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Education, Income Distribution and Innovation

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Education, Income Distribution and Innovation

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Abstract

In this paper we study the impact of the income distribution on innovation through the demand for quality goods. For simplicity, we assume that there are two types of consumers, rich and poor. The income distribution is measured by the population share of the poor and the relative income of the poor. Contrary to the literature, we assume that both are interdependent through education. The larger the income difference between the poor and the rich, the more individuals undergo education, because individuals can become rich through education. Quality goods are first invented, and then produced by oligopolists. Rich consumers have a higher willingness to pay for the better quality than the poor. Hence, the firms' profit depends on the income distribution of consumers. We focus on the separating equilibrium, where goods of different qualities are sold to different consumers. In this equilibrium, a lower relative income of the poor is good for innovation, and a larger population share of the poor is bad for innovation.

JEL Classification: D31, D43, I20, O12, O15, O31.

Keywords: inequality, growth.

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1. Introduction

The relationship between a country's income distribution and its economic growth is a permanent topic which sparks debates not only among economists but also policy-makers. In the last fifteen years, most cross-country studies (e.g., Berg and Sachs 1987, Persson and Tabellini 1994, Alesina and Rodrik 1994, Clarke 1995) show that if there is a relationship at all, inequality has a negative impact on long run growth rates. Nonetheless, there also is evidence that inequality has a positive impact on short or medium run growth rates (Forbes 2000), or that the relationship between the income distribution and the long run growth rate is non-linear (Chen 2003, Banerjee and Duflo 2003). In this article we provide a theoretical model to shed light on the ambiguous relationship between income distribution and the economic growth.[2] We argue that inequality, which is measured by the Gini-coefficient, includes many variables, which may have a different impact on the economic growth.

For simplicity, we assume that there are two types of individuals, poor people and rich people. The income distribution can be measured by two variables, the income of the poor relative to the average income, and the population share of the poor. Both an increase in the relative income of the poor and a decline in the population share of the poor indicate a decrease in inequality. The minimal wage level, social insurance and so on could be considered as policies to improve the income of the poor, whereas mandatory education is easily understood as one to reduce the population share of the poor. If they have a different impact on growth, above cross-country evidence, which is based on the simple regression of the Gini-coefficient on the economic growth rate, could be ambiguous. In particular, we may be unable to draw from such simple empirical studies recommendations on redistribution policies for achieving a higher economic growth rate as well as a more equal income distribution.

The present model consists of three parts: first, we assume an overlapping-generations economy, where individuals live for two periods: young and old. Every individual faces a different decision problem in each period. They decide whether to undergo education when young, and how to consume when old. Hence, if we neglect the young period and its education decision, the model reduces to the benchmark model by Zweimüller and Brunner (1998).

[2] There are many different theoretical models to explain these different empirical results. E.g., Bénabou (1996) summarized three points of view to explain the negative impact of inequality on growth. Bénabou (2002) provides a model to illustrate the non-linear relationship between redistribution and growth.

Second, we discuss the impact of the income distribution on the firms' profits in a vertically differentiated goods market in a model originally introduced by Shaked and Sutton (1982, 1983). However, our analysis focuses on the general equilibrium, whereas those papers are interested in issues of competition in a partial equilibrium framework. Rich consumers can afford more high quality goods than the poor and are willing to pay more for them. Hence, firms supply different qualities to different consumers in order to reduce the competitive pressure on prices. Therefore, the firms' profit depends on the income distribution. Since we focus on the impact of the income distribution on the demand for quality goods, the labour market and production are assumed as simple as possible: the labour force is the single production factor, which is allocated among the production sectors and the research activity. Everybody inelastically supplies one unit of labour and the productivity is also same for all individuals.

Third, economic growth is achieved through innovation. The high quality good is firstly invented, and then produced by oligopolists. Innovation is assumed to follow a Poisson process. However, an inventor can increase the Poisson arrival rate through employing more workers. The inventor's incentive to innovate depends on the profit of production after taking the cost of innovating into account. Hence, the income distribution can affect innovation through profits. If we consider the pooling case, where the oligopolistic market reduces to a monopolistic one, we are back to the case of Aghion and Howitt (1992).

Inequality may give rise to quality differentiation and a higher incentive for firms to innovate because rich consumers can pay more for high quality goods than the poor. But on the other side, the relatively small market share of high quality goods implied by inequality impedes the spread of better quality goods. Hence, the effect of the income distribution on innovation is *a priori* unclear. The result of Zweimüller and Brunner (1998) is that, in general, inequality has a negative impact on innovation both through a larger population share of the poor and a lower relative income of the poor, under the assumption, that the population share of the poor and the relative income of the poor are independent. This seems unrealistic. The improvement of relative income of the rich may increase the incentive of the poor to become the rich. Of course, we can also argue that this incentive will decrease if the poor find that the rich are so rich that they can't catch up. If there are suitable channels in our society through which the poor are able to become the rich, for example, through education, immigration, or winning a lottery, we may find that the population share of the poor is an endogenous variable given exogenous relative income of the poor. In this paper we assume that individuals can become rich through education. If we increase the relative income of the poor, individuals have less of an incentive to undergo education. Hence, the population share of the poor increases. It reflects the idea of "social mobility" in political economy, which describes the movement of individuals among different income classes. It is helpful to understand why in a democratic society a relatively poor majority does not support an expropriating redistribution scheme, because they expect rationally that they will become the rich in the future. (For a discussion, see Bénabou and Ok 2001.)

This paper shows that assuming interdependence between relative income and population share is crucial in the study of the impact of inequality on innovation. The result of Zweimüller and Brunner (1998), that the redistribution from the rich to the poor raises the innovation rate, does not hold under the assumption of interdependence. The main results of this paper are that there is a separating equilibrium, in which the high quality good is sold only to the rich and the low quality good only to the poor. In this equilibrium, a lower relative income of the poor is good for innovation, and a larger population share of the poor is bad for innovation. This result is consistent with Foellmi and Zweimüller (2002). But there they introduce hierarchic preferences[3], and the innovation induces new goods but not the improvement of quality.

The other novel result is that education enrollment is always positively associated with the innovation rate, although we have not assumed that education can increase productivity. In literature, economists focus on the impact of education on economic growth through increasing productivity. However, the current paper argues that education is able to influence economic growth through the demand for better quality. Education generates not only higher productivity, but also richer consumers. The latter is almost neglected by most economic studies.

The paper is organized as follows. In section 2, the set-up of the model is introduced. In section 3 and 4, we study equilibrium and simulate the model to show the impact of the relative income of the poor on innovation given the endogenous population share of the poor. In section 5, we discuss the case, where the relative income of the poor is endogenous and the

[3] "A hierarchy of wants implies that goods can be ranked according to their priority in consumption" (Foellmi and Zweimüller 2002)

population share of the poor is exogenous. Section 6 presents the dynamics of the model. The main results are summarized in section 7.

2. The Model

We have two types of agents, consumers and firms. Consumers live for two periods: young and old. The young people decide whether to undergo education, and the old people decide how to consume. Firms produce two kinds of goods, the standard good and the quality good. In order to produce quality goods, they must first invent them.

2.1 The Environment

We consider an overlapping-generations economy populated by a continuum of consumers, who live for two periods: young and old. The population size is constant. We normalize the population size of young people to measure one. Then the population of the old also has measure one at any time. In the first period, young individuals receive an amount of money from old individuals which can cover their education cost denoted by e. This transfer is exogenous. We can imagine that it can be implemented through a lump-sum education tax on the old. This transfer e is the single source of income for the young.

The old individual works and owns firms. Our model focuses on the demand for consumption in the period "old". Hence, we assume a simplistic view regarding the production of consumption goods. Labour is the single productive factor, and every old individual inelastically supplies one unit of labour to the competitive labour market. Consequently, wage income (w) is the same for all old individuals. Besides the wage income, the old individual i can also earn interest income θA_i, with θ as the interest rate, and A_i taking the value of firms owned by the individual i, we call it wealth. Hence, the total income of the old individual i is $y_i = w + \theta A_i$. For simplicity, we assume that there are only two groups of old individuals, the poor (p) and the rich (r), distinguished by wealth, $A_r > A_p$, and, consequently, by income $y_r > y_p$. Their consumption expenditure is the total income net of the education cost e. (In Table 1 below, $y_p - e$ and $y_r - e$, respectively.) We assume $A_p = dV$, where d ($0 < d < 1$) measures the wealth of the old poor relative to the average level of old people, V is the average wealth per capita in period "old". Hence, $V = \beta A_p + (1-\beta) A_r$, where β ($0 < \beta < 1$)

denotes the population share of the poor in period "old". We get $A_r = \frac{1-d\beta}{1-\beta}V$. Hence, the more young people undergo education, the lower the relative wealth of the old rich people ($\frac{1-d\beta}{1-\beta}$), holding d constant. The average wealth V is accumulated by the profit of firms after netting research costs and interest payment. For the definition of profits see section 2.3 and for that of research costs see section 2.4.

Young individuals can decide whether or not to go to school. If they go, they pay e as tuition. Thus, they have nothing left to spend on consumption (in Table 1 below, the consumption expenditure of a student is 0). Otherwise they can consume with their budget e (in Table 1 below, there is consumption expenditure e for non-students). Without education, a young person is confined to a poor position in society upon reaching old age and gets wealth A_p, otherwise they can have the wealth of a rich person A_r. In other words, the population share of students in period "young" is $1-\beta$.

Table 1: Expenditure and population size of different individuals

	Young		Old	
	Student	Non-student	Poor	Rich
Consumption expenditure	0	e	$y_p - e$	$y_r - e$
Population size	$1-\beta$	β	β	$1-\beta$

There are two kinds of goods, referred to as standard good and quality good, respectively. Let x be the quantity of the standard good, which has a constant quality (normalized to 1) and is traded in a competitive market. Hence, the price P_x is equal to its marginal cost, which is also normalized to 1. The marginal cost of the standard good can be expressed as wb, where w is the wage and the unit labour demand is b, which measures how many units of labour are needed to produce one unit of the standard good. We get $P_x = wb = 1$.

The quality goods are traded in an oligopolistic market. At any time there are many qualities q_j, $j = 0,-1,-2,...$ available in the market, the high quality good is k times better than the next lower one: $q_j = kq_{j-1}$. But marginal costs are same, denoted by wa, where $a < 1$ is again the unit labour demand. Every quality good is first invented through research and then produced by one firm. After a successful innovation, the new inventor can produce a k-times better quality good than the existing best one in the next period. Innovation is a random process, which will be introduced in section 2.4. Hence, the life-time of the oligopolistic firm is uncertain. The firm which sells the highest quality good q_0 can keep its position until the successful inventor enters, after which its good becomes the second best good q_{-1} until the next new inventor enters and so on. Since in the original model of Zweimüller and Brunner (1998) only two firms can exist in their vertical differentiation competition, we take their results to mean that the third best quality is driven out of the market. This third best quality supplier can be considered as the potential competitor, who sets the price at the marginal cost. Nonetheless its demand is still zero.

2.2 The household's decision problem

Every individual faces a two-stage decision problem. At the beginning of young period she decides whether or not to go to school, i.e., she allocates the consumption expenditure over time. When she is old, the individual decides how to allocate her instantaneous consumption expenditure between standard good and quality good. It doesn't mean that young people have no consumption decision. They consume with budget e if they do not go to school. But we assume for simplicity that they can only consume the standard good.[4] Hence, they simply spend all of their income e on the standard good in order to maximize their instantaneous utility.

We begin our analysis from the second stage of the individual's decision problem. There is no saving. All income of old people except for the education lump-sum tax is spent either on the consumption of the standard good or the quality good. Every old individual can consume one and only one unit of the quality good q_j. There is no limitation to the consumption of the standard good x_i except for the budget constraint, i.e., $y_i - e = 1 \cdot x_i + P_j \cdot 1 \quad j = 0, -1$, where the price of standard goods is 1, the price of the best quality is denoted by P_0 and the second best one's price is P_{-1}. The preference for consumption of the standard good and the quality good is given by the following instantaneous utility function:

[4] This assumption is made to ensure that the potential consumers of quality goods have only two types (old poor and old rich). It guarantees that there are only two qualities in the market (see Zweimüller and Brunner1998).

$$u_{old,i}(x_i, q_j) = \ln x_i + \ln q_j \qquad i = p, r \text{ and } j = 0, -1 \tag{1}$$

which can also be expressed as: $u_{old,i} = \ln(y_i - e - P_j) + \ln q_j$.

Analogously, we assume the instantaneous utility of young individuals as follows:

$$u_{young,i} = \begin{cases} \ln e & i = p \\ 0 & i = r \end{cases} \tag{2}$$

Young individuals maximize their life-time utility U_i at the beginning of their young period:

$$U_i = u_{young,i} + \rho u_{old,i} \tag{3}$$

where ρ is the discount factor. If they decide not to undergo education, their life-time utility is $U_p = \ln e + \rho u_{old,p}$. Otherwise, they have $U_r = \rho u_{old,r}$. Suppose $U_p > U_r$, all young people are not willing to undergo education. Hence, $\beta \to 1$. Recall that the relative wealth of the rich ($\frac{1-d\beta}{1-\beta}$) increases in β, $\beta \to 1$ implies $y_r \to \infty$. We assume that the instantaneous utility increases in the income[5], hence, $U_r \to \infty$, which contradicts $U_p > U_r$. It implies that $U_p > U_r$ cannot be an equilibrium. Analogy, $U_p < U_r$ cannot be an equilibrium, either. Hence, in equilibrium we must have $U_p = U_r$, where individuals are indifferent between education and non-education. This leads to:

$$\ln e = \rho(u_{old,r} - u_{old,p}) \tag{4}$$

The left hand side of (4) is the cost of education, while the right hand side is the benefit. In equilibrium both should be equal. Hence, the heterogeneity among old consumers (poor and rich) comes from the indifference between education and non-education for the young. In this heterogeneous steady state, β is determined by exogenous parameters, although individuals are randomly divided between the poor and the rich.

[5] Since we don't know the price of firms, we can only assume the monotonous relationship between the utility and the income. In next section, we analyze price decision of firms, the monotony expressed through equations (18) and (19) can then be proven.

2.3 The Pricing Decision of Oligopolists

Firms have all the above information but they are unable to distinguish between individuals by income. The strategy which firms can pursue is to choose a price while quality is fixed. We concentrate only on the steady state where prices are constant over time. The whole market size of oligopolists is 1 while only the old individuals can buy quality goods. We differentiate between rich and poor consumers of quality goods respectively, dropping "old" below. For simplicity, we assume that the consumer prefers better quality goods if both quality goods yield the same utility.

Lemma 1: There are only two kinds of pricing equilibria: In the pooling equilibrium the best quality good is sold to both the rich and the poor, the second best quality good can't be sold; in the separating case the best quality good is consumed by the rich and the second best quality good is consumed by the poor.

Proof: see Appendix 1.

The second best quality supplier considers only how to attract the poor to purchase her good while the rich never buy the second best good in equilibrium, according to Lemma 1. Because of the existence of potential competitors which offer the price at marginal cost, the highest price which the second best firm offers satisfies:

$$\ln(y_p - e - \overline{P}_{-1}) + \ln q_{-1} = \ln(y_p - e - wa) + \ln q_{-2} \qquad (5)$$

The left hand side of this equation is the utility when poor individuals buy the second best quality good q_{-1} and the right hand side is the utility when they consume the third best quality good q_{-2}. Only if the second best quality good can yield at least the same utility as the third best quality good to consumers, the consumer prefers buying it. Substituting $q_{-1} = kq_{-2}$ and rearranging the equation, we get the highest price of the second best quality good:

$$\overline{P}_{-1} = (1 - \frac{1}{k})(y_p - e) + \frac{wa}{k} \qquad (6)$$

The lowest price which the second best quality firm can offer is at marginal cost wa. Analogously, the best firm can set its highest price satisfying:

$$\ln(y_i - e - \overline{P}_0) + \ln q_0 = \ln(y_i - e - P_{-1}) + \ln q_{-1} \qquad (7)$$

leading to $$\overline{P}_0 = (1-\frac{1}{k})(y_i - e) + \frac{P_{-1}}{k}, \quad i = p, r \qquad (8)$$

These two reaction functions are depicted in Figure 1. In order to attract the poor to buy its products the best firm sets its price as high as $(1-\frac{1}{k})(y_p - e) + \frac{P_{-1}}{k}$ (it is the line CD in Figure 1). Because the rich can afford more good quality goods than the poor, they are willing to buy the best good too if the poor prefer the best good to the second best good. Hence, the area below CD (including CD) is the pooling strategy case, where the best quality good captures the entire market and the second best quality good is not sold. Above CD the poor don't purchase the best quality good. The line AB in Figure 1 expresses the highest price of the best good, given P_{-1}, if the best firm wants to attract only the rich. Hence, the area ABCD excluding the line CD is the separating strategy.

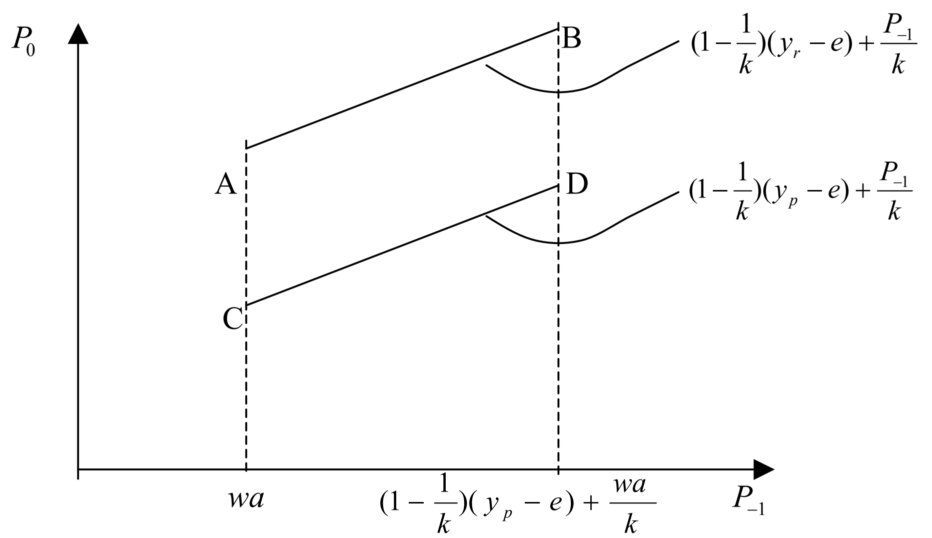

Figure 1: Pricing decision of quality goods firms

We define the profit of firms as $\pi_j = (P_j - wa)(market\ share\ of\ firm\ j)$, $j = 0, -1$. The firms set their prices as high as possible given the market share of their goods. The two possible equilibria in price competition are summarized below:

1. Pooling: q_0 is sold to all consumers. In this case the second best firm becomes the potential competitor and its price and profit are respectively:

$$P_{-1} = wa \quad (9)$$

$$\pi_{-1} = 0 \quad (10)$$

The best firm can set its price at:

$$P_0 = (1-\frac{1}{k})(y_p - e) + \frac{wa}{k} \quad (11)$$

and earn profit $$\pi_0 = (1-\frac{1}{k})(y_p - e - wa) \quad (12)$$

2. Separating: q_0 is sold to the rich and q_{-1} to the poor. Because this is a repeated game until a new inventor comes in, many possible equilibria could exist. In order to get a unique result, we assume that no player is punished if she changes her price without affecting the other player's profit. Then the single separating equilibrium is point B:

$$P_{-1} = (1-\frac{1}{k})(y_p - e) + \frac{wa}{k} \quad (13)$$

$$P_0 = (1-\frac{1}{k})(y_r - e) + (\frac{k-1}{k^2})(y_p - e) + \frac{wa}{k^2} \quad (14)$$

$$\pi_{-1} = \beta(P_{-1} - wa) \quad (15)$$

$$\pi_0 = (1-\beta)(P_0 - wa) \quad (16)$$

Proof: see Appendix 2.

2.4 Innovation

As mentioned before, the new entrant of this oligopolistic market should do research before production. Only after the successful innovation it can produce a quality k-times better than the currently best. Following the work by Aghion and Howitt (1992), we assume that the innovation is random and arrives according to a Poisson process with parameter ϕ. The researcher can employ n workers to reach the Poisson arrival rate ϕ, i.e., $\phi = \lambda n$, where λ is the productivity of workers in research, which is given by the technology of research. This assumption of innovation means that the success of research depends only on current input, not upon past research. The flow of research cost is wn. And the flow of research benefit is ϕB, where B is the present value of future profits when innovation takes place:

$$B = \sum_{t=0}^{\infty}\left(\frac{\pi_0}{(1+\theta)^t} prob\{0\ innovation\ before\ t\} + \frac{\pi_{-1}}{(1+\theta)^t} prob\{1\ innovation\ before\ t\}\right)$$

$$= \sum_{t=1}^{\infty}\left(\frac{\pi_0(1-\phi^e)^{t-1}}{(1+\theta)^t} + \frac{\pi_{-1}(t-1)(1-\phi^e)^{t-2}\phi^e}{(1+\theta)^t}\right)$$

leads to

$$B = \frac{\pi_0}{\phi^e+\theta} + \frac{\phi^e \pi_{-1}}{(\phi^e+\theta)^2} \tag{17}$$

where t is a time index, $\phi^e = \lambda n^e$ is the expected future arrival rate of innovation, and n^e is the expected future number of workers in the research sector.

We are now in a position to define V. We assume that the firms' profits net of interest payments and research costs consist of average wealth, i.e., $\Delta V = \pi_0 + \pi_{-1} - wn - \theta V$, where ΔV presents the difference of the average wealth between two subsequent periods. V can be interpreted as the aggregate value of firms. According to our assumption this wealth is distributed among old individuals according to their education level.

3. Equilibria

The general equilibrium, which consists of three conditions, is presented in this section. Substituting the price decisions of firms into equilibrium conditions, we obtain two possible equilibria: the pooling and the separating.

3.1 Equilibrium Conditions

The equilibrium condition of the education decision is given by equation (4). Substituting the pooling price (11) and the separating prices (13), (14) in (4), respectively, we get the same form as follows:

$$\ln e = \rho \ln\left(\frac{k(y_r - y_p)}{y_p - e - wa} + 1\right) \tag{18}$$

leads to $$\ln e = \rho \ln\left(\frac{k\theta V \frac{1-d\beta}{1-\beta}}{w+\theta dV - e - wa} + 1 \right) \qquad (19)$$

From (19) we know the interdependence between β and d. The left hand side of (19) is the education cost and the right hand side is the benefit from education. If we improve the relative wealth of the poor (d rises), y_p increases and y_r decrease. In other words, the benefit of education declines. Therefore, young people have less of an incentive to undergo education. This means *ceteris paribus* a higher population share of the poor. We assume now that d is exogenous and β is endogenous. In section 5, we discuss the impact of an exogenous β on the innovation rate, given that d is endogenous.

Lower time preference ρ indicates more impatience. Hence, fewer individuals invest in education, which means a higher β. The effect of e on β is not so obvious. At first, e is the education cost. The increase in the education cost decreases the incentive of education for the young. Hence, β increases. This is the effect of e on the left hand side of equation (19). On the other hand, e is also a social transfer from the old to the young. The old becomes poorer if e increases. Hence, the marginal utility of education increases, i.e., $y_r - y_p$ yields more utility if y is lower. This induces a lower β. If e is not so large, the latter effect is dominated by the former. We can proof that β increases in e if it satisfies the following sufficient condition:

Assumption 1 $$e \le \frac{w - wa}{1+\rho} \qquad (20)$$

Proof: see Appendix 3.

For the research sector we assume free entry and perfect foresight for firms in equilibrium, which is analogous to Aghion and Howitt (1992). Hence, the innovation equilibrium condition means that the flow of research costs should be equal to the flow of expected profits, i.e. $wn = \phi B = \lambda n B$ and $\phi = \phi^e$ (or, $n = n^e$) due to perfect foresight. This leads to:

$$\frac{w}{\lambda}=\frac{\pi_0}{\phi+\theta}+\frac{\phi\pi_{-1}}{(\phi+\theta)^2} \tag{21}$$

The underlying intuition is similar to Aghion and Howitt (1992). The left hand side of equation (21) represents the flow cost of research per efficient worker, which decreases in the productivity of research workers λ. The effect of λ on ϕ is positive, because the researcher employs more workers to do research, if the productivity of workers increases. The interest rate affects the innovation rate via two channels: first it is a discount factor, hence, the higher θ, the lower is the benefit of research. Therefore, the innovation rate decreases in the interest rate. On the other hand, higher θ means more interest income of individuals, which increases the profit of firms. Hence, the benefit of research increases. This implies the positive impact on the innovation rate. The main difference between our model and that of Aghion and Howitt (1992) lies in the market structure. They assume a monopoly market, hence, firms can survive only until a successful inventor comes in. Hence, there is no π_{-1}. In our oligopoly model firms can exist in two stages: best quality supplier and second best supplier. Note that in the pooling case $\pi_{-1}=0$, which is equivalent to the case of the monopoly in Aghion and Howitt (1992).

The income of the education sector $(1-\beta)e$ originates from students, and the education sector employs workers to supply courses. We assume the labour demand of the education sector is S, thus, the cost of the education sector is wS. In equilibrium the budget of the education sector should be balanced: $(1-\beta)e = wS$.

The labour market equilibrium condition is standard, i.e., at any point in time the labour supply should be equal to the labour demand:

$$1=n+a+b(e\beta+x_p\beta+x_r(1-\beta))+S \tag{22}$$

which implies:

$$\frac{w\phi}{\lambda}=\pi_0+\pi_{-1}-\theta V \tag{23}$$

By assumption, each old individual supplies one unit of labour, so the total labour supply is 1. The total labour demand is illustrated by the right hand side of equation (22): n is the labour demand in the research sector. The quality good sector needs labour a, because the total demand for quality goods is 1 and the unit labour demand of quality goods is a. $e\beta + x_p\beta + x_r(1-\beta)$ is the total demand for standard goods, which consists of three parts: the non-students' demand, and the demand of the poor and the rich. Recall that b is the unit labour demand of standard goods, hence, the third item of the right hand side of equation (22) measures the demand for labour in the standard goods sector. Finally S is the labour demand of the education sector. Recall that $\Delta V = \pi_0 + \pi_{-1} - \theta V - \frac{w\phi}{\lambda}$. Hence, (23) implies that in a stationary equilibrium, the average wealth V remains constant ($\Delta V = 0$). Now we have three equations (19, 21, 23) in the three variables β, ϕ and V. We omit the discussion of equilibrium existence condition, because it is similar to that in Zweimüller and Brunner (1998).

3.2 The Pooling Equilibrium

Substituting price and profit equations (9, 10, 11, 12) and (13, 14, 15, 16), respectively, in the above equilibria conditions (21, 23) leads to two different equilibria, namely "Pooling" and "Separating". We discuss the simple case first.

Pooling equilibrium:

$$\ln e = \rho \ln\left(\frac{k\theta V \frac{1-d\beta}{1-\beta}}{w + \theta dV - e - wa} + 1\right) \tag{19}$$

$$\frac{w}{\lambda} = \frac{(w + \theta dV - e - wa)(1-\frac{1}{k})}{\phi + \theta} \tag{24}$$

$$\frac{w\phi}{\lambda} = (w + \theta dV - e - wa)(1-\frac{1}{k}) - \theta V \tag{25}$$

It is of interest to see the impact of e, ρ, d on the equilibrium value of variables β, ϕ and V. If we substitute (24) in (25), then $V^* = \frac{w}{\lambda}$. Hence, e, ρ, d have no impact on V^*. We depict these equations below in Figure 2. In the pooling equilibrium (24) and (25) are independent of β. This simplifies the analysis. In fact the right hand side of Figure 2 shows just the model of Zweimüller and Brunner (1998) with an exogenous β, which is the special case in my model without (19).

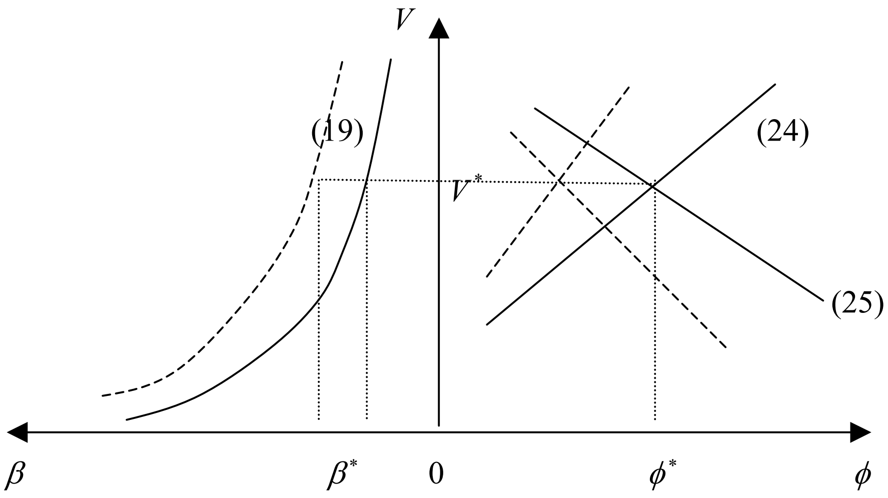

Figure 2: The pooling equilibrium

When e increases, the opportunity cost of education increases (dotted line in Figure 2). This means that fewer young people undergo education. Hence, β^* increases under the assumption 1. On the other hand, e is a transfer from the old to the young, viz. to those who cannot buy quality goods. It is equivalent to say that the consumer of the quality good becomes poorer. A lower willingness to pay translates into a reduced price and less profit from quality goods. This, in turn, leads to a lower incentive to innovate.

If ρ increases, then β^* decreases, because the young are more patient and thus more of them invest in education. However, it does not necessarily imply a higher innovation rate, because in the pooling equilibrium the market of the best quality goods is made up of the whole population of old people, the change of β^* does not change the market share and profits of the best quality good supplier. Hence, there is no impact on the incentive to invent.

From equation (24) and (25) we get $\phi^* = (\lambda - a\lambda - e\lambda / w + \theta d)(1 - 1/k) - \theta$. When d increases, the benefit of education decreases. This leads to fewer students, in addition to fewer rich consumers of quality goods. But in the pooling equilibrium the market of quality goods is the whole population of the old. The profit of firms is independent on the population share of the poor. Moreover, the decisive consumer in the price decision is the poor. The improvement of their budgets means that they can pay more for quality goods. The firm can charge a higher price and earn a larger profit, which increases the incentive to innovate. In this sense we can say that redistribution (increase in d) is good for the long run growth rate (here, a higher innovation rate ϕ).

We summarise the relationship between inequality and innovation as follows:

Result 1:

In the pooling equilibrium, the relative wealth of the poor has a positive impact on the innovation rate; and the innovation rate is independent on the population share of the poor. A higher education cost leads to a lower innovation rate. The time preference ρ has no impact on the innovation rate.

3.3 The Separating Equilibrium

Now we turn to the separating equilibrium. According to the vertical differentiation model of Shaked and Sutton (1982, 1983), maximization of quality differentiation is the optimal choice for firms in order to reduce price competition. The target of quality differentiation is to separate the whole market into several parts. The firms can then play in some sense a monopolistic role. The pooling equilibrium appears only because we assume for simplicity that there are two types of consumers. If income is continuously distributed among individuals, the pooling equilibrium cannot exist any more. Hence, the separating equilibrium is more general and more important than the pooling case. We will concentrate on the separating equilibrium in following discussion.

In the separating equilibrium q_0 is sold only to the rich and q_{-1} is sold only to the poor. Hence, β enters the profit function of firms and the equilibrium equations of innovation and labour market. So the analysis is more involved. In order to show the impact, we simulate the model in the next section.

$$\ln e = \rho \ln\left(\frac{k\theta V \frac{1-d\beta}{1-\beta}}{w+\theta dV - e - wa} + 1 \right) \qquad (19)$$

$$\frac{w}{\lambda}=\frac{\left(\left(w-e+\frac{1-d\beta}{1-\beta}\theta V\right)\left(1-\frac{1}{k}\right)+\left(w-e+d\theta V\right)\left(\frac{k-1}{k^2}\right)+\frac{wa}{k^2}-wa\right)(1-\beta)}{\phi+\theta}+\frac{\phi\left(\left(w-e+d\theta V\right)\left(1-\frac{1}{k}\right)+\frac{wa}{k}-wa\right)\beta}{(\phi+\theta)^2} \tag{26}$$

$$\frac{w\phi}{\lambda}=\left(\left(w-e+\frac{1-d\beta}{1-\beta}\theta V\right)\left(1-\frac{1}{k}\right)+\left(w-e+d\theta V\right)\left(\frac{k-1}{k^2}\right)+\frac{wa}{k^2}-wa\right)(1-\beta)+\left(\left(w-e+d\theta V\right)\left(1-\frac{1}{k}\right)+\frac{wa}{k}-wa\right)\beta-\theta V \tag{27}$$

4. Simulation

As equations (19), (26), (27) show, the separating equilibrium is not as easy to analyze as the pooling case, because the population share of the poor is able to influence the firms' profits. Our main purpose is to study the impact of the income distribution (here measured by d) on the innovation rate ϕ and the population share β, as well as the impact of the education cost e and the time preference ρ on ϕ and β. For the purpose of simplification, we show the impact by numerically simulating the model.

4.1 Simulation Procedure

First, we assume d to be exogenous and β to be endogenous. (The reverse case is considered in the next section.) In order to show the impact of parameter changes and exogenous variables on the endogenous variables, we analyze them *ceteris paribus*. E.g., in order to show the impact of d on ϕ, β, V, we let d move away from the benchmark value 0.4 to 0.2 and 0.6, respectively, holding the other parameters at the fixed benchmark value (the value of d from 0.2 to 0.6 and the according values of endogenous variables are shown in Table 2, first part).

We set all parameters and exogenous variables at the following values as a benchmark. $\theta=0.5, \rho=0.5, w=10, e=2, a=0.3, \lambda=1, k=4, d=0.4$. They are chosen for the following reasons. 50% is the suitable interest rate per period, because the period in the current paper reflects the generation. In reality it is about 20-30 years. We also assume the subjective discount rate $\rho = 0.5$, which reflects the time preference between young and old periods. The wage rate and the education cost are set to satisfy Assumption 1. Another reason why we set the education cost so low is that the model of Zweimüller and Brunner (1998) does not consider education, and we do not want to deviate too much. We choose $a = 0.3$ so that total labour supply is almost equally allocated among research, quality good production and the standard good sector. The other two parameters characterizing research and innovation -- λ, k -- are chosen only for simplicity, because we know very little about such characteristics in this pure theoretical model.

4.2 Simulation results

The simulation result is summarized as below Result 2 and Table 2:

Result 2:

In the separating equilibrium, redistribution from the rich to the poor (i.e., d increases) has a negative impact on the innovation rate; the population share of the poor increases with the relative wealth of the poor. A higher education cost and a lower time preference leads to a lower innovation rate.

Table 2: Simulation results of the separating equilibrium

with $\theta = 0.5, \rho = 0.5, w = 10, e = 2, a = 0.3, \lambda = 1, k = 4, d = 0.4$ as benchmark

d	0.2	0.3	0.4	0.5	0.55	0.6
ϕ	0.35	0.33	0.31	0.29	0.28	0.26
β	0.08	0.21	0.34	0.46	0.52	0.57
V	10.43	11.28	12.34	13.67	14.49	15.45
$\frac{\pi_0^{po}(V^{se})}{\phi^{se} + \theta}$	5.33	6.05	6.91	7.99	8.64	9.51

ρ	0.25	0.3	0.4	0.5	0.6	0.65
ϕ	0.18	0.21	0.27	0.31	0.35	0.37
β	0.83	0.74	0.54	0.34	0.16	0.07
V	17.91	16.46	14.09	12.34	11.0	10.44

$\frac{\pi_0^{po}(V^{se})}{\phi^{se}+\theta}$	9.47	8.76	7.61	6.91	6.35	6.11

e	1.75	2	2.25	2.5	2.75	3
ϕ	0.38	0.31	0.26	0.22	0.18	0.15
β	0.15	0.34	0.47	0.56	0.62	0.67
V	10.92	12.34	13.41	14.26	14.95	15.51
$\frac{\pi_0^{po}(V^{se})}{\phi^{se}+\theta}$	6.34	6.91	7.33	7.66	7.99	8.19

We first check whether the above solution of separating price strategy can yield at least as much benefit as under pooling. Otherwise the firm would switch to pooling. The benefit from separating is $\frac{\pi_0^{se}}{\phi^{se}+\theta}+\frac{\phi^{se}\pi_{-1}^{se}}{(\phi^{se}+\theta)^2}$, which is equal to $\frac{w}{\lambda}$ in equilibrium. The benefit from switching to pooling given the average wealth level V in separating equilibrium is $\frac{\pi_0^{po}(V^{se})}{\phi^{se}+\theta}$, where indices "se" and "po" refer to separating and pooling, respectively. The necessary condition for separating to occur is: $\frac{\pi_0^{po}(V^{se})}{\phi^{se}+\theta}\leq\frac{w}{\lambda}=10$. Parameter values satisfy this condition, see above Table.

As opposed to the pooling case we have two different results in a separating equilibrium:

1) An increase in d still implies less education, but no longer large incentives to innovate, because the consumer determining the profit of the best quality good is the rich. The lower education, the fewer rich individuals exist. d has a direct negative impact on the relative wealth of the rich $A_r=\frac{1-d\beta}{1-\beta}V$, and an indirect positive impact on it via β and V. According to this simulation, the net effect is positive. Hence, the best quality supplier faces a smaller market share and a higher willingness to pay. However, the positive effect on the consumers' willingness to pay is dominated by the negative effect on the market share. Thus, the profit of the best quality supplier decreases in d. On the other hand, the second best quality supplier has a higher demand. The population share of the poor increases and their willingness to pay has been improved. Intuitively, the

profit of the best quality is more important, because it has a bigger weight than that of second best quality in the equation of present value calculation (21). Hence, the relative wealth of the poor has a net negative impact on the innovation rate. For the simulation results of the impact of d on A_r, π_0, π_{-1} and their weights see Appendix 4. The increase in d delays the realization of profits (π_0 decreases and π_{-1} increases). Hence, less research is undergone. It implies the lower research cost and the higher V, according to (23).

2) An increase in ρ results in more education in the young generation, and therefore increases the share of the rich in the old generation. Hence, the profit of the best quality increases and the profit of the second best quality declines. Again because the profit of the best quality good has a large weight in the valuation of innovation, firms have thus more incentives to innovate when they face an increasing population share of the rich. This impact of the population share on innovation does not appear in the pooling case, where the best quality good is sold to the entire population of the old generation. More incentive to innovate implies more labour in research sector. Hence, the research cost increases and the value of oligopolists (V) decreases.

The effect of the education cost e is similar as in the case of pooling, but the reason is different. An increase in e impedes individuals to accept education, so that the population share of the rich decreases. Moreover, the consumer of the quality good becomes poorer when she has to pay higher transfers to young people. Both of them decrease the profit from selling the best quality good. But the impact on the second best quality good is *a priori* unclear, because the market share increases and the consumers' willingness to pay decreases. The simulation results suggest that the net effect of education cost on innovation is negative.

When comparing our results to those of Zweimüller and Brunner (1998), some differences become apparent. In their proposition 3 (ii) they argue that "redistribution from the rich to the poor (i.e., an increase in d), holding population shares constant, increases the rate of innovation if and only if $\phi^* > \theta(2\beta - 1)/(1-\beta)$." As mentioned above, our model is equivalent to theirs without equation (19). Then we lose an equilibrium condition of education, hence β is given exogenously. For the sake of comparison we show the simulation results of their model in Table 3 setting $\beta = 0.34$, which is the equilibrium value in our simulation benchmark:

Table 3: The impact of d on ϕ given $\beta = 0.34$

d	0.2	0.3	0.4	0.5	0.55	0.6
ϕ	0.302	0.308	0.313	0.319	0.322	0.325

This simulation satisfies the condition $\phi^* > \theta(2\beta - 1)/(1-\beta)$ so that redistribution has a positive effect on innovation although it is very weak. An increase in d improves the willingness to pay for the second best quality by the poor. So π_{-1} is increasing in d. This in turn allows the best quality producer to charge a higher price. On the other hand, the rich becomes poorer with an increase of d. If the population share of the rich is big enough (satisfying above condition), their wealth doesn't reduce strongly when d increases. The negative effect on π_0 is dominated by the positive effect on π_{-1}. In sum, less inequality in the sense of a higher d is good for innovation. But this effect is weak because π_{-1} is lower weighted than π_0 in (21). From the supply side, the sum of π_{-1} and π_0 is increasing in d ($\frac{\partial(\pi_0 + \pi_{-1})}{\partial d} = (1-\beta)\theta V\left(\frac{1}{k} - \frac{1}{k^2}\right) > 0$). There is less expenditure for the standard goods. Hence, more labour units can be allocated in the research sector, which induces a higher innovation rate.

Contrary to Zweimüller et al. (1998), population shares in our model are not constant. The redistribution (d increases) cannot only improve the budget constraint of the poor, but also decrease the incentive to education, thus decrease the population share of the rich. Hence, a higher d has two effects on π_0: the direct effect is that it decreases the wealth of the rich, which is considered in the model of Zweimüller et al. (1998); the indirect effect is that it reduces the market share of the best quality, which is neglected by Zweimüller et al.. In other words, the negative effect on π_0 is strengthened by the endogenous population share. Although the positive effect on π_{-1} is also strengthened, its weight in (21) is smaller than that of best quality good. Hence, as we have seen in the simulation results, the net effect of redistribution on innovation is negative when we assume the population share is endogenous through education.

Another interesting result from our model is that the education enrollment $1-\beta$ and the innovation rate ϕ are positively correlated although we don't assume that education can increase the productivity. In this sense, education looks much more like a tool of distribution, but not like a production factor in this paper. However, it can still increase the innovation rate (or growth rate in some sense, see section 5) because it produces richer consumers, who induce society to allocate more recourse in the research sector. Normally economists discuss the impact of education only through the supply side, i.e., education increases the productivity. Hence, the supply increases and the economy will grow. However, Bils and Klenow (2000) supply empirical evidence that the human capital (as a production factor) which is produced by a higher education level cannot explain the higher long run growth rate which is associated with this higher education level. Their explanation is that education is much more like consumption than productive investment. Hence, individuals will increase education in the young period, if they expect that in the future they will have a higher income level. In other words, it is the expected higher long run growth rate that leads to a higher education level today, but not vice versa. Our model supplies another explanation for the empirical results of Bils and Klenow (2000). If individuals can become richer through education, a higher growth rate can be achieved regardless of whether education increases productivity. The higher education level is associated with the higher growth rate through the demand for better quality goods.

5. Exogenous Population Shares and Endogenous Relative Income

The above analysis assumes that the income of the poor relative to the average level is an exogenous variable and the population share of the poor is endogenous. We can also assume that the population share of the poor is exogenous and the relative income of the poor is endogenous. These different assumptions could imply different policies. We can interpret it as follows. If the government wants to decrease the income inequality by setting a higher minimum wage or by giving more social transfers, it is similarly to say, d increases exogenously with respect to our model's analysis. Then we should consider its impact on the innovation rate not only through its direct effects on the willingness to pay, but also indirect effects through the population share. On the other hand, if government sets up a mandatory education law to improve the population share of the student, it decreases β exogenously.

Now we discuss what the impact of the exogenous β is on the innovation rate by assuming an endogenous d. The simulation results are as follows:

Table 4: The impact of the exogenous β on ϕ, d, and V

where $\theta = 0.5, \rho = 0.5, w = 10, e = 2, a = 0.3, \lambda = 1, k = 4$

β	0.2	0.3	0.34	0.4	0.5	0.6
ϕ	0.33	0.32	0.31	0.30	0.28	0.25
d	0.29	0.37	0.40	0.45	0.54	0.63
V	11.19	11.98	12.34	12.96	14.24	16.0
$\frac{\pi_0^{po}(V^{se})}{\phi^{se} + \theta}$	5.98	6.60	6.91	7.42	8.19	10

Result 3:

In separating equilibrium, a higher population share of the poor has a negative impact on the innovation rate. The relative wealth of the poor is positively associated with the population share of the poor.

In this example, if the government increases the education opportunity (β decreases), and in equilibrium all such education opportunities are used by individuals, the relative wealth of the poor has to decline in order to push individuals to enter school. Because of the increasing population share of the rich, the profit of best quality good increases. This in turn raises the incentive to invent.

What is the impact of inequality (through β or d, respectively) on utility? From (1) we have:

$$\Delta u = \frac{\Delta x}{x} + \frac{\Delta q}{q} \qquad (28)$$

In a steady state, the consumption of standard goods is constant ($\Delta x = 0$), and $\Delta q = \phi(k-1)q$. Hence, we have $\Delta u = \phi(k-1)$. The higher the innovation rate, the larger is the increase in the utility. Redistribution from the rich to the poor (d increases exogenously, β is endogenous) increases the price and profit of the quality good, and the average wealth, too. Hence, consumers become richer through redistribution and consume more standard goods. The production resource, labor, is shifted from the research sector to the standard good sector.

Consumers enjoy a higher utility level in the short run, but the long run growth rate of the utility is lower than before because of a lower innovation rate. In contrast to redistribution, the decrease of the population share of the poor can induce a higher innovation rate. Hence, in a new equilibrium consumers have lower consumption of standard goods, but the long run growth rate of the utility becomes higher.

6. The Model Dynamics

So far, we have only discussed the steady state. The comparative statics show us the long run impact of parameters on the equilibrium. In this section we discuss the effect of parameters on the endogenous variables in the short run and the medium run, i.e., the path to equilibrium. We focus on the case where d is exogenous and β is endogenous (the case in section 3 and 4).

From section 2.4, we know that the accumulation function of average wealth is:

$$\Delta V = \pi_0 + \pi_{-1} - \theta V - \frac{w\phi}{\lambda} \qquad (29)$$

The total profits net of the interests payment ($\pi_0 + \pi_{-1} - \theta V$) is the net income of firms. $\frac{w\phi}{\lambda}$ is the research cost. If the net income of firms is higher than the research cost, the economy can accumulate the average wealth.

From equation (19) we know:

$$\Delta\beta = -\frac{(1-d\beta)(1-\beta)(w-e-wa)}{(1-d)(w+\theta dV-e-wa)V}\Delta V \qquad (30)$$

According to Assumption 1, $w-e-wa>0$. Hence, the accumulation of the average wealth ($\Delta V > 0$) enlarges the income gap between the poor and the rich. More young people are attracted to education. Hence, the population share of the poor declines ($\Delta\beta < 0$).

The innovation rate $\phi = \lambda n$ is determined by the number of workers in the research sector (n). From (22) we have:

$$n = 1 - a - eb - b(x_p \beta + x_r (1-\beta)) \tag{31}$$

where $x_i, i \in \{p, r\}$ is the consumption of the standard good by the poor and the rich, respectively. Because $1 - a - eb$ is constant, n depends only on the aggregate consumption of standard goods, i.e., $\Delta n = -b\Delta(x_p \beta + x_r (1-\beta))$.

We concentrate on the impact of d on the dynamics of V, β, ϕ. Suppose there is a shock (d increases). We define the short run as the time just after the shock and before any other endogenous change of variables, and the medium run as the period when all endogenous variables move simultaneously to the long run equilibrium values. Distinguishing the short run from the medium run enables us to study the different effects of the shock. The direct effect of d on the endogenous variables can be observed in the short run. The indirect effect of d through the interaction among the endogenous variables takes place in the medium run.

In the short run, when d increases and V is still unchanged, β increases (from equation (19)) because the benefit of education decreases.

The total profits of firms are as follows:

$$\begin{aligned}\sum_i \pi_i &= \beta(P_{-1} - wa) + (1-\beta)(P_0 - wa) \\ &= \beta[(1-1/k)(w + \theta dV - e) + \frac{wa}{k} - wa] + (1-\beta)[(1-1/k)(w + \theta \frac{1-d\beta}{1-\beta} V - e) \\ &\quad + (\frac{k-1}{k^2})(w + \theta dV - e) + \frac{wa}{k^2} - wa]\end{aligned}$$

Since $\frac{\partial \sum \pi_i}{\partial d} = \frac{k-1}{k^2}(1-\beta)\theta V > 0$, the total profit of firms increases in d. The intuition is as follows: if d increases, the poor become richer. Thus the firm producing the second-best quality goods is able to raise the price without losing consumers, which enables the supplier of the best quality good to increase her price, too. Hence, an increase in d raises the profit from quality goods. Therefore, the net income of firms is above the total cost of research, and $\Delta V > 0$ (from (29)).

From (31) we get $\frac{\partial n}{\partial d} = b\frac{k-1}{k^2}(1-\beta)rV > 0$, i.e., in the short run, the effect of d on the innovation rate is positive. Because V is kept unchanged in the short run, the total income of all consumers doesn't change. However, we know from above that the price of the quality goods increases in d. Hence, consumers have to decrease the consumption of the standard goods. This leads to more labor input in the research sector and a higher innovation rate.

Result 4:

In short run, the relative wealth of the poor (d) has a positive impact on the population share of the poor, the accumulation of the average wealth, and the innovation rate.

In the medium run, d reaches the new level. But the accumulation of the average wealth has just started. From (29) we get:

$$\frac{\partial \Delta V}{\partial V} = \frac{\partial \sum \pi_i}{\partial V} - \theta = (1-\frac{1}{k})[1+(1-\beta)\frac{d}{k}]\theta - \theta = \frac{\theta}{k}[(1-\beta)(1-\frac{1}{k})d - 1] < 0$$

It indicates that the average wealth does increase, but the change of the average wealth is diminishing. According to (30), β decreases in the average wealth. Hence, after the immediate jump in the short run the population share of the poor declines with the accumulation of the average wealth.

From (29) we know also:

$$\begin{aligned}\frac{\partial \sum \pi_i}{\partial \beta} &= (1-1/k)(w+\theta dV - e) + wa/k - wa - [(1-1/k)(w-e) + (\frac{k-1}{k^2})(w+\theta dV - e) + \frac{wa}{K^2} - wa] \\ &\quad - (1-1/k)\theta dV \\ &= (\frac{k-1}{k^2})(wa - w - \theta dV + e) \\ &= (\frac{k-1}{k^2})(wa - P_{-1}) < 0\end{aligned}$$

Hence, at first ΔV decreases because of the immediate jump of β in the short run, then increases because β declines in the medium run. The average wealth V increases before it reaches the equilibrium value in long term.

From (31) we get $\frac{\partial n}{\partial V} = -\frac{b\theta}{k}[1-d(1-\beta)(1-\frac{1}{k})] < 0$. The intuition is as follows: Because of the accumulation of the average wealth all consumers become richer than before, and the demand for standard goods is increasing. Hence, more labor units have to be allocated into the standard sector and less into research. The innovation rate begins to decrease after the immediate increase in the short run. Additionally, we have $\frac{\partial n}{\partial \beta} = -\frac{b(k-1)}{k^2}(w+\theta dV - e - wa) < 0$. This implies that the decrease of the innovation rate in the medium run is accelerated by the immediate jump of β in the short run.

To sum up, after the shock the average wealth increases slowly until it reaches a new higher equilibrium value. The population share of the poor increases in the short run but then decreases in the accumulation of the average wealth. However, our simulation results show that the long run equilibrium value is still higher than before. Because the demand for the standard good sinks in the short run, the innovation rate achieves a higher level. In the medium run, the innovation rate decreases because both the average wealth and the population share of the poor increase.

7. Conclusions

In this paper we distinguish between two measures of income inequality, the population share of the poor and the relative wealth of the poor. We discuss their different impact on the rate of innovation. Our results are established on the basis of a model by Zweimüller and Brunner (1998), but we do not assume the independence between the population share and the relative income. The relaxation of this assumption leads to the novel result that in separating equilibrium, the improvement of the relative income of the poor impedes the innovation rate, and a decrease of the population share of the poor accelerates the rate of innovation.

There are some important implications regarding our result. First, since the Gini-coefficient does not differentiate between the relative income of the poor and the population share of the poor, it is not suitable for policy recommendations. Each different measure of inequality has a different impact on economic growth. Second, the interdependent relationship between relative income and the population share is very important when considering the impact of inequality on growth; we can achieve quite different results compared to Zweimüller and Brunner (1998). Finally, the effect of education on growth or innovation is not only due to an increase of productivity, which is discussed by most economists, but also due to an increase of the demand for better quality. The latter is almost neglected by most economists.

We believe that future research should be directed to empirical work bringing to focus the relationship between the relative income of the poor and the education enrollment rate. We also need evidence to support the argument that education could produce rich consumers, through which the education enrollment is positively associated with the growth rate. Moreover, the current paper points out that there are possible different polices with different effects on the economic growth, e.g., the redistribution from the rich to the poor, and the public school. However, the more important question is under what conditions society would choose the one, which can achieve a higher economic growth rate, as well as a fair income distribution.

Appendix

Appendix 1

First, it is easy to see that both qualities cannot be sold to the same consumer because of the assumption that consumers will choose the better quality if both generate the same utility. There are two possible cases: the best quality good is sold either to the rich or to the poor. In the first case the second best good cannot be sold to the rich but to the poor. This is the separating equilibrium. The other case is the pooling equilibrium, in which the best quality good is accepted by the poor, i.e., the utility of the poor from consuming the best quality good is larger than that of the second best quality:

$$\ln(y_p - e - P_0) + \ln q_0 \geq \ln(y_p - e - P_{-1}) + \ln q_{-1}$$

$$\Leftrightarrow (k-1)(y_p - e) - P_0 k + P_{-1} \geq 0 \qquad \text{and} \quad y_r > y_p$$

$$\Rightarrow (k-1)(y_r - e) - P_0 k + P_{-1} > 0$$

$$\Leftrightarrow \ln(y_r - e - P_0) + \ln q_0 > \ln(y_r - e - P_{-1}) + \ln q_{-1}$$

The rich prefer the best quality good to the second best one, too. Hence, in the pooling equilibrium the second best quality good is not sold.

Appendix 2

1. Pooling: Given the price of q_{-1} the firm of q_0 will charge his price as high as possible. Hence, the possible equilibria lie on the line CD of Figure 1. Suppose P_0 is higher than $\left(1-\frac{1}{k}\right)(y_p - e) + \frac{wa}{k}$, then firm q_{-1} can charge a price higher than marginal cost and attract all poor consumers. It is the separating case. This contradicts the pooling assumption. Hence, the single stage pooling equilibrium is $P_0 = \left(1-\frac{1}{k}\right)(y_p - e) + \frac{wa}{k}, P_{-1} = wa$. We should also consider if other possible equilibria can be sustained through any punishment in a repeated game. Because here the lowest profit which firm q_{-1} can earn is zero, it is impossible to punish him because what the firm has in equilibrium is also zero. Hence, above stage equilibrium is also the equilibrium for the whole repeated game.

2. Separating: The best-reply function of the best quality firm is $\overline{P}_0 = \left(1-\frac{1}{k}\right)(y_r - e) + \frac{P_{-1}}{k}$, which is the line AB in Figure 1. For the poor the utility if he consumes q_{-1} is strictly greater than that if he consumes q_0 given above best-reply function. It implies that firm q_{-1} has an incentive to increase its price without losing its consumers. Hence, the single stage equilibrium is point B in Figure 1. However, for the whole repeated game, other points on AB can also be sustained as equilibria because the firm q_0 can punish the other to set the pooling price (then firm q_{-1} can earn only zero profit) in future if firm q_{-1} increases its price in this stage. Hence, theoretically there are many possible separating equilibria. But such punishment is in some sense unrealistic because the deviation in P_{-1} is not able to decrease the profit of the firm q_0. Hence, under the assumption that the deviation will not be punished if such deviation does not affect other's profit, we have a single separating equilibrium B.

Appendix 3

Denote ED as follows: $ED = \rho \ln\left(\frac{k\theta V \frac{1-d\beta}{1-\beta}}{w+\theta dV - e - wa} + 1 \right) - \ln e$. Hence, $\left.\frac{\partial \beta}{\partial e}\right|_{ED=0} = -\frac{\partial ED / \partial e}{\partial ED / \partial \beta}$.

We have: $\left.\frac{\partial \beta}{\partial e}\right|_{ED=0} = \frac{1-\beta}{1-d}\left[\frac{(1-d\beta)(w+\theta dV - e - wa - e\rho)}{e\rho(w+\theta dV - e - wa)} + \frac{(1-\beta)(w+\theta dV - e - wa)}{e\rho k\theta V} \right]$. The sufficient condition of $\left.\frac{\partial \beta}{\partial e}\right|_{ED=0} = -\frac{\partial ED / \partial e}{\partial ED / \partial \beta} \geq 0$ is that $w - e - wa - e\rho \geq 0$.

Appendix 4

Table 5: The impact of d on A_r, π_0, π_{-1}, and their weights

d	0.2	0.3	0.4	0.5	0.55	0.6
A_r	11.16	13.38	16.15	19.49	21.55	23.64
π_0	8.34	7.92	7.40	6.82	6.49	6.20
$1/\phi+\theta$	1.18	1.20	1.23	1.27	1.28	1.32
π_{-1}	0.36	1.05	1.90	2.90	3.50	4.12
$\phi/(\phi+\theta)^2$	0.48	0.48	0.47	0.46	0.46	0.45

When d increases, the relative wealth of the rich increases. However, the profit of the best quality good decreases because of the decreasing market share $1-\beta$. π_{-1} increases because both the market share and the income of the poor increase. But in the present value of innovation π_0 has a higher weight factor $1/\phi+\theta$ than π_{-1} ($\phi/(\phi+\theta)^2$). Hence, the net effect of d on the present value of innovation is negative, which impedes the innovation rate.

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