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An Elementary Approach to the Hold-Up Problem with Renegotiation

by

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An Elementary Approach to the Hold-Up Problem with Renegotiation

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Abstract

So far, the existing literature on the hold-up problem with renegotiation has imposed assumptions such that the post-renegotiation payoffs are absolutely continuous functions. Since payoffs may fail to be differentiable at the investment profile to be sustained, first order conditions for incentives to invest must be handled with care. To avoid these difficulties, the present paper propagates a more elementary approach. A general condition is provided which necessarily must hold for an investment profile to be sustainable by a message contingent contract. If only one of the parties invests or, more generally, if investments can be aggregated into one dimension then the paper introduces assumptions leading to conditions which are necessary and sufficient for an investment profile to be sustainable.

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1 Introduction

This paper deals with the mechanism design approach to the hold-up problem. There are two parties undertaking relationship-specific investments. After uncertainty has unraveled, some decision must be taken. Ex ante, the parties can sign any contract out of the general class of all message contingent contracts. Yet, if the chosen messages would lead to an inefficient decision, renegotiations are assumed to take place. By assumption, investments and the state of the world can be observed by both parties such that renegotiation takes place under complete information but they fail to be verifiable. The paper provides a simple but general condition which necessarily must hold for an investment profile to be sustainable by message contingent contracts with renegotiation. The condition works for one-dimensional as well as multi-dimensional investments. For the case of investments which can be aggregated into one dimension, the paper introduces conditions which are necessary and sufficient for an investment profile to be sustainable.

Maskin and Moore [1999] were the first to formulate the problem of mechanim design with renegotiation in general. They give a characterization in terms of incentive compatibility constraints which, however, remain difficult to check. The present paper comes closer in spirit to Edlin and Reichelstein [1996] and, in particular, to Che and Hausch [1999] who have studied buyer-seller relationships with a one-dimensional quantity decision. Notice, however, that our results allow for much broader interpretations. The paper also touches some of those results of Segal and Whinston [1999] which concern the hold-up problem but without making use of the Mirrlees [1971] approach to mechanism design.

So far, the existing literature has relied on asumptions such that, for any message contingent contract, the post-renegotiation payoffs are absolutely continuous functions and, hence, must be differentiable but only almost everywhere. Moreover, incentives to invest are investigated by means of their first order conditions. Since payoffs may fail to be differentiable at, of all, the investment profile to be sustained, rigorous arguments require some extra effort. The present paper propagates a more elementary approach. No use of the theory of absolutely continuous functions and, except for the last proposition of the paper, not even of calculus is made. Moreover, for most of the results, the set neither of decisions nor of investment choices need be connected, i.e. the situation of discrete choices is also included. As a consequence, the results can be applied to hold-up problems as they arise in the property rights approach to the firm (see, e.g., Grossman and Hart [1986] and Hart[1995]). In this approach, parties may choose from a finite set of governance or ownership structures. The situation, if expressed in our framework, leads to a set of feasible decisions which fails to be connected. The reader is also referred to Roider [2000] who combines the choice of ownership structures with a continuous quantity choice. In this context, to justify the property rights approach, it is important to know conditions under which it is sufficient to consider non-contingent contracts. The present paper provides such conditions.

The paper is organized as follows. The next section introduces the model. Section 3 contains the main results for the case of one-dimensional investments. A condition is introduced which necessarily must hold for an investment profile to be sustainable by a message contingent contract. The condition is shown to generalize the measures of cooperativeness as introduced by Che and Hausch [1999]. Section 4 deals with the case of one-dimensional investments which are one-sided. Under some additional assumption, a condition is established which is both necessary and sufficient for an investment profile to be sustainable. To sustain such profiles, option contracts may be needed where the option must be exercised after investments can be observed but before the state of the world becomes known. Section 5 extends some of the findings of section 3 to multi-dimensional investments. For a given party, a direction of investment is called non-harmful if the other party would benefit from increased investments in that direction to a positive but possibly small extent. Our necessary condition if applied to the efficient investment profile establishes that this profile fails to be sustainable if, for one party at least, there exists at least one direction of investment which is non-harmful to the other party. Section 6 considers the case where investments can be aggregated into one dimension. The results generalize some of the findings on one-sided investments of Section 4. Again, a condition is introduced which is both necessary and sufficient for an investment profile to be sustainable. This section also revisits some of the findings of Segal and Whinston [1999] without, however, making use of the theory of absolutely continuous functions.

Section 7 concludes.

2 The model

Two parties $i = 1, 2$ choose investments $e_i \in E_i$. The sets E_i of feasible choices are assumed to be closed subsets of finitely dimensional Euclidean spaces $\Re^{l_i}$. Investment profiles are denoted by $e = (e_1, e_2) \in E = E_1 \times E_2$. Investment costs of party i are denoted by $c_i(e_i)$. After investment decisions have been taken, uncertainty $\omega \in \Omega$ unravels. For simplicity, the set Ω of states of the world is assumed to be finite ($\#\Omega = m$). The probabilities with which the different states occur are exogenously given. The expectation operator with respect to the uncertain state of the world is denoted by $E_\omega[\ ]$ [1]. Any vector $\beta = (e, \omega) \in B = E \times \Omega$ is called a *history* of the hold-up problem. Histories can be observed by the two parties but fail to be verifiable in front of courts.

After the parties have learned the history, a decision $x \in X$ must be taken. The set of feasible decisions is assumed to be a subset of some Euclidean space, i.e. $X \subset \Re^n$. In some of the existing literature, this decision is assumed to concern a quantity choice, i.e. $X = [0, \infty)$ or $X = [0, 1]$. Yet, to also capture versions of the hold-up problem as studied in the property-rights approach to the theory of the firm (see introduction), our setting does require the set X neither to be one-dimensional nor to be connected.

Profits (excluding investment costs and transfer payments) of party i amount to $p_i(\beta, x)$. The maximum *social surplus* after history β

$$s(\beta) = \max_{x \in X} p_1(\beta, x) + p_2(\beta, x)$$

is assumed to exist and to be finite for all histories. The *efficient investment profile* which also is assumed to exist is denoted by

$$e^* \in \arg\max_{\beta \in B} E_\omega [s(\beta)] - c_1(e_1) - c_2(e_2).$$

Ex ante, i.e. before investment decisions are due, the parties sign a *message contingent contract* $\gamma = [M_1, M_2, x(m), t_1(m), t_2(m)]$ where $x(m) \in X$ and $t_1(m) + t_2(m) = 0$ denote the decision and the transfer payments at

[1] The approach can easily be extended to infinite state spaces. All we require is that the expected value E_ω exists whenever use is made of the expectation operator.

message profile $m \in M = M_1 \times M_2$. M_i is the set of messages which party i can send after having observed the history of the hold-up problem. Let Γ denote the set of all message contingent contracts. This class is very general. It includes, in particular, all non-contingent contracts ($\#M_1 = \#M_2 = 1$) which play an important role in the property rights approach to the firm as well as all party i option contracts where only party i has a true choice as far as messages are concerned, i.e. $\#M_j = 1$ for party $j \neq i$ [2].

If, at history β and messages m, the contractual decision $x(m)$ fails to be efficient, renegotiations are assumed to take place and to lead to an efficient post-renegotiation solution. Post-renegotiation payoffs are denoted by

$$r_i(\beta, x(m)) + t_i(m) \tag{1}$$

where

$$r_i(\beta, x) = p_i(\beta, x) + \alpha_i(\omega)\left[s(\beta) - p_1(\beta, x) - p_2(\beta, x)\right]$$

is the *post-renegotiation profit function* of player i. The term in square brackets denotes the maximum social gain from renegotiation, out of which party i is assumed to get the share $\alpha_i(\omega)$ ($\alpha_i(\omega) \geq 0$, $\alpha_1(\omega) + \alpha_2(\omega) \equiv 1$). While the bargaining power may depend on the state of the world, the parties' shares are exogenously fixed. Rearranging terms leads to

$$r_i(\beta, x) = (1 - \alpha_i(\omega))p_i(\beta, x) + \alpha_i(\omega)\left[s(\beta) - p_j(\beta, x)\right]. \tag{2}$$

Notice that, for all histories β and all decisions x, it holds that

$$r_1(\beta, x) + r_2(\beta, x) = s(\beta). \tag{3}$$

Hence the message game with payoff functions (1) is a fixed-sum game such that, according to the Min-Max-Theorem, all its Nash equilibria are payoff equivalent. Let $m^\gamma(\beta) \in M$ denote one of these Nash equilibria (if there are several) after history β has been observed.

The payoff frontier $[R_1\beta), R_2(\beta)]$ is called *implementable* if there exists a message contingent contract $\gamma \in \Gamma$ leading to a Nash equilibrium $m^\gamma(\beta)$ such that

$$R_i(\beta) = r_i(\beta, x(m^\gamma(\beta))) + t_i(m^\gamma(\beta))$$

[2] See Segal and Whinston [1999] for this general notion of option contracts.

holds for both parties $i = 1, 2$. An *incentive compatible mechanism* asks parties separately to reveal the history, i.e. has message sets $M_1 = M_2 = B$ and it has telling the truth as a Nash equilibrium. It follows from the revelation principle (see Maskin and Moore [1999]) that the payoff frontier $[R_1\beta), R_2(\beta)]$ is implementable if, and only if, there is an incentive compatible mechanism $[x(), t_1(), t_2()]$ such that

$$R_i(\beta) = r_i(\beta, x(\beta, \beta)) + t_i(\beta, \beta)$$

holds for all histories $\beta \in B$.

Finally, an investment profile $e^N \in E$ is called *sustainable* if there exists an implementable payoff frontier $[R_1\beta), R_2(\beta)]$ such that

$$e_i^N \in \arg\max_{e_i \in E_i} E_\omega \left[R_i(e_i, e_j^N, \omega) \right] - c_i(e_i) \tag{4}$$

holds for $\{i, j\} = \{1, 2\}$. Let E^N denote the set of sustainable investment profiles. Our main focus will be on this set of sustainable profiles.

3 One-dimensional investments

In this section, it is assumed that parties have one-dimensional investment choices only, i.e. $E_1, E_2 \subset \Re$. Let $\Phi = \{\phi : \Omega \to X\} \subset \Re^{mn}$ denote the set of all *state contingent decisions* ϕ and let

$$\nu_i(e, \phi) = E_\omega \left[r_i(e, \omega, \phi(\omega)) \right] - c_i(e_i)$$

denote the net profit which party i would make if the investment profile were $e \in E$ and if the state contingent decision $\phi \in \Phi$ were implemented. For all $e_j \in E_j$ and $\phi \in \Phi$, let

$$\varepsilon_i^{\phi+}(e_j) = \inf_{e_i \in E_i} e_i$$

such that $\nu_i(e, \phi)$ is strictly monotonically decreasing for all $e_i' \geq e_i$ and

$$\varepsilon_i^{\phi-}(e_j) = \sup_{e_i \in E_i} e_i$$

such that $\nu_i(e, \phi)$ is strictly monotonically increasing for all $e_i' \leq e_i$. Moreover, for all $e_j \in E_j$, let

$$\varepsilon_i^+(e_j) = \sup_{\phi \in \Phi} \varepsilon_i^{\phi+}(e_j) \text{ and } \varepsilon_i^-(e_j) = \inf_{\phi \in \Phi} \varepsilon_i^{\phi-}(e_j).$$

It follows that, for $e_i \geq \varepsilon_i^+(e_j)$, $\nu_i(e,\phi)$ is strictly monotonically decreasing for all $\phi \in \Phi$. Similarly, for $e_i \leq \varepsilon_i^-(e_j)$, $\nu_i(e,\phi)$ is strictly monotonically increasing for all $\phi \in \Phi$. The following proposition can be established which makes use of this notation and which provides a necessary condition for an investment profile to be sustainable.

Proposition 1 *If the investment profile e^N is sustainable, i.e. if $e^N \in E^N$, then*

$$\varepsilon_i^-(e_j^N) \leq e_i^N \leq \varepsilon_i^+(e_j^N)$$

must hold for both parties.

Proof. A choice function $f : B \to X$ is said to *induce* incentives e^N if

$$c_i(e_i^N) - c_i(e_i) \leq E_\omega \left[r_i(e^N, \omega, f(e_i, e_j^N, \omega)) - r_i(e_{i,} e_j^N, \omega, f(e_i, e_j^N, \omega)) \right] \quad (5)$$

holds for all investments $e_i \in E_i$. We *claim* that, if $e^N \in E^N$, then there exists a choice function which induces e^N. In fact, it is well-known (see, e.g., Maskin and Moore [1999] or Segal and Whinston [1999]) that the payoff frontier $[R_1(\beta), R_2(\beta)]$ is implementable iff there exists a function $x : B \times B \to X$ such that

$$r_i(\beta', x(\beta;\beta')) - r_i(\beta, x(\beta;\beta')) \leq R_i(\beta') - R_i(\beta) \leq r_i(\beta', x(\beta';\beta)) - r_i(\beta, x(\beta';\beta))$$

holds for all histories β, $\beta' \in B$. To prove the claim, assume that $[R_1(\beta), R_2(\beta)]$ is implementable and provides incentives to invest e^N, i.e. (4) must hold for both parties. Then the *claim* is easily seen to hold for the choice function $f(e,\omega) = x(e^N, \omega; e, \omega)$.

To prove the proposition, assume first that $e_i^{\sup} = \varepsilon_i^+(e_j^N) < e_i^N$. Let $e^{\sup} = (e_i^{\sup}, e_j^N)$. It then follows from the definition of $\varepsilon_i^+(e_j^N)$ that $\nu_i(e^{\sup},\phi) > \nu_i(e^N,\phi)$ must hold for all $\phi \in \Phi$, in particular for $\phi = f(e^{\sup},\omega)$. But this contradicts (5). Therefore $\varepsilon_i^+(e_j^N) \geq e_i^N$ must hold for both parties.

Assume second that $\varepsilon_i^-(e_j^N) > e_i^N$. Then this leads to a contradiction in the same way as above. Therefore, $\varepsilon_i^-(e_j^N) \leq e_i^N$ must hold for both parties as well. The proposition is established. ■

At first glance, the necessary condition of the proposition for an investment profile to be sustainable looks quite abstract. However, as we now want to show, the condition is related to the measures of cooperativeness as introduced by Che and Hausch [1999]. In order to establish this claim, for any state contingent decision $\phi \in \Phi$ and any investment profile $e \in E$, let us define

$$\begin{aligned} \pi_i(e,\phi) &= E_\omega\left[p_i(e,\omega,\phi(\omega))\right], \\ \pi_j(e,\phi) &= E_\omega\left[p_j(e,\omega,\phi(\omega))\right], \\ \rho_i(e,\phi) &= E_\omega\left[r_i(e,\omega,\phi(\omega))\right] \text{ and} \\ \sigma(e) &= E_\omega\left[s(e,\omega)\right]. \end{aligned}$$

The Greek letter simply expresses the expected value of the function with the corresponding Latin letter. To simplify, it is assumed that the bargaining power does not depend on the state of the world and that the expected net social surplus is a single-peaked function of both its arguments. More precisely, we assume the following:

Assumption SP

1. $\alpha_i(\omega) \equiv \alpha_i$, $0 < \alpha_i$, *and* $\alpha_1 + \alpha_2 = 1$

2. $\forall \mu \in [0,1]$, $\mu\sigma(e_i,e_j) - c_i(e_i)$ *is strictly single-peaked* [3] *as a function of* e_i, *its peak being denoted by*

$$B_i^\mu(e_j) = \arg\max_{e_i \in E_i} \mu\sigma(e_i,e_j) - c_i(e_i)$$

3. $B_i^\mu(e_j) < B_i^1(e_j)$ *for all* $\mu < 1$

Suppose party i would receive the fixed share $\mu < 1$ of the social surplus. Then 2. requires that its best response would come from maximizing a strictly single-peaked function whereas 3. requires that it would underinvest relative to the efficient response $B_i^1(e_j)$.

Since we have imposed very little structure, the following remark may be in order. Assumption SP implies that $\sigma(e_i,e_j)$ as a function of e_i must be strictly increasing in the range $[B_i^\mu(e_j), B_i^1(e_j)]$ at least. Moreover, Assumption SP would follow from the following more familiar but also more restrictive assumption:

[3] The function is assumed to be strictly monotonically increasing to the left and strictly monotonically decreasing to the right of its peak.

The sets E_i of feasible investments are intervals of the real line, $s(e,\omega)$ and $c_i(e_i)$ are differentiable functions of e_i, $s(e,\omega)$ is a strictly concave and strictly increasing function of e_i and $c_i(e_i)$ a quasi convex function of e_i, for all states ω, and the appropriate Inada conditions hold.

We now consider the following measures of cooperativeness. Party i's investments are called *cooperative* (at the other party's investment level e_j) if

$$\rho_i(e',\phi) - \rho_i(e,\phi) \leq \alpha_i \left[\sigma(e') - \sigma(e)\right] \tag{6}$$

or, equivalently (as follows from the fixed-sum property (3)), if

$$\rho_j(e',\phi) - \rho_j(e,\phi) \geq \alpha_j \left[\sigma(e') - \sigma(e)\right]$$

holds for all $e' = (e_i', e_j)$ and $e = (e_i, e_j) \in E$ where $e_i' > e_i$ and all $\phi \in \Phi$. In other words, a party's investments are called cooperative if the other party would benefit from increased investments by the first party to a degree not below its own bargaining power. Notice that, under suitable differentiability, condition (6) would follow from the slightly stronger assumption that

$$\alpha_i \frac{\partial p_j(e,\omega,x)}{\partial e_i} \geq (1-\alpha_i) \frac{\partial p_i(e,\omega,x)}{\partial e_i}$$

must hold for all histories (e,ω) and all decisions x (c.f. (2)). In this form, the condition corresponds to conditions (6) and (8) in Che and Hausch [1999].

Party i's investment are called *non-harmful* (at the other party's investment level e_j) if

$$\rho_i(e',\phi) - \rho_i(e,\phi) \leq (1-\varepsilon) \left[\sigma(e') - \sigma(e)\right] \tag{7}$$

or, equivalently (as follows from the fixed-sum property (3)), if

$$\rho_j(e',\phi) - \rho_j(e,\phi) \geq \varepsilon \left[\sigma(e') - \sigma(e)\right]$$

holds for all $e' = (e_i', e_j)$ and $e = (e_i, e_j) \in E$ where $e_i' > e_i$ and all $\phi \in \Phi$. The ε in the above conditions is an arbitrarily small but positive real number. Under non-harmful investments, if one party would increase its investments then the other party would benefit from that increase by a small but positive share. Notice that, under suitable differentiability, condition (7) would follow from the slightly stronger assumption (c.f. again (2)) that

$$\alpha_i \frac{\partial p_j(e,\omega,x)}{\partial e_i} + (1-\alpha_i-\varepsilon)\frac{\partial s(e,\omega)}{\partial e_i} \geq (1-\alpha_i) \frac{\partial p_i(e,\omega,x)}{\partial e_i}$$

must hold for all histories (e, ω) and all decisions x. This condition corresponds, up to ε, to conditions (7) and (9) in Che and Hausch [1999] [4].

Proposition 2 *Under Assumption SP, if, for one party i at least, investments are non-harmful at the other party's efficient level of investments e_j^* then the efficient investment profile cannot be sustained, i.e. $e^* \notin E^N$.*[5]

This proposition easily follows from our Proposition 1. In order to establish this claim, use of the following lemma will be made.

Lemma 1 *Suppose*

1. *$g(e_i) - f(e_i)$ is a monotonically increasing function of e_i,*
2. *$g(e_i) - c_i(e_i)$ is strictly single-peaked as a function of e_i,*
3. *$e_i^g = \arg\max g(e_i) - c_i(e_i)$ and*
4. *$e_i^g \leq e_i < e_i'$.*

Then $f(e_i) - c_i(e_i) > f(e_i') - c_i(e_i')$.

Notice that $f(e_i) - c_i(e_i)$ need not be single-peaked for the lemma to hold. First, we prove the lemma.

Proof. Due to strict single-peakedness, it holds that $g(e_i) - c_i(e_i) > g(e_i') - c_i(e_i')$. Moreover, due to monotonicity, it holds that $g(e_i) - f(e_i) \leq g(e_i') - f(e_i')$ from which the lemma follows immediately. ■

Second, we prove the proposition.

Proof. Let us apply the lemma to the functions $g(e_i) = (1-\varepsilon)\sigma(e_i, e_j^N)$ and $f(e_i) = \rho_i(e_i, e_j^N, \phi)$. It follows from the above Lemma, (7) and Assumption SP that $\rho_i(e_i, e_j^N, \phi) - c_i(e_i)$ is strictly monotonically decreasing to the right of $B_i^{1-\varepsilon}(e_j^*)$. Since this holds for all state contingent decisions ϕ, it follows that $\varepsilon_i^+(e_j^*) \leq B_i^{1-\varepsilon}(e_j^*) < B_i^1(e_j^*) = e_i^*$. Therefore, according to Proposition 1, e^* cannot be sustained as was to be shown. ■

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[4]The ε is needed because we have imposed so little structure and because, in contrast to Che and Hausch, we consider the bargaining power of parties to be fixed.

[5]This proposition generalizes Proposition 3(i) of Che and Hausch.

Under Assumption SP and if investments of party i are non-harmful then, as follows from the above proof, no state contingent decision provides incentives sufficient enough to reach the efficient response. This is the main message of the proposition.

In order to generalize Proposition 3(ii) of Che and Hausch, let us introduce the following assumption which requires a decision to exist at which pre-renegotiation profits are vanishing. In Che and Hausch, this decision would correspond to the zero quantity which, at the same time, they identify with the *Williamson contract*, i.e. no ex-ante contract at all. We allow for interpretations beyond zero quantities.

Assumption 0

There exists a decision $x^0 \in X$ such that pre-renegotiation profits are nil, i.e. $p_i(\beta, x^0) \equiv 0$ for all histories. Hence, for post-renegotiation profits, it must hold that $r_i(\beta, x^0) = \alpha_i s(\beta)$.

Finally, let us define the following set of investment profiles

$$E^\alpha = \{e \in E : e_i \leq B_i^{\alpha_i}(e_j) \text{ } holds \text{ } for \text{ } i = 1, 2\}$$

and let e^{**} be an investment profile such that

$$e_i^{**} = B_i^{\alpha_i}(e_j^{**}) \tag{8}$$

holds for both parties. Notice that, under Assumptions SP and 0, the expected payoff $\nu_i(e, x^0)$ is a strictly single-peaked function of e_i. If its peak is denoted by $\widehat{e}_i(e_j, x^0)$ then condition (8) can equivalently be expressed as $e_i^{**} = \widehat{e}_i(e_j^{**}, x^0)$.

Proposition 3 *Under Assumptions SP and 0, if, for both parties, investments are cooperative at all investment levels of the other party then $E^N \subset E^\alpha$. Moreover, if reaction curves (8) are increasing and have a unique point of intersection e^{**} then the investment profile e^{**} can be sustained by a non-contingent contract and must be the solution to the hold-up problem in the sense that it maximizes the expected net social surplus over all sustainable profiles.*

Proof. Apply Lemma 1 to the functions $g(e_i) = \alpha_i \sigma(e_i, e_j^N)$ and $f(e_i) = \rho_i(e_i, e_j^N, \phi)$. It follows from the lemma, (6) and Assumptions SP and 0 that $\rho_i(e_i, e_j^N, \phi) - c_i(e_i)$ is strictly monotonically decreasing to the right of $B_i^{\alpha_i}(e_j)$. Since this holds for all state contingent decisions ϕ and since $\rho_i(e, x_i^0) = \alpha_i \sigma(e)$, it follows that $B_i^{\alpha_i}(e_j) = \varepsilon_i^+(e_j)$. Therefore, it follows from Proposition 1 that $E^N \subset E^\alpha$ and the first part of the proposition is established.

As for the second part, let us assume that

$$e^0 \in \arg\max_{e \in E^\alpha} \sigma(e) - c_1(e_1) - c_2(e_2). \tag{9}$$

If, for one party i, it were the case that $e_i^0 < B_i^{\alpha_i}(e_j^0) < B_i^1(e_j^0)$ then, as follows from Assumption SP, the social surplus at $e' = (B_i^{\alpha_i}(e_j^0), e_j^0)$ would strictly exceed the one at e^0. Since $e^{'} \in E^\alpha$ as follows from the assumption that reaction functions are increasing, this leads to a contradiction. Therefore, e^0 must be on both reaction curves. Since a unique point of intersection is assumed to exist, it must coincide with e^{**}. Finally, since $\rho_i(e, x^0) = \alpha_i \sigma(e)$, it follows that

$$e_i^0 = \arg\max_{e_i \in E_i} \rho_i(e, x^0) - c_i(e_i)$$

such that any non-contingent contract prescribing decision x^0 sustains the investment profile e^{**}. ■

Under Assumptions 0 and SP and if, for both parties, investments are cooperative then the non-contingent decision x^0 provides the highest incentives to invest. Yet these incentives are not high enough to reach the efficient response. This is the main message of the above proposition.

4 One-sided and one-dimensional investments

Let us consider the case where only one of the parties, say party $i = 1$, invests, its choice being one-dimensional, i.e. $E_1 = E \subset \Re$. We investigate again which investment levels can be sustained. Proposition 1 implies that the following condition necessarily must hold

$$\varepsilon_1^- \leq e_1^N \leq \varepsilon_1^+ \tag{10}$$

for e_1^N to be sustainable. Remember that ε_1^+ (ε_1^-) is defined as $\inf e_1$ $(\sup e_1)$ such that, for all state contingent decisions $\phi \in \Phi$, $v_1(e_1, \phi)$ is strictly monotonically decreasing (increasing) for all investments $e_1 \geq \varepsilon_1^+$ $(e_1 \leq \varepsilon_1^-$, respectively). In this section, we look for conditions which are sufficient for an investment level to be sustainable. The sufficiency part of the problem turns out to be more complicated. In any case, further assumptions have to be imposed.

Assumption SPX:

For any decision $x \in X$, the net expected payoff $\nu_1(e_1, x)$ is strictly single-peaked as a function of e_1, its peak being denoted by $\widehat{e_1}(x)$.

If $r_1(e_1, \omega, x) - c_1(e_1)$ is a strictly concave function of e_1, as is usually assumed, then Assumption SPX would obviously be met.

State contingent decisions enter condition (10). Such decisions may be difficult to implement. Therefore let us assume that non-contingent decisions $x_L, x_H \in X$ exist such that

$$\widehat{e_1}(x_L) \leq e_1^N \leq \widehat{e_1}(x_H) \tag{11}$$

holds for some given investment level $e_1^N \in E_1$. This condition, while in general more restrictive than (10), is now shown to be sufficient for the investment level e_1^N to be sustainable provided that Assumption SPX is met. In fact, consider the following contract which specifies the decision x_H ex ante but which gives party 2 the option to decision x_L at strike price $S = \rho_1(e_1^N, x_H) - \rho_1(e_1^N, x_L)$. The option must be exercised before ω unravels but after investments can be observed. The option if exercised affects the bargaining positions of both parties.

Proposition 4 *Under Assumption SPX* [6]*, if there exist decisions x_L and x_H, such that condition (11) is met then the above party 2 option contract sustains the investment level e_1^N.*

Proof. Party 2 exercises the option iff $\rho_2(e_1, x_L) - S \geq \rho_2(e_1, x_H)$ which, as follows from the fixed-sum property (3), is equivalent to

$$\rho_1(e_1, x_H) - \rho_1(e_1, x_L) \geq S. \tag{12}$$

Notice that, by assumption, the option is exercised if party 2 is indifferent between exercising and abandoning it. If (12) holds and party 2 exercises the option then party 1 receives $\rho_1(e_1, x_L) - c_1(e_1) + S$ whereas if (12) is violated

[6] It would be sufficient to require that $\nu_1(e_1, x)$ is strictly single-peaked for the two decisions x_L and x_H.

then party 1 receives $\rho_1(e_1, x_H) - c_1(e_1)$. Therefore, party 1's net payoff can equivalently be summarized by either

$$\rho_1(e_1, x_L) - c_1(e_1) + \min\left[S, \rho_1(e_1, x_H) - \rho_1(e_1, x_L)\right] \tag{13}$$

or

$$\rho_1(e_1, x_H) - c_1(e_1) + \min\left[S + \rho_1(e_1, x_L) - \rho_1(e_1, x_H), 0\right]. \tag{14}$$

If $e_1 \geq e_1^N \geq \widehat{e_1}(x_L)$ then, as follows from Assumptions SPX, (11) and (13), party 1's net payoff does not exceed $\rho_1(e_1, x_L) - c_1(e_1) + S$ such that, in this range, the optimum choice must be e_1^N leading to net payoff $\rho_1(e_1^N, x_L) - c_1(e_1^N) + S = \rho_1(e_1^N, x_H) - c_1(e_1^N)$. If $e_1 \leq e_1^N \leq \widehat{e_1}(x_H)$ then, as follows from (11) and (14), party 1's net payoff does not exceed $\rho_1(e_1, x_H) - c_1(e_1)$ such that, in this range, the optimum payoff cannot be higher than under the optimum in the other range. This establishes the proposition. ■

Edlin and Reichelstein [1996] provide conditions under which the efficient investment level e_1^* can be sustained by a non-contingent contract. They consider a setting where the decision $x^0 \in X$ (c.f. Assumption 0) provides the lowest incentives to invest. They further assume another decision x^{ph} to exist for which

$$\rho_1(e_1', x^{ph}) - \rho_1(e_1, x^{ph}) \geq \sigma(e_1') - \sigma(e_1) \tag{15}$$

holds for all $e_1' > e_1 \in E_1$. Due to the fixed-sum property (3), the condition (15) is equivalent to $\rho_2(e_1', x^{ph}) - \rho_2(e_1, x^{ph}) \leq 0$, i.e. party 2 would never benefit if party 1 were to increase its investments and if the decision x^{ph} were taken. In the spirit of our earlier terminology, such an investment is called *potentially harmful* for the other party. This decision, as can be shown in the usual way, provides incentives to invest at least as much as under the efficient response. Therefore, if $\nu_1(e_1, x)$ is strictly single-peaked as a function of e_1 for both $x = x^0$ and $x = x^{ph}$ then

$$\widehat{e_1}(x^0) \leq e_1^* \leq \widehat{e_1}(x^{ph}) \tag{16}$$

must hold. By making use of the intermediate value theorem, Edlin and Reichelstein conclude that a decision $\overline{x} \in X$ has to exist such that

$$\arg\max_{e_1 \in E_1} \nu_1(e_1, \overline{x}) = e_1^*.$$

It then follows that any non-contingent contract prescribing decision $\overline{x}$ provides the efficient incentives to invest. Their approach requires further assumptions which we have not imposed [7]. In any case, if (16) is met then the efficient investment level can be sustained by an option contract as follows from the above proposition.

To conclude this section an additional assumption is introduced which guarantees that the condition (11) is not only sufficient but also necessary for the investment level e_1^N to be sustainable.

Assumption LH:

1. The set E_1 of feasible investment levels is a connected interval of the real line $\Re$.

2. For all $e_1^N \in E_1$, there exists some decision $x_L = x_L(e_1^N) \in X$ such that

   $$E_\omega\left[r_1(e_1,\omega,x_L) - r_1(e_1^N,\omega,x_L)\right] \leq E_\omega\left[r_1(e_1,\omega,\phi(\omega)) - r_1(e_1^N,\omega,\phi(\omega))\right]$$

   holds for all state contingent decisions $\phi \in \Phi$ and all investment levels $e_1 > e_1^N \in E_1$ sufficiently close to e_1^N.

3. For all $e_1^N \in E_1$, there exists some decision $x_H = x_H(e_1^N) \in X$ such that

   $$E_\omega\left[r_1(e_1^N,\omega,\phi(\omega)) - r_1(e_1,\omega,\phi(\omega))\right] \leq E_\omega\left[r_1(e_1^N,\omega,x_H) - r_1(e_1,\omega,x_H)\right]$$

   holds for all state contingent decisions $\phi \in \Phi$ and all investment levels $e_1 < e_1^N \in E_1$ sufficiently close to e_1^N.

Notice that, under suitable differentiability, Assumption LH would imply that

$$\frac{\partial E_\omega\left[r_1(e_1,\omega,x_L)\right]}{\partial e_1} \leq \frac{\partial E_\omega\left[r_1(e_1,\omega,\phi(\omega))\right]}{\partial e_1} \leq \frac{\partial E_\omega\left[r_1(e_1,\omega,x_H)\right]}{\partial e_1} \tag{17}$$

must hold for all state contingent decisions $\phi \in \Phi$ and all investment levels $e_1 \in E_1$. In other words, marginal returns form investments are highest under the non-contingent decision $x_H = x_H(e_1)$ whereas they are lowest

[7] In particular, they assume that X is a one-dimensional and connected quantity choice, investments enter the pre-renegotiation profit function of the investing party only and that $\arg\max_{e_1} \nu_1(e_1, x)$ is a continuous function of x.

under $x_L = x_L(e_1)$. While Assumption LH is slightly more restrictive than (17), it allows to simplify the proof of the following proposition.

Assumption LH extends the measures of cooperativeness in the following sense. If party 1's investments were cooperative and if Assumption 0 is met then the non-contingent decision x^0 provides the highest incentives to invest. In this case, irrespective of e_1^N, we can choose $x_H(e_1^N) \equiv x^0$ in condition 3. of the assumption. The assumption, however, allows that the non-contingent decision may depend on the level of investments it has to sustain. In the setting of Edlin and Reichelstein [1996], the non-contingent decision x^0 provides the lowest incentives to invest. In this case, we could choose $x_L(e_1^N) \equiv x^0$ in condition 2. of Assumption LH.

Proposition 5 *Suppose that only party 1 invests and Assumptions SPX and LH are met. If the investment level e_1^N can be sustained, i.e. if $e_1^N \in E^N$ then there exist non-contingent decisions $x_L, x_H \in X$ such that (11) holds.*

Proof. Since e_1^N is sustainable, as in the proof of Proposition 1, a choice function $f : B = E_1 \times \Omega \to X$ must exist which induces incentives e_1^N, i.e. for which

$$c_1(e_1^N) - c_1(e_1) \leq E_\omega \left[r_1(e_1^N, \omega, f(e_1, \omega)) - r_1(e_1, \omega, f(e_1, \omega)) \right]$$

holds for all investments $e_1 \in E_1$.

To establish the proposition, take the decisions $x_L = x_L(e_1^N)$ and $x_H = x_H(e_1^N)$ such that Assumptions LH 2. and 3. are met. We then claim that, for these decisions, (11) holds. Assume the contrary, e.g. $\widehat{e_1}(x_L) > e_1^N$. It then follows from Assumption SPX that $\nu_1(e_1^N, x_L) < \nu_1(e_1, x_L)$ must hold for all $e_1^N < e_1 < \widehat{e_1}(x_L)$. In particular,

$$E_\omega \left[r_1(e_1^N, \omega, x_L) - r_1(e_1, \omega, x_L) \right] < c_1(e_1^N) - c_1(e_1)$$

must hold for all $e_1 > e_1^N$ sufficiently close to e_1. It then follows from Assumption LH 2. that

$$E_\omega \left[r_1(e_1, \omega, x_L) - r_1(e_1^N, \omega, x_L) \right] \leq E_\omega \left[r_1(e_1, \omega, f(e_1, \omega)) - r_1(e_1^N, \omega, f(e_1, \omega)) \right]$$

must hold which, however, leads to a contradiction. Therefore, $\widehat{e_1}(x_L) \leq e_1^N$ is established. The other inequality of (11) can be shown to hold by a similar argument. ■

The above proposition shows that condition (11) is necessary for an investment level to be sustainable. If, on the other side, condition (11) is met then the investment level can be sustained by an option contract as follows from the previous proposition. In this sense, condition (11) is necessary and sufficient for the investment level e_1^N to be sustainable, provided of course that the assumptions needed for the corresponding propositions are met.

5 Multi-dimensional investments

Some of the results on one-dimensional investments can easily be extended to the case of multi-dimensional investments as we now want to show. It is assumed that both parties have multi-dimensional investment choices $e_i \in E_i \subset \Re^{l_i}$. Let $\Phi = \{\phi : \Omega \to X\} \subset \Re^{mn}$ again denote the set of all state contingent decisions ϕ and let $\nu_i(e, \phi) = E_\omega[r_i(e, \omega, \phi(\omega))] - c_i(e_i)$ denote, as before, the net profit which party i would make if the investment profile were $e \in E$ and if the state contingent decision $\phi \in \Phi$ were implemented.

In order to extend the analysis to multi-dimensional investments, the concepts of section 3, Assumption SP in particular, must be adapted. Single-peakedness is a one-dimensional concept. To make use of it in the multi-dimensional setting, we introduce the following notion of investment directions. Let $\Delta_i = \{(d_1, d_2) \in \Re^{l_1} \times \Re^{l_2} : d_j = 0\}$ denote the set of such *investment directions* for player i and let $\Lambda_i(e_j, d_i) = \{\lambda \in \Re : e + \lambda d_i \in E\}$ denote the set of feasible investment intensities at investment profile $e \in E$ in direction of $d_i \in \Delta_i$. Since E is assumed to be closed it follows that $\Lambda_i(e_j, d_i)$ must be closed as well. Moreover, since $e \in E$, it follows that $\Lambda_i(e_j, d_i)$ contains 0 and, hence, must be non-empty. For all $e_j \in E_j$, $d_i \in \Delta_i$ and $\phi \in \Phi$, let

$$\varepsilon_i^\phi(e_j, d_i, \phi) = \inf_{\lambda \in \Lambda_i(e_j, d_i)} \lambda$$

such that $\nu_i(e + \lambda' d_i, \phi)$ is strictly monotonically decreasing for all $\lambda' \geq \lambda$. Moreover, let

$$\varepsilon_i(e_j, d_i) = \sup_{\phi \in \Phi} \varepsilon_i^\phi(e_j, d_i, \phi).$$

Using this notation, the following result, which extends Proposition 1 to multi-dimensional investments, can be established.

Proposition 6 *If the investment profile e^N is sustainable, i.e. if $e^N \in E^N$ then*

$$\inf_{d_i \in \Delta_i} \varepsilon_i(e_j, d_i) \geq 0$$

must hold for both parties.

Proof. Since e^N is sustainable a choice function $f : B \to X$ must exist which induces incentives e^N such that (5) holds for all investment choices $e_i \in E_i$.

To establish the proposition, assume the contrary which means that an investment direction $d_i \in \Delta_i$ must exist such that $\varepsilon_i^{\sup} = \varepsilon_i(e_j^N, d_i) < 0$. Let $e^{\sup} = e^N + \varepsilon_i^{\sup} d_i$. It follows from the definition of $e^{\sup}$ that $\nu_i(e^{\sup}, \phi) > \nu_i(e^N, \phi)$ must hold for all $\phi \in \Phi$, in particular for $\phi = f(e^{\sup}, \omega)$. But this contradicts (5). Therefore $\varepsilon_i^{\sup} \geq 0$ must hold as was to be shown. ■

The proposition provides a necessary condition for the efficient investment profile e^* to be sustainable. In fact, if there exists a single direction of investment d_i for only one of the parties such that $\lambda_i(e_j^*, d_i, \phi) < 0$ holds for all state contingent decisions $\phi \in \Phi$ then e^* cannot be sustained by a message contingent contract, no matter how sophisticated this contract might be. To elaborate on this idea, one of the measures of cooperativeness as introduced in the previous section is extended to multi-dimensional investments in the following way. For simplicity, we restrict the analysis to the efficient investment profile e^*.

For some party i, fix a direction $d_i \in \Delta_i$, $d_i \neq 0$. For all $\lambda \in \Lambda_i = \Lambda_i(e_j^*, d_i)$ consider histories of the form $\beta = (e^* + \lambda_i d_i, \omega)$. Define

$$\begin{aligned} \pi_i(\lambda, \phi) &= E_\omega \left[p_i(\beta, \phi(\omega)) \right], \\ \pi_j(\lambda, \phi) &= E_\omega \left[p_j(\beta, \phi(\omega)) \right], \\ \rho_i(\lambda, \phi) &= E_\omega \left[r_i(\beta, \phi(\omega)) \right], \\ \gamma_i(\lambda) &= c_i(e + \lambda_i d_i) \text{ and} \\ \sigma(\lambda) &= E_\omega \left[s(\beta) \right]. \end{aligned}$$

The Greek letter expresses the expected value of the function with the corresponding Latin letter in the direction of some given d_i. Using this notation, Assumption SP can be extended in the following way:

Assumption SPM

1. $\alpha_i(\omega) \equiv \alpha_i$, $0 < \alpha_i$, *and* $\alpha_1 + \alpha_2 = 1$

2. *For all* $\mu \in [0,1]$, $\mu\sigma(\lambda) - \gamma_i(\lambda)$ *is strictly single-peaked as a function of* λ, *its peak being denoted by*

$$B_i^\mu = \arg \max_{\lambda \in \Lambda_i(e_j, d_i)} \mu\sigma(\lambda) - \gamma_i(\lambda)$$

3. $B_i^\mu < B_i^1$ *for all* $\mu < 1$.

Suppose party i would receive the fixed share $\mu < 1$ of the social surplus. Then 2. requires that its best response in the given direction would come from maximizing a strictly single-peaked function whereas 3. requires that it would underinvest relative to the efficient investment intensity $B_i^1(e_j)$ (in the given direction). Similar to the one-dimensional case, Assumption SPM implies that $\sigma(\lambda)$ must be strictly increasing in the range $[B_i^\mu, B_i^1]$ at least. Moreover, Assumption SPM would follow from the following more familiar but also more restrictive assumption:

The sets E_i *of feasible investments are convex subsets,* $s(e,\omega)$ *and* $c_i(e_i)$ *are differentiable functions,* $s(e,\omega)$ *is a strictly concave and* $c_i(e_i)$ *a convex function of* e_i, *for all states* ω, *and the appropriate Inada conditions hold. Moreover, the given direction must be positive in the sense that*

$$\frac{d\sigma}{d\lambda} = E_w\left[\sum_{k=1}^{l_i} \frac{\partial s(e^* + B_i^\mu d_i, \omega)}{\partial e_{ik}} d_{ik}\right] > 0$$

holds for $\mu \in [0,1]$.

Assumption SPM extends Assumption SP in an obvious way. We now call the investment direction d_i *non-harmful* if

$$\rho_i(\lambda', \phi) - \rho_i(\lambda, \phi) \leq (1-\varepsilon)\left[\sigma(\lambda') - \sigma(\lambda)\right],$$

holds for all $\lambda' > \lambda \in \Lambda_i(e_j, d_i)$ (c.f. (7)). As before, ε is some arbitrarily small but positive real number. Proposition 2 can now be extended to the multi-dimensional case as follows:

Proposition 7 *If, for one of the parties, an investment direction exists such that Assumption SPM holds and such that this direction is not harmful to the other party then the efficient profile cannot be sustained, i.e.* $e^* \notin E^N$.

The proof makes use of Proposition 6 in exactly the same way as Proposition 2 does of Proposition 1. Therefore, the argument need not be repeated here. In principle, Proposition 3 could also be extended to the present case. Due to the great variety of investment directions, however, the analysis becomes more intricate and will not be pursued.

6 Investments which can be aggregated

So far, conditions which are necessary and sufficient for an investment profile to be sustainable have only been derived for the case of one-sided and one-dimensional investments (see section 4). In the present section, it is shown that similar results can be obtained if investments are two-sided and multi-dimensional but can be aggregated into one dimension. The following assumption is due to Segal and Whinston [1999].

Assumption A

There exists an aggregation function $\overline{a} : E = E_1 \times E_2 \to A \subset \Re$ *such that the post-renegotiation profit functions can be written as*

$$r_i(e, \omega, x) = g_i(e, \omega) + h_i(\overline{a}(e), \omega, x).$$

Notice, if Assumption A holds for one of the parties then it also does so for the other party as follows from the fixed-sum property (3). Moreover, without further loss of generality, the following zero-sum property can be imposed:

$$h_1(a, \omega, x) + h_2(a, \omega, x) \equiv 0. \tag{18}$$

For later reference, we introduce the set of aggregated investments which party i can generate. For given investments $e_j^N \in E_j$ of the other party, this set is defined as

$$A_i(e_j^N) = \{a \in A : \exists e_i \in E_i \text{ such that } \overline{a}(e_i, e_j^N) = a\}.$$

The next assumption is closely related to Assumption H$\pm$ of Segal and Whinston. However, to simplify the analysis, we express the assumption in terms of differences instead of derivatives. The assumption extends our earlier Assumption LH to the case of two-sided investments which can be aggregated into one dimension.

Assumption LHA

1. *The sets* $A_i(e_j^N)$ *are connected subsets of the real line* $\Re$.

2. *For all* $a^N \in A$*, there exists some decision* $x_{iL} = x_{iL}(a^N) \in X$ *such that*

$$E_\omega\left[h_i(a,\omega,x_{iL}) - h_i(a^N,\omega,x_{iL})\right] \leq E_\omega\left[h_i(a,\omega,\phi(\omega)) - h_i(a^N,\omega,\phi(\omega))\right]$$

holds for all state contingent decisions $\phi \in \Phi$ *and all* $a > a^N \in A_i(e_j^N)$ *sufficiently close to* a^N.

3. *For all* $a^N \in A$*, there exists some decision* $x_{iH} = x_{iH}(a^N) \in X$ *such that*

$$E_\omega\left[h_i(a^N,\omega,\phi(\omega)) - h_i(a,\omega,\phi(\omega))\right] \leq E_{Gw}\left[h_i(a^N,\omega,x_{iH}) - h_i(a,\omega,x_{iH})\right]$$

holds for all state contingent decisions $\phi \in \Phi$ and all $a < a^N \in A_i(e_j^N)$ *sufficiently close to* a.

Notice, if Assumption LHA holds for one of the parties then it also does so for the other party as follows from the zero-sum property (18). Moreover, under differentiability, it would follow from Assumption LHA that

$$\frac{\partial E_\omega\left[h_i(a^N,\omega,x_{iL}(a^N))\right]}{\partial a} \leq \frac{\partial E_\omega\left[h_i(a^N,\omega,x)\right]}{\partial a} \leq \frac{\partial E_\omega\left[h_i(a^N,\omega,x_{iH}(a^N))\right]}{\partial a}$$

must hold for all $x \in X$ and $\omega \in \Omega$. In other words, the marginal revenues from aggregated investments are lowest (highest) at decision x_{iL} (x_{iH}). While Assumption LHA is slightly more restrictive than the condition on marginal revenues, Assumption LHA allows again to simplify proofs [8].

Segal and Whinston, in their Assumption C, assume strict concavity with respect to investments in order to ensure that first order conditions of incentives to invest are sufficient. Our next assumption achieves the same by imposing single-peakedness with respect to aggregated investments. Concerning disaggregated investments, no further assumption will be needed. In order to formulate the assumption, the following definition is introduced. For

[8] Segal and Whinston impose $\frac{\partial h_i(a^N,\omega,x_{iL}(a^N))}{\partial a} \leq \frac{\partial h_i(a^N,\omega,x)}{\partial a} \leq \frac{\partial h_i(a^N,\omega,x_{iH}(a^N))}{\partial a}$ from which our condition on marginal revenues would follow by integration (see their assumption H±).

any aggregated investment which party i can generate, i.e. for all $a \in A_i(e_j^N)$, consider the function

$$G_i(a; e_j^N) = \max_{e_i \in E_i} E_\omega \left[g_i(e_i, e_j^N, \omega) \right] - c_i(e_i)$$

subject to $\overline{a}(e_i, e_j^N) = a$. This function captures that part of the post-renegotiation profit function, including investment costs, which does not depend on the decision $x \in X$. The following assumption corresponds to Assumption SPX which was introduced in the section on one-sided investments.

Assumption SPXA

For all decisions $x \in X$, party i's expected net profit

$$G_i(a; e_j^N) + E_\omega \left[h_i(a, \omega, x) \right]$$

is a strictly single-peaked function of $a \in A_i$, its peak being denoted by $\widehat{a}_i(x; e_j^N)$.

The next proposition introduces two conditions which are sufficient for an investment profile to be sustainable. The first condition (19) is in the spirit of Propositions 1 and 6. Notice that, due to Assumption LHA and as in Proposition 5, only fixed instead of state contingent decisions must be taken into account to restrict the set of sustainable investment profiles. The second condition (20) simply follows from the zero-sum property (18).

Proposition 8 *Suppose we are given a sustainable investment profile $e^N \in E$ such that Assumptions A, LHA and SPXA* [9] *are met. Then there exist non-contingent decisions x_{iL} and $x_{iH} \in X$ such that, at these decisions, party i would under- and overinvest, respectively, i.e.*

$$\widehat{a}_i(x_{iL}, e_j^N) \leq a^N = \overline{a}(e^N) \leq \widehat{a}_i(x_{iH}, e_j^N). \tag{19}$$

Moreover,

$$a^N \in \arg \max_{a \in A_1(e_2^N) \cap A_2(e_1^N)} G_1(a; e_2^N) + G_2(a; e_1^N) \tag{20}$$

must also hold.

[9] It would be sufficient to require that Assumption SPXA holds for the two decisions $x_{iL}(\overline{a}(e^N))$ and $x_{iL}(\overline{a}(e^N))$.

Proof. Since e^N is sustainable, there exists a message contingent contract $\gamma = [M_1, M_2, x(m), t_1(m), t_2(m)]$ and a Nash equilibrium $m = m^\gamma(e, \omega)$ of the message game such that

$$e_i^N \in \arg\max_{e_i \in E_i} E_\omega \left[R_i(e_i, e_j^N, \omega)\right] - c_i(e_i) \tag{21}$$

must hold for both parties where

$$R_i(e, \omega) = g_i(e, \omega) + h_i(\overline{a}(e), \omega, x(m^\gamma(e, \omega))) + t_i(m^\gamma(e, \omega))$$

denotes the payoff excluding investment costs under the given contract. It follows from the Min-Max-Theorem, that this payoff can be written in the form

$$R_i(e, \omega) = g_i(e, \omega) + H_i(\overline{a}(e), \omega)$$

where $H_i : A \times \Omega \to \Re$ is a function properly chosen. Moreover, since $[R_1(), R_2()]$ is implementable it follows that, for all histories $\beta = (a, \omega)$ and $\beta' = (a', \omega') \in A \times \Omega$, there exist decisions $x(\beta, \beta')$ and $x(\beta', \beta) \in X$ such that

$$h_i(\beta', x(\beta; \beta')) - h_i(\beta, x(\beta; \beta')) \leq H_i(\beta') - H_i(\beta) \leq h_i(\beta', x(\beta'; \beta)) - h_i(\beta, x(\beta'; \beta)) \tag{22}$$

must hold.

Condition (21) implies that

$$e_i^N \in \arg\max_{e_i \in E_i} G_i(\overline{a}(e_i, e_j^N), e_j^N) + E_\omega \left[H_i(\overline{a}(e_i, e_j^N), \omega)\right]$$

and, hence, that

$$a^N \in \arg\max_{a \in A_i} G_i(a, e_j^N) + E_\omega \left[H_i(a, \omega)\right]. \tag{23}$$

By making use of the zero-sum property (18), condition (23) leads to (20).

To establish condition (19), it is now shown that

$$\widehat{a}_i(x_{iL}(a^N), e_j^N) \leq a^N = \overline{a}(e^N) \leq \widehat{a}_i(x_{iH}(a^N), e_j^N)$$

must hold. Assume the contrary, e.g. $\widehat{a}_i(x_{iL}(a^N)) > a^N$. It then follows from Assumption SPXA that

$$G_i(a^N; e_j^N) + E_\omega \left[h_i(a^N, \omega, x_{iL}(a^N)\right] < G_i(a; e_j^N) + E_\omega \left[h_i(a, \omega, x_{iL}(a^N)\right]$$

hods for all $a > a^N$ sufficiently close to a^N. Moreover, as follows from Assumption LHA and (22),

$$E_\omega\left[h_i(a,\omega,x_{iL}(a^N)) - h_i(a^N,\omega,x_{iL}(a^N))\right] \leq E_\omega\left[H_i(a,\omega) - H_i(a^N,\omega)\right]$$

which contradicts (23). Therefore $\widehat{a}_i(x_{iL}(a^N), e_j^N) \leq a^N$ must be true. That $a^N \leq \widehat{a}_i(x_{iH}(a^N), e_j^N)$ also holds can be shown analogously. The proposition is established. ■

The final proposition establishes that, in essence, the two conditions (19) and (20) are sufficient as well for an investment profile to be sustainable. The proposition is the only one in this paper which requires continuity and differentiability of certain functions. Assumption D spells out the details [10].

Assumption D

1. For the given *decision* e_j^N *of the other party, the functions* $G_i(a; e_j^N)$ *and* $E_\omega[h_i(a,\omega,x)]$ *are differentiable with respect to aggregated investments* $a \in A$ *(for all decisions* $x \in X$*).*

2. *The set of decisions* X *is pathwise connected.*

3. For the given *decision* e_j^N *of the other party, the peak* $\widehat{a}_i(x, e_j^N)$ *is a continuous function of* x*.*

Proposition 9 *Suppose we are given an investment profile* $e^N \in E$ *such that Assumptions A, SPXA and D are met and suppose there exist for one of the players, say player* $i = 1$*, decisions* x_L *and* $x_H \in X$ *such that*

$$\widehat{a_1}(x_L, e_2^N) < a^N = \overline{a}(e^N) < \widehat{a_1}(x_H, e_2^N) \tag{24}$$

holds [11]*. Moreover, condition (20) is also assumed to be met. Then the investment profile* e^N *is sustainable and there even exists a non-contingent contract which sustains the profile.*

[10] Notice that Segal and Whinston, in essence, impose similar assumptions.

[11] The strict inequalities ensure that a^N is in the interior of $A_1 \cap A_2$. If the inequalities fail to be strict but if a^N is nevertheless in the interior of $A_1 \cap A_2$ then the proof still works. Notice that Segal and Whinston have to assume that investments, not just aggregated investments are in the interior.

Proof. The proof can easily be adapted from Segal and Whinston. In fact, it follows from (24) that a^N is in the interior of A_i and, hence, from (20) that

$$\frac{dG_1(a^N, e_2^N)}{da} + \frac{dG_2(a^N, e_1^N)}{da} = 0. \tag{25}$$

Moreover, by Assumption D, there exists a decision $\widehat{x} \in X$ such that, for $i = 1$,

$$\widehat{a}_i(\widehat{x}, e_j^N) = a^N = \overline{a}(e^N, e_j^N). \tag{26}$$

Hence, for $i = 1$,

$$\frac{dG_i(a^N, e_j^N)}{da} + \frac{d}{da}E_\omega\left[h_i(a^N, \omega, \widehat{x})\right] = 0. \tag{27}$$

Since, according to the zero-sum property (18),

$$\frac{d}{da}E_\omega\left[h_1(a^N, \omega, \widehat{x})\right] + \frac{d}{da}E_\omega\left[h_2(a^N, \omega, \widehat{x})\right] = 0,$$

it follows from (25) and (27) that (27) must hold for $i = 2$ as well. Assumption SPXA implies that the first-order conditions are sufficient and, hence, that (26) must hold for $i = 2$ as well. Therefore, any non-contingent contract prescribing the fixed decision $\widehat{x}$ provides incentives to invest e^N. The proposition is established. ■

Under the assumptions made in the present section, neither option contracts nor more general message contingent contracts allow to provide incentives to invest beyond what already can be achieved by simple non-contingent contracts [12]. In other words, it is enough for parties ex ante to agree on a suitable but fixed decision and to rely on renegotiations after they both have observed investment decisions and the state of the world. This result is due to the fact that investment decisions are simultaneous. If they were sequential, option contracts could be of use as Nöldeke and Schmidt [1998] have shown.

7 Concluding remarks

If an investment profile can be sustained by a message contingent contract then a choice function must exist which induces the post-renegotiation payoff

[12] See also Proposition 6 in Segal and Whinston [1999].

frontier in the sense of equation (5). The existence of such a choice function has led to a simple condition which nessecarily must hold for an investment profile to be sustainable. While the condition can be formulated in very general terms, it will typically fail to be sufficient (except for the cases as spelled out in sections 4 and 6 of the paper). It would be desirable to find sufficient conditions, in particular for those situations in which truly message contingent contracts cannot be dispensed with.

One might try to proceed as follows. If the set X of decisions is pathwise connected then any sustainable payoff frontier $[R_1(\beta), R_2(\beta)]$ can be *generated* by a choice function $f : B \to X$ in the sense that, for a given base β^0, $R_i(\beta) = R_i(\beta^0) + r_i(\beta, f(\beta)) - r_i(\beta^0, f(\beta))$ must hold for all histories β [13]. Moreover, it is quite easy to characterize the choice functions which induce a given profile of investments. The only remaining problem concerns conditions which are sufficient for a payoff frontier, generated by a choice function in the above sense, to be sustainable by some message contingent contract. While the incentive constraints would easily lead to such a condition, namely, for any two histories β and β', two decisions x and x' must exist such that

$$\begin{aligned} r_1(\beta', x) - r_1(\beta, x) &\leq \Big[r_1(\beta', f(\beta')) - r_1(\beta^0, f(\beta'))\Big] - \\ \Big[r_1(\beta, f(\beta)) - r_1(\beta^0, f(\beta))\Big] &\leq r_1(\beta', x') - r_1(\beta, x') \end{aligned}$$

holds, this condition is cumbersome and difficult to handle. How to make this condition more easily accessible has to remain the subject of future research.

8 References

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[13] Notice that our notion of generating payoff frontiers must carefully be distinguished from the notion which Segal and Whinston rely on and which parallels the one which Mirrlees [1971] had pioneered in a setting of one-dimensional private information.

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