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by

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Sandbagging*

Matthias Kräkel†

Abstract

Participants of dynamic competition games may prefer to play with the rules of the game by systematically withholding effort in the beginning. Such behavior is referred to as sandbagging. I consider a two-period contest between heterogeneous players and analyze potential sandbagging of high-ability participants in the first period. Such sandbagging can be beneficial to avoid second-period matches against other high-ability opponents. I characterize the conditions under which sandbagging leads to a coordination problem, similar to that of the battle-of-the sexes game. Moreover, if players' abilities have a stronger impact on the outcome of the first-period contest than effort choices, mutual sandbagging by all high-ability players can arise.

Key Words: coordination problem; dynamic contest; heterogeneous contestants; withholding effort.

JEL Classification: C72; D72.

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1 Introduction

In dynamic contests, sandbagging can take two different forms. First, a player may withhold effort[1] in a current round in order to hide his strength. As a result, the opponents' beliefs about the player's ability are adjusted downwards, which leads to a reduction of the opponents' effort choices in future rounds. This form of sandbagging belongs to situations, where the same players meet in repeated contests. Second, a player can withhold effort to exploit the rules of the dynamic contest. In particular, the player can voluntarily lose a current contest in order to be matched with weaker opponents in the future. This kind of sandbagging is aimed at the contest designer and not at the current opponents. From the viewpoint of society, sandbagging is detrimental for two reasons – it leads to an underprovision of effort and to a possible misallocation of players to higher-order contests.

There are several real-world examples for either kind of sandbagging. The first class of sandbagging problems is based on signal jamming, where one player chooses an unobservable action (e.g., effort) to manipulate the beliefs of other players regarding his true ability. In the case of sandbagging, a player tries to appear as a weak contestant to make his opponents choose only low efforts in the next round. Examples can be observed in sports when individual athletes or teams try to win only with a small lead at the beginning of a tournament to hide their true abilities until meeting stronger opponents in later rounds. At the beginning of a military conflict, one party can withhold military resources to make the opposing party believe that one is easy prey in future battles. In poker, a player that has a strong hand bets weakly to convince the other players to stay in the game. In a litigation contest, one lawyer starts with rather weak arguments to deter the lawyer of the other party from further collecting evidence.

The second class of sandbagging problems is not based on signal jamming. Instead, the sandbagger withholds effort to play with the rules of the game or

[1] Effort can take different forms. In sports, "effort" consists of training intensity as well as effort during the match. In military conflicts, "effort" describes the military resources used by the conflicting parties. In business and litigation contests, "effort" is given by money and time invested in the competition.

even manipulate them. In a sales contest, individuals sometimes hold aside realized sales when clearly having outperformed their opponents or when being a clear loser in order to use these realized sales in the next-period contest. In many (professional) sports, handicap systems are used for leveling out the playing field. Here, sandbagging happens when individuals deliberately perform poorly in less important contests to accumulate additional handicap for key contests in the future. Such practice can be frequently observed in bowling and golf. In certain racing series, drivers may prefer to underperform in qualifying rounds to obtain a better starting position as handicap. Soccer tournaments like the FIFA soccer world championship often consist of two parts – the group phase and the subsequent knock-out phase. Typically, in the round of sixteen, group winners are matched with second-best teams of other groups. A soccer team that wants to be matched with the first instead of the second-best team of another group can gain a strategic advantage by deliberately losing the final group match and becoming second instead of first in its respective group. In games like professional chess, Go, billiards or BMX racing, players can voluntarily lose unimportant contests to decrease their average rating and, thus, to start in the next contest in a lower class. In this class, the sandbagger can more easily win against less stronger opponents. However, such sandbagging has the drawback that winner prizes in lower classes are typically smaller than prizes in higher classes. In dynamic career contests, workers may prefer to underperform at some time to be assigned to a less demanding career track in which they can easily outperform their opponents. Similarly to the previous example, sandbagging leads to the disadvantage that the player forgoes larger incomes at higher career tracks.

Whereas several contest papers have discussed the signal-jamming argument of sandbagging,[2] to the best of my knowledge there do not exist contest papers on sandbagging as means of playing with the rules of the game. My note aims at narrowing this gap. The analysis will proceed in two steps. I start by considering a heterogeneous contest between a high-ability player and a low-ability one. The contest winner is assigned to a high-prize contest in the second period (*major*

[2]See the conjecture by Rosen (1986), p. 714, as well as the papers by Amegashie (2006), Hörner and Sahuguet (2007), Zhang and Wang (2009), Münster (2009).

contest), whereas the contest loser is relegated to a low-prize contest (*consolation contest*). In both second-period contests, there will be random matching with an unknown opponent. The two first-period players only know a discrete type distribution over the second-period opponents for each kind of match.

In the benchmark case of an isolated first-period contest without the existence of subsequent second-period contests, both the high-ability player and his low-ability opponent would choose identical equilibrium efforts. I show that the assignment to different second-period contests can make the high-ability player sandbag by withholding effort in the first round. Such sandbagging will be rational if saved effort costs from avoiding another high-ability player in a possibly homogeneous major contest exceed expected income losses from being assigned to the consolation match. The corresponding sandbagging condition also shows how a contest designer can choose the characteristics of the contest to avoid sandbagging. A high winner prize for the major contest does not necessarily solve the sandbagging problem since this winner prize also influences equilibrium efforts and, thus, effort costs in equilibrium. However, optimally designing the distributions over the second-period opponents' types is a very effective device against sandbagging.

In a second step, I consider a situation with two heterogeneous contests that simultaneously take place in period one. This setting is used to address the problem of endogenous matching in the second period. While first-period winners are assigned to the major contest, the first-period losers are relegated to a consolation contest. This more complicated setting renders a complete solution of the game impossible but I can still characterize situations in which sandbagging happens. In particular, there are cases in which the high-ability players face a coordination problem similar to that of the battle-of-the sexes, since each high-ability player can only profit from sandbagging if the other one prefers not to sandbag. Moreover, if the impact of players' abilities dominate the impact of effort on the outcome of the first-period contests, even mutual sandbagging by both high-ability players can be an equilibrium.

One immediate implication of my results is that the current rules for the soccer world championship's group phase should be redesigned in order to avoid sandbagging. According to these rules, the final matches within a certain group have to take place simultaneously, which should restore suspense. However, in order to prevent sandbagging, the final matches of *each two* adjacent groups whose best and second best teams are seeded together within the round of sixteen should take place at the same time. This redesign would come at a cost since customers would like to visit several of these matches which is only possible if the matches proceed sequentially.

2 The Model

I consider a contest game that lasts two periods.[3] In the first period, there is a heterogeneous match between a player with low ability a_L and a player with high ability a_H ($> a_L$). Let $\Delta a := a_H - a_L$ denote the ability difference. Each player i ($i = H, L$) exerts non-negative effort e_i to influence his score function

$$x_i(e_i) = h(e_i) + a_i + \varepsilon_i \tag{1}$$

with h being monotonically increasing and concave and ε_i describing random noise. Following the seminal paper by Lazear and Rosen (1981), I assume that ε_H and ε_L are stochastically independent and identically distributed (i.i.d.). The cumulative distribution function of $\varepsilon_H - \varepsilon_L$ is denoted by G and the corresponding density by g. Let g be unimodal and symmetric about zero.[4] Exerting effort e_i leads to cost $c(e_i)$ for player i with $c(0) = c'(0) = 0$ and $c'(e_i), c''(e_i) > 0$ for $e_i > 0$. Player H (L) is declared contest winner if $x_H(e_H) > x_L(e_L)$ ($x_H(e_H) < x_L(e_L)$). The contest winner receives income $Y > 0$, whereas the loser gets zero.

In the second period, the two players H and L are assigned to two different contests – depending on the outcome of the first-period contest. The winner of the first-period contest participates in the major contest M where he is randomly matched with another player. This opponent has ability a_H with probability $p_M \in$

[3] The game is based on the difference-form contest-success function used, e.g., by Lazear and Rosen (1981), Dixit (1987), Baik (1998), Che and Gale (2000).

[4] E.g., if ε_H and ε_L are normally (uniformly) distributed, the convolution g will be normal (triangular). If ε_H and ε_L are exponential, g follows a Laplace distribution.

$(0, 1)$ and ability a_L with probability $1 - p_M$. After having entered contest M but before choosing efforts, both players observe individual abilities. The two players have the same score function (1) as in the first-period contest. The contest winner (i.e., the player with the higher score) earns income $Y_M = \alpha \cdot Y$ with $\alpha > 1$, and the loser gets zero.

The loser of the first-period contest is assigned to a consolation contest C. Here, he meets an opponent of ability a_H with probability $p_C \in (0, 1)$ and an opponent of ability a_L with probability $1 - p_C$. Analogously to the major contest, individual abilities become common knowledge before both players exert efforts, each of the two players has the same score function (1), and the player with the higher score is declared contest winner. This player gets income $Y_C = \beta \cdot Y$ with $\beta < \alpha$, whereas the loser of contest C earns zero.

Finally, let all players have zero reservation values in both periods, which guarantees participation in each contest. The game is solved by backward induction, starting with the second-period contests M and C. The main concern is with the players' equilibrium efforts (e_H^*, e_L^*) in the first-period contest. Since without the existence of second-period contests both players would exert identical efforts in period one,[5] the equilibrium (e_H^*, e_L^*) will be called a sandbagging equilibrium, if the high-ability player exerts less effort than his low-ability opponent (i.e., $e_H^* < e_L^*$).

3 Solution to the Game

In the second period, two players (the first-round winner and a randomly matched opponent) meet in the major contest and two players (the first-round loser and a randomly matched opponent) have to participate in the consolation match. Consider either of these two contests $\kappa \in \{C, M\}$ with participants i and j having talents a_i and a_j $(i, j \in \{H, L\})$ competing for prize $Y_\kappa \in \{Y_C, Y_M\}$ by choosing efforts e_i and e_j, respectively. Let EU_{ij}^κ (EU_{ji}^κ) denote the second-round expected

[5] This claim can be immediately checked by the solution of the second-period contests, because the game ends after these contests.

utility of player i (j) from competing against opponent j (i) in contest $\kappa \in \{C, M\}$:

$$\begin{aligned} EU_{ij}^{\kappa} &= Y_{\kappa} \cdot G\left(h\left(e_i\right) + a_i - h\left(e_j\right) - a_j\right) - c\left(e_i\right) \\ \text{and} \quad EU_{ji}^{\kappa} &= Y_{\kappa} \cdot \left[1 - G\left(h\left(e_i\right) + a_i - h\left(e_j\right) - a_j\right)\right] - c\left(e_j\right). \end{aligned}$$

I assume that an equilibrium in pure strategies exists and that it is characterized by the first-order conditions[6]

$$Y_{\kappa} g\left(h\left(e_i\right) + a_i - h\left(e_j\right) - a_j\right) = \frac{c'\left(e_i\right)}{h'\left(e_i\right)} = \frac{c'\left(e_j\right)}{h'\left(e_j\right)}.$$

Thus, if a pure-strategy equilibrium exists, it will be unique and symmetric with both players choosing effort level $\Psi\left(Y_{\kappa} g\left(a_i - a_j\right)\right)$ where Ψ denotes the (monotonically increasing) inverse of c'/h'. Note that, in equilibrium, a player has effort costs $c\left(\Psi\left(Y_{\kappa} g\left(0\right)\right)\right)$ when competing in a homogeneous match and $c\left(\Psi\left(Y_{\kappa} g\left(\Delta a\right)\right)\right)$ when competing in a heterogeneous match. Let

$$\Delta C_{\kappa} := c\left(\Psi\left(Y_{\kappa} g\left(0\right)\right)\right) - c\left(\Psi\left(Y_{\kappa} g\left(\Delta a\right)\right)\right) > 0$$

denote a player's additional equilibrium effort costs in contest κ from being matched with an equally strong opponent instead of a weaker or stronger opponent, where $\Delta C_{\kappa} > 0$ follows from the unimodality of g.

In the first-period contest, both players H and L anticipate the possible outcomes in the second period when choosing optimal efforts. Let $EU_{ij}^{\kappa *}$ denote the equilibrium expected utility of player i from competing against player j in second-period contest κ. Then, player H maximizes

$$\begin{aligned} &\left(Y + p_M \cdot EU_{HH}^{M*} + \left(1 - p_M\right) \cdot EU_{HL}^{M*}\right) \cdot G\left(h\left(e_H\right) + \Delta a - h\left(e_L\right)\right) \\ &+ \left(p_C \cdot EU_{HH}^{C*} + \left(1 - p_C\right) \cdot EU_{HL}^{C*}\right) \cdot \left[1 - G\left(h\left(e_H\right) + \Delta a - h\left(e_L\right)\right)\right] - c\left(e_H\right) \end{aligned}$$

[6]The problem that the existence of pure-strategy equilibria cannot be guaranteed in general is well-known in the contest literature; see, e.g., Lazear and Rosen (1981), p. 845, and Nalebuff and Stiglitz (1983), p. 29. Instead of using a certain specification for f, c and g so that an existence condition can be fixed, I follow the usual procedure by assuming existence of pure-strategy equilibria and then describing their characteristics.

and player L

$$\left(Y + p_M \cdot EU_{LH}^{M*} + (1 - p_M) \cdot EU_{LL}^{M*}\right) \cdot \left[1 - G\left(h\left(e_H\right) + \Delta a - h\left(e_L\right)\right)\right]$$
$$+ \left(p_C \cdot EU_{LH}^{C*} + (1 - p_C) \cdot EU_{LL}^{C*}\right) \cdot G\left(h\left(e_H\right) + \Delta a - h\left(e_L\right)\right) - c\left(e_L\right),$$

yielding the following result:

Proposition 1 *In the first-period contest, a pure-strategy equilibrium will be a sandbagging equilibrium* $\left(e_H^*, e_L^*\right)$ *with* $e_H^* < e_L^*$*, if and only if*

$$(1 - 2p_C)\,\Delta C_C + (2p_M - 1)\,\Delta C_M > \left(G\left(\Delta a\right) - \frac{1}{2}\right)(\alpha - \beta)\,Y. \tag{2}$$

Proof. See Appendix. ■

The right-hand side of condition (2) is strictly positive. It describes player H's *worst-case* opportunity costs from sandbagging. If sandbagging is successful and player H is assigned to contest C instead of contest M, the worst thing that can happen is to meet another a_H-player in contest C whereas player L is matched with another a_L-player in contest M. Now, player H would deeply regret not to be assigned to contest M, because switching to contest M in this situation would increase his winning probability by $G\left(\Delta a\right) - \frac{1}{2}$ and the winner prize by $(\alpha - \beta)\,Y$.

The left-hand side of condition (2) may be positive or negative. It is negative if $p_C > 1/2$ and $p_M < 1/2$, which makes (2) impossible to be satisfied. The intuition for this finding is clear, because the exclusive aim of sandbagging is to avoid another a_H-player. If, on the contrary, $p_C = 0$ and $p_M = 1$ then sandbagging can be quite attractive for H, since he would avoid (meet) another a_H-player for sure by entering contest C (contest M). In that case, player H realizes gains from sandbagging in form of saved effort costs – he directly realizes a relative cost advantage ΔC_C from being matched with an a_L-player in contest C and indirectly gains cost savings ΔC_M from not being matched with an a_H-player in contest M.

Interestingly, the winner prize of the first-period contest does not play a role for the sandbagging condition (2). At first sight, this observation seems puzzling, because not winning the first-period contest due to sandbagging is directly associated with the loss of this winner prize. However, the proof of Proposition 1 shows that the winner prize of the first-period contest influences the effort choices of H and L in the same way so that this influence cancels out when comparing e_H^* and e_L^*.

Condition (2) shows that a sufficiently high winner prize in the major contest (i.e., a large value of α) does not necessarily work against sandbagging. On the one hand, a high Y_M makes participation in the major contest quite attractive for player H: the right-hand side of (2) increases in α, which works against sandbagging. On the other hand, α also influences ΔC_M on the left-hand side of (2) so that the overall effect of α is not clear. The influence of the ability difference Δa is ambiguous, too. The higher Δa the larger are the expected opportunity costs of sandbagging (via $G(\Delta a)$). However, saved effort costs from avoiding a match with another a_H-player also increase in Δa (i.e., $\partial \Delta C_\kappa / \partial \Delta a > 0$, $\kappa = C, M$, by the unimodality of g).

We obtain clear-cut results for the influence of the probabilities p_C and p_M, as highlighted above. The impact of the shape of the cost function c on the left-hand side of (2) is twofold. On the one hand, the steeper the cost function, the larger are the cost savings from sandbagging, ΔC_κ ($\kappa = C, M$), for *given* equilibrium efforts. On the other hand, the steeper the cost function the flatter is Ψ, and hence the smaller are the equilibrium efforts in each match. If equilibrium effort levels are very small anyway, the cost-saving advantage of sandbagging will be negligible. For a wide range of convex cost functions, this second effect dominates the first effect. Let, for simplicity, $h(e_i) = e_i$ and c belong to the family of cost functions $c(e_i) = \hat{c} \cdot e_i^m / m$ with $\hat{c} > 0$ and $m \geq 2$. Then, according to (2), there exists a cut-off value so that a sandbagging equilibrium exists if and only if $\hat{c}$ is smaller than this cut-off:

$$\hat{c} < \frac{\left(g(0)^{\frac{m}{m-1}} - g(\Delta a)^{\frac{m}{m-1}}\right)^{m-1} \left[(1-2p_C)\,\beta^{\frac{m}{m-1}} + (2p_M - 1)\,\alpha^{\frac{m}{m-1}}\right]^{m-1} Y}{\left[m(\alpha - \beta)\left(G(\Delta a) - \frac{1}{2}\right)\right]^{m-1}}.$$

Intuitively, only if the cost function is not too steep, equilibrium effort levels will sufficiently differ in homogeneous and heterogeneous matches, and the cost-saving advantage for player H from avoiding another high-ability player via sandbagging can be sufficiently large.

4 Endogenous Matching

The comparative static results have shown that a contest designer has several possibilities to influence sandbagging behavior. Following condition (2), the most successful way to prevent sandbagging seems to be an appropriate choice of p_C and p_M. For example, the contest designer could create maximal uncertainty about the opponents' types in contests C and M (i.e., condition (2) cannot be satisfied if $p_C = p_M = 1/2$). However, such policy is not possible in situations with *two* simultaneous contests in period one combined with the natural rule that the first-period winners are both assigned to the major contest M, whereas the two first-period losers are matched together in the consolation contest C.[7] In that case, matching is generated endogenously, which can lead back to the problem of sandbagging.

Consider such situation with two heterogeneous first-period contests A and B between one a_H-player and one a_L-player, respectively. Let e_{ij} denote the effort choice of the a_i-player in contest j ($i = H, L$; $j = A, B$) and $\bar{G}_j := G(h(e_{Hj}) + \Delta a - h(e_{Lj}))$ the winning probability of the a_H-player in contest j. The objective function of the a_H-player in contest k ($k = A, B$; $k \neq j$) can be written as

$$\begin{aligned}&\left(Y + \bar{G}_j \cdot EU_{HH}^{M*} + \left(1 - \bar{G}_j\right) \cdot EU_{HL}^{M*}\right) \cdot G\left(h\left(e_{Hk}\right) + \Delta a - h\left(e_{Lk}\right)\right) \\ &+ \left(\left(1 - \bar{G}_j\right) \cdot EU_{HH}^{C*} + \bar{G}_j \cdot EU_{HL}^{C*}\right) \cdot \left[1 - G\left(h\left(e_{Hk}\right) + \Delta a - h\left(e_{Lk}\right)\right)\right] - c\left(e_{Hk}\right)\end{aligned}$$

whereas the a_L-player in contest k maximizes

$$\begin{aligned}&\left(Y + \bar{G}_j \cdot EU_{LH}^{M*} + \left(1 - \bar{G}_j\right) \cdot EU_{LL}^{M*}\right) \cdot \left[1 - G\left(h\left(e_{Hk}\right) + \Delta a - h\left(e_{Lk}\right)\right)\right] \\ &+ \left(\left(1 - \bar{G}_j\right) \cdot EU_{LH}^{C*} + \bar{G}_j \cdot EU_{LL}^{C*}\right) \cdot G\left(h\left(e_{Hk}\right) + \Delta a - h\left(e_{Lk}\right)\right) - c\left(e_{Lk}\right).\end{aligned}$$

[7] This scenario captures the career-contest example from the introduction as well as the sports cases where a sandbagger wants to decrease his average rating to start in the next contest in a lower class.

The two first-order conditions read as follows:

$$\left(Y + \left[EU_{HH}^{M*} - EU_{HL}^{C*}\right]\bar{G}_j + \left[EU_{HL}^{M*} - EU_{HH}^{C*}\right]\left(1 - \bar{G}_j\right)\right)\cdot \bar{g}_k = \frac{c'\left(e_{Hk}\right)}{h'\left(e_{Hk}\right)}$$

$$\left(Y + \left[EU_{LH}^{M*} - EU_{LL}^{C*}\right]\bar{G}_j + \left[EU_{LL}^{M*} - EU_{LH}^{C*}\right]\left(1 - \bar{G}_j\right)\right)\cdot \bar{g}_k = \frac{c'\left(e_{Lk}\right)}{h'\left(e_{Lk}\right)}$$

with $\bar{g}_k := g\left(h\left(e_{Hk}\right) + \Delta a - h\left(e_{Lk}\right)\right)$. As c'/h' is a monotonically increasing function, in a pure-strategy equilibrium we will have $e_{Hk} < e_{Lk}$ if and only if

$$Y + \left[EU_{HH}^{M*} - EU_{HL}^{C*}\right]\bar{G}_j + \left[EU_{HL}^{M*} - EU_{HH}^{C*}\right]\left(1 - \bar{G}_j\right) <$$
$$Y + \left[EU_{LH}^{M*} - EU_{LL}^{C*}\right]\bar{G}_j + \left[EU_{LL}^{M*} - EU_{LH}^{C*}\right]\left(1 - \bar{G}_j\right),$$

which – by inserting for the second-period expected utilities from the proof of Proposition 1 – can be rewritten, so that we obtain the following result:

Proposition 2 *A pure-strategy equilibrium in contest k ($k = A, B$) will exhibit sandbagging by the a_H-player (i.e., $e_{Hk}^* < e_{Lk}^*$) if and only if*

$$\bar{G}_j > \frac{1}{2} + \frac{\left[G\left(\Delta a\right) - \frac{1}{2}\right]\left(\alpha - \beta\right)Y}{2\left(\Delta C_M + \Delta C_C\right)} \qquad (j = A, B;\ j \neq k). \tag{3}$$

Proposition 2 shows that an extreme sandbagging result $h\left(e_{LA}^*\right) > h\left(e_{HA}^*\right) + \Delta a$ in combination with $h\left(e_{LB}^*\right) > h\left(e_{HB}^*\right) + \Delta a$ is impossible. Suppose that $\bar{G}_j < \frac{1}{2}$, i.e., $h\left(e_{Lj}\right) > h\left(e_{Hj}\right) + \Delta a$. In this situation, the a_l-player is more likely to win than the a_h-player in contest j, but the same cannot be true in contest k, because here the a_h-player already leads by Δa and, in addition, chooses more effort than his opponent according to (3). Thus, it is impossible that in both first-period contests the two a_l-players have higher winning probabilities than their respective opponents. Intuitively, if an a_h-player anticipates that in the other contest the less able player has a higher winning probability than the more able one, for three reasons he prefers to enter the major contest and, thus, to win the first round. First, by winning the first-period contest the a_h-player would earn Y, whereas losing is associated with zero first-period income. Second, the a_h-player wants to avoid being matched with the other a_h-player in the next round, which would imply high effort costs, due to the homogeneous competition, and a rather low winning probability. Third, in the major contest he can win αY but in the consolation contest only $\beta Y < \alpha Y$.

The proposition also points to a non-trivial coordination problem of the players in period one. Condition (3) shows that if in contest j ($j = A, B$) the high-ability player has a high winning probability, it will be quite attractive for the high-ability player in the other contest $k \neq j$ to sandbag. Inequality (3) is more easily satisfied if, for given $\bar{G}_j$, the expression $\left[G(\Delta a) - \frac{1}{2}\right](\alpha - \beta) Y$ takes small values and $\Delta C_M + \Delta C_C$ large values.[8] If, for example, $Y > 0$ but $\alpha \to \beta$ both a_H-players will face a serious coordination problem since winning either second-period contest is equally attractive. Now a major aim of the a_H-players is to avoid meeting one another in period two. The coordination problem is comparable to that of the battle-of-the sexes game since achieving a heterogeneous match in contest M and directly earning first-period income Y is more preferable for an a_H-player than being matched with an a_L-player in contest C. Note that Proposition 2 does not rule out the possibility that even *both* high-ability players sandbag in period one. In particular, if the right-hand side of condition (3) is close to 1/2, the condition can be satisfied for $e^*_{Hj} < e^*_{Lj}$.

However, even if parameterized functions are used to specify h, c, and G, the complicated structure of the two simultaneous first-period contests does not allow for an exact computation of the equilibria. Note also that, aside from the a_H-players, an a_L-player may be interested as well to be matched with a weak a_L-player instead of a strong a_H-player in period two. For these reasons, I refer to the tractable case of binary efforts to check the robustness of the previous conjectures. I do not consider all $2^4 = 16$ effort combinations but focus on the two possibilities of sandbagging equilibria sketched in the paragraph before:[9]

Proposition 3 *Suppose that, in the first-period contests, players can only decide between* e_1 *and* e_0 *with* $h(e_1) := h_1 > 0 =: h(e_0)$ *and* $c(e_1) := c_1 > 0 =: c(e_0)$.

[8] The intuition for this finding is analogous to the intuition for the finding of Proposition 1.
[9] The proof is relegated to the "Additional material for the referees".

(a) If

$$G\left(-h_1+\Delta a\right)\leq\frac{1}{2}+\frac{\left[G\left(\Delta a\right)-\frac{1}{2}\right]\left(\alpha-\beta\right)Y}{2\left(\Delta C_M+\Delta C_C\right)}\leq G\left(h_1+\Delta a\right),$$

there exist feasible parameter constellations leading to multiple sandbagging equilibria $(e_{HA}^*,e_{LA}^*,e_{HB}^*,e_{LB}^*)=(e_0,e_1,e_1,e_0)$ *and* $(e_{HA}^*,e_{LA}^*,e_{HB}^*,e_{LB}^*)=(e_1,e_0,e_0,e_1)$.
(b) If

$$G\left(-h_1+\Delta a\right)\geq\frac{1}{2}+\frac{\left[G\left(\Delta a\right)-\frac{1}{2}\right]\left(\alpha-\beta\right)Y}{2\left(\Delta C_C+\Delta C_M\right)},$$

there exist feasible values of c_1 *so that both* a_H*-players sandbag in equilibrium (i.e.,* $(e_{HA}^*,e_{LA}^*,e_{HB}^*,e_{LB}^*)=(e_0,e_1,e_0,e_1)$*).*

The proposition confirms that indeed there exist feasible parameter values leading to a coordination problem of the a_H-players in period one. In these cases, the impact of higher effort (i.e., h_1) must be sufficiently large so that the players can effectively influence the outcome of the first-period contests. If, however, the impact of higher effort is dominated by player heterogeneity (i.e., $\Delta a > h_1$), even both a_H-players may prefer to sandbag and rest on their lead Δa, while both a_L-players choose higher effort to assure themselves a sufficiently high winning probability.

Appendix: Proof of Proposition 1

Computing the players' first-order conditions gives

$$\begin{aligned}&\left[\left(Y+p_M EU_{HH}^{M*}+\left(1-p_M\right)EU_{HL}^{M*}\right)\right.\\&\qquad\left.-\left(p_C EU_{HH}^{C*}+\left(1-p_C\right)EU_{HL}^{C*}\right)\right]h'\left(e_H\right)g\left(h\left(e_H\right)+\Delta a-h\left(e_L\right)\right)\\&=c'\left(e_H\right)\end{aligned}$$

and

$$\begin{aligned}&\left[\left(Y+p_M EU_{LH}^{M*}+\left(1-p_M\right)EU_{LL}^{M*}\right)\right.\\&\qquad\left.-\left(p_C EU_{LH}^{C*}+\left(1-p_C\right)EU_{LL}^{C*}\right)\right]h'\left(e_L\right)g\left(h\left(e_H\right)+\Delta a-h\left(e_L\right)\right)\\&=c'\left(e_L\right).\end{aligned}$$

Thus, both players have the same marginal costs of exerting effort, but potentially different marginal returns, as indicated by the expressions in square brackets. Comparing these expressions shows that player H will choose less effort than player L in the first-period contest, if

$$\begin{aligned}&\left(Y + p_M EU_{HH}^{M*} + (1 - p_M)\, EU_{HL}^{M*}\right) - \left(p_C EU_{HH}^{C*} + (1 - p_C)\, EU_{HL}^{C*}\right) < \\ &\left(Y + p_M EU_{LH}^{M*} + (1 - p_M)\, EU_{LL}^{M*}\right) - \left(p_C EU_{LH}^{C*} + (1 - p_C)\, EU_{LL}^{C*}\right).\end{aligned}$$

Inserting for

$$\begin{aligned}EU_{HH}^{M*} &= EU_{LL}^{M*} = \frac{\alpha Y}{2} - c\left(\Psi\left(\alpha Y g\left(0\right)\right)\right)\\ EU_{HL}^{M*} &= \alpha Y \cdot G\left(\Delta a\right) - c\left(\Psi\left(\alpha Y g\left(\Delta a\right)\right)\right)\\ EU_{HH}^{C*} &= EU_{LL}^{C*} = \frac{\beta Y}{2} - c\left(\Psi\left(\beta Y g\left(0\right)\right)\right)\\ EU_{HL}^{C*} &= \beta Y \cdot G\left(\Delta a\right) - c\left(\Psi\left(\beta Y g\left(\Delta a\right)\right)\right)\\ EU_{LH}^{M*} &= \alpha Y \cdot \left[1 - G\left(\Delta a\right)\right] - c\left(\Psi\left(\alpha Y g\left(\Delta a\right)\right)\right)\\ EU_{LH}^{C*} &= \beta Y \cdot \left[1 - G\left(\Delta a\right)\right] - c\left(\Psi\left(\beta Y g\left(\Delta a\right)\right)\right)\end{aligned}$$

and rewriting yields condition (2).

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Additional material for the referees:

(a) Multiple sandbagging equilibria:

Note that the same conditions for $(e_{HA}, e_{LA}, e_{HB}, e_{LB}) = (e_0, e_1, e_1, e_0)$ to be an equilibrium must also hold for an equilibrium $(e_{HA}, e_{LA}, e_{HB}, e_{LB}) = (e_1, e_0, e_0, e_1)$. Therefore, we have to consider only one of them. The expected utility of the a_H-player in contest A under $(e_{HA}, e_{LA}, e_{HB}, e_{LB}) = (e_0, e_1, e_1, e_0)$ is given by

$$\begin{aligned}
EU_{HA}(e_0) &= \left(Y + EU_{HH}^{M*} G(h_1 + \Delta a) + EU_{HL}^{M*}\left[1 - G(h_1 + \Delta a)\right]\right) G(-h_1 + \Delta a) \\
&\quad + \left(EU_{HH}^{C*}\left[1 - G(h_1 + \Delta a)\right] + EU_{HL}^{C*} G(h_1 + \Delta a)\right)\left[1 - G(-h_1 + \Delta a)\right] \\
&= \left(Y + EU_{HL}^{M*} - \left(EU_{HL}^{M*} - EU_{HH}^{M*}\right) G(h_1 + \Delta a)\right) G(-h_1 + \Delta a) \\
&\quad + \left(EU_{HH}^{C*} + \left(EU_{HL}^{C*} - EU_{HH}^{C*}\right) G(h_1 + \Delta a)\right)\left[1 - G(-h_1 + \Delta a)\right] \\
&= [Y + EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{HL}^{\kappa*} - EU_{HH}^{\kappa*}\right) G(h_1 + \Delta a)] G(-h_1 + \Delta a) \\
&\quad + EU_{HH}^{C*} + \left(EU_{HL}^{C*} - EU_{HH}^{C*}\right) G(h_1 + \Delta a).
\end{aligned}$$

If this player deviates to $e_{HA} = e_1$ his payoff switches to

$$\begin{aligned}
EU_{HA}(e_1) &= [Y + EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{HL}^{\kappa*} - EU_{HH}^{\kappa*}\right) G(h_1 + \Delta a)] G(\Delta a) \\
&\quad + EU_{HH}^{C*} + \left(EU_{HL}^{C*} - EU_{HH}^{C*}\right) G(h_1 + \Delta a) - c_1.
\end{aligned}$$

Hence, he will not deviate if

$$c_1 \geq \Omega_{HA} \cdot \left[G(\Delta a) - G(-h_1 + \Delta a)\right] \tag{4}$$

$$\text{with } \Omega_{HA} := [Y + EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{HL}^{\kappa*} - EU_{HH}^{\kappa*}\right) G(h_1 + \Delta a)].$$

Under $(e_{HA}, e_{LA}, e_{HB}, e_{LB}) = (e_0, e_1, e_1, e_0)$ the a_L-player in contest A obtains

$$\begin{aligned}
EU_{LA}(e_1) &= [Y + EU_{LH}^{M*} G(h_1 + \Delta a) \\
&\qquad + EU_{LL}^{M*}\left(1 - G(h_1 + \Delta a)\right)]\left(1 - G(-h_1 + \Delta a)\right) \\
&\quad + \left(EU_{LH}^{C*}\left(1 - G(h_1 + \Delta a)\right) + EU_{LL}^{C*} G(h_1 + \Delta a)\right) G(-h_1 + \Delta a) - c_1
\end{aligned}$$

$$= \quad Y + EU_{LL}^{M*} - \left(EU_{LL}^{M*} - EU_{LH}^{M*}\right) G\left(h_1 + \Delta a\right) - c_1$$
$$+[EU_{LH}^{C*} - EU_{LL}^{M*} - Y + \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}\right) G\left(h_1 + \Delta a\right)]G\left(-h_1 + \Delta a\right).$$

Deviation to $e_{LA} = e_0$ leads to

$$EU_{LA}\left(e_0\right) = Y + EU_{LL}^{M*} - \left(EU_{LL}^{M*} - EU_{LH}^{M*}\right) G\left(h_1 + \Delta a\right)$$
$$+[EU_{LH}^{C*} - EU_{LL}^{M*} - Y + \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}\right) G\left(h_1 + \Delta a\right)]G\left(\Delta a\right).$$

The a_L-player will not deviate if

$$c_1 \leq \Omega_{LA} \cdot \left[G\left(\Delta a\right) - G\left(-h_1 + \Delta a\right)\right] \tag{5}$$
$$\text{with } \Omega_{LA} := [Y + EU_{LL}^{M*} - EU_{LH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}\right) G\left(h_1 + \Delta a\right)].$$

Conditions (4) and (5) show that for an equilibrium (e_0, e_1, e_1, e_0) we must have

$$\Omega_{HA} \cdot \left[G\left(\Delta a\right) - G\left(-h_1 + \Delta a\right)\right] \leq c_1 \leq \Omega_{LA} \cdot \left[G\left(\Delta a\right) - G\left(-h_1 + \Delta a\right)\right]. \tag{6}$$

A non-empty interval for feasible values of c_1 is guaranteed, if

$$\Omega_{HA} < \Omega_{LA} \Leftrightarrow$$
$$EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{HL}^{\kappa *} - EU_{HH}^{\kappa *}\right) G\left(h_1 + \Delta a\right) \leq$$
$$EU_{LL}^{M*} - EU_{LH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}\right) G\left(h_1 + \Delta a\right). \tag{7}$$

Inserting for the second-period expected utilities from the proof of Proposition 1 yields

$$G\left(h_1 + \Delta a\right) \geq \frac{1}{2} + \frac{\left[G\left(\Delta a\right) - \frac{1}{2}\right]\left(\alpha - \beta\right) Y}{2\left(\Delta C_C + \Delta C_M\right)}. \tag{8}$$

Next, we have to ensure that the players in contest B do not want to deviate from $(e_{HA}, e_{LA}, e_{HB}, e_{LB}) = (e_0, e_1, e_1, e_0)$ either. The payoff of the a_H-player in contest B under (e_0, e_1, e_1, e_0) is given by

$$EU_{HB}(e_1) = \left(Y + EU_{HH}^{M*} G(-h_1 + \Delta a) + EU_{HL}^{M*}(1 - G(-h_1 + \Delta a))\right) G(h_1 + \Delta a)$$
$$+ \left(EU_{HH}^{C*}(1 - G(-h_1 + \Delta a)) + EU_{HL}^{C*} G(-h_1 + \Delta a)\right)(1 - G(h_1 + \Delta a)) - c_1$$

$$= [Y + EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} (EU_{HL}^{\kappa*} - EU_{HH}^{\kappa*}) G(-h_1 + \Delta a)] G(h_1 + \Delta a)$$
$$+ \left[EU_{HH}^{C*}(1 - G(-h_1 + \Delta a)) + EU_{HL}^{C*} G(-h_1 + \Delta a)\right] - c_1.$$

If the player deviates he will realize

$$EU_{HB}(e_0) = [Y + EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} (EU_{HL}^{\kappa*} - EU_{HH}^{\kappa*}) G(-h_1 + \Delta a)] G(\Delta a)$$
$$+ \left[EU_{HH}^{C*}(1 - G(-h_1 + \Delta a)) + EU_{HL}^{C*} G(-h_1 + \Delta a)\right].$$

He will not deviate if

$$c_1 \leq \Omega_{HB} \cdot [G(h_1 + \Delta a) - G(\Delta a)] \tag{9}$$
$$\text{with } \Omega_{HB} := [Y + EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} (EU_{HL}^{\kappa*} - EU_{HH}^{\kappa*}) G(-h_1 + \Delta a)].$$

The expected utility of the a_L-player in contest B under $(e_{HA}, e_{LA}, e_{HB}, e_{LB}) = (e_0, e_1, e_1, e_0)$ is described by

$$EU_{LB}(e_0) = Y + EU_{LH}^{M*} G(-h_1 + \Delta a) + EU_{LL}^{M*}(1 - G(-h_1 + \Delta a))$$
$$+ [EU_{LH}^{C*} - EU_{LL}^{M*} - Y + \sum_{\kappa \in \{M,C\}} (EU_{LL}^{\kappa*} - EU_{LH}^{\kappa*}) G(-h_1 + \Delta a)] G(h_1 + \Delta a).$$

If the player deviates to $e_{LB} = e_1$ he obtains

$$EU_{LB}(e_1) = Y + EU_{LH}^{M*} G(-h_1 + \Delta a) + EU_{LL}^{M*}(1 - G(-h_1 + \Delta a)) - c_1$$
$$+ [EU_{LH}^{C*} - EU_{LL}^{M*} - Y + \sum_{\kappa \in \{M,C\}} (EU_{LL}^{\kappa*} - EU_{LH}^{\kappa*}) G(-h_1 + \Delta a)] G(\Delta a).$$

Deviation is not profitable if

$$c_1 \geq \Omega_{LB} \cdot [G(h_1 + \Delta a) - G(\Delta a)] \tag{10}$$
$$\text{with } \Omega_{LB} := [Y + EU_{LL}^{M*} - EU_{LH}^{C*} - \sum_{\kappa \in \{M,C\}} (EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}) G(-h_1 + \Delta a)].$$

According to conditions (9) and (10), an equilibrium $(e_{HA}^*, e_{LA}^*, e_{HB}^*, e_{LB}^*) = (e_0, e_1, e_1, e_0)$ requires

$$\Omega_{LB} \cdot [G(h_1 + \Delta a) - G(\Delta a)] \leq c_1 \leq \Omega_{HB} \cdot [G(h_1 + \Delta a) - G(\Delta a)]. \tag{11}$$

Thus, we must have that

$$\begin{aligned} & \Omega_{LB} < \Omega_{HB} \Leftrightarrow \\ & EU_{LL}^{M*} - EU_{LH}^{C*} - \sum_{\kappa \in \{M,C\}} (EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}) G(-h_1 + \Delta a) \leq \\ & EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} (EU_{HL}^{\kappa *} - EU_{HH}^{\kappa *}) G(-h_1 + \Delta a). \end{aligned} \tag{12}$$

Comparison with (7) immediately shows that (12) is equivalent to

$$G(-h_1 + \Delta a) \leq \frac{1}{2} + \frac{\left[G(\Delta a) - \frac{1}{2}\right](\alpha - \beta) Y}{2(\Delta C_C + \Delta C_M)}. \tag{13}$$

Combining (8) with (13) gives

$$G(-h_1 + \Delta a) \leq \frac{1}{2} + \frac{\left[G(\Delta a) - \frac{1}{2}\right](\alpha - \beta) Y}{2(\Delta C_M + \Delta C_C)} \leq G(h_1 + \Delta a), \tag{14}$$

which is satisfied as long as $\left[G(\Delta a) - \frac{1}{2}\right](\alpha - \beta) Y < \Delta C_M + \Delta C_C$ and h_1 is sufficiently large.

Recall that (14) only guarantees that $\Omega_{HA} < \Omega_{LA}$ and $\Omega_{LB} < \Omega_{HB}$. However, we must still ensure that conditions (6) and (11) hold for the same values of c_1. Thus, the intervals $(\Omega_{HA}[G(\Delta a) - G(-h_1 + \Delta a)], \Omega_{LA}[G(\Delta a) - G(-h_1 + \Delta a)])$ and $(\Omega_{LB}[G(h_1 + \Delta a) - G(\Delta a)], \Omega_{HB}[G(h_1 + \Delta a) - G(\Delta a)])$ must overlap. Since

$\Omega_{HA} < \Omega_{HB}$ and $\Omega_{LA} < \Omega_{LB}$ – implying $\Omega_{HA} < \Omega_{LA} < \Omega_{LB} < \Omega_{HB}$ – the probability mass $G(\Delta a) - G(-h_1 + \Delta a)$ must be sufficiently large and $G(h_1 + \Delta a) - G(\Delta a)$ sufficiently small to make the intervals overlap, i.e.,

$$\begin{aligned}
&\Omega_{LA}[G(\Delta a) - G(-h_1 + \Delta a)] > \Omega_{LB}[G(h_1 + \Delta a) - G(\Delta a)] \Leftrightarrow \\
&\left(Y + EU_{LL}^{M*} - EU_{LH}^{C*}\right)([G(\Delta a) - G(-h_1 + \Delta a)] - [G(h_1 + \Delta a) - G(\Delta a)]) \\
&\qquad > \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}\right) \times \qquad (15) \\
&\{G(h_1 + \Delta a)[G(\Delta a) - G(-h_1 + \Delta a)] - G(-h_1 + \Delta a)[G(h_1 + \Delta a) - G(\Delta a)]\}.
\end{aligned}$$

There are several ways to construct parameter constellations so that this condition holds without violating (14). Suppose, for example, that G has a bounded support and Δa is near the upper bound so that $G(\Delta a) \approx 1$ and $g(\Delta a) \approx 0$ (recall that g has a unique mode at zero).[10] Then, condition (15) boils down to

$$\begin{aligned}
&\left(Y + EU_{LL}^{M*} - EU_{LH}^{C*}\right)([1 - G(-h_1 + \Delta a)]) \\
&> \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}\right)\{[1 - G(-h_1 + \Delta a)]\}.
\end{aligned}$$

Inserting for

$$\begin{aligned}
\left(Y + EU_{LL}^{M*} - EU_{LH}^{C*}\right) &= \left(1 + \frac{\alpha}{2} - \beta[1 - G(\Delta a)]\right) Y \\
&\quad - [c(\Psi(\alpha Y g(0))) - c(\Psi(\beta Y g(\Delta a)))] \\
&\approx \left(1 + \frac{\alpha}{2}\right) Y - c(\Psi(\alpha Y g(0)))
\end{aligned}$$

and

$$\begin{aligned}
\sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa *} - EU_{LH}^{\kappa *}\right) &= \left(G(\Delta a) - \frac{1}{2}\right)(\alpha + \beta) Y - (\Delta C_M + \Delta C_C) \\
&\approx \frac{(\alpha + \beta) Y}{2} - [c(\Psi(\alpha Y g(0))) + c(\Psi(\beta Y g(0)))]
\end{aligned}$$

[10]Note also that condition (14) becomes $G(-h_1 + \Delta a) \leq \frac{1}{2} + \frac{(\alpha - \beta) Y}{4(\Delta C_M + \Delta C_C)} \leq 1$, which can still be satisfied.

yields

$$\frac{(2-\beta)\,Y}{2} > -c\left(\Psi\left(\beta Y g\left(0\right)\right)\right),$$

which is clearly satisfied for $\beta < 2$. To sum up, there exist feasible parameter constellations for which multiple equilibria $(e^*_{HA}, e^*_{LA}, e^*_{HB}, e^*_{LB}) = (e_0, e_1, e_1, e_0)$ and $(e^*_{HA}, e^*_{LA}, e^*_{HB}, e^*_{LB}) = (e_1, e_0, e_0, e_1)$ exist.

(b) Both high-ability players sandbag:

The expected utility of either a_H-player in the two first-period contests under $(e_{HA}, e_{LA}, e_{HB}, e_{LB}) = (e_0, e_1, e_0, e_1)$ is given by

$$\begin{aligned} EU_H\left(e_0\right) = {} & [Y + EU^{M*}_{HL} - EU^{C*}_{HH} - \sum_{\kappa\in\{M,C\}} \left(EU^{\kappa*}_{HL} - EU^{\kappa*}_{HH}\right) G\left(-h_1+\Delta a\right)]G\left(-h_1+\Delta a\right) \\ & + EU^{C*}_{HH} + \left(EU^{C*}_{HL} - EU^{C*}_{HH}\right) G\left(-h_1+\Delta a\right). \end{aligned}$$

If an a_H-player deviates to e_1, he will obtain

$$\begin{aligned} EU_H\left(e_1\right) = {} & [Y + EU^{M*}_{HL} - EU^{C*}_{HH} - \sum_{\kappa\in\{M,C\}} \left(EU^{\kappa*}_{HL} - EU^{\kappa*}_{HH}\right) G\left(-h_1+\Delta a\right)]G\left(\Delta a\right) \\ & + EU^{C*}_{HH} + \left(EU^{C*}_{HL} - EU^{C*}_{HH}\right) G\left(-h_1+\Delta a\right) - c_1. \end{aligned}$$

Thus, deviation does not pay if

$$\begin{aligned} c_1 \geq {} & [Y + EU^{M*}_{HL} - EU^{C*}_{HH} - \sum_{\kappa\in\{M,C\}} \left(EU^{\kappa*}_{HL} - EU^{\kappa*}_{HH}\right) G\left(-h_1+\Delta a\right)] \\ & \cdot \left[G\left(\Delta a\right) - G\left(-h_1+\Delta a\right)\right]. \end{aligned} \tag{16}$$

The expected utility of either a_L-player in the two first-period contests under $(e_{HA}, e_{LA}, e_{HB}, e_{LB}) = (e_0, e_1, e_0, e_1)$ is described by

$$\begin{aligned} & EU_L\left(e_1\right) = Y + EU^{M*}_{LL} - \left(EU^{M*}_{LL} - EU^{M*}_{LH}\right) G\left(-h_1+\Delta a\right) - c_1 \\ & + [EU^{C*}_{LH} - EU^{M*}_{LL} - Y + \sum_{\kappa\in\{M,C\}} \left(EU^{\kappa*}_{LL} - EU^{\kappa*}_{LH}\right) G\left(-h_1+\Delta a\right)]G\left(-h_1+\Delta a\right). \end{aligned}$$

Deviation to e_0, yielding

$$EU_L(e_0) = Y + EU_{LL}^{M*} - \left(EU_{LL}^{M*} - EU_{LH}^{M*}\right) G(-h_1 + \Delta a)$$
$$+[EU_{LH}^{C*} - EU_{LL}^{M*} - Y + \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa*} - EU_{LH}^{\kappa*}\right) G(-h_1 + \Delta a)]G(\Delta a)$$

is not profitable if

$$\begin{aligned} c_1 \;\leq\; & [Y + EU_{LL}^{M*} - EU_{LH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa*} - EU_{LH}^{\kappa*}\right) G(-h_1 + \Delta a)] \\ & \cdot [G(\Delta a) - G(-h_1 + \Delta a)] . \end{aligned} \tag{17}$$

Both conditions (16) and (17) together require that[11]

$$EU_{HL}^{M*} - EU_{HH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{HL}^{\kappa*} - EU_{HH}^{\kappa*}\right) G(-h_1 + \Delta a) \leq$$
$$EU_{LL}^{M*} - EU_{LH}^{C*} - \sum_{\kappa \in \{M,C\}} \left(EU_{LL}^{\kappa*} - EU_{LH}^{\kappa*}\right) G(-h_1 + \Delta a) \Leftrightarrow$$
$$G(-h_1 + \Delta a) \geq \frac{1}{2} + \frac{\left[G(\Delta a) - \frac{1}{2}\right](\alpha - \beta) Y}{2(\Delta C_C + \Delta C_M)}.$$

[11] Note that the final inequality analogously follows from (7) and (8).