

# BONN ECON DISCUSSION PAPERS

Discussion Paper 12/2000

**On the Foundations of the Property Rights  
Theory of the Firm: Cooperative Investments and  
Message-Dependent Contracts**

by

**Andreas Roider**

October 2000



Bonn Graduate School of Economics  
Department of Economics  
University of Bonn  
Adenauerallee 24 - 42  
D-53113 Bonn

The Bonn Graduate School of Economics is  
sponsored by the

Deutsche Post  World Net  
*MAIL EXPRESS LOGISTICS FINANCE*

# On the Foundations of the Property Rights Theory of the Firm: Cooperative Investments and Message-Dependent Contracts

Andreas Roider<sup>1</sup>  
University of Bonn<sup>2</sup>

First Version: April 2000  
This Version: October 2000

<sup>1</sup>I am very grateful to Urs Schweizer, Georg Nöldeke and Christoph Lüllesmann for their comments and advice. I would also like to thank Yeon-Koo Che, Antoine Faure-Grimaud, Oliver Hart, Gilat Levy, John Moore, Gerd Mühlheuß, Sönje Reiche, Frank Thierbach, and seminar participants at LSE and the Jamboree 2000 of the European Doctoral Program. All remaining errors are my own. This research was conducted while the author spent the advanced year of the European Doctoral Program at the London School of Economics. The hospitality of LSE and financial support by the German Academic Exchange Service (DAAD) are gratefully acknowledged.

<sup>2</sup>*Mailing address:* Department of Economics, Wirtschaftspolitische Abteilung, University of Bonn, Adenauerallee 24-26, 53113 Bonn, Germany; *phone:* +49-(0)228-733918; *fax:* +49-(0)228-739221; *email:* andreas.roider@wiwi.uni-bonn.de.

## **Abstract**

The property-rights theory assumes that trade is non-contractible ex-ante and focusses exclusively on the allocation of property-rights. We derive foundations for this focus on property-rights by identifying scenarios where only one of the simple ownership structures is optimal even though trade is contractible. In these scenarios it is optimal: (1) not to sign a trade contract; (2a) to sign a partially enforced trade contract; (2b) such a combination of asset ownership and a trade contract might even achieve the first-best. For the purpose of identifying the optimal simple ownership structure, trade contracts can be neglected in scenarios (2a) and (2b).

*Keywords:* Property Rights, Incomplete Contracts, Specific Investments.

*JEL-Classification:* D23, D82, L14, L22.

## 1 Introduction

The property-rights theory of the firm tries to give an answer to a very basic question which was initially raised by Coase (1937). Namely, why are certain transactions conducted within firms and not in markets and, therefore, what determines the boundaries of the firm? The seminal contributions by Grossman and Hart (1986), Hart and Moore (1990) and Hart (1995) addressed this question by defining a firm as a collection of non-human assets asserting that ownership of an asset implies residual control rights over the asset. These papers look at the problem to induce relationship-specific investments in an incomplete contract setting: when ex-post renegotiations over the use of the assets break down the ownership structure becomes important because the residual control rights over the assets determine the threatpoint payoffs of the parties. In turn, the threatpoint payoffs influence the ex-ante incentives of the parties to make specific investments which raise the surplus of the relationship. The optimal ownership structure induces investment incentives such that the surplus is maximized. Thus, these models are able to derive the boundaries of the firm endogenously.

While the property-rights theory makes an important contribution to the theory of the firm, its foundations have been debated (see for example: Tirole (1999)): an assumption which is made throughout the property-rights literature is that trade which takes place between the parties at a later stage is *non-contractible ex-ante*. This assumption is made in order to justify the exclusive focus on property-rights in these models. If trade were contractible the parties would be able to write (possibly message-dependent) trade contracts (specifying trade quantities and payments) which could determine threatpoint payoffs and, thereby, investment incentives. Several papers<sup>1</sup> have shown in bilateral trade settings without assets that trade contracts can be an effective tool for achieving the first-best. If one would introduce assets in these models, the boundaries of the firm would be *indeterminate* because the parties would be able to sign the optimal trade contract independent of the underlying ownership structure. Therefore, the question arises whether the assumption of non-contractible trade is crucial for the property-rights theory to be able to explain the boundaries of the firm. Furthermore, is it possible that both the ownership structure and trade contracts are important at the same time?

This paper gives a foundation for the exclusive focus on property-rights when determining the boundaries of the firm: we study a model where trade between a buyer (she) and a seller (he) is contractible ex-ante. Only the seller makes an investment in his human capital, and the parties agree on a general, message-dependent contract which specifies the ownership structure of a non-human asset, the trade decision regarding one, describable good and a payment as functions of messages of the parties. In this setting, we provide

---

<sup>1</sup>For example: Chung (1991), Hermalin and Katz (1993), Aghion, Dewatripont, and Rey (1994), Nöldeke and Schmidt (1995b) and Edlin and Reichelstein (1996).

conditions such that the ownership structure plays a role even though trade is contractible ex-ante. We identify three scenarios: (1) If the investment is sufficiently cooperative in the sense of Che and Hausch (1999, p. 129), i.e. if it directly benefits the trading partner, it is optimal for the parties to write no trade contract at all, and they only choose among the various ownership structures as in the property-rights approach.<sup>2</sup> (2) In the next two cases trade contracts are useful but the parties find it optimal to write a trade contract which is only enforced in some ex-post states of nature: the optimal, simple ownership structure determines threatpoint payoffs when the trade contract is not enforced. The optimal combination of an ownership structure and a partially enforced trade contract either (2a) gets closer to efficiency than when no trade contract is signed, but does not achieve the first-best, or (2b) does even achieve the first-best. Under the conditions of (1), (2a), and (2b) the same simple ownership structure as in the property-rights approach turns out to be optimal, and the boundaries of the firm are determined endogenously even though trade is contractible. Therefore, *for the purpose of identifying the optimal simple ownership structure*, the exclusive focus on property rights is justified in all three scenarios.

The structure of our model is closely related to Che and Hausch (1999) who show in a bilateral trading model without assets that the value of trade contracts for generating investment incentives is reduced when investments are cooperative:<sup>3</sup> a trade contract specifies a threatpoint in possible renegotiations between the buyer and the seller. A cooperative investment by one party increases the threatpoint payoff of the trading partner and, thereby, reduces the available renegotiation surplus. This reduces the marginal investment return for the investor. If the investment creates a very strong externality it may, therefore, be optimal to specify no threatpoint at all. We introduce asset ownership in the model of Che and Hausch (1999). We assume that the seller's investment in his human capital may have a direct positive effect on the buyer when the buyer and the seller trade with each other, but when the parties trade with an outside-market, the seller's investment has no influence on the buyer's valuation.<sup>4</sup> Hence, even though trade is contractible asset ownership can play a role in this framework because the negative incentive effect of a cooperative investment is not present when trade with the outside-market forms the threatpoint in renegotiations. In order to establish our main result on the interaction between asset ownership and trade contracts, we strengthen a result of Che and Hausch (1999) regarding the

---

<sup>2</sup>This result generalizes a finding of Che and Hausch (1999).

<sup>3</sup>The model of Che and Hausch (1999) is a generalized version of the model of Edlin and Reichelstein (1996). Che and Hausch (1999) study the value of trade contracts for solving the hold-up problem which was initially formalized by Hart and Moore (1988). Other papers that point out the reduced value of trade contracts when investments are cooperative are Che and Chung (1999) and in a more general, non-differentiable setting Schweizer (2000).

<sup>4</sup>Even though Che and Hausch (1999, Footnote 33) only briefly touch upon the issue of asset ownership, they make the same assumption. However, Che and Hausch (1999) do not explore the possible interaction of asset ownership and trade contracts.

value of trade contracts which are always enforced (=full-enforcement contracts). We show that even when no full-enforcement contract exists which induces the first-best, trade contracts might still generate private investment returns above social returns in *some* ex-post states of the world.<sup>5</sup> By relying on trade with the outside-market as threatpoint only in ex-post states where the negative effect of the externality would be especially strong (and private investment returns under a trade contract low), it might be possible to increase the investment incentives relative to a full-enforcement contract. Hence, a combination of a simple ownership structure and a partially enforced contract might induce the first-best, even when no full-enforcement contract can achieve that.

The foundations of the property-rights theory of the firm have previously been studied by Maskin and Tirole (1999a): based on work by Hart and Moore (1999) and Segal (1999), they consider a model in which a buyer and a seller want to trade one unit of a widget when there exist a large number of possible, *ex-ante describable* widgets. Only one special widget creates a surplus ex-post, and it is ex-ante uncertain which widget turns out to be special. Maskin and Tirole's (1999a) result is based on the complexity of the environment: they show that as the number of possible widgets goes to infinity it is optimal for the parties to write no trade contract at all, and the exclusive focus on property-rights is justified. Our result (1) above describes the same irrelevance of trade contracts in a different setting: in our model the parties are certain which good they want to trade but decide on the trade quantity. Whereas Maskin and Tirole (1999a) and most of the literature on property rights restrict attention to selfish investments,<sup>6</sup> in our model the possibly cooperative nature of investments plays a crucial role. In addition to these irrelevance results our results (2a) and (2b) provide conditions such that, *even when the parties find it optimal to write trade contracts*, (i) only one of the simple ownership structures is optimal, and (ii) for the purpose of identifying the optimal simple ownership structure the exclusive focus on property rights is justified.<sup>7</sup> Another related paper is Segal and Whinston (1999, Section 3). They allow for message-dependent ownership structures but they maintain the assumption that trade is non-contractible ex-ante. In this setting, they characterize circumstances when an unconditional (but possibly probabilistic) ownership structure is optimal.

Two further implications arise from our model. *First*, in the property

---

<sup>5</sup>Che and Hausch (1999) prove the failure of full-enforcement contracts to achieve the first-best for the case that every possible full-enforcement contract generates private returns below social returns in *all* ex-post states of the world.

<sup>6</sup>I.e. investments that *directly* benefit only the investor. One exception to this is Rosenkranz and Schmitz (1999).

<sup>7</sup>Two other papers which relax the asymmetry between ex-ante and ex-post contractability of trade are Aghion, Dewatripont, and Rey (2000) and Aghion and Rey (1999): they study the role of the allocation of control rights when some of the actions of the parties are both ex-ante and ex-post contractible and other actions of the parties are neither ex-ante nor ex-post contractible.

rights literature simple ownership structures<sup>8</sup> generally fail to achieve the first-best. This is problematic because on the one hand one frequently observes simple ownership structures, but on the other hand several papers (for example: Nöldeke and Schmidt (1995a), Nöldeke and Schmidt (1998) and Maskin and Tirole (1999a)) have shown in property-rights models that under certain circumstances slightly more complicated, conditional ownership structures are superior to simple ownership structures and induce the first-best.<sup>9</sup> Our results (2a) and (2b) address this problem: in these two scenarios the same simple ownership structure as in the property-rights approach turns out to be optimal. If one is only interested in identifying the optimal simple ownership structure, (2a) and (2b) provide conditions when, *for this purpose*, trade contracts can be neglected and the exclusive focus on property-rights is justified. But at the same time, the parties *are going to sign* a partially enforced trade contract in addition to the simple ownership structure, and these trade contracts either reduce (2a) or eliminate (2b) the inefficiency that would result if the parties would only agree on the simple ownership structure. *Second*, in our model asset ownership and trade contracts interact in a non-trivial way in creating investment incentives: in the results (2a) and (2b) the optimal simple ownership structure is unique and the parties sign a partially enforced trade contract at the same time, i.e. under these circumstances it is suboptimal for the parties either to write a trade contract which is always enforced or to sign no trade contract at all. This interaction is a new feature compared to the previous literature on property rights and bilateral trading: previously either only asset ownership (for example: Grossman and Hart (1986), Hart and Moore (1990), Hart (1995), DeMeza and Lockwood (1998), Chiu (1998), Maskin and Tirole (1999a)) or only trade contracts (for example: Aghion, Dewatripont, and Rey (1994), Nöldeke and Schmidt (1995b), Edlin and Reichelstein (1996)) mattered for creating investment incentives.<sup>10</sup>

The structure of the paper is as follows. The next section describes a general framework to study the possible interaction of asset ownership and trade contracts. In Section 3 the outcome under the property-rights approach is derived. In Section 4 we establish our results on the foundations of the property rights theory of the firm. In Section 5 we discuss possible extensions of our model. Section 6 concludes.

---

<sup>8</sup>For example, the seller rather than the buyer owns the asset, or vice versa.

<sup>9</sup>Nöldeke and Schmidt (1995a) consider collateralized debt contracts which allocate ownership conditional on repayment. Nöldeke and Schmidt (1998) show that an option-to-own an asset might induce the first-best in a sequential investment problem. In Maskin and Tirole (1999a) an option-to-sell an asset combined with a payment to the public achieves the first-best.

<sup>10</sup>Either by assumption or in equilibrium.



## 2 The Model

### 2.1 Description of the Model

A buyer and a seller, both of whom are risk-neutral, want to trade a variable quantity of an ex-ante describable good. In a first stage only the seller makes a relationship-specific investment in his human capital. In a second stage, after a random variable has been realised, the level of trade  $q \in [0, \bar{q}] \equiv Q$  is determined. The buyer and the seller may either trade with each other or with an outside-market. For simplicity we assume that they cannot trade with each other and the outside-market simultaneously. The parties use their human capital as well as a non-human asset  $A$  for production and trade. The structure of the game is as follows:

*Timing.* Figure 1 depicts the sequence of events. At *date 1* the parties sign a contract  $[x(\cdot), t(\cdot)]$  which specifies a three-dimensional decision variable  $x(\cdot) \equiv [OS(\cdot), q(\cdot), e(\cdot)]$  and a payment  $t(\cdot)$  from the buyer to the seller. The functions  $x(\cdot)$  and  $t(\cdot)$  possibly depend on messages sent by the parties. The function  $OS(\cdot)$  determines who owns the asset after the messages have been sent.<sup>11</sup> Two different realizations of  $OS$  are possible:  $OS \in \{SO, BO\}$ , where:  $SO$ =the seller owns the asset, and  $BO$ =the buyer owns the asset.<sup>12</sup> As we assume that the good can be costlessly described at date 1 the level of trade  $q(\cdot)$  between the buyer and the seller is verifiable by a court and can therefore be specified in a contract. Additionally, the parties agree on a function  $e(\cdot)$  which can take on the values 0 and 1.  $e(\cdot)$  determines whether ex-post  $q(\cdot)$  is indeed enforced ( $e = 1$ ) or not ( $e = 0$ ) if renegotiations between the parties break down.<sup>13</sup> When  $q(\cdot)$  is not enforced the parties are free to trade with the outside-market. We make enforcement of  $q(\cdot)$  a decision variable to allow interaction between asset ownership and trade contracts (which specify trade quantities and payments) to emerge.<sup>14</sup> In general one could imagine that

---

<sup>11</sup>As will become obvious later, for investment incentives it only plays a role who owns the asset *after that point in time*, if it plays a role at all.

<sup>12</sup>Joint ownership (JO) of the asset would be an additional possibility. Under JO a party is only able to use the asset if the other party agrees. As will become obvious later, in our setting the investment incentives under JO are equivalent to those under BO, and, therefore, we omit an explicit treatment of JO.

<sup>13</sup>When the buyer and the seller want to trade with the outside-market in the case that certain messages are sent, they specify  $e(\cdot) = 0$  for these messages in the contract. This is *not* equivalent to  $[OS(\cdot), q(\cdot) = 0, t(\cdot), e(\cdot) = 1]$  because the latter means that the parties preclude *any* trade (even with the outside-market). This latter contract can be interpreted as an "exclusive contract." Exclusive contracts are studied in detail by Segal and Whinston (1998).

<sup>14</sup>In the related papers by Edlin and Reichelstein (1996) and Che and Hausch (1999) the question of not enforcing  $q(\cdot)$  does not arise: Edlin and Reichelstein's (1996) model does not include assets, and they consider investments which directly benefit only the investor: in their model a simple trade contract specifying a fixed quantity and a fixed price achieves the first best in the one-sided investment case. In the model of Che and Hausch (1999) there are also no assets, and it is assumed that no trade between the buyer and the seller leads to no trade at all and zero surplus: but this is equivalent to a trade contract specifying zero trade and zero payments.

the parties send messages ex-post, and a court enforces  $OS()$ ,  $q()$  and  $t()$  depending on the value of  $e()$ . The following definitions will be useful:

**Definition 1** A *full-enforcement contract* specifies  $e() \equiv 1$  independent of the messages which are sent.

**Definition 2** A *partial-enforcement contract* specifies  $e() = 0$  for some messages and  $e() = 1$  for others.

At *date 2* the seller makes a relationship-specific investment  $s \in [0, \bar{s}] \equiv S$  in his human capital,<sup>15,16</sup> and at *date 3* a random variable  $\epsilon \in \{\epsilon^0, \dots, \epsilon^I\} \equiv E$  with  $Prob(\epsilon^i) \equiv p^i$  for  $i = 0, \dots, I$  is realized. We assume that  $0 < p^i < 1 \forall i$ . It is useful to denote the true ex-post state of the world by  $\theta \equiv (s, \epsilon) \in \Theta \equiv S \times E$ . At *date 4* the buyer and seller send messages  $\theta^j \equiv (s^j, \epsilon^j) \in \Theta$  for  $j = B, S$  if the contract is message-dependent. At *date 5* renegotiation takes place. At *date 6* the good is produced and traded.

*Information.* Throughout we assume that the buyer and the seller have symmetric information. They both observe the relevant variables and functions. However, only the ownership structure, the quantity of trade, payments between the parties, the enforcement decision and messages sent by the parties are assumed to be verifiable by a court. Therefore, the most general contract is registered with the courts,<sup>17</sup> and specifies the ownership structure, the level of trade, the payment and the enforcement decision as functions of the messages sent by the parties after the state of the world has been realized.

*Renegotiation.* We assume that renegotiation results in an ex-post efficient outcome. The seller and the buyer divide the resulting renegotiation surplus in Nash-Bargaining with exogenously given bargaining power  $\alpha^S$  for the seller and  $\alpha^B$  for the buyer, where  $0 \leq \alpha^S < 1$  and  $\alpha^S + \alpha^B = 1$ .<sup>18</sup> The threatpoint is determined by  $[q(), t()]$  if  $e() = 1$ , and by the underlying ownership structure  $OS()$  and  $t()$  if  $e() = 0$ . If  $q()$  is enforced when renegotiations break down the buyer and the seller have to trade with each other. If  $q()$  is *not* enforced both parties have the opportunity to trade with the outside-market. We assume that the parties are unable to commit not to renegotiate.<sup>19</sup>

In the following we describe the payoff functions of the buyer and the seller, and subsequently we state the formal assumptions regarding these functions:

<sup>15</sup>The case of an one-sided investment by the buyer is analogous.

<sup>16</sup>An investment in human capital is *not* embedded in the asset. For a detailed discussion on the distinction between investments in *human* capital and in *physical* capital see Hart (1995, Ch. 3).

<sup>17</sup>I.e. the parties include a *specific performance damage clause* in their contract and register it with the courts (see Edlin and Reichelstein (1996)).

<sup>18</sup>DeMeza and Lockwood (1998) and Chiu (1998) have shown that the optimal ownership structure might depend on the nature of the bargaining game. In our paper the interaction of asset ownership and trade contracts is at the centre of attention and, therefore, we neglect this aspect for simplicity.

<sup>19</sup>We do not deny that under certain circumstances the parties might be able to commit not to renegotiate, but we want to consider renegotiation as a practical possibility.

*Payoff functions when the buyer and the seller trade with each other.* When the parties trade with each other the seller has a cost function  $C(q, s, \epsilon)$ , and the buyer has a valuation function  $V(q, s, \epsilon)$ , and both have access to the non-human asset for production and trade. We assume that  $C(\cdot)$  and  $V(\cdot)$  are twice continuously differentiable in  $q$  and  $s$ . Before we proceed to describe the payoff functions in more detail it is useful to classify the investment of the seller according to the identity of the parties who directly benefit from it:<sup>20</sup>

**Definition 3** *The investment  $s$  of the seller is said to be:*<sup>21</sup>

- (i) **selfish** if  $C_s(q, s, \epsilon) < 0$  for  $q > 0$ , and
- (ii) **cooperative** if  $V_s(q, s, \epsilon) > 0$  for  $q > 0$ .

Our specification of  $V(\cdot)$  allows for the possibility that, for some realizations of  $\epsilon$ , the investment of the seller directly increases the buyer's valuation of trade. We have in mind that, in principle, the seller can produce a tailor-made product for the buyer but only for some realizations of  $\epsilon$  he has acquired the right skills to do so. For these realizations of  $\epsilon$ ,  $V(\cdot)$  is increasing in  $s$ , and  $s$  is cooperative in the above sense.

*Payoff functions when the buyer and the seller trade with the outside-market.* In accordance with Maskin and Tirole (1999a, p. 140) we assume that the payoffs that accrue to the parties when they trade with the outside-market have the nature of private benefits, and, therefore, that the trades with outsiders are non-contractible. When the seller and the buyer trade with the outside-market their payoffs depend on who owns the asset, and they have payoffs  $\tilde{u}^S(OS, s, \epsilon)$  and  $\tilde{u}^B(OS, \epsilon)$  respectively. We assume that  $\tilde{u}^S(\cdot)$  is twice continuously differentiable in  $s$ .  $\tilde{u}^B(OS, \epsilon)$  is independent of  $s$  for all realizations of  $\epsilon$  because when the buyer acquires the good on the outside-market she does not profit from the value-enhancing, relationship-specific investment of the seller.<sup>22</sup>

**Assumption 1 (A1)**

- (i)  $C(0, \theta) = V(0, \theta) \equiv 0 \forall \theta$ .
- (ii)  $C(\cdot)$  ( $V(\cdot)$ ) is strictly increasing and strictly convex (concave) in  $q$ .
- (iii)  $C(\cdot)$  ( $\tilde{u}^S(\cdot)$ ) is strictly decreasing (increasing) and strictly convex (concave) in  $s$  for all  $\epsilon \in E^S$ , where  $E^S$  is a non-empty subset of  $E$ ; and independent of  $s$  otherwise.

<sup>20</sup>In the whole paper subscripts denote partial derivatives.

<sup>21</sup>See Che and Hausch (1999, p. 129).

<sup>22</sup>The following is an *alternative motivation of the assumption* that  $V(\cdot)$  (but not  $\tilde{u}^B(\cdot)$ ) might depend on  $s$ : for some realizations of  $\epsilon$  the buyer by working with the seller learns from the seller, and she is thereby able to improve her own human capital which in turn increases her valuation of trade with the seller. Even though the buyer always observes the level of investment  $s$  and is able to think of the potential benefits from learning from the seller (i.e. the effect of  $s$  on  $V(\cdot)$ ) she needs to work closely with the seller to realize this benefit. From outside (i.e. when trading with the outside-market) she is not able to capture the details of the potential improvement and is unable to implement it.

- (iv)  $V(\cdot)$  is strictly increasing and strictly concave in  $s$  for all  $\epsilon \in E^B$ , where  $E^B$  is a subset of  $E$ ; and independent of  $s$  otherwise.
- (v)  $C_{qs} < 0$  for all  $\epsilon \in E^S$ , and  $V_{qs} \geq 0$  for all  $\epsilon \in E$ .

## 2.2 The First-Best

We assume that trade between the buyer and the seller is always efficient ex-post. Therefore, the ex-post efficient quantity maximizes the ex-post surplus of trade between the buyer and the seller:<sup>23</sup>

$$q^*(\theta) \in \arg \max_q [V(q, \theta) - C(q, \theta)]. \quad (1)$$

The ex-post surplus  $\phi(\theta)$  and the expected gross surplus  $\Phi(s)$  are then defined in the following way:

$$\phi(\theta) \equiv V(q^*(\theta), \theta) - C(q^*(\theta), \theta) \quad \text{and} \quad \Phi(s) \equiv E[\phi(s, \epsilon)]. \quad (2)$$

**Assumption 2 (A2)**  $\Phi(s)$  is strictly concave in  $s$ .

To achieve ex-ante efficiency the investment of the seller has to maximize the expected net surplus of trade. Therefore, the first-best investment  $s^*$  solves:

$$s^* \in \arg \max_s [\Phi(s) - s]. \quad (3)$$

## 2.3 Payoffs of the Buyer and the Seller

The ex-post payoffs of the parties depend on whether or not  $q(\theta^B, \theta^S)$  is enforced if renegotiations break down (i.e. whether  $e(\theta^B, \theta^S)$  equals 0 or 1):

### 2.3.1 Threatpoint: Trade with the Outside Market

If renegotiations break down and  $e(\theta^B, \theta^S) = 0$  then  $q(\theta^B, \theta^S)$  is *not enforced* and the underlying ownership structure  $OS$  and the transfer payment  $t$  determine threatpoint payoffs. Therefore, the *threatpoint payoffs* of the seller and the buyer respectively are:

$$\hat{u}^S(\theta^B, \theta^S, \theta) \equiv t(\theta^B, \theta^S) + \tilde{u}^S(OS(\theta^B, \theta^S), \theta), \quad (4)$$

$$\hat{u}^B(\theta^B, \theta^S, \theta) \equiv \tilde{u}^B(OS(\theta^B, \theta^S), \epsilon) - t(\theta^B, \theta^S). \quad (5)$$

In accordance with the property-rights theory of the firm (see for example Hart (1995, p. 37)) we assume that the marginal investment return for the seller when he trades with the outside-market is relatively low, and that the investment is more effective when the seller has control over the asset  $A$ .<sup>24</sup>

<sup>23</sup>In the whole paper we assume interior solutions.

<sup>24</sup>I.e. joint ownership of the asset would imply the same marginal payoff as under buyer-ownership.

**Assumption 3 (A3)**  $\tilde{u}_s^S(BO, \theta) < \tilde{u}_s^S(SO, \theta) < -C_s(q^*(\theta), \theta) \forall \epsilon \in E^S, \forall s$ .

Hence, for  $e(\theta^B, \theta^S) = 0$ , the following *renegotiation surplus* results:

$$rs(\theta^B, \theta^S, \theta) \equiv \phi(\theta) - \tilde{u}^S(OS(\theta^B, \theta^S), \theta) - \tilde{u}^B(OS(\theta^B, \theta^S), \epsilon). \quad (6)$$

The *ex-post payoffs* consist of the threatpoint payoff and the respective share of the renegotiation surplus:

$$u^j(\theta^B, \theta^S, \theta) \equiv \hat{u}^j(\theta^B, \theta^S, \theta) + \alpha^j \cdot rs(\theta^B, \theta^S, \theta) \quad (7)$$

for  $j = S, B$ .

### 2.3.2 Threatpoint: Trade between the Buyer and the Seller

If renegotiations break down and  $e(\theta^B, \theta^S) = 1$  then  $q(\theta^B, \theta^S)$  is *enforced*. In this case the *threatpoint payoffs* of the seller and the buyer respectively are:

$$\hat{U}^S(\theta^B, \theta^S, \theta) \equiv t(\theta^B, \theta^S) - C(q(\theta^B, \theta^S), \theta), \quad (8)$$

$$\hat{U}^B(\theta^B, \theta^S, \theta) \equiv V(q(\theta^B, \theta^S), \theta) - t(\theta^B, \theta^S), \quad (9)$$

and the *renegotiation surplus* is:

$$RS(\theta^B, \theta^S, \theta) \equiv \phi(\theta) - [V(q(\theta^B, \theta^S), \theta) - C(q(\theta^B, \theta^S), \theta)]. \quad (10)$$

Hence, when  $e(\theta^B, \theta^S) = 1$  the *ex-post payoffs* of the seller and the buyer respectively are:

$$U^j(\theta^B, \theta^S, \theta) \equiv \hat{U}^j(\theta^B, \theta^S, \theta) + \alpha^j \cdot RS(\theta^B, \theta^S, \theta) \quad (11)$$

for  $j = S, B$ .

### 2.3.3 Ex-Post Payoffs of the Buyer and the Seller

In the previous two subsection we derived the ex-post payoffs of the buyer and the seller for the two possible values of  $e(\theta^B, \theta^S)$ . Hence, combining (7) and (11) yields the ex-post payoffs of the seller and the buyer as functions of the messages and the true ex-post state of the world:

$$R^j(\theta^B, \theta^S, \theta) \equiv u^j(\theta^B, \theta^S, \theta) - e(\theta^B, \theta^S) \cdot [u^j(\theta^B, \theta^S, \theta) - U^j(\theta^B, \theta^S, \theta)] \quad (12)$$

for  $j = S, B$ . Define:  $\overline{R}^j(\theta) \equiv R^j(\theta, \theta, \theta)$  for  $j = S, B$ , and  $e(\theta) \equiv e(\theta, \theta)$ .

## 2.4 The Optimization Problem

An optimal contract induces an investment by the seller such that the expected net surplus of the relationship between the buyer and the seller is maximized. Throughout we rely on subgame perfect equilibrium as the solution concept.

The revelation principle allows us to restrict attention to direct revelation mechanisms which specify the ownership structure, the trade quantity, the enforcement decision and the transfer payment as functions of the messages  $\theta^B$  of the buyer and  $\theta^S$  of the seller about the true state of the world  $\theta$  in a way such that the parties have an incentive to report truthfully. Formally, a direct revelation mechanism is defined as a mapping  $[x, t] : \Theta^2 \rightarrow \{SO, BO\} \times Q \times \{0, 1\} \times \mathbb{R}$ .<sup>25</sup> The optimal direct revelation mechanism solves the following problem:<sup>26</sup>

$$\underset{x(\cdot), t(\cdot)}{Max} \{ \Phi(s) - s \}, \quad (C)$$

subject to:

$$\overline{R}^S(\theta) \geq R^S(\theta, \theta^S, \theta) \quad \forall \theta, \forall \theta^S, \quad (SIC)$$

$$\overline{R}^B(\theta) \geq R^B(\theta^B, \theta, \theta) \quad \forall \theta, \forall \theta^B, \quad (BIC)$$

$$s \in \arg \max_{\tilde{s}} \left\{ E \left[ \overline{R}^S(\tilde{s}, \epsilon) \right] - \tilde{s} \right\}. \quad (I)$$

Conditions (SIC) and (BIC) ensure that truth-telling is a Nash-equilibrium on and off the equilibrium path.

## 3 The Property-Rights Approach

Because the property-rights approach is nested in our model, we derive its prediction as a benchmark. The property-rights theory of the firm considers a model where the relevant good is non-describable ex-ante. It is *impossible* for the parties to write a trade contract because the object of trade is not verifiable, and, therefore, a trade contract could not be enforced. In our

---

<sup>25</sup>We do not consider random mechanisms. Allowing for random mechanisms would not change our results qualitatively since even when we consider random mechanisms situations arise where no full-enforcement contract can achieve the first-best or where trade contracts have no value (see Che and Hausch (1999, Footnote 23)). Maskin and Tirole (1999a) consider random mechanisms in their foundation of the property-rights approach because in their model only *one* unit of a good is traded. Random mechanisms play a crucial role when the parties are risk-averse (see Maskin and Tirole (1999b)).

<sup>26</sup>This program does not require participation constraints because ex-ante payments exist which induce participation by both parties.

model this corresponds to a situation where the underlying ownership structure determines threatpoint payoffs independent of the messages which are sent. Therefore, we assume in this section that:

$$e(\theta^B, \theta^S) \equiv 0 \quad \forall \theta^B, \theta^S. \quad (13)$$

Because some papers in the property-rights literature consider options on asset ownership we still allow for message-dependent ownership structures, i.e. the ownership structure and the transfer payment might depend on the messages of the parties. The following result can be established:

**Lemma 1** *Assume (A1)-(A3). If  $e(\theta^B, \theta^S) \equiv 0 \quad \forall \theta^B, \forall \theta^S$ , then the following is true:*

- (i) *Unconditional seller-ownership (i.e.  $OS(\theta^B, \theta^S) = SO \quad \forall \theta^B, \forall \theta^S$ ) is optimal,*
- (ii) *Unconditional buyer-ownership (i.e.  $OS(\theta^B, \theta^S) = BO \quad \forall \theta^B, \forall \theta^S$ ) is not optimal,*
- (iii) *The seller underinvests.*

**Proof.** See Appendix A. ■

The property-rights theory suggests that only one of the unconditional ownership structures is optimal, and that, in the one-sided investment problem, message-dependent ownership structures cannot improve upon unconditional seller-ownership.<sup>27</sup> Hence, in this setting the restriction of attention to unconditional ownership structures in itself does not carry an efficiency loss.

#### 4 Foundations of the Property-Rights Theory

In this section we state our results on the foundations of the property rights theory of the firm: *first*, we show that, if the investment of the seller is sufficiently cooperative, it is optimal for the parties to rely on *unconditional seller-ownership as threatpoint* in renegotiations independent of the messages of the parties (i.e.  $e(\theta^B, \theta^S) = 0 \quad \forall \theta^B, \forall \theta^S$ ). In this first scenario a full-enforcement contract is not optimal, and unconditional buyer-ownership is never part of an optimal contract.<sup>28</sup> *Second*, when this condition is not fulfilled an interesting interaction of asset ownership and partial-enforcement contracts might emerge, and, even though the parties sign a trade contract, the choice of the right ownership structure might play an important role in order to achieve the first-best. We focus on this novel observation in the second part of this section: we describe two scenarios where *unconditional seller-ownership in combination with a partial-enforcement contract is optimal*: unconditional

<sup>27</sup>Given the large set of possible message-dependent ownership structure which we allow for, unconditional seller-ownership is in general not uniquely optimal. However, as Hart and Moore (1999, p. 133) argue, more important than uniqueness is the ability of the property-rights approach to show that certain ownership structures are suboptimal.

<sup>28</sup>Thereby, we generalize a finding of Che and Hausch (1999, Footnote 33) who discuss the irrelevance of contracting in the presence of asset ownership in a special example.

asset ownership alone, unconditional buyer-ownership in combination with a partial-enforcement contract, or a full-enforcement contract are *not* optimal.

In all three scenarios the choice of the right ownership structure is important *even though trade is contractible*: in all three scenarios, as in the property-rights approach (see Lemma 1), unconditional seller-ownership is optimal but unconditional buyer-ownership is not optimal. Therefore, *if one is only interested in the optimal unconditional ownership structure*, the exclusive focus on property-rights is justified (i.e. for this purpose *one can assume*  $e(\theta^B, \theta^S) \equiv 0 \quad \forall \theta^B, \theta^S$  from the outset). At the same time, in the last two scenarios, the inefficiency which the property-rights approach suggests is reduced or even completely eliminated because one knows that in fact the parties are going to sign a partial-enforcement contract (i.e. *the parties find a contract optimal* where  $e(\cdot)$  is sometimes equal to one and sometimes equal to zero).

#### 4.1 The Irrelevance of Trade Contracts

The value of a trade contract for generating investment incentives is reduced when investments are cooperative: if the investment of the seller is cooperative and  $e(\theta) = 1$ , the seller, by investing, worsens his bargaining position because the threatpoint payoff of the buyer  $\hat{U}^B$  is increased and, thereby, the renegotiation surplus is reduced. This effect reduces the seller's investment incentive. The larger the bargaining power of the seller the more of this reduction of the renegotiation surplus is internalized by the seller, and the smaller are his investment incentives.<sup>29</sup> As seller-ownership is an optimal ownership structure when  $e(\theta^B, \theta^S) \equiv 0 \quad \forall \theta^B, \theta^S$  (see Lemma 1), the investment incentives created under this ownership structure form the benchmark for the value of trade contracts in our model: if, for each realization of  $\epsilon$ , no contract exists which can generate higher marginal investment returns than unconditional seller-ownership alone, then it is optimal for the parties to agree on unconditional seller-ownership and to refrain from specifying trade quantities in a contract, i.e.  $e(\theta^B, \theta^S) = 0 \quad \forall \theta^B, \theta^S$ .

To prove this formally we define for a given  $OS$  and for each realization of  $\epsilon$  critical values  $\beta(\epsilon, OS)$ : for all values of the bargaining power of the seller above this threshold, the private ex-post returns which are generated when  $OS$  determines threatpoint payoffs are no less than the private ex-post return which any trade quantity could generate. To simplify notation, we define:

$$M(k, q, \theta) \equiv -kV(q, \theta) - (1 - k)C(q, \theta), \quad (14)$$

---

<sup>29</sup>If the buyer and the seller are able to commit *not* to renegotiate an initial contract, Che and Hausch (1999) and Maskin and Tirole (1999b) have shown that a full-enforcement contract exists which induces the first-best. In this case the ownership structure is irrelevant for generating investment incentives.



where  $k \in [0; 1]$ . Now the thresholds are defined as:<sup>30</sup>

$$\beta(\epsilon, OS) \equiv \inf\{k \in [0; 1] / M_s(k, q, s, \epsilon) \leq (1 - k) \tilde{u}_s^S(OS, s, \epsilon) \quad \forall q \in Q, \forall s \in S\}, \quad (15)$$

if the respective set is nonempty, and  $\beta(\epsilon, OS)$  equals one if the respective set is empty. A low value of  $\beta(\epsilon, OS)$  means that the seller's investment is highly cooperative when a certain  $\epsilon$  has been realised. The  $\beta()$  might depend on  $\bar{q}$  and  $\bar{s}$ : as  $\bar{q}$  and  $\bar{s}$  increase, the condition in (15) has to hold for a larger set of quantities and investments. Therefore, the  $\beta(\epsilon, OS)$  are non-decreasing in  $\bar{q}$  and  $\bar{s}$ . In the following we suppress this dependency for ease of notation. For each  $OS \in \{BO, SO\}$ , we define:

$$\bar{\beta}(OS) \equiv \max_{\epsilon} \beta(\epsilon, OS). \quad (16)$$

**Proposition 1** *Assume (A1)-(A3). If  $\alpha^S > \bar{\beta}(SO)$  then the following is true:*

- (i) *Unconditional seller-ownership,  $e(\theta^B, \theta^S) = 0$  and  $t(\theta^B, \theta^S) = t \forall \theta^B, \forall \theta^S$  is optimal,*
- (ii) *A full-enforcement contract is not optimal,*
- (iii) *Unconditional buyer-ownership is never part of an optimal contract.*

**Proof.** See Appendix B. ■

## 4.2 The Interaction of Asset Ownership and Partial Enforcement Contracts

In this section we state our main results on the foundation of the property-rights theory of the firm. In order to show that a combination of seller-ownership and a partial-enforcement contract might achieve the first-best when no full-enforcement contract can do so, we begin this section by strengthening a result of Che and Hausch (1999) regarding the value of full-enforcement contracts:

### 4.2.1 No Full-Enforcement Contract can Induce the First-Best

Che and Hausch (1999) derive a sufficient condition such that *independent of the probability distribution over  $\epsilon$*  no full-enforcement contract can achieve the first-best: if the bargaining power  $\alpha^S$  of the seller is above a certain threshold no full-enforcement contract achieves efficiency because above this threshold no full-enforcement contract can induce private ex-post returns above

---

<sup>30</sup>In order to provide an intuition for this definition compare the marginal (ex-post) investment returns when an unconditional ownership structure  $OS$  and a payment  $t$  determine threatpoint payoffs with the marginal returns when a fixed-quantity/fixed-payment  $(q, t)$ -trade contract determines threatpoint payoffs:  $(\alpha^S \phi_s(s) + (1 - \alpha^S) \tilde{u}_s^S(OS, s, \epsilon))$  versus  $(\alpha^S \phi_s(s) + [-\alpha^S V_s(q, s, \epsilon) - (1 - \alpha^S) C_s(q, s, \epsilon)])$ . The  $(q, t)$ -trade contract generates a (weakly) lower marginal investment return than  $OS$  when the condition in the definition holds.  $\beta(\epsilon, OS)$  is defined in a way to allow statements about all possible message-dependent contracts.

social ex-post returns *for any realization of  $\epsilon$* . We reformulate Che and Hausch's (1999) sufficient condition for the case of *a given probability distribution over  $\epsilon$* , and show that even when trade contracts can still induce private ex-post returns above social ex-post returns *for some realization of  $\epsilon$* , no full-enforcement contract might be able to generate *expected private returns* above *expected social returns*.<sup>31</sup> Hence, a combination of seller-ownership and a trade contract which is only enforced for a realization of  $\epsilon$  where private ex-post returns above social ex-post returns can be generated might improve upon the investment incentives which any full-enforcement contract can induce.

We begin our analysis by looking at individual realizations of  $\epsilon$ . *For a given realization of  $\epsilon$*  the respective threshold  $\gamma(\epsilon)$  is defined as the lowest value of the bargaining power of the seller such that the private ex-post returns which any trade contract can generate are less or equal to the social ex-post returns for this realization of  $\epsilon$ . Formally, the thresholds  $\gamma(\epsilon)$  are defined as:<sup>32</sup>

$$\gamma(\epsilon) \equiv \inf\{k \in [0; 1] \mid M_s(k, q, s, \epsilon) \leq (1 - k) \phi_s(s, \epsilon) \quad \forall q \in Q, \forall s \in S\}, \quad (17)$$

if the sets are nonempty, and each measure equals one if the respective set is empty. A low value of a certain  $\gamma(\epsilon)$  means that the seller's investment is highly cooperative for this  $\epsilon$ . By the same argument as above the  $\gamma(\cdot)$  are non-decreasing in  $\bar{q}$  and  $\bar{s}$ . Again, we suppress this dependency for ease of notation. It follows from (17), (15), (A3) and (A1)(iii),(iv),(v) that:

$$\gamma(\epsilon) \leq \beta(\epsilon, SO) \leq \beta(\epsilon, BO) \quad \forall \epsilon. \quad (18)$$

Che and Hausch (1999, (B4)) have shown that if  $\alpha^S > \gamma(\epsilon)$  no trade contract can generate private ex-post returns for the seller equal to or above social ex-post returns for that  $\epsilon$ . They prove that if  $\alpha^S > \max_{\epsilon} \gamma(\epsilon)$ , and if the parties cannot commit not to renegotiate, then no full-enforcement contract can induce the first-best.<sup>33</sup> This obviously holds true independent of the probability distribution of  $\epsilon$ .

---

<sup>31</sup>I.e. whereas Che and Hausch's (1999) result holds independent of the probability distribution over  $\epsilon$ , our focus on a given probability distribution allows us to establish the same result for a wider parameter range. Note, that our condition in Lemma 2 below is again *sufficient* because for some values of  $\alpha^S$  below our threshold the functions  $q(\cdot)$  and  $t(\cdot)$  which in principle induce the first-best might not satisfy (BIC) and (SIC), i.e.  $[q(\cdot), t(\cdot)]$  might not be implementable.

<sup>32</sup>In order to provide an intuition for this definition consider the private ex-post returns when a fixed-quantity/fixed-payment  $(q, t)$ -trade contract determines treatpoint payoffs:  $\alpha^S \phi_s(s) + [-\alpha^S V_s(q, s, \epsilon) - (1 - \alpha^S) C_s(q, s, \epsilon)]$ . The  $(q, t)$ -trade contract generates a private ex-post return (weakly) below the social ex-post return ( $= \phi_s(s)$ ) when the condition in the definition holds.  $\gamma(\epsilon)$  is again defined in a way to allow statements about all possible message-dependent contracts.

<sup>33</sup>Their  $\underline{\alpha}^*$  (defined in Che and Hausch (1999, p. 139)) is identical to  $\max_{\epsilon} \gamma(\epsilon)$  in our model.

For a given probability distribution over  $E$  we define a critical value  $\gamma^*$  such that for levels of the bargaining power of the seller above this critical value no full-enforcement contract can induce private returns above social returns *in expected terms*. Formally:

$$\gamma^* \equiv \inf\{k \in [0; 1] / \sum_{i=0}^1 p^i \cdot M_s(k, q^i, s, \epsilon^i) \leq (1-k) \Phi(s) \ \forall (q^0, q^1, s)\}, \quad (19)$$

if the set is nonempty, and  $\gamma^*$  equals one if the set is empty. A low value of  $\gamma^*$  means that the seller's investment is highly cooperative in expected terms. By the same argument as above  $\gamma^*$  is non-decreasing in  $\bar{q}$  and  $\bar{s}$ . Moreover,  $\gamma^*$  might depend on the probability distribution of  $\epsilon$ , and we are going to exploit this fact in subsequent sections. Again, we suppress the arguments of the function  $\gamma^*$  for ease of notation. It follows from (17) and (19) that:

$$\min_{\epsilon} \gamma(\epsilon) \leq \gamma^* \leq \max_{\epsilon} \gamma(\epsilon). \quad (20)$$

**Lemma 2** *Assume (A1)-(A2). If  $\alpha^S > \gamma^*$  then no full-enforcement contract can induce the first-best, and it is possible that  $\gamma^* < \max_{\epsilon} \gamma(\epsilon)$ .*

**Proof.** See Appendix C. ■

Because a  $\gamma^* < \max_{\epsilon} \gamma(\epsilon)$  is possible, our Lemma 2 indeed strengthens the result of Che and Hausch (1999, Prop. 3 (i)).<sup>34</sup> This is important in order to establish our results below because it shows that  $\gamma^* < \alpha^S < \max_{\epsilon} \gamma(\epsilon)$  is possible: for such values of  $\alpha^S$  no full-enforcement contract can induce the first-best but specifying trade quantities in a contract can still induce private ex-post returns above social ex-post returns for some realizations of  $\epsilon$ . Therefore, a partial-enforcement contract might achieve the first-best. For  $\alpha^S > \max_{\epsilon} \gamma(\epsilon)$  this possibility does not arise.

#### 4.2.2 Seller-Ownership and Partial-Enforcement Contracts

In order to illustrate the important role which combinations of asset-ownership and partial-enforcement contracts might play we simplify our model in that we assume that  $\epsilon$  is a pure demand shock: suppose that the cost function of the seller and the payoffs of the parties from trading with the outside-market are independent of  $\epsilon$ . In the following the random variable  $\epsilon$  can take on two different values and it only determines whether or not the investment  $s$  of the seller has an influence on  $V(\cdot)$ : for  $\epsilon^1$  the investment is purely selfish, but for  $\epsilon^0$  the investment is cooperative and selfish at the same time. Under slight abuse of notation we denote the cost function of the seller with  $C(q, s)$  and

<sup>34</sup>In contrast to Che and Hausch (1999) we assume the existence of upper bounds for  $q$  and  $s$ . However, these assumptions are not necessary for an  $\gamma^* < \max_{\epsilon} \gamma(\epsilon)$  to emerge. What is important is that the investment returns as a function of  $q$  and  $s$  are bounded.

the payoffs of the buyer and the seller from trading with the outside-market with  $\tilde{u}^B(OS)$  and  $\tilde{u}^S(OS, s)$  respectively. Formally, we make the following assumption:

**Assumption 4 (A4)**

- (i)  $\tilde{u}^B(\cdot)$ ,  $\tilde{u}^S(\cdot)$  and  $C(\cdot)$  are independent of  $\epsilon$ ,
- (ii)  $E = E^S = \{\epsilon^0, \epsilon^1\}$  and  $E^B = \{\epsilon^0\}$ ,
- (iii)  $V(q, \epsilon^1) \equiv V(q, 0, \epsilon^0) \quad \forall q$ .

Define:  $\theta^0 \equiv (s, \epsilon^0)$  and  $\theta^1 \equiv (s, \epsilon^1)$ . Given (A4), it follows immediately from (17) that:

$$\gamma(\epsilon^1) = 1 \quad \text{and} \quad \gamma(\epsilon^0) \leq 1, \quad (21)$$

i.e. for  $\epsilon^0$  no trade contract is able to induce private ex-post returns above social ex-post returns if  $\alpha^S$  is sufficiently large. Besides  $\bar{q}$  and  $\bar{s}$  the overall critical value  $\gamma^*$  might also depend on the probability distribution of  $\epsilon$  (i.e. the probability  $p^0$  of the cooperative state): as  $p^0$  becomes bigger the expected effect of the externality under a full-enforcement contract becomes more severe, and, therefore,  $\gamma^*$  is non-increasing in  $p^0$ . Note, that (20) and (18) imply:

$$\gamma(\epsilon^0) \leq \gamma^* \quad \text{and} \quad \gamma(\epsilon^0) \leq \beta(\epsilon^0, BO). \quad (22)$$

These observations and the discussion in Section 4.2.1 hint at the possibility that a combination of asset ownership and a trade contract, which is enforced when  $\epsilon^1$  has been realized (i.e. when the investment is purely selfish) but which is not enforced when the investment is cooperative, might achieve the first-best even when no full-enforcement contract can do so (i.e. when  $\alpha^S > \gamma^*$ ). To allow for this possibility we focus in the following on situations where  $\gamma^* < 1$ .

**Remark 1** *Even in this setting the ownership structure may be irrelevant and the boundaries of the firm may be indeterminate: in principle, all possible ownership structures in combination with a partial-enforcement contract might achieve the first-best. As we are interested in situations where unconditional seller-ownership but not unconditional buyer-ownership is optimal, the above setting is non-trivial because we additionally have to characterize circumstances where unconditional buyer-ownership is never part of an optimal contract.*

When the parties specify trade quantities in the contract which are *only enforced* when  $\epsilon^1$  has been realized three different incentive effects relative to a full-enforcement contract are at work: (i) For  $\epsilon^0$  the externality of the investment would be relatively strong. When the underlying ownership structure determines threatpoint payoffs for this realization of  $\epsilon$  the externality is avoided. This raises the investment incentives of the seller relative to a

full-enforcement contract. (ii) For  $\epsilon^0$  the marginal increase in the seller's threatpoint payoff does not depend on the function  $C_s(q, \epsilon^0, \cdot)$  but it depends on the function  $\tilde{u}_s^S(OS, \epsilon^0, \cdot)$ . The sign of this effect is ambiguous. (iii) For  $\epsilon^1$  there is no externality. Hence, for  $\epsilon^1$  it is possible to induce private ex-post returns above social ex-post returns by specifying a high enough trade quantity in the contract (note:  $C_{qs} < 0$ ). This increases the incentives of the seller. However, this effect is limited because we assume  $q \in [0, \bar{q}]$ , i.e. it is technologically not feasible to produce or trade an unlimited quantity.

If it is possible to specify a high enough trade quantity which exactly induces first-best investment incentives in the above way, then a combination of unconditional seller-ownership and a partial-enforcement contract can in principle achieve the first-best: however, such a combination has also to be implementable, i.e. decisions and payments which the parties specify in a  $[x(\cdot), t(\cdot)]$ -contract have to be incentive-compatible (i.e. satisfy (SIC) and (BIC)): given a mild assumption on the absolute payoffs of the seller we are going to show that a contract exist which implements the desired partial-enforcement. Whereas in Grossman and Hart (1986), Hart and Moore (1990) and Hart (1995) the absolute cost and valuation levels do not play a role for generating investment incentives,<sup>35</sup> in our model the absolute levels are important because they determine whether a partial-enforcement contract which might induce an efficient investment fulfills (BIC) and (SIC), and hence can be implemented.<sup>36</sup>

We now proceed by deriving our results formally. Consider an unconditional ownership structure  $OS$  and payment  $t$ , and assume that the parties agree on a fixed trade quantity  $q^{OS}$  which is only enforced when  $\epsilon^1$  has been realized. When  $\epsilon = \epsilon^0$  the underlying ownership structure  $OS$  determines threatpoint payoffs. Using (A4), (12), (10), (9), (8), (6) and (4) we derive the marginal expected investment return of the seller for this case:<sup>37</sup>

$$\begin{aligned} \frac{\delta E[\bar{R}^S(OS, q^{OS}, s, \epsilon)]}{\delta s} &= p^0 \cdot [\alpha^S \cdot \phi_s(s, \epsilon^0) + (1 - \alpha^S) \cdot \tilde{u}_s^S(OS, s)] \\ &\quad + p^1 \cdot [\alpha^S \cdot \phi_s(s, \epsilon^1) - (1 - \alpha^S) \cdot C_s(q^{OS}, s)] \\ &= \alpha^S \cdot \Phi_s(s) \\ &\quad + (1 - \alpha^S) \cdot [p^0 \cdot \tilde{u}_s^S(OS, s) - p^1 \cdot C_s(q^{OS}, s)]. \end{aligned} \quad (23)$$

<sup>35</sup>In DeMeza and Lockwood (1998) and Chiu (1998) the absolute cost and valuation levels influence the outcome of the bargaining game, and, thereby, they have an influence on the investment incentives of the parties.

<sup>36</sup>An alternative to making assumptions on the absolute cost and valuation levels would be to assume that  $\epsilon$  is verifiable: if  $\epsilon$  was verifiable the parties could specify different functions  $x^i(\cdot)$ ,  $t^i(\cdot)$  for each realization  $\epsilon^i$ ,  $i = 0, 1$ , and the parties would only send reports regarding the investment level: Lemma 1, Lemma 2 and Proposition 1 would continue to hold. Moreover, no assumptions on the absolute cost and valuation levels would be needed to guarantee that the optimal partial-enforcement contracts in Proposition 2 and Proposition 3 below could be implemented because the parties could simply specify the desired enforcement for each realization of  $\epsilon$  in the initial contract.

<sup>37</sup>Again under slight abuse of notation.

It follows from (3) and (A2) that a  $q^{OS}$  which in combination with an ownership structure  $OS$  induces first-best investment incentives is implicitly defined by:<sup>38</sup>

$$\frac{\delta E \left[ \bar{R}^S (OS, q^{OS}, s^*, \epsilon) \right]}{\delta s} \equiv 1. \quad (24)$$

Using (23) and (3), we rewrite (24):

$$[p^0 \cdot \tilde{u}_s^S(OS, s^*) - (1 - p^0) \cdot C_s(q^{OS}, s^*)] \equiv 1. \quad (25)$$

The optimal quantities  $q^{OS}$  are functions of  $p^0$ : (25), (3), (A3), (A1)(v) and the Implicit-Function Theorem imply that the  $q^{OS}$  are strictly increasing functions of  $p^0$ : as  $p^0$  becomes larger it becomes more likely that the investment is cooperative and that the parties do not want to enforce the trade quantity. If, however, the underlying ownership structure determines threatpoint pay-offs the private ex-post returns are below the social ex-post returns, and  $q^{OS}$  has to rise in order to compensate for the reduction in expected investment returns. We suppress the dependency of  $q^{OS}$  on  $p^0$  for ease of notation. Because  $\tilde{u}_s^S(\cdot, s^*)$  is larger when the seller owns the asset and because of (A1)(v) it follows that:

$$q^{SO} < q^{BO}. \quad (26)$$

Note, that the  $q^{OS}$  are independent of the distribution of bargaining power between the seller and the buyer.

A combination of an unconditional ownership structure and a partial-enforcement contract with trade quantity  $q^{OS}$  can achieve efficiency only if  $q^{OS}$  is weakly below  $\bar{q}$ : if, for a given unconditional ownership structure  $OS$ ,  $q^{OS}$  is above  $\bar{q}$  then first-best investment incentives cannot be generated in this way because a  $q^{OS}$  above  $\bar{q}$  is technologically not feasible. It is obvious from (26) that seller-ownership is the most likely ownership structure to satisfy  $q^{SO} \leq \bar{q}$ .

We impose a mild condition on the absolute payoffs of the seller in order to guarantee that the above described partial-enforcement contract can be implemented when the seller owns the asset: for a given trade quantity  $q^{SO}$ , we assume that for the seller the potential direct effect of investing is larger *inside* the relationship with the buyer:

**Assumption 5 (A5)**  $- [C(q^{SO}, \bar{s}) - C(q^{SO}, 0)] \geq [\tilde{u}^S(SO, \bar{s}) - \tilde{u}^S(SO, 0)]$ .

We are now able to state our main result on the optimality of unconditional seller-ownership and partial-enforcement contracts:

---

<sup>38</sup>If a  $q^{OS}$  which satisfies (24) does not exist we (arbitrarily) define:  $q^{OS} \equiv 2\bar{q}$ .

**Proposition 2** Assume (A1)-(A5) and  $q^{SO} \leq \bar{q} < q^{BO}$ . Then the following is true for all  $\alpha^S > \max \{ \beta(\epsilon^0, BO), \gamma^* \}$ :

- (i) A combination of unconditional seller-ownership and a partial-enforcement contract achieves the first-best,
- (ii) Unconditional seller-ownership in combination with  $e(\theta^B, \theta^S) = 0 \forall \theta^B, \forall \theta^S$  is not optimal,
- (iii) A full-enforcement contract is not optimal,
- (iv) Unconditional buyer-ownership is not part of an optimal contract.

**Proof.** See Appendix D. ■

There exists a quite simple partial-enforcement contract which in combination with unconditional seller-ownership induces the first-best:

**Corollary 1** The partial-enforcement contract (55) in the proof of Proposition 2 (i) can be interpreted in the following way: unconditional seller-ownership and giving the seller an option-to-sell  $q^{SO}$  units of the good at strike price  $\frac{N}{q^{SO}}$ , where

$$N = \alpha^S [V(q^{SO}, \epsilon^1) - \tilde{u}^B(SO)] + (1 - \alpha^S) [C(q^{SO}, 0) + \tilde{u}^S(SO, 0)]$$

induces the first-best.

**Proof.** This follows directly from the definition in (55) and the proof of Proposition 2 (i). ■

Proposition 2 describes a situation where, even though trade is contractible ex-ante, asset ownership plays a crucial role *for achieving the first-best*: a combination of unconditional seller-ownership and a partial-enforcement contract induces an efficient investment of the seller, even when no full-enforcement contract can do so. Unconditional buyer-ownership is not optimal because the investment incentives which this ownership structure generates when  $\epsilon = \epsilon^0$  are sufficiently weak (i.e.  $q^{BO} > \bar{q}$ ) and because the externality which the investment causes is sufficiently strong (i.e.  $\alpha^S > \beta(\epsilon^0, BO)$ ).

The next proposition describes a similar situation as above, but in contrast to Proposition 2 the investment incentives which seller-ownership generates are now assumed to be relatively weak (i.e.  $q^{SO} > \bar{q}$ ): unconditional seller-ownership is still optimal, the parties sign a partial-enforcement contract, but the first-best cannot be achieved in this case:

**Proposition 3** Assume (A1)-(A4) and  $\bar{q} < q^{SO}$ . Then the following is true for all  $\alpha^S > \beta(\epsilon^0, BO)$ :

- (i) A combination of unconditional seller-ownership and a partial-enforcement contract is optimal but the first-best cannot be achieved,
- (ii) Unconditional seller-ownership in combination with  $e(\theta^B, \theta^S) = 0 \forall \theta^B, \theta^S$  is not optimal,
- (iii) A full-enforcement contract is not optimal,
- (iv) Unconditional buyer-ownership is not part of an optimal contract.

**Proof.** See Appendix E. ■

The optimal partial-enforcement contract (61) in the proof of Proposition 3 can again be interpreted as an option-to-sell the good:

**Corollary 2** *The partial-enforcement contract (61) in the proof of Proposition 3 can be interpreted in the following way: unconditional seller-ownership and giving the seller an option-to-sell  $\bar{q}$  units of the good at strike price  $\frac{N}{\bar{q}}$ , where*

$$N = \alpha^S [V(\bar{q}, \epsilon^1) - \hat{u}^B(SO)] + (1 - \alpha^S) [C(\bar{q}, 0) + \hat{u}^S(SO, 0)]$$

*is optimal.*

**Proof.** This follows directly from the definition of (61), the proof of Proposition 3 (i) and the proof of Proposition 2 (i). ■

## 5 Possible Extensions

Our model could be extended in various ways:

1. A generalization of Proposition 2 and Proposition 3 to more than two possible realizations of  $\epsilon$  would complicate the analysis but it would not change our results qualitatively: if no full-enforcement contract can induce the first-best, but if it is still possible to generate private ex-post returns above social ex-post returns for more than one realization of  $\epsilon$  the scope for asset ownership and partial-enforcement contracts should increase: for a given ownership structure several different partial-enforcement contracts which are enforced in different states of nature might raise investment incentives relative to a full-enforcement contract. Therefore, for a given ownership structure and for given absolute cost and valuation levels it is more likely that a partial-enforcement contract, which can be implemented and which raises investment incentives relative to a full-enforcement contract, exists. If, in addition, the investment incentives under buyer-ownership are sufficiently low the results of Proposition 2 and Proposition 3 should go through.
2. In Section 4.2, we would obtain the same results if the investment of the seller was selfish and cooperative at the same time for both realizations of  $\epsilon$  and if the cooperative component of the investment were sufficiently weak for  $\epsilon = \epsilon^1$ .
3. The case of two-sided investments (i.e. both the seller and the buyer invest simultaneously) would be another interesting extension. If both parties invest simultaneously in their human capital the optimal ownership structure, which the property rights approach prescribes, depends on the relative importance of the two investments for increasing the total surplus and on the effect of a certain ownership structure on the



investment incentives of both parties (see for example: Grossman and Hart (1986), Hart and Moore (1990) and Hart (1995)). In this case partial-enforcement contracts seem to be relevant if the investments of *both* parties are highly cooperative for some realizations of  $\epsilon$  but highly selfish for others. For the implementation of an optimal partial-enforcement contract it seems to suffice that in the highly cooperative states trade with the outside-market compared to trade with the partner is relatively attractive for *one* of the parties because they could design the partial-enforcement contract in a way such that it only depends on the messages of this party.

## 6 Conclusion

The property-rights theory of the firm provides a framework to determine the boundaries of the firm endogenously. The property-rights approach justifies its exclusive focus on the allocation of ownership rights by assuming that trade between the parties is non-contractible ex-ante. This paper provides foundations for the exclusive focus on property-rights based on the cooperative nature of an ex-ante investment. We allow for general, message-dependent contracts which include the possibility that the parties might want to write a trade contract which is only enforced in some ex-post states of nature. We consider an investment of the seller in his human capital which has an asymmetric effect on the buyer depending on whether the buyer and the seller trade with each other or with an outside-market: when both parties work together the seller is sometimes able to produce a tailor-made product for the buyer, and then the buyer directly benefits from the seller's investment in his human capital. Hence, under these circumstances the investment is cooperative in the sense of Che and Hausch (1999). When the parties trade with an outside-market this externality is not present.

When trade contracts are possible the parties can in principle specify a trade contract in a way such that it is enforced in every ex-post state of nature. If such a full-enforcement contract turns out to be optimal the ownership structure of assets does not play a role for generating investment incentives because the same full-enforcement contract is possible under each ownership structure. In such a case the optimal ownership structure and the boundaries of the firm(s) are indeterminate.

We allow for message-dependent ownership structures and derive three scenarios where *only one* of the simple ownership structures - the same as under the property-rights approach - turns out to be optimal: when the externality that the cooperative investment generates is sufficiently large it is optimal for the parties to write no trade contract at all, and, therefore, the same ownership structures as under the property-rights approach are optimal (Proposition 1). Our most novel results relate to the interaction of asset ownership and partial-enforcement contracts in generating investment incentives. We define a partial-enforcement contract as a contract which specifies trade quantities and payments but which is not always enforced ex-post.

Proposition 2 and Proposition 3 provide conditions when unconditional seller-ownership (but not unconditional buyer-ownership) is optimal but the parties also sign a partial-enforcement contract: under these conditions both a full-enforcement contract and asset ownership alone are suboptimal. The property-rights approach (where trade contracts are impossible) also states that unconditional seller-ownership but not unconditional buyer-ownership is optimal. Hence, under the conditions of Proposition 2 and Proposition 3, if one is only interested in determining the optimal simple ownership structure, the exclusive focus on property-rights is justified and trade contracts can be neglected. However, at the same time one is assured that in addition to seller-ownership the buyer and the seller are going to sign a partial-enforcement contract which reduces or even eliminates the inefficiency which the property-rights approach suggests. The absolute cost and valuation levels play a role in this setting because, given the non-verifiability of the ex-post state of the world, they determine whether the optimal partial-enforcement contract can be implemented.

## APPENDIX

### A Proof of Lemma 1

*Step 1.* As a preliminary step consider the problem of the seller when the ownership structure and the payment are unconditional, i.e.  $OS(\theta^B, \theta^S) \equiv OS$  and  $t(\theta^B, \theta^S) \equiv t \forall \theta^B, \theta^S$ : then, (7), (6) and (4) imply that the seller will choose his investment  $s^{OS}$  such that:

$$s^{OS} \in \arg \max_s \{ E[t + \alpha^S \phi(\theta) + (1 - \alpha^S) \tilde{u}^S(OS, \theta) - \alpha^S \tilde{u}^B(OS, \epsilon)] - s \}. \quad (27)$$

It then follows from (A1)(iii) and (A2) that  $s^{OS}$  satisfies the following first-order condition:

$$\alpha^S \cdot \Phi_s(s^{OS}) + (1 - \alpha^S) E[\tilde{u}_s^S(OS, s^{OS}, \epsilon)] \equiv 1 \quad \forall OS. \quad (28)$$

*Step 2.* We now return to the general setting. Note the following fixed-sum game property:

$$R^B(\theta^B, \theta^S, \theta) + R^S(\theta^B, \theta^S, \theta) = \phi(\theta) \quad \forall \theta^B, \forall \theta^S, \forall \theta. \quad (29)$$

For some arbitrary  $\theta', \theta \in \Theta$  (SIC) implies that  $\bar{R}^S(\theta) \geq R^S(\theta, \theta', \theta)$ , and (BIC) implies that  $\bar{R}^B(\theta) \geq R^B(\theta', \theta, \theta)$ . Combining these two observations with (29) yields:

$$R^S(\theta, \theta', \theta) \leq \bar{R}^S(\theta) \leq R^S(\theta', \theta, \theta). \quad (30)$$

Now assume that  $\theta'$  is true, and apply the same method as above to arrive at:

$$R^S(\theta', \theta, \theta') \leq \bar{R}^S(\theta') \leq R^S(\theta, \theta', \theta'). \quad (31)$$

Combining (30), (31), (12) and (13) implies:<sup>39</sup>

$$\begin{aligned} & \bar{R}^S(\theta) - \bar{R}^S(\theta') \\ & \leq R^S(\theta', \theta, \theta) - R^S(\theta', \theta, \theta') = \\ & = \alpha^S [\phi(\theta) - \phi(\theta')] + (1 - \alpha^S) [\tilde{u}^S(OS(\theta', \theta), \theta) - \tilde{u}^S(OS(\theta', \theta), \theta')]. \end{aligned} \quad (32)$$

Now, fix  $\theta \equiv (s, \epsilon)$  and  $\theta' \equiv (s', \epsilon)$ . Because  $\tilde{u}^S(\cdot)$  is not differentiable in  $OS$  we now derive an *upper bound* for the private ex-post return of the seller by

---

<sup>39</sup>The inequality in (32) is a well-established relationship (see for example: Maskin and Moore (1999))

using (32):

$$\begin{aligned}
\frac{\overline{\delta \bar{R}^S(\theta)}}{\delta s} &\equiv \limsup_{s' \rightarrow s} \frac{\bar{R}^S(\theta) - \bar{R}^S(\theta')}{s - s'} \leq \\
&\leq \alpha^S \cdot \phi_s(\theta) + \limsup_{s' \rightarrow s} \{ (1 - \alpha^S) \cdot \tilde{u}_s^S(OS(\theta', \theta), \theta) \} \\
&\leq \alpha^S \cdot \phi_s(\theta) + (1 - \alpha^S) \cdot \tilde{u}_s^S(SO, \theta) \\
&< \phi_s(\theta) \quad \forall \theta,
\end{aligned} \tag{33}$$

where the second inequality follows trivially because  $\tilde{u}_s^S(OS, \theta) \leq \tilde{u}_s^S(SO, \theta) \forall OS$ , and where the third inequality follows from (A3) and (2). (33), (28) and (A1)(iii) imply that the seller underinvests and that the investment incentives are maximal when  $OS(\theta^B, \theta^S) \equiv SO \forall \theta^B, \theta^S$ . (33), (A3) and (28) imply that  $OS(\theta^B, \theta^S) \equiv BO \forall \theta^B, \theta^S$  is not optimal. This proves the lemma.

## B Proof of Proposition 1

The same reasoning as in the proof of Lemma 1 (see Appendix A, Step 2) and using (12) and (14) imply for arbitrary  $\theta$  and  $\theta'$ :

$$\begin{aligned}
\bar{R}^S(\theta) - \bar{R}^S(\theta') &\leq R^S(\theta', \theta, \theta) - R^S(\theta', \theta, \theta') = \\
&= \alpha^S [\phi(\theta) - \phi(\theta')] \\
&\quad + (1 - e(\theta', \theta)) \cdot (1 - \alpha^S) [\tilde{u}^S(OS(\theta', \theta), \theta) - \tilde{u}^S(OS(\theta', \theta), \theta')] \\
&\quad + e(\theta', \theta) \cdot [M(\alpha^S, q(\theta', \theta), \theta) - M(\alpha^S, q(\theta', \theta), \theta')]
\end{aligned} \tag{34}$$

Now, fix  $\theta \equiv (s, \epsilon)$  and  $\theta' \equiv (s', \epsilon)$ . Because  $q(\cdot)$  need not be differentiable we again derive an *upper bound* for the private ex-post return of the seller:

$$\begin{aligned}
\frac{\overline{\delta \bar{R}^S(\theta)}}{\delta s} &\equiv \limsup_{s' \rightarrow s} \frac{\bar{R}^S(\theta) - \bar{R}^S(\theta')}{s - s'} \\
&\leq \alpha^S \cdot \phi_s(\theta) + \\
&\quad + \limsup_{s' \rightarrow s} \{ e(\theta', \theta) \cdot [M_s(\alpha^S, q(\theta', \theta), \theta) - (1 - \alpha^S) \cdot \tilde{u}_s^S(OS(\theta', \theta), \theta)] \\
&\quad \quad + (1 - \alpha^S) \cdot \tilde{u}_s^S(OS(\theta', \theta), \theta) \} \\
&\leq \alpha^S \cdot \phi_s(\theta) + (1 - \alpha^S) \cdot \tilde{u}_s^S(SO, \theta) \\
&< \phi_s(\theta) \quad \forall \theta.
\end{aligned} \tag{35}$$

where the first inequality follows from (34), the second inequality follows because  $\tilde{u}_s^S(OS, \theta) \leq \tilde{u}_s^S(SO, \theta) \forall OS$  and  $\alpha^S > \bar{\beta}(SO)$ , and where the third inequality follows from (A3) and (2).

(i) (35) together with Appendix A, Step 1 implies that unconditional seller-ownership and  $e(\theta^B, \theta^S) \equiv 0 \forall \theta^B, \theta^S$  are optimal. (35) shows that the payment does not play a role for investment incentives and, therefore one can fix  $t(\theta^B, \theta^S) \equiv t \forall \theta^B, \theta^S$ . (ii)  $\alpha^S > \bar{\beta}(SO)$  implies:

$$M_s(\alpha^S, q, \theta) - (1 - \alpha^S) \cdot \tilde{u}_s^S(SO, \theta) < 0 \quad \forall q, \forall \theta. \tag{36}$$

(35) and (36) imply that  $e(\theta^B, \theta^S) \equiv 1 \forall \theta^B, \theta^S$  is not optimal. Hence, a full enforcement contract is not optimal. (iii) If one plugs  $OS(\theta', \theta) \equiv BO \forall \theta', \forall \theta$  into (35) then it follows from (36) and (A3) that unconditional buyer-ownership is never part of an optimal contract.

## C Proof of Lemma 2

Condition (34) and  $e(\theta^B, \theta^S) \equiv 1 \forall \theta^B, \theta^S$  (full-enforcement) imply:

$$\begin{aligned} \bar{R}^S(\theta) - \bar{R}^S(\theta') &\leq R^S(\theta', \theta, \theta) - R^S(\theta', \theta, \theta') = \\ &= \alpha^S [\phi(\theta) - \phi(\theta')] + [M(\alpha^S, q(\theta', \theta), \theta) - M(\alpha^S, q(\theta', \theta), \theta')] \end{aligned} \quad (37)$$

Now, fix  $\theta^i \equiv (s, \epsilon^i)$  and  $\theta^{i'} \equiv (s', \epsilon^i)$  for  $i = 0, 1$ . Using this notation, we rewrite (37) for a certain  $\epsilon^i$ , weight it with its respective probability  $p^i$  and sum over all  $i$  to arrive at:

$$\begin{aligned} E[\bar{R}^S(s, \epsilon)] - E[\bar{R}^S(s', \epsilon)] &\leq \alpha^S \cdot [\Phi(s) - \Phi(s')] \\ &\quad + \sum_{i=0}^1 p^i \cdot \{M(\alpha^S, q(\theta^{i'}, \theta^i), \theta^i) - M(\alpha^S, q(\theta^{i'}, \theta^i), \theta^{i'})\}. \end{aligned} \quad (38)$$

Because  $q(\cdot)$  need not be differentiable we again derive an *upper bound* for the expected private investment return by using (38):

$$\begin{aligned} \frac{\delta E[\bar{R}^S(\theta)]}{\delta s} &\equiv \limsup_{s' \rightarrow s} \frac{E[\bar{R}^S(s, \epsilon)] - E[\bar{R}^S(s', \epsilon)]}{s - s'} \\ &\leq \alpha^S \cdot \Phi_s(s) \\ &\quad + \limsup_{s' \rightarrow s} \left\{ \sum_{i=0}^1 p^i \cdot M_s(\alpha^S, q(\theta^{i'}, \theta^i), s, \epsilon^i) \right\} \\ &< \Phi_s(s), \quad \text{if } \alpha^S > \gamma^*. \end{aligned} \quad (39)$$

Hence, for  $\alpha^S > \gamma^*$  the expected private investment return for the seller is always below the expected social return and, therefore, no full-enforcement contract can induce the first-best.

In the following, we provide an example that a  $\gamma^* < \max_{\epsilon} \gamma(\epsilon)$  is possible: assume that  $\epsilon$  is drawn from  $\{0, 1\}$  with probabilities  $p^0$  and  $p^1$  respectively and that  $\bar{q} > \frac{1}{4}$ . Suppose that the seller and the buyer have the following cost and valuation functions when they trade with each other:<sup>40</sup>

$$C(q, s) \equiv 10(q)^2 \left[ 1 + \frac{1}{\sqrt{1+s}} \right], \quad (40)$$

$$V(q, s, \epsilon) \equiv 10q \left[ 1 + (1 - \epsilon) \sqrt{1+s} \right], \quad (41)$$

---

<sup>40</sup>Under slight abuse of notation.

i.e. the investment of the seller is purely selfish for  $\epsilon = 1$ , but selfish and cooperative at the same time for  $\epsilon = 0$ . In this case, the following critical values  $\gamma(0)$ ,  $\gamma(1)$  and  $\gamma^*$  can be obtained (these results are derived below):

$$\gamma(0) = \begin{cases} 0 & \text{if } (\bar{q})^2 \leq \frac{12}{16} \\ \frac{4(\bar{q})^2 - 3}{4(\bar{q})^2 - 3 + 4\bar{q}} & \text{if } (\bar{q})^2 > \frac{12}{16} \end{cases}, \quad \gamma(1) = 1 \quad \text{and} \quad (42)$$

$$\gamma^* = \begin{cases} 0 & \text{if } (\bar{q})^2 \leq X(p^0), \\ \frac{16(\bar{q})^2 - 11p^0 - 1}{16(\bar{q})^2 - 11p^0 - 1 + 16\bar{q}p^0} & \text{if } X(p^0) < (\bar{q})^2 \leq \frac{1}{1-p^0} \cdot X(p^0), \\ 1 & \text{if } (\bar{q})^2 > \frac{1}{1-p^0} \cdot X(p^0), \end{cases} \quad (43)$$

where:  $X(p^0) \equiv \frac{11}{16}p^0 + \frac{1}{16}$ .

It is obvious from (43) that for a certain parameter range our Lemma 2 indeed strenghtens the result of Che and Hausch (1999, Prop. 3 (i)): if  $(\bar{q})^2 \leq \frac{1}{1-p^0}X(p^0)$ , then  $\gamma^* < \gamma(1)$ .<sup>41</sup> In this case, for  $\alpha^S > \gamma^*$ , no full-enforcement contract can induce the first-best, even though it is still possible to induce marginal private returns above social returns when  $\epsilon = 1$ . Note, that both  $\gamma(0)$  and  $\gamma^*$  are non-decreasing in  $\bar{q}$ : as  $\bar{q}$  gets bigger the marginal cost savings that the seller can possibly appropriate by specifying a large trade quantity in a trade contract become bigger since by (A1)(v) we have  $C_{qs} < 0$ : only if the bargaining power of the seller is relatively large and, thereby, the negative effect of the externality on his investment incentives is relatively strong, no contract can induce private returns above social returns when  $\epsilon = 0$  and in expected terms respectively. The overall critical value  $\gamma^*$  is non-increasing in  $p^0$ : if the probability of  $\epsilon = 0$  is very high then even for a relatively low bargaining power of the seller no full-enforcement contract can induce the first-best.

### Derivation of $\gamma(0)$ , $\gamma(1)$ and $\gamma^*$

It follows from (40) and (41) that:

$$C_s(q, s) = -5(q)^2 \frac{1}{\sqrt[3]{1+s}} \quad \text{and} \quad V_s(q, s, \epsilon) = 5q(1 - \epsilon) \frac{1}{\sqrt{1+s}}. \quad (44)$$

We use (40), (41), (1) and (2) to derive the ex-post social surplus:

$$\phi(s, 0) = 2.5(1 + \sqrt{1+s} + s) \quad \text{and} \quad \phi(s, 1) = 2.5 \frac{\sqrt{1+s}}{\sqrt{1+s}+1}, \quad (45)$$

and by differentiating:

$$\phi_s(s, 0) = 2.5 + 1.25 \frac{1}{\sqrt{1+s}} \quad \text{and} \quad \phi_s(s, 1) = 1.25 \frac{1}{\sqrt{1+s} + 2(1+s) + \sqrt[3]{1+s}}. \quad (46)$$

---

<sup>41</sup>For example, this condition is fulfilled if  $p^0 = 0.5$  and  $\bar{q} \leq 0.90$ .

(i) To derive  $\gamma(0)$  and  $\gamma(1)$  we substitute (44) and (46) into the condition in (17). For  $\epsilon = 1$  we arrive at:

$$4(1-k)(q)^2 \leq (1-k) \frac{1}{1 + 2\frac{1}{\sqrt{1+s}} + \frac{1}{1+s}}. \quad (47)$$

Since (47) has to hold for all  $q$  and  $s$  we assume without loss of generality that  $s = 0$  and  $q = \bar{q}$ . Then (47) simplifies to:

$$16(1-k)(\bar{q})^2 \leq (1-k). \quad (48)$$

As  $\bar{q} > \frac{1}{4}$ , (48) only holds if  $k = 1$ . It then follows from (17) that:

$$\gamma(1) = 1. \quad (49)$$

If we apply the same method in the case  $\epsilon = 0$  we can again assume  $s = 0$  without loss of generality, and arrive at:

$$4kq + (1-k)[3 - 4(q)^2] \geq 0. \quad (50)$$

If the term in square brackets is positive for  $q = \bar{q}$  then (50) holds for all  $k$ . If this is not the case, note that for a given  $k$  the left-hand side of (50) is first increasing and then possibly decreasing in  $q$  for  $q \leq \bar{q}$ . Therefore, (50) holds for a certain  $k$ , if it holds for  $q = \bar{q}$ . These observations finally lead to:

$$\gamma(0) = \begin{cases} 0 & \text{if } (\bar{q})^2 \leq \frac{3}{4} \\ \frac{4(\bar{q})^2 - 3}{4(\bar{q})^2 - 3 + 4\bar{q}} & \text{if } (\bar{q})^2 > \frac{3}{4} \end{cases}. \quad (51)$$

(ii) To derive  $\gamma^*$  we substitute (44) and (46) into the condition in (19). Rearranging this condition yields:

$$\begin{aligned} & 4(1-k) \left( p^0 (q^0)^2 + p^1 (q^1)^2 \right) - 4p^0 k (1+s) q^0 \\ & \leq (1-k) \left[ 2p^0 \sqrt[3]{1+s} + p^0 (1+s) + p^1 \frac{1}{1 + 2\frac{1}{\sqrt{1+s}} + \frac{1}{1+s}} \right]. \end{aligned} \quad (52)$$

It is obvious from (52) that we can assume  $s = 0$  and  $q^1 = \bar{q}$  without loss of generality. Using this observation and rearranging (52) yields:

$$kq_0 + (1-k) \left[ \frac{12}{16} + \frac{p^1}{p^0} \left( \frac{1}{16} - (\bar{q})^2 \right) - (q^0)^2 \right] \geq 0. \quad (53)$$

If the term in square brackets is positive for  $q^0 = \bar{q}$  then (53) holds for all  $k$ . If this is not the case, note that for a given  $k$  the left-hand side of (53) is first increasing and then possibly decreasing in  $q^0$  for  $q^0 \leq \bar{q}$ . Two cases are possible: if the left-hand side of (53) is negative for  $q^0 = 0$  then (53) only holds for  $k = 1$ . If, on the other hand, the left-hand side of (53) is positive for  $q^0 = 0$  then it follows from the discussion above that it holds for a certain  $k$ , if it holds for  $q^0 = \bar{q}$ . These observations finally lead to:

$$\gamma^* = \begin{cases} 0 & \text{if } (\bar{q})^2 \leq X(p^0) \\ \frac{16(\bar{q})^2 - 11p^0 - 1}{16(\bar{q})^2 - 11p^0 - 1 + 16\bar{q}p^0} & \text{if } X(p^0) < (\bar{q})^2 \leq \frac{1}{1-p^0} X(p^0) \\ 1 & \text{if } (\bar{q})^2 > \frac{1}{1-p^0} X(p^0), \end{cases} \quad (54)$$

where:  $X(p^0) \equiv \frac{11}{16}p^0 + \frac{1}{16}$ .

## D Proof of Proposition 2

(i) It follows from our discussion above Proposition 2 that unconditional seller-ownership in combination with a quantity  $q^{SO}$  which is *only* enforced when  $\epsilon^1$  has been realized (i.e.  $e(s, \epsilon^1) \equiv 1 \forall s$  and  $e(s, \epsilon^0) \equiv 0 \forall s$ ) induces first-best investment incentives. However, it remains to be shown that the message-dependent functions  $x(\cdot, \cdot)$  and  $t(\cdot, \cdot)$  can be specified in a way such that the above partial-enforcement is achieved and the resulting contract  $[x(\theta^B, \theta^S), t(\theta^B, \theta^S) \forall \theta^B, \forall \theta^S \in \Theta]$  induces truth-telling as a Nash-Equilibrium at date 4 (i.e. satisfies (SIC) and (BIC)). In the following we show that a contract exists that satisfies (BIC) and (SIC) and induces the desired enforcement: let  $N$  be a real number:  $N \in \mathbb{R}$ . Consider the following contract:

$$\begin{aligned} OS(\theta^B, \theta^S) &\equiv SO, \quad q(\theta^B, \theta^S) \equiv q^{SO} \quad \text{and} \quad t(\theta^B, \theta^S) \equiv N \quad \forall \theta^B, \forall \theta^S, \\ e(\theta^B, (s, \epsilon^0)^S) &\equiv 0 \quad \forall s, \forall \theta^B \quad \text{and} \quad e(\theta^B, (s, \epsilon^1)^S) \equiv 1 \quad \forall s, \forall \theta^B, \end{aligned} \quad (55)$$

i.e. the function  $e(\cdot, \cdot)$  only depends on whether the seller reports  $\epsilon^0$  or  $\epsilon^1$ . Under this contract, (BIC) is always satisfied because the message of the buyer has no influence on the contractual outcome. It remains to be shown that (SIC) is satisfied. Assume that  $\epsilon^0$  has been realized at date 3: (SIC) is satisfied if the seller never has an incentive to report  $\epsilon^1$ . For this case, using (12), (10), (9), (8), (6), (A4) and (55), we can rewrite (SIC) in the following way:

$$N \leq \alpha^S [V(q^{SO}, s, \epsilon^0) - \tilde{u}^B(SO)] + (1 - \alpha^S) [C(q^{SO}, s) + \tilde{u}^S(SO, s)] \quad \forall s. \quad (56)$$

Now assume that  $\epsilon^1$  has been realized at date 3. (SIC) is satisfied if the seller never has an incentive to report  $\epsilon^0$ . For  $\epsilon = \epsilon^1$ , (SIC) simplifies to:

$$N \geq \alpha^S [V(q^{SO}, \epsilon^1) - \tilde{u}^B(SO)] + (1 - \alpha^S) [C(q^{SO}, s) + \tilde{u}^S(SO, s)] \quad \forall s. \quad (57)$$

In order for (SIC) to hold,  $N$  has to satisfy (56) and (57) simultaneously for all  $s$ : it follows from the definition of  $\beta(\epsilon^0, SO)$ ,  $\alpha^S > \beta(\epsilon^0, SO)$  and (18) that the right-hand side of (56) is increasing in  $s$ . Therefore, (56) holds for all  $s$  if it holds for  $s = 0$ . It follows from (4), (A3), (A1)(v),  $q^{SO} > q^*(s^*, \epsilon^1)$  and  $\frac{\delta q^*}{\delta s} > 0$  that the right-hand side of (57) is first decreasing and then possibly increasing in  $s$ . This observation together with (A5) implies that (57) holds for all  $s$  if it holds for  $s = 0$ . Substituting  $s = 0$  into (56) and (57) reveals that a  $N$ , such that (SIC) is satisfied, exists if  $V(q^{SO}, 0, \epsilon^0) \geq V(q^{SO}, \epsilon^1)$ . Assumption (A4)(iii) implies that this is true. This proves that unconditional seller-ownership in combination with the partial-enforcement contract (55) induces the first-best.



- (ii) This follows from Lemma 1 (iii) and the fact that the contract in (i) achieves the first-best.
- (iii) This follows from  $\alpha^S > \gamma^*$  and Lemma 2 above.
- (iv) Assume that  $OS(\theta^B, \theta^S) \equiv BO \forall \theta^B, \forall \theta^S$ . Suppose that  $\epsilon^0$  has been realised at date 3. It follows from  $\alpha^S > \beta(\epsilon^0, BO)$ , (18), (35) and (15) that for  $OS = BO$  and for all  $\theta^0 \in \Theta$ :

$$\frac{\overline{\delta R^S(BO, \theta^0)}}{\delta_s} \leq u_s^S(BO, \theta^0) < \phi_s(\theta^0), \quad (58)$$

i.e. when  $\epsilon = \epsilon^0$  investment incentives are maximized by relying on the underlying ownership structure as threatpoint in renegotiations. But this is exactly what is underlying our derivation of  $q^{BO}$ . Now suppose that  $\epsilon^1$  has been realised at date 3. Assumption (A3),  $\bar{q} > q^*(\theta^1) \forall \theta^1$  and (A1)(v) imply:

$$\tilde{u}_s^S(BO, \theta^1) < -C_s(\bar{q}, \theta^1) \quad \forall \theta^1. \quad (59)$$

It now follows from (35), (14) and (A4)(i),(ii) that for  $OS = BO$  and for all  $\theta^1, \theta' \in \Theta$ :

$$\begin{aligned} & \frac{\overline{\delta R^S(BO, \theta^1)}}{\delta_s} \\ & \leq u_s^S(BO, s) \\ & \quad - \liminf_{s' \rightarrow s} \{e(\theta', \theta^1) (1 - \alpha^S) [C_s(q(\theta', \theta^1), s) + \tilde{u}_s^S(BO, s)]\} \\ & \leq u_s^S(BO, s) \\ & \quad - \liminf_{s' \rightarrow s} \{e(\theta', \theta^1) (1 - \alpha^S) [C_s(\bar{q}, s) + \tilde{u}_s^S(BO, s)]\} \\ & < \alpha^S \phi_s(\theta^1) - (1 - \alpha^S) C_s(q^{BO}, s), \end{aligned} \quad (60)$$

where the second inequality follows from (A1)(v) and  $q(\theta', \theta^1) \leq \bar{q} \forall \theta^1, \forall \theta' \in \Theta$ , and where the third inequality follows from (59),  $q^{BO} > \bar{q}$  and (A1)(v). To sum up, (58) and (60) together with (23) and (24) imply that every contract where  $q(\theta^B, \theta^S) \leq \bar{q} \forall \theta^B, \forall \theta^S \in \Theta$  induces underinvestment when  $OS(\theta^B, \theta^S) \equiv BO \forall \theta^B, \forall \theta^S$ .

### E Proof of Proposition 3

- (i) Consider the following contract:

$$\begin{aligned} OS(\theta^B, \theta^S) & \equiv SO, \quad q(\theta^B, \theta^S) \equiv \bar{q} \quad \text{and} \quad t(\theta^B, \theta^S) \equiv N \quad \forall \theta^B, \forall \theta^S, \\ e(\theta^B, (s, \epsilon^0)^S) & \equiv 0 \quad \forall s, \forall \theta^S \quad \text{and} \quad e(\theta^B, (s, \epsilon^1)^S) \equiv 1 \quad \forall s, \forall \theta^S. \end{aligned} \quad (61)$$

It follows from the reasoning in the proof of Proposition 2 (i) that the partial-enforcement contract (61) satisfies (BIC) and (SIC). However, note that assumption (A5) is *not* required for (61) to be implementable: the analogue of (57) for contract (61) is always decreasing in  $s$  because  $\bar{q} > q^*(s, \epsilon^1) \forall s$ .

- (ii) A similar argument as for (59) implies:

$$\tilde{u}_s^S(SO, \theta^1) < -C_s(\bar{q}, \theta^1) \quad \forall \theta^1. \quad (62)$$

(62) and (61) imply that unconditional seller-ownership and  $e(\theta^B, \theta^S) = 0$   $\forall \theta^B, \theta^S$  generate strictly lower investment incentives than the contract (61). Because the optimal contract displays underinvestment this proves the claim.  
 (iii) It follows from (15) that  $\alpha^S > \beta(\epsilon^0, BO)$  implies:

$$M_s(\alpha^S, q, s, \epsilon^0) < (1 - \alpha^S) \tilde{u}_s^S(OS, s, \epsilon^0) \quad \forall q \in Q, \forall s \in S. \quad (63)$$

This observation and (35) imply that, for  $\epsilon = \epsilon^0$ , unconditional seller-ownership creates strictly higher investment incentives than a full-enforcement contract. Now, we turn to the case that  $\epsilon^1$  has been realized: the first inequality in (35), (A3), (A4)(ii) and (A1)(v) imply that when  $\epsilon^1$  has been realized investment incentives are maximized by specifying a trade quantity  $\bar{q}$  as threatpoint in renegotiations. Combining these observation for  $\epsilon^0$  and  $\epsilon^1$  shows that a full-enforcement contract is not optimal.

(iv) Assume that  $OS(\theta^B, \theta^S) \equiv BO \quad \forall \theta^B, \forall \theta^S$ . Suppose that  $\epsilon^1$  has been realised: the argument in (iii) above shows that when  $\epsilon^1$  has been realized investment incentives are maximized by specifying a trade quantity  $\bar{q}$  as threatpoint in renegotiations. Now, suppose that  $\epsilon^0$  has been realised:  $\alpha^S > \beta(\epsilon^0, BO)$  implies that investment incentives are maximal when the underlying ownership structure determines threatpoint payoffs. Combining these observation for  $\epsilon^0$  and  $\epsilon^1$ , it follows from (61) and (A3) that unconditional buyer-ownership is never part of an optimal contract.

## References

- AGHION, P., M. DEWATRIPONT, AND P. REY (1994): "Renegotiation design with unverifiable information," *Econometrica*, 62(2), 257–282.
- (2000): "Partial contracting, control allocation and cooperation," *mimeo, UCL and IDEI*.
- AGHION, P., AND P. REY (1999): "Allocating decision rights under liquidity constraints: a simple framework," *mimeo, UCL and IDEI*.
- CHE, Y. K., AND T. Y. CHUNG (1999): "Contract damages and cooperative investments," *Rand Journal of Economics*, 30(1), 84–105.
- CHE, Y. K., AND D. B. HAUSCH (1999): "Cooperative investments and the value of contracting," *American Economic Review*, 89(1), 125–147.
- CHIU, Y. S. (1998): "Noncooperative bargaining, hostages, and optimal asset ownership," *American Economic Review*, 88(4), 882–901.
- CHUNG, T. Y. (1991): "Incomplete contracts, specific investments, and risk sharing," *Review of Economic Studies*, 58(5), 1031–1042.
- COASE, R. H. (1937): "The Nature of the Firm," *Economica*, 4, 386–405.
- DEMEZA, D., AND B. LOCKWOOD (1998): "Does asset ownership always motivate managers? Outside options and the property rights theory of the firm," *Quarterly Journal of Economics*, 113(2), 361–386.
- EDLIN, A. S., AND S. REICHELSTEIN (1996): "Holdups, standard breach remedies, and optimal investment," *American Economic Review*, 86(3), 478–501.
- GROSSMAN, S. J., AND O. D. HART (1986): "The costs and benefits of ownership - A theory of vertical and lateral integration," *Journal of Political Economy*, 94(4), 691–719.
- HART, O., AND J. MOORE (1988): "Incomplete contracts and renegotiation," *Econometrica*, 56(4), 755–785.
- (1990): "Property-rights and the nature of the firm," *Journal of Political Economy*, 98(6), 1119–1158.
- (1999): "Foundations of incomplete contracts," *Review of Economic Studies*, 66(1), 115–138.
- HART, O. D. (1995): *Firms, contracts, and financial structure*, Clarendon Lectures in Economics. Clarendon Press, New York.
- HERMALIN, B. E., AND M. L. KATZ (1993): "Judicial modification of contracts between sophisticated parties - a more complete view of incomplete contracts and their breach," *Journal of Law Economics and Organization*, 9(2), 230–255.
- MASKIN, E., AND J. MOORE (1999): "Implementation and renegotiation," *Review of Economic Studies*, 66(1), 39–56.
- MASKIN, E., AND J. TIROLE (1999a): "Two remarks on the property-rights literature," *Review of Economic Studies*, 66(1), 139–149.
- (1999b): "Unforeseen contingencies and incomplete contracts," *Review of Economic Studies*, 66(1), 83–114.
- NÖLDEKE, G., AND K. M. SCHMIDT (1995a): "Debt as an option to own in

- the theory of ownership rights,” in *Perspectives on contract theory*, ed. by A. Picot, and E. Schlicht, pp. 1–15. Physica Verlag, Berlin.
- (1995b): “Option contracts and renegotiation - a solution to the hold-up problem,” *Rand Journal of Economics*, 26(2), 163–179.
- (1998): “Sequential investments and options to own,” *Rand Journal of Economics*, 29(4), 633–653.
- ROSENKRANZ, S., AND P. W. SCHMITZ (1999): “Know-how disclosure and incomplete contracts,” *Economics Letters*, 63(2), 181–185.
- SCHWEIZER, U. (2000): “An Elementary Approach to the Hold-Up Problem with Renegotiation,” *Bonn Econ Discussion Paper, University of Bonn*.
- SEGAL, I. (1999): “Complexity and renegotiation: A foundation for incomplete contracts,” *Review of Economic Studies*, 66(1), 57–82.
- SEGAL, I., AND M. WHINSTON (1998): “Exclusive contracts and protection of investments,” *mimeo, Northwestern University*.
- (1999): “The Mirrlees approach to mechanism design with renegotiation (with applications to hold-up and risk sharing),” *mimeo, Northwestern University*.
- TIROLE, J. (1999): “Incomplete contracts: Where do we stand?,” *Econometrica*, 67(4), 741–781.

# FIGURES

**Figure 1: The Sequence of Events**

