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Abstract: We investigate a duopsonistic wage-setting game in which the firms have a limited number of workplaces. We assume that the firms have heterogeneous productivity, that there are two types of workers with different reservation wages and that a worker's productivity is independent of his type. We show that equilibrium unemployment arises in the wage-setting game under certain conditions, although the efficient allocation of workers would result in full employment.

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1 Introduction

In the literature we can find various micro-theoretic models of explaining unemployment in the market, see for example, Weiss (1980), Shapiro and Stiglitz (1984), Ma and Weiss (1993), Rebitzer and Taylor (1995) among many others. These works have the common feature that they neglect the strategic interaction between wage-setting firms competing for workers. In recent papers Hamilton, Thisse and Zenou (2000), Thisse and Zenou (2000) and Wauthy and Zenou (2002) showed that unemployment may arise as an

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equilibrium of an oligopsonistic wage-setting game. Hamilton, Thisse and Zenou (2000) and Thisse and Zenou (2000) based their analysis on Salop's (1979) circular city. Wauthy and Zenou (2002) considered a duopsonistic wage-setting game in which the labour force is heterogeneous with respect to education cost and in which to work for the high-technology firm requires more education. In this paper we present another type of wage-setting game to explain unemployment. Our model may be regarded as an adaptation of Bertrand-Edgeworth's competition to the labour market.

To keep our model as simple as possible we distinguish only between two types of workers, which differ in their reservation wages. However, both types of workers have the same productivity. Thus, the workers are vertically differentiated, rather than horizontally. Moreover, there is a fixed finite number of workers of each type. We assume that the two firms are heterogeneous with respect to their productivity, but homogeneous with respect to the workers' types. In addition, the firms have a limited number of workplaces. In this market we will establish that under certain conditions equilibrium unemployment emerges.

We have to emphasise that there are situations in real markets in which the assumption that workers are equally productive but have different reservation wages is satisfied. One example would be to consider male and female workers as the two different types of workers. Supposing they have the same level of education, they can be regarded as equally productive. However, female workers may have smaller reservation wages because of possible discrimination or different opportunity cost of time. Another example would be to distinguish between native and ethnic minority workers. Again even if these two types of workers are equally productive, workers belonging to ethnic minorities may have lower reservation wages due to possible discrimination.

There is some relation between the Harris and Todaro (1970) model, which also gives us a third example satisfying our assumption of equally productive workers with different reservation wages, and the model presented in this paper. The two types of workers can be interpreted as rural and urban workers. The low-productivity firm operates in the rural area while the high-productivity firm operates in the urban area. Rural and urban workers both satisfy equally the requirements of the firms. However, urban workers have higher reservation wages, which may be caused by higher unemployment benefits or by higher costs of living. Thus, the emerging equilibrium unemployment results from the inflow of workers from the rural area into the urban area.[1] The main difference between the present model and that

[1] In order to maintain the differences in reservation wages in a dynamic context, one might think of rural workers employed in the urban area as commuters.

of Harris and Todaro (1970) lies in the wage determination process since we have endowed the firms with strategic wage-setting power.

The equilibrium of the wage-setting game predicts to us how many workers of each type will be assigned to a particular firm. In this respect our model can be regarded as an assignment model which has the following interesting feature: Unemployment in the market may exist though the workers have the same productivity (skills) and the total number of jobs is equal to the total number of workers. For an overview of assignment models in the job market we refer to Sattinger (1993).

The remainder of the paper is organized as follows. Section 2 contains the formal description of our model. Section 3 analyzes the capacity-constrained wage-setting duopsonistic game and identifies those conditions under which unemployment exists in the market. Section 4 concludes our paper. The more technical part on the mixed-strategy equilibrium is contained in the Appendix.

2 The framework

Our labour market will be very simple. There are two different types of workers denoted by α and β. We assume that workers belonging to the same type have all equal reservation wages. Let us denote these values by r_α and r_β. We shall assume that $r_\alpha < r_\beta$. Suppose that the market contains m_α and m_β workers of type α and β respectively. For simplicity we assume that there are only two firms denoted by A and B. We assume that, independently of the worker's type, a worker employed by firm A generates ρ_A and a worker employed by firm B generates ρ_B revenue. This assumption means that the firms do not care which type of worker they employ. We assume that firm B has a higher productivity, that is, $\rho_B > \rho_A$. In addition, we assume that $r_\alpha \leq \rho_A$ and $r_\beta \leq \rho_B$, which implies that both types of workers can generate a surplus at a certain firm. Suppose that the firms have a limited number of workplaces denoted by n_A and n_B. The wages set by the firms are w_A and w_B. We say that there is *unemployment* in the market, if there are workers who have reservation wages less or equal to the higher wage offer, and did not get a job in the market specified in the remainder of this section. Since we do not want to consider 'structural' unemployment, we assume that $m_\alpha = n_A$ and $m_\beta = n_B$. Under these circumstances an efficient allocation of workers would be if all α-type workers were assigned to firm A at wage r_α and all β-type workers were assigned to firm B at wage r_β.

We will consider the following wage-setting game: First, the firms make a wage offer. Next, the workers are trying to get a job with the firm making the higher wage offer if the offer exceeds or equals their reservation wages. If there are no more vacancies at the high-wage firm, the workers turn to the firm with the lower wage offer. We have to specify the strategic game describing the situation in the market. Let the firms' strategy sets be $W_A := [0, \infty)$ and $W_B := [0, \infty)$ respectively. Clearly, nobody will apply at a firm setting a wage lower than r_α. It is also obvious that a firm setting a wage greater or equal to r_β can fill all its workplaces since even if its opponent is setting a higher wage, the workers not obtaining a job with the high-wage firm will apply to the low-wage firm.

If at least one firm picks a wage from the interval $[r_\alpha, r_\beta)$ and the other firm from the interval $[0, r_\beta)$, then only α-type workers will apply. Moreover, if they set the same wage, we assume that the two firms share in expected value the α-type workers in proportion to the size of their workplaces. If firm A sets the higher wage, then it will employ all the α-type workers, while if firm B sets the higher wage, it will employ $\min \{m_\alpha, m_\beta\}$ workers.

Suppose that firm B sets a wage greater or equal to r_β and that firm A sets a wage in $[r_\alpha, r_\beta)$. We assume that the number of α-type workers employed by firm B is determined through a random sample. In particular, each worker obtains a lottery ticket and m_β tickets are drawn (without replacement) out of an urn filled with $m_\alpha + m_\beta$ tickets. This means that the number of α-type workers employed at firm B, henceforth denoted by X, has a hypergeometric distribution. Hence, the probability of hiring $k \in \{0, 1, \ldots, m_\beta\}$ α-type workers equals

$$\Pr(X = k) = \frac{\binom{m_\alpha}{k}\binom{m_\beta}{m_\beta - k}}{\binom{m_\alpha + m_\beta}{m_\beta}}.$$

It is also reasonable to assume that the number of α-type workers employed at firm B is hypergeometrically distributed if both types of workers are equally eager and able to obtain a job with the high-wage firm B. Note that we have unemployment in the market with the exception of the low probability event that $X = 0$, because β-type workers will not apply for a job with firm A. Expected unemployment EX will be $m_\beta \frac{m\alpha}{m_\alpha + m_\beta}$.

In a similar way as in the previous paragraph we can determine the expected profits of the firms for the case when firm A sets a wage greater or equal to r_β and firm B sets a wage in $[r_\alpha, r_\beta)$. We assume that the number of α-type workers employed by firm A, denoted by Y, is determined through a random sample of size m_α, where the sampling is done without replacement from an urn containing $m_\alpha + m_\beta$ workers. Then Y has also a hypergeometric distribution, i.e., the probability of hiring $k \in \{0, 1, \ldots, m_\alpha\}$ α-type workers by firm A equals

$$\Pr(Y=k)=\frac{\binom{m_\alpha}{k}\binom{m_\beta}{m_\alpha-k}}{\binom{m_\alpha+m_\beta}{m_\alpha}}.$$

Now we have unemployment in the market with the exception of the low probability event that $m_\alpha - Y = m_\beta$, because β-type workers will not apply for a job with firm B. Note that $\Pr(m_\alpha - Y > m_\beta) = 0$ and expected unemployment equals $m_\beta - (m_\alpha - EY) = m_\beta \frac{m_\beta}{m_\alpha+m_\beta}$.

Summarizing the cases described in the preceding paragraphs, firm A has an expected profit function $E\pi_A(w_A, w_B) :=$

$$\begin{cases} (\rho_A - w_A)\, m_\alpha, & \text{if } w_A \geq r_\beta; \\ (\rho_A - w_A)\, m_\alpha \frac{m_\alpha}{m_\alpha+m_\beta}, & \text{if } w_A \in [r_\alpha, r_\beta) \text{ and } w_B \geq r_\beta; \\ (\rho_A - w_A) \max\{m_\alpha - m_\beta, 0\}, & \text{if } w_A, w_B \in [r_\alpha, r_\beta) \text{ and } w_A < w_B; \\ (\rho_A - w_A)\, m_\alpha \frac{m_\alpha}{m_\alpha+m_\beta}, & \text{if } w_A, w_B \in [r_\alpha, r_\beta) \text{ and } w_A = w_B; \\ (\rho_A - w_A)\, m_\alpha, & \text{if } w_A \in [r_\alpha, r_\beta) \text{ and } w_A > w_B; \\ 0, & \text{if } w_A < r_\alpha. \end{cases}$$

and firm B has expected profit function $E\pi_B(w_A, w_B) :=$

$$\begin{cases} (\rho_B - w_B)\, m_\beta, & \text{if } w_B \geq r_\beta; \\ (\rho_B - w_B)\, m_\alpha \frac{m_\beta}{m_\alpha+m_\beta}, & \text{if } w_B \in [r_\alpha, r_\beta) \text{ and } w_A \geq r_\beta; \\ 0, & \text{if } w_A, w_B \in [r_\alpha, r_\beta) \text{ and } w_A > w_B; \\ (\rho_B - w_B)\, m_\alpha \frac{m_\beta}{m_\alpha+m_\beta}, & \text{if } w_A, w_B \in [r_\alpha, r_\beta) \text{ and } w_A = w_B; \\ (\rho_B - w_B) \min\{m_\alpha, m_\beta\}, & \text{if } w_B \in [r_\alpha, r_\beta) \text{ and } w_A < w_B; \\ 0, & \text{if } w_B < r_\alpha. \end{cases}$$

Assuming that the firms are risk neutral, they will play game

$$\Gamma := \langle \{A, B\}, (W_A, W_B), (E\pi_A, E\pi_B) \rangle.$$

Notice that we have not included the workers themselves as strategic players, but we have included their behaviour in the specification of $(E\pi_A, E\pi_B)$.

3 The equilibrium of the wage-setting game

Our aim is to determine the equilibrium of game Γ and those conditions in the market under which unemployment exists. First, we investigate the case in which firm A's productivity allows firm A to make profits even through hiring β-type workers, that is, in the following we shall assume $\rho_A > r_\beta$. Supposing that firm B sets wage r_β we shall denote by w_A^* the wage at which firm A is indifferent to whether it sets wage r_β or $w_A^* \in (-\infty, r_\beta)$, that is, w_A^* is the solution of equation

$$(\rho_A - r_\beta)\, m_\alpha = (\rho_A - w_A^*)\, m_\alpha \frac{m_\alpha}{m_\alpha + m_\beta}.$$

By solving this equation we obtain that $w_A^* = \frac{1}{m_\alpha}(r_\beta (m_\alpha + m_\beta) - \rho_A m_\beta)$. Clearly, w_A^* may be even less than r_α, but we allow this to simplify our analysis. In an analogous way we define the value $w_B^* \in (-\infty, r_\beta)$ as the solution of equation

$$(\rho_B - r_\beta)\, m_\beta = (\rho_B - w_B^*)\, m_\alpha \frac{m_\beta}{m_\alpha + m_\beta},$$

which results in $w_B^* = \frac{1}{m_\alpha}(r_\beta (m_\alpha + m_\beta) - \rho_B m_\beta)$. Observe that we have $r_\beta > w_A^* > w_B^*$ because of $\rho_B > \rho_A > r_\beta$.

The following proposition describes the outcome of game Γ in case of $\rho_A > r_\beta$.

Proposition 1. *Suppose that $\rho_A > r_\beta$. Then in game Γ we have the following cases:*

1. *If $w_A^* < r_\alpha$, then the unique equilibrium equals $(w_A, w_B) = (r_\beta, r_\beta)$ and there is no unemployment.*
2. *If $w_A^* > r_\alpha$ and $w_B^* \le r_\alpha$, then the unique equilibrium equals $(w_A, w_B) = (r_\alpha, r_\beta)$ and expected unemployment equals $m_\beta \frac{m_\alpha}{m_\alpha + m_\beta}$.*
3. *If $w_A^* = r_\alpha$, then $(w_A, w_B) = (r_\alpha, r_\beta)$ and $(w_A, w_B) = (r_\beta, r_\beta)$ are both equilibria.*
4. *If $w_B^* > r_\alpha$, then an equilibrium in pure strategies does not exist.*

Proof. First, observe that neither of the two firms will set a wage above r_β. In addition, any wage below r_α is dominated by wage r_β. A strategy profile $(w_A, w_B) \in [r_\alpha, r_\beta) \times [r_\alpha, r_\beta)$ cannot be an equilibrium profile because; if $w_A = w_B$, then both firms have the incentive to unilaterally increase their wages slightly, and if $w_A \neq w_B$, then the firm setting the higher wage can increase its profit by reducing its wage slightly. Hence, in an equilibrium at least one firm has to set wage r_β.

Case (1): Suppose that $w_A^* < r_\alpha$. We already know that at least one firm, say firm A, sets wage r_β. Then from $w_B^* < r_\alpha$ it follows that every wage

$w_B \in [r_\alpha, r_\beta)$ is dominated by wage r_β. The same argument can be repeated if we assume that firm B sets wage r_β.

Cases (2) and (3): We will split our analysis into two subcases: (i) $w_B^* < r_\alpha$ and (ii) $w_B^* = r_\alpha$. We start with (i). Suppose that $w_A^* \geq r_\alpha$ and $w_B^* < r_\alpha$. We know that in a possible equilibrium at least one firm sets wage r_β. If $w_A = r_\beta$, then $w_B = r_\beta$ follows since firm B realizes less profit by setting a wage below r_β than by setting wage r_β. However, if $w_B = r_\beta$, then wage r_α is a best reply for firm A because $E\pi_A(r_\alpha, r_\beta) \geq E\pi_A(w_A^*, r_\beta) = E\pi_A(r_\beta, r_\beta)$. In addition, $E\pi_B(r_\alpha, r_\beta) = E\pi_B(r_\beta, r_\beta) > E\pi_B(r_\beta, r_\alpha) = E\pi_B(r_\alpha, r_\alpha)$. Thus, $(w_A, w_B) = (r_\alpha, r_\beta)$ is an equilibrium. Observe that $(w_A, w_B) = (r_\beta, r_\beta)$ is another equilibrium if $w_A^* = r_\alpha$. Now we turn to subcase (ii). Suppose that $w_B = r_\beta$. But then firm A sets wage r_α since $E\pi_A(r_\beta, r_\beta) = E\pi_A(w_A^*, r_\beta) < E\pi_A(r_\alpha, r_\beta)$ because $w_B^* = r_\alpha$ implies $w_A^* > r_\alpha$. We obtain that (r_α, r_β) is an equilibrium strategy profile since $E\pi_B(r_\alpha, r_\beta) = E\pi_B(r_\beta, r_\beta) = E\pi_B(r_\beta, r_\alpha) = E\pi_B(r_\alpha, r_\alpha)$. Now suppose that $w_A = r_\beta$. But then firm B has two best replies: $w_B = r_\beta$ and $w_B = r_\alpha$, where in the first case (r_β, r_β) cannot be an equilibrium strategy profile since firm A would deviate to wage r_α. Consider the second possibility of $w_B = r_\alpha$. However, this is in contradiction with $w_A = r_\beta$ being an equilibrium strategy of firm A since $E\pi_A(r_\beta, r_\alpha) = E\pi_A(r_\beta, r_\beta) = E\pi_A(w_A^*, r_\beta) < E\pi_A(w_A^*, r_\alpha)$. Thus, we conclude that (r_α, r_β) is the unique equilibrium strategy profile in subcase (ii).

Case (4): Suppose that $w_B^* > r_\alpha$. As was shown in the first paragraph of this proof, in an eventual pure-strategy equilibrium at least one firm has to set wage r_β. Suppose that $w_A = r_\beta$. But then firm B sets wage r_α since $E\pi_B(r_\beta, r_\beta) = E\pi_B(r_\beta, w_B^*) < E\pi_B(r_\beta, r_\alpha)$. However, this is in contradiction with $w_A = r_\beta$ being an equilibrium strategy of firm A since $E\pi_A(r_\beta, r_\alpha) = E\pi_A(r_\beta, r_\beta) = E\pi_A(w_A^*, r_\beta) < E\pi_A(w_A^*, r_\alpha)$. The same argumentation can be repeated if we assume that $w_B = r_\beta$. Hence, we conclude that a pure-strategy equilibrium does not exist. □

In case (1) of Proposition 1 wage r_α is high enough to prevent the firms from setting low wages. Therefore, we have full employment in the market. Let us remark that in the full employment case α-type workers may be employed by firm B and β-type workers may be employed by firm A.

Case (2) of Proposition 1 occurs if only firm B does not strictly prefer setting wage r_α to r_β whenever its opponent sets wage r_β. In this case firm B has no vacancies but firm A cannot find enough workers since β-type workers will not apply to firm A and only those α-type workers will apply to firm A who could not obtain a job with firm B. Hence, all α-type workers get employed and there are β-type workers seeking for a job with firm B. In particular, we can expect that $m_\beta \frac{m_\alpha}{m_\alpha + m_\beta}$ β-type workers will not get a job. Thus, unemployment exists in the market, which arises because of the inefficient allocation of workers to firm B. All workers apply first to the high-wage firm B and workers are hired through a first-come, first-employed mechanism, where each order of arrival is assumed to be equally probable.[2] Unemployment is caused by a mismatching between firms and workers. However, there is a serious reason why we have to worry about matching workers with firms; in particular, the high-wage firm cannot employ all the workers who want to be employed with the high-wage firm, since the firm has only a limited number of workplaces. Hence, competition is relaxed by the introduction of capacity constraints, as is usually the case in Bertrand-Edgeworth type games. Among other reasons this makes our model behave differently from Waughty and Zenou (2002).

Unemployment could also be explained by a lack of coordination between firms. However, to avoid the emerging unemployment firm B has to introduce a different selection procedure. Clearly, firm B has no incentive to employ a different kind of selection procedure, since this might imply additional costs. Hence, one cannot expect that this type of unemployment disappears if the game is repeated infinitely.

Now turning to case (3) we can observe that either case (1) or case (2) emerges. Finally, case (4) of Proposition 1 occurs if both firms set wage r_α whenever they believe that their opponent sets wage r_β. Unfortunately, in this case an equilibrium in pure strategies does not exist. However, in a mixed-strategy equilibrium a non-efficient assignment will arise with positive probability, that is, either there will be unemployed β-type workers or β-type workers will not apply for a job at all, since (r_β, r_β) cannot be an equilibrium in pure strategies and (r_β, r_β) is the only undominated outcome leading to an efficient assignment of workers. The mixed-strategy equilibrium can be found in the Appendix.

We still have to investigate the case of $\rho_A \leq r_\beta$. Clearly, if even $\rho_A < r_\beta$, the workers will not be assigned to the firms efficiently, since even if firm B sets wage r_β, α-type workers will be employed by firm B with the exception of the low-probability event of $X = 0$.

The following proposition determines the Nash equilibrium of the capacity-constrained wage-setting game Γ for the case of $\rho_A \leq r_\beta$.

Proposition 2. *Suppose that $\rho_A \leq r_\beta$. Then in game Γ we have the following cases:*

[2]This results in the expected assignment of α-type workers to firm B described in Section 2.

1. *If* $E\pi_B(r_\alpha, r_\beta) \geq E\pi_B(0, r_\alpha)$, *then the unique equilibrium is* (r_α, r_β) *and expected unemployment equals* $m_\beta \frac{m_\alpha}{m_\alpha + m_\beta}$.

2. *If* $E\pi_B(r_\alpha, r_\beta) < E\pi_B(0, r_\alpha)$, *then an equilibrium in pure strategies does not exist.*

Proof. Clearly, firm A will never set its wage above ρ_A while firm B will never set its wage above r_β. In addition, any wage below r_α is dominated by wage r_β for firm B and at least weakly dominated by wage r_α for firm A.

First, suppose that $\rho_A < r_\beta$. Then a strategy profile $(w_A, w_B) \in [r_\alpha, \rho_A] \times [r_\alpha, \rho_A]$ cannot be an equilibrium profile because; if $w_A = w_B$, then at least firm B has the incentive to unilaterally increase its wage slightly, and if $w_A \neq w_B$, then the firm setting the higher wage can increase its profit by reducing its wage. Hence, in a possible pure-strategy equilibrium firm B has to set its wage in $(\rho_A, r_\beta]$. However, a strategy $w_B \in (\rho_A, r_\beta)$ cannot be an equilibrium strategy of firm B since $E\pi_B(w_A, w_B)$ is strictly decreasing on (ρ_A, r_β) in w_B for any fixed $w_A \in [0, \rho_A]$. Thus, in a possible pure-strategy equilibrium firm B has to set wage r_β. This implies that firm A has to set wage r_α.

Second, in case of $\rho_A = r_\beta$ strategy profile (r_β, r_β) cannot be an equilibrium profile since then $E\pi_A(r_\beta, r_\beta) = 0$, while $E\pi_A(r_\alpha, r_\beta) > 0$. Through repeating the argumentation of the previous paragraph one can show that a strategy profile $(w_A, w_B) \in [r_\alpha, \rho_A) \times [r_\alpha, \rho_A)$ cannot be an equilibrium profile. Hence, we obtain that profile (r_α, r_β) is the only one which can still be a pure-strategy equilibrium.

Finally, we have to determine the condition under which (r_α, r_β) is a Nash equilibrium. First, it can be easily checked that $E\pi_A(r_\alpha, r_\beta) \geq E\pi_A(w_A, r_\beta)$ for all $w_A \in W_A$. Second, we need $E\pi_B(r_\alpha, r_\beta) \geq E\pi_B(r_\alpha, w_B)$ for all $w_B \in W_B$. Taking into consideration that $E\pi_B(r_\alpha, w_B)$ is strictly decreasing on (r_α, r_β) in w_B we obtain that

$$E\pi_B(r_\alpha, r_\beta) \geq \lim_{w_B \searrow r_\alpha} E\pi_B(r_\alpha, w_B) = E\pi_B(0, r_\alpha)$$

is a sufficient condition for (r_α, r_β) being a Nash equilibrium. In addition, if $E\pi_B(r_\alpha, r_\beta) < E\pi_B(0, r_\alpha)$, there exists a sufficiently small $\varepsilon > 0$ such that $E\pi_B(r_\alpha, r_\beta) < E\pi_B(r_\alpha, r_\alpha + \varepsilon)$. We conclude that $E\pi_B(r_\alpha, r_\beta) \geq E\pi_B(0, r_\alpha)$ is a necessary and sufficient condition for (r_α, r_β) being a Nash equilibrium strategy profile. □

Condition $E\pi_B(r_\alpha, r_\beta) \geq E\pi_B(0, r_\alpha)$ is equivalent to

$$m_\beta(\rho_B - r_\beta) \geq \min\{m_\alpha, m_\beta\}(\rho_B - r_\alpha).$$

Thus, clearly $m_\beta > m_\alpha$ is a necessary condition for $E\pi_B(r_\alpha, r_\beta) \geq E\pi_B(0, r_\alpha)$. Moreover, if m_β is increased sufficiently while m_α, r_α, r_β, ρ_A and ρ_B are kept fixed, then (r_α, r_β) will become a pure-strategy equilibrium.

Although in case (2) of Proposition 2 we did not determine the outcome of game Γ we know that an efficient outcome with full employment is not possible in a mixed-strategy equilibrium, since firm A will never set a wage above ρ_A. Thus, if $\rho_A < r_\beta$, we have either unemployment with vacancies at firm A and unemployed β-type workers, or a total of m_β vacancies and β-type workers will not apply for a job.

4 Concluding remarks

In this paper we considered a wage-setting duopsonistic game in which the firms differ in their productivity and the workers in their reservation wages. To simplify the analysis we assumed that there are only two firms and two possible levels of reservation wages. It would be interesting to determine the outcome of a more general setting with n firms in the market and m different levels of reservation wages. However, the number of cases to be investigated increases rapidly as m or n increases and therefore, this generalization would take much space.

Under certain conditions we pointed out the existence of unemployment (Propositions 1 and 2). In particular, unemployment emerges because α-type workers may occupy better paid jobs, which would be acceptable even for β-type workers. Thus, we explain unemployment through a non-efficient assignment of workers to firms. An interesting feature of the model is that unemployment may emerge although the workers have the same productivity (skills). In addition, in case of unemployment there are also unfilled vacancies at the firm setting the lower wage. The coexistence of unemployment and unfilled vacancies has been demonstrated, for example, by Gottfries and McCormick (1995) in a different setting.

To demonstrate the existence of unemployment in our job market we have applied random rationing of α-type workers. This resulted in the application of the input market equivalent of the so-called random rationing rule (at least in expected value), which is well-known in the literature of price-setting games in output markets. We refer to Vives (1999) for a description of rationing rules in product markets.

An appealing way to resolve the assumption of equally productive workers would be to consider a model like Wauthy and Zenou (2002). In particular, consider the α-type workers as low-skilled workers and the β-type workers as high-skilled workers. Now suppose that α-type workers face education costs

E_α^A and E_α^B if they want to work for firms A and B respectively. Define E_β^A and E_β^B in an analogous way. Given that the α-type workers are the low-skilled ones and firm A is the low-productivity firm it would be natural to assume that $E_\alpha^A > E_\beta^A$, $E_\alpha^B > E_\beta^B$, $E_\alpha^A < E_\alpha^B$ and $E_\beta^A < E_\beta^B$. Now even if we maintain our assumptions imposed on the number of workers and workplaces (that is, $m_\alpha = n_A$ and $m_\beta = n_B$), it can be verified that there is a range of parameter values such that we have full employment and the firms set different wages in contrast to Proposition 1. A more complete analysis of this modified model deserves attention in future research.

Appendix

In the Appendix we consider case (4) of Proposition 1 in detail. We know that in this case an equilibrium in pure strategies does not exist and we start with pointing out that we cannot apply the existence theorems on games with discontinuous payoffs established by Dasgupta and Maskin (1986), Simon (1987) and Reny (1999) for all parameter values. To verify this latter statement we can restrict ourselves to Reny's (1999) Corollary 5.2, since the other existence theorems on discontinuous games all follow from Reny's corollary. In particular, for the case of $m_\alpha \leq m_\beta$ we will check that the mixed extension of $\Gamma' := \langle \{A, B\}, [0, \rho_B]^2, (E\pi_A, E\pi_B) \rangle$ is not better-reply secure at (r_β, r_β).[3] It can be easily verified that game Γ' itself is not better-reply secure at (r_β, r_β), since firm $i \in \{A, B\}$ could only increase its profit by setting a wage below w_i^*. Now if firm i's opponent reduces its wage slightly, then firm i makes zero profit. Hence, firm i cannot secure payoffs higher than $E\pi_i(r_\beta, r_\beta)$. However, we have to show that the mixed extension of Γ' is not better-reply secure at the profile in which both firms are setting wage r_β with probability one. Suppose that firm i deviates by playing a mixed strategy resulting in higher payoffs than $E\pi_i(r_\beta, r_\beta)$. Now if its opponent sets wage $r_\beta - \varepsilon$ with probability one, where ε is sufficiently small, then firm i makes less profit than $E\pi_i(r_\beta, r_\beta)$, since it could only have slightly higher profit with a very low probability while it makes zero profit with a very large probability.

In the following a mixed strategy is a probability measure defined on the σ-algebra of Borel measurable sets on $[0, r_\beta]$. A mixed-strategy equilibrium (μ_A, μ_B) is determined by the following two conditions:

$$E\pi_A(w_A, \mu_B) \leq \pi_A^*, \qquad E\pi_B(\mu_A, w_B) \leq \pi_B^* \tag{1}$$

[3] Game Γ' is *better-reply secure* if whenever $(w^*, E\pi^*)$ is in the closure of the graph of its vector payoff function and w^* is not an equilibrium profile, then there exist an $\varepsilon > 0$, a player $i \in \{A, B\}$ and a strategy $w_i \in [0, \rho_B]$ such that $E\pi_i(w_i, w'_{-i}) \geq E\pi_i^* + \varepsilon$ for all $w'_{-i} \in N(w^*_{-i})$ for some open neighborhood $N(w^*_{-i})$ of w^*_{-i}.

holds true for all $w_A, w_B \in [0, r_\beta]$, and

$$E\pi_A (w_A, \mu_B) = \pi_A^*, \qquad E\pi_B (\mu_A, w_B) = \pi_B^* \tag{2}$$

holds true μ_A-almost everywhere and μ_B-almost everywhere, where π_A^*, π_B^* stand for the equilibrium profits corresponding to (μ_A, μ_B). We shall denote the distribution functions associated with μ_A and μ_B by F_A and F_B, respectively.[4]

We start with the case of $m_\alpha \leq m_\beta$. Throughout the proofs it will be helpful to rewrite assumption $w_B^* > r_\alpha$ in form of $r_\beta (m_\alpha + m_\beta) > \rho_B m_\beta + r_\alpha m_\alpha$ or

$$m_\alpha (r_\beta - r_\alpha) > m_\beta (\rho_B - r_\beta). \tag{3}$$

For the case of $m_\alpha \leq m_\beta$ we can have two different types of equilibria. The first one is described by the following Proposition.

Proposition 3. *If $\rho_A > r_\beta$, $w_B^* > r_\alpha$, $m_\alpha \leq m_\beta$ and*

$$\frac{(\rho_B - r_\beta) m_\beta}{\left(\rho_B - \rho_A + \frac{(\rho_A - r_\alpha)(\rho_A - r_\beta) m_\alpha}{(\rho_A - r_\alpha) m_\alpha - (\rho_A - r_\beta) m_\beta}\right) m_\alpha} < 1, \tag{4}$$

then (μ_A, μ_B) given by

$$\begin{aligned}
\mu_B (\{r_\beta\}) &= \frac{\rho_A - r_\beta}{\rho_A - r_\alpha} \frac{m_\alpha + m_\beta}{m_\alpha}, \quad \overline{w} = \rho_A - \frac{(\rho_A - r_\alpha)(\rho_A - r_\beta) m_\alpha}{(\rho_A - r_\alpha) m_\alpha - (\rho_A - r_\beta) m_\beta}, \\
\mu_A ([\overline{w}, r_\beta)) &= 0, \quad \mu_B ([\overline{w}, r_\beta)) = 0, \quad \mu_B (\{r_\alpha\}) = 0, \\
\mu_A (\{r_\beta\}) &= \frac{m_\alpha + m_\beta}{m_\alpha} \left(1 - \frac{(\rho_B - r_\beta) m_\beta}{(\rho_B - \overline{w}) m_\alpha}\right), \\
\mu_A (\{r_\alpha\}) &= \frac{\rho_B - r_\beta}{\rho_B - r_\alpha} \frac{m_\beta}{m_\alpha} - \frac{m_\beta}{m_\alpha + m_\beta} \mu_A (\{r_\beta\}), \\
F_A(w) &= \frac{\rho_B - r_\beta}{\rho_B - w} \frac{m_\beta}{m_\alpha} - \frac{m_\beta}{m_\alpha + m_\beta} \mu_A (\{r_\beta\}) \text{ for all } w \in (r_\alpha, \overline{w}], \text{ and} \\
F_B(w) &= \frac{\rho_A - r_\beta}{\rho_A - w} - \frac{m_\alpha}{m_\alpha + m_\beta} \mu_B (\{r_\beta\}) \text{ for all } w \in (r_\alpha, \overline{w}]
\end{aligned}$$

is an equilibrium in mixed strategies in which the corresponding equilibrium profits equal $\pi_A^* = (\rho_A - r_\beta) m_\alpha$ and $\pi_B^* = (\rho_B - r_\beta) m_\beta$.

[4] We follow the convention that the distribution functions are left-continuous. Hence, $F_A (w) = \mu_A ([0, w))$ and $F_B (w) = \mu_B ([0, w))$ for all $w \in [0, r_\beta]$.

Proof. It is straightforward to check that $\overline{w} > r_\alpha$ follows from $w_B^* > r_\alpha$, while $\overline{w} < r_\beta$ follows from $\rho_A > r_\beta$. Hence, $\overline{w} \in (r_\alpha, r_\beta)$. In addition, $w_B^* > r_\alpha$, implies $\mu_B(\{r_\beta\}) < 1$. Obviously, $0 < \mu_B(\{r_\beta\})$. Observe that condition (4) is equivalent to $\mu_A(\{r_\beta\}) > 0$. Of course, F_A and F_B are both increasing in w on $\in (r_\alpha, \overline{w}]$. In addition, one can check that $\mu_A(\{r_\beta\}) = 1 - F_A(\overline{w})$, $\mu_B(\{r_\beta\}) = 1 - F_B(\overline{w})$, $\mu_A(\{r_\alpha\}) = \lim_{w \searrow r_\alpha} F_A(w)$, $\lim_{w \searrow r_\alpha} F_B(w) = 0$,

$$\pi_A^* = (\rho_A - w)\, m_\alpha F_B(w) + (\rho_A - w)\, m_\alpha \frac{m_\alpha}{m_\alpha + m_\beta} \mu_B(\{r_\beta\}) \tag{5}$$

and

$$\pi_B^* = (\rho_B - w)\, m_\alpha F_A(w) + (\rho_B - w)\, m_\alpha \frac{m_\beta}{m_\alpha + m_\beta} \mu_A(\{r_\beta\}) \tag{6}$$

for all $w \in (r_\alpha, \overline{w}]$. From the equations above one immediately sees that setting wages $w \in (\overline{w}, r_\beta)$ result in lower profits, since

$$\pi_A^* > (\rho_A - w)\, m_\alpha F_B(\overline{w}) + (\rho_A - w)\, m_\alpha \frac{m_\alpha}{m_\alpha + m_\beta} \mu_B(\{r_\beta\})$$

and

$$\pi_B^* > (\rho_B - w)\, m_\alpha F_A(\overline{w}) + (\rho_B - w)\, m_\alpha \frac{m_\beta}{m_\alpha + m_\beta} \mu_A(\{r_\beta\})$$

for all $w \in (\overline{w}, r_\beta)$.

We still have to verify whether $0 < \mu_A(\{r_\alpha\}) < 1$ and $\mu_A(\{r_\beta\}) < 1$. Observe that we have $\mu_A(\{r_\alpha\}) + \mu_A(\{r_\beta\}) < 1$ by $\mu_A(\{r_\alpha\}) = \lim_{w \searrow r_\alpha} F_A(w) < F_A(\overline{w}) = 1 - \mu_A(\{r_\beta\})$. Hence, it is sufficient to establish $0 < \mu_A(\{r_\alpha\})$, which turns out to be the least obvious case, so we include here some steps of the calculations. Now $0 < \mu_A(\{r_\alpha\})$ is equivalent to

$$\begin{aligned}
\frac{(\rho_B - r_\beta)\, m_\beta}{(r_\beta - r_\alpha)\, m_\alpha} > \frac{\rho_B - \overline{w}}{\rho_B - r_\alpha} &= \frac{\rho_B - r_\beta - (\rho_A - r_\beta) + \frac{(\rho_A - r_\alpha)(\rho_A - r_\beta) m_\alpha}{(\rho_A - r_\alpha) m_\alpha - (\rho_A - r_\beta) m_\beta}}{\rho_B - r_\alpha} = \\
&= \frac{\rho_B - r_\beta + \frac{(\rho_A - r_\beta)^2 m_\beta}{(\rho_A - r_\alpha) m_\alpha - (\rho_A - r_\beta) m_\beta}}{\rho_B - r_\alpha} = \\
&= \frac{(\rho_B - r_\beta)(\rho_A - r_\alpha)\, m_\alpha - (\rho_B - r_\beta)(\rho_A - r_\beta)\, m_\beta + (\rho_A - r_\beta)^2 m_\beta}{(\rho_B - r_\alpha)\left((\rho_A - r_\alpha)\, m_\alpha - (\rho_A - r_\beta)\, m_\beta\right)}.
\end{aligned}$$

After simple rearrangements we obtain

$$\begin{aligned}
(\rho_B - r_\alpha)(\rho_B - r_\beta)\left((\rho_A - r_\alpha)\, m_\alpha - (\rho_A - r_\beta)\, m_\beta\right) m_\beta > {} & (r_\beta - r_\alpha)(\rho_B - r_\beta)(\rho_A - r_\alpha)\, m_\alpha^2 - \\
& (\rho_B - \rho_A)(\rho_A - r_\beta)(r_\beta - r_\alpha)\, m_\alpha m_\beta.
\end{aligned}$$

We increase the right hand side by employing (3), and we show that this increased expression is still smaller than the left hand side. More specifically, we prove

$$(\rho_B - r_\alpha)(\rho_B - r_\beta)((\rho_A - r_\alpha) m_\alpha - (\rho_A - r_\beta) m_\beta) m_\beta > (r_\beta - r_\alpha)(\rho_B - r_\beta)(\rho_A - r_\alpha) m_\alpha^2 - (\rho_B - \rho_A)(\rho_A - r_\beta)(\rho_B - r_\beta) m_\beta^2.$$

Now carrying out the necessary simplifications and rearrangements one obtains

$$(\rho_B - r_\alpha) m_\alpha m_\beta - (r_\beta - r_\alpha) m_\alpha^2 > (\rho_A - r_\beta) m_\beta^2. \tag{7}$$

To verify that equation (7) is indeed satisfied, first we check that even if we have $m_\alpha (r_\beta - r_\alpha) = m_\beta (\rho_B - r_\beta)$, then (7) is fulfilled; and second we check that an increase in r_β does not reduce the difference between the two sides in (7), since

$$\frac{\partial}{\partial r_\beta}\left((\rho_B - r_\alpha) m_\alpha m_\beta - (r_\beta - r_\alpha) m_\alpha^2\right) = -m_\alpha^2 \geq -m_\beta^2 = \frac{\partial}{\partial r_\beta}(\rho_A - r_\beta) m_\beta^2,$$

Hence, μ_A and μ_B are indeed probability measures and therefore, the firms' profits equal $\pi_A^* = (\rho_A - r_\beta) m_\alpha$ and $\pi_B^* = (\rho_B - r_\beta) m_\beta$. Now we can get the stated formulas for F_A and F_B by simply rearranging equations (6) and (5), respectively, which completes the proof. □

The following Proposition considers the other possible equilibrium that might arise in case of $m_\alpha \leq m_\beta$.

Proposition 4. *Suppose that $\rho_A > r_\beta$, $w_B^* > r_\alpha$, $m_\alpha \leq m_\beta$ and*

$$\frac{(\rho_B - r_\beta) m_\beta}{\left(\rho_B - \rho_A + \frac{(\rho_A - r_\alpha)(\rho_A - r_\beta) m_\alpha}{(\rho_A - r_\alpha) m_\alpha - (\rho_A - r_\beta) m_\beta}\right) m_\alpha} \geq 1. \tag{8}$$

Then (μ_A, μ_B) given by

$$\begin{aligned}
\mu_B(\{r_\alpha\}) &= 0, \quad \overline{w} = \frac{1}{m_\alpha}(\rho_B m_\alpha - \rho_B m_\beta + r_\beta m_\beta) \in (r_\alpha, r_\beta], \\
\mu_A([\overline{w}, r_\beta]) &= 0, \quad \mu_B([\overline{w}, r_\beta)) = 0, \\
\mu_A(\{r_\alpha\}) &= \frac{(\rho_B - r_\beta) m_\beta}{(\rho_B - r_\alpha) m_\alpha}, \quad \mu_B(\{r_\beta\}) = \frac{m_\alpha + m_\beta}{m_\beta + \frac{\rho_A - r_\alpha}{\rho_A - \overline{w}} m_\alpha}, \\
F_A(w) &= \frac{(\rho_B - r_\beta) m_\beta}{(\rho_B - w) m_\alpha} \quad \textit{for all } w \in (r_\alpha, \overline{w}], \textit{ and} \\
F_B(w) &= \frac{m_\alpha}{m_\alpha + m_\beta}\left(\frac{\rho_A - r_\alpha}{\rho_A - w} - 1\right)\mu_B(\{r_\beta\}) \quad \textit{for all } w \in (r_\alpha, \overline{w}]
\end{aligned}$$

is an equilibrium in mixed strategies with $\pi_A^* = (\rho_A - r_\alpha)\, m_\alpha \frac{m_\alpha}{m_\alpha + m_\beta} \mu_B(\{r_\beta\})$ *and* $\pi_B^* = (\rho_B - r_\beta)\, m_\beta$.

Proof. Of course, $0 < \mu_A(\{r_\alpha\})$ and it is not difficult to verify that $\mu_A(\{r_\alpha\}) < 1$ by $w_B^* > r_\alpha$ and $\rho_B > r_\beta$. One can check easily that $\overline{w} > r_\alpha$ follows from $w_B^* > r_\alpha$ and that $\overline{w} \leq r_\beta$ follows from $\rho_B > r_\beta$. In addition, we can have $\overline{w} = r_\beta$ only if $m_\alpha = m_\beta$. Thus, we also must have $0 < \mu_B(\{r_\beta\}) < 1$. Clearly, F_A and F_B are both increasing in w on $\in (r_\alpha, \overline{w}]$. Moreover, by simple calculations one obtains $F_A(\overline{w}) = 1$, $\mu_B(\{r_\beta\}) = 1 - F_B(\overline{w})$, $\mu_A(\{r_\alpha\}) = \lim_{w \searrow r_\alpha} F_A(w)$, $\lim_{w \searrow r_\alpha} F_B(w) = 0$,

$$\pi_A^* = (\rho_A - w)\, m_\alpha F_B(w) + (\rho_A - w)\, m_\alpha \frac{m_\alpha}{m_\alpha + m_\beta} \mu_B(\{r_\beta\}) \tag{9}$$

and

$$\pi_B^* = (\rho_B - w)\, m_\alpha F_A(w) \tag{10}$$

for all $w \in (r_\alpha, \overline{w}]$. Clearly, setting wages $w \in (\overline{w}, r_\beta)$ result in lower profits. Since μ_A and μ_B are probability measures, the firms' profits equal $\pi_A^* = (\rho_A - r_\alpha)\, m_\alpha \frac{m_\alpha}{m_\alpha + m_\beta} \mu_B(\{r_\beta\})$ and $\pi_B^* = (\rho_B - r_\beta)\, m_\beta$. Therefore, and by the equations (9) and (10) we can determine F_B and F_A.

We still must verify whether firm A could gain from setting wage r_β. In fact, as it can be verified, this cannot be the case, since condition (8) is equivalent to $\pi_A^* \geq (\rho_A - r_\beta)\, m_\alpha$, which completes the proof. □

Now we turn to the case of $m_\alpha > m_\beta$. For this case we also have two different types of equilibria.

Proposition 5. *If* $\rho_A > r_\beta$, $w_B^* > r_\alpha$, $m_\alpha > m_\beta$ *and*

$$(\rho_A - r_\alpha)\, m_\beta > (r_\beta - r_\alpha)\, m_\alpha, \tag{11}$$

then a mixed-strategy equilibrium (μ_A, μ_B) *is given by*

$$\begin{aligned}
\mu_B(\{r_\beta\}) &= \frac{\rho_A - r_\beta}{\rho_A - r_\alpha} \frac{m_\alpha (m_\alpha + m_\beta)}{m_\beta^2} - \frac{m_\alpha^2}{m_\beta^2} + 1, \\
\mu_B(\{r_\alpha\}) &= 0, \quad \overline{w} = \rho_A - \frac{(\rho_A - r_\alpha)(\rho_A - r_\beta)\, m_\beta}{(r_\beta - r_\alpha)\, m_\alpha}, \quad \mu_B([\overline{w}, r_\beta)) = 0, \\
\mu_A(\{r_\beta\}) &= \frac{m_\alpha + m_\beta}{m_\beta} \left(1 - \frac{\rho_B - r_\beta}{\rho_B - \overline{w}}\right), \quad \mu_A([\overline{w}, r_\beta)) = 0, \\
\mu_A(\{r_\alpha\}) &= \frac{\rho_B - r_\beta}{\rho_B - r_\alpha} - \frac{m_\alpha}{m_\alpha + m_\beta} \mu_A(\{r_\beta\}), \\
F_A(w) &= \frac{\rho_B - r_\beta}{\rho_B - w} - \frac{m_\alpha}{m_\alpha + m_\beta} \mu_A(\{r_\beta\}) \text{ for all } w \in (r_\alpha, \overline{w}], \text{ and} \\
F_B(w) &= \frac{(\rho_A - r_\beta)\, m_\alpha}{(\rho_A - w)\, m_\beta} - \frac{(\rho_A - r_\beta)\, m_\alpha}{(\rho_A - r_\alpha)\, m_\beta} \text{ for all } w \in (r_\alpha, \overline{w}],
\end{aligned}$$

where $\pi_A^* = (\rho_A - r_\beta)\, m_\alpha$ *and* $\pi_B^* = (\rho_B - r_\beta)\, m_\beta$.

Proof. Now $r_\alpha < \overline{w}$ follows from $w_B^* > r_\alpha$ and $\rho_A > r_\beta$, while $\overline{w} < r_\beta$ follows from condition (11). Hence, $\mu_A(\{r_\beta\}) > 0$. By carrying out the necessary rearrangements it can be checked that $\mu_B(\{r_\beta\}) > 0$ is equivalent to condition (11). In addition, $\mu_B(\{r_\beta\}) < 1$ just follows from $w_B^* > r_\alpha$. Clearly, F_A and F_B are both increasing in w on $\in (r_\alpha, \overline{w}]$. Furthermore, one can check that $\mu_A(\{r_\beta\}) = 1 - F_A(\overline{w})$, $\mu_B(\{r_\beta\}) = 1 - F_B(\overline{w})$, $\mu_A(\{r_\alpha\}) = \lim_{w \searrow r_\alpha} F_A(w)$, $\lim_{w \searrow r_\alpha} F_B(w) = 0$,

$$\pi_A^* = (\rho_A - w)\left(m_\beta F_B(w) + m_\alpha - m_\beta + \frac{m_\beta^2}{m_\alpha + m_\beta}\mu_B(\{r_\beta\})\right) \tag{12}$$

and

$$\pi_B^* = (\rho_B - w)\left(m_\beta F_A(w) + m_\alpha \frac{m_\beta}{m_\alpha + m_\beta}\mu_A(\{r_\beta\})\right) \tag{13}$$

for all $w \in (r_\alpha, \overline{w}]$. From $\mu_A([\overline{w}, r_\beta)) = 0$, $\mu_B([\overline{w}, r_\beta)) = 0$, (12) and (13) we can see that setting wages $w \in (\overline{w}, r_\beta)$ result in lower profits.

We still have to verify that $\mu_A(\{r_\beta\}) < 1$ and $0 < \mu_A(\{r_\alpha\}) < 1$. Observe that since we have $\mu_A(\{r_\alpha\}) + \mu_A(\{r_\beta\}) < 1$ by $\mu_A(\{r_\alpha\}) = \lim_{w \searrow r_\alpha} F_A(w) < F_A(\overline{w}) = 1 - \mu_A(\{r_\beta\})$, it suffices to check $\mu_A(\{r_\alpha\}) > 0$. However, verifying this inequality is not straightforward, so we include here some steps of the following calculations. Again, it will be helpful to employ $w_B^* > r_\alpha$ in form of (3). Now, $0 < \mu_A(\{r_\alpha\})$ is equivalent to

$$\begin{aligned}\frac{(\rho_B - r_\alpha)\, m_\alpha}{(\rho_B - r_\alpha)\, m_\alpha - (\rho_B - r_\beta)\, m_\beta} &> \frac{\rho_B - \overline{w}}{\rho_B - r_\beta} = \frac{\rho_B - r_\beta - (\rho_A - r_\beta) + \frac{(\rho_A - r_\alpha)(\rho_A - r_\beta) m_\beta}{(r_\beta - r_\alpha) m_\alpha}}{\rho_B - r_\beta} = \\ &= 1 - \frac{(\rho_A - r_\beta)\,(r_\beta - r_\alpha)\, m_\alpha - (\rho_A - r_\alpha)\,(\rho_A - r_\beta)\, m_\beta}{(\rho_B - r_\beta)\,(r_\beta - r_\alpha)\, m_\alpha}.\end{aligned}$$

The above inequality is equivalent to

$$\frac{(\rho_A - r_\alpha)\,(\rho_A - r_\beta)\, m_\beta - (\rho_A - r_\beta)\,(r_\beta - r_\alpha)\, m_\alpha}{(\rho_B - r_\beta)\,(r_\beta - r_\alpha)\, m_\alpha} < \frac{(\rho_B - r_\beta)\, m_\beta}{(\rho_B - r_\alpha)\, m_\alpha - (\rho_B - r_\beta)\, m_\beta}.$$

After simple rearrangements we obtain

$$\begin{aligned}(\rho_B - \rho_A)\,(\rho_B - r_\beta)\,(r_\beta - r_\alpha)\, m_\alpha m_\beta &> (\rho_A - r_\alpha)\,(\rho_A - r_\beta)\,(\rho_B - r_\alpha)\, m_\alpha m_\beta - \\ &\quad (\rho_A - r_\alpha)\,(\rho_A - r_\beta)\,(\rho_B - r_\beta)\, m_\beta^2 - \\ &\quad (\rho_A - r_\beta)\,(r_\beta - r_\alpha)\,(\rho_B - r_\alpha)\, m_\alpha^2.\end{aligned}$$

We decrease the left hand side by employing (3) and $\rho_B > \rho_A$, and we show that this decreased expression is still larger than the right hand side. In particular, we prove

$$\begin{aligned}(\rho_B - \rho_A)(\rho_B - r_\beta)(\rho_A - r_\beta)\, m_\beta^2 \quad > \quad & (\rho_A - r_\alpha)(\rho_A - r_\beta)(\rho_B - r_\alpha)\, m_\alpha m_\beta - \\ & (\rho_A - r_\alpha)(\rho_A - r_\beta)(\rho_B - r_\beta)\, m_\beta^2 - \\ & (\rho_A - r_\beta)(r_\beta - r_\alpha)(\rho_B - r_\alpha)\, m_\alpha^2.\end{aligned}$$

Now carrying out the necessary simplifications and rearrangements one obtains

$$(\rho_A - r_\alpha)\, m_\alpha m_\beta - (r_\beta - r_\alpha)\, m_\alpha^2 < (\rho_B - r_\beta)\, m_\beta^2. \tag{14}$$

To verify (14), first it can be checked that even if we have $m_\alpha (r_\beta - r_\alpha) = m_\beta (\rho_B - r_\beta)$, then (14) is satisfied; and second it can be checked that an increase in r_β does not reduce the difference between the two sides in (14), since

$$\frac{\partial}{\partial r_\beta}\left((\rho_A - r_\alpha)\, m_\alpha m_\beta - (r_\beta - r_\alpha)\, m_\alpha^2\right) = -m_\alpha^2 < -m_\beta^2 = \frac{\partial}{\partial r_\beta}(\rho_B - r_\beta)\, m_\beta^2.$$

Thus, μ_A and μ_B are indeed probability measures. Hence, $\pi_A^* = (\rho_A - r_\beta)\, m_\alpha$ and $\pi_B^* = (\rho_B - r_\beta)\, m_\beta$. Now one can obtain the stated expressions for F_B and F_A in the Proposition by rearranging (12) and (13), respectively, and we are done. □

Finally, we have to consider the other possible equilibrium that might arise in case of $m_\alpha > m_\beta$.

Proposition 6. *Assume that $\rho_A > r_\beta$, $w_B^* > r_\alpha$, $m_\alpha > m_\beta$ and*

$$(\rho_A - r_\alpha)\, m_\beta \leq (r_\beta - r_\alpha)\, m_\alpha. \tag{15}$$

Then (μ_A, μ_B) given by

$$\begin{aligned}\mu_B(\{r_\alpha\}) &= 0, \quad \overline{w} = \frac{1}{m_\alpha}(\rho_A m_\beta + r_\alpha m_\alpha - r_\alpha m_\beta), \\ \mu_A([\overline{w}, r_\beta]) &= 0, \quad \mu_B([\overline{w}, r_\beta]) = 0, \quad \mu_A(\{r_\alpha\}) = \frac{\rho_B - \overline{w}}{\rho_B - r_\alpha}, \\ F_A(w) &= \frac{\rho_B - \overline{w}}{\rho_B - w} \quad \text{for all } w \in (r_\alpha, \overline{w}], \text{ and} \\ F_B(w) &= \frac{\rho_A - \overline{w}}{\rho_A - w}\frac{m_\alpha}{m_\beta} - \frac{m_\alpha}{m_\beta} + 1 \quad \text{for all } w \in (r_\alpha, \overline{w}]\end{aligned}$$

is an equilibrium in mixed strategies in which the equilibrium profits equal $\pi_A^* = (\rho_A - \overline{w})\, m_\alpha$ and $\pi_B^* = (\rho_B - \overline{w})\, m_\beta$.

Proof. One can check easily that $\rho_A > r_\alpha$ implies $\overline{w} > r_\alpha$, while (15) is equivalent to $\overline{w} \leq r_\beta$. Therefore, it follows that $0 < \mu_A(\{r_\alpha\}) < 1$. Obviously, F_A and F_B are both increasing in w on $\in (r_\alpha, \overline{w}]$. One immediately sees that $\lim_{w \nearrow \overline{w}} F_A(w) = 1$, $\lim_{w \nearrow \overline{w}} F_B(w) = 1$ and $\mu_A(\{r_\alpha\}) = \lim_{w \searrow r_\alpha} F_A(w)$. In addition, by simple calculations one obtains $\lim_{w \searrow r_\alpha} F_B(w) = 0$,

$$\pi_A^* = (\rho_A - w)(m_\alpha F_B(w) + (m_\alpha - m_\beta)(1 - F_B(w))) \tag{16}$$

and

$$\pi_B^* = (\rho_B - w) m_\beta F_A(w) \tag{17}$$

for all $w \in (r_\alpha, \overline{w}]$. Obviously, the equations above imply that setting wages $w \in (\overline{w}, r_\beta)$ result in lower profits. The firms' profits equal $\pi_A^* = (\rho_A - \overline{w}) m_\alpha$ and $\pi_B^* = (\rho_B - \overline{w}) m_\beta$. Thus, we can express F_A and F_B from (17) and (16), respectively.

It still could happen that firm A gains from setting wage r_β. However, as can be verified, this cannot happen, since condition (15) is equivalent to $\pi_A^* \geq (\rho_A - r_\beta) m_\alpha$, which completes the proof. □

Propositions 3, 4, 5 and 6 provide a complete mixed-strategy solution for case (4) of Proposition 1. Moreover, this equilibrium is a unique one, however, we omit here the very tedious calculations demonstrating uniqueness.[5]

References

[1] Dasgupta, P., Maskin, E., 1986. The existence of equilibria in discontinuous games, I: Theory. Review of Economic Studies 53, 1-26.

[2] Gottfries, N., McCormick, B., 1995. Discrimination and open unemployment in a segmented labour market. European Economic Review 39, 1-15.

[3] Hamilton, J., Thisse, J.-F., Zenou, Y., 2000. Wage Competition with Heterogeneous Workers and Firms. Journal of Labor Economics 18, 453-472.

[4] Harris, J.R., Todaro, M.P., 1970. Migration, Unemployment and Development: A Two-Sector Analysis. American Economic Review 60, 126-142.

[5] Ma, Ching-to A., Weiss, A.M., 1993. A signaling theory of unemployment. European Economic Review 37, 135-157.

[5] The proof of uniqueness is available upon request from the author.

[6] Rebitzer, J.B., Taylor, L.J., 1995. The consequences of minimum wage laws: Some new theoretical ideas. Journal of Public Economics 56, 245-255.

[7] Reny, P.J, 1999. On the Existence of Pure and Mixed Strategy Nash Equilibria in Discontinuous Games. Econometrica 67, 1029-1056.

[8] Salop, S.C., 1979. Monopolistic Competition with Outside Goods. Bell Journal of Economics 10, 141-156.

[9] Sattinger, M., 1993. Assignment Models of the Distribution of Earnings. Journal of Economic Literature 31, 831-880.

[10] Shapiro, C., Stiglitz, J. E., 1984. Equilibrium Unemployment as a Worker Discipline Device. American Economic Review 74, 433-444.

[11] Simon, L., 1987. Games with Discontinuous Payoffs. Review of Economic Studies 54, 569-597.

[12] Thisse, J.-F., Zenou, Y., 2000. Skill mismatch and unemployment. Economics Letters 69, 415-420.

[13] Wauthy, X., Zenou, Y., 2002. How Does Imperfect Competition in the Labour Market Affect Unemployment Policies? Journal of Public Economic Theory 4, 417-436.

[14] Weiss, A.M., 1980. Job Queues and Layoffs in Labor Markets with Flexible Wages. Journal of Political Economy 88, 526-538.

Additional calculations (not intended for publication)

The following proposition points out how we derived the equilibrium in mixed strategies contained in the Appendix, and why this equilibrium is unique. In particular, we can make the following statement about the form of the mixed-strategy equilibrium if there exist such at all.

Proposition 7. *Assume that $\rho_A > r_\beta$ and $w_B^* > r_\alpha$. If (μ_A, μ_B) is a mixed-strategy equilibrium of Γ, then there exists a wage $\overline{w} \in (r_\alpha, r_\beta]$ such that $\mu_A([\overline{w}, r_\beta)) = 0$, $\mu_B([\overline{w}, r_\beta)) = 0$,*

$$F_A(w) = \frac{\pi_B^*}{(\rho_B - w)\min\{m_\alpha, m_\beta\}} - \frac{m_\alpha}{\min\{m_\alpha, m_\beta\}} \frac{m_\beta}{m_\alpha + m_\beta} \mu_A(\{r_\beta\}), \quad (18)$$

and $F_B(w) =$

$$\frac{\pi_A^*}{(\rho_A - w)\min\{m_\alpha, m_\beta\}} - \frac{\min\{m_\alpha, m_\beta\}}{m_\alpha + m_\beta} \mu_B(\{r_\beta\}) - \max\left\{\frac{m_\alpha - m_\beta}{m_\beta}, 0\right\} \quad (19)$$

for all $w \in (r_\alpha, \overline{w}]$, where π_A^*, π_B^* stand for the equilibrium profits corresponding to (μ_A, μ_B).

Proof. Assume that (μ_A, μ_B) is an equilibrium in mixed strategies. We shall denote the supremum of wages smaller than r_β that might be set by firm $i \in \{A, B\}$ by $\overline{w}_i := \inf\{w \in [0, r_\beta] \mid \mu_i([w, r_\beta)) = 0\}$. In an analogous way we define the infimum of wages that might be set by firm $i \in \{A, B\}$ by $\underline{w}_i := \sup\{w \in [0, r_\beta] \mid \mu_i([0, w)) = 0\}$.
Step 1: Both firms cannot have atoms at the same wage $w \in [0, r_\beta)$, that is, $\mu_A(\{w\}) = 0$ or $\mu_B(\{w\}) = 0$ for all $w \in [0, r_\beta)$. If this is not the case, then both firms could increase their profits by switching unilaterally to pure strategy $w + \varepsilon$, where ε is a sufficiently small positive value; a contradiction.
Step 2: We show that $\underline{w}_A = \underline{w}_B = r_\alpha$. Clearly, $\underline{w}_A < r_\beta$ and $\underline{w}_B < r_\beta$. Suppose that $\underline{w}_A < \underline{w}_B$. Then we must have $\mu_A((\underline{w}_A, \underline{w}_B)) = 0$ and $\mu_A(\{\underline{w}_A\}) > 0$. By Step 1 we know that only one of the two firms can have an atom at $\underline{w}_B$. If only firm A has an atom at $\underline{w}_B$, then it will make higher profits by setting a wage $w \in (\underline{w}_A, \underline{w}_B)$ instead of setting wage $\underline{w}_B$, a contradiciton. The same would hold true for firm B if only firm B had an atom at wage $\underline{w}_B$. Hence, we must have $\underline{w}_A \geq \underline{w}_B$. In an analogous way we can show that $\underline{w}_B \geq \underline{w}_A$. Thus, $\underline{w}_A = \underline{w}_B$. Henceforth we can write $\underline{w} := \underline{w}_A = \underline{w}_B$. Now if $r_\alpha < \underline{w}$, then Step 1 and $w_B^* > r_\alpha$ imply that at least one firm can increase its profit by setting wage r_α; a contradiciton.

Step 3: We must have $\overline{w}_A = \overline{w}_B$. Suppose that $\overline{w}_A < \overline{w}_B < r_\beta$. But then in contradiction with (μ_A, μ_B) being an equilibrium profile, firm B would make higher profit by switching to pure strategy $\overline{w}_A + \varepsilon$, where ε is a sufficiently small positive value. Obviously, for similar reasons we cannot have $\overline{w}_B < \overline{w}_A < r_\beta$. Hence, $\overline{w}_A = \overline{w}_B$ whenever $\overline{w}_A < r_\beta$. Now if, for instance, $\overline{w}_A < \overline{w}_B = r_\beta$ is the case, then firm B can gain from playing a wage $w_B \in (\overline{w}_A, r_\beta)$ with probability $\mu_B([w_B, r_\beta))$ instead of having the same mass distributed over the interval $[w_B, r_\beta)$. Again for similar reasons we cannot have $\overline{w}_B < \overline{w}_A = r_\beta$. Thus, $\overline{w}_A = \overline{w}_B$ in any case.
Step 4: There are no atoms in $(r_\alpha, \overline{w})$. Suppose firm A has an atom at $w_A \in (r_\alpha, \overline{w})$. Then firm B certainly will not set a wage in interval $(w_A - \varepsilon, w_A)$, where ε is a sufficiently small positive value, since even wage w_A for firm B would dominate wages in $(w_A - \varepsilon, w_A)$. Therefore, firm A could do better by setting some wage slightly below w_A because firm B does not have an atom at wage w_A by Step 1; a contradicition. Of course, we can establish in an analogous way that firm B does not have an atom in $(r_\alpha, \overline{w})$.
Step 5: There are no atoms at $\overline{w}$ whenever $\overline{w} < r_\beta$. This statement can be verified by mimicking the argument appearing in Step 4, where we only have to replace w_A with $\overline{w}$.
Step 6: We can now determine the form of the equilibrium wage distribution functions on $(r_\alpha, \overline{w}]$ whenever there exists an equilibrium in mixed strategies. By the previous steps we must have

$$\begin{aligned} \pi_A^* = \; & (\rho_A - w)\, m_\alpha F_B(w) + \\ & (\rho_A - w)\, m_\alpha \frac{m_\alpha}{m_\alpha + m_\beta} \mu_B(\{r_\beta\}) + \\ & (\rho_A - w) \max\{m_\alpha - m_\beta, 0\} \left(1 - F_B(w) - \mu_B(\{r_\beta\})\right) \end{aligned} \tag{20}$$

and

$$\pi_B^* = (\rho_B - w) \min\{m_\alpha, m_\beta\} F_A(w) + (\rho_B - w)\, m_\alpha \frac{m_\beta}{m_\alpha + m_\beta} \mu_A(\{r_\beta\}) \tag{21}$$

for all $w \in (r_\alpha, \overline{w}]$. Rearranging equations (21) and (20) we can derive (18) and (19). □

Proposition 7 reduces the number of possible atoms to four. Hence, altogether there are sixteen possible cases. Since both firms cannot simultaneously have atoms at r_α by Step 1, the number of cases reduces immediately to twelve. Going through all these cases only the solution given in the Appendix survives. In particular, the other cases lead to negative values for the atoms or to contradictory equations.

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