path, the other player could just propose a price that is slightly better for her (slightly higher for the seller and slightly lower for the buyer) and be sure that the other player will still compromise whenever uncommitted. Since this would increase the payoff of the player considering the deviation both players have to be indifferent between compromising and insisting on the equilibrium path.

This completes the sketch of the proof that the equilibria have the form described in the theorem. Let us emphasize once more that in the above sketch we ignored the fact that proposed prices could be close or equal to $\underline{p}$ and $\bar{p}$, which leads to complications and more subtle comparisons of the advantages and disadvantages of the considered deviations. This is also the reason why step (iii) does not fully generalize if we consider the extended framework from section 2 and, in that framework, there may exist equilibria in which one player attempts commitment and the other does not.

4. Conclusion

In this paper we propose an alternative non-cooperative foundation for the Nash bargaining solution in a model in which each player can attempt to precommit to his or her proposal but the commitment abilities of both players are extremely weak.

Following Nash (1950), many non-cooperative foundations for the Nash bargaining solution have been proposed, starting with the "smoothed" Nash demand game considered in Nash (1953). We think that one of the reasons why this "Nash Program" is interesting is that it allows us to gain a better sense of the different factors that can influence bargaining outcomes or - to put it differently - can help us understand what affects the "weights" in Nash's axiomatic derivation of his bargaining solution. For example, Rubinstein (1982) sheds light on how bargaining outcomes may be related to opportunity costs of waiting (either due to impatience or an exogenous chance of termination). We hope that, similarly, our results not only provide another foundation for the generalized Nash bargaining solution, but also do shed some light on a particular factor that can affect bargaining outcomes: access to low-quality commitment devices.

Mathematical Appendix

Notation. To be able to explore the symmetry between buyers and sellers, it is convenient to define $\alpha_b = \alpha$ and $\alpha_s = (1 - \alpha)$. This means that the compromise price at which trade is conducted whenever both players propose prices p_b and p_s such that $p_b < p_s$ and then both compromise in stage 2 is given by

$$p_{comp} = \alpha_b \cdot p_b + \alpha_s \cdot p_s.$$

Trivially, the above definition implies that $\alpha_b, \alpha_s \in (0, 1)$, $\alpha_b = 1 - \alpha_s$, and $\alpha_s = 1 - \alpha_b$.

Proof of Theorem 1. The condition from the theorem that equilibrium prices p_b and p_s have the property that on the equilibrium path both players are indifferent between compromising and insisting in stage 2 means that for the seller:

$$(1) \qquad (1-\varepsilon_b)\cdot v_s(p_s) = \varepsilon_b \cdot v_s(p_b) + (1-\varepsilon_b)\cdot v_s(\alpha_b \cdot p_b + \alpha_s \cdot p_s)$$

and for the buyer:

$$(2) \qquad (1-\varepsilon_s)\cdot v_b(p_b) = \varepsilon_s \cdot v_b(p_s) + (1-\varepsilon_s)\cdot v_b(\alpha_b \cdot p_b + \alpha_s \cdot p_s).$$

We first note that for any $\varepsilon_s = k_s \cdot \varepsilon$ and $\varepsilon_b = k_b \cdot \varepsilon$ there is a pair of prices p_s and p_b solving the above system of equations.

Lemma 1. *For any $\varepsilon \in (0,1)$, there exists a pair of prices p_s and p_b with $p_b < p_s$ such that (1) and (2) both hold.*

Proof. Note first that if $p_b \geq p_s$ then the fact that v_s is strictly increasing implies that the right hand side in (1) is strictly larger than the left hand side. Thus, any pair of prices that solves the considered system of equations must have the property that $p_b < p_s$.

Note next that, for any fixed $p_s \in [\underline{p}, \bar{p}]$, there is a unique p_b that solves equation (1). Indeed, consider the right hand side of (1). The expression on the right hand side of (1) is clearly continuous and strictly increasing as a function of p_b, is strictly smaller than the left hand side for p_b equal to $\underline{p}$, and strictly larger than the left hand side for p_b equal to p_s. Thus, for any p_s, there exists a unique $p_b \in (\underline{p}, p_s)$ such that (1) holds with equality.

For any $p_s \in [\underline{p}, \bar{p}]$ let us denote the unique p_b that solves (1) by $\tilde{p}_b(p_s)$. Note next that the function $\tilde{p}_b : [\underline{p}, \bar{p}] \to [\underline{p}, \bar{p}]$ is continuous. Indeed, to prove continuity we need to show that for any sequence $p_s^n \in [\underline{p}, \bar{p}]$ converging to some $p_s \in [\underline{p}, \bar{p}]$ it is the case that $\tilde{p}_b(p_s^n)$ converges to $\tilde{p}_b(p_s)$. Assume that this is not the case. Since the interval $[\underline{p}, \bar{p}]$ is clearly compact, there exists a sub-sequence $p_s^{n_k}$ such that $\tilde{p}_b(p_s^{n_k})$ converges to some $p_b \neq \tilde{p}_b(p_s)$. However, the definition of $\tilde{p}_b$ implies that

$$(1-\varepsilon_b)\cdot v_s(p_s^{n_k}) = \varepsilon_b \cdot v_s(\tilde{p}_b(p_s^{n_k})) + (1-\varepsilon_b)\cdot v_s(\alpha_b \cdot \tilde{p}_b(p_s^{n_k}) + \alpha_s \cdot p_s^{n_k})$$

and therefore - since v_b and v_s are both continuous - it must be that in the limit

$$(1-\varepsilon_b)\cdot v_s(p_s) = \varepsilon_b \cdot v_s(p_b) + (1-\varepsilon_b)\cdot v_s(\alpha_b \cdot p_b + \alpha_s \cdot p_s).$$

Since we have already shown above that for any $p_s \in [\underline{p}, \bar{p}]$ there is a unique p_b that solves equation (1) the last equation implies that $p_b = \tilde{p}_b(p_s)$. We obtained a contradiction. Thus, $\tilde{p}_b$ must be continuous.

An analogous reasoning shows that for any $p_b \in [\underline{p}, \bar{p}]$ there is a unique p_s that solves equation (2) and the function $\tilde{p}_s : [\underline{p}, \bar{p}] \to [\underline{p}, \bar{p}]$ which for each $p_b \in [\underline{p}, \bar{p}]$ returns the unique p_s that solves equation (2) is continuous.

Let $F : [\underline{p}, \bar{p}]^2 \to [\underline{p}, \bar{p}]^2$ be given by $F(p_s, p_b) = (\tilde{p}_s(p_b), \tilde{p}_b(p_s))$. Clearly, the continuity of the functions $\tilde{p}_s$ and $\tilde{p}_b$ implies that F is continuous as well. Thus, by Brouwer's fixed point theorem, there exists a pair (p_s, p_b) such that $F(p_s, p_b) = (p_s, p_b)$. Note that $F(p_s, p_b) = (p_s, p_b)$ means that for those p_s and p_b, equations (1) and (2) both hold. □

Lemma 2. *As $\varepsilon \to 0$, any pair of prices p_s and p_b satisfying (1) and (2) for $\varepsilon_b = k_b \cdot \varepsilon$ and $\varepsilon_s = k_s \cdot \varepsilon$ both converge to the price p^{NBS} maximizing*

$$v_b(p)^{\frac{k_b}{\alpha_b}} \cdot v_s(p)^{\frac{k_s}{\alpha_s}}$$

over the set of all prices $p \in [\underline{p}, \bar{p}]$.

Proof. As a preliminary observation note first that the price p^{NBS} maximizing

$$v_b(p)^{\frac{k_b}{\alpha_b}} \cdot v_s(p)^{\frac{k_s}{\alpha_s}}$$

over $p \in [\underline{p}, \bar{p}]$ is also the unique price $p \in [\underline{p}, \bar{p}]$ for which the first order condition

$$\frac{k_b}{\alpha_b} \cdot \frac{v_b'(p)}{v_b(p)} + \frac{k_s}{\alpha_s} \cdot \frac{v_s'(p)}{v_s(p)} = 0 \tag{3}$$

holds.[19]

Imagine now, that we have a sequence of positive probabilities ε^n converging to zero and a corresponding sequence of prices p_b^n and p_s^n satisfying equations (1) and (2). We want to show that it must be the case that both p_b^n and p_s^n converge to p^{NBS}. Assume that is not the case. Choosing a subsequence if necessary, we can assume that p_b^n converges to some $\lim_{n\to\infty} p_b^n$, p_s^n converges to some $\lim_{n\to\infty} p_b^n$, and at least one of those limits is not equal to p^{NBS}.

Taking the limit $\varepsilon^n \to 0$ in equation (2) and using the fact that v_b is a continuous function we obtain

$$v_b(\lim_{k\to\infty} p_b^n) = v_b(\alpha_b \cdot \lim_{k\to\infty} p_b^n + \alpha_s \cdot \lim_{k\to\infty} p_s^n).$$

Since v_b was assumed to be strictly increasing this immediately implies that

$$\lim_{k\to\infty} p_b^n = \alpha_b \cdot \lim_{k\to\infty} p_b^n + \alpha_s \cdot \lim_{k\to\infty} p_s^n$$

but since $\alpha_b + \alpha_s = 1$ this immediately implies

$$\lim_{k\to\infty} p_b^n = \lim_{k\to\infty} p_s^n.$$

[19]Note that since for $p = \underline{p}$ and $p = \bar{p}$ the objective function is equal to zero, the maximum must be an interior one, i.e. $p^{NBS} \in (\underline{p}, \bar{p})$. Since a monotone transformation will not affect the location of the maximum, p^{NBS} must also be an interior maximum of $\ln(v_b(p)^{\frac{k_b}{\alpha_b}} \cdot v_s(p)^{\frac{k_s}{\alpha_s}}) = \frac{k_b}{\alpha_b} \cdot \ln v_b(p) + \frac{k_s}{\alpha_s} \cdot \ln v_s(p)$. Calculating the first order condition, we see that as an interior maximum p^{NBS} must satisfy $\frac{k_b}{\alpha_b} \cdot \frac{v_b'(p^{NBS})}{v_b(p^{NBS})} + \frac{k_s}{\alpha_s} \cdot \frac{v_s'(p^{NBS})}{v_s(p^{NBS})} = 0$. Since $\frac{k_b}{\alpha_b} \cdot \ln v_b(p) + \frac{k_s}{\alpha_s} \cdot \ln v_s(p)$ is clearly concave in p as the sum of two concave functions, there is only one point satisfying the first order condition. (The functions $\ln v_s(p)$ and $\ln v_b(p)$ are concave since the ln function is increasing and concave and v_b and v_s are concave.)

Let us denote the value to which both p_b^n and p_s^n converge by p^*. To obtain the desired contradiction and thereby complete the proof, it is therefore enough to show that $p^* = p^{NBS}$.

Note that, by the mean value theorem, there exist $\zeta_s^n \in [\alpha_b \cdot p_b^n + \alpha_s \cdot p_s^n, p_s^n]$ such that

$$v_s'(\zeta_s^n) = \frac{v_s(p_s^n) - v_s(\alpha_b \cdot p_b^n + \alpha_s \cdot p_s^n)}{p_s^n - (\alpha_b \cdot p_b^n + \alpha_s \cdot p_s^n)}$$

and $\zeta_b^k \in [p_b^n, \alpha_b \cdot p_b^n + \alpha_s \cdot p_s^n]$ such that

$$v_b'(\zeta_b^n) = \frac{v_b(\alpha_b \cdot p_b^n + \alpha_s \cdot p_s^n) - v_b(p_b^n)}{\alpha_b \cdot p_b^n + \alpha_s \cdot p_s^n - p_b^n}.$$

Rearranging terms, we see that the last two equations are equivalent to

$$v_s(\alpha_b \cdot p_b^n + \alpha_s \cdot p_s^n) = v_s(p_s^n) - v_s'(\zeta_s^n) \cdot \alpha_b \cdot (p_s^n - p_b^n)$$

$$v_b(\alpha_b \cdot p_b^n + \alpha_s \cdot p_s^n) = v_b(p_b^n) + v_b'(\zeta_b^n) \cdot \alpha_s \cdot (p_s^n - p_b^n).$$

Plugging in from those two equations into (1) and (2) and rearranging terms we obtain:

$$\varepsilon_b^n \cdot v_s(p_b^n) = (1 - \varepsilon_b^n) \cdot v_s'(\zeta_s^n) \cdot \alpha_b \cdot (p_s^n - p_b^n)$$

$$\varepsilon_s^n \cdot v_b(p_s^n) = -(1 - \varepsilon_s^n) \cdot v_b'(\zeta_b^n) \cdot \alpha_s \cdot (p_s^n - p_b^n).$$

Together the last two equalities imply that:

$$\frac{\varepsilon_b^n \cdot v_s(p_b^n)}{\varepsilon_s^n \cdot v_b(p_s^n)} = -\frac{(1 - \varepsilon_b^n) \cdot v_s'(\zeta_s^n) \cdot \alpha_b}{(1 - \varepsilon_s^n) \cdot v_b'(\zeta_b^n) \cdot \alpha_s}.$$

Recalling now that $p_b^n \leq \zeta_b^n \leq \zeta_s^n \leq p_s^n$ and both p_b^n and p_s^n converge to the same value p^* as $n \to \infty$ and that $\frac{\varepsilon_b^n}{\varepsilon_s^n}$ converge to $\frac{k_b}{k_s}$ as $n \to \infty$ we obtain that in the limit

$$\frac{k_b}{k_s} \cdot \frac{v_s(p^*)}{v_b(p^*)} = -\frac{v_s'(p^*)}{v_b'(p^*)} \cdot \frac{\alpha_b}{\alpha_s}.$$

Rearranging terms, we see that the last equation means that p^* satisfies (3). Given our preliminary observation at the beginning of this proof, this implies that $p^* = p^{NBS}$. □

Consider a pair of prices satisfying equations (1) and (2). We want to construct an equilibrium where whenever the seller deviates and attempts to commit to a higher price p'_s, in stage 2 the buyer will insist and the seller compromise (whenever uncommitted). Note that, since v_b is strictly decreasing in prices, equation (2) immediately implies that for $p'_s > p_s$:

$$(1 - \varepsilon_s) \cdot v_b(p_b) > \varepsilon_s \cdot v_b(p'_s) + (1 - \varepsilon_s) \cdot v_b(\alpha_b \cdot p_b + \alpha_s \cdot p'_s).$$

That however, means that the buyer would prefer to insist if she would know that the seller will compromise whenever uncommitted and, therefore, that there will indeed be an equilibrium of the stage 2 game where the buyer insists and the seller compromises.

What if the seller deviates and attempts to commit to a price $p'_s < p'_s$? We want to construct an equilibrium where, after such a deviation, in stage 2 both players compromise whenever uncommitted. (This, of course, implies that offering a lower price will never be appealing for the seller.) The next lemma shows that this can be done, if ε is small enough.

Lemma 3. *For any sufficiently small ε, any pair of prices p_s and p_b satisfying equations (1) and (2) has the following property.*

For for any $p'_b \in [p_b, p_s]$ it is the case that:

$$(1 - \varepsilon_b) \cdot v_s(p_s) < \varepsilon_b \cdot v_s(p'_b) + (1 - \varepsilon_b) \cdot v_s(\alpha_b \cdot p'_b + \alpha_s \cdot p_s)$$

$$(1 - \varepsilon_s) \cdot v_b(p'_b) < \varepsilon_s \cdot v_b(p_s) + (1 - \varepsilon_s) \cdot v_b(\alpha_b \cdot p'_b + \alpha_s \cdot p_s)$$

and for any $p'_s \in [p_b, p_s]$ it is the case that:

$$(1-\varepsilon_s)\cdot v_b(p_b) < \varepsilon_s \cdot v_b(p'_s) + (1-\varepsilon_s)\cdot v_b(\alpha_b \cdot p_b + \alpha_s \cdot p'_s)$$

$$(1-\varepsilon_b)\cdot v_s(p'_s) < \varepsilon_b \cdot v_s(p_b) + (1-\varepsilon_b)\cdot v_s(\alpha_b \cdot p_b + \alpha_s \cdot p'_s).$$

Proof. Note that, since v_b is strictly decreasing in prices, equation (2) immediately implies that for $p'_b \in [p_b, p_s]$ the first inequality from the lemma holds for any ε.

Since v_b is continuously differentiable, $v'_b < 0$ on $[\underline{p}, \bar{p}]$, and $\alpha_b < 1$, Lemma 2 implies that, for sufficiently small ε, it will be the case that, for any $p'_b \in [p_b, p_s]$,

$$|v'_b(p'_b)| > \alpha_b \cdot |v'_b(\alpha_b \cdot p'_b + \alpha_s \cdot p_s)|.$$

The second inequality in the lemma now follows immediately from the above bound and equation (1).

We have proven that, for sufficiently small ε, the first two inequalities from the lemma have to hold for $p'_b \in [p_b, p_s]$. The proof that the last two inequalities hold for $p'_s \in [p_b, p_s]$ is analogous. □

We will now prove the existence part of Theorem 1. Let ε be small enough so that Lemma 3 can be applied and let p^*_s and p^*_b be a pair of prices such that equations (1) and (2) both hold for this ε. Now, consider the following strategy profile. In stage 1 the seller proposes p^*_s, the buyer proposes p^*_b and both attempt to commit. Behavior in stage 2 is as follows:

- After any history with $p_b < p_s$, where neither player attempted to commit, the buyer insists and the seller compromises.
- After any history with $p_b < p_s$, where one player attempted to commit and the other did not, the player who attempted to commit insists and the other compromises.

- After any history with $p_b < p_s$, where both players attempted to commit and $p_b = p_b^*$, behavior in stage 2 is as follows:
  - both players compromise if $p_s \leq p_s^*$
  - the buyer insists and the seller compromises if $p_s > p_s^*$.
- After any history with $p_b < p_s$, where both players attempted to commit and $p_s = p_s^*$, behavior in stage 2 is as follows:
  - both players compromise if $p_b \geq p_b^*$
  - the seller insists and the buyer compromises if $p_b < p_s^*$.
- After any other history behavior in stage 2 is as follows:
  - if it is the case that for the seller
    $(1 - \varepsilon_b) \cdot v_s(p_s) \leq \varepsilon_b \cdot v_s(p_b) + (1 - \varepsilon_b) \cdot v_s(\alpha_b \cdot p_b + \alpha_s \cdot p_s)$
    and for the buyer
    $(1 - \varepsilon_s) \cdot v_b(p_b) \leq \varepsilon_s \cdot v_b(p_s) + (1 - \varepsilon_s) \cdot v_b(\alpha_b \cdot p_b + \alpha_s \cdot p_s)$
    then both players compromise.
  - if one of the last two inequalities does not hold then one player insists and the other player compromises where the insisting player is the one for which the above inequality did not hold.[20]

It is straightforward to verify that the above strategy profile is indeed an equilibrium. Since perhaps the most non-obvious part in this verification is that neither player i has a beneficial deviation in which she simply proposes the price p_j proposed in equilibrium by her opponent, let us check this property for the seller.[21]

Note that, since the prices p_b and p_s in the constructed strategy profile satisfy quations (1) and (2), the seller will be indifferent between insisting and compromising in stage 2 if both players follow the strategy profile. Therefore, the payoff of the seller if he follows the strategy profile is the same as if he

[20]If both inequalities do not hold, just select one of the two equilibria.

[21]The calculation for the buyer is analogous.

would alsways insist in stage 2, i.e. equal to $(1-\varepsilon_b)\cdot v_s(p_s)$. If, on the other hand, the seller would deviate and simply propose a price equal to p_b, her payoff clearly would be $v_s(p_b)$. Thus, to show that the considered deviation is not profitable, it is enough to show that

$$v_s(p_b) \le (1-\varepsilon_b)\cdot v_s(p_s). \tag{4}$$

Now note that equation (1) together with the concavity of v_s implies that

$$(1-\varepsilon_b)\cdot v_s(p_s) \ge \varepsilon_b \cdot v_s(p_j) + (1-\varepsilon_b)\cdot(\alpha_b \cdot v_s(p_b) + \alpha_s \cdot v_s(p_s))$$

or, equivalently,

$$(1-\varepsilon_b)\cdot v_s(p_s)\cdot(1-\alpha_s) \ge (\varepsilon_b + \alpha_b\cdot(1-\varepsilon_b))\cdot v_s(p_b).$$

Plugging in α_b for $1-\alpha_s$ in the left hand side and using that $\varepsilon_b+\alpha_b\cdot(1-\varepsilon_b) < \alpha_b\cdot(1-\varepsilon_b)$ we get that

$$(1-\varepsilon_j)\cdot v_i(p_i)\cdot\alpha_j > \alpha_j\cdot(1-\varepsilon_j)\cdot v_i(p_j).$$

Dividing both sides by α_j we see that (4) actually holds with strict inequality and therefore the considered deviation is not profitable.

We showed that equilibria having the structure described in the theorem exist and for all equilibria with that structure it is the case that the prices p_b and p_s converge to the generalized Nash bargaining solution as ε becomes small.

Remark 4. Note also that the constructed equilibrium has the property that, for any player, any deviation in stage 1 by that player results in a strictly lower payoff.

All that remains is for us to show that, for small enough ε, all equilibria must have the properties described in the theorem.

Lemma 4. *Consider any history in which player i proposes some p_i (either attempting commitment or not) and the other player j attempts to commit to a $p_j \in (\underline{p}, \bar{p})$ such that*

$$v_i(p_i) - v_i(p_j) < \frac{\varepsilon_j}{\alpha_j + \varepsilon_j \cdot \alpha_i} \cdot v_i(p_i)$$

and $p_b \leq p_s$.

In any equilibrium, after such a history, player i will compromise whenever uncommitted.

Proof. Clearly, the inequality in the lemma is equivalent to

$$v_i(p_i) \cdot \frac{\alpha_j + \varepsilon_j \cdot \alpha_i - \varepsilon_j}{\alpha_j + \varepsilon_j \cdot \alpha_i} < v_i(p_j).$$

Plugging in $1-\alpha_i$ for α_j in the numerator and $1-\alpha_j$ for α_i in the denominator and rearranging terms yields

$$v_i(p_i) \cdot \frac{(1-\varepsilon_j) \cdot (1-\alpha_i)}{\varepsilon_j + \alpha_j \cdot (1-\varepsilon_j)} < v_i(p_j).$$

Now, multiplying both sides with $\varepsilon_j + \alpha_j \cdot (1-\varepsilon_j)$ and rearranging terms we obtain

$$(1-\varepsilon_j) \cdot v_i(p_i) < \varepsilon_j \cdot v_i(p_j) + (1-\varepsilon_j) \cdot (\alpha_i \cdot v_i(p_i) + \alpha_j \cdot v_i(p_j)).$$

Note that since v_i is concave it must be that $v_i(\alpha_i \cdot p_i + \alpha_j \cdot p_j) \geq \alpha_i \cdot v_i(p_i) + \alpha_j \cdot v_i(p_j)$ and therefore the last inequality implies that

$$(1-\varepsilon_j) \cdot v_i(p_i) < \varepsilon_j \cdot v_i(p_j) + (1-\varepsilon_j) \cdot v_i(\alpha_i \cdot p_i + \alpha_j \cdot p_j).$$

This, however, means that if player j compromises whenever uncommitted, player i will strictly prefer to compromise. Since $p_j \in (\underline{p}, \bar{p})$ player i also strictly prefers to compromise when player j always insists. Thus, after such a history, compromising always yields strictly higher payoffs for player i than insisting and, therefore, player i must be compromising after that history in equilibrium. □

As an immediate corollary we get the following result.

Lemma 5. *In any equilibrium, the expected payoff of each player is positive.*

Proof. We will show that the expected equilibrium payoff of the buyer is positive. The argument for the seller is analogous.

Note that if $p_s < \bar{p}$ the buyer's payoff must be positive since she could deviate and propose the price p_s herself thereby guaranteeing herself a payoff of $v_b(p_s)$, but $v_b(p_s) > 0$ since $p_s < \bar{p}$.

However, if $p_s = \bar{p}$, then, by Lemma 4, the buyer could achieve a positive payoff by proposing a price slightly below p_s. □

Lemma 6. *Fix $k_b, k_s > 0$. For sufficiently small ε , in any equilibrium players propose different prices and the proposed prices satisfy $p_b > p_s$.*

Proof. Note first that, for any $\varepsilon > 0$, it cannot be the case that $p_b < p_s$. Indeed, if $p_b < p_s$, either player would be better off proposing the opponents price instead of her own.

Thus it must be the case that $p_b \geq p_s$. To prove the lemma, we just need to show that, for sufficiently small ε there are no equilibria such that $p_b = p_s$.

Assume the last statement is not true. Then, there exists a sequence $(\varepsilon_1^n, \varepsilon_2^n) = \varepsilon^n \cdot (k_s, k_b)$ such that ε^n converges to zero and a corresponding sequence of equilibria in which both players propose the same price p^n. Choosing a sub-sequence if necessary, we can assume without loss of generality that p^n converges to some $p^* \in [\underline{p}, \bar{p}]$.

Consider first the case where $p^* \neq \bar{p}$. For any n, define $\hat{p}_s^n$ by

$$\hat{p}_s^n = p^n - \frac{1}{2}\frac{\varepsilon_s^n \cdot v_b(p^*)}{v_b'(p^*)}$$

and note that $\hat{p}_s^n \in (p^n, \bar{p})$ for large n.[22]

By the mean value theorem, for any n, there exists $\zeta^n \in [p^n, \hat{p}_s^n]$ such that

$$v_b(p^n) - v_b(\hat{p}_s^n) = v_b'(\zeta^n) \cdot (p^n - \hat{p}_s^n).$$

Since, by construction, $\hat{p}_s^n - p^n = -\frac{1}{2}\frac{\varepsilon_s^n \cdot v_b(p^*)}{v_b'(p^*)}$, we obtain

$$v_b(p^n) - v_b(\hat{p}_s^n) = v_b'(\zeta^n) \cdot \frac{1}{2}\frac{\varepsilon_s^n \cdot v_b(p^*)}{v_b'(p^*)}$$

or, equivalently,

$$\frac{v_b(p^n) - v_b(\hat{p}_s^n)}{\varepsilon_s^n} = v_b'(\zeta^n) \cdot \frac{1}{2} \cdot \frac{v_b(p^*)}{v_b'(p^*)}.$$

Note that since $\zeta^n \in [p^n, \hat{p}_s^n]$ and both p^n and $\hat{p}_s^n$ converges to p^* the right hand side of the last equality converges to $\frac{1}{2} \cdot v_b(p^*)$. Since $p^* < \bar{p}$ we know that $v_b(p^*) > 0$ and, therefore, $v_b(p^*) > \frac{1}{2} \cdot v_b(p^*)$. Thus, for sufficiently large n it will be the case that

$$\frac{v_b(p^n) - v_b(\hat{p}_s^n)}{\varepsilon_s^n} < v_b(p^*)$$

or, equivalently,

$$v_b(p^n) - v_b(\hat{p}_s^n) < \varepsilon_s^n \cdot v_i(p_i).$$

Thus, for all sufficiently large n, we can apply Lemma 4 to conclude that the buyer will compromise in stage 2 if she proposes p^n and the seller attempts to commit to $\hat{p}_s^n$.[23]

Consider now, for all n that are sufficiently large so that Lemma 4 can be applied, a deviation where the seller instead of proposing p^n proposes $\hat{p}_s^n$ and after a proposal of the buyer of p^n compromises in stage 2 whenever not committed. Note that, for all such n, it cannot be the case that the buyer does not attempt to commit in stage 1. Indeed, if the buyer proposed p^n

[22] $\hat{p}_s^n > p^n$ follows from $v_b' < 0$ and $v_b(p^*) > 0'$, where $v_b(p^*) > 0'$ in turn follows from $p^* < \bar{p}$. $\hat{p}_s^n < \bar{p}$ for large n follows from $p^n \to p^*$, $p^* < \bar{p}$ and $\varepsilon_s^n \to 0$.

[23] The inequality required in Lemma 4 follows from the last inequality since $\alpha_j + \varepsilon_j \cdot \alpha_i < 1$, where $\alpha_j + \varepsilon_j \cdot \alpha_i < 1$ is a direct consequence of $\alpha_j = 1 - \alpha_i$ and $\varepsilon \in (0, 1)$.

without attempting to commit, then the considered deviation would give the seller payoff of $\varepsilon_s \cdot v_s(\hat{p}_s^n) + (1-\varepsilon_s) \cdot v_s(\alpha_s \cdot \hat{p}_s^n + \alpha_b \cdot p^n)$ which clearly is bigger than the equilibrium payoff of $v_s(p^n)$.[24] Thus it must be that, for all such n, the buyer does attempt to commit. But then the seller's expected payoff from the deviation is

$$S^n = \varepsilon_s^n \cdot (1-\varepsilon_b^n) \cdot v_s(\hat{p}_s^n) + (1-\varepsilon_s^n) \cdot \varepsilon_b \cdot v_s(p_b^n) + (1-\varepsilon_s^n) \cdot (1-\varepsilon_b^n) \cdot v_s(\alpha_s \cdot \hat{p}_s^n + \alpha_b \cdot p_b^n).$$

To obtain a contradiction it is enough to show that $\frac{S^n - v_s(p^n)}{\varepsilon_s^n} > 0$ for sufficiently large n, since this would mean that there exists a number n for which the payoff from the deviation is bigger than the equilibrium payoff. Note, however, that since $\hat{p}_s^n = p^n - \frac{1}{2}\frac{\varepsilon_s^n \cdot v_b(p^*)}{v_b'(p^*)}$ the expression $\frac{S^n - v_s(p^n)}{\varepsilon_s^n}$ converges to

$$-\alpha_s \cdot \frac{v_s'(p^*)}{2} \cdot \frac{v_b(p^*)}{v_b'(p^*)},$$

as n goes to infinity. Since $v_b(p^*) > 0$[25], $v_s' > 0$, and $v_b' < 0$, the last expression is positive. Of course, if $\frac{S^n - v_s(p^n)}{\varepsilon_s^n}$ converges to a positive number as n goes to infinity, it must be positive for some sufficiently large n, which is the contradiction we have been looking for.

The argument for the case $p^* = \bar{p}$ is analogous to the above argument (which included the case $p^* = \underline{p}$) except that an analogous deviation for the buyer is considered. □

Lemma 7. *Fix $k_b, k_s > 0$. For sufficiently small ε, there are no equilibria in which neither player attempts to commit.*

Proof. Assume there is a sequence of equilibria in which neither player attempts to commit for a sequence $(\varepsilon_s^n, \varepsilon_b^n)$ such that both ε_s^n and ε_b^n converge to zero. Based on Lemma 6 we know that for sufficiently large n it is the case

[24]This follows trivially from $\hat{p}_s^n > p^n$ and the fact that v_s is increasing.

[25]This follows from the fact that p^* was assumed to be strictly smaller than $\bar{p}$

that the equilibrium prices satisfy $p_b^n < p_s^n$. Without loss of generality assume $p_b^n < p_s^n$ for all n.[26]

Note next that for any n, it cannot be that on the equilibrium path both players insist. Indeed, if that was the case, both players would receive a payoff of zero which would contradict Lemma 5. Also, it cannot be that on the equilibrium path both players compromise. Indeed, if that was the case, then $p_b^n < p_s^n$ implies that either player would be better off insisting instead. Therefore, it must be the case that on the equilibrium path exactly one player insists and one player compromises. This means that for each n either trade occurs with probability 1 at the price p_b or trade occurs with probability 1 at the price p_s. Let p^n be the price at which trade occurs.

Consider a fixed n. For the sake of concreteness assume that it is the buyer who proposes p^n; the argument for the case where it is the seller is analogous. Since, by Lemma 5, payoffs in equilibrium have to be positive for both players, $p^n < \bar{p}$. Then, however, Lemma 4 implies that the seller could achieve a higher payoff by attempting to commit to a price p'_s slightly above the price p^n proposed by the seller. □

Lemma 8. *Fix $k_b, k_s > 0$. For sufficiently small ε, in any equilibrium both players attempt to commit.*

Proof. By the last lemma, for sufficiently small ε, there are no equilibria in which neither player attempts to commit. It therefore remains to be shown, that for sufficiently small ε, there are no equilibria in which exactly one player does not attempt to commit. We will show that, for sufficiently small ε, there are no equilibria in which the buyer attempts to commit and the seller does not. The argument that there are no equilibria in which the seller attempts to commit and the buyer does not is analogous.

[26] If that is not the case, just choose an appropriate sub-sequence.

Assume there is a sequence of such equilibria for a sequence $(\varepsilon_s^n, \varepsilon_b^n) = \varepsilon^n \cdot (k_s, k_b)$ where ε^n converges to zero. By Lemma 6 we can without loss of generality assume that the prices for the corresponding equilibria satisfy $p_s^n > p_b^n$.

Note first that, for any n, it has to be that on the equilibrium path the seller compromises. Indeed, assume this is not true, i.e. assume that the seller insists. Since by Lemma 5, the equilibrium payoff of the buyer must be positive, it must be the case that the buyer compromises whenever she is not committed. That means that with probability $1 - \varepsilon_s^n$ trade is conducted at the price proposed by the seller and with probability ε_s^n both players get a payoff of zero. Now, since by Lemma 5 the expected payoffs of both players are non-negative, it must be that $v_b(p_s^n) > 0$. Then, however, the buyer would be better off off just proposing p_s^n and getting an expected payoff of $v_b(p_s^n)$ instead of an expected payoff of $(1 - \varepsilon_b^n) \cdot v_b(p_s^n)$.

Note next that, the fact that the seller compromises on the equilibrium path implies that the buyer will also insist on the equilibrium path when not committed. Therefore, trade occurs with probability 1 at price $p^n = p_b^n$.

Choosing a sub-sequence if necessary we can without loss of generality assume that $\lim_{n\to\infty} p^n$ exists and denote this limit by p^*.

For the case where $p^* < \bar{p}$, we can now simply complete the proof by obtaining a contradiction using an analogous deviation strategy for the seller as in the proof of Lemma 6.

However, for the case where $p^* = \bar{p}$, we need a more subtle argument. To derive this argument, let us first understand a bit better how equilibria have to look like for a fixed n. Note first that, by Lemma 4 it must be that

$$v_s(p_s^n) - v_s(p_b^n) \geq \frac{\varepsilon_b^n}{\alpha_b + \varepsilon_b^n \cdot \alpha_s} \cdot v_s(p_s^n)$$

since otherwise the seller would also compromise if the buyer would propose a slightly lower price which would mean that such a slightly lower price would constitute a profitable deviation. Since the function v_s is concave it must be that

$$v_s(p_s^n) - v_s(p_b^n) \leq (p_s^n - p_b^n) \cdot v'_s(p_b^n).$$

Combining the last two inequalities we obtain

$$(p_s^n - p_b^n) \cdot v'_s(p_b^n) \geq \frac{\varepsilon_b^n}{\alpha_b + \varepsilon_b^n \cdot \alpha_s} \cdot v_s(p_s^n)$$

or, equivalently,

$$(5) \qquad (p_s^n - p_b^n) \geq \frac{\varepsilon_b^n}{\alpha_b + \varepsilon_b^n \cdot \alpha_s} \cdot \frac{v_s(p_s^n)}{v'_s(p_b^n)}.$$

Since $p_s^n \leq \bar{p}$ and $v_b(\bar{p}) = 0$ the above bound on $p_s^n - p_b^n$ together with the concavity of v_b yields that

$$(6) \qquad v_b(p_b^n) \geq -v'_b(p_b^n) \cdot (p_s^n - p_b^n) \geq -v'_b(p_s^n) \cdot \frac{\varepsilon_b^n}{\alpha_b + \varepsilon_b^n \cdot \alpha_s} \cdot \frac{v_s(p_s^n)}{v'_s(p_b^n)}.$$

Now, fix a constant $\gamma \in (\alpha_b, 1)$ and let

$$(7) \qquad \tilde{p}_s^n = p_b^n + \gamma \cdot \frac{\varepsilon_s^n}{\alpha_s + \varepsilon_s^n \cdot \alpha_b} \cdot \frac{\varepsilon_b^n}{\alpha_b + \varepsilon_b^n \cdot \alpha_s} \cdot \frac{v_s(p_s^n)}{v'_s(p_b^n)}.$$

Note that, for sufficiently large n, $\tilde{p}_s^n \in (p_b^n, \bar{p})$. Indeed, $\tilde{p}_s^n > p_b^n$ follows immediately from the definition. Inequality (5), on the other hand, implies that, for sufficiently large n, it must be that $\tilde{p}_s^n < p_s^n$ and, therefore, $\tilde{p}_s^n < \bar{p}$.[27]

Next, note that

$$v_b(p_b) - v_b(\tilde{p}_s^n) \leq -v_b'(\tilde{p}_s^n) \cdot (p_s^n - p_b^n) \leq$$

$$\leq -v_b'(\tilde{p}_s^n) \cdot \gamma \cdot \frac{\varepsilon_s^n}{\alpha_s + \varepsilon_s^n \cdot \alpha_b} \cdot \frac{\varepsilon_b^n}{\alpha_b + \varepsilon_b^n \cdot \alpha_s} \cdot \frac{v_s(p_s^n)}{v'_s(p_b^n)} \leq$$

[27]This follows immediately from equation (7) and inequality (5), given that ε_s^n goes to zero as $n \to \infty$.

$$\leq \frac{v_b'(\tilde{p}_s^n)}{v_b'(p_b^n)} \cdot \gamma \cdot \frac{\varepsilon_s^n}{\alpha_s + \varepsilon_s^n \cdot \alpha_b} \cdot v_b(p_b^n).$$

where the first inequality follows from the concavity of v_b, the second from the definition of $\tilde{p}_s^n$ (equation (7)), and the third from inequality (6).

Finally, note that, since $\frac{v_b'(\tilde{p}_s^n)}{v_b'(p_b^n)}$ converges to 1 as n goes to infinity and $\gamma < 1$, the above implies that

$$v_b(p_b) - v_b(\tilde{p}_s^n) < \frac{\varepsilon_s^n}{\alpha_s + \varepsilon_s^n \cdot \alpha_b} \cdot v_b(p_b^n)$$

for sufficiently large n. Thus, by Lemma 4, for those large n, if the seller attempts to commit to $\tilde{p}_s^n$, the buyer will compromise whenever uncommitted. As a result, if the seller attempts to commit to $\tilde{p}_s^n$ and then compromises in stage 2, her payoff is equal to

$$S^n = \varepsilon_s^n \cdot (1-\varepsilon_b^n) \cdot v_s(\hat{p}_s^n) + (1-\varepsilon_s^n) \cdot \varepsilon_b \cdot v_s(p_b^n) + (1-\varepsilon_s^n) \cdot (1-\varepsilon_b^n) \cdot v_s(\alpha_s \cdot \hat{p}_s^n + \alpha_b \cdot p_b^n).$$

Of course, in equilibrium, it must be that the payoff from the above deviation is lower or equal then the equilibrium payoff of $v_s(p_b^n)$ and therefore $\frac{S^n - v_s(p_b^n)}{\varepsilon_b^n \cdot \varepsilon_s^n} \leq 0$.

Note, however, that equation (7) immediately implies that

$$\lim_{n\to\infty} \frac{S^n - v_s(p_b^n)}{\varepsilon_b^n \cdot \varepsilon_s^n} =$$

$$= \lim_{n\to\infty} \frac{(1-\varepsilon_s^n) \cdot (1-\varepsilon_b^n) \cdot (v_s(\alpha_s \cdot \hat{p}_s^n + \alpha_b \cdot p_b^n) - v_s(p_b^n)) - \varepsilon_b^n \cdot \varepsilon_s^n \cdot v_s(p_b^n)}{\varepsilon_b^n \cdot \varepsilon_s^n} =$$

$$= \alpha_s \cdot \frac{\gamma}{\alpha_s \cdot \alpha_b} \cdot v_s(p^*) - v_s(p^*)$$

Since $\gamma > \alpha_b$, this means that $\lim_{n\to\infty} \frac{S^n - v_s(p_b^n)}{\varepsilon_b^n \cdot \varepsilon_s^n} = (\frac{\gamma}{\alpha_b} - 1) \cdot v_s(p^*) > 0$, which of course yields the desired contradiction since, as we saw above, it must be that $\frac{S^n - v_s(p_b^n)}{\varepsilon_b^n \cdot \varepsilon_s^n} \leq 0$ for all n. □

Lemma 9. *For sufficiently small ε, in any equilibrium players attempt to commit to prices $p_b < p_s$, are indifferent between compromising and insisting* *on the equilibrium path, and compromise on the equilibrium path whenever uncommitted.*

Proof. The fact that players must attempt to commit to prices $p_b < p_s$ has been shown in the previous lemmas. Note that it cannot be that a player insists whenever uncommitted on the equilibrium path since then the other player would be better off proposing the price of her opponent.

Thus, both players must compromise on the equilibrium path whenever uncommitted and, therefore, compromising must yield a payoff that is at least as high as insisting. The only thing that is left is to show that the payoff of compromising is not strictly higher than the payoff from insisting. Note, however, that if that was the case, the other player could increase her payoff by attempting to commit to a slightly lower or slightly higher price, since her opponent would still compromise after that.

Thus, on the equilibrium path, each player must be indifferent between insisting and compromising. □

We have proven that, for ε small enough, all equilibria have the structure described in Theorem 1. This concludes the proof of Theorem 1.

Proof of Theorem 2. Note first that Lemmas 1 and 2 are still true since they have nothing do with the considered modification of the basic model.

Lemma 10. *Assume ε is sufficiently small. For any stage 1 history where both players attempted to commit to prices $p_b < p_s$, consider the stage 2 sub-game after that history. If for some player i*

$$(1-\varepsilon_j)\cdot v_i(p_i) > \varepsilon_j \cdot v_i(p_j) + (1-\varepsilon_j)\cdot v_i(\alpha_i \cdot p_i + \alpha_j \cdot p_j) \tag{8}$$

where j stands for player i's opponent, then there exists a pure-strategy Bayesian equilibrium of the stage 2 game in which player i always insists and player j insists if and only if she is committed and $c_j > v_j(p_i)$.

Moreover, if (8) does not hold for both players, then there is a pure-strategy Bayesian equilibrium of the stage 2 game in which both players compromise whenever uncommitted and insist whenever committed (i.e. when they face some positive cost of breaking their commitment).

Proof. Let $\underline{c} > 0$ be such that all points in the support of F_b and F_s are bigger than $\underline{c}$. Since the supports of F_b and F_s are finite such a $\underline{c}$ always exists. Assume ε is small enough so that

(9)
$$\varepsilon < \min(\frac{\underline{c}}{k_s \cdot v_b(\underline{p})}, \frac{\underline{c}}{k_b \cdot v_s(\bar{p})}).$$

To prove the first part of the lemma, assume that for some player i inequality (8) holds. Let $q = 1 - F_i(v_i(p_j))$. Note that (8) implies

$$(1 - q \cdot \varepsilon_j) \cdot v_i(p_i) > q \cdot \varepsilon_j \cdot v_i(p_j) + (1 - q \cdot \varepsilon_j) \cdot v_i(\alpha_i \cdot p_i + \alpha_j \cdot p_j).$$

Indeed, mathematically, this follows trivially from $v_i(p_i) \geq -v_i(p_j) + v_i(\alpha_i \cdot p_i + \alpha_j \cdot p_j)$.[28] On a more intuitive level, this corresponds to the observation that if player i prefers to insist if his opponent insists with a given probability, she will also prefer to insist if her opponent insists with a lower probability.

Consider the strategy profile in which player i always insists and player j insists if and only if she is committed and $c_j > v_j(p_i)$. Note that the last inequality guarantees that player i's payoff from insisting when she is uncommitted is higher than her payoff from compromising. Since for a committed type of player i compromising is associated with an additional cost this implies that insisting is strictly optimal whenever player i is committed.

[28] $v_i(p_i) \geq -v_i(p_j) + v_i(\alpha_i \cdot p_i + \alpha_j \cdot p_j)$ in turn follows immediately from $v_i(p_j) \geq 0$ and $p_b > p_s$ (the latter implies $v_i(p_i) > v_i(\alpha_i \cdot p_i + \alpha_j \cdot p_j)$).

Now consider player j. Since player i always insists, compromising is clearly a best response for the uncommitted type of player j. For the committed type of player j insisting is strictly better than compromising if and only if $c_j > v_j(p_i)$. Thus, those types of player j, also do not have a beneficial deviation.

We have proven the first part of the lemma.

To prove the second part of the lemma, assume that for both players inequality (8) does not hold, i.e.

$$(1-\varepsilon_b)\cdot v_s(p_s) \leq \varepsilon_b \cdot v_s(p_b) + (1-\varepsilon_b)\cdot v_s(\alpha_b \cdot p_b + \alpha_s \cdot p_s)$$

and

$$(1-\varepsilon_s)\cdot v_b(p_b) \leq \varepsilon_s \cdot v_b(p_s) + (1-\varepsilon_s)\cdot v_b(\alpha_b \cdot p_b + \alpha_s \cdot p_s).$$

Consider a strategy profile in which the seller and buyer compromise if and only if they are uncommitted. Note that the above two inequalities imply that compromising is indeed optimal for the uncommitted types of both buyer and seller.

To check that committed seller types cannot achieve higher payoffs by compromising it is enough to verify that

$$(1-\varepsilon_b)\cdot v_s(p_s) \geq \varepsilon_b \cdot v_s(p_b) + (1-\varepsilon_b)\cdot v_s(\alpha_b \cdot p_b + \alpha_s \cdot p_s) - \underline{c}.$$

Note, however, that the above inequality is equivalent to

$$\underline{c} \geq \varepsilon_b \cdot v_s(p_b) + (1-\varepsilon_b)\cdot (v_s(\alpha_b \cdot p_b + \alpha_s \cdot p_s) - v_s(p_s)).$$

This inequality, however, follows from $v_s(p_s) > v_s(\alpha_b \cdot p_b + \alpha_s \cdot p_s)$, $\varepsilon_b = k_b \cdot \varepsilon$ and inequality (9).

An analogous argument shows the committed buyer types cannot achieve higher payoffs by compromising. This means the considered strategy profile is a pure-strategy Bayesian equilibrium. We have proven the second part of the lemma. □

Consider any ε small enough so that Lemma 3 and 10 can both be applied. Let p_s^* and p_b^* be a pair of prices such that equations (1) and (2) both hold and consider the following strategy profile. In stage 1 the seller attempts to commit to p_s^* and the buyer attempts to commit to p_b^*. Behavior in stage 2 is as follows:

- After any history with $p_b < p_s$ where neither player attempted to commit, the buyer insists and the seller compromises.
- After any history with $p_b < p_s$ where one player attempted to commit and the other did not, the player who attempted to commit always insists and the other compromises.
- After any history where the buyer attempts to commit to p_b^* and the seller attempts to commit to some $p_s > p_b^*$ use
  - the equilibrium from Lemma 10 in which the buyer always insists if $p_s > p_s^*$ (and, therefore, inequality (8) from Lemma 10 holds for the buyer)
  - the second equilibrium from Lemma 10 if $p_s \leq p_s^*$ (and, therefore, by Lemma 3, inequality (8) does not hold for both players)
- After any history where the seller attempts to commit to p_s^* and the buyer attempts to commit to some $p_b < p_s^*$ use
  - the equilibrium from Lemma 10 in which the seller always insists if $p_b < p_b^*$ (and, therefore, the inequality (8) from Lemma 10 holds for the seller)
  - the second equilibrium from Lemma 10 if $p_b \in [p_b^*, p_s^*]$ (and, therefore, by Lemma 3, inequality (8) does not hold for both players)

- After any other history with $p_b < p_s$ where both players attempted to commit using some commitment devices use any equilibrium from Lemma 10. Note that since either (8) holds for at least one player or does not hold for both, it is always the case that at least one of the three equilibria described in Lemma 10 exists.

It is again straightforward to check that the above strategy profile is indeed a perfect Bayesian equilibrium of the sub-game starting with stage 1.

To prove the second part of the theorem, define

$$q = \min(1 - F_b(v_b(\underline{p})), 1 - F_s(v_s(\bar{p})))$$

and note that given the assumptions on F_b and F_s it has to be that $q > 0$.

The proof of the second part of Theorem 2 follows the corresponding part of the proof of Theorem 1. Let us start by observing that while Lemma 4 does not hold in the modified framework, the following lemma does hold.

Lemma 11. *Consider any history in which player i proposes some p_i (either attempting commitment or not) and the other player j attempts to commit to a $p_j \in (\underline{p}, \bar{p})$ such that*

$$v_i(p_i) - v_i(p_j) < \frac{q \cdot \varepsilon_j}{\alpha_j + q \cdot \varepsilon_j \cdot \alpha_i} \cdot v_i(p_i)$$

and $p_b > p_s$.

In any equilibrium, after such a history, player i will compromise whenever uncommitted.

Proof. The proof is essentially identical to the proof of Lemma 4 for the basic model. □

Using Lemma 11 instead of Lemma 4 the same arguments as in the proof of Lemmas 5, 6, 7, and 8 show that those lemmas still hold, the only exception being the calculation at the end of Lemma 8 used to exclude equilibria described in Theorem 2 under (ii) and (iii).

We can therefore conlude that, for sufficiently small ε, any equilibrium that does not have the structure described in (ii) or (iii) has the property that players attempt to commit to prices $p_b < p_s$.

Note now that, in such an equilibrium, it cannot be that one of the players insists on the equilibrium path when she is uncommitted. Indeed, if a player would find it optimal to insist when she is uncommitted, she would also find it optimal to insist when facing an additional cost of $c_i > 0$ whenever she compromises. However, if one player always insists, then the other player would have a profitable deviation: instead of attempting to commit to her own price, she could just propose the same price that the other player is proposing in equilibrium.[29]

Thus, both players must compromise on the equilibrium path whenever uncommitted. Since each player compromises on the equilibrium path when uncommitted, compromising must yield a payoff that is at least as high as the payoff from insisting. The only thing that is left for us to show, therefore, is that, on the equilibrium path, for small ε, uncommitted players do not have a strict preference for compromising, i.e. the payoff of compromising is not strictly higher than the payoff from insisting.

[29]If player i attempts to commit to p_i and then always insists, the best expected payoff the other player can get given that she attempted to commit to some other price is bounded from above by $(1 - q \cdot \varepsilon_j) \cdot v_j(p_i)$. Since both players have positive expected payoffs in equilibrium (see Lemma 5) it must be that $v_j(p_i) > 0$. In that case, however, player j would be strictly better off just proposing p_i and receiving an expected payoff of $v_j(p_i)$.

To see that this cannot be the case, notice that, for small ε, all the committed types with $c_i > 0$ have a strict preference for insisting.[30] Now, if it was the case that, for those small enough ε, some player i when uncommitted has a strict preference for compromising then her opponent could change the price slightly and increase her profits since if the change was small enough player i would still compromise whenever committed and insist whenever uncommitted. We can therefore conclude that, for small enough ε on the equilibrium path uncommitted types are indifferent between compromising and insisting, uncommitted types compromise, and committed types insist. This completes our proof that equilibria have the desired structure.

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[30] The advantage of insisting is that the cost c_i does not need to be payed and that prices are better if the opponent compromises. The disadvantage is that whenever the opponent insists, there will be no trade. Since the uncommitted type always compromises the disadvantages are of order ε while the advantages coming from the fact that the cost c_i does not need to be payed are bounded from bellow by $\underline{c}_i$, the lowest cost value in the support of F_i. Since $\underline{c}_i$ is fixed, for ε which is sufficiently small the advantages of insisting will strictly outweigh the disadvantages.

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