Bonn Econ Discussion Papers

Discussion Paper 10/2000

Anarchy, Efficiency, and Redistribution

by

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September 2000

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September 22, 2000

Abstract

The purpose of this paper is twofold. We first develop a contractarian theory of redistribution. The existence of rules of redistribution is explained without any recourse to the risk-aversion of individuals. Hence, we depart from the standard legitimization of redistribution as fundamental insurance and interpret it as stemming from a principle of reciprocity in trade. The second purpose of the paper is to develop a theory of institutions that implement optimal allocations. We depart from the assumption of an exogenous enforcement of constitutional rules. Hence, the self-enforcement of constitutional rules is crucial for the implementability of allocations. This approach implies that there is no allocative difference between constitutional and ordinary rules. What makes constitutions different from ordinary rules is their potential ability to create a focal point that conditions the expectations of individuals on a certain equilibrium strategy. Hence, constitutions help to solve coordination problems, not cooperation problems.

Keywords: Anarchy, Constitution, Redistribution

JEL classification: D23, D30, D74, H10

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1 Introduction

There is a broad consensus among economists that the definition and enforcement of property rights leads to welfare improvements that justify the existence of a monopolistic agency called state. These normative theories of the state typically start from an initial situation of anarchy. An anarchic society is a society of conflict where individuals can neither rely on the voluntary respect for individual possession nor on the fulfillment of bilateral or multilateral arrangements (Bush and Mayer 1974). Hence, anarchy is seen as a prisoner's dilemma with individual incentives for overinvestment in defense and aggression, and underinvestment in directly productive activities (Hirshleifer 1995). Credibly enforced property rights are a means to overcome this dilemma.

The consensus among economists is weaker when it comes to other fields of government activities, especially redistribution. Given a certain set of property rights, individual activities define a primary distribution of goods and resources. If this distribution fails to fulfill certain normative criteria of justice, the state should have the power to redistribute in order to get closer to the desired allocation. Following a contractarian theory of justice, redistribution of this type can be seen as insurance against risks which cannot be privately insured because they have already been realized before the individuals become legally capable. Rawls's theory of justice (1971) as well as Harsanyi's theories of utilitarianism (1953,1955) are examples for this line of argumentation. In the present paper we shall develop a theory where redistribution is not explained as insurance against risk, but from differences in individual productivity. In contrast to the insurance theory, where initially equal individuals agree on redistribution *in order to avoid the adverse consequences of becoming unequal*, our theory legitimizes redistribution because individuals are initially unequal. It is exactly the *inequality* of individuals in the initial situation that creates gains from redistribution. Therefore, redistribution is conceived as compensation for the Pareto-improving choice of property rights. Thus, the normative legitimation of redistribution changes from insurance toward a principle of reciprocity in trade. The model that we use to analyze these questions is based on Skaperdas (1992).

Irrespective of its popularity, the usual contractarian models suffer from at least three conceptual weaknesses. First, the insurance argument requires risk aversion of the individuals in the initial situation, under the veil of ignorance. In its most extreme version, the Rawlsian maximin principle assumes infinitely risk-averse individuals in the initial situation. The initial situation, however, is no practical situation, where the adequacy of this assumption can be tested empirically. It is a normative construction that reflects intuitive ideas of justice and fairness. There is no reason to accept the assumption that risk aversion is a reasonable justification of fairness. Second, *real* individuals are not bound by contracts signed by *hypothetical* individuals in an initial situation. Hence, there is an inconsistency between the idea of legitimation underlying contractarian theories and the concept of a veil of ignorance. The theory loses its obligatory power.

The third methodological weakness of contractarian models is the presupposition of the enforcement of constitutional rules. However, in the absence of outside enforcement agencies, constitutional rules cannot be in conflict with the distribution of power within the society because otherwise they would be abolished by the most powerful groups in society. This observation focuses on the self-enforcement of rules, as accentuated by Binmore (1998). He argues that the *" ... principal role* [of fairness norms] *is to single out one of the many equilibria typically available as Pareto-improvements on the status-quo ... ."* (p. 209). Hence, in Binmore's view morality exists because it helps to solve the equilibrium-selection problem,[1] and the only way to meet this requirement with egoistic individuals is by repeated interaction in an indefinitely repeated game.

We share Binmore's basic views. In our paper rules of redistribution can only be implemented if it is guaranteed that they will be voluntarily respected. The analysis of self-enforcing rules allows us to gather further insight into the nature of constitutions. First, we can define conditions under which penal codes can be implemented. Second, we get a better understanding of the nature of constitutions in comparison to ordinary rules: if a rule of redistribution can be implemented by the establishment of a penal code in repeated interaction, this equilibrium is not unique; the class of these folk-theorem equilibria in general is very large. Hence, we get an equilibrium-selection problem. The result of the requirement of self-enforcing rules is striking: since constitutional rules cannot be distinguished from other rules because of their better enforcement capacity, they can only be understood with respect to their potential ability to condition expectations with respect to a certain equilibrium. Hence, denoting a specific rule a constitution is an act of communication that – if successful – creates a focal point. Constitutions cannot be understood with respect to their ability to solve cooperation problems, but only with respect to their ability to solve coordination problems.

Recently there has been an increased interest in the analysis of anarchy and its allocative consequences. Most of this literature uses structures that are equivalent to an all-pay auction that has, for instance, been used to analyze rent-seeking contests. Particularly well-known are Hirshleifer (1995) who focuses on the dynamic stability of anarchy, and Skaperdas (1992) who analyzes technological prerequisites for the existence of cooperation

[1]Binmore distinguishes between small-group problems within the familiy where morality may change the game by changing the payoff functions of the individuals because of mutual sympathy and large-group problems where morality solves an equilibrium-selection problem for a fixed game.

in anarchy. Skaperdas and Syropoulos (1997) compare the income distributions which result from anarchy and from a perfectly competitive equilibrium, respectively. The paper most closely related to our approach is Grossman (1997). He uses a model of production and predation to explain when a government Pareto-improves anarchy. In his model, anarchy has a cost because a fraction of the population specializes in predation. This cost has to be compared to the costs of a government which result from its misuse of coercive power. Bureaucrats will use their power to extract some fraction of tax revenues for their own purposes. However, Leviathan's hands are bound because individuals will avoid taxation by specializing in predation if an insufficient amount of tax revenues is used to secure property rights. In contrast to Grossman (1997), in the present paper we are interested in the explanation and justification of rules of voluntary redistribution that Pareto-improve an anarchic initial situation.

Several additional papers, which are related to our approach, are the following: Grossman and Kim (1996a,b) analyze predator-prey relationships where the predator can invest in production and appropriation, whereas the prey can invest in production and defense. It turns out that different types of equilibria can occur in this model, ranging from nonaggressive to aggressive ones. Anderson and Marcouiller (1997) adopt the idea of anarchy to analyze insecure property rights in international trade.[2] In their model, there are gains from specialization of the agents, but specialization incurs a risk if property rights are not credibly enforced: if the agents decide to specialize and trade, they may be predated by their trading partners. Hence, equilibria tend to be inefficient because (i) some resources are wasted for predatory activities and (ii) the degree of specialization is inefficiently low. Sutter (1995) analyzes the emergence of private and competing defense agencies that offer protection in anarchy. He is interested in the incentives of these agencies to exploit their clients in this situation of anarcho capitalism. It turns out that competition between agencies can secure "rights" of the individuals.

The paper proceeds as follows: In Section 2 we discuss the methodological problems of existing theories of constitutions. In Section 3 we set up the model. Then, in section 4 we analyze the equilibria of the model, in Section 5 the optimal allocation of land and the emergence of rules of redistribution. In section 6 we turn to the question of coercive power and self enforcement. Section 7 analyzes the normative consequences of the model.

[2]See also Skaperdas and Syropoulos (1996).

2 Contractarianism

Constitutional contractarianism explores the relationship between a set of initial conditions - the initial situation or natural equilibrium - and institutions. The working hypothesis is that the initial situation has a tendency to transform itself into a certain institution (for example by unanimous consent of all individuals). Hence, the initial situation is the *explanans*, the institution is the *explanandum* of the theory. Contractarian theories are used to *explain* the emergence or to *justify* the legitimacy of institutions. The first branch will be called *positive* contractarianism, whereas the latter will be called *normative* contractarianism.

Both normative and positive contractarianism assume that the rationality of individuals drives their decisions in the initial situation. This is a situation without institutions, which can be called *anarchy*. Normative contractarianism characterizes the initial situation of anarchy by special value judgments: an institution is just if it can be derived from an initial situation which reflects particular intuitions of fairness. In contrast, positive contractarianism describes the initial situation by plausible intuitions about a society without institutions.

Let us present a set of seven characteristics which define a large field of possible contractarian theories, positive as well as normative, depending on the interpretation of the initial situation. The acceptance of any such theory depends on the plausibility of the specification of the initial situation because in all cases the theory starts from a hypothetical initial situation. This situation has to specify:

(a) the economic background or allocation problem to be solved (private or public goods, resource constraints, technological constraints (economies of scale or scope)),
(b) the objective functions and abilities of the individuals (risk neutrality or aversion, egoistic, envious, or altruistic preferences, ...),
(c) the distribution of information across individuals (complete information, uncertainty, asymmetric information),
(d) restrictions on the set of contractible variables in the initial situation (ex ante) (private property already exists or does not exist, binding commitments are possible or impossible, contracts are complete or incomplete, ...),
(e) restrictions on the set of contractible variables thereafter (ex post) (as above),
(f) the equilibrium concept (Nash, dominant strategy, maxmin, bargaining, ...),
(g) a rule for the establishment of a constitution (unanimity, qualified majority, power, ...).

A specification of (a) to (g) leads to an initial equilibrium. Compared to the equilibrium that results if all ex-post contractual opportunities are exhausted, this equilibrium is in general inefficient, either for all individuals, or for the decisive majority or for the most powerful group of individuals. The emergence of institutions from this equilibrium is motivated by Pareto-improvements of the relevant reference group. The difference in the set of contractible variables ex post and ex ante characterizes the institutions to be justified or explained, and may refer to rights, obligations, rules etc.

Most attention has been devoted to the interrelation between the specification of (a) to (d) and (f) to (g), and the resulting institutions. Such an analysis is severely incomplete for a number of reasons. It is the specification of (e) that is crucial for the determination of the constitution. The concept of two-stage constitutions with general and specific rules (Buchanan's idea of constitutional architecture) as well as concepts like property rights and redistribution cannot be deduced without the precise specification of the ex-post contractual opportunities.

Let us now consider the benchmark of complete ex-post contracts because it has an interesting consequence for the institutional structure of an economy. If one allows for unrestricted complexity, the optimal constitution will specify a list of *state-contingent* activities irrespective of the specification of the rest of the initial situation. This list specifies with sufficient precision what individuals have to do under what contingencies and a set of penalties for deviations from this list. If these penalties are credible, such a list implements a Pareto-efficient allocation. The point along the Pareto frontier that is implemented by such a constitution is determined by the exact specification of (a) to (d) and (f) to (g) of the initial equilibrium. Therefore, the individuals simply carry out the plan specified in the list. The constitution can be seen as a specification of *specific* rights of control. There is neither a meaningful way to talk about property nor to talk about *re*distribution in this context. Property as defined by Hart and Moore (1990) is a set of discretionary rights: the owner can decide on those contingencies for which the ex-ante contract is silent. Hence, the whole concept of property becomes meaningless in the case of specific rights.[3] By the same token, the concept of redistribution becomes meaningless because everything that can be implemented by redistributing goods given constitution c_1 can be achieved by the specification of a constitution c_2 that directly implements the resulting allocation. Every allocation that can be reached by a complicated set of constitutional rules can also be reached by a constitution that directly specifies the allocation by a list of state-contingent activities.

To summarize, with complete ex-post contracts there is no reason to specify residual

[3]This logic is also employed by Rajan and Zingales (2000) who analyze the consequences of transfers in a world with imperfect property rights and poorly developed credit markets.

rights or general rules in a constitution. Contractarian theories that seek to explain the existence of property, general rules of conduct like majority voting, or redistribution, must therefore depart from the assumption of complete ex-post contracts irrespective of how the initial situation otherwise is defined. So far this observation has not reached much attention in the literature on constitutional contractarianism.[4] Even Buchanan is vague about this point. His main argument in favor of a constitution that specifies general rules or procedural rights is to overcome the complete paralyzation of political activity in cases of conflict of interest. However, he does not touch the underlying question of why general instead of state-contingent rules should be used if state-contingent constitutions are feasible. It is the *source* of contractual incompleteness that ultimately explains the specific structure of optimal constitutions. General remarks on transaction costs and enforcement can be misleading unless they are defined in an operational way.

In this paper we will analyze the following contractarian situation: there is a private good (corn) that can be produced by two egoistic and risk-neutral individuals by the use of an external resource (land) and an internal resource (time). The internal and external resources can also be used for the defense of the initial possession of goods. Both individuals share the same information but there are no initial rights or institutions. This restricts the set of contractible variables ex ante. Only enforceable variables can be contracted upon, for instance distributions of corn that are compatible with the real allocation of power that is determined by the investments in defense. An initial anarchic Nash equilibrium is characterized by the restriction to enforceable variables. Contracts which replicate the anarchic equilibrium can be written, but are without material consequence.

We assume that one of the central features of a society is the repetition of interactions. Hence, the set of contractible variables ex post does not change because there is an outside enforcement agency, but because there is repeated interaction. Any institution that is to be explained or justified relies on penal codes that become effective because of the repetition of the game. There is no qualitative difference between anarchy and other forms of organization because both depend on the balance of power between the members of the society. Rules that deviate from this allocation of power cannot be sustained as an equilibrium. This point of view has decisive consequences for the nature of constitutions: they cannot solve *cooperation problems*, but only *coordination problems*. The emergence of institutions is a manifestation of the willingness to cooperate. However, the mere willingness to cooperate is not sufficient to pin down the expectations of the individuals to a certain equilibrium strategy; individuals have to be able to solve the equilibrium-selection

[4]A recent paper by Gersbach (1999) stresses the importance to limit the set of ex-post contractible variables.

problem. Calling a certain rule of cooperation "the constitution," therefore, is a means to pin down expectations to a certain equilibrium and to solve the coordination problem. Calling a rule a constitution is an act of communication that creates a focal point. Constitutions are successful (a) if they manage to coordinate expectations with respect to an efficient equilibrium and (b) if the act of communication is accepted by the individuals.

An example may further clarify this point. The 1789 Declaration of Rights rests on a conception of the nature of the individual. Individuals are entitled to their internal resources because they have been allocated to them by nature (naturalistic argument) or god's will (metaphysical argument) (Kolm 1996). This conception of self ownership extends to ownership of the external world, for instance if elements of the external world are combined with labor (John Locke). Hence, there is a metaphysical or naturalistic foundation of the concept of property. Let us assume the society begins with a metaphysical foundation of property. As long as individuals believe in the validity of the metaphysical argument, the conception of property rights defined in a constitution is successful in solving the coordination problem because nobody doubts the legitimization of the equilibrium that results from private ownership. If the legitimating power of metaphysical arguments is no longer undisputed, other forms of legitimization have to take their place, for example naturalistic arguments. If the individuals believe in the validity of the naturalistic foundation, this adapted act of communication may supersede the metaphysical argument as a successful way to solve the coordination problem, and so forth.

3 The model

We consider an economy in complete anarchy, which means that there is no coercive power that could enforce any formal rules. Hence, there is no property but only possession of goods and resources. In this section we develop the idea that the allocation of resources may have an impact on production possibilities and on the individuals' bargaining power.

There is a single consumption good, called corn, and individual utility depends only on the quantities of corn consumed. Corn is produced from a basic resource – for the sake of simplicity called land – and from working time of two individuals. Any individual invests x_i, $i = 1, 2$, units of time in the production of corn and y_i units of time in the accumulation of power, where $x_i + y_i = 1$. The investment of time for production determines the total amount of corn that can be distributed across individuals and the investment in power determines the final distribution of corn.

The possession of corn is the ultimate source of utility for the individuals. However, the possession of land has decisive impacts in our model. First, it influences the total production of corn. We denote by $a \in [0,1]$ the fraction of land that is possessed by individual 1. Now assume that individual 1 is more talented in production, whence it is plausible to assume that total production increases if individual 1 possesses more land. By way of an example, the total production of corn could be $F = a\gamma x_1 + (1-a)\eta x_2$, with $\gamma > \eta$. Note, however, that the production function need not be additively linear. Individual 1's higher productivity may in part be due to his skilful deployment of fertilizer, pest control and weed control, and these activities spill over to individual 2's part of the field. Accordingly, we consider the following *production technology:*

> **Assumption 1 (production technology):** The amount of corn $F(x_1, x_2, a)$ is a function of the working-time investments in production, x_1 and x_2, and of the distribution of land a. F is twice continuously differentiable. It has positive but decreasing marginal products, $F_1 > 0$, $F_2 > 0$, $F_{11} \leq 0$ and $F_{22} \leq 0$ and constant returns to scale. Subscripts denote partial derivatives. W.l.o.g. we assume that the first individual has a (weak) advantage in production, $F_a \geq 0$ and $\lim_{a \to 0} F_a > F$ if $F_a > 0$.[5]

Second, the possession of land influences the ability to appropriate corn. Assume that corn grows on a field and that the final distribution of corn is determined by the possession of the field: the two players get $\widehat{V}_1 = aF$ and $\widehat{V}_2 = (1-a)F$ of the crop. However, in a state of anarchy corn can be taken away from the other individual. This can be captured by a *conflict* or *bargaining function* which depends on the relative strength of the individuals which, in turn, depends on their investments in power. Denote this conflict function by $\widetilde{p}(y_1, y_2)$. The simplest model would assume a linear specification, that is, a final distribution of the harvest according to $\widetilde{V}_1 = (a + \widetilde{p})F$ and $\widetilde{V}_2 = (1 - a - \widetilde{p})F$, with $\widetilde{V}_1, \widetilde{V}_2 \geq 0$. However, the linear specification is unlikely to describe the practice of conflicts and bargaining. American football is a good example of what we have in mind. The closer the offense gets to the end zone, the more difficult it becomes for them to gain yards. First, the players are crowded into a physically smaller area – the end zone, and thus it becomes more difficult for them to maneuver themselves. Second, the defensive line becomes increasingly aggressive in its defense of the final few yards. This example shows that the initial distribution of land has an influence on the final possession of goods and that this relationship need not be linear.

[5] The assumption $F_a \geq 0$ may raise problems with respect to the interpretation of possession. In particular, if F is non-decreasing in a, even when a is close to unity, then it seems as if person 2 has to be working on person 1's land. All qualitative results of our paper can also be derived if the assumption $F_a \geq 0 \; \forall \; a$ is replaced by the assumption that F_a is increasing up to a threshold $\widetilde{a}$, and decreasing afterward.

Assumption 2 (conflict technology): $p(y_1, y_2, a)$ is the fraction of corn that is finally possessed by individual 1, a fraction $(1-p)$ is possessed by individual 2. The function p is twice continuously differentiable and has the following properties:

- $p(y_1, y_2, a) \in [0,1] \quad \forall \quad y_1, y_2, a \in [0,1]$ (probability of winning),
- $p_1 > 0$, $p_2 < 0$, $p_{11} < 0$, $p_{22} > 0$ (investments y_i have positive but diminishing marginal productivities),
- $p_a \gtreqless 0$ (possession of land may change the bargaining position),
- $p(y, y, a) = a$ (equal investments imply unchanged possession).

The relationship of the conflict function of this paper and the standard theory of conflict functions remains to be shown. In contrast to this paper, the standard conflict function only describes how individual bargaining powers influence the probability of winning in a conflict. In our terminology this would be a conflict function $p(y_1, y_2)$.[6] In the lobbying and rent-seeking literature, the following two specifications of the standard conflict function have most widely been applied: first, Hirshleifer's (1989) logistic contest-success function $p = 1/(1+exp(k(y_2-y_1))$ which implies convexity of p if $y_1 < y_2$ and concavity thereafter. This function is, for example, applied in Skaperdas (1992). Second, Tullock's (1980) ratio model $p = y_1/(y_1+y_2)$ which exhibits decreasing marginal effectiveness of investments in power.

Assumption 2 of this paper can be fulfilled by the Tullock ratio model if it is adequately modified, as will be shown at the end of section 5 below. However, it is not fulfilled by the Hirshleifer model. In our setting it is not sure, and in fact impossible for a large class of problems, to guarantee the existence of interior equilibria in the case of the Hirshleifer function, whereas the modified Tullock function leads to robust interior solutions.[7]

There are several ways to interpret the conflict or bargaining function p. The most literal interpretation refers to the success in the appropriation of corn in an open conflict or war. In this scenario anarchy is interpreted as a war of all against all where individuals will have to fight for their final consumption of corn. In a second interpretation, the distribution of corn is determined by a "cold war," where we do not have to bother about the partial or total destruction of crop which would inevitably result in the cases of open conflict or war. The cold-war scenario is chosen in the present paper.

[6]See Hillman and Riley (1989), Hillman and Samet (1987), Hirshleifer (1989), Körber and Kolmar (1996), Nitzan (1994), and Tullock (1980) for economic interpretations of this function and Amann and Leininger (1996), Baye et al. (1993), Esteban and Ray (1999), and Krishna and Morgan (1997) for their general structure.

[7]See Körber and Kolmar (1996).

Note that $F(0, x_2, 0)$ or $F(x_1, 0, 1)$ need not be equal to zero. Hence, situations where an individual who does not possess any land specializes in conflict ($y_i = 1$) are potential equilibria of the game with non-zero production. This has the straightforward interpretation of a society where one group specializes in production and defense whereas the other group specializes in predation without being productive.[8]

We assume that both individuals are risk neutral. With these specifications the final utility of the individuals is given by

$$V_1(y_1, y_2, a) = p(y_1, y_2, a)\, F(1 - y_1, 1 - y_2, a), \tag{1}$$

$$V_2(y_1, y_2, a) = (1 - p(y_1, y_2, a))\, F(1 - y_1, 1 - y_2, a). \tag{2}$$

Let us conclude this section by presenting the *optimum benchmark*. Given the specifications of the model, the first-best optimum is given by the allocation $y_1 = y_2 = 0$ and $a = 1$, as can easily be verified by solving the respective maximization problem. Note that the first-best optimum leads to a distribution $V_1(0, 0, 1) = F(1, 1, 1)$, $V_2(0, 0, 1) = 0$.

4 Equilibrium with given possession of land

We consider a stage game with the following timing of events:

- At stage 0 individual 1 possesses a fraction a^o of the land, individual 2 the remaining fraction $1 - a^o$. This initial distribution of land is determined by chance or history (positive contractarianism) or by moral considerations (normative contractarianism). Both individuals can successfully defend their land before production.[9]

- At stage 1 the individuals can voluntarily agree on a redistribution of land (and only land). They will do so if this improves every agent's individual utility.

- At stage 2 the individuals can voluntarily agree on a further redistribution of land coupled with a compensating distribution of corn. They will do so if the combined redistribution of land and corn leads to further utility improvements for every individual, beyond the utility achieved by pure redistribution of land. To guarantee the actual payment of corn, certain rules will be codified in a "constitution."

- At stage 3 the individuals invest time in production and in the attainment of bargaining power. The crop F and the bargaining strength p are determined.

[8]This situation is the starting point of the analysis in Grossman (1997), Grossman and Kim (1996a,b, 1997).

[9]This assumption is a short cut for a more complex situation where a^o is determined by initial bargaining of the individuals.

- At stage 4 the crop is distributed according to the bargaining strength and consumption takes place.

The timing of events characterizes our theory of constitutional rules which guarantee the completion of voluntary redistribution. If individual 2 gives up land at stage 2, he cannot be compensated before stage 4, because the compensation is to be made in crop. The ex-post changes in the bargaining power of the players may allow individual 1 to shirk from his obligation to compensate the other player. Therefore, individual 2 will only agree to give up land if there are explicit rules of redistribution contained within the constitution. Accordingly, our paper differs decisively from Buchanan (1975) who recognizes the reciprocity of trade, but does not consider any temporal sequence of events.

We will now determine the Nash equilibrium of stage 3 for a given possession of land a. Both individuals maximize their utility specified in (1) and (2) and obtain the following first-order conditions:

$$\partial V_1/\partial y_1 \quad = \quad p_1 F - p\, F_1 \quad \begin{cases} = 0 & \wedge \quad y_1 \in [0,1] \\ < 0 & \wedge \quad y_1 = 0 \\ > 0 & \wedge \quad y_1 = 1 \end{cases} , \tag{3}$$

$$\partial V_2/\partial y_2 \quad = \quad -p_2 F - (1-p) F_2 \quad \begin{cases} = 0 & \wedge \quad y_2 \in [0,1] \\ < 0 & \wedge \quad y_2 = 0 \\ > 0 & \wedge \quad y_2 = 1 \end{cases} , \tag{4}$$

abbreviating $p_1(y_1, y_2, a)$ by p_1 etc. These conditions determine reaction functions $y_1(y_2, a)$, $y_2(y_1, a)$. A Nash equilibrium $\overline{y}_1(a)$, $\overline{y}_2(a)$ of the game is a fixed point $\overline{y}_1 = y_1(y_2(\overline{y}_1, a), a) \wedge \overline{y}_2 = y_2(y_1(\overline{y}_2, a), a)$. The associated levels of utility are given by $\overline{V}_1(a)$, $\overline{V}_2(a)$.

> **Proposition 1:** For any value of $a \in [0,1]$, there exists a Nash equilibrium $\overline{y}_1(a), \overline{y}_2(a)$ of the game in stage 3.

The proof of this proposition and of all other results of this paper can be found in an appendix which is sent to the reader on request. Since a Nash equilibrium exists for all $a \in [0,1]$, it also exists for the initial distribution a^o. Equilibria of the game can be different in nature. It can either be that $\overline{y}_i \in (0,1)$ or that $\overline{y}_i = \{0,1\}$. In the latter case, it can either be characterized by $\partial V_i/\partial y_i = 0$ or $\partial V_i/\partial y_i \neq 0$. Equilibria in which the inequality condition is fulfilled for at least one individual will henceforth be called boundary equilibria, whereas all other equilibria will be called interior equilibria.

Since we are concerned with equilibria of the game at stage 3 for different possessions of land a, the uniqueness of equilibria has to be guaranteed in order to make any comparative-static analysis meaningful.

Proposition 2: An equilibrium $\overline{y}_1, \overline{y}_2$ is unique if

$$\begin{aligned} & \Big(F(p_{11}p(1-p) - 2p_1^2(1-p)) + p^2(1-p)F_{11}\Big) \\ & \Big(-F(p_{22}p(1-p) + 2p_2^2 p) + (1-p)^2 pF_{22}\Big) \\ > \;& \Big(F(p_{12}p(1-p) + (2p-1)p_1p_2) + p^2(1-p)F_{12}\Big) \\ & \Big(-F(p_{12}p(1-p) + (2p-1)p_2p_1) + (1-p)^2 pF_{12}\Big) . \end{aligned}$$

We will assume that this condition is fulfilled throughout the text.

5 The possibility of voluntary redistribution of land in anarchy

Given the levels of utility for an initial possession of land we can now analyze the effects of a change in this initial distribution. Since the assumptions of the model guarantee that $\overline{V}_i$ is continuous, we can apply the envelope theorem and obtain the following derivatives, evaluated at the initial distribution a^o:[10]

$$\partial \overline{V}_1 / \partial a = p_a F + pF_a - F_2 \frac{dy_2}{da}, \tag{5}$$

$$\partial \overline{V}_2 / \partial a = -p_a F + (1-p)F_a - F_1 \frac{dy_1}{da}. \tag{6}$$

The total effect of a change in a can be decomposed into a *bargaining effect* given by the first term, a *direct production effect* given by the second term, and an *indirect production effect* given by the third term on the right-hand sides of (5) and (6). The bargaining effect measures the change in the bargaining power of any individual, and the direct production effect measures the change in production due to a change in a for given y_1 and y_2. Both effects are positive for individual 1. Individual 2, however, faces a trade-off: he benefits from the increase in the total crop which is induced if individual 1 possesses a larger fraction of land, but he loses because his relative bargaining strength is reduced. In other

[10]The effects $F_i \cdot dy_i/da$ vanish if $\partial \overline{V}_i / \partial a \neq 0$, because this implies a corner solution $y_i = \{0, 1\}$ which does not respond to changes in a. Note that we have to perform a comparative-static analysis which refers to changes of Nash equilibria; it is not enough to move along any single individual's reaction function. This is the reason why in $\partial \overline{V}_i / \partial a$ only the cross effect $F_j \cdot dy_j/da$, $j \neq i$, is of relevance. For details see the appendix which is sent to the reader on request.

words: the interplay of the bargaining effect and the direct production effect implies that individual 2 gets a smaller share of a larger cake.

The indirect production effect measures the effect of a change in a on consumption due to a reallocation of time between production and bargaining. Changing a changes the marginal productivities of investments in production and bargaining. This change causes a reallocation of time which may either increase or decrease production. By way of an example, if dy_2/da is positive, an increase in possession of individual 1 makes individual 2 more aggressive, that is, he reallocates time so as to increase his bargaining power. This reallocation has a negative effect on the utility of individual 1. The opposite case has an analogous interpretation.

We are now in the position to establish our results with respect to the allocation of and. We begin with a simple benchmark where we assume that no individual has a comparative advantage in conflict technology or production technology.

> **Proposition 3:** If p and F are independent of $a \in [0, 1]$, the institutional structure is irrelevant for the coordination of individual behavior in the Nash equilibrium.

Proposition 3 has a straightforward interpretation: if the distribution of productive resources has no influence on production and conflict activities, the resulting anarchic equilibrium is independent of the distribution of these resources. Hence, the individuals cannot improve upon anarchy by setting rules in the stage game.

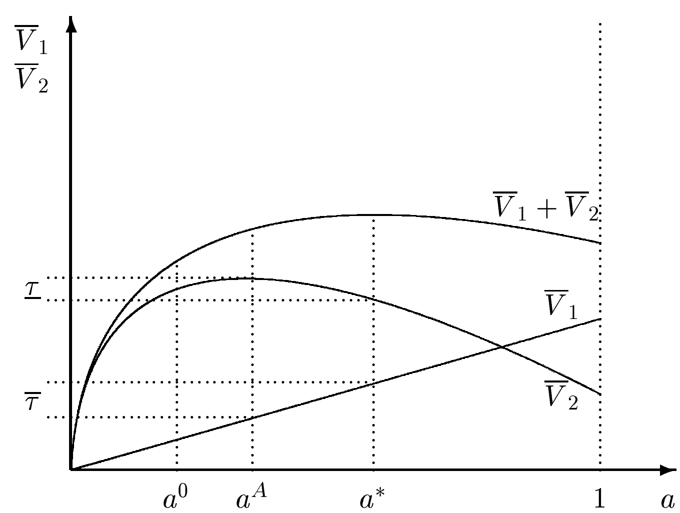

Figure 1: Gains from redistribution of land

To motivate the *general analysis* of the redistribution of land and of crop, let us consider an example which is illustrated in figure 1.

We assume that V_1 increases monotonically in a, whereas V_2 first increases and then decreases. Assume that we start from a situation a^0. In this case both individuals will voluntarily agree to redistribute land until a^A is reached which maximizes individual 2's utility. However, a^A does not maximize the sum of the individual utilities. This is rather attained by a distribution of land a^*. However, this distribution cannot be reached without rules defining the redistribution of crop because in a^* individual 2's utility is lower than in a^A (taking into account the bargaining powers $p, (1-p)$ associated with a^* and a^A, respectively). Therefore, rules of redistribution of crop are necessary to reach an agreement on redistributions of land beyond a^A.

For a concise analysis consider first the possibility of voluntary redistribution as illustrated by a^A in figure 1. Given any initial possession of land a^o, individuals will voluntarily agree on a distribution of land $a^A \neq a^o$ if $\overline{V}_1(a^A) \geq \overline{V}_1(a^o) \wedge \overline{V}_2(a^A) \geq \overline{V}_2(a^o)$, and one inequality is strict. Voluntary redistribution occurs until there is no other $a \in [0,1]$ which improves at least one individual's utility without reducing the other's utility. We denote the set of undominated a's by $A^A(a^o)$, $a^A(a^o) \in A^A(a^o)$. It is the set of distributions of land that is Pareto efficient given the initial distribution a^o and given Nash behavior of the individuals. If there are multiple solutions $a^A(a^o)$ which yield different utilities of the two individuals, then we suppose that one of the solutions is chosen by an arbitrary bargaining scheme that we do not model explicitly. This assumption does not influence any of the qualitative results of this paper.

Definition 1: A distribution $a^A(a^o)$ of land is called *cooperative anarchy.*

Proposition 4: Given an arbitrary initial distribution of land a^o, it is possible that in anarchy land is voluntarily redistributed.

At first glance Proposition 4 might be surprising because it states that anarchy is not characterized by a situation where every individual grabs as much as possible of both corn and land. Rational individuals anticipate the effect of the distribution of land on production and are therefore willing to dispense with land as long as this has a positive effect on the final appropriation of corn. This is the case as long as i) the direct and indirect production effects are positive and ii) the bargaining position is not weakened in a way that overcompensates the production effects. Only if $a^0 \in A^A(a^o)$, there will be no voluntary redistribution of land. But in this case the initial distribution has been Pareto efficient taking as given the lack of institutions in anarchy.

Unfortunately, however, there may be potential gains that cannot be realized in a situation of cooperative anarchy. The reader might recall the move from a^A to a^* in figure 1 above, which is a *potential* Pareto improvement: since total utility increases, both individuals may gain if the surplus of the move is shared by appropriate redistribution. Without redistribution, however, individual 2 will face a utility reduction and, therefore, veto the move.

Due to the uniqueness of equilibria there exists a pair $y_1(a), y_2(a)$ of equilibrium strategies for any value of a, and therefore a value of production $F(a) = F(y_1(a), y_2(a), a)$. The subset $A^* \subset [0,1]$ of a's that maximize $F(a)$ is the set of potentially Pareto-efficient distributions of land given Nash behavior of the individuals. We denote by a^* an element of A^*. It can be determined by the maximization of the sum of the individual utilities and is therefore characterized by the following first-order condition:

$$\frac{\partial \overline{V}_1}{\partial a} + \frac{\partial \overline{V}_2}{\partial a} = F_a - F_1 \frac{dy_1}{da} - F_2 \frac{dy_2}{da} \quad \begin{cases} = 0 & \wedge \quad a \in [0,1] \\ < 0 & \wedge \quad a = 0 \\ > 0 & \wedge \quad a = 1 \end{cases} . \tag{7}$$

If A^* contains more than one element, one can pick any of them because by the definition of A^* they entail the same quantity of F and, thus, the same total utility of the two agents. If we compare the individual Nash conditions for a^A, equations (5) and (6), and the condition for a^*, equation (7), it is evident that a^A does not necessarily maximize the sum of individual utilities. This cooperation failure is due to the fact that every individual takes into account the effect a change in a has on his bargaining position, $p_a F$, whereas this effect is irrelevant for the determination of potential Pareto optima. If a^A does not maximize total utility, this creates an externality: maximization of corn requires a redistribution of land that would weaken the bargaining position of one of the individuals in a way that reduces his share of corn below the level that he could guarantee himself with a^A.

A redistribution of land beyond cooperative anarchy is worthwhile to be made if the sum of the individual utilities increases. By construction, the move from a^A to a^* increases the utility of individual 1 at the expense of individual 2 or vice versa. Assume w.l.o.g., that it is individual 1 that benefits. Then it is obvious that at stage 2 individual 2 will only agree to a redistribution of land from a^A to a^* if both individuals can credibly commit to redistribute corn deviating from the level which is determined by $(1 - p(a^*))$ at stage 4. In principle this can be done by the introduction of a credible scheme τ of redistribution from 1 to 2. The maximum individual 1 is willing to pay is the amount of corn he gains, that is, his utility increase. The minimum individual 2 requires is compensation of his utility loss. Therefore, both individuals will agree on a distribution of land a^* if

$$\overline{V}_1(a^*) - \overline{V}_1(a^A) =: \overline{\tau} \geq \tau \geq \underline{\tau} := -\left(\overline{V}_2(a^*) - \overline{V}_2(a^A)\right). \tag{8}$$

The redistribution scheme τ turns potential Pareto improvements into actual Pareto improvements.

> **Proposition 5:** (i) A necessary condition for the realization of gains from the redistribution of land beyond a cooperative anarchy is the credible implementation of a redistribution scheme $\tau \in [\underline{\tau}, \overline{\tau}]$. (ii) Gains from trade beyond a cooperative anarchy exist if and only if a^A does not maximize the sum of individual utilities, that is, if $A^A(a^0) \cap A^* = \emptyset$.

This proposition is a central piece in the *explanation* or *justification* of redistribution in the absence of risk aversion: rules of redistribution for corn are an institutional prerequisite for the realization of gains that stem from the reallocation of land. Hence, in a normative interpretation an unequal possession of land creates an obligation for the possessors. Participation of other individuals in the returns on land beyond their bargaining power is not legitimized by private charity of land possessors, but by a normative claim stemming from the initial willingness to relinquish individual possession of land.

Proposition 5 links gains from trade to the probability that a^A does not maximize the sum of utilities. It must be stressed that this is not a serious restriction. As long as the number of elements in A^* is finite, gains from redistribution exist generically. Therefore, it can be expected that rules of redistribution improve the efficiency of the allocation unless there is an interval $[a', a'']$ for which condition (7) always holds as an equality. This, however, can be ruled out because both individuals' maximization problems are strictly convex. In figure 1, for example, this condition is fulfilled for all $a \neq a^*$. Hence, gains from redistribution exist almost certainly.[11]

[11] An interesting special case might occur if there are multiple solutions, for instance $a_1^A(a^o)$ and $a_2^A(a^o)$, where both individuals gain in both cooperative-anarchic situations compared to the initial situation a^o. However, there might be a conflict of interest between both individuals: $a_1^A(a^o)$ might be better for individual 1, $a_2^A(a^o)$ for individual 2. Note that all elements $a^* \in A^*$ have the same sum of utilities and that the set A^* is not influenced by the initial distribution a^o. Hence, an agreement on $a^A(a^o) \in A^A(a^o)$ may influence the direction of transfers (either individual 1 may pay individual 2 or vice versa) but not the efficiency-enhancing role of transfers sui generis. Therefore, the specification of the bargaining concept that determines $a^A(a^o) \in A^A(a^o)$ may allow interesting insights in the specific structure of transfers, but is not central to the primary goal of this paper. Conflicting interests in the case of multiple solutions have a straightforward economic interpretation: a relatively equal distribution of land is inefficient because of its large potential for conflict but neither individual has a large comparative advantage in production. In this case it is important to concentrate possession in the hands of one individual but the identity of the

Before turning to a systematic discussion of the positive and normative implications of propositions 4 and 5 let us present an example for specified conflict and production functions.[12] As conflict function we consider a modified Tullock function $p = 2ay_1/(y_1+y_2)$, where, without restriction of generality we have normalized $a \in [0, 1/2]$. The production function is specified as $F = 100000(1 + a - ay_1 - y_2)$. Given these specifications, the individuals maximize their utilities with respect to their time investments in conflict y_i. We obtain interior solutions which are characterized by the following first-order conditions:[13]

$$(1 + a - ay_1 - y_2) = \frac{ay_1(y_1 + y_2)}{y_2},$$
$$(1 + a - ay_1 - y_2) = \frac{(y_1 + y_2)^2 - 2ay_1(y_1 + y_2)}{2ay_1}. \tag{9}$$

Solving for the Nash equilibrium values of $y_1(a), y_2(a)$ we obtain the individual conflict investments and utility levels as presented in figures 2 and 3.[14]

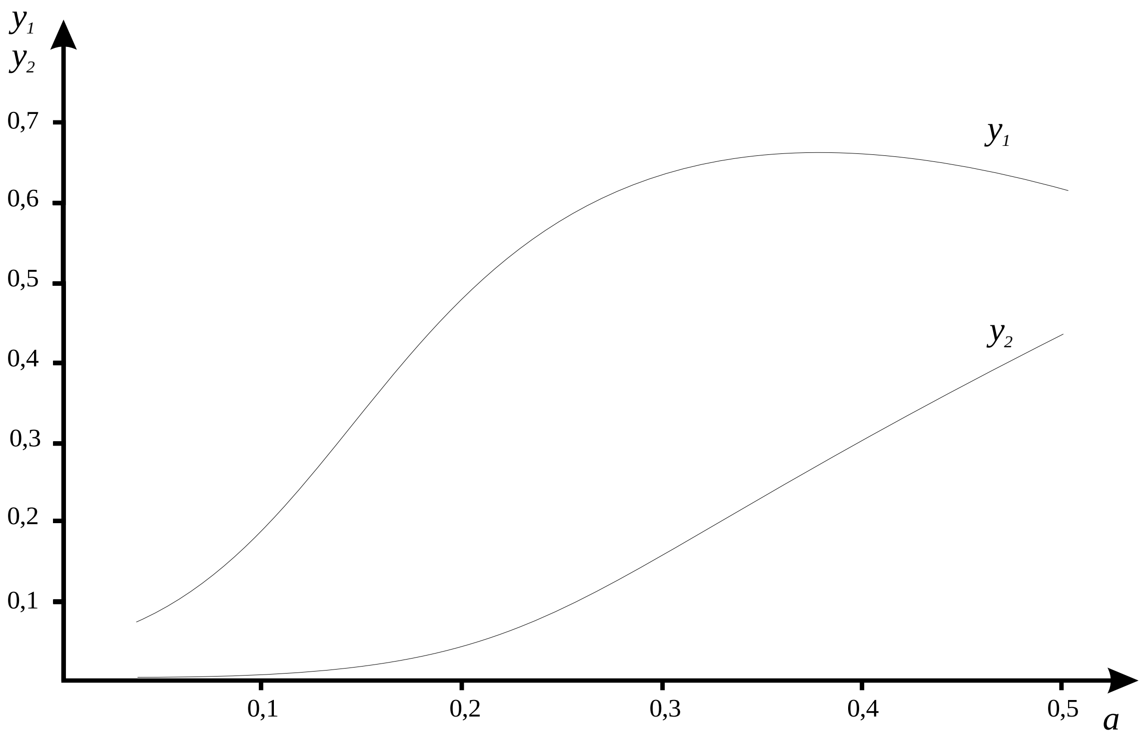

Figure 2: Optimal investments in conflict

individual does not matter. We are grateful to Sam Bucovetsky who has drawn our special attention to the case of multiple solutions of our model.

[12]Details of the calculation of this example are given in appendix A.3 which is sent to the reader on request.

[13]These conditions are special cases of the equations (3) and (4) above.

[14]These figures present the results of a simulation analysis whose precise data are given in a table in appendix A.3.

For the interpretation recall our normalization of a: if, say, $a = 0.4$, this means that 80 percent of the land belong to the more able individual 1. Figure 2 reveals the high potential for conflict inherent in the modified Tullock function.

If the unable individual possesses nearly all of the land ($a \approx 0$), both agents are very peaceful. However, if the more able individual owns the land, he becomes increasingly aggressive: if he owns, say, half of the land ($a = 0.25$), 60 percent of his time is occupied by conflict. And he continues to invest even more than 60 percent into conflict if a increases further. The less able person always is less aggressive, never investing more than 45 percent into conflict. Figure 3 presents the individual utilities.

In spite of his large investments in conflict, the more able individual 1 gains all the way. (The direct production and the bargaining effect dominate the indirect production effect.) For the less able individual 2 the direct production effect dominates if a is low: giving more of the land to 1 is also beneficial for 2. However, pretty soon the weakening of 2's bargaining position becomes decisive, resulting in a sharp decline of utility if more and more of the land is given to individual 1. Note that the individual utilities in our example follow the same pattern as illustrated in figure 1: for a low initial a^o, there is voluntary redistribution until the cooperative-anarchy solution a^A is reached. Further, a potential Pareto improvement from a^A to a^* could be achieved by redistribution.

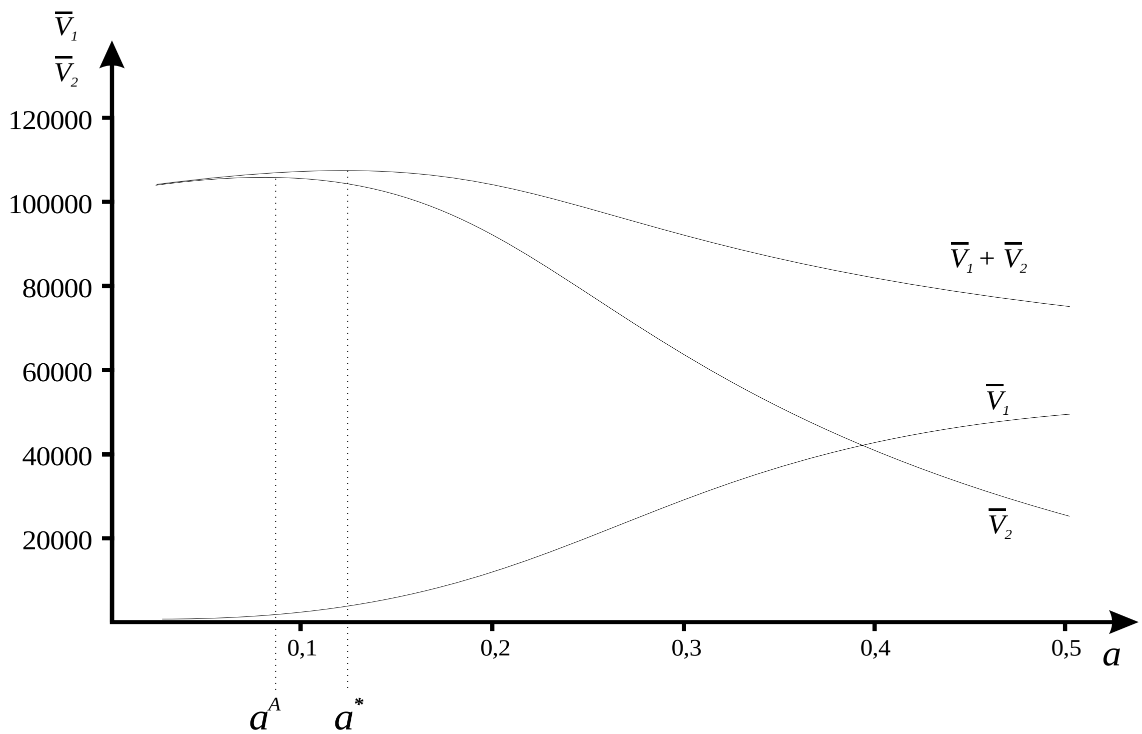

Figure 3: Individual utilities for different values of a

Finally, our example exhibits the social costs of conflict. If both individuals decide to refrain from investing in conflict, total utility becomes $\overline{V}_1 + \overline{V}_2 = F = 150000$. In our example, however, F only reaches a bliss point of slightly over 100000.

6 The nature of constitutions

6.1 Voluntary enforcement of rules

The pair $\{a^*, \tau\}$ can be interpreted as a rudimentary system of rules that introduces the formal concepts of "property rights" and "rules of redistribution." From this point of view, possession becomes property if it is mutually accepted. Calling a certain possession "property," therefore, is an act of communication that signals its mutual acceptance. Rules of redistribution are a means to guarantee this mutual acceptance. Hence, the concepts of property and redistribution are indissoluble. Note that redistribution can only be meaningfully defined if two prerequisites are met. First, the allocation of property has to have an influence on the efficiency of the allocation. Second, property rights alone have to be a *coarser* instrument to allocate specific rights on goods than property rights together with rules of redistribution.[15] This view of redistribution does not explain or justify rules of redistribution of goods as an insurance contract that stems from risk aversion and uncertainty under a veil of ignorance, but as an ex-post compensation for the redistribution of resources in situations where ex-ante compensations are not feasible.

However, the crucial element in the formulation of Proposition 5 is the *credibility* of the implementation of the redistribution scheme. In the static model of the preceding section and without any enforcement mechanism, the individual who receives land does not have an incentive to actually pay τ. If this is anticipated at stage 2, the individual who gives up land will not agree on any distribution of land other than the cooperative-anarchy distribution. Rules of redistribution require coercive power in order to be credible, and coercive power has its material basis in the bargaining power of the individuals. This power, in turn, is determined by a special balance between investments in conflict and possession of land – the determinants of power p. Rules that deviate from this special balance cannot be enforced.

To illustrate this point assume that both individuals agree to the following set of rules: first, the cooperative-anarchy distribution a^A is called property. Second, the owner

[15]They are *coarser* because the set of distributions of F that can be attained using property rights alone is smaller than the set of distributions that can be attained using property rights together with rules of redistribution.

of some fraction of land is also owner of the goods that are produced by means of that fraction of land (principle of returns[16] ω: who owns a fraction a of land also owns aF of the crop). This set of rules is codified as a "constitution" $\{a^A, \omega\}$. Whenever $V_1(a^A, \omega) = a^A F(a^A) \neq p(a^A)F(a^A) = V_1(a^A)$, one individual will break the constitutional contract at stage 4. Therefore, rules of redistribution must be used to correct for deviations between the constitutional principles (in this example the principle of returns) and the distribution of power.

Therefore, the enforcement of rules cannot be taken for granted, as it is done in most of the literature on constitutional economics. Brennan and Buchanan (1985), for example, argue that one of the major reasons for rules is *"that without them we would surely fight."* (p.3) They clearly recognize the problem of enforcement (*"In the absence of effective enforcement procedures, adherence to rules rather than departure from them requires that individuals forswear expected utility maximization, ..."*), but nevertheless treat the problem as being solved for the rest of their analysis. It is the purpose of this section to explicitly deal with the problem of enforceability of rules in the absence of an exogenous enforcement agency. The basic new insight of this section is not the way we solve the problem of enforceability (by transforming the stage game into an indefinitely repeated game), but the interpretation of a constitution that follows from this argument.

If it is impossible to establish cooperative behavior in a one-shot game, it is possible to establish it in a large number of cases if the game is indefinitely repeated. This is the basic logic of the folk theorem. In the context of constitutions the adoption of this argument allows valuable insights into the nature of constitutions. Let us assume that the subgame beginning at stage 2 is infinitely repeated, $t = 0, ..., \infty$, and that $\delta \leq 1$ is the discount factor on future consumption.[17] In this case a rule r can be established that generates a^* as an equilibrium if certain requirements are met.[18] For the formulation of this rule we assume w.l.o.g. that a Pareto-improving move from a^A to a^* requires more land to be given to the more efficient individual 1. Individual 2 gives up land and receives a compensation in terms of corn.

Rule r: Both individuals agree to distribute land according to a^* at stage 2. This distribution is fixed in all future periods. Individual 2 invests $y_2(a^*)$

[16] Wärneryd (1993) calls this the homestead principle.

[17] There are two possibilities to interpret this assumption. First, one could assume the existence of dynasties where the children do the same as the parents. Second, one could motivate the model with infinite time horizon by an equivalent model with finite time horizon, where the discount factor is a measure of the probability of survival.

[18] For a detailed discussion of the term 'rule' see Ostrom (1986).

as long as individual 1 pays the transfer τ. If individual 1 does not pay the transfer in period t, individual 2 punishes individual 1 by applying a minmax strategy for n periods.

Definition 2: A rule r that implements $\{a^*, \tau\}$ as an equilibrium in repeated interaction is called a *civil society.*

In a formal way, rule r can be characterized as follows. The minmax value for individual 1 is given by

$$\min_{y_2} \max_{y_1} p(y_1, y_2, a^*) F(1 - y_1, 1 - y_2, a^*). \tag{10}$$

Maximization with respect to y_1 yields the reaction function $y_1(y_2, a^*)$. We substitute this reaction function into (10) and differentiate with respect to y_2. Using the envelope theorem, we obtain $p_2 F - p F_2$, which is smaller than zero for all values of y_2. Hence, $y_2 = 1$ is the minmax strategy of individual 2. The rule can thus be specified as the tuple $r = \{a^*, \tau, \{1, n\}\}$.[19] The punishment strategy $\{1, n\}$ has a straightforward interpretation as a *penal code* of the society.

Three requirements must be met to sustain a civil society under rule r:

• The transfer must be large enough to induce agreement of individual 2, that is $\tau \geq \overline{V}_2(a^A) - \overline{V}_2(a^*)$.

• Individual 2 must be able to punish individual 1. Recall the definition of a^* which implies

$$\overline{V}_1(a^*) \geq V_1(y_1(1, a^*), 1, a^*). \tag{11}$$

If (11) holds with equality, individual 2 is not able to punish individual 1 for not paying the transfer. In this case it is impossible to improve upon cooperative anarchy despite the fact that potential Pareto-improvements exist. There is simply no way to overcome the problem of self-enforcement of constitutional rules. If, however, the inequality in (11) holds, individual 2 is able to punish individual 1 for deviations from the transfer scheme.

• Individual 1 must be interested in not cheating but paying the compensation τ, otherwise the redistributional component of $r = \{a^*, \tau, \{1, n\}\}$ breaks down. Therefore, we have to compare two situations: given the acceptance of r, if individual 1 decides to pay τ, his discounted payoff is

$$\mathcal{V}(\tau) = \sum_{t=0}^{\infty} \delta^t \left(\overline{V}_1(a^*) - \tau \right).$$

[19] Note that it is a-priori unclear whether $\overline{V}_2(a^A) \gtreqless V_2(y_1(1, a^*), 1, a^*)$. If the left-hand side is larger then the right-hand side, individual 2 can lose from agreeing on the distribution a^*.

In contrast, if individual 1 decides to cheat, his discounted payoff is

$$\mathcal{V}(0) = \overline{V}_1(a^*) + \sum_{t=1}^{n} \delta^t V_1(y_1(1,a^*),1,a^*) + \sum_{t=n+1}^{\infty} \delta^t \left(\overline{V}_1(a^*) - \tau\right).$$

The civil society can only be implemented if $\mathcal{V}(\tau)$ exceeds $\mathcal{V}(0)$, that is,[20]

$$\mathcal{V}(\tau) \geq \mathcal{V}(0) \Leftrightarrow \frac{\delta(1-\delta^n)}{1-\delta^{n+1}} \left(\overline{V}_1(a^*) - V_1(y_1(1,a^*),1,a^*)\right) \geq \tau. \tag{12}$$

Taken together, the above three requirements imply the following proposition:

Proposition 6:

(i) A necessary condition for the deviation from cooperative anarchy is that individual 2 can punish individual 1 for deviations from the transfer scheme, $\overline{V}_1(a^*) > V_1(y_1(1,a^*),1,a^*)$.

(ii) A civil society can be sustained as an equilibrium with penal code $\{1,n\}$ if

$$\delta \geq \frac{\overline{V}_2(a^A) - \overline{V}_2(a^*)}{\overline{V}_1(a^*) - V_1(y_1(1,a^*),1,a^*)}.$$

(iii) If $\overline{V}_1(a^*) + \overline{V}_2(a^*) < V_1(y_1(1,a^*),1,a^*) + \overline{V}_2(a^A)$, a civil society can never be implemented.

Therefore, in order to make a civil society implementable by repeated interaction, the threat of punishment for individual 1, $\overline{V}_1(a^*) - V_1(y_1(1,a^*),1,a^*)$, has to exceed the uncompensated loss for individual 2 that results from the redistribution of land, $\overline{V}_2(a^A) - \overline{V}_2(a^*)$. If this condition is met we have established a way to support $\{a^*, \tau\}$ as an equilibrium in the repeated subgame. Rules that are intended to turn potential Pareto improvements into actual Pareto improvements have to be supplemented by a system of penalties. Hence, repeated interaction is a way to enforce institutions that would otherwise be unenforceable.[21]

A large class of distributions a and rules of redistribution τ can be sustained as equilibria. In fact, any pair $\{a, \tau\}$ for which

$$\overline{V}_1(a) \geq V_1(y_1(1,a),1,a), \tag{13}$$

$$\delta \geq \frac{\overline{V}_2(a^A) - \overline{V}_2(a)}{\overline{V}_1(a) - V_1(y_1(1,a),1,a)} \tag{14}$$

can be sustained as an equilibrium of the repeated subgame given an adequately defined penal code $\{1,n\}$. It is well known that the multiplicity of folk-theorem equilibria creates

[20] Note that the condition (12) gives an implicit definition of the length of the period of punishments n.

[21] See Axelrod (1984, 1997).

an equilibrium-selection problem. Therefore, any mechanism that leads to a Pareto improvement must necessarily incorporate an equilibrium-selection criterion. According to Binmore (1998) a fairness norm is the required criterion that completes the mechanism. We argue that similar to fairness norms, constitutions can play the role of an equilibrium-selection criterion. For this purpose we will argue along the lines of the theory of focal points (Schelling 1960).

6.2 Constitutions as coordination device

It has been argued that starting from anarchy, constitutions are established because they entail higher enforcement capacity (Azariadis and Galasso 1999, Ostrom 1986). Alternatively, Brennan and Buchanan (1985) argue that constitutions are established because the population wants special rules with high intrinsic commitment that is guaranteed by the unanimity rule. However, the unanimity rule will only be respected if the population can rely on its enforcement. Accordingly, Azariadis and Galasso (1999), Ostrom (1986), and Brennan and Buchanan (1985) consider two sides of the same coin. Buchanan's two-stage theory of constitutions derives the increased commitment capacity of constitutions from the unanimity that is required for their change. Our analysis has demonstrated that this point of view is ill-conceived. It is impossible to deviate from the real allocation of power in the implementation of rules. Rules that deviate from the individuals' balance of power cannot survive in equilibrium, thus they will be abolished by the powerful individual(s) irrespective of any formal requirements on unanimity etc.[22] The acceptance of the unanimity rule *presupposes* enforcement power that is not endogenized within the model. By the same token our analysis has shown that assuming an exogenous superiority with respect to enforcement is ill-conceived. If one tries to endogenize enforcement, one finds that there is no difference between ordinary and constitutional rules.

According to Kliemt (1993), *"we should think of rights primarily as social facts that are brought into existence by the rule-observing and rule-enforcing behavior of human actors."* Methodological individualists cannot refer to natural rights[23] ("it is a matter of fact") or religious ("it is god's will") lines of argumentation to justify the self evidence and

[22] The "quiet revolution" in the former German Democratic Republic is a good example of this point.

[23] Normative theories in the natural-rights tradition take the view that value judgments have the same objective status as, for example, laws in physics. Moral rules can be discovered and deduced from facts in the same way as, for example, the law of gravity in physics has been discovered. Since Hume ("naturalistic fallacy": the ought cannot be deduced from the is), naturalistic premises are no longer accepted as valid foundations of normative theories in mainstream practical philosophy. Early utilitarianism is an example of a naturalistic theory. It is Harsanyi's merit to transform utilitarianism from a naturalistic to a contractarian theory in his 1953 and 1955 papers.

enforcement of constitutional rules. It must be the adequately defined majority of the population that supports constitutional rules in order to justify them.

Constitutions are distinguished from ordinary rules by their potential ability to *condition individual expectations* with respect to a certain equilibrium. This potential ability stems from the mere fact that a rule is called a "constitution." It is the specific name that makes a rule an outstanding basis of the coordination of individual beliefs. In other words, naming a rule a constitution makes it a focal point. By the same token, religions, fairness norms, and theories of justice can be seen as competing "stories," trying to coordinate beliefs in a certain equilibrium.[24] If the majority believes in the moral superiority of a specific rule, the economic equilibrium that is sustained by this rule becomes focal. Vice versa, the limits of moral values in our model are revealed whenever an efficient solution cannot be implemented as a folk-theorem equilibrium.

Therefore, the exact story that is told in connection with a "constitution" becomes essential for its ability to coordinate behavior, not for its allocative properties. Different moral rules or religious beliefs might give rise to different constitutions that are equivalent from an efficiency point of view. If the population finds a constitution morally more appealing than any other, then it may be accepted and operate as an instrument to coordinate beliefs such that the equilibrium is attained. The decisive difference between rules and constitutions cannot be understood by reference to their different commitment and enforcement properties, but only by reference to their appropriateness as a coordination device. In this setting, constitutions are codified social norms that help to overcome the equilibrium-selection problem.

Our analysis leads to a conclusion that is similar to the one in Hardin (1989) who argues that constitutions differ qualitatively from contracts. According to Hardin, contracts typically govern prisoner's dilemma situations, whereas constitutions typically govern coordination problems. Our findings support Hardin's conception with respect to this aspect, and in fact both concepts are close. They differ, however, with respect to the logical relationship that is presumed between contracts and constitutions. Hardin sees a constitution as logically prior to contracts because *"it creates the institution of contracting"* (p.101) in the sociological sense of creating a culture of accepting rules of conduct. Hence, a constitution is not a meta-contract that is necessary to credibly enforce contracts. According to our concept, rules or contracts are logically prior to constitutions because a constitution is a rule plus the act of communication necessary to coordinate beliefs in this rule.[25]

[24]This conclusion holds as long as preferences are not influenced by moral codes. If one assumes that moral codes change preferences, their economic function might exceed the coordination of beliefs.

[25]One could argue that the requirement of self-enforcement is the distinguishing feature between consti-

6.3 Disarmament Treaties

Until now we have assumed that individuals restrict coordination to the distribution of land and crop. The two players never chose to coordinate their investments in conflict, but rather chose the Nash-equilibrium investments which were optimal in the subgame-perfect equilibrium. In a repeated-game context, however, it is no longer necessary to restrict coordination to the distribution of land and crop. It might instead be possible to reach disarmament treaties according to which individuals agree on $y_i < y_i(a)$ for a given value of a.

A first step in this direction would be to redistribute land from a^A to a^* and to abolish any non-productive activity, that is, to choose $y_1 = 0$ and $y_2 = 0$. Suppose that conflict has prisoner's dilemma aspects, $\overline{V}_i(a^*) < V_i(0,0,a^*)$ and assume a redistribution scheme of $\tau = \overline{V}_k(a^A) - V_k(0,0,a^*)$, where k is the person who gets the transfer. Such a conflict-free society is sustainable whenever a civil society is sustainable – and Pareto dominates it.

There may even exist cases where the individuals could choose a conflict-free society with $a > a^*$. In the extreme case, it may even be possible to find a rule which supports an equilibrium with $y_1 = y_2 = 0, a = 1$, which would imply the realization of the first best. (Recall our treatment of the first-best benchmark at the end of section 3.)

> **Definition 3:** A rule r that implements $y_1 = y_2 = 0, a = 1$ as an equilibrium in repeated interaction is called a *perfect civil society.*[26]

A transition from cooperative anarchy to a perfect civil society requires that more land has to be given to the more efficient individual 1 ($a^A \leq 1$). Individual 2 receives a transfer of corn and punishes 1 if he does not pay the transfer. Accordingly, in the perfect civil society redistribution is bound by the limits $\underline{\tau} = \overline{V}_2(a^A)$ and $\overline{\tau} = V_1(0,0,1) - \overline{V}_1(a^A)$, because without any transfers we would have

$$V_1(0,0,1) = F(0,0,1) > \overline{V}_1(a^A), \tag{15}$$

$$V_2(0,0,1) = 0 < \overline{V}_2(a^A). \tag{16}$$

To implement a perfect civil society as an equilibrium, a penal code has to be established. The penality strategy is derived as before and is denoted by $y_1 = y_1(1,1), y_2 = 1$.[27]

tutions and contracts. This view overlooks that many business contracts rely on self-enforcement because of transaction costs or nonverifiable contingencies.

[26]The modified Tullock function that we have used as an example has a discontinuity at $y_1 = y_2 = 0$. This discontinuity, however, does not matter here because the penalty strategy implies a discrete change in strategies from $(0,0)$ to $(y_1(1), 1)$.

[27]This is the solution of a minmax problem analogous to (10).

The associated payoffs are $V_1(y_1(1,1),1,1)$ and $V_2(y_1(1,1),1,1)$. Therefore, we obtain the following condition for the implementability of a perfect civil society:

$$\frac{\delta(1-\delta^n)}{1-\delta^{n+1}}\Big(V_1(0,0,1)-V_1(y_1(1,1),1,1)\Big)\geq\tau. \tag{17}$$

This allows to formulate the following proposition:

Proposition 7: (i) A perfect civil society can be sustained as an equilibrium with penal code $\{1,n\}$ if

$$\delta\geq\frac{\overline{V}_2(a^A)}{V_1(0,0,1)-V_1(y_1(1,1),1,1)}.$$

(ii) If $V_1(0,0,1)-V_1(y_1(1,1),1,1)<\overline{V}_2(a^A)$, a perfect civil society can never be implemented.

6.4 The role of synergies in group formation

A perfect civil society can only be implemented if the threat of punishment for individual 1, $V_1(0,0,1)-V_1(y_1(1,1),1,1)$, is sufficiently large. If the value of this threat does not exceed the minimum compensation that has to be paid to compensate individual 2 for the loss of land, penal codes that rely on the repeated interaction of individuals are insufficient. If the production function requires time investments of both players, $F(0,x_2,a)=F(x_1,0,a)=0$, individual 2 can always prevent the production of corn by playing $y_2=1$. It is easy to check that in this case a perfect civil society can always be implemented.

A similar argument holds for the case of a civil society. Propositions 6 and 7 show that a civil and a perfect civil society are easier to implement the lower individual 1's minmax value.[28] This value decreases with the degree of complementarity between both individuals' time inputs in production. This observation can be generalized to a hypothesis for the formation of societies. Credible penal codes are easier to define the more complementary the individuals' time investments in production. Since penal codes are necessary for the implementation of civil societies, the formation of civil societies is more likely the more complementary the individuals' time investments for the production of corn.

This rationalizes the wide-spread belief that group formation is more likely the more synergies exist between individuals. If these synergies are low, individuals organize as a co-operative anarchy, if the synergies are large, a civil or a perfect civil society might emerge.

[28]The reader should consider individual 1's minmax value $V_1(y_1(1,a^*),1,a^*)=p(y_1(1,a^*),1,a^*)F(1-y_1(1,a^*),0,a^*)$ in the conditions (i) and (ii) of Proposition 6 and the value of $V_1(y_1(1,1),1,1)$ in condition (i) of Proposition 7.

In the light of this result, the formation of national states, the process of globalization, and the formation of supranational organizations like the European Union can be attributed to the technological progress that makes individual skills more complementary. The degree of economic integration should be positively correlated with some measure of the complementarity of technological skill: this is a falsifiable hypothesis which is supported by our model. The reader familiar with the theory of the firm developed by Grossman and Hart (1986) and Hart and Moore (1990) will recognize the similarity between our result and their results on vertical integration.

Another implication of the model is the following: since the change from cooperative anarchy toward civil societies is due to gains from the redistribution of resources, increasing complementarity yields more concentration of property. The idea is as follows: assume that production and conflict technologies are such that the restrictions in Proposition 6 (ii) or 7 (i) are binding. An increase in complementarity between both individuals' productive inputs reduces individual 1's minmax value. This implies that the restriction is no longer binding, or analogously, that there is a larger number of cases where a civil or perfect civil society can be implemented. Therefore, individual 2 is willing to redistribute land to individual 1 more often or in excess of the previous level. This leads to more concentration of property. This implication is roughly compatible with empirical findings.[29]

7 Normative implications of the model

Most of the analysis so far has focused on the positive interpretation of the model. The model, however, has straightforward normative implications if it is adequately interpreted. Following the standard contractarian philosophy of the social contract, the design of the initial situation of anarchy should reflect moral intuitions about fairness. For Rawls (1971), the initial situation is fair if individuals abstract from their real-life identities. Thus, he develops the idea of a 'filter,' the veil of ignorance, that withholds from the individuals all information that is seen as morally relevant. Thus, the veil of ignorance creates a situation of moral impartiality in which individual rationality and collective justice coincide.[30]

Despite its intellectual attractiveness, the veil of ignorance suffers from a conceptual problem that is well known in political philosophy and that we will therefore mention only

[29] See IMF Fiscal Affairs Department (1998).

[30] The treatment of collective conflicts by the introduction of a veil of ignorance has great intellectual elegance that makes it attractive for the normative analysis of institutions. This model has often been used, for example by Harsanyi (1953, 1955) for his justification of utilitarianism and Gauthier (1986) in his game-theoretic reconstruction of just institutions.

briefly. Contractarianism is attractive because the voluntary acceptance of a contractual obligation creates a situation of mutual obligation without reference to any naturalistic or metaphysical arguments: one can expect the fulfillment of contractual obligations without reference to the specific situation the individuals live in, because they accepted the obligation on a voluntary basis. It is precisely this feature that gets lost if a veil of ignorance is introduced: real individuals are in no way bound by decisions made by hypothetical individuals under a veil of ignorance. Thus, if justice requires the abstraction from real-life circumstances, contractarianism loses its obligational power.[31] [32]

Let us now interpret our model as a normative contractarian approach. Then we recognize that it is a major advantage of our model that we either do not need any veil of ignorance or, in an alternative interpretation, only a "thin" veil of ignorance. Note first that in the initial situation of our model, individuals know their preferences and the productivities they will have at the post-constitutional stage. At stage 2 the individuals have to agree on the rules according to which they will live at the post-constitutional stages 3 and 4. Therefore, the obligational power of mutual agreements is not diluted due to a change in "identities" of the individuals between the pre- and post-constitutional stage: we never need a veil of ignorance with respect to individual preferences and productivities. However, with respect to the possession of land, two diverging interpretations of our model are possible.

If we follow Nozick (1974), we do not need any veil of ignorance to justify redistribution in our model. It is just fine to start from the individuals' actual possession of land at date 0 as long as this distribution of land meets Nozick's principle of just acquisition of goods and resources that are not yet privately possessed. Nozick does not postulate a starting position of an equal initial possession of land. According to his view, the appropriation of unowned resources is "just" as long as it makes no one worse off than he would have been without the appropriation. Hence, the utilization of resources plays a major role in the evaluation of just appropriation. This conception respects a first-come-first-serve principle:

[31]Rawls himself seems to be well aware of this conceptual conflict of contractarianism because he embedded the contractarian argument in the broader concept of a "reflective" equilibrium. Individuals are bound to the consequences of the contract not because this contract created any obligation *by itself* but because of an intrinsic, Kantian urge for logical consistency of the actual individuals who accepted the use of the contractarian argument as a model to shape their moral intuitions: one cannot accept a situation as being fair and at the same time reject its consequences. This is why Rawls – especially in his later writings – does not see himself in a contractarian tradition but in the tradition of Kant.

[32]Buchanan in his various writings uses veils of different thickness in order to clarify the interrelation of the respective veil and the resulting institutions. For example, he does not take away the individual identities from the individuals in the veil of ignorance. Buchanan's playing around with different specifications of the initial situation has been criticized for its apparent explanatory emptiness.

justice of appropriation does not require that the individual with the highest productivity possesses the resource. An individual who creates a positive but low public value from the use of a resource but who is lucky enough to stumble over the resource first is the legitimate owner of the resource. Given such a legitimate initial distribution there is, in general, room for Pareto-improving reallocations of land as long as they are accompanied by rules of redistribution of corn. From this point of view, the redistribution of corn is a means to legitimize certain distributions of land even in a radical liberal conception of justice as given by Nozick. Moreover, since Pareto improvements are only possible if the rules of redistribution are supported by a penal code, even a constitution in the sense of Nozick would entail both redistribution and coercive power.[33]

If, on the other hand, we follow Kolm (1996), we have to assume a veil of ignorance. However, this veil is "thin," because it refers only to the possession of land and not to any other personal attributes like preferences or productivities. This thin veil becomes necessary because any theory of just redistribution according to Kolm requires an equal initial endowment of land, $a^o = 1/2$, and since this is not the actual distribution of land, the agents face a veil of ignorance with respect to the initial possession of land. Kolm argues as follows: veils of ignorance of different thickness involve different judgments about attributes that are morally relevant.[34] The basic principle of morality is equality. Theories of justice seek to justify why one allocation is better than another. To justify means to give a reason. It is only the equal distribution of all relevant variables that passes this check.[35] Therefore, if one agrees on a set of morally relevant variables, all individuals should ideally have equal access to these attributes.

An interpretion of our model à la Kolm has to assume that only the initial possession of land is morally relevant. In contrast, the individual differences between productive and bargaining productivities would be seen as morally irrelevant. Therefore, it follows from Kolm's basic principle of justice that the initial situation of anarchy would be fair if at stage 0 both individuals have an equal initial endowment of land, $a^o = 1/2$.[36] The

[33]Nozick postulates three principles of just entitlement: (1) the just acquisition of goods and resources that are not yet privately possessed, (2) the voluntary transfer of goods and resources, and (3) the rectification of violations against (1) and (2). The principle of voluntariness requires redistributive rules, the principle of rectification requires the existence of coercive power.

[34]See Kolm (1996), chapters 3 to 9, who develops the relationship between morally relevant attributes and institutional consequences in great detail.

[35]See Kolm (1996), section 2.3.

[36]There is a close connection between this interpretation of our model and the *theory of equity* developed by Tinbergen (1946), Foley (1967) and Kolm (1997). In their theory, inequality is justified as long as it Pareto-improves the situation of ideal equality of the morally relevant variables. The main difference between the theory of equity and the Kolm-type interpretation of our model is the following: the charac-

institutional consequences that result from this specification operationalize a just society. Most important for this normative evaluation is our Proposition 5: rules of redistribution can be normatively justified for a large number of cases. This finding is complementary to the standard justification of rules of redistribution as an insurance contract under the veil of ignorance. This justification assumes that individuals are risk averse under the veil of ignorance. The hypothetical identity in the initial situation solves the problem of ex-post diverging interests, and thus guarantees unanimous support. Risk aversion urges the individuals to seek insurance against the hypothetical risk of becoming a "bad" type. Hence, the veil of ignorance makes individuals *identical* with respect to the morally relevant variables. *Redistribution is legitimized because hypothetical individuals are identical and fear to become different in real life.*

In contrast, in our model redistribution is legitimized because individuals are *different* in the initial situation. This reverses the logic of legitimization. Individuals in the initial situation know that they differ with respect to their productivities. Redistribution of *land* is a means to exploit these differences. Redistribution of land and redistribution of corn are two sides of the the same coin: the individual who gives up land has a *legitimate claim* for compensation. Since he cannot be paid by the other individual at the pre-production stage, rules of redistribution serve as an institutional substitute for direct payments. Therefore, redistribution is neither legitimized by insurance motives nor by private charity, but follows from the principle of reciprocity in trade.

Summarizing, our analysis has demonstrated that redistributive rules are not only in accordance with, but are in general a distinctive feature of, liberal conceptions of society. That this result can be derived without recourse to any veil of ignorance, at least without recourse to "thick" veils of ignorance, avoids obligational problems inherent in most other contractarian conceptions of justice, and that it can be derived without risk aversion gives an alternative perspective for the evaluation of existing rules of redistribution.

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terization of just allocations is at the heart of the theory of equity whereas the structure of just institutions is central to our model. Consequently, the theory of equity takes as given the perfect enforcement of complete property rights. For reasons that are given in section 2 above the theory of equity, therefore, is not able to derive institutional consequences, for example a concept of redistribution; all potential Pareto improvements are at the same time actual Pareto improvements because of the existence of perfect property rights.

Acknowledgements

We gratefully acknowledge helpful comments by Friedrich Breyer, Sam Bucovetsky, Louis Gevers, Mathias Kifmann und Carlo Schultheiss and various participants in the ISPE Conference "Public finance and redistribution", CORE, June 22-24, 2000, where a preliminary version of the paper was presented.

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Appendix (not for publication)

A.1 The initial Nash equilibrium

A.1.1 Proof of Proposition 1:

The second-order conditions of V_1, V_2 with respect to y_1, y_2 are

$$\partial^2 V_1/\partial y_1^2 = p_{11}F - 2p_1F_1 + pF_{11} < 0, \tag{A.1}$$

$$\partial^2 V_2/\partial y_2^2 = -p_{22}F + 2p_2F_2 + (1-p)F_{22} < 0. \tag{A.2}$$

The strict inequalities follow from assumptions 1 and 2. Both individuals' optimization problems are strictly convex, which implies that a maximizer exists and is unique. This guarantees the existence of reaction functions $y_1(y_2, a) : [0,1] \times [0,1] \to [0,1], y_2(y_1, a) : [0,1] \times [0,1] \to [0,1]$. Furthermore, $p, p_1, p_2, F, F_1, F_2, F_{11}, F_{22}$ are all continuous which implies that the reaction functions are continuous as well. The existence of an equilibrium is therefore a direct consequence of Brouwer's fixed-point theorem. □

A.1.2 Proof of Proposition 2:

In order to guarantee uniqueness, we introduce the notion of local stability of an equilibrium. An equilibrium $\overline{y}_1, \overline{y}_2$ is said to be locally stable if $\partial y_1(y_2, a)/\partial y_2 \cdot \partial y_2(y_1, a)/\partial y_1 < 1$.

> **Lemma 1:** If there exists no equilibrium that is locally unstable, then the equilibrium $\overline{y}_1, \overline{y}_2$ is unique.

Proof: See Skaperdas (1992), proof of Theorem 2. □

Hence, it suffices to rule out local instability of equilibria in order to make the comparative-static analysis meaningful.

> **Lemma 2:** Every boundary equilibrium $\overline{y}_1, \overline{y}_2$ is locally stable.

Proof: Local stability requires $\partial y_1(y_2, a)/\partial y_2 \cdot \partial y_2(y_1, a)/\partial y_1 < 1$. At a boundary equilibrium it must be that $\partial y_i(y_j, a)/\partial y_j = 0, i, j = 1, 2, i \neq j$ for at least one individual $i = 1, 2$.
□

> **Lemma 3:** An interior equilibrium $\overline{y}_1, \overline{y}_2$ is locally stable if and only if

$$\begin{aligned} & \left(F(p_{11}p(1-p) - 2p_1^2(1-p)) + p^2(1-p)F_{11}\right) \\ & \left(-F(p_{22}p(1-p) + 2p_2^2p) + (1-p)^2pF_{22}\right) \\ > & \left(F(p_{12}p(1-p) + (2p-1)p_1p_2) + p^2(1-p)F_{12}\right) \\ & \left(-F(p_{12}p(1-p) + (2p-1)p_2p_1) + (1-p)^2pF_{12}\right) \end{aligned} \tag{A.3}$$

is fulfilled at the equilibrium value.

Proof: Local stability requires $\partial y_1(y_2, a)/\partial y_2 \cdot \partial y_2(y_1, a)/\partial y_1 < 1$. The derivative of the reaction function of individual i with respect to a change in y_j can be derived by totally differentiating the first-order condition of individual i with respect to y_i and y_j (remember that we are in an interior equilibrium, hence $\partial V_i/\partial y_i = 0$). This yields

$$\frac{\partial y_i(y_j, a)}{\partial y_j} = -\frac{\dfrac{\partial^2 V_i}{\partial y_i \partial y_j}}{\dfrac{\partial^2 V_i}{\partial y_i^2}}, \qquad i, j = 1, 2, i \neq j. \tag{A.4}$$

Hence, local stability requires that

$$\frac{\dfrac{\partial^2 V_1}{\partial y_1 \partial y_2}}{\dfrac{\partial^2 V_1}{\partial y_1^2}} \frac{\dfrac{\partial^2 V_2}{\partial y_2 \partial y_1}}{\dfrac{\partial^2 V_2}{\partial y_2^2}} < 1. \tag{A.5}$$

The second derivatives in (A.5) are $\partial^2 V_1/\partial y_1^2 = p_{11}F - 2p_1 F_1 + pF_{11}$, $\partial^2 V_1/(\partial y_1 \partial y_2) = p_{12}F - p_1 F_2 - p_2 F_1 + pF_{12}$, $\partial^2 V_2/\partial y_2^2 = -p_{22}F + 2p_2 F_2 + (1-p)F_{22}$, and $\partial^2 V_2/(\partial y_2 \partial y_1) = -p_{12}F + p_1 F_2 + p_2 F_1 + (1-p)F_{12}$. Note that the first-order conditions imply that $F_1 = p_1 F/p$, and $F_2 = -p_2 F/(1-p)$ and, therefore, $2p_1 F_1 = 2p_1^2 F/p$, $2p_2 F_2 = -2p_2^2 F/(1-p)$. Substituting in the above expressions and rearranging yields Lemma 3. $\square$

Lemmas 1 to 3 imply Proposition 2. $\square$

Note that Skaperdas (1992) assumes $p(1-p)p_{12} + (2p-1)p_1 p_2 = 0$. However, Skaperdas's assumption and the assumptions on the derivatives of the production function constitute a very strong sufficient condition for local stability in our model. Therefore, we prefer the much weaker assumption (A.3).

A.1.3 Types of equilibria and conditions for their existence

Several types of equilibria can exist in anarchy ranging from full cooperation ($y_1 = y_2 = 0$) to full conflict ($y_1 = y_2 = 1$). The first-order conditions for an equilibrium are given in (3) and (4). We will derive the conditions for the existence of a fully cooperative equilibrium. The derivation for all other types of equilibria runs analogously and is summarized in table A.1.[37]

A fully cooperative equilibrium exists iff

$$p_1(0,0,a)F(1,1,a) - p(0,0,a)F_1(1,1,a) \leq 0, \tag{A.6}$$

$$-p_2(0,0,a)F(1,1,a) - (1 - p(0,0,a))F_2(1,1,a) \leq 0. \tag{A.7}$$

[37] The complete proof of table A.1 can be received from the authors upon request.

The assumption of constant returns to scale of F in x_1 and x_2 implies $F(x_1, x_2, a) = x_1 F_1(x_1, x_2, a) + x_2 F_2(x_1, x_2, a)$. Hence, $F(1,1,a) = F_1(1,1,a) + F_2(1,1,a)$. Rearranging (A.6) yields[38]

$$\frac{p_1(0,0,a)}{p(0,0,a) - p_1(0,0,a)} \leq \frac{F_1(1,1,a)}{F_2(1,1,a)}. \tag{A.8}$$

For (A.7) a similar procedure yields

$$\frac{1 - p(0,0,a) + p_2(0,0,a)}{-p_2(0,0,a)} \geq \frac{F_1(1,1,a)}{F_2(1,1,a)}. \tag{A.9}$$

Since $p(y, y, a) = a$, we can combine (A.8) and (A.9) to obtain

$$\frac{p_1(0,0,a)}{a - p_1(0,0,a)} \leq \frac{F_1(1,1,a)}{F_2(1,1,a)} \leq \frac{1 - a + p_2(0,0,a)}{-p_2(0,0,a)}. \tag{A.10}$$

As can be seen in table A.1, nine types of equilibria may occur in this model depending on the conflict and production technologies. *Partial cooperation* is the situation where both individuals decide to invest part of their time in conflict and part of their time in production. If both individuals invest all (none) of their time in conflict, the resulting equilibrium is called *full conflict (full cooperation).* There are, however, a number of asymmetric equilibria. If one individual decides not to invest in conflict and the other does, the equilibrium is called *cooperative submission*, and finally, if one of the individuals invests all of his time in conflict and the other one only part of it, the equilibrium is called *conflict submission.* In the extreme case where one individual invests all of his time in conflict and the other none, the equilibrium is called *polarized.*

Not all types of equilibria can always occur. In particular, full conflict can only occur if $F_1(0, x_2, a) = 0$ and $F_2(x_1, 0, a) = 0$. This implies complementarity between both individuals' productive inputs; only if both individual labor inputs are needed to produce a minimum amount of output, full conflict can be sustained as an equilibrium.

[38]This condition is derived under the assumption $p_1 < p$. Otherwise, (A.6), $-(p - p_1)F_1 + p_1 F_2 \leq 0$, cannot hold.

	eq. strategies		FOC		condition for existence
	y_1	y_2	ind. 1	ind. 2	
partial coop.:	$(0,1)$	$(0,1)$	$V_1^1 = 0$	$V_2^2 = 0$	$\frac{(1-y_2)p_1}{p-(1-y_1)p_1} = \frac{F_1}{F_2} = \frac{(1-p)+(1-y_2)p_2}{-(1-y_1)p_2}$ $\wedge p_2(1-y_2)+(1-p) > 0, p - p_1(1-y_1) > 0$
full conflict:	1	1	$V_1^1 \geq 0$	$V_2^2 \geq 0$	$aF_1 \leq 0$ $\wedge (1-a)F_2 \leq 0$
full coop.:	0	0	$V_1^1 \leq 0$	$V_2^2 \leq 0$	$\frac{p_1}{a-p_1} \leq \frac{F_1}{F_2} \leq \frac{(1-a)+p_2}{-p_2}$ $\wedge p_2 + (1-a) > 0, a - p_1 > 0$
coop. subm.1:	0	$(0,1)$	$V_1^1 \leq 0$	$V_2^2 = 0$	$\frac{(1-y_2)p_1}{p-p_1} \leq \frac{F_1}{F_2} = \frac{(1-p)+(1-y_2)p_2}{-p_2}$ $\wedge p_2(1-y_2)+(1-p) > 0, p - p_1 > 0$
coop. subm.2:	$(0,1)$	0	$V_1^1 = 0$	$V_2^2 \leq 0$	$\frac{p_1}{p-(1-y_1)p_1} = \frac{F_1}{F_2} \leq \frac{(1-p)+p_2}{-(1-y_1)p_2}$ $\wedge p_2 + (1-p) > 0, p - p_1(1-y_1) > 0$
conflict subm.1:	$(0,1)$	1	$V_1^1 = 0$	$V_2^2 \geq 0$	$\frac{F_1}{F_2} \geq \frac{(1-p)}{-(1-y_1)p_2}$ $\wedge \epsilon_1^p := p_1 \frac{(1-y_1)}{p} = 1, -p_1(1-y_1) + p > 0$
conflict subm.2:	1	$(0,1)$	$V_1^1 \geq 0$	$V_2^2 = 0$	$\frac{(1-y_2)p_1}{p} \geq \frac{F_1}{F_2}$ $\wedge \epsilon_2^{1-p} := -p_2 \frac{1-y_2}{1-p} = 1, p_2(1-y_2) + (1-p) > 0$
polarization 1:	0	1	$V_1^1 \leq 0$	$V_2^2 \geq 0$	$\frac{F_1}{F_2} \geq \frac{(1-p)}{-p_2}$ $\wedge p_1 < p, p - p_1 > 0$
polarization 2:	1	0	$V_1^1 \geq 0$	$V_2^2 \leq 0$	$\frac{p_1}{p} \geq \frac{F_1}{F_2}$ $\wedge -p_2 < (1-p), p_2 + (1-p) > 0$

Table A.1: Equilibrium strategies and conditions for their existence. (All functions are evaluated at the values given in columns y_1, y_2; $V_i^i := \partial V_i / \partial y_i, i = 1, 2$; subm. = submission.)

A.1.4 Comparative-static analysis of the Nash equilibrium

A.1.4.1 Changes in $\overline{V}_i$ due to changes in a

The simultaneous solution of (3) and (4) results in Nash-equilibrium functions $y_1(a), y_2(a)$. Substituting in V_1 and V_2 yields $\overline{V}_1(a), \overline{V}_2(a)$. Assume first that the first-order conditions (3) and (4) are fulfilled as equalities, $\partial V_1/\partial y_1 = 0, \partial V_2/\partial y_2 = 0$ (the derivatives are evaluated at the Nash-equilibrium values of y_1 and y_2). Differentiation of $\overline{V}_1$ with respect to a yields

$$\begin{aligned}\frac{\partial \overline{V}_1}{\partial a} &= \left(p_1 \frac{dy_1}{da} + p_2 \frac{dy_2}{da} + p_a\right) F + \left(-F_1 \frac{dy_1}{da} - F_2 \frac{dy_2}{da} + F_a\right) p \\ &= \underbrace{(p_1 F - pF_1)}_{=0} \frac{dy_1}{da} + \underbrace{(p_2 F - pF_2)}_{-F_2} \frac{dy_2}{da} + p_a F + pF_a. \end{aligned} \tag{A.11}$$

The first term is equal to zero because of (3). Expansion of the second term in brackets by $+F_2 - F_2$ gives $p_2 F + (1-p)F_2 - F_2$. The first two terms of this expression are equal to zero because of (4). This yields (5). An analogous argument holds for the derivation of (6).

Next we look at the comparative-static effects of equilibria where at least one of the conditions (3), (4) is fulfilled as equality. First assume that $\partial V_1/\partial y_1 = 0, \partial V_2/\partial y_2 \neq 0$. In this case, $dy_2/da = 0$ and (5) and (6) simplify to

$$\partial \overline{V}_1/\partial a = p_a F + pF_a, \tag{A.12}$$

$$\partial \overline{V}_2/\partial a = -p_a F + (1-p)F_a - F_1 \frac{dy_1}{da}. \tag{A.13}$$

By the same token, with $\partial V_1/\partial y_1 \neq 0, \partial V_2/\partial y_2 = 0$ we obtain

$$\partial \overline{V}_1/\partial a = p_a F + pF_a - F_2 \frac{dy_2}{da}, \tag{A.14}$$

$$\partial \overline{V}_2/\partial a = -p_a F + (1-p)F_a. \tag{A.15}$$

Finally, for $\partial V_1/\partial y_1 \neq 0, \partial V_2/\partial y_2 \neq 0$ we obtain

$$\partial \overline{V}_1/\partial a = p_a F + pF_a, \tag{A.16}$$

$$\partial \overline{V}_2/\partial a = -p_a F + (1-p)F_a. \tag{A.17}$$

A.1.4.2 Changes in y_i due to changes in a

For an explicit solution of (A.11) to (A.17), we solve for dy_i/da, $i = 1, 2$. We start with the case of $\partial V_1/\partial y_1 = 0, \partial V_2/\partial y_2 = 0$ which are the first-order conditions for an interior Nash equilibrium. These conditions have to be fulfilled for any feasible realization of a. We differentiate both equations with respect to y_1, y_2 and a and obtain (using matrix notation):

$$\begin{bmatrix} \frac{\partial V_1^1}{\partial y_1} & \frac{\partial V_1^1}{\partial y_2} \\ \frac{\partial V_2^2}{\partial y_1} & \frac{\partial V_2^2}{\partial y_2} \end{bmatrix} \begin{bmatrix} \frac{dy_1}{da} \\ \frac{dy_2}{da} \end{bmatrix} = - \begin{bmatrix} \frac{\partial V_1^1}{\partial a} \\ \frac{\partial V_2^2}{\partial a} \end{bmatrix}.$$

Using Cramer's rule we obtain

$$\frac{dy_1}{da} = \frac{\frac{\partial V_2^2}{\partial a}\frac{\partial V_1^1}{\partial y_2} - \frac{\partial V_1^1}{\partial a}\frac{\partial V_2^2}{\partial y_2}}{\frac{\partial V_1^1}{\partial y_1}\frac{\partial V_2^2}{\partial y_2} - \frac{\partial V_1^1}{\partial y_2}\frac{\partial V_2^2}{\partial y_1}}, \tag{A.18}$$

$$\frac{dy_2}{da} = \frac{\frac{\partial V_1^1}{\partial a}\frac{\partial V_2^2}{\partial y_1} - \frac{\partial V_2^2}{\partial a}\frac{\partial V_1^1}{\partial y_1}}{\frac{\partial V_1^1}{\partial y_1}\frac{\partial V_2^2}{\partial y_2} - \frac{\partial V_1^1}{\partial y_2}\frac{\partial V_2^2}{\partial y_1}}. \tag{A.19}$$

The denominators of (A.18) and (A.19) are identical. The first term is the product of both individuals' second-order conditions which have to be smaller than zero for a maximum. The second term is the product of both individuals' cross effects. If we assume that the direct effect on marginal utility of an increase in y exceeds the effect on the other individual, the denominator is unambiguously positive.

For the calculation of the numerators recall the specifications of the individual first-order conditions:

$$\begin{aligned} V_1^1 &= \quad p_1(y_1, y_2, a)\, F(1-y_1, 1-y_2, a) \\ &\quad - \; p(y_1, y_2, a)\, F_1(1-y_1, 1-y_2, a) = 0, \end{aligned} \tag{A.20}$$

$$\begin{aligned} V_2^2 &= - \; p_2(y_1, y_2, a)\, F(1-y_1, 1-y_2, a) \\ &\quad - \; (1 - p(y_1, y_2, a))\, F_2(1-y_1, 1-y_2, a) = 0. \end{aligned} \tag{A.21}$$

Accordingly, the numerator of (A.18) is equal to

$$\begin{aligned} &\underbrace{(-p_{2a}F - p_2F_a + p_aF_2 - (1-p)F_{2a})}_{A1}\underbrace{(p_{12}F - p_1F_2 - p_2F_1 + pF_{12})}_{B1} \\ - \; &\underbrace{(p_{1a}F + p_1F_a - p_aF_1 - pF_{1a})}_{A2}\underbrace{(-p_{22}F + (1-p)F_{22} + 2p_2F_2)}_{B2}, \end{aligned} \tag{A.22}$$

whereas the numerator of (A.19) is equal to

$$\begin{aligned} &\underbrace{\left(p_{1a}F + p_1F_a - p_aF_1 - pF_{1a}\right)}_{A2}\underbrace{\left(-p_{21}F + p_2F_1 + p_1F_2 + (1-p)F_{21}\right)}_{C1} \\ - &\underbrace{\left(-p_{2a}F - p_2F_a + p_aF_2 - (1-p)F_{2a}\right)}_{A1}\underbrace{\left(p_{11}F + pF_{11} - 2p_1F_1\right)}_{C2}. \end{aligned} \tag{A.23}$$

Next we will derive the comparative-static effects for the case $\partial V_1/\partial y_1 = 0, \partial V_2/\partial y_2 \neq 0$. We know that in this case we have $dy_2/da = 0$. Therefore, total differentiation of the equation $p_1F - pF_1 = 0\ \forall a$ is sufficient for the derivation of dy_1/da. We solve for dy_1/da and obtain

$$\frac{dy_1}{da} = -\frac{p_{1a}F + p_1F_a - p_aF_1 - pF_{1a}}{p_{11}F - 2p_1F_1 + pF_{11}}. \tag{A.24}$$

By the same token we get for $\partial V_1/\partial y_1 \neq 0, \partial V_2/\partial y_2 = 0$ that $dy_1/da = 0$ and

$$\frac{dy_2}{da} = -\frac{-p_{2a}F - p_2F_a + p_aF_2 - (1-p)F_{2a}}{-p_{22}F + 2p_2F_2 + (1-p)F_{22}}. \tag{A.25}$$

If $\partial V_1/\partial y_1 \neq 0, \partial V_2/\partial y_2 \neq 0$ it follows immediately that $dy_1/da = dy_2/da = 0$. Table A.2 summarizes the findings.

$\partial V_1/\partial y_1$	$\partial V_2/\partial y_2$	$\partial \overline{V}_1/\partial a$	$\partial \overline{V}_2/\partial a$
$=0$	$=0$	$p_aF + pF_a - F_2 \cdot (A.19)$	$-p_aF + (1-p)F_a - F_1 \cdot (A.18)$
$\neq 0$	$=0$	$p_aF + pF_a - F_2 \cdot (A.25)$	$-p_aF + (1-p)F_a$
$=0$	$\neq 0$	$p_aF + pF_a$	$-p_aF + (1-p)F_a - F_1 \cdot (A.24)$
$\neq 0$	$\neq 0$	$p_aF + pF_a$	$-p_aF + (1-p)F_a$

Table A.2: Comparative-static effects given the Nash equilibrium. (All functions are evaluated at the Nash-equilibrium values.)

Recall that in equation (7) we have shown that

$$\frac{dF}{da} = \frac{\partial \overline{V}_1}{\partial a} + \frac{\partial \overline{V}_2}{\partial a}. \tag{A.26}$$

This allows to calculate dF/da on the basis of table A.2.

A.2 Proofs of Propositions 3 to 7:

Proof of Proposition 3:

If F and p are independent of a, (A.18), (A.19) and (A.24), (A.25) are all equal to zero, that is, $\partial\overline{V}_i/\partial a = 0, i = 1, 2$. This implies that (5) and (6) are equal to zero. Hence, the allocation of land is irrelevant for allocative purposes. This, however, implies that one cannot improve upon anarchy in the stage game. □

Proof of Proposition 4:

We will prove Proposition 4 by way of an example. Assume that there is a fully-cooperative boundary Nash equilibrium at a^0, $y_1 = y_2 = 0$, where both individuals' first-order conditions are fulfilled as inequalities. A voluntary redistribution of land starting from a^0 requires that a change in a increases both individual utilities. In our example we assume that more land should be given to individual 1 and, therefore, land is voluntarily redistributed if (compare the last line of table A.2):

$$p_a(0,0,a^0)F(1,1,a^0) + p(0,0,a^0)F_a(1,1,a^0) > 0 \tag{A.27}$$

and

$$\begin{aligned} & -p_a(0,0,a^0)F(1,1,a^0) + (1 - p(0,0,a^0))F_a(1,1,a^0) > 0 \\ \Leftrightarrow \quad & (1 - p(0,0,a^0))F_a(1,1,a^0) > p_a(0,0,a^0)F(1,1,a^0). \end{aligned} \tag{A.28}$$

Since $p(y,y,a) = a$, we have $p_a(y,y,a) = 1$. Therefore,

$$\begin{aligned} & (1 - p(0,0,a^0))F_a(1,1,a^0) > p_a(0,0,a^0)F(1,1,a^0) \\ \Leftrightarrow \quad & \frac{F_a(1,1,a^0)}{F(1,1,a^0)} > \frac{1}{1-a^0}. \end{aligned} \tag{A.29}$$

(A.29) holds for $a^o \in [0, 1/2)$, since $F(1,1,a^o) = a^o$ and, accordingly, $F_a(1,1,a^o) = 1$. (If $a^0 \to 0$, the left-hand side tends to a value larger than one by assumption 1, whereas the right-hand side tends to 1.)

In order to complete the proof we have to check that full cooperation is compatible with $a^o \in [0, 1/2)$. From table A.1 we know that this is the case if

$$\frac{p_1}{a^o - p_1} \le \frac{F_1}{F_2} \le \frac{(1-a^o) + p_2}{-p_2} \qquad \wedge \quad p_1 < a^o \quad \wedge \quad -p_2 < 1 - a^o. \tag{A.30}$$

Therefore, a full-cooperation equilibrium can be attained if
(i) $p_1 < a^o$,
(ii)$-p_2 < 1 - a^o$. □

Proof of Proposition 6:

(ii) The minimum transfer that individual 2 is willing to accept is $\overline{V}_2(a^A) - \overline{V}_2(a^*)$. Substituting for τ in (12), letting n converge to infinity and reformulating gives the expression. If the expression is fulfilled for $n \to \infty$, one can find a minimal n such that (12) is still fulfilled. (iii) The higher δ, the easier (12) can be fulfilled. Hence, if it can be fulfilled at all, it must be fulfilled for $\delta = 1$. Substituting into the inequality of part (ii) yields the inequality of part (iii). □

Proof of Proposition 7:

Parallels the proof of Proposition 6. □

A.3 The example at the end of section 5

We deal with the following modified Tullock function:

$$p = 2ay_1/(y_1 + y_2), \tag{A.31}$$

where we have rescaled a such that $a \in [0, 1/2]$. As can easily be seen, this modified Tullock function fulfills Assumption 2 of our paper because it has the following properties: $p_1 = 2ay_2/(y_1 + y_2)^2 \geq 0$, $p_2 = -2ay_1/(y_1 + y_2)^2 \leq 0$, $p_{11} = -4ay_2/(y_1 + y_2)^3 \leq 0$, $p_{22} = 4ay_1/(y_1 + y_2)^3 \geq 0$, $p_a = 2y_1/(y_1 + y_2) > 0$, and $p(y, y, a) = 2a/2 = a$. Furthermore, $p(0, 1, a) = 1$ and $p(1, 0, a) = 2a \leq 1$.

a	y_1	y_2	$\overline{V}_1$	$\overline{V}_2$	$\overline{V}_1 + \overline{V}_2$
0,025	0,054	0,000	14,778	102343,972	102358,75
0,050	0,114	0,001	136,353	104233,647	104370,00
0,100	0,253	0,006	1400,574	105469,426	106870,00
0,150	0,397	0,023	5339,598	101405,402	106745,00
0,200	0,523	0,058	12609,099	91130,901	103740,00
0,250	0,610	0,111	21693,628	76956,372	98650,00
0,300	0,655	0,178	30298,001	62251,999	92550,00
0,350	0,668	0,245	37193,166	49926,834	87120,00
0,400	0,661	0,314	42360,053	39799,947	82160,00
0,450	0,643	0,378	46243,087	32021,913	78265,00
0,500	0,623	0,439	49588,869	25361,131	74950,00

Table A.3: Equilibrium strategies and utilities for the modified Tullock function.

The calculations have been done with the help of Maple. The solution algorithm is given below:

```
R1 := (1+a-a*y1-y2) = (a*y1+(y1+y2))/y2;
```

```
R2 := (1+a-a*y1-y2) = ((y1+y2)^2 - 2*a*y1*(y1+y2))/(2*a*y1);

R1a := solve(R1, y1);

R2a := solve(R2, y2);

NGG := solve({R1,R2}, {y1,y2});
```

If the various values for a are inserted one gets the values in Table A.3.