Can Media Pluralism Be Harmful to News Quality?

Federico Innocenti

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1 University of Mannheim, Department of Economics and CRC. email: finnocen@mail.uni-mannheim.de

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Abstract

I study the effect of polarization and competition on information provision. With a single expert who faces decision-makers with heterogeneous priors, the expert solves a trade-off between persuading sceptics and retaining believers. With high polarization, an expert has incentives to supply low-quality information to leverage believers’ credulity. With multiple experts with opposite biases, competition is harmful if attention is limited. Unbiased and Bayesian decision-makers rationally devote attention to like-minded experts. Echo chambers arise endogenously, whereas decision-makers would be better informed in monopoly. My model can rationalize the spread and persistence of conspiracy theories and fake news.

Keywords: Bayesian Persuasion, Competition, Echo Chambers, Heterogeneous Priors, Limited Attention, Media Pluralism.

JEL Classification: D82, D83, L82

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†University of Mannheim, Department of Economics and CRC. email: finnocen@mail.uni-mannheim.de
1. Introduction

Does competition in information provision improve individual decision-making? Receiving information from many different sources is usually considered a desirable goal for a society or an organization. The more information a decision-maker receives, the more she knows about an issue, and the less she is influenced by the bias of a particular expert. At the beginning of the new century, the Internet has been considered a very effective way to guarantee political pluralism in the information, and to counteract media ownership concentration (Keen, 2015). However, several empirical facts point to a deterioration of the quality of the information available. Examples that come to mind are the spread of conspiracy theories and fake news. Public opinion is significantly affected by misperceptions in the digital age (Allcott and Gentzkow, 2017; Allcott et al., 2020).

I uncover a novel rationalization for competition in information provision being harmful. I use the COVID-19 vaccination as my leading example for illustrative purposes, and my results can be framed as follows. There are two possible states of the world: either a vaccine is safe or not. Nobody knows a priori the true state of the world. The government wants to persuade citizens to vaccinate, to reach herd immunity. An anti-vax leader opposes vaccinations to make profits with alternative treatments (Ghoneim et al., 2020). Citizens are unbiased as they want to vaccinate themselves if and only the vaccine is safe. However, citizens have heterogeneous prior beliefs. Some of them are sceptical about vaccinations being safe and are not willing to vaccinate a priori (Paul et al., 2020). All the information produced is accessible by each citizen. However, citizens receive too much information and are not able (or do not want) to process it all. I show that competition in information provision is harmful when citizens strategically allocate their limited attention.

I develop a theoretical model that allows comparing a setting with a single expert (e.g., a government-controlled media) to one with two experts (e.g., a government-controlled media and one controlled by the anti-vax leader).

To begin, I consider a single expert who faces decision-makers (citizens) with heterogeneous priors. The expert’s objective is to design information to maximize the probability that decision-makers take the expert’s favourite action (either to vaccinate or not). In the baseline model, the expert faces two decision-makers: a sceptic and a believer. Ex-ante the believer would choose the expert’s favourite action, whereas the sceptic would not. Hence,

1Priors are heterogeneous among different decision-makers, and potentially between any decision-maker and the expert.
the latter needs to be persuaded. All the information is public. Thus, any attempt to change the sceptic’s beliefs affects the believer’s beliefs as well. The expert solves a trade-off between persuading the sceptic and retaining the believer.

The expert can use two strategies. One strategy focuses on persuading the sceptic. I label it hard-news strategy. To persuade the sceptic a message must be credible: it must be hard evidence, such as scientific data, which can be misleading only to a limited extent. Therefore, this hard evidence is supplied at the cost of revealing the unfavourable state with positive probability to both decision-makers. Then, ex-post the believer could take the expert’s adverse action.

The alternative strategy focuses on retaining the believer. I label it soft-news strategy. The expert mixes between a message that persuades the sceptic (hard evidence as before) and a message that just retains the believer. With the latter, the expert leverages the believer’s credulity. Therefore, I interpret such a message as soft evidence, for instance, anecdotal facts. This strategy ensures that the believer chooses the expert’s favourite action with probability one. I show that the hard-news strategy is more informative than the soft-news strategy according to the Blackwell (1953)’s criterion.

The expert prefers the soft-news strategy if the decision-makers are sufficiently polarized. A proxy for the polarization of the decision-makers is the difference in their priors of the favourable state. Higher polarization is equivalent to one of the two priors becoming more extreme. If the prior of the sceptic becomes more extreme (closer to 0), then it is more costly to persuade the sceptic. To be credible, the expert has to reveal the unfavourable state with a higher probability. Instead, if the prior of the believer becomes more extreme (closer to 1), then it is easier to leverage the believer’s credulity using soft evidence. A second channel is given by the expert’s prior. The soft-news strategy is more appealing the lower is the expert’s confidence in his favourite action being the right one. Intuitively, the expert values more his ability to be misleading.

In a monopoly, the supply of news about the COVID-19 vaccine by the government depends on its level of confidence about vaccinations. If the government is very confident, then it provides hard evidence. For instance, the evaluations by the European Medicines Agency (EMA) based on clinical trials. The government attempts to persuade sceptics to vaccinate because it expects persuasion to be very likely. However, if the government is not confident enough, high polarization makes it optimal to supply also soft evidence. For instance, weaker statements such as “benefits are higher than risks”. In this way, the government makes sure to retain those citizens who were already willing to vaccinate.
Next, I show how competition makes decision-makers worse off. I assume that decision-makers have limited attention. Two experts (a government-controlled media and an anti-vax media) compete to persuade decision-makers (citizens). However, the latter can devote attention to just one expert. Therefore, each expert persuades as a monopolist in his market (as before). The novelty here is the interaction between optimal persuasion and the endogenous allocation of attention. I assume that the two take place simultaneously. The allocation of attention affects the optimal strategies of the experts. Indeed, the allocation of attention affects the distribution of priors each expert has to deal with. In equilibrium, the decision-makers must not have an incentive to change their allocation of attention, given the strategies of the experts.

Each decision-maker allocates her attention to maximize her subjective probability to take the correct action. This probability is minimized without information, whereas persuasion requires revealing some information, and makes the decision-maker (weakly) better off. Therefore, the problem can be expressed as each decision-maker maximizing her (subjective) information gain from persuasion. It makes a difference whether a decision-maker is being targeted by an expert. Being targeted means that the expert’s strategy is designed explicitly to persuade that particular decision-maker. For example, the sceptic is targeted if the expert uses the hard-news strategy. Any decision-maker who is targeted by a given expert will receive zero information gain when devoting attention to this particular expert. Indeed, it is never optimal for an expert to reveal more information than what is necessary to persuade a targeted decision-maker. This result shapes the incentives of the decision-makers about the allocation of attention.

I show that echo chambers with babbling is the unique pure strategy equilibrium of this game where both experts are active. Within echo chambers, each decision-maker devotes attention to an expert who finds it optimal to confirm the priors of such a decision-maker. Indeed, each expert is facing only his believer. Thus, babbling is the optimal strategy for each expert. Given babbling by both experts, the decision-makers have no incentive to deviate, as the information gain is zero in any case.

In a monopoly, decision-makers are better informed than within echo chambers. Indeed, a monopolist has to trade-off between persuading the sceptic and retaining the believer. Thus, the expert uses either the hard-news strategy or the soft-news strategy, which are both more informative than babbling according to the Blackwell (1953)’s criterion. Therefore, limited attention makes competition harmful, as single-homing decision-makers may cluster into echo chambers. Ubiquitous information could make all information useless.
The endogenous allocation of attention makes competition harmful. Consider the case where both experts are using the hard-news strategy. Each decision-maker has incentives to become the believer of an expert to get a positive information gain. However, the strategic response of the experts traps decision-makers into echo chambers. I show that a "neutral" platform can allocate the attention in a way that induces both experts to use the hard-news strategy. In this way, competition can be beneficial.

I study the mixed strategy equilibria of the game with an endogenous allocation of attention. In terms of informativeness, these are intermediate between a monopoly and echo chambers. These equilibria rationalize the coexistence of informative and babbling experts. For instance, the government produces hard news about the COVID-19 vaccine, whereas the anti-vax leader does babbling within its echo chamber. Citizens who are sceptical about vaccinations rationally consume their limited attention with confirmatory news. These citizens cannot be persuaded by the government to vaccinate. Therefore, herd immunity can be achieved only if the size of the anti-vax echo chamber is not too large.

In a model with arbitrary many decision-makers endowed with different priors, I show that the optimal persuasion strategy is still either a hard-news strategy or a soft-news strategy. With competition and limited attention, any equilibrium is such that at least one expert is babbling.

The COVID-19 vaccination is just one of many examples which fit my model. Other examples are climate change, political issue campaigns and persuasive advertisement of products.

1.1. Related Literature

I contribute to the literature by exploring how the endogenous supply of misleading information to decision-makers with heterogeneous beliefs interacts with limited attention. Therefore, my paper connects with the following streams in the literature.

Limited attention

"...information consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention." Simon (1971)

Decision-makers process only a subset of the available information because of high cognitive cost (Gabaix et al. 2006). Empirical evidence shows growing segregation in news consumption, despite the greater availability of content from heterogeneous sources (Flaxman et al. 2016; Schmidt et al. 2017).
Falkinger (2008) analyses the supply and demand determinants of limited attention. The latter can explain many seemingly irrational empirical patterns, for instance, asset-price dynamics (Peng and Xiong, 2006), the attraction effect (Masatlioglu et al., 2012) and the superstar effect (Hefti and Lareida, 2021). Limited attention influences price competition and advertising within and across industries (Anderson and de Palma, 2012; De Clippel et al., 2014; Hefti and Liu, 2020). I assume that the allocation of attention is under the control of rational decision-makers (Sims, 2003), and I offer new insights about the effects of limited attention.

Bayesian persuasion. A standard assumption in this literature - pioneered by Kamenica and Gentzkow (2011) - is the existence of a common prior. By contrast, I examine the problem of a sender (expert) who faces many receivers (decision-makers) endowed with heterogeneous priors. In Guo and Shimaya (2019), a separating (soft-news) strategy yields a higher payoff to the sender than a pooling (hard-news) strategy if the receiver has sufficiently accurate private information. Each realization of private information can be interpreted as a receiver holding heterogeneous priors. From this perspective, I show that more accurate private information corresponds to less accurate public information. Indeed, higher polarization lowers the quality of the information provided by the sender. A similar effect arises in Gitmez and Molavi (2020). However, these authors focus on the ability of a sender to gather attention from receivers with heterogeneous priors. Gentzkow and Kamenica (2017a,b) argue that competition among senders weakly increases information provision and benefits receivers. I show that this conclusion fails if receivers have heterogeneous priors and limited attention. My model incorporates endogenous allocation of attention between competing senders and endogenous persuasion. In Knoepfle (2020), senders compete to gather the attention of a receiver. By contrast, senders are concerned about receivers’ actions in my model. This difference leads to opposite results: endogenous echo chambers in my model, whereas full revelation is reached in Knoepfle (2020).

Alonso and Camara (2016) study a setting where a sender persuades a single receiver, and the two have heterogeneous priors. Laclau and Renou (2016) and Shimoh (2016) study general frameworks with heterogeneous priors. Instead, I supply a richer characterization of optimal persuasion and provide new insights. Priors are exogenous to the model, and it is beyond the purpose of this paper to analyse the origin of priors (Flynn et al., 2017). Che and Mierendorf (2019) and Leung (2020) study the problem of a receiver who has to allocate her limited attention between biased senders. In these papers, the information design is exogenous. Bloedel and Segal (2020), Gitmez and Molavi (2020), Lipnowski et al. (2020) and Wei (2020) study how limited attention by the receiver(s) affects optimal persuasion by a single sender.
Echo chambers. Jann and Schottmüller (2019) rationalize echo chambers in a many-to-many cheap talk model with biased decision-makers. By contrast, even unbiased decision-makers may cluster into echo chambers in my model: the existence of a bias is not necessary to support an equilibrium with echo chambers. Martinez and Tenev (2020) study a model where experts are unbiased but endowed with different technologies. A decision-maker rationally prefers an expert with a signal closer to the decision-maker’s prior, as she infers that such expert has higher quality. By contrast, in my model experts are endowed with the same technology, but supply different slants of the information. Each decision-maker maximizes her information gain, and the strategic interaction between decision-makers and experts plays a crucial role in the formation of echo chambers.

Unravelling and harmful competition. The “unravelling” effect of competition has been disputed along many dimensions. With vertically differentiated firms, only high-quality firms have incentives to disclose (Board 2009), or there is no disclosure at all (Janssen and Roy 2014). By contrast, unravelling can work in the contexts of comparative and price advertising (Janssen and Teteryatnikova 2016) and political competition (Schipper and Wool 2019). Generally speaking, competition can backfire. Chen and Riordan (2008) show that price-increasing competition occurs when products are sufficiently differentiated. In the insurance market, competition can increase distortions when agents have heterogeneous perceptions of risk (Spinnewijn 2013). Information overload does not allow decision-makers to identify high-quality experts (Persson 2018), and implies higher prices because consumers get lost in diversity (Hetti 2018). Costly information acquisition or communication reduce each expert’s effort in the presence of other experts; Free-riding harms decision-makers (Kartik et al. 2017; Emons and Fluet 2019). I provide a novel rationalization for harmful competition in information provision.

1.2. Structure of the Paper

I present the theoretical model in Section 2. I devote Section 3 to the characterization of optimal persuasion by a monopolistic expert. In Section 4 I describe the effects of competition when decision-makers have limited attention. I examine some extensions in Section 5. Finally, in Section 6 I present some conclusive remarks. Appendix A contains the most technical proofs, whereas further extensions are presented in Appendix B.

4See also Nimark and Sundaresan (2019), where echo chambers arise because the cost of processing information is increasing in its precision.
2. Model

There are two states of the world and two corresponding actions. That is, I denote with \( \Omega := \{ \omega_1, \omega_2 \} \) the set of states and with \( A \) the set of actions, where \( A \equiv \Omega \). I denote with \( I \) the set of decision-makers, whereas \( J \) is the set of experts. Each agent \( l \in I \cup J \) has distinct prior beliefs with full support: \( \mu_0^l(\cdot) \in \Delta_+^\Omega \), where \( \mu_0^l(\omega) \) is agent \( l \)'s belief that the state is \( \omega \).

The message space \( S_j \) for any expert \( j \) contains at least two elements. For any expert \( j \) (he) there exists a unique favourite action \( a_j \in A \). His payoff from a decision-maker who takes action \( a \in A \) given state \( \omega \in \Omega \) is:

\[
    u_j(a, \omega) = u_j(a) := 1 \{ a = a_j \}
\]

Each expert has state-independent preferences, and his payoff is 1 if and only if the action chosen by a decision-maker is the expert's favourite action.

Instead, each decision-maker \( i \in I \) (she) wants to match her action \( a \) with the state \( \omega \):

\[
    u_i(a, \omega) := 1 \{ a = \omega \} \tag{1}
\]

The game has the following timing:

1. Each expert \( j \in J \) commits to a strategy \( \pi_j : \Omega \rightarrow \Delta(S_j) \). In words, each expert commits to the probability \( \pi_j(s_j | \omega) \) to send message \( s_j \) given state \( \omega \), for any message \( s_j \in S_j \) and any state \( \omega \in \Omega \). At the same time as the commitment of each expert, each decision-maker \( i \in I \) devotes attention to a non-empty subset \( J_i \subseteq J \) of experts. The allocation problem is analysed in greater details in Section 4.

2. Each decision-maker \( i \) observes \( \pi_j(s_j | \omega) = \Pi_{j \in J_i} \pi_j(s_j | \omega) \) for any \( s \in S_{J_i} := \bigtimes_{j \in J_i} S_j \) and any \( \omega \in \Omega \), and a realization \( s \in S_{J_i} \) chosen by Nature. Based on that, she constructs posterior beliefs \( \mu_i := \{ \mu_i^\pi_j(\omega | s) \}_{\omega \in \Omega} \) using Bayesian Updating:

\[
    \mu_i^\pi_j(\omega | s) := \frac{\pi_j(s_j | \omega) \mu_0^i(\omega)}{\pi_j(s_j | \omega_1) \mu_0^i(\omega_1) + \pi_j(s_j | \omega_2) \mu_0^i(\omega_2)} \tag{2}
\]

3. Given any posterior beliefs \( \mu_i \), each decision-maker \( i \) takes an optimal action \( \sigma : [0,1]^2 \rightarrow A \).

\[\text{In Section B.3 I discuss an extension with more than two states.}\]

\[\text{In Section B.2 I consider a sequential version of the game. The effect of competition is different when experts implicitly become attention-seekers, as in Knoepfel (2020).}\]
The game is solved by backward induction, and the equilibrium notion is Subgame Perfection. By (1), the optimal action by decision-maker \( i \) with posterior beliefs \( \mu_i \) is given by the following function:

\[
\sigma(\mu_i) := \begin{cases} 
\omega_1 & \text{if } \mu_i^\pi_i(\omega_1 | s) \geq \mu_i^\pi_i(\omega_2 | s) \\
\omega_2 & \text{otherwise}
\end{cases}
\]

Thus, if an expert \( j \in J_i \) wants to persuade decision-maker \( i \) to take action \( \omega_1 \) (without loss of generality), the following condition must hold:

\[
\mu_i^\pi_i(\omega_1 | s) \geq \mu_i^\pi_i(\omega_2 | s) \iff \pi_j(s | \omega_1) \mu_i^0(\omega_1) \geq \pi_j(s | \omega_2) \mu_i^0(\omega_2) \tag{3}
\]

**Definition 1** (Persuasion constraints). By (3), the persuasion constraint for a decision-maker \( i \in I \), who devotes attention to \( J_i \) and observes \( s \), in order for her to take action \( \omega_1 \) (without loss of generality), is:

\[
\pi_j(s_j | \omega_1) \leq \frac{\mu_i^0(\omega_1)}{\mu_i^0(\omega_2)} \frac{\pi_j(s_j | \omega_2)}{\pi_j(s_j | \omega_2)} \pi_j(s_j | \omega_1) \quad \text{for any } j \in J_i \tag{4}
\]

In the following let \( \phi_{ij}(\omega_1, \omega_2, s_j) := \frac{\mu_i^0(\omega_1)}{\mu_i^0(\omega_2)} \frac{\pi_j(s_j | \omega_1)}{\pi_j(s_j | \omega_2)}. \)

For any \( i \in I \), I define \( g_i \in (0,1) \) as the relative "importance" of decision-maker \( i \) for any expert. \(^7\) I assume \( \sum_{i \in I} g_i = 1 \). Therefore, the payoff of expert \( j \) from decision-maker \( i \) who devote attention to \( J_i \) and observes \( s \) is:

\[
v_{ij}(\pi_j, s) := g_i u_j(\sigma(\mu_i(\pi_j, s)))
\]

The expert \( j \) solves the following problem:

\[
\max_{\pi_j} \sum_{i \in I} \sum_{s \in S_i} \sum_{\omega \in \Omega} \pi_j(s | \omega) \mu_i^0(\omega) v_{ij}(\pi_j, s) \tag{5}
\]

The expert takes as given \( J_i \) for each decision-maker \( i \). Therefore, (5) is a best-response problem in a simultaneous-move game, where each decision-maker \( i \) chooses her \( J_i \) and each expert \( j \) chooses his \( \pi_j \).

This problem entails a trade-off: on the one hand, the expected payoff increases in the probability of observing a persuading message; on the other hand, the message must be "credible" to persuade a decision-maker. Thus, the persuasion constraint has to hold. I can restrict the set of strategies to consider as best responses. First, "redundant" messages cannot help the expert to persuade (Lemma 5). Second, at least one persuasion constraint must be binding (Lemma 6). The formal lemmas are in Appendix A. In the following section, I use these insights to find candidate optimal strategies.

\(^7\) An alternative interpretation is the following: \( I \) is the set of decision-makers’ types, and \( g_i \) is the mass of decision-makers of type \( i \). See Sections 5.1 and 5.2.
3. Monopolistic Expert

The objective of an expert is to persuade decision-makers to take his favourite action. An expert can unilaterally disclose the true state, hence inducing his favourite action when it is optimal for decision-makers. An expert has persuasion power if he can persuade decision-makers to take his favourite action, even when it is not optimal for them. In this section, I show that a monopolistic expert has persuasion power and that the way he wields it depends on decision-makers’ priors. I assume without loss of generality that the expert’s favourite action is $\omega_1$, and I omit the index $j$ for simplicity. By (3), a message $s$ persuades a decision-maker $i$ to take action $\omega_1$ if and only if

$$\pi(s|\omega_2) \leq \frac{\mu_i^0(\omega_1)}{\mu_i^0(\omega_2)} \pi(s|\omega_1) \equiv \phi_i(\omega_1,\omega_2) \pi(s|\omega_1)$$

From the perspective of the expert, there are two categories of decision-makers:

**Definition 2** (Believers and sceptics). A decision-maker $i$ is a believer of state $\omega_1$ relative to $\omega_2$ if $\phi_i(\omega_1,\omega_2) \geq 1$. Let $I_1 \subset I$ be the corresponding set of believers. A decision-maker $i$ is a sceptic of state $\omega_1$ relative to $\omega_2$ if $\phi_i(\omega_1,\omega_2) < 1$. Let $I_2 \subset I$ be the corresponding set of sceptics.

The function $\phi_i(\cdot)$ is the ratio of priors for decision-maker $i$. It characterizes her ex-ante behaviour. A priori, a believer chooses the expert’s favourite action, whereas a sceptic does not. Therefore, a sceptic requires persuasion. The expert manipulates a sceptic’s beliefs through his strategy $\pi$, to induce such a sceptic to take action $\omega_1$. However, the expert must account for the indirect effect that persuasion of a sceptic has on the behaviour of a believer, as all the information is public.

In this section, I assume $I = \{1, 2\}$ where decision-maker 1 is a believer i.e. $\phi_1(\omega_1,\omega_2) > 1$, and decision-maker 2 is a sceptic i.e. $\phi_2(\omega_1,\omega_2) < 1$. Thus, the expert can use a message to persuade either both decision-makers or only the believer (or nobody) to take action $\omega_1$. By Lemma 6, there is at most one message for each of these purposes. By Lemma 6, the optimal strategy must contain (at least) one message which makes one decision-maker just indifferent between $\omega_1$ and $\omega_2$. Hence, I identify two potentially optimal strategies:

**Definition 3** (Hard-news strategy). The hard-news strategy $\pi_h$ consists of a persuading message $s$ and a residual message $s'$ such that

$$\pi_h(s|\omega_1) = 1, \quad \pi_h(s'|\omega_1) = 0,$$

In Section 5.3 I consider the case of many decision-makers.
\[ \pi_h(s | \omega_2) = \phi_2(\omega_1, \omega_2), \quad \pi_h(s' | \omega_2) = 1 - \phi_2(\omega_1, \omega_2) \]

with posterior beliefs

\[ \mu_1^{\pi_h}(\omega_1 | s) = \frac{\phi_1(\omega_1, \omega_2)}{\phi_1(\omega_1, \omega_2) + \phi_2(\omega_1, \omega_2)} > \mu_2^{\pi_h}(\omega_1 | s) = \frac{1}{2}, \quad \mu_1^{\pi_h}(\omega_1 | s') = \mu_2^{\pi_h}(\omega_1 | s') = 0 \]

**Definition 4 (Soft-news strategy).** The soft-news strategy \( \pi_s \) consists of two persuading messages \( s, s' \) such that

\[ \pi_s(s | \omega_1) = k, \quad \pi_s(s' | \omega_1) = 1 - k, \]

\[ \pi_s(s | \omega_2) = \phi_2(\omega_1, \omega_2)k, \quad \pi_s(s' | \omega_2) = \phi_1(\omega_1, \omega_2)(1 - k) \]

where

\[ k := \frac{\phi_1(\omega_1, \omega_2) - 1}{\phi_1(\omega_1, \omega_2) - \phi_2(\omega_1, \omega_2)} \]

with posterior beliefs

\[ \mu_1^{\pi_s}(\omega_1 | s) = \frac{\phi_1(\omega_1, \omega_2)}{\phi_1(\omega_1, \omega_2) + \phi_2(\omega_1, \omega_2)} > \mu_2^{\pi_s}(\omega_1 | s) = \frac{1}{2}, \]

\[ \mu_1^{\pi_s}(\omega_1 | s') = \frac{1}{2} > \mu_2^{\pi_s}(\omega_1 | s') = \frac{\phi_2(\omega_1, \omega_2)}{\phi_1(\omega_1, \omega_2) + \phi_2(\omega_1, \omega_2)} \]

The hard-news strategy persuades both decision-makers after seeing \( s \), and nobody after seeing \( s' \). Thus, both decision-makers are persuaded in state \( \omega_1 \), and sometimes in state \( \omega_2 \). The hard-news strategy can be interpreted as providing well-grounded information able to influence the sceptic. However, this comes at a high cost to make the persuading message \( s \) credible. The residual message \( s' \) reveals the unfavourable state \( \omega_2 \), inducing both decision-makers to choose the expert’s adverse action.

The soft-news strategy persuades both decision-makers after seeing \( s \), and the believer after seeing \( s' \). Thus, the believer is always persuaded, whereas the sceptic is persuaded with a positive probability (but smaller than one) in either state. The soft-news strategy is a compromise between the incentives to persuade the sceptic and to retain the believer. It ensures that the believer chooses the favourite action \( \omega_1 \) with probability one. The expert mixes well-grounded information \( s \) with more gossip-style news \( s' \) able to persuade only the believer. The expert leverages the believer’s credulity without completely giving up on the persuasion of the sceptic. The value of \( k \) is the maximal extent of persuasion of the sceptic, which is possible without affecting the believer’s behaviour.
Lemma 1 (Blackwell’s criterion). The hard-news strategy is more informative than the soft-news strategy, using Blackwell’s (1953) ordering over distributions of posterior beliefs.

A strategy $\pi$ is more informative than $\pi'$ according to Blackwell (1953) if the distribution of posteriors induced by $\pi$ constitutes a mean preserving spread of the distribution of posteriors induced by $\pi'$. Following this definition, truth-telling is the most informative strategy, as posteriors are either 0 or 1. Instead, babbling leaves beliefs unchanged, and then it is the least informative strategy. The hard-news strategy is more informative than the soft-news strategy, for all decision-makers. Indeed, it induces more dispersion in the posteriors through the residual message, which reveals the unfavourable state for the expert.

Proposition 1 (Optimal persuasion). Let $I = \{1, 2\}$, with $\phi_1(\omega_1, \omega_2) > 1$ and $\phi_2(\omega_1, \omega_2) < 1$. A monopolistic expert has persuasion power, and the unique optimal strategy is either the hard-news strategy or the soft-news strategy. The hard-news strategy is optimal if 1.) the polarization is sufficiently small or 2.) the weight of the believer is sufficiently small or 3.) the expert’s favourable state is sufficiently likely from his perspective.

By Proposition 1, three parameters influence optimal persuasion:

1. Decision-makers’ polarization: The larger is $\phi_1(\omega_1, \omega_2)$, the higher is the incentive to use the soft-news strategy. Indeed, it is easier to leverage the believer’s credulity using message $s'$. The smaller is $\phi_2(\omega_1, \omega_2)$, the higher is the incentive to use the soft-news strategy. Indeed, it is more costly to persuade the sceptic using the message $s$. The difference $\phi_1(\omega_1, \omega_2) - \phi_2(\omega_1, \omega_2)$ is a proxy for polarization, as the underlying priors become more extreme as the difference grows. Therefore, the larger polarization, the higher the incentive to use the soft-news strategy;

2. Believer’s importance: The more relevant is the believer in the expert’s payoff (the higher $g_1$), the higher is the incentive to retain the believer (and the lower the incentive to persuade the sceptic). This implies a higher incentive to use the soft-news strategy;

3. Expert’s priors: The lower the expert’s prior of the favourable state $\mu_j^0(\omega_1)$, the higher the cost of revealing the unfavourable state to both decision-makers with the hard-news strategy. This implies a higher incentive to use the soft-news strategy.

Proposition 1 relates to Kamenica and Gentzkow (2011) in the following way. Kamenica and Gentzkow (2011) assume a common prior and, if
the decision-maker is a sceptic, the hard-news strategy is optimal. Heterogeneous priors give rise to a new type of optimal strategy - the soft-news strategy - pointing out the importance of decision-makers’ polarization for optimal persuasion. Moreover, Kamenica and Gentzkow (2011) argue that if a decision-maker chooses the expert’s adverse action, then it must be the case that the state is one where such action is optimal. However, this holds only if the expert uses the hard-news strategy. With the soft-news strategy, the sceptic may choose the expert’s adverse action even if it is not optimal for her. Finally, persuasion is always optimal when decision-makers have heterogeneous priors. The expert uses either the hard-news strategy or the soft-news strategy. Babbling is never optimal.

Example - I consider the example from the introduction and take the perspective of the government. Its objective is to persuade that the vaccine is safe. There are two equally sized groups of citizens, 1 and 2, and say group 1 are believers whereas group 2 are sceptics. I assume priors $\mu_1^{0}(\text{safe}) = 0.7$ and $\mu_2^{0}(\text{safe}) = 0.2$. Each citizen decides whether to vaccinate or not. The hard-news strategy is as follows:

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>Safe</th>
<th>Not Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi(s \mid \omega)$</td>
<td>$\text{safe}$</td>
<td>$\text{not safe}$</td>
</tr>
<tr>
<td>safe</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>not safe</td>
<td>0.75</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Message safe is designed to persuade sceptics. To be credible, the government needs to commit to revealing the unfavourable state with the residual message not safe.

The soft-news strategy consists of two persuading messages. Message safe (hard evidence such as clinical trials) is designed to persuade sceptics but has a low chance to be misleading. Message anecdotal safe (soft evidence such as vague comparisons benefits-risks) has a higher chance to be misleading but can persuade only believers.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>Safe</th>
<th>Not Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi(s \mid \omega)$</td>
<td>$\text{safe}$</td>
<td>$\text{anecdotal safe}$</td>
</tr>
<tr>
<td>safe</td>
<td>0.64</td>
<td>0.36</td>
</tr>
<tr>
<td>anecdotal safe</td>
<td>0.16</td>
<td>0.84</td>
</tr>
</tbody>
</table>

The advantage of the soft-news strategy is that the believers vaccinate themselves with probability one. With anecdotal safe the government leverages believers’ credulity. Meanwhile, it does not give up entirely from the
persuasion of sceptics (message safe). Whether the soft-news strategy is better than the hard-news strategy depends on the government’s prior. In the following table, I compare the expected payoff for the government from the two strategies for different priors.

<table>
<thead>
<tr>
<th>Government’s prior (safe)</th>
<th>Hard-news</th>
<th>Soft-news</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.625</td>
<td>0.7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.85</td>
<td>0.772</td>
</tr>
</tbody>
</table>

Therefore, the government uses the soft-news strategy if citizens’ priors are sufficiently polarized, and it is not very confident that the vaccine is safe. I rationalize this behaviour with the need to reach herd immunity to reopen economic activities.

4. Competing Experts

In this section, I examine how competition affects persuasion. I restrict attention to competition between two experts with different favourite actions. The next lemma establishes the effect of competition with unlimited attention, namely \( J_i = J \) for any \( i \in I \).

**Lemma 2** (Competition). Let \( J = \{j_1, j_2\} \) with \( a_{j_1} = \omega_1 \) and \( a_{j_2} = \omega_2 \). For any \( s_{j_1} \in S_{j_1} \) and any \( s_{j_2} \in S_{j_2} \) such that \( \pi_{j_1}(s_{j_1} | \omega_1), \pi_{j_2}(s_{j_2} | \omega_2) > 0 \), then \( \phi_{ij_1}(\omega_1, \omega_2, s_{j_2}) = \phi_{ij_2}(\omega_2, \omega_1, s_{j_1}) = 0 \) for any \( s = (s_{j_1}, s_{j_2}) \) and any \( i \in I \).

By Lemma 6, any best response by any expert is such that a persuasion constraint must hold with equality for at least one decision-maker. Such decision-maker is just indifferent between either favourite action. Thus, the rival has incentives to undercut the expert with respect to such a decision-maker. Therefore, there cannot be an equilibrium unless any expert refrains from persuading under each other’s favourable state. Thus, full revelation is the equilibrium when decision-makers have unlimited attention, and this result is coherent with the literature (Gentzkow and Kamenica 2017a,b; Ravindran and Cui 2020). In the following, I introduce limited attention and show that full revelation is not an equilibrium: Competition is actually harmful for decision-makers.

---

\(^9\) The entry of experts with the same preferences as the incumbent does not affect information provision. Indeed, the entrant cannot refine optimal persuasion of the incumbent. See Section B.3 in the Appendix.
Limited attention implies that each decision-maker \( i \in I \) can devote attention to just one expert. In other words, either \( J_i = \{j_1\} \) or \( J_i = \{j_2\} \). This makes the problem for each expert equivalent to the one he would face if he were a monopolist in his own market. Remarkably, each expert’s market is given by a subset of decision-makers. The distribution of priors each expert faces becomes endogenous, and it is the result of the optimal attention choices made by decision-makers.

In order to study the effect of competition with limited attention, I consider a simultaneous move game. The allocation of attention and the optimal persuasion strategy are chosen simultaneously, by each decision-maker and each expert respectively.

The objective function for each decision-maker is her subjective probability to choose the correct action (that is, her expected payoff). Suppose that a decision-maker \( i \in I \) devotes attention to the expert \( j \in J \). The resulting probability is:

\[
\lambda_i(\pi_j) := \sum_{s \in S_j} \sum_{\omega \in \Omega} \pi_j(s | \omega) \mu_i^0(\omega) \mathbb{1}\{\sigma(\mu_i(\pi_j, s)) = \omega\}
\]

**Lemma 3** (Decision-maker’s payoff). \( \lambda_i(\pi_j) \leq 1 \) and \( \lambda_i(\pi_j) = 1 \) if and only if \( \pi_j \) is truth-telling. If \( \pi_j \) is babbling, then \( \lambda_i(\pi_j) = \mu_i^0(\hat{\omega}) \), where \( \hat{\omega} = \arg\max_{\omega \in \Omega} \mu_i^0(\omega) \). It holds that \( \lambda_i(\pi_j) \in [\mu_i^0(\hat{\omega}), 1] \).

Intuitively, the subjective probability to take the correct action is maximized when an expert reveals the true state. Persuasion cannot decrease such a probability compared to the no information case. An expert can persuade a decision-maker to take an action which is ex ante suboptimal for her. However, this requires to reveal some information, and makes the decision-maker (weakly) better off. Note that \( \mu_i^0(\hat{\omega}) \) is the subjective probability to take the correct action without information. Therefore, \( \Delta_{ij} := \lambda_i(\pi_j) - \mu_i^0(\hat{\omega}) \geq 0 \) is the subjective information gain from persuasion.

**Definition 5** (Target). Consider any expert \( j \in J \). A target is a decision-maker whose persuasion constraint holds with equality given the expert \( j \)'s strategy. Let \( T_j \) be the set of targets for expert \( j \).

By Lemma 3 the set of targets is non-empty. The hard-news strategy targets a sceptic, whereas the soft-news strategy targets a sceptic and a believer. A decision-maker being targeted means that the strategy of the
expert is designed to persuade marginally such decision-maker, making her just indifferent between the two actions.

**Proposition 2** (Zero information gain for a target). For each expert \( j \in J \) and each \( i \in T_j \), it holds that \( \Delta_{ij} = 0 \).

Proposition 2 states that for a decision-maker being targeted by an expert is equivalent to receive zero information gain when devoting attention to this expert. Intuitively, an expert reveals only the information that is strictly necessary to persuade a targeted decision-maker. Being targeted is a sufficient (but not necessary) condition for zero information gain from persuasion.\(^{11}\)

Proposition 2 is shaping decision-makers’ incentives regarding the allocation of attention. The optimal allocation of attention for a decision-maker \( i \) is given by \( J_i(\pi_j^1, \pi_j^2) \), and \( J_i(\cdot) = j \) requires that \( j \in \arg \max_{j \in J} \Delta_{ij} \). In other words, each decision-maker devotes attention to an expert that grants her the highest information gain. The set of decision-makers who pay attention to the expert \( j \) (his market) is \( H_j := \{ i \in I \mid J_i(\cdot) = j \} \subseteq I \). Each expert \( j \) uses the optimal strategy \( \pi_j(H_j) \), which is a function of his market.

Any equilibrium is characterized by the following set: \( (\pi_j^1, \pi_j^2, J_1, \ldots, J_I) \).

Each strategy must be a best response for the corresponding expert. For each expert \( j \in J \), it must hold \( \pi_j = \pi_j(H_j) \). In other words, each expert designs his strategy given the distribution of priors implied by \( H_j \). At the same time, the allocation of attention must be consistent with decision-makers’ incentives. In particular, for any expert \( j \in J \) and any decision-maker \( i \in H_j \), it must hold that \( J_i(\pi_j(H_j), \pi_{-j}(H_{-j})) = j \).

Here, I assume \( I = \{1, 2\} \) with \( \phi_1(\omega_1, \omega_2) > 1 \) and \( \phi_2(\omega_1, \omega_2) < 1 \).\(^{12}\) Remarkably, decision-maker \( i = 1 \) (\( i = 2 \)) is a believer (sceptic) of \( \omega_1 \) and a sceptic (believer) of \( \omega_2 \). There are three pure strategy candidate equilibria:\(^{13}\)

1. **Monopoly.** One expert collects the whole attention, and his optimal strategy is described by Proposition 1. The other expert is not active, and then he can use any strategy;

2. **Echo chambers.** Each expert collects attention by his believer only. Therefore, the optimal strategy is babbling;

3. **Opposite-bias learning.** Each expert collects attention by his sceptic only. Therefore, the optimal strategy is the hard-news strategy.\(^{14}\)

\(^{11}\)Not-targeted decision-makers whose behaviour is not affected by beliefs updating have zero information gain as well.

\(^{12}\)In Section 5.3 I consider the case of more than two decision-makers.

\(^{13}\)Mixed strategy equilibria are analysed in Section 5.2.

\(^{14}\)The soft-news strategy solves the trade-off between persuading the sceptic and retaining the believer. It cannot be optimal if only the sceptic is devoting attention.
**Proposition 3 (Equilibrium).** Let $J = \{j_1, j_2\}$ and $I = \{1, 2\}$, where decision-maker $1$ ($2$) is a believer from the perspective of expert $j_1$ ($j_2$). Echo chambers is the unique symmetric pure strategy equilibrium. Decision-makers are better informed with a monopoly.

Within echo chambers, each expert’s market is given by his believer and babbling is the best response. Given babbling by both experts, decision-makers have no incentive to deviate, because each expert provides zero information gain. Therefore, echo chambers is an equilibrium. The equilibria with a monopolist require that the non-active expert (who is indifferent between any strategy) provides zero information gain. Otherwise, the decision-maker(s) targeted by the monopolist would find it beneficial to deviate. By Lemma 1, opposite-bias learning would be desirable as both experts would use the hard-news strategy. However, opposite-bias learning cannot be an equilibrium because it is not compatible with each decision-maker’s incentives: each targeted sceptic can get a strictly positive information gain becoming the believer of the other expert. Indeed, the believer is not targeted by the hard-news strategy, but her behaviour is affected by such strategy.

Proposition 3 implies that echo chambers arise endogenously with competition. Echo chambers are harmful because it is optimal for each expert to do babbling. Both decision-makers can strictly benefit from a monopoly. Indeed, in a monopoly the expert uses either the hard-news strategy or the soft-news strategy, which are both more informative than babbling according to the Blackwell’s (1953)’s criterion.

The main take-away is the following: Competition in information provision can harm the decision-makers. This happens if the latter have limited attention, and they can freely allocate their attention across experts. Each decision-maker attempts to get a positive information gain from persuasion by avoiding to be the target of an expert. However, this leads decision-makers to cluster into echo chambers. Echo chambers are harmful because each expert faces only his believers, and the best response is babbling. Instead, a monopolist faces decision-makers with heterogeneous priors, and solves the implied trade-off in a way that benefits the decision-makers.

Within echo chambers, citizens who are sceptical about vaccinations get confirmatory news. There is no chance that their worldview can change, which means that they would never be willing to vaccinate. If there exists a sufficiently large mass of sceptics, herd immunity is unachievable. If the government were the monopolist, sceptics could be persuaded to vaccinate. Moreover, decision-makers would get more accurate information, because the latter are designed to persuade the sceptics.
5. Extensions

5.1. Neutral Platform

The negative effect of competition is related to the endogenous allocation of attention. It is natural to ask if there exists an exogenous allocation of attention which does a better job than a monopoly. Now, I assume that there exists a third agent (a platform) which chooses this allocation in order to maximize the informativeness of news. Let $\hat{g}_{ij}$ be the mass of decision-makers of type $i \in I$ that the expert $j \in J$ is facing, induced by the allocation of attention chosen by the platform. Each expert $j$ solves (5) given $g_j = \{\hat{g}_{ij}\}_{i \in I}$.

Let $J = \{j_1, j_2\}$, $a_{j_1} = \omega_1$, $a_{j_2} = \omega_2$ and $I = \{1, 2\}$, where decision-maker 1 (2) is a believer of state $\omega_1$ ($\omega_2$). By Lemma 1, the most informative strategy, among those that are compatible with each expert’s incentives, is the hard-news strategy. By Proposition 1 (in particular equation (7) in the Appendix), each expert uses the hard-news strategy if there are not too many believers in his audience:

\[
\hat{g}_{1j_1} \leq \frac{\mu^0_{j_1}(\omega_1) + \mu^0_{j_1}(\omega_2)\phi_2(\omega_1, \omega_2)}{\mu^0_{j_1}(\omega_1) + \mu^0_{j_1}(\omega_2)\phi_1(\omega_1, \omega_2)} \quad \hat{g}_{2j_2} \leq \frac{\mu^0_{j_2}(\omega_2) + \mu^0_{j_2}(\omega_1)\phi_1(\omega_2, \omega_1)}{\mu^0_{j_2}(\omega_2) + \mu^0_{j_2}(\omega_1)\phi_2(\omega_2, \omega_1)}
\]

This constraint is missing in the endogenous allocation of attention, and this explains echo chambers. Indeed, given that both experts are using the hard-news strategy, all decision-makers have incentives to become believers. However, this makes the hard-news strategy suboptimal for each expert, and traps decision-makers into echo chambers.

**Proposition 4** (Platform). For each decision-maker, a monopoly with the hard-news strategy is more informative than opposite-bias learning, which in turn is more informative than a monopoly with the soft-news strategy.

The hard-news strategy is more informative for a believer than for a sceptic. Therefore, if there exists a monopolist willing to use the hard-news strategy, such outcome is better than opposite-bias learning for the believers (whereas the sceptics are indifferent). Note that the platform can do better than opposite-bias learning: some believers can be allocated to each expert without affecting his incentives to use the hard-news strategy. A platform would like to allocate decision-makers to like-minded experts ($\hat{g}_{1j_1}, \hat{g}_{2j_2}$ ↑). However, this is effective only if each expert is using the hard-news strategy, and this requires the presence of enough sceptics ($\hat{g}_{1j_1}, \hat{g}_{2j_2}$ ↓). There could be allocations of attention which outperform a monopoly (with the hard-news strategy) in terms of aggregate informativeness. However, some decision-makers would be less informed than in monopoly.
5.2. Mixed Strategy Equilibria

Proposition 3 shows that echo chambers are the unique pure strategy equilibrium where both experts are active, and therefore competition is harmful for decision-makers. Now, I investigate whether there exists a mixed strategy equilibrium which makes competition beneficial\footnote{I assume that only the decision-makers are mixing or, equivalently, that there exists a positive mass of decision-makers for each type $i \in I$. In the following, I use the latter interpretation. Given any allocation of attention, the optimal persuasion strategy is unique (Proposition 1). Therefore, there is no room for mixed strategies by the experts.} Optimal persuasion depends on the mass of each type of decision-maker devoting attention. A first class of mixed strategy equilibria is such that both types are indifferent between the two experts. This is true only if each expert uses the soft-news strategy. Indeed, both types are targeted independently of who they pay attention to, and hence each decision-maker experiences zero information gain in any case. Any allocation of attention which makes the soft-news strategy optimal for each expert constitutes an equilibrium.

A second class of mixed strategy equilibria is such that only one type is indifferent between the two experts, whereas the other one strictly prefers one expert. The expert who collects attention by both types uses the hard-news strategy. Therefore, his believers must all devote attention, whereas his sceptics are indifferent because each sceptic is either targeted or receives babbling from the other expert. Any allocation of attention which makes the hard-news strategy optimal for only one expert constitutes an equilibrium.

Finally, there is no mixed strategy equilibrium where both experts use the hard-news strategy. In that case, there is no indifference, as each decision-maker can get a positive information gain being the believer of an expert. This makes impossible for decision-makers to replicate endogenously the most informative outcome, as in Section 5.1.

The mixed strategy equilibria are in between a monopoly and echo chambers in terms of informativeness. Moreover, all the equilibria of this game are different in terms of persuasion: different allocations of attention lead to different optimal actions by decision-makers. An expert’s expected payoff is maximized (minimized) when he is a monopolist (non-active). Echo chambers, as well as the mixed strategy equilibria, provide a more balanced distribution of persuasion. Indeed, this is a zero-sum game: if an expert persuades, the rival cannot. Each expert enjoys to get attention by an additional decision-maker, because such decision-maker’s action would be affected by the expert’s optimal persuasion strategy. If a decision-maker does not devote attention to an expert, such decision-maker takes the expert’s favourite action only residually (when persuasion by the rival fails).
In the example from the introduction, the equilibrium that is implemented determines how many citizens can be persuaded to vaccinate. If the government is a monopolist all the citizens can be persuaded, whereas with echo chambers only the believers vaccinate. The mixed strategy equilibria involve echo chamber(s) of different sizes. Herd immunity can be achieved only if the size of the anti-vax echo chamber is sufficiently small.

5.3. Many Decision-makers

In this section, I show that my results continue to hold with any arbitrary set of decision-makers. I assume the existence of multiple sceptics and believers, each one endowed with different priors. Let $|I| = R > 2$ and $|I_2| = R_2 < R$. In the following, I order the decision-makers from the most sceptical to the least:

$$
\phi_1(\omega_1, \omega_2) < \cdots < \phi_{R_2}(\omega_1, \omega_2) < 1 < \cdots < \phi_R(\omega_1, \omega_2)
$$

**Proposition 5 (Optimal Persuasion).** A monopolistic expert has persuasion power and the unique optimal strategy is either a hard-news strategy or a soft-news strategy. A hard-news (soft-news) strategy is optimal if a sceptic (believer) has the largest value of being persuaded marginally.

Proposition 5 shows that optimal persuasion is robust to heterogeneity within believers and sceptics. I use the insights from Proposition 5 to extend the analysis to a continuous distribution of decision-makers’ priors.

**Proposition 6 (Optimal persuasion).** Let $F(x)$ be a distribution with support $[0, 1]$ and density $f(x) > 0 \ \forall x$. Let $\mu_i^0(\omega_1) \sim F(\cdot)$. Then, the expert uses a hard-news strategy targeting sceptics with prior $x \in [0, 0.5]$ if $x$ solves

$$
h(x) = \left[ (1 - x)^2 \frac{\mu_i^0(\omega_1)}{\mu_i^0(\omega_2)} + \frac{x}{1 - x} \right]^{-1}
$$

and condition (9) holds. Note that $h(x) \equiv \frac{f(x)}{1 - F(x)}$ is the hazard rate.

Condition (6) is a very tractable test for the quality of information given the distribution of priors (and the prior of the expert). Gitmez and Molavi (2020) find a similar characterization of the optimal strategy in a setting where the expert is trading-off between an extensive margin (how many decision-makers devote attention) and an intensive margin (how many decision-makers are persuaded). By contrast, in my setting devoting attention to one expert is costless, thus all decision-makers devote attention.

In the following figure, two examples of density functions for decision-makers’ priors with the corresponding optimal strategies.
If the majority of decision-makers has intermediate priors, then a hard-news strategy is optimal. By contrast, a soft-news strategy is optimal when extreme priors are dominant.

**Proposition 7 (Competition with limited attention).** In any equilibrium, at least one expert is babbling.

The key mechanism behind this result is the following: for any allocation of attention and corresponding optimal persuasion strategies, there exists at least one targeted decision-maker who can deviate and get a positive information gain, unless at least one expert is babbling.

Therefore, the results of Proposition 3 extend. Echo chambers are an equilibrium, and all decision-makers are better informed with a monopoly according to the Blackwell (1953)’s criterion. Indeed, by Proposition 5 the monopolist uses either a hard-news strategy or a soft-news strategy.

The existence of more than two decision-makers generates additional pure strategy equilibria, which I call partial echo chambers. In these equilibria, an ordered subset of believers (those with the most extreme priors) join the echo chamber of one expert. The other expert gets attention from the remaining decision-makers, including some sceptics. Given babbling, nobody outside the echo chamber wants to join it. Any believer within the echo chamber would become the most sceptical decision-maker of the other expert in case of a deviation, and she would not be persuaded. Therefore, this deviation would yield zero information gain, and this supports the equilibrium.

In the case of partial echo chambers, it is less obvious to determine whether decision-makers would be better informed with a monopoly. The following lemma is instrumental for any comparison.

**Lemma 4 (Blackwell’s criterion).** A strategy $\pi$ is more informative the more extreme are the priors of its target(s).

More extreme targets induce a more disperse distribution of posteriors. Indeed, the optimal persuasion strategy moves closer to the truth-telling
strategy. To fix ideas, I consider a simplified example. Let $I = \{1, 2, 3, 4\}$ with $R_2 = 2$, and consider partial echo chambers with $H_{j_1} = \{2, 3, 4\}$ and $H_{j_2} = \{1\}$. In words, the expert $j_2$ gets attention only from his believer 1, whereas the other decision-makers pay attention to the expert $j_1$. I compare this case with $j_1$ being a monopolist. Within partial echo chambers, $j_1$ might use a hard-news strategy which targets sceptic 2. If with a monopoly $j_1$ uses a hard-news strategy which targets sceptic 1, then by Lemma 4 all decision-makers are better off with a monopoly. However, there could exist cases where the transition has ambiguous effects. Assume that within partial echo chambers $j_1$ uses a soft-news strategy which targets sceptic 2 and believer 4. Instead, with a monopoly $j_1$ uses a soft-news strategy which targets sceptic 1 and believer 3. Sceptic 1 is obviously better off because in partial echo chambers she gets babbling. Instead by Lemma 4 the effect on the other decision-makers is ambiguous. 

5.4. Applications

Throughout the paper, I have considered the COVID-19 vaccination as an example to illustrate my results. Such an example has some caveats. Perhaps it is controversial to assume that the government has state-independent preferences. There is a trade-off between economic outcomes and the time needed to eradicate COVID-19, thus herd immunity is a goal. However, a government can also be concerned about safety. Moreover, many citizens are irrational and cannot be persuaded. Hence, my model applies to a subset of the population. Nevertheless, endogenous echo chambers can explain why many rational citizens are still sceptical about vaccinations, and can be a threat to reaching herd immunity.

In this section, I argue that the applicability of my results goes beyond the previous example. Limited attention is a well-established fact. Heterogeneous priors are also very likely to exist in all situations where the objective probability for a claim to be true is ambiguous. Whenever the true state of the world is disputed, there are likely competing interpretations of the current state of events. If this is true, then the last requirement to apply my insights, namely competition between experts, is fulfilled. The following list of examples is not exhaustive.

My model can apply to the design of information about political issues. A politician wants to persuade voters to support a particular point of view. The optimal design of information trade-off the desire of persuading sceptical voters and the goal of keeping loyalists. As a result, some information is...
provided. With competition and limited attention, some voters will belong to echo chamber(s) and get no useful information.

A recent example is Trump’s claim that the US Presidential election was fraudulent. The United States show increasing political polarization (Finkel et al., 2020). My model can explain why Republicans believe Biden won because of a ‘rigged’ election, even though Trump has failed to provide any evidence about that (Rutenberg et al., 2020). The effectiveness of social distancing during the COVID-19 pandemic is another example where my insights can apply. Democrats are more concerned about COVID-19 than Republicans. Consistently, Democrats comply with social distancing more than Republicans (Allcott et al., 2020). This happens even if Democrats and Republicans can access the same information about COVID-19, and it is in line with the existence of echo chambers.

Climate change is another relevant example. A vast majority of scientists claim that climate change is real. Many NGOs warn that immediate intervention is necessary to avoid a sharp increase in mass disasters, whereas corporations (especially coal and oil producers) try to defuse such warnings. Endogenous echo chambers can explain the existence of climate change deniers. Similarly, believers of a long list of debunked conspiracy theories can survive within echo chambers. The common root is widespread scepticism about Science (Achenbach, 2015).

My model can apply to the advertising of differentiated products. A firm wants to persuade consumers to buy a product with uncertain value. Some consumers believe the product has a high value, whereas others believe it has low value. Each consumer buys if and only if she believes the product has high value. The firm designs the advertisement to maximize sales and then optimally provides some information about the product’s value. With competition and limited attention, each consumer believes one product has a higher value than the other and may devote her attention only to the producer of such product. Echo chambers make it optimal for the firms to provide no information. My model can also rationalize asymmetric equilibria where one firm invest in informative advertising, whereas the other enjoys its echo chamber. If both firms design informative advertising, consumers rationally want to learn about their favoured products. But then providing informative advertising is not optimal for the firms.

The applicability of my model extends to everyday discussions. For example, fans that discuss the referee’s decisions. A priori, a fan believes that the right decision should be in favour of her team. Each team’s supporters cluster into echo chambers, where partisan news are just confirming priors. This can explain the persistent disagreement between fans of different teams.
6. Conclusion

Heterogeneous priors affect optimal persuasion. When an expert faces decision-makers with heterogeneous beliefs, the former solves a trade-off between persuading sceptics and retaining believers. Two strategies can be optimal for the expert: the hard-news strategy and the soft-news strategy. The latter is optimal in a polarized society, or if the expert is not sufficiently confident that his favourable state is the true one. The expert mixes well-grounded information with gossip-style information, where the latter allows to leverage believers’ credulity. The soft-news strategy may resemble the low-quality news that one experiences when searching on Google or social media.

Limited attention makes competition harmful. I study a setting with an endogenous allocation of attention. Each decision-maker devotes attention to one expert to maximize her information gain. At the same time, each expert chooses his optimal persuasion strategy. In this setting, echo chambers arise endogenously as the unique symmetric pure strategy equilibrium. Decision-makers are less informed than with a monopoly. My findings therefore provide a sobering insight into the effects of media pluralism: Under media users’ limited attention and heterogeneous priors, media pluralism leads to worse-informed media users.

Jann and Schottmüller (2019) and Martinez and Tenev (2020) argue that echo chambers can be helpful, either to enhance communication in a network or to separate high-quality and low-quality news. Instead, echo chambers have a negative interpretation in my paper. The reason is the endogenous supply of information by biased experts. Even if echo chambers make decision-makers more willing to communicate, as in Jann and Schottmüller (2019), the information provided is useless. Echo chambers arise because each decision-maker devotes attention to the expert with the highest perceived quality (that is, the information gain), as in Martinez and Tenev (2020). Each decision-maker ends up paying attention to a like-minded expert, who then optimally provides no information.

Information disintermediation is not necessarily beneficial. I show that even unbiased decision-makers end up devoting attention to like-minded experts. Therefore, my paper supplies a novel rationalization for confirmation bias. Goette et al. (2020) provide experimental evidence which confirms that preferences are not the unique driver of confirmation bias.

A policymaker should support competition in information provision but only to the extent that the attention budget is not binding. When attention

\[ \text{Goette et al. (2020)} \] provide experimental evidence which confirms that preferences are not the unique driver of confirmation bias.

A policymaker should support competition in information provision but only to the extent that the attention budget is not binding. When attention
is limited, increasing the diversity of news sources leads decision-makers to cluster into echo chambers. As a consequence, the incentives for experts to provide information vanish. Vaccinations are a leading example of the negative effects of an excessive supply of news. Sceptics devote their attention to confirmatory news, and then it is impossible to persuade them to vaccinate.

My paper leaves an open question that requires further research. How can we mitigate confirmation bias? One approach is to enhance attention, but it is unclear how to do this. An alternative is to manipulate the allocation of attention to increase information provision. In Section 5.1 I have shown how a platform that wants to maximize the informativeness of news should allocate attention. Such a platform would design each expert’s audience to give him incentives to use the hard-news strategy. For instance, opposite-bias learning is an appropriate allocation of attention (but perhaps a platform can do better). Platforms such as news aggregators seem to have the ability to induce a particular allocation of attention. However, there is no guarantee that such platforms behave as a social planner would do.
References


A. Appendix A

Lemma 5 (Redundancy). Consider any expert $j$ and any strategy $\pi_j$. Let $s \in S_j$. Split $s$ into two new messages $s'$ and $s''$, that is, consider the message space $S'_j = (S_j \setminus \{s\}) \cup \{s', s''\}$ and any strategy $\pi'_j$ such that $\pi'_j(s' | \omega) + \pi'_j(s'' | \omega) = \pi_j(s | \omega)$ and $\pi'_j(s'' | \omega) = \pi_j(s'' | \omega)$ for any $s'' \in S_j \cap S'_j$ and any $\omega \in \Omega$. Consider strategies $\pi_{-j}$ such that any decision-maker $i$ takes the same action before and after the split, that is, $\sigma(\mu_i((\pi_j, \pi_{-j}), (s, s_{-j}))) = \sigma(\mu_i((\pi'_j, \pi_{-j}), (s', s_{-j}))) = \sigma(\mu_i((\pi'_j, \pi_{-j}), (s'', s_{-j})))$ for every $s_{-j}$. Then, $(\pi_j, \pi_{-j})$ and $(\pi'_j, \pi_{-j})$ are payoff-equivalent.

Proof of Lemma 5

Proof. Pick any $i \in I$. If $j \notin J_i$, then the strategy of expert $j$ does not affect decision-maker $i$. Let $j \in J_i$ and consider any $s_{-j} \in S_{-j}$. By assumption, $v_{ij}(\pi_{J_i}, s, s_{-j}) = v_{ij}(\pi'_j, s', s_{-j}) = v_{ij}(\pi'_j, s'', s_{-j})$. By (5), it holds that

$$\sum_{\omega \in \Omega} \pi_j(s, s_{-j} | \omega) \mu_j^0(\omega) v_{ij}(\pi_j, s, s_{-j}) = \sum_{s \in \{s', s''\}} \sum_{\omega \in \Omega} \pi'_j(s, s_{-j} | \omega) \mu_j^0(\omega) v_{ij}(\pi'_j, s, s_{-j})$$

Note that $\sum_{\omega \in \Omega} \pi_j(s, s_{-j} | \omega) \mu_j^0(\omega) = \sum_{s \in \{s', s''\}} \sum_{\omega \in \Omega} \pi'_j(s, s_{-j} | \omega) \mu_j^0(\omega)$ follows because the messages are independent conditionally on $\omega$. \qed

Lemma 6 (Persuasion constraint). Consider any expert $j$. Given $\pi_{-j}$, in any best-response $\pi_j$ at least one persuasion constraint holds with equality. Let $a_j = \omega_1$ without loss of generality. Then, there exist a decision-maker $i$ with $j \in J_i$, $s_j \in S_j$ and $s_{-j} \in S_{-j}$ such that $\pi_j(s_j | \omega_2) = \phi_{ij}(\omega_1, \omega_2, s_{-j}) \pi_j(s_j | \omega_1)$.

Proof of Lemma 6

Proof. I assume there exists $i \in I$ with $j \in J_i$ and $s_{-j} \in S_{-j}$ such that $\phi_{ij}(\omega_1, \omega_2, s_{-j}) < 1$. Otherwise, persuasion is not necessary and babbling is the only optimal strategy. I assume by contradiction that $\exists s_j \in S_j$ such that $\pi_j(s_j | \omega_2) = \phi_{ij}(\omega_1, \omega_2, s_{-j}) \pi_j(s_j | \omega_1)$ for some $i \in I$ and $s_{-j} \in S_{-j}$. Let $\{\phi_{ij}(\omega_1, \omega_2, s_{-j})\}$ be the ordered (in ascending order) set of constraints for any couple $(i, s_{-j})$ the expert faces. If the $n$-th constraint holds for a message $s_j \in S_j$, then the $m$-th constraint holds too, for any $m > n$. Therefore, if $n$-th constraint holds there is more persuasion than if only the $m$-th constraint were holding, ceteris paribus. Thus, if the $n$-th constraint is slack, it is beneficial for the expert to increase the probability of the corresponding message, at the expense of the probability of a message which satisfy only the $m$-th constraint. \qed
Proof of Lemma 1

Proof. Let $\pi_h$ be the hard-news strategy whereas $\pi_s$ is the soft-news strategy. First of all, the distributions of posteriors induced by these two strategies have the same mean, which coincides with $\mu_i^0(\omega_1)$ for any $i \in I$, following Bayesian plausibility. Given the definitions of the two strategies, it follows:

$$
\mu_1^h(\omega_1 | s) = \frac{\phi_1(\omega_1, \omega_2)}{\phi_1(\omega_1, \omega_2) + \phi_2(\omega_1, \omega_2)} > 0.5 \quad \mu_2^h(\omega_1 | s) = 0.5, \quad \pi = \pi_h, \pi_s
$$

$$
\mu_1^s(\omega_1 | s') = \mu_2^s(\omega_1 | s') = 0 \quad \mu_1^s(\omega_1 | s') = 0.5 \quad \mu_2^s(\omega_1 | s') = \frac{\phi_2(\omega_1, \omega_2)}{\phi_1(\omega_1, \omega_2) + \phi_2(\omega_1, \omega_2)} \in (0, 0.5)
$$

Therefore, $\pi_h$ is characterized by more dispersion than $\pi_s$. $\square$

Proof of Proposition 1

Proof. The payoff for Babbling is $V_u := g_1$, whereas the payoff for the Truth-telling strategy is $V_t := \mu_j^0(\omega_1)$. The Hard-news strategy is as follows:

$$
\begin{array}{c|c|c|c}
\omega & \omega_1 & \omega_2 \\
\hline
s & s & s' \\
\pi_j(s | \omega) & 1 & \phi_2(\omega_1, \omega_2)
\end{array}
$$

$$
\implies V_h := \mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_2(\omega_1, \omega_2)
$$

The Soft-news strategy is as follows:

$$
\begin{array}{c|c|c|c|c|c|c|c|c}
\omega & \omega_1 & \omega_2 \\
\hline
s & s & s' & s' \\
\pi_j(s | \omega) & k & 1-k & \phi_2(\omega_1, \omega_2)k & \phi_1(\omega_1, \omega_2)(1-k)
\end{array}
$$

$$
\implies V_s := kV_h + (1-k)\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_1(\omega_1, \omega_2)\right] g_1
$$

where

$$
1 - \phi_2(\omega_1, \omega_2)k = \phi_1(\omega_1, \omega_2)(1-k) \iff k = \frac{\phi_1(\omega_1, \omega_2) - 1}{\phi_1(\omega_1, \omega_2) - \phi_2(\omega_1, \omega_2)}
$$

Any alternative strategy with $\pi(s | \omega_1) < k$ is suboptimal, because the soft-news strategy increases the probability to persuade the sceptic without affecting the behaviour of the believer.
Note that $V_h \geq V_i$. Hence, the expert does not use the truth-telling strategy, that is expert \( j \) has some persuasion power. Moreover, $V_i > V_h$ for any $g_1 \in (0, 1)$. The hard-news strategy is optimal if:

$$V_h \geq V_i \iff \mu_j^0(\omega_1) + \mu_j^0(\omega_2) \phi_2(\omega_1, \omega_2) \geq (\mu_j^0(\omega_1) + \mu_j^0(\omega_2) \phi_1(\omega_1, \omega_2)) g_1$$

$$\iff \mu_j^0(\omega_1)(1 - g_1) \geq \mu_j^0(\omega_2) (\phi_1(\omega_1, \omega_2) g_1 - \phi_2(\omega_1, \omega_2)) \quad (7)$$

Note that the RHS of (7) is increasing in $\phi_1(\omega_1, \omega_2)$ and decreasing in $\phi_2(\omega_1, \omega_2)$. The difference of these two values is a proxy for decision-makers’ polarization in terms of priors. The RHS (LHS) of (7) is increasing (decreasing) in $g_1$, the importance of the believer. Finally, the RHS (LHS) of (7) is decreasing (increasing) in $\mu_j^0(\omega_1)$, the expert’s prior of his favourable state.

**Proof of Lemma 2**

Proof. Note that if $\pi_{j_1}(s_{j_1} | \omega_1), \pi_{j_2}(s_{j_2} | \omega_2) = 0$ then $s_{j_1}, s_{j_2}$ cannot persuade. I assume by contradiction that $\pi_{j_2}^0(s_{j_2} | \omega_2) > 0$ and $\pi_{j_2}^0(s_{j_2} | \omega_1) > 0$ for some $s_{j_2}$. Thus, it holds $\phi_{ij_1}(\omega_1, \omega_2, s_{j_2}) > 0$ for any $i \in I$, and by Lemma 6 $\pi_{j_1}(s_{j_1} | \omega_2) = \phi_{ij_1}(\omega_1, \omega_2, s_{j_2}) \pi_{j_1}(s_{j_1} | \omega_1)$ for some $i' \in I$. It follows that $\phi_{ij_2}(\omega_1, \omega_2, s_{j_2}) > 0$ for any $i \in I$, and $\phi_{ij_2}(\omega_1, \omega_2, s_{j_1}) = \pi_{j_2}^0(s_{j_1} | \omega_1)$. I assume without loss of generality that decision-makers break the ties in favour of expert $j_1$. In order to persuade $i'$, $s_{j_2}$ has to satisfy the following persuasion constraint:

$$\pi_{j_2}(s_{j_2} | \omega_1) < \phi_{ij_2}(\omega_1, \omega_2, s_{j_1}) \pi_{j_2}(s_{j_2} | \omega_2)$$

which requires simply to set $\pi_{j_2}(s_{j_2} | \omega_1) = \pi_{j_2}^0(s_{j_2} | \omega_1) - \epsilon$ with $\epsilon > 0$ and small. This is a beneficial deviation because $j_2$ persuades an additional decision-maker ($i'$) with a negligible reduction in the probability of persuasion. By 4 it follows that the persuasion constraint for expert $j_1$ becomes:

$$\pi_{j_1}(s_{j_1} | \omega_2) \leq \frac{\pi_{j_2}^0(\omega_2)}{\mu_{ij}^0(\omega_2)} \pi_{j_2}(s_{j_2} | \omega_2) \pi_{j_1}(s_{j_1} | \omega_1) < \phi_{ij_1}(\omega_1, \omega_2, s_{j_2}) \pi_{j_1}(s_{j_1} | \omega_1)$$

that is a contradiction, which follows from the fact that this is a zero-sum game for the experts.

**Proof of Lemma 3**

Proof. Assume that $\pi_j$ is truth-telling. Hence, $\pi_j(s | \omega_1) = \pi_j(s' | \omega_2) = 1$ and $\pi_j(s | \omega_2) = \pi_j(s' | \omega_1) = 0$. This implies that $\lambda_i(\pi_j) = 1$. Assume that $\pi_j$ is not truth-telling, and without loss of generality $\pi_j(s | \omega_2) > 0$. Note that either $\sigma(\mu_i(\pi_j, s)) = \omega_1$ or $\sigma(\mu_i(\pi_j, s)) = \omega_2$. It follows that $\lambda_i(\pi_j) < 1$. 

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If \( \pi_j \) is babbling then, for any \( s \in S_j \), \( \sigma(\mu_i(\pi_j, s)) = \hat{\omega} \). It follows that \( \lambda_i(\pi_j) = \mu_i^0(\hat{\omega}) \). Assume that there exists \( s \in S_j \) such that \( \pi_j(s|\omega) \neq \pi_j(s|\hat{\omega}) \). Note that \( \sigma(\mu_i(\pi_j, s)) = \omega \) if \( \pi_j(s|\omega) \geq \frac{\mu_i^0(\omega)}{\mu_i^0(\hat{\omega})} \pi_j(s|\hat{\omega}) \), and this implies that \( \lambda_i(\pi_j) \geq \mu_i^0(\hat{\omega}) \).

**Proof of Proposition 2**

*Proof.* Assume without loss of generality \( a_j = \omega_1 \). If \( \pi_j \) is a hard-news strategy then \( T_j = \{i\} \) and \( \phi_i(\omega_1, \omega_2) < 1 \). This implies \( \lambda_i(\pi_j) = \mu_i^0(\omega_1) + \mu_i^0(\omega_2)[1 - \phi_i(\omega_1, \omega_2)] = \mu_i^0(\omega_2) \). If \( \pi_j \) is the soft-news strategy then \( T_j = \{i, i'\} \) and without loss of generality \( \phi_{i'}(\omega_1, \omega_2) > 1 > \phi_i(\omega_1, \omega_2) \). Therefore, \( \lambda_i(\pi_j) = \mu_i^0(\omega_1)k + \mu_i^0(\omega_2)[1 - \phi_i(\omega_1, \omega_2)k] = \mu_i^0(\omega_2) \) and \( \lambda_{i'}(\pi_j) = \mu_i^0(\omega_1) \). \( \square \)

**Proof of Proposition 4**

*Proof.* The following table summarizes the posteriors belief of the decision-makers that the state is \( \omega_1 \), following the different (incentive-compatible) strategies that the experts can use:

<table>
<thead>
<tr>
<th>( i )</th>
<th>Hard-news ( j_1 )</th>
<th>Hard-news ( j_2 )</th>
<th>Soft-news ( j_1 )</th>
<th>Soft-news ( j_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>( s' )</td>
<td>( s )</td>
<td>( s' )</td>
<td>( s )</td>
</tr>
<tr>
<td>1</td>
<td>( \epsilon (0.5, 1) )</td>
<td>0</td>
<td>( 0.5 )</td>
<td>( \epsilon (0.5, 1) )</td>
</tr>
<tr>
<td>2</td>
<td>( 0.5 )</td>
<td>0</td>
<td>( \epsilon (0, 0.5) )</td>
<td>1</td>
</tr>
</tbody>
</table>

With Opposite-bias learning, each decision-maker is a sceptic, and each expert uses the hard-news strategy (the payoffs are in bold in the table). Comparing it with a monopolist using the hard-news strategy, it is immediate to see that the sceptic is indifferent whereas the believer is better informed with a monopoly, according to the Blackwell (1953)’s criterion. Instead, the comparison with a monopolist using the soft-news strategy shows that each decision-maker is worse off with a monopoly. \( \square \)

**Proof of Proposition 5**

*Proof.* For any decision-maker \( r \in I \), I define the value of being persuaded marginally as

\[
E_r := \left[ \mu_{i'}^0(\omega_1) + \mu_{i'}^0(\omega_2)\phi_{i'}(\omega_1, \omega_2) \right] \sum_{i \in r} g_i \tag{8}
\]

For any \( r, r' \in I \), it is possible to define the following strategies:

**Definition 6** (Hard-news strategy). A hard-news strategy, with target \( T = \{r\} \) such that \( r \leq R_2 \), consists of a persuading message \( s \) and a residual...
message $s'$ such that

$$\pi_h(s | \omega_1) = 1 \quad \pi_h(s' | \omega_1) = 0$$

$$\pi_h(s | \omega_2) = \phi_r(\omega_1, \omega_2) \quad \pi_h(s' | \omega_2) = 1 - \phi_r(\omega_1, \omega_2)$$

with posterior beliefs

$$\mu_i^{\pi_h}(\omega_1 | s) = \frac{\phi_i(\omega_1, \omega_2)}{\phi_i(\omega_1, \omega_2) + \phi_r(\omega_1, \omega_2)}, \quad \mu_i^{\pi_h}(\omega_1 | s') = 0 \quad \forall i \in I$$

**Definition 7** (Soft-news strategy). A soft-news strategy, with targets $T = \{r, r'\}$ such that $r \leq R_2$ and $r' > R_2$, consists of two persuading messages $s, s'$ such that

$$\pi_s(s | \omega_1) = k \quad \pi_s(s' | \omega_1) = 1 - k$$

$$\pi_s(s | \omega_2) = \phi_r(\omega_1, \omega_2)k \quad \pi_s(s' | \omega_2) = \phi_r(\omega_1, \omega_2)(1 - k)$$

where

$$k := \frac{\phi_r(\omega_1, \omega_2) - 1}{\phi_r(\omega_1, \omega_2) - \phi_r(\omega_1, \omega_2)}$$

with posterior beliefs

$$\mu_i^{\pi_s}(\omega_1 | s) = \frac{\phi_i(\omega_1, \omega_2)}{\phi_i(\omega_1, \omega_2) + \phi_r(\omega_1, \omega_2)}, \quad \mu_i^{\pi_s}(\omega_1 | s') = \frac{\phi_i(\omega_1, \omega_2)}{\phi_i(\omega_1, \omega_2) + \phi_r(\omega_1, \omega_2)}$$

The payoff of a hard-news strategy is

$$V_r := E_r$$

whereas the payoff of a soft-news strategy is

$$V_{(r, r')} := kE_r + (1 - k)E_{r'}$$

The payoff from the truth-telling strategy is $V_i = \mu_j^0(\omega_1)$ and $V_1 > V_i$. Hence, a monopolistic expert has persuasion power. The payoff from babbling is $V_a = G_1 := \sum_{r=0}^{R_2+1} g_j$. Note that $V_{(r, R_2+1)} > V_a$. Therefore, babbling is not optimal. I assume that there exist a unique $r^* = \arg\max_r E_r$. It follows that a monopolistic expert is optimally using either a hard-news strategy or a soft-news strategy. This assumption rules out, for instance, any linear combination of hard-news strategies targeting different sceptics. If $r^* \leq R_2$, a hard-news strategy with $T = \{r^*\}$ is optimal. Clearly $V_{r^*} > V_r$ for any $r \leq R_2$ and $r \neq r^*$. Moreover $V_{r^*} > V_{(r, r^*)}$ as $E_{r^*} \geq E_r$ and $E_{r^*} > E_{r'}$ for any $r \leq R_2$ and any $r' > R_2$. If $r^* > R_2$, clearly $V_{(r, r^*)} > V_r$ for any $r \leq R_2$. Therefore, a soft-news strategy is optimal. However, $r^*$ is not necessarily the target. $\square$
Proof of Proposition [6]

Proof. The value of being persuaded marginally - a generalization of expression (8) - is:

\[ E_x := \left[ \mu^0_i(\omega_1) + \mu^0_j(\omega_2) \frac{x}{1-x} \right] [1 - F(x)] \]

As suggested by Proposition [3], the expert uses a hard-news strategy or a soft-news strategy depending on whether the solution to \( \max_x E_x \) belongs to \([0, 0.5]\) or to \([0.5, 1]\), respectively. The F.O.C. is:

\[ \frac{\mu^0_i(\omega_2)}{(1-x)^2} [1 - F(x)] - f(x) \left[ \mu^0_j(\omega_1) + \mu^0_j(\omega_2) \frac{x}{1-x} \right] = 0 \]

and implies condition (6), whereas the S.O.C. is:

\[ \frac{2\mu^0_i(\omega_2)}{(1-x)^2} [1 - F(x)] - \frac{2\mu^0_j(\omega_2)}{(1-x)^2} f(x) - f'(x) \left[ \mu^0_j(\omega_1) + \mu^0_j(\omega_2) \frac{x}{1-x} \right] < 0 \]

which implies

\[ h(x) > \frac{2}{(1-x)^3} + \frac{\partial \ln f(x)}{\partial x} \left[ \frac{\mu^0_i(\omega_1)}{\mu^0_j(\omega_2)} + \frac{x}{1-x} \right] \]

Clearly, if the F.O.C. is always negative/positive (or the S.O.C. is violated) there exist a corner solution, namely the most valuable decision-maker is \( x = 0 \) or \( x = 1 \). Following Proposition [5], \( x = 0 \) implies the truth-telling strategy, which is a special case of a hard-news strategy in this setting. Instead, \( x = 1 \) does not imply necessarily that such decision-maker is targeted. The actual targets of the soft-news strategy depends on the shape of \( F(\cdot) \). \( \square \)

Proof of Proposition [7]

Proof. If at least one expert gathers attention exclusively from believers, then his best response is babbling. This supports the existence of an equilibrium in some cases. More details in the main text. Here, I focus on showing that this is a necessary condition. I assume that both experts gathers attention from some sceptics and some believers. By Proposition [5], each expert \( j \) uses either a hard-news strategy with target \( r_j \) or a soft-news strategy with targets \( \{r_j, r'_j\} \). Consider a hard-news strategy. It follows:

\[ \lambda_i(\pi_j) = \begin{cases} 
\mu^0_i(\omega_2) & \text{if } i \leq r_j \\
\frac{\mu^0_i(\omega_2)}{\mu^0_j(\omega_2)} \left[ \mu^0_j(\omega_2) - \mu^0_j(\omega_1) \right] > \mu^0_i(\omega_2) & \text{if } i \in (r_j, R_2] \\
\frac{\mu^0_i(\omega_2)}{\mu^0_j(\omega_2)} \left[ \mu^0_j(\omega_2) - \mu^0_j(\omega_1) \right] > \mu^0_i(\omega_1) & \text{if } i > R_2 
\end{cases} \]
Therefore, $\Delta_{ij} > 0 \iff i > r_j$.

Consider a soft-news strategy. It follows:

$$\lambda_i(\pi_j) = \begin{cases} 
\mu_i^0(\omega_2) & \text{if } i \leq r_j \\
\mu_i^0(\omega_1)k + \frac{\mu_i^0(\omega_2)}{\mu_j^0(\omega_2)}[\mu_j^0(\omega_2) - \mu_j^0(\omega_1)k] > \mu_i^0(\omega_2) & \text{if } i \in (r_j, R_2] \\
\mu_i^0(\omega_1) + \mu_j^0(\omega_2)\mu_j^0(\omega_1)(1-k) > \mu_i^0(\omega_1) & \text{if } i \in (R_2, r'_j) \\
\mu_i^0(\omega_1) & \text{if } i \geq r'_j 
\end{cases}$$

Therefore, $\Delta_{ij} > 0 \iff i \in (r_j, r'_j)$.

There are three cases to analyse:

1. Both experts use hard-news strategies. It follows that for each expert one sceptic is targeted, and she gets zero information gain. Such sceptic can deviate, become a believer of the other expert, and get a positive information gain.

2. One expert uses a soft-news strategy whereas the other uses a hard-news strategy. The sceptic targeted by the soft-news strategy can deviate, become a believer of the other expert, and get a positive information gain.

3. Both experts use soft-news strategies. Let $T_{j_1} = \{r_{j_1}, r'_{j_1}\}$ and $T_{j_2} = \{r_{j_2}, r'_{j_2}\}$ be the set of targets for the experts $j_1$ and $j_2$ respectively. I assume without loss of generality that $r_{j_1} < r'_{j_2} \leq R_2 < r_{j_2} < r'_{j_1}$. By Proposition 2 each target experiences zero information gain. Those targets who have intermediate priors (in this case, $r'_{j_2}$ and $r_{j_2}$) have incentive to deviate, in order to get a positive information gain.

\[\square\]

**Proof of Lemma 4**

*Proof. Let us consider two hard-news strategies $\pi_r$ and $\pi_{r'}$, with targets $T = \{r\}$ and $T = \{r'\}$ respectively, such that $r < r'$. Then, $\pi_r$ is more informative than $\pi_{r'}$ according to the Blackwell (1953)’s criterion. Indeed, given the definitions of the two strategies, it follows that for any $i \in I$:

$$\phi_r(\omega_1, \omega_2) < \phi_{r'}(\omega_1, \omega_2) \implies \mu_i^{\pi_r}(\omega_1 | s) > \mu_i^{\pi_{r'}}(\omega_1 | s)$$

$$\mu_i^{\pi_r}(\omega_1 | s') = \mu_i^{\pi_{r'}}(\omega_1 | s') = 0$$

Now, let us consider two soft-news strategies $\pi_{r''}$ and $\pi_{r'''}$, with targets $T = \{r, r''\}$ and $T = \{r, r''\}$ respectively, such that $r' > r''$. Then, $\pi_{r''}$ is more
informative than $\pi_{r''}$ according to the Blackwell (1953)'s criterion. Indeed, given the definitions of the two strategies, it follows that for any $i \in I$:

$$\mu_{r'}(\omega_1 | s) = \mu_{r''}(\omega_1 | s) > \mu_{r''}(\omega_1 | s')$$

$$\phi_{r'}(\omega_1, \omega_2) > \phi_{r''}(\omega_1, \omega_2) \implies \mu_{r'}(\omega_1 | s') < \mu_{r''}(\omega_1 | s')$$
B. Appendix B

B.1. Costly Attention

The results in my paper are derived under the assumption that each decision-maker can devote attention to just one expert. Now, I endogenize this decision by allowing each decision-maker to devote attention to a second expert at a cost $c \geq 0$.

**Proposition 8.** Truth-telling is an equilibrium if and only if $c = 0$.

Assume that $\pi_{j_1}$ and $\pi_{j_2}$ are truth-telling strategies. It follows that $\lambda_i(\pi_{j_1}) = \lambda_i(\pi_{j_2}) = \lambda_i(\pi_J) = 1$ for any $i \in I$. Therefore, it is sufficient to devote attention to one expert in order to maximize the subjective probability to take the correct action. If $c = 0$, decision-makers can pay attention to both experts without any cost. This is equivalent to unlimited attention. By Lemma 2 truth-telling is indeed the equilibrium in such a setting. If $c > 0$, each decision-maker strictly prefers to devote attention to just one expert, as she gains no additional information from the second one. However, it is not optimal for the experts to reveal the true state when decision-makers pay attention to only one expert.

The equilibria of the game are robust for any $c \geq 0$. Given any equilibrium, it follows by Proposition 7 that there is no incentive to devote attention to a second expert. Multi-homing is not optimal because at least one expert does babbling. For instance, consider partial echo chambers with $j_2$ babbling. For any $i \in H_{j_1}$, it holds $\lambda_i(\pi_{j_1}) = \lambda_i(\pi_J)$ because $\pi_{j_2}$ does not affect posteriors, hence optimal actions. For any $i \in H_{j_2}$ it must be the case that both experts are providing zero information gains, and $\lambda_i(\pi_{j_1}) = \lambda_i(\pi_{j_2}) = \lambda_i(\pi_J) = \mu_i^0(\hat{\omega})$. Therefore, decision-makers are not willing to pay $c \geq 0$ to devote attention to a second expert.

B.2. Alternative Timing

In the main text, I assume that optimal persuasion and the allocation of attention are simultaneous. Now, I examine the possibility that the two are sequential.

If the allocation of attention is chosen before persuasion takes place, my results extend. Remarkably, a monopoly is a much more credible equilibrium in this case. The allocation of attention cannot react to optimal persuasion by a monopolist. Therefore, it does not matter what is the strategy of the non-active expert in the second stage of the game.
If the allocation of attention is chosen after persuasion takes place, babbling by both experts (with any allocation of attention) is not an equilibrium. Suppose, by contradiction, the opposite. Believers take each expert’s favourite action, but any expert can deviate and persuade also his sceptic with positive probability (for instance, with the soft-news strategy). In order to do so, it is sufficient to provide a strictly positive information gain, which requires to avoid targeting the sceptic.

At the same time, truth-telling is an equilibrium. If any expert deviates, he does not collect attention. Therefore, he is not able to persuade, and indifference follows. This result is in line with Knoepfel (2020). Experts are implicitly attention-seekers: persuasion is effective only if an expert gets attention in the second stage. Optimal persuasion involves targeting of some decision-makers. However, by Proposition 2 a targeted decision-maker gets zero information gain from persuasion. Therefore, she is unlikely to devote attention in the second stage of the game.

The latter setting is in line with the literature on media bias, where consumers buy news knowing the media’s reputation or slant (Gentzkow et al., 2015). In turn, the latter is influenced by the incentive to steal consumers from the rival, and this is likely to generate beneficial competition. My approach is different because I assume that persuasion is rather flexible compared to the attention habits. Experts behave strategically taking as given the allocation of attention, and this is a source of persuasion power.\footnote{There exist empirical evidence that biased experts, for example politicians, respond strategically to attention habits. See for instance Eisensee and Strömberg (2007).}

### B.3. Competition with Homogenous Experts

With unlimited attention, having two experts with the same preferences does not affect information provision compared to a monopoly.

**Proposition 9** (Homogeneous experts). Consider $J = \{j_1, j_2\}$ and assume $a_{j_1} = a_{j_2}$ and $\mu_{j_1}(\omega_1) = \mu_{j_2}(\omega_1)$ for any $\omega \in \Omega$. In the equilibrium one expert (say $j_1$) behaves as a monopolist whereas the other one (say $j_2$) does babbling.

Given babbling by $j_2$, $j_1$ uses the optimal strategy as monopolist (Proposition 1). The two experts have the same preferences and the same priors. Therefore, the strategy of $j_1$ is optimal also for $j_2$. There is no incentive to change the posterior beliefs by providing further information. Hence, babbling is optimal for $j_2$.

The entry of (potentially many) experts with the same preferences and priors as the incumbent is not affecting information provision. The intu-
ition is that the entrant cannot refine the optimal persuasion strategy of the incumbent.\footnote{Experts with heterogeneous priors can have different optimal strategies (in monopoly). However, differently from Lemma \ref{lem:no undercut}, there is no incentive to undercut the rival because the favourite actions coincide.}

With limited attention, two experts using the same strategy can be active. Indeed, each decision-maker is indifferent about her allocation of attention (and can randomize), as each expert provides her the same information gain.\footnote{If the experts use different strategies, then decision-makers have incentive to devote attention to the most informative one.} This allows to extend the prediction of my model beyond a duopoly. The existence of additional experts has the effect of splitting attention, but it does not affect the equilibria of the game qualitatively.

With costly attention, a decision-maker could rationally pay attention to multiple experts providing her a positive information gain. However, single-homing triggers a strategic response by the experts (Proposition \ref{thm:single-homing}). In this setting, the unique equilibrium is a monopoly.

\textbf{B.4. Micro-targeting}

In the paper, persuasion is public. By contrast here, I assume that decision makers are micro-targeted: an expert uses a specific strategy for each decision-maker. Let $\pi_j^i$ be the strategy of expert $j \in J$ which targets decision-maker $i \in I$. In a monopoly, $\pi_j^i$ is babbling if the decision-maker $i$ is a believer, whereas it is the hard-news strategy if the decision-maker $i$ is a sceptic. This follows from Kamenica and Gentzkow (2011). With competition and single-homing, $\lambda_i(\pi_j^i) = \mu_i^i(\omega)$ for any $i \in I$ and any $j \in J$. In words, there cannot be a positive information gain from persuasion, for any decision-maker. This follows from Lemma \ref{lem:info gain} and Proposition \ref{prop:no info gain}. Therefore, decision-makers are indifferent about the allocation of attention.

An expert benefits from the possibility to target many different decision-makers. By contrast, the effect of micro-targeting on decision-makers is ambiguous: the believers are always worse off, but the sceptics might benefit. For instance, assume that public persuasion is given by a soft-news strategy. With micro-targeting, a sceptic is tailored with a specific hard-news strategy, and she could be better informed by Lemma \ref{lem:info gain}.

Here, the equivalence between public and private persuasion (Kolotilin et al., 2017) fails because the expert knows the priors of each decision-maker.
B.5. Many States

In this section, I examine how my model can be extended allowing for more than two states of the world.

A first approach is to consider a continuous state space while keeping the action binary. Here, I adopt a setting similar to Guo and Shmya (2019), and illustrate with the help of the COVID-19 vaccination example. I keep assuming that each citizen decides whether to vaccinate or not. However, the state of the world is now given by the safety level of a vaccine. I assume \( \Omega = [0, 1] \), where \( \omega = 1 \) means the vaccine is fully safe, and vice-versa \( \omega = 0 \) means the vaccine is not safe at all. I assume that each citizen follows a threshold rule: she vaccinates herself if and only if the safety level is above a threshold \( \bar{\omega} \). It follows that the optimal action for decision maker \( i \) becomes:

\[
\sigma(\mu_i) = \begin{cases} 
\text{vaccinate} & \text{if } \int_{\omega}^{1} \pi_i^j(\omega \mid s) d\omega \geq \int_{0}^{\bar{\omega}} \mu_i^0(\omega \mid s) d\omega \\
\text{not vaccinate} & \text{otherwise}
\end{cases}
\]

and the implied persuasion constraint is

\[
\int_{\omega}^{1} \pi_i(s \mid \omega) \mu_i^0(\omega) d\omega \geq \int_{0}^{\bar{\omega}} \pi_i(s \mid \omega) \mu_i^0(\omega) d\omega
\]

In such a setting, I can keep the restriction of two decision makers only, a believer \( (i = 1) \) and a sceptic \( (i = 2) \). The believer is such that \( \int_{\omega}^{1} \mu_i^0(\omega) d\omega > \frac{1}{2} \), whereas the sceptic is such that \( \int_{0}^{\bar{\omega}} \mu_i^0(\omega) d\omega < \frac{1}{2} \). As in the baseline model, the optimal strategy focuses either on persuading the sceptic or on retaining the believer. However, the structure of the optimal strategy changes.

If the focus is to persuade the sceptic (hard-news strategy), then a candidate optimal strategy must satisfy the following constraint:

\[
\int_{\omega}^{1} \mu_2^0(\omega) d\omega = \int_{0}^{\bar{\omega}} \pi(s \mid \omega) \mu_2^0(\omega) d\omega \tag{10}
\]

I denote with \( \Pi_H \) the subset of strategies such that (10) holds. Note that in the baseline model \( \Pi_H \) is singleton, whereas here the expert has degrees of freedom on the distribution of probability for each state \( \omega \in [0, \bar{\omega}] \). By (5), the incentive of the expert is to pool states with high \( \mu_j^0(\omega) \), while fully revealing others.

If the focus is to retain the believer (soft-news strategy), then a candidate optimal strategy must satisfy the following constraints:

\[
\int_{\omega}^{1} \pi(s \mid \omega) \mu_2^0(\omega) d\omega = \int_{0}^{\bar{\omega}} \pi(s \mid \omega) \mu_2^0(\omega) d\omega \tag{11}
\]

\[
\int_{\omega}^{1} \pi(s' \mid \omega) \mu_1^0(\omega) d\omega = \int_{0}^{\bar{\omega}} \pi(s' \mid \omega) \mu_1^0(\omega) d\omega \tag{12}
\]
I denote with $\Pi$ the subset of strategies such that (11)-(12) hold, and note that in the baseline model $\Pi$ is singleton. In this case, the goal of the expert is to maximize the probability to persuade the sceptic subject to the constraint that the believer chooses the favourite action with probability one. The incentives of the expert are difficult to disentangle, as these depend on $\mu_0^0(\omega)$, $\mu_0^1(\omega)$ and $\mu_0^2(\omega)$.

However, even if the structure of the optimal strategy changes, my results are not affected. In particular, Proposition 2 generalizes to this setting. Note that

$$\int_0^{\bar{\omega}} \mu_2^0(\omega)d\omega = \int_0^{\bar{\omega}} \pi(s|\omega)\mu_2^0(\omega)d\omega + \int_0^{\bar{\omega}} \pi(s'|\omega)\mu_2^0(\omega)d\omega$$

which implies

$$\int_0^{\bar{\omega}} \pi(s'|\omega)\mu_2^0(\omega)d\omega = \int_0^{\bar{\omega}} \mu_2^0(\omega)d\omega - \int_0^{\bar{\omega}} \pi(s|\omega)\mu_2^0(\omega)d\omega$$

It follows that the sceptic gets a zero information gain. By (11),

$$\lambda_2(\pi) = \int_0^{\bar{\omega}} \pi(s|\omega)\mu_2^0(\omega)d\omega + \int_0^{\bar{\omega}} \pi(s'|\omega)\mu_2^0(\omega)d\omega = \int_0^{\bar{\omega}} \mu_2^0(\omega)d\omega$$

Hence, $\Delta_2 = 0$. Proposition 2 characterizes the incentives of decision makers about the allocation of attention. Therefore, the effect of competition with limited attention is unchanged.

The analysis of optimal persuasion becomes generally intractable if $\Omega \equiv A$, that is in a setting with many states and corresponding actions. A message $s$ persuades a decision-maker $i$ that the state is $\omega$ if $\pi(s|\omega') \leq \phi_i(\omega, \omega') \pi(s|\omega)$ for any $\omega' \in \Omega$. A decision-maker is a true believer (sceptic) of state $\omega$ if $\phi_i(\omega, \omega') \geq 1$ ($< 1$) for any $\omega' \in \Omega$. A hard-news strategy can target true sceptics. A soft-news strategy can solve the trade-off between persuading true sceptics and retaining true believers. Therefore, if an expert faces only true sceptics and true believers, the result of Proposition 5 extends. However, different strategies could be optimal if there exist decision-makers who believe that some states are a priori more plausible than $\omega$, whereas others are not.

**Example** - I consider the COVID-19 vaccination example, and I assume that there exists a third state of the world: safe but with caution (simply caution now on). Therefore $\Omega = \{\omega_1, \omega_2, \omega_3\} = \{\text{caution, safe, not safe}\}$. I assume that the monopolistic expert (say a politician) is biased towards caution. For instance, the politician might want to vaccinate only the elderly.

$^{21}$A full characterization of priors requires $|\Omega|$ decision-makers. Unlike Section 5.3 there is no intuitive ordering of decision-makers. Optimal persuasion cannot be studied generically without restrictive assumptions on the distribution of priors.
There are two decision-makers as before: the believer and the sceptic, respectively, about the vaccine being safe. I assume $\phi_1(\omega_1, \omega_2) > 1 > \phi_1(\omega_1, \omega_3)$ and $\phi_2(\omega_1, \omega_2) > 1 > \phi_2(\omega_1, \omega_3)$. A soft-news strategy is not useful because there are not true believers. Let $\pi_h$ be a hard-news strategy:

$$
\pi_h(s|\omega_1) = 1 \quad \pi_h(s'|\omega_1) = 0 \\
\pi_h(s|\omega_2) = \phi_1(\omega_1, \omega_2) \quad \pi_h(s'|\omega_2) = 1 - \phi_1(\omega_1, \omega_2) \\
\pi_h(s|\omega_3) = \phi_2(\omega_1, \omega_3) \quad \pi_h(s'|\omega_3) = 1 - \phi_2(\omega_1, \omega_3)
$$

Let us consider as alternative $\pi_s$:

$$
\pi_s(s|\omega_1) = k \quad \pi_s(s'|\omega_1) = 1 - k \\
\pi_s(s|\omega_2) = \phi_1(\omega_1, \omega_2)k \quad \pi_s(s'|\omega_2) \leq \phi_2(\omega_1, \omega_2)(1 - k) \\
\pi_s(s|\omega_3) = \phi_2(\omega_1, \omega_3)(1 - k) \quad \pi_s(s'|\omega_3) \leq \phi_1(\omega_1, \omega_3)k
$$

The favourable state of the politician is caution, that is a compromise between opposite decision-makers’ beliefs. If priors are sufficiently polarized (and the politician is sufficiently uncertain about the true state), then it is optimal to use $\pi_s$. The intuition is similar to Proposition 1. With $\pi_s$, the politician mixes between messages that either support one extreme state or the other. In other words, in order to persuade citizens that the best option is to take caution, a politician mixes between positive and negative news about vaccinations. These news are not designed to move one group from one extreme to the other, but just from one extreme to a compromise. The alternative is to provide hard evidence that vaccinations are safe given precautions. This is extremely costly with high polarization, as both extreme views have to be contrasted at the same time. Note that $\pi_s$ is not a soft-news strategy, but it works similarly: the goal is to leverage believers’ credulity.

The intractability of optimal persuasion does not allow to study the whole game. However, intuitively my results should not be affected by the existence of many states of the world and corresponding actions. For instance, let us consider Proposition 3. True believers clustering into echo chambers is an equilibrium. Indeed, no information is provided and hence the decision-makers do not have incentives to deviate. Decision-makers are better informed with a monopoly, because the existence of heterogeneous priors makes optimal for the expert to use some informative strategy, where informativeness is defined according to the Blackwell (1953)’s criterion.
B.6. Biased Decision-makers

In the paper, decision-makers were assumed to be unbiased in their utilities. All the results were driven exclusively by heterogeneous priors. Now, I show that the same results can be obtained in a setting where decision-makers share a common prior \( \mu^0(\omega_1) \), but each decision-maker \( i \) is endowed with a vector of biases \( \{b^\omega_i\}_{\omega \in \Omega} \). The utility of decision-maker \( i \) is \( u_i(a, \omega) := \mathbb{1}\{a = \omega\}b^\omega_i \). See (1) for a comparison. The corresponding optimal action is as follows:

\[
\sigma(\mu_i) = \begin{cases} 
\omega_1 & \text{if } \mu^{\pi_i}(\omega_1 | s)b^\omega_{i1} \geq \mu^{\pi_i}(\omega_2 | s)b^\omega_{i2} \\
\omega_2 & \text{otherwise}
\end{cases}
\]

Therefore, action \( \omega_1 \) is chosen if and only if:

\[
\mu^{\pi_i}(\omega_1 | s)b^\omega_{i1} \geq \mu^{\pi_i}(\omega_2 | s)b^\omega_{i2} \iff \pi_i(s | \omega_2) \leq \frac{\mu^0(\omega_1)b^\omega_{i1}}{\mu^0(\omega_2)b^\omega_{i2}} \pi_i(s | \omega_1) \tag{13}
\]

A model with unbiased decision-makers and heterogeneous priors is equivalent to a model with biased decision-makers and a common prior only if, for any \( i \in I \) and any \( \omega \in \Omega \), \( b^\omega_i = \frac{\mu^0(\omega)}{\mu^0(\omega)} \). This follows immediately from the comparison of conditions (3) and (13). Note that \( b^\omega_i > 1 \) if and only if \( \mu^0(\omega) > \mu^0(\omega) \). Hence, a larger bias is equivalent to a decision-maker having a higher prior belief that the state \( \omega \) is the true state. Remarkably, this multiplicative bias is different from the common definition of bias. In the literature, the utility of biased decision-makers depends on the action, but not on the state. By contrast here, each decision-maker has a strict preference to take the correct action given the state. The bias is limited to each decision-maker valuing some states more than others ex ante.