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Consumer Misperceptions, Uncertain Fundamentals, and the Business Cycle

by

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Consumer Misperceptions, Uncertain Fundamentals, and the Business Cycle*

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Abstract

This paper explores the importance of shocks to consumer misperceptions, or "noise shocks", in a quantitative business cycle model. I embed imperfect information as in Lorenzoni (2009) into a new Keynesian model with price and wage rigidities. Agents learn about the components of labor productivity by only observing aggregate productivity and a noisy signal. Noise shocks lead to expectational errors about the true fundamentals triggering aggregate fluctuations. Estimating the model with Bayesian methods on US data shows that noise shocks contribute to 20 percent of consumption fluctuations at short horizons. Wage rigidity is pivotal for the importance of noise shocks.

Keywords: Imperfect Information, Noise Shocks, Aggregate Fluctuations, Bayesian Estimation.

JEL Classification: D83, E32.

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1 Introduction

Do episodes of consumer optimism or pessimism cause business cycle fluctuations? The role of psychological factors and expectations in explaining business cycle fluctuations has long been emphasized by economists. The idea dates at least back to Pigou (1927), who believed that "errors of undue optimism or undue pessimism in their business forecasts" caused industrial fluctuations, and Keynes (1936), who assigned a large role to "animal spirit shocks" in explaining business cycle fluctuations. Recent studies have re-emphasized the idea of expectation-driven cycles (see Lorenzoni, 2009; Blanchard et al., 2009; Beaudry and Portier, 2004, 2006). Lorenzoni (2009) presents a model where noise shocks or "animal spirit shocks" induce business cycle fluctuations. Consumers temporarily misperceive the true productive capacity of the economy and, hence, over- or underestimate actual productivity. Noise shocks induce fluctuations in consumers' beliefs unrelated to fundamental changes and share the features of demand shocks, i.e. output, employment, and inflation increase temporarily.[1]

The empirical literature on the actual importance of noise shocks is still inconclusive. Blanchard et al. (2009) employ a maximum likelihood estimation of a highly stylized new Keynesian model with noise shocks using only consumption and productivity data. Their intriguing finding is that 75 percent of consumption fluctuations on impact and still more than 50 percent after four quarters are due to noise shocks, while technology shocks account for the remaining fraction. However, their estimation yields virtually fixed prices running counter to microeconomic evidence on price adjustments (Bils and Klenow, 2004) and macroeconomic evidence from estimated DSGE models (Smets and Wouters, 2007). Barsky and Sims (forthcoming) estimate a DSGE model featuring price rigidity, habit formation, and adjustment costs with impulse response function matching and find that noise shocks explain virtually no aggregate fluctuations due to general equilibrium effects.

Given these ambiguous results, the present paper sheds light on the actual importance of noise shocks by estimating a richer variant of the imperfect information model by Blanchard et al. (2009). For this purpose, I use a new Keynesian model with price and wage stickiness similar to Rabanal and Rubio-Ramirez (2005) without imposing any restrictions on the importance of noise shocks. I estimate the model with Bayesian methods using quarterly US data from 1970 to 2009. A full-information structural estimation strategy is required to avoid identification problems regarding noise shocks arising from the consumers' signal extraction problem.[2] Moreover, I use data for inflation, nominal

[1] Blanchard (1993) argues that the US recession in 1990/91 was mainly driven by a severe negative consumption shock that was accompanied by an exogenous shift in pessimism, i.e. a negative noise shock, inducing a contractionary demand effect.

[2] Blanchard et al. (2009) show that if agents solve a signal extraction problem of the type considered in this paper, the structural shocks of the model cannot be identified with any identification scheme using a vector autoregression (VAR). The nature of the signal extraction problem induces a problem of non-fundamentalness, which means that the reduced form residuals in a VAR cannot be mapped

interest rates and real wages to better estimate nominal rigidities. I also add monetary policy shocks to ensure that noise shocks do not mechanically capture all demand effects in the data per se.

The main results are that noise shocks contribute to 20 percent of consumption fluctuations on impact and about 15 percent after four quarters. Noise shocks also account for 20 percent of consumption per capita growth after 20 quarters. The estimate for the precision of the noisy signal indicates that consumers take about eight quarters to disentangle noise from fundamental shocks. Regarding nominal rigidities the estimation reveals an average price and wage duration of three and a half and four quarters, respectively.[3] Price and wage rigidities are pivotal for the actual importance of noise shocks. The decisive role of wage rigidity is remarkable, as previous papers either found noise shocks to be important for economic fluctuations only when prices were almost fixed (Blanchard et al., 2009) or irrelevant for plausible values of price stickiness (Barsky and Sims, forthcoming). In line with the latter study, I also find noise shocks to be negligible in generating business cycles when considering the case of flexible wages in a counterfactual analysis.

The underlying intuition for why noise shocks explain a moderate fraction of consumption fluctuations in the presence of wage rigidity is as follows: Sticky wages imply that firms rationally anticipate reduced fluctuations in their real marginal costs. Hence, inflation variability decreases as compared to the flexible wage case. But less variation in inflation reduces the responsiveness of the real interest rate through the Fisher equation and therefore reduces consumers' willingness to postpone consumption to later periods. Thus, when a positive noise shock induces a positive wealth effect, households increase consumption more under sticky wages than under flexible wages due to a dampened real interest rate response.

The present paper is also related to research articles that explore the role of anticipated shocks as a source of business cycle fluctuations such as Beaudry and Portier (2004, 2006), Jaimovich and Rebelo (2009), and Schmitt-Grohé and Uribe (2009). These papers assume that agents observe future changes ("news") about productivity affecting today's decision. Current and future shocks are both perfectly observed by consumers and firms. These studies provide empirical evidence that a large fraction of macroeconomic volatility may be due to anticipated, or news, shocks. In news-driven business cycle models agents perfectly observe current and future productivity shocks, whereas in noise-driven models agents disentangle fundamental shocks from pure noise shocks.[4] In the latter, correlated erroneous beliefs about the true state of the economy arise naturally because agents

into the structural shocks of the model.

[3] The estimates for the Calvo-parameters are $\theta_p = 0.71$ and $\theta_w = 0.75$, which is in line with the ones from other estimated new Keynesian models such as Smets and Wouters (2007).

[4] In principle, information about future changes may be offset by a new observation in the next period, e.g. a positive news shock to be realized in three periods from today may be offset in period two. However, as news shocks are typically assumed to be i.i.d., no systematic/correlated erroneous beliefs can arise.

gradually learn about the productive capacity of the economy. Learning gives rise to temporary deviations regarding agents' decisions as compared to the full-information equilibrium.

The paper proceeds as follows. Section 2 presents the model and formalizes the imperfect information environment. Section 3 outlines the estimation strategy and presents the results of the Bayesian estimation. Section 4 provides impulse responses, forecast error variance decompositions as well as counterfactual experiments. Section 5 concludes.

2 The Model

The model economy is structured as follows: It is inhabited by a continuum of households that each offer differentiated labor services to intermediate firms, a continuum of monopolistically competitive intermediate good firms and a final good firm that bundles the intermediate goods, and a central bank that sets monetary policy. All agents in the economy face imperfect information regarding the economy's productive capacity, i.e. private agents cannot observe the permanent and temporary component of aggregate productivity; instead they form beliefs about the actual state of the economy by observing noisy signals.

2.1 Households

The economy is inhabited by a continuum of households indexed by $h \in [0,1]$. Preferences are additively separable in consumption and labor supply

$$U(C_t(h), N_t(h)) = \frac{C_t(h)^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\varphi}}{1+\varphi} \ , \tag{1}$$

where $C_t(h)$ denotes household h's consumption and $N_t(h)$ the amount of hours worked. The aggregate consumption index C_t is a composite of differentiated goods $C_t(i)$ indexed by $i \in [0,1]$

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right]^{\frac{\epsilon_p}{\epsilon_p - 1}} \ , \tag{2}$$

where ϵ_p denotes the intratemporal elasticity of substitution across different varieties of consumption goods. Maximizing the consumption index C_t for a given level of expenditures yields the demand schedule for consumption goods

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_p} C_t \ . \tag{3}$$

The minimum costs of a bundle of intermediate goods that yield one unit of composite good amounts to the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon_p} di\right]^{\frac{1}{1-\epsilon_p}} . \tag{4}$$

Each household h supplies a different type of labor $N_t(h)$ and has some monopoly power in the labor market, posting the nominal wage $W_t(h)$ at which it is willing to supply specialized labor services to firms that demand them (see Erceg et al., 2000). A typical household chooses nominal wages in a staggered fashion. In every period a fraction $(1-\theta_w)$ of households is randomly drawn from the population and is allowed to reset its wage.

Differentiated labor services are bundled to a homogenous labor good N_t according to a Dixit-Stiglitz aggregator

$$N_t = \left[\int_0^1 N_t(h)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dh\right]^{\frac{\varepsilon_w}{\varepsilon_w-1}} , \tag{5}$$

where ϵ_w denotes the intratemporal elasticity of substitution across different varieties of labor types. Cost minimization yields the optimal bundling of differentiated labor services which gives rise to the labor demand schedule

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\varepsilon_w} N_t . \tag{6}$$

The aggregate wage index W_t is a composite of all labor type specific wage rates

$$W_t = \left[\int_0^1 W_t(h)^{1-\varepsilon_w} dh\right]^{\frac{1}{1-\varepsilon_w}} . \tag{7}$$

Households have access to a complete set of state-contingent Arrow-Debreu securities to fully insure against idiosyncratic income risk that derives from the limited ability to adjust wages in each period. Under complete markets, consumption and the marginal utility of consumption are equalized across households and states at all times in equilibrium (given identical endowments). Let $D_{t+1}(h)$ denote the payoff in period $t+1$ of the portfolio of state-contingent securities held by household h at the end of period t and let $Q_{t,t+1}$ denote the stochastic discount factor. The budget constraint of household h is given by

$$W_t(h)N_t(h) - T_t - P_tC_t(h) = E_t\left\{Q_{t,t+1}D_{t+1}(h)\right\} - D_t(h) , \tag{8}$$

where T_t are nominal lump-sum payments including taxes and dividends.

A representative household h maximizes the expected discounted lifetime utility with respect to $C_t(h)$ and $D_{t+1}(h)$ subject to the budget constraint (8) and a standard no-Ponzi-game condition. The resulting first-order conditions are

$$C_t(h)^{-\sigma} = \lambda_t(h) P_t \tag{9}$$

$$Q_t = \beta E_t \frac{\lambda_{t+1}(h)}{\lambda_t(h)}, \tag{10}$$

where $\lambda_t(h)$ is the multiplier on the household's budget constraint. Under complete markets the standard result emerges that the stochastic discount factor is given by (see Chari et al. (2002) for a detailed treatment)

$$Q_{t,t+1} = \beta E_t \frac{U_{C_{t+1}}}{U_{C_t}} \frac{P_t}{P_{t+1}} , \tag{11}$$

where U_{C_t} denotes the marginal utility of consumption in period t. As consumers have access to complete financial markets they insure their idiosyncratic income risk, such that the first-order conditions for each household are identical.

2.2 Optimal Wage Setting

Similar to Calvo-pricing only a fraction $(1-\theta_w)$ of households can adjust their posted nominal wage. Wage inflation and infrequent wage adjustments induce relative wage distortions that facilitate an inefficient allocation of labor. Each period, the optimizing households choose their wage $W_t^\star(h) = W_t(h)$ for their labor type in order to maximize the expected discounted lifetime utility subject to the labor demand schedule. Considering only the relevant terms of the optimization problem

$$\max_{W_t^\star(h)} E_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \left[\lambda_{t+s} N_{t+s}(h) \frac{W_t(h)}{P_{t+s}} - \frac{N_{t+s}(h)^{1+\varphi}}{1+\varphi} \right] \quad \text{s.t.} \tag{12}$$

$$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\varepsilon_w} N_t \ . \tag{13}$$

The resulting first-order condition reads

$$E_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \left[\lambda_{t+s} N_{t+s}(h) \left(\frac{W_t^\star(h)}{P_{t+s}} - \mathcal{M}_w \mathrm{MRS}_{t+s} \right) \right] = 0 \ , \tag{14}$$

where $\mathcal{M}_w = \frac{\epsilon_w}{\epsilon_w - 1}$ is the steady state wage markup and $\mathrm{MRS} = -\frac{U_N}{U_C}$ denotes the marginal rate of substitution between consumption and hours (where U_i is the derivative with respect to $i = C, N$). The evolution of the aggregate wage index is

$$W_t = \left[\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w) \left(W_t^\star \right)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}} \ . \tag{15}$$

Given the assumption of complete markets (assuming identical initial conditions) and separable utility in labor (see Erceg et al., 2000), I consider a symmetric equilibrium where $C_t(h) = C_t, \lambda_t(h) = \lambda_t$ and $W_t^\star(h) = W_t^\star$.

2.3 Firms

There is a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$, where each firm produces a differentiated good using the same technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \ . \tag{16}$$

A competitive final good firm bundles intermediate goods to a final good Y_t following a Dixit-Stiglitz aggregation technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right]^{\frac{\epsilon_p}{\epsilon_p - 1}} \ . \tag{17}$$

The aggregate level of technology is given by $A_t = X_t Z_t$, where X_t and Z_t denote the permanent and temporary component, respectively. The growth rate of the permanent component follows a first-order autoregressive process, which implies that the level X_t builds up gradually over time. The stochastic processes are

$$\frac{X_t}{X_{t-1}} = \left(\frac{X_{t-1}}{X_{t-2}} \right)^{\rho_x} \exp(\epsilon_t) \ , \qquad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2) \tag{18}$$

$$Z_t = Z_{t-1}^{\rho_z} \exp(\eta_t) \ , \qquad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2) \ . \tag{19}$$

Throughout the paper it is assumed that agents only observe aggregate productivity A_t, but neither the exact realization of its permanent nor its temporary component. In addition, consumers observe a noisy signal about the permanent component

$$S_t = X_t \exp(\nu_t) \ , \qquad \nu \sim \mathcal{N}(0, \sigma_\nu^2) \ , \tag{20}$$

where σ_ν measures the precision of the signal. The signal represents information that help consumers to infer the actual level of permanent productivity. The additional information comprises, for example, consumer sentiment studies, financial market prices, or sector statistics of the economy. As to how exactly consumers form beliefs about unobserved variables is given in Section 2.6.

Firms set prices in a staggered fashion à la Calvo (1983), i.e. firms can reoptimize prices with probability $(1 - \theta_p)$ each period and therefore take into account that they may not be able to adjust prices in the next period. Firms set prices $P^\star = P(i)$ to maximize expected profits subject to the demand schedule (3)

$$\max_{P_t^\star} E_t \sum_{s=0}^{\infty} \theta_p^s Q_{t,t+s} Y_{t+s}(i) \left(\frac{P_t(i)}{P_{t+s}} - MC_{t+s}(i) \right) \qquad \text{s.t.} \tag{21}$$

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_p} C_t \ , \tag{22}$$

where $Q_{t,t+s}$ is the households' stochastic discount factor as defined before and $MC_t(i)$ is firm i's real marginal cost. Market clearing in goods markets implies that $C_t(i) = Y_t(i)\ \forall i, t$, which was substituted in the demand schedule above. Note that firms which are able to reset their price face an identical optimization problem. The optimal price-setting condition is

$$\sum_{s=0}^{\infty} \theta^k E_t \left[Q_{t,t+s} Y_{t+s,t} \left(\frac{P_t^\star}{P_{t-1}} - \mathcal{M}_p MC_{t+s,t} \Pi_{t-1,t+s} \right) \right] = 0 \ , \tag{23}$$

where $\mathcal{M}_p = \frac{\epsilon_p}{\epsilon_p - 1}$ is the steady state price markup and $\Pi_{t,t+s} = \frac{P_{t+s}}{P_t}$. The economy's overall price level is

$$P_t = \left[\theta_p P_{t-1}^{1-\epsilon_p} + (1-\theta_p) \left(P_t^\star \right)^{1-\epsilon_p} \right]^{\frac{1}{1-\epsilon_p}} \ . \tag{24}$$

2.4 Monetary Policy

To close the model a rule for monetary policy is specified. The central bank uses a Taylor rule where the nominal interest rate R_t reacts to contemporaneous changes in inflation

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \exp(\varpi_t) \tag{25}$$

$$\varpi_t = \rho_m \varpi_{t-1} + m_t \ , \qquad m_t \sim \mathcal{N}(0, \sigma_m^2) \ , \tag{26}$$

where ϕ_π denotes the Taylor rule cofficient.

2.5 Linearization

To solve the model with standard perturbation methods, all non-stationary variables are detrended by dividing through lagged productivity A_{t-1}. Detrended variables are denoted with a 'hat', e.g. $\hat{A}_t = A_t / A_{t-1}$. Using standard solution methods, I log-linearize the first-order conditions around the zero price and wage inflation steady state. Henceforth, lower case variables denote log-linear deviations from their steady state value. The following equations are the linearized first-order conditions that describe the model dynamics

$$
\begin{aligned}
\hat{w}_t^r &= \hat{w}_{t-1}^r + \pi_t^w - \pi_t^p - \hat{a}_{t-1} && (27)\\
\pi_t^p &= \beta E_t \pi_{t+1}^p + \lambda_p \frac{\alpha}{1-\alpha}\hat{y}_t + \lambda_p \hat{w}_t^r - \lambda_p \frac{1}{1-\alpha}\hat{a}_t && (28)\\
\pi_t^w &= \beta E_t \pi_{t+1}^w + \lambda_w \left(\sigma + \frac{\varphi}{1-\alpha}\right)\hat{y}_t - \lambda_w \hat{w}_t^r - \lambda_w \frac{\varphi}{1-\alpha}\hat{a}_t && (29)\\
\sigma \hat{y}_t + r_t &= \sigma E_t \hat{y}_{t+1} + E_t \pi_{t+1}^p + \hat{a}_t && (30)\\
r_t &= \phi_\pi \pi_t + \varpi_t && (31)\\
\hat{y}_t &= \hat{c}_t \ , && (32)
\end{aligned}
$$

where $\hat{w}_t^r$ refers to the real wage.

Equation (27) describes the link between real wage growth, nominal wage inflation and price inflation. The new Keynesian Phillips curve (NKPC), equation (28), relates current inflation to next period's expected inflation. The linearized forward-looking first-order condition for wage inflation has a similar form and interpretation as the NKPC, i.e. if the average wage in the economy is below the level consistent with maintaining (on average) the desired markup, households resetting their nominal wage will tend to increase the latter, and thereby generate positive wage inflation. Imperfect adjustment of nominal wages will generally drive a wedge between the real wage and the marginal rate of substitution (MRS) of each household. This link translates into a wedge between the average real wage and the average marginal rate of substitution which induces variations in the average wage markup and thus in wage inflation according to the wage Phillips curve. The dynamic IS curve, equation (30), constitutes the forward-looking Euler equation. The Taylor rule in equation (31) closes the model. The last equation is the linearized goods market clearing condition.

Aggregate productivity equals the sum of permanent and temporary productivity

$$
\hat{a}_t = \hat{x}_t + z_t \ . \qquad (33)
$$

The permanent and temporary productivity component are, respectively, given by

$$
\begin{aligned}
\hat{x}_t &= \rho_x \hat{x}_{t-1} - \rho_x \hat{x}_{t-2} - z_{t-1} + \rho_x \hat{a}_{t-2} + \epsilon_t \ , && \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2) && (34)\\
z_t &= \rho_z z_{t-1} + \eta_t \ , && \eta_t \sim \mathcal{N}(0, \sigma_\eta^2) \ . && (35)
\end{aligned}
$$

Finally, the noisy signal $\hat{s}_t$ is defined as

$$
\hat{s}_t = \hat{x}_t + \nu_t \ , \qquad \nu_t \sim \mathcal{N}(0, \sigma_\nu^2) \ . \qquad (36)
$$

2.6 Information structure

Consumers imperfectly observe the state of the economy, which allows incorporating noise shocks. It is assumed that agents observe aggregate productivity $\hat{a}_t$ and a noisy signal $\hat{s}_t$ about the permanent component of productivity. The signal represents additional information that improves the consumers' estimate about the true permanent productivity. A noise shock ν_t affects the private sector's beliefs about aggregate productivity that are uncorrelated with productivity shocks and leads consumers to temporarily over- or underestimate the actual productivity of the economy.

The information structure captures the notion that agents make expectational errors about the fundamentals of the economy and thereby induce short-run fluctuations. Support for this notion is also provided by Lorenzoni (2009), who points out that a shock to the signal has the characteristics of a demand shock, i.e. consumption, output, inflation and hours worked temporarily increase. Having observed aggregate labor productivity and the signal, consumers update their beliefs about the permanent and the temporary component via the Kalman filter. As the system of equations is linear and all shocks are Gaussian, using the Kalman filter implies that consumers process information in the most efficient way (see Hamilton, 1994, Chapter 13).[5] Consumers' beliefs about the unobserved variables follow the law of motion (see Appendix A)

$$\begin{pmatrix} \hat{x}_{t|t} \\ \hat{x}_{t-1|t} \\ z_{t|t} \\ \hat{a}_{t-1|t-1} \end{pmatrix} = A \begin{pmatrix} \hat{x}_{t-1|t-1} \\ \hat{x}_{t-2|t-1} \\ z_{t-1|t-1} \\ \hat{a}_{t-2|t-2} \end{pmatrix} + B \begin{pmatrix} \hat{a}_t \\ \hat{s}_t \end{pmatrix} , \qquad (37)$$

where, to clarify notation, $\hat{x}_{t-1|t}$ denotes the consumers' belief about the unobserved state $\hat{x}_{t-1}$ at time t or equivalently $\hat{x}_{t-1|t} = E\left[\hat{x}_{t-1}|\mathcal{I}_t\right]$, where $\mathcal{I}_t$ denotes the consumers' information set comprising all observables up to period t. Solving the filtering problem numerically yields the elements of matrix A and B which are nonlinear functions of the parameters $\rho_x, \rho_z, \sigma_\epsilon, \sigma_\eta$ and σ_ν. The coefficients in matrix B indicate how strongly consumers weight the respective observables. The elements of matrix A provide the information by how much the beliefs of the previous period are weighted in the current beliefs. For example, the more precise the signal (low σ_ν), the more weight do agents give to this observable when updating their beliefs about the unobserved variables.

[5] In contrast to the research on rational inattention (e.g. Sims, 2003; Mackowiak and Wiederholt, 2009), where agents dynamically choose which variables they observe given that they are restricted in the amount of information they can process, and thus learn actively, the present paper assumes that agents learn passively.

2.7 Solving the Imperfect Information Model

First, I solve the full-information log-linearized model by a first-order approximation around the steady state using standard solution methods for rational expectations models (e.g. Klein, 2000; Sims, 2002). However, solving DSGE models where agents receive noisy information and learn about unobserved state variables necessitates an adjustment of these methods. Agents are assumed to behave fully rational given their information set. This means agents optimally form their expectations about the unobserved states from the set of observables by employing the Kalman filter to solve the signal extraction problem. Certainty-equivalence applies as I consider the linearized model equilibrium. Consequently agents behave as if their optimal forecast of an unobserved state was the same as the true state variable. Hence, consumers' beliefs, $(\hat{x}_{t|t}, \hat{x}_{t-1|t}, z_{t|t})$, about the unobserved state variables subsequently replace the respective actual state variables in the log-linearized state-space representation under perfect information.[6] Baxter et al. (2011) provide a general overview on how to solve rational expectation models with various informational frictions. The general idea is to first solve the full information model for which the state space is given by

$$X_{1,t} = \Pi X_{2,t-1} , \tag{38}$$

$$X_{2,t} = M X_{2,t-1} + \tilde{R}\mu_t , \tag{39}$$

where $X_{1,t}$ is the vector of state variables, $X_{2,t}$ contains all control variables, and $\tilde{R}$ is a matrix that scales the shock vector μ_t. The unobserved states are then replaced by their estimated counterparts such that the control variables $X_{1,t}$ are a linear function of the estimated states, i.e.

$$X_{1,t} = \Pi X_{2,t-1|t-1} . \tag{40}$$

Appendix B describes the solution method in more detail.

3 Estimation Methodology

To investigate how much noise shocks contribute to US business cycle fluctuations I estimate the model using Bayesian methods. A full-information structural estimation technique is required to avoid identification problems that arise due to the consumers' signal extraction problem. Blanchard et al. (2009) show that if agents face a signal extraction problem of the type considered in the present paper, the DSGE model exhibits a non-invertible VAR representation in the observables, i.e. there exists no mapping from

[6] The term $\hat{a}_{t-1|t-1}$ is not needed as it can be perfectly observed; however it is needed to derive the detrended process for the permanent productivity component $\hat{x}_{t|t}$.

the reduced form residuals into the structural shocks of the model. Given that the model is not invertible one cannot identify the structural shocks via any identification scheme in a structural VAR. However, the authors as well as Leeper et al. (2009) show that it is possible to use a full-information estimation approach such as maximum likelihood or Bayesian methods in order to identify the structural shocks. I pursue the latter approach and use prior information about certain parameter values.

The estimation in Blanchard et al. (2009) attributes all demand effects to noise shocks as the remaining shocks are two supply shocks. In this sense, the estimation is in the spirit of generating as much noise-driven volatility as possible. The same idea is pursued by Lorenzoni (2009), who calibrates the precision of the signal such that it generates as much demand-side volatility as possible. The spirit of the estimation performed here is different. I add a monetary policy shock that shares the features of a demand shock, i.e. output, inflation and employment increase. A simultaneous estimation of both demand shocks ensures that I identify the effects of both shocks separately. Thus, the estimation employed here sheds light on the importance of noise shocks in presence of monetary policy shocks.

The vector of parameters $\Theta = \{\rho_x, \rho_z, \rho_m, \sigma_\epsilon, \sigma_\eta, \sigma_\nu, \sigma_m, \sigma_{rw}^{me}, \theta_p, \theta_w, \phi_\pi\}$ is estimated with Bayesian methods following the steps in An and Schorfheide (2007) and Fernández-Villaverde (2010). Denote the observed data series by $\{\mathcal{Y}_t\}_{t=1}^T$. Using the Kalman filter I obtain the likehood $\mathcal{L}\left(\{\mathcal{Y}_t\}_{t=1}^T|\Theta\right)$ from the state space representation of the model. The object of interest is the posterior distribution which is proportional to the likelihood times the prior, i.e.

$$P\left(\Theta|\{\mathcal{Y}_t\}_{t=1}^T\right) \propto \mathcal{L}\left(\{\mathcal{Y}_t\}_{t=1}^T|\Theta\right) P\left(\Theta\right). \tag{41}$$

Since there is no closed-form solution for the posterior distribution, I use numerical methods. With a Monte-Carlo based optimization routine, I compute the posterior mode and the Hessian evaluated at the posterior mode. Given the posterior mode, I use the random-walk Metropolis-Hastings algorithm to sample from the posterior density. The scale parameter for the jumping distribution is chosen to match an acceptance rate of 0.33. I generate two chains with $500,000$ draws each and I keep the last $50,000$ draws of each chain to generate posterior statistics.

3.1 Data

The model is estimated using quarterly US data for the sample period 1970:1 to 2009:4. The data set contains labor productivity growth, consumption per capita growth, real wage growth, inflation, and the effective federal funds rate.[7] All variables are demeaned

[7] The data sources and the construction of the series in the observation equation are given in Table 3 and Table 4 in Appendix C.

prior to estimation. The observation equation relates the observed data to the respective counterparts in the model, i.e.

$$\mathcal{Y}_t = \begin{pmatrix} \Delta \log(\mathrm{A}_t) \\ \Delta \log(\mathrm{C}_t) \\ \Delta \log(\mathrm{W}_t^r) \\ \Delta \log(\mathrm{GDPDEF}_t) \\ \mathrm{FFR}_t \end{pmatrix} = \begin{pmatrix} \hat{a}_t \\ \hat{c}_t - \hat{c}_{t-1} + \hat{a}_{t-1} \\ \hat{w}_t^r - \hat{w}_{t-1}^r + \hat{a}_{t-1} \\ \pi_t \\ r_t \end{pmatrix} + 100 \times \begin{pmatrix} 0 \\ 0 \\ \sigma_{rw}^{me} \\ 0 \\ 0 \end{pmatrix}. \tag{42}$$

The model dynamics are driven by four structural shocks while I use five observables. To avoid the problem of stochastic singularity measurement error is assumed for one observable following Sargent (1989). Therefore, I choose measurement error for the real wage data series denoted by σ_{rw}^{me} in the observation equation.

3.2 Fixed Parameters

Prior to estimation a set of parameters is fixed (see Table 1). The model is matched to quarterly data. The discount factor β is set to 0.99. The production function is linear in labor, determined by $\alpha = 0$. Agents derive utility from a log-linear additively separable utility function, $\varphi = 1$ and $\sigma = 1$, consistent with balanced growth. A value of one for the Frisch elasticity of labor supply is in line with the estimate in Kimball and Shapiro (2008). In steady state the price and wage markup are each set to 10 percent.

Table 1: Parameters fixed prior to estimation

Parameter	Value	Description
β	0.99	discount factor
α	0	linear production function
φ	1	Frisch elasticity
σ	1	log utility in consumption
ϵ_p	11	price markup of 10%
ϵ_w	11	wage markup of 10%

3.3 Priors

A summary of the prior choices is given in Table 2. With regard to the three autocorrelation parameters I use a diffuse beta distribution with mean 0.7 and standard deviation 0.2. Concerning the prior assumption for the standard errors of the structural shocks, I assume an inverse gamma distribution with mean 0.5 and infinite variance, except for permanent productivity where I choose a smaller mean of 0.1. This choice is motivated by the observation that only small shocks to the permanent component are required to induce large level effects. The measurement error is inverse gamma distributed with mean 0.05 and infinite variance.[8] Concerning the price and wage stickiness parameters I select a beta distribution with a prior mean value that implies an average price and wage duration of 2.85 quarters and standard deviation 0.2. According to the terminology in Del Negro and Schorfheide (2008), who perform a thorough analysis of choosing prior distributions for these two parameters, my choice would be in the middle of their categories *agnostic* and *high rigidities*. Regarding the Taylor rule coefficient I select a gamma distribution with mean 1.5 and standard deviation 1.5.

3.4 Posterior Distributions

Table 2 shows the estimated means of the posterior distribution and the 5 and 95 percentiles.[9] The estimation results regarding the autocorrelation parameters for the productivity processes imply that the permanent component increases gradually and the temporary component decreases slowly. The standard deviation for permanent productivity is much smaller than for temporary productivity. The autocorrelation parameter for monetary policy shocks is about 0.77.

Table 2: Estimation Results

		Prior			Posterior		
Parameter	Description	Distr.	Mean	Std	Mean	5%	95%
ρ_x	autocorr. perm.	$\mathcal{B}$	0.7	0.2	0.92	0.91	0.93
ρ_z	autocorr. temp.	$\mathcal{B}$	0.7	0.2	0.92	0.91	0.93
ρ_m	autocorr. MP	$\mathcal{B}$	0.7	0.2	0.774	0.77	0.78
$100\sigma_\epsilon$	permanent prod.	$\mathcal{IG}$	0.1	∞	0.14	0.12	0.16
$100\sigma_\eta$	temporary prod.	$\mathcal{IG}$	0.5	∞	0.88	0.79	0.96
$100\sigma_\nu$	noise shock	$\mathcal{IG}$	0.5	∞	1.21	0.67	1.78
$100\sigma_{mp}$	monetary policy	$\mathcal{IG}$	0.5	∞	0.46	0.41	0.50
$100\sigma_{rw}^{me}$	measurement error	$\mathcal{IG}$	0.05	∞	0.20	0.20	0.20
θ_p	price stickiness	$\mathcal{B}$	0.65	0.2	0.705	0.703	0.708
θ_w	wage stickiness	$\mathcal{B}$	0.65	0.2	0.75	0.746	0.757
ϕ_π	Taylor rule	$\mathcal{G}$	1.5	1.5	1.47	1.46	1.50

Notes: $\mathcal{B}$ is beta distribution, $\mathcal{G}$ is gamma distribution, $\mathcal{IG}$ is inverse gamma distribution.

With regard to the noise shock I find that when admitting sticky prices and wages, as measured by the Calvo parameters, the precision of the signal is smaller than the estimate

[8] The upper bound is set to 25 percent of the standard deviation of the real wage data series.
[9] The estimation is carried out in Dynare 4.2.1. for which I modified the code to incorporate the solution method as described in Section 2.7.

in Blanchard et al. (2009). Their estimate is 0.89% and lies within the 5 and 95 percentile of the posterior distribution. The signal, according to my estimation, remains sufficiently noisy indicating substantial misperceptions of consumers about the true underlying productivity processes. The measurement error for real wage growth is close to its upper bound. This could be due to the fact that the model does not exhibit all the frictions of large-scale DSGE models and misses some structural shocks whose effects are now captured by the measurement error. The estimates of the Calvo parameters for price and wage stickiness show a considerable degree of nominal rigidities that imply average price and wage changes every three and a half to four quarters, respectively. Both estimates are in line with other empirical studies in the literature such as Smets and Wouters (2007), Rabanal and Rubio-Ramirez (2005), and Christiano et al. (2005). Given the estimates of the posterior mean, I simulate the model responses to the structural shocks.

4 Results

4.1 Impulse Responses

Figure 1 depicts the impulse response functions to each one standard deviation shock for the estimated model. The first row shows the observed aggregate productivity as well as consumers' beliefs about all unobserved variables. The first column displays the response to a noise shock which leaves the fundamentals of the economy, productivity in this case, unaltered. When a positive noise shock materializes, consumers believe that permanent productivity has increased; however, as they have not yet seen a change in aggregate productivity, they believe that there is a negative temporary technology shock that offsets the permanent shock. It takes about eight quarters until agents have learned that the productive capacity of the economy has actually not changed. The noise shock resembles a perceived wealth effect that strongly increases consumption (and output). Inflation, real wages and the real interest rate also increase. Sticky nominal wages induce a hump-shaped response in the real wage. Monetary policy reacts more than one-for-one to inflation, which implies a slightly positive response in the real rate. Employment also rises strongly as output increases while productivity remains constant.[10]

The consumption response is driven by the following mechanism of the model: A dampened response in real wages due to nominal rigidities translates into less variability in real marginal costs of firms in response to a noise shock. Iterating the NKPC forward shows that inflation equals the discounted sum of current and future deviations of real marginal cost from steady state. By the Fisher equation, which relates the real rate to the nominal interest rate and future inflation expectations, less variability in the latter mutes the real interest rate. Given the consumers' intertemporal Euler equation, households

[10] Output is linear in labor, such that in log-linearized terms labor supply is given by $n_t = \hat{y}_t - \hat{a}_t$.

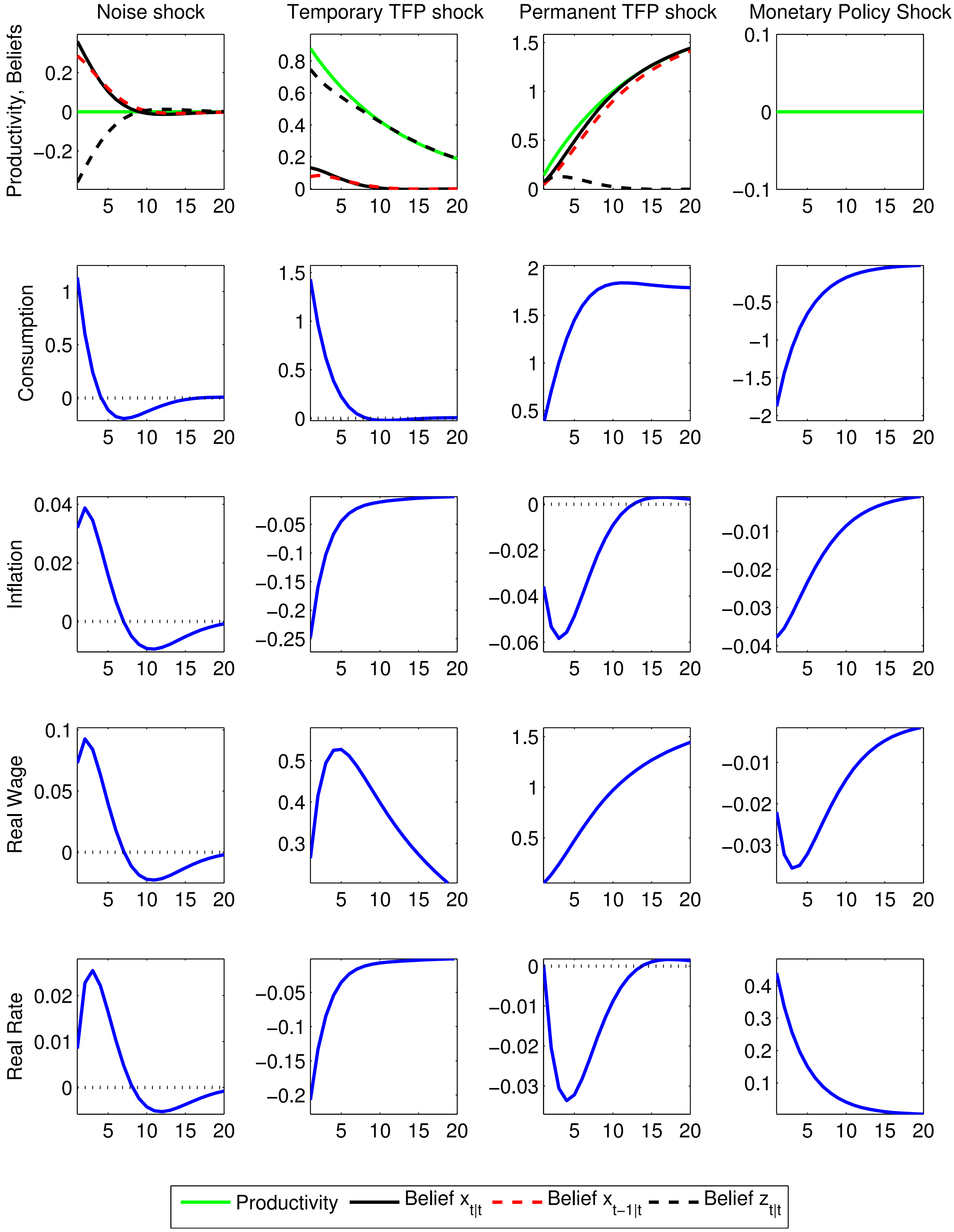

Figure 1: Impulse responses to all shocks

Notes: Impulse responses to each one standard deviation shock at posterior mean parameter values. All variables are measured in percentage deviations from steady state.

increase consumption substantially since they do not have a strong incentive to postpone consumption to later periods. As more and more observations arrive, private agents learn that the productive capacity of the economy has not changed and thus all variables gradually return to their steady state value.

In response to a temporary technology shock, consumption and output increase whereas inflation falls. As agents observe economic conditions imperfectly, they place some probability on having observed a permanent technology increase such that consumption increases by more than in the full information model. Consumers temporarily overestimate the productive capacity of the economy. The real wage increases temporarily while monetary policy is accommodative due to a decrease in inflation.

Consumption adjusts gradually in response to a permanent productivity shock (third column) as agents do not observe the pure shock itself. Therefore, agents' beliefs incorporate the possibility of having observed a temporary shock. As the number of observations increases over time, agents put more and more probability on having observed a permanent shock. If consumers had observed the permanent shock without delay, consumption would have jumped to the new consumption level immediately. Due to noisy information, consumers underestimate the true productive capacity in the first quarters until they realize that the fundamentals of the economy have actually changed. The permanent productivity shocks leads also to a permanent increase in the real wage while inflation and the real rate eventually return to their initial values.

In contrast to all other shocks, the monetary policy shock is perfectly observed by consumers. Hence, a one standard deviation shock to the nominal interest rate has the well-documented features of a negative demand shock, i.e. consumption/output, inflation, labor and the real wage fall temporarily. As compared to the noise shock, two important differences emerge: First, the noise shock causes stronger hump-shapes, especially regarding consumption which even turns slightly negative five quarters after the noise shock. Second, a positive noise shock induces an economic expansion accompanied by an increase in the nominal interest rate to reduce inflation; a surprise increase in the nominal interest rate decreases demand and has thus contractionary effects on the observed variables. These differences in the conditional moments ensure that both shocks can be separately identified in the estimation.

4.2 Variance Decomposition

I conduct forecast error variance decompositions to assess the quantitative importance of supply, demand and noise shocks in explaining business cycles. Figure 2 shows the conditional forecast error variance decomposition for productivity, consumption and consumption growth. The noise shock explains about 20 percent of consumption fluctuations on impact and still accounts for about 15 percent after four quarters. Eventually the permanent productivity shock explains the highest fraction of consumption volatility because it is the only source that permanently changes consumption. The monetary policy shock explains almost 50 percent of consumption volatility on impact. Consumption growth is mainly driven by monetary policy, while noise shocks explain 20 percent of consumption per capita growth fluctuations even after 20 quarters.

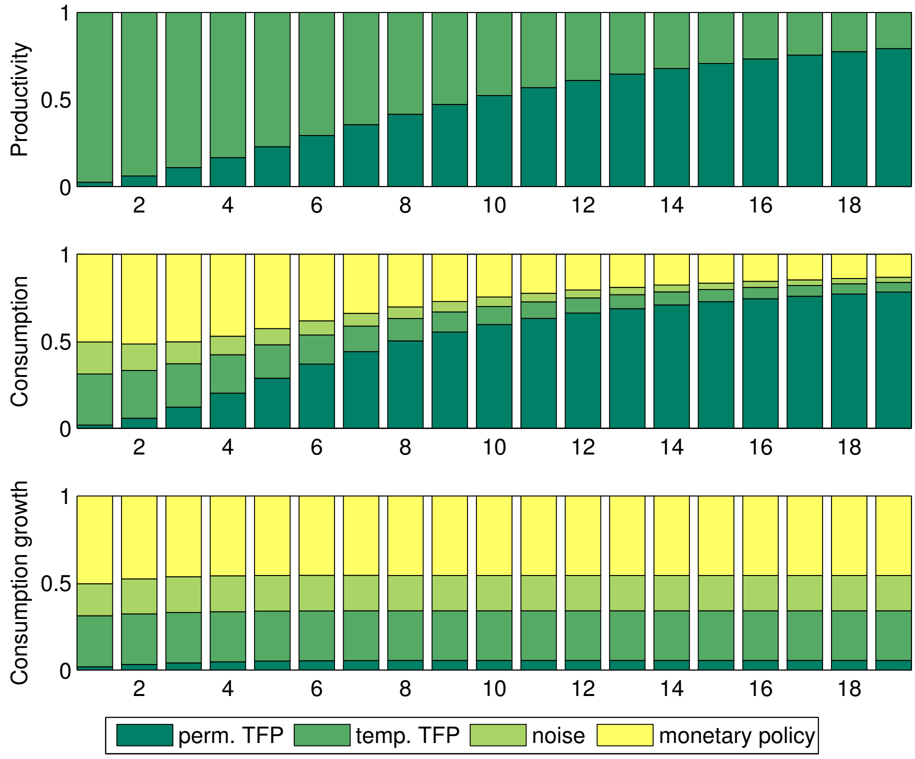

Figure 2: Forecast error variance decompositions

Notes: Forecast error variance decompositions of the estimated model where the parameters are evaluated at their posterior mean.

In the more extensive perfect information new Keynesian model with capital of Justiniano et al. (2011), preference shocks are found to explain more than 50 percent of consumption fluctuations. Preference shocks affect the economy via the intertemporal Euler equation, whereas noise shocks have similar features but offer a different interpretation. While preference shocks are difficult to interpret, noise shocks emerge naturally in a model of imperfect information and square well with the notion that consumer sentiments partially drive cyclical fluctuations.

4.3 Interaction of Nominal Rigidities and Noise Shocks

To build up further intuition regarding the propagation of noise shocks and to assess their role in the model, I perform counterfactual experiments. Therefore, I vary the degree of price and wage rigidity including the extreme case of fixed prices as well as sticky prices while keeping nominal wages flexible. The consumers' Euler equation is found to play an important role for the transmission of noise shocks.

The first column of Figure 3 shows the impulse responses to a one standard deviation noise shock for varying degrees of price rigidity. First, I briefly turn to the case of fixed prices, i.e. $\theta_p = 1$. In this case, inflation does not change and thus the nominal interest rate is zero at all times. Consequently, the real interest rate is also constant and zero, which implies perfect consumption smoothing if consumers were perfectly informed about the components of productivity. In the fixed-price model the intertemporal substitution effect is effectively shut off and thus the model turns into a partial equilibrium model as the intertemporal price, i.e. the real interest rate, is constant. Hence, if prices are fixed, quantities fully adjust to the temporary wealth effect inducing a strong response in consumption and output (by the full amount of expected long-run movement in productivity). In this scenario noise shocks explain a large fraction of consumption volatility.

Allowing for sticky prices adds an intertemporal substitution effect which substantially alters the importance of noise shocks in explaining business cycles. Increasing the frequency of price adjustment into the region of typical estimates where price stickiness takes more realistic values, i.e. $\theta_p \in (0.6, 0.8)$, mutes the propagation of noise shocks with regard to consumption.[11] As inflation and consequently the nominal interest rate increase, the real interest rate also rises. Thus consumers prefer to postpone consumption to later periods, but eventually learn that the fundamentals of the economy have actually not changed, which explains the relatively small response in consumption.

Adding nominal wage rigidity increases the role of noise shocks in explaining consumption volatility as compared to flexible wages. The second column of Figure 3 presents the impulse responses to a noise shock for various degrees of wage rigidity while price stickiness is fixed to $\theta_p = 0.71$ (the posterior mean value). Importantly, the figure illustrates that the higher the degree of wage rigidity the stronger the consumption response to noise shocks. Consumption demand increases due to the perceived wealth effect. Firms increase prices, inducing moderate inflation. Moreover, firms demand more labor in order to satisfy increased demand, which translates into increases in the real wage. Qualitatively the transmission described above is the same in the sticky price model with and without wage stickiness. However, in presence of sticky wages the inflation response is muted inducing a less pronounced increase in the real interest rate. With sticky wages the interest rate channel is substantially weakened such that consumption increases much more on impact than with flexible wages. Given the consumers' Euler equation, less consumption is postponed to later periods, which implies a more pronounced consumption

[11] Estimated new Keynesian models for the US typically find intermediate degrees of price stickiness. Estimates of the Calvo parameter are usually in the range of 0.6 to 0.85, which implies that price changes occur on average every three to five quarters (see Smets and Wouters, 2007; Christiano et al., 2005).

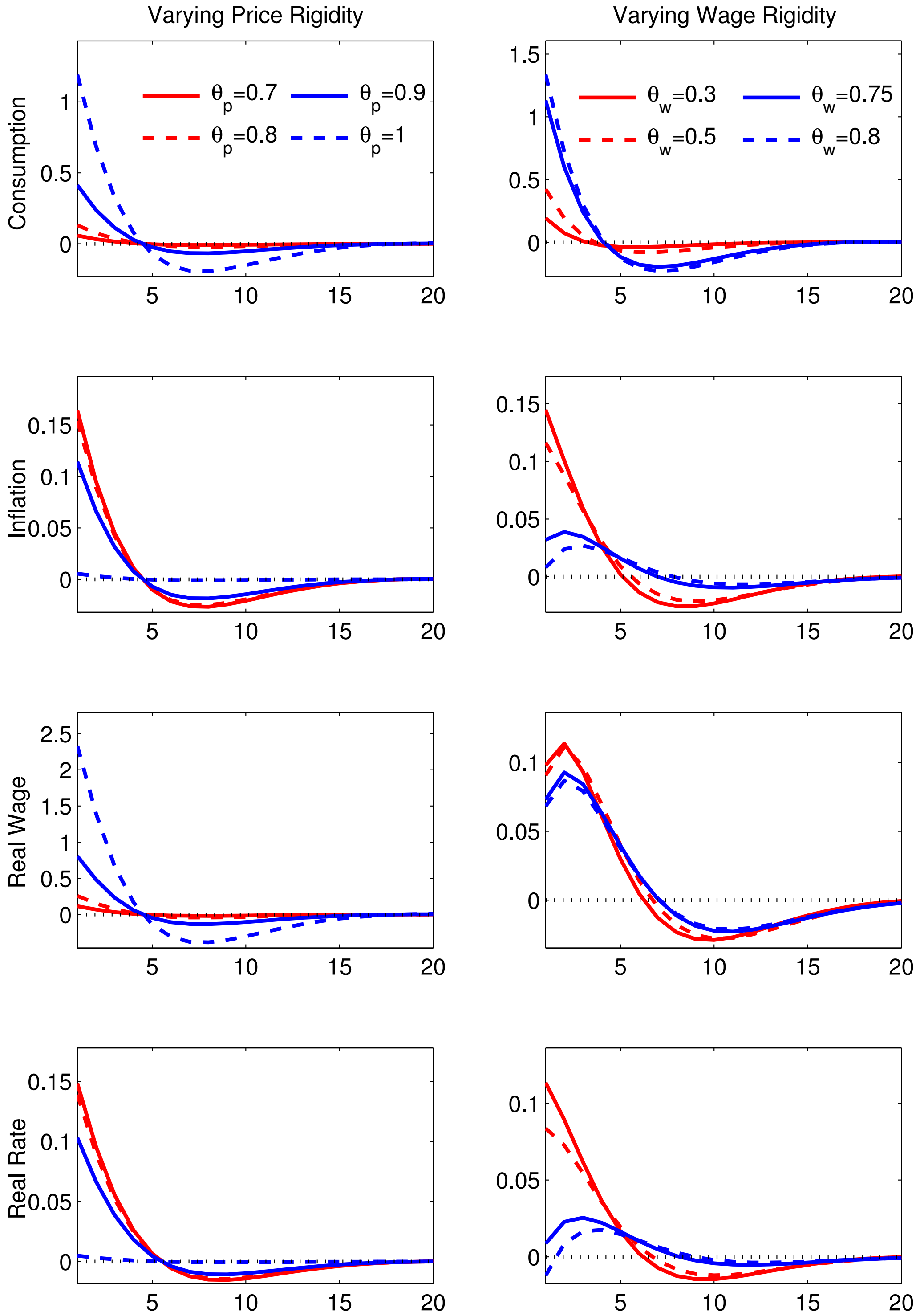

Figure 3: Impulse responses to noise shock for varying degree of nominal rigidities

Notes: Impulse responses to a noise shock with all parameters at posterior mean and varying degrees of price stickiness (first column) and wage stickiness (second column). Each legend refers to the respective column. All variables are measured in percentage deviations from steady state.

response on impact as compared to the case of flexible wages. Hence, incorporating sticky nominal wages amplifies the role of noise shocks as a driving force of consumption and output fluctuations.

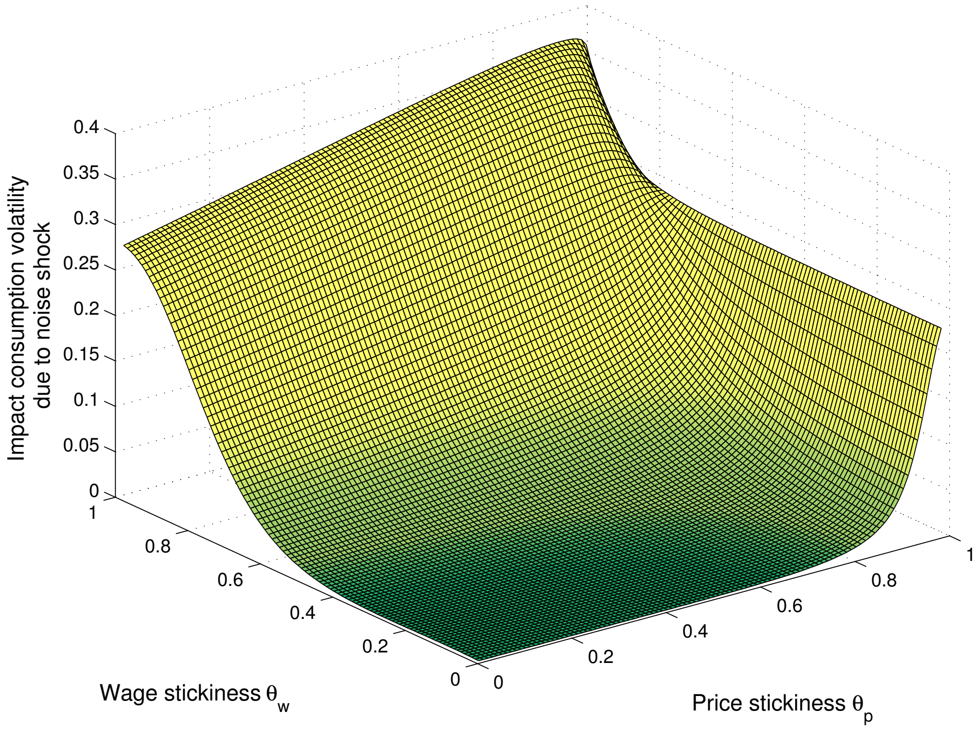

Figure 4: Interaction of nominal rigidities and noise shocks

Notes: Fraction of consumption volatility explained by noise shock on impact at posterior mean where price and wage stickiness are varied over the admissible parameter space.

Having highlighted the channels through which noise shocks affect the real variables of the economy I evaluate their importance in a variance decomposition for various degrees of nominal rigidity. Figure 4 depicts the fraction that is explained by noise shocks in a variance decomposition for consumption on impact. The higher the degree of nominal rigidity, the stronger the effect of noise shocks on consumption volatility. The counterfactual experiment reveals that if wages are assumed to be fully flexible while the remaining parameters are at their posterior mean value, the noise shock has virtually no real effects on consumption for reasonable degrees of price stickiness.

The experiments permit the following conclusion: The importance of noise shocks depends strongly on the degree of nominal rigidity. The fixed-price model of Blanchard et al. (2009) may have overestimated the importance of noise shocks. The results in Barsky and Sims (forthcoming) are confirmed regarding the diminishing role of noise shocks for plausible values of price stickiness. However, neither of the two studies incorporates sticky wages, which have a strong effect on the importance of noise shocks. The estimation confirms that US data square well with a fairly noisy signal and intermediate values of price and wage stickiness implying that noise shocks explain a sizeable fraction of US business cycles.

5 Conclusion

In this paper, I analyzed the importance of noise shocks in generating cyclical fluctuations in an estimated new Keynesian model. Based on forecast error variance decompositions noise shocks contribute to 20 percent of consumption fluctuations on impact, while the monetary policy shock explains about 50 percent. Whereas the importance of noise shocks vanishes after 16 quarters in explaining consumption fluctuations, they explain about 20 percent of consumption growth even at longer horizons. Thus, although the fixed-price model of Blanchard et al. (2009) overestimated the importance of consumer misperceptions about the true state of the economy, they are nevertheless a significant factor in explaining US consumption fluctuations.

Nominal frictions were identified to play a major role in determining the importance of noise shocks for consumption fluctuations. Sticky wages dampen the response in the real interest rate such that consumption increases strongly after a perceived wealth effect. As emphasized by Lorenzoni (2009), noise shocks have the same properties as a demand shock. Counterfactual experiments confirm the result in Barsky and Sims (forthcoming) that for intermediate degrees of price stickiness and flexible wages the noise shock explains virtually no consumption fluctuations. Strikingly, allowing for both types of nominal rigidity revives the importance of noise shocks in driving business cycle fluctuations.

Large-scale DSGE models such as Smets and Wouters (2007), which are widely applied by central bankers and policy makers, assume that business cycles are due to structural shocks that induce fundamental changes in the economy. This paper contributes to the literature by providing empirical evidence that if agents perceive the economy imperfectly and learn about the state of the economy gradually, shocks to consumer misperceptions also contribute substantially to business cycle fluctuations. Hence, future models should take into account that part of economic fluctuations may be driven by noise shocks.

A Consumers' Kalman Filter

Define the matrices

$$C = \begin{bmatrix} \rho_x & -\rho_x & -1 & \rho_x \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \rho_z & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \Sigma_1 = \begin{bmatrix} \sigma_\varepsilon^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \Sigma_2 = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_\nu^2 \end{bmatrix}.$$

The process for $\xi_t = (\hat{x}_t, \hat{x}_{t-1}, z_t, \hat{a}_{t-1})$ is described compactly as

$$\xi_t = C\xi_{t-1} + R\mu_t \ , \tag{43}$$

and the observation equation for consumers is

$$y_t = (\hat{a}_t, \hat{s}_t)' = D\xi_t + S\mu_t \ , \tag{44}$$

where y_t is the vector of observables, $\mu_t = (\varepsilon_t, \nu_t, \eta_t, m_t)', E[R\mu_t\mu_t'R'] = \Sigma_1$ and $E[S\mu_t\mu_t'S'] = \Sigma_2$. Let $P = \mathrm{Var}_{t-1}[\xi_t]$. The value of P is found by solving this equation

$$P = C\left[P - PD'(DPD' + \Sigma_2)^{-1}DP\right]C' + \Sigma_1. \tag{45}$$

According to the updating equation of a linear projection (see Hamilton (1994), equation 13.2.15)

$$\begin{aligned} \xi_{t|t} &= \xi_{t|t-1} + PD(DPD' + \Sigma_2)^{-1}(y_t - D\xi_{t|t-1}) && (46) \\ &= (I - BD)\xi_{t|t-1} + PD(DPD' + \Sigma_2)^{-1}y_t && (47) \\ &= A\xi_{t-1|t-1} + BDC\xi_{t-1} + B(DR + S)\mu_t \ . && (48) \end{aligned}$$

The last step uses $\xi_{t|t-1} = C\xi_{t-1|t-1}, B = PD(DPD' + \Sigma_2)^{-1}$ and $A = (I - BD)C$. Equation (37) in the main text uses the notation with matrices A and B.

B Model Solution

The solution to the full information log-linearized model can be obtained using standard methods, e.g. Klein (2000). The vector of control variables is $X_{1,t}$ and the vector of state variables is denoted by $X_{2,t}$. The full information model solution is given in recursive form by the policy and transition function respectively

$$X_{1,t} = \Pi X_{2,t-1} \ , \tag{49}$$

$$X_{2,t} = M X_{2,t-1} + \tilde{R}\mu_t \ , \tag{50}$$

where $X_{2,t} = [\hat{x}_t \ \ \hat{x}_{t-1} \ \ z_t \ \ \hat{w}_{t-1} \ \ \varpi_t]'$ and $\mu_t = [\epsilon_t \ \ \eta_t \ \ \nu_t \ \ \omega_t]'$.

Introducing imperfect information necessitates an adjustment of solution methods as proposed in Baxter et al. (2011). In this case private agents cannot directly observe the components of labor productivity, i.e. $\hat{x}_t$ and z_t. Define the vector of unobserved state as $\xi_t = [\hat{x}_t \ \ \hat{x}_{t-1} \ \ z_t \ \ \hat{a}_{t-1}]'$ which is a subset of all state variables $X_{2,t}$.[12] Agents form contemporaneous estimates about the state, i.e. $\xi_{t|t}$, stemming from solving the Kalman filtering problem (Appendix A contains a detailed derivation). The following system describes the evolution of the actual states and the beliefs of the agents

$$\begin{bmatrix} \xi_t \\ \xi_{t|t} \end{bmatrix} = \begin{bmatrix} N_{11} & 0 \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \xi_{t-1} \\ \xi_{t-1|t-1} \end{bmatrix} + \begin{bmatrix} R \\ B(DR+S) \end{bmatrix} \mu_t \ ,$$

where $N_{11} = C$. Solving he consumers' Kalman filtering problem yields a recursive solution for the contemporaneous beliefs (see equation (48)), i.e.

$$\xi_{t|t} = A\xi_{t-1|t-1} + B\xi_t = A\xi_{t-1|t-1} + BDC\xi_{t-1} + B(DR+S)\mu_t \ , \tag{51}$$

such that $N_{21} = BC$ and $N_{22} = A$. The matrices A, B, C and D where already introduced in the filtering problem (Appendix A). Given the contemporaneous estimates about the unobserved state $\xi_{t-1|t-1}$ and the linearity of the model, certainty equivalence applies (see Baxter et al., 2011) and hence

$$X_{1,t} = \Pi X_{2,t-1|t-1} \ , \tag{52}$$

where $X_{2,t-1|t-1} = \left[\xi_{t-1|t-1} \ \ \hat{w}_{t-1} \ \ \varpi_{t-1}\right]'$ and it is assumed that the real wage and the monetary policy shock are perfectly observed, i.e. $\hat{w}_{t|t} = \hat{w}_t$ and $\varpi_{t|t} = \varpi_t \ \forall t$.[13] In this model certainty equivalence implies that even though consumers know that they

[12] Note that a_{t-1} is perfectly observed, however we need to track a_{t-1} due to the detrending of the model to pin down $\hat{x}_t$.

[13] For example, Pearlman et al. (1986), Pearlman (1992), Svensson and Woodford (2004) and Lorenzoni (2009) also use certainty equivalence in a linear model with partial information.

imperfectly observe the fundamentals of the economy, their decisions are as if they knew the true value of the unobserved state variable (i.e. under full information).

The solution of the model under imperfect information is given by the system

$$\begin{bmatrix} \xi_t \\ \xi_{t|t} \\ w_t \\ \varpi_t \end{bmatrix} = \begin{bmatrix} C & 0 & 0 & 0 \\ BC & A & 0 & 0 \\ 0 & Q_{32} & m_{44} & 0 \\ 0 & 0 & 0 & m_{55} \end{bmatrix} \begin{bmatrix} \xi_{t-1} \\ \xi_{t-1|t-1} \\ w_{t-1} \\ \varpi_{t-1} \end{bmatrix} + \begin{bmatrix} R \\ B(DR+S) \\ 0 \\ \tilde{r}_{5.} \end{bmatrix} \mu_t \quad ,$$

where $Q_{32} = [m_{41} \quad m_{42} \quad m_{43}]$ contains specific elements from matrix M, i.e. the coefficients for $\hat{x}_t, \hat{x}_{t-1}$ and z_t (obtained in the full-information solution) are removed to their estimated counterparts $\hat{x}_{t|t}, \hat{x}_{t-1|t-1}$ and $z_{t|t}$. $\tilde{r}_{5.}$ denotes the fifth row of matrix $\tilde{R}$.

C Data Appendix

Table 3: Data Sources

Label	Frequ.	Description	Source
GDP	Q	Gross domestic product	BEA (Table 1.1.5, Line 1)
GDPQ	Q	Real gross domestic product	BEA (Table 1.1.6, Line 1)
GCD	Q	Personal consumption expenditures	BEA (Table 1.1.5, Line 3)
P16	Q	Civilian non-institutional pop. over 16	BLS (LNU00000000Q)
E16	Q	Civilian employment (S.A.)	BLS (LNS12000000)
LBCPU	Q	Hourly non-farm business compensation	BLS (PRS85006103)
FYFF	M	Federal funds rate	St. Louis FRED

Table 4: Data Construction

Label	Description	Construction
GDPDEF	GDP deflator	GDPQ/GDP
A	Labor productivity	GDPQ/E16
C	Real per-capita consumption	GCD/P16/GDPDEF
W^r	Real wages	LBCPU/GDPDEF
FFR	Effective Federal funds rate	quarterly average of FYFF

Notes: The data set constructed with US data is transformed to match the model equivalents in the observation equation (42) in the main text.

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