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The Delegation Perspective on Representative Democracy*

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Abstract

Why do political constituencies delegate decision power to representative assemblies? And how is the size of such assemblies determined? We analyze these questions of constitutional design in a model with voters learnin their preferred alternative only after engaging in costly information gathering. We show that there is an optimal assembly size that would be chosen at a constitutional stage. This implies a relationship between assembly and constituency size. We then compared this relationship to actual data. Fitting a single parameter to the data, we show that our model can explain the actual relationship between assembly and constituency size quite well.

Keywords: Constitutional Design, Representative Democracy, Parliament Size, Information Costs.

JEL-Classification: D72, D82, P16.

1 Introduction

"The smallness of the proportion of representatives had been considered by many members of the Convention, an insufficient security for the rights and interests of the people." [James Madison 1787, [9, p.579]

Why are there representative assemblies like parliaments or councils in almost all democracies? We suggest an answer to this question by focussing on the delegation perspective on representation; that is, we focus on the fact that representative democracy is a way of governing by delegating most decisions to the representative assembly. Hence we ask: under what circumstances would a rational electorate decide, on a constitutional stage, to install government by representative assembly? And does this tell us anything about the right interrelation between electorate and assembly size? If so, how does this compare with the interrelation we find in real world democracies?

The emphasis in our model is on the informational asymmetry between voters and assembly members. The empirical literature is quite unambiguous on the fact that the average voter is very poorly informed about most political issues (cf. Althaus [1], Bartels [3] and the literature surveyed there). By contrast, assembly members hold their office as a full-time job and they also have subordinates who provide them with information.[1] So they are in a much better position to predict the way they themselves will be affected by

[1] For example, a survey in the US showed that only half of the interviewed could name the majority party in the House of Representatives and less than 60% knew what a "recession" was. However, an assembly member would have to know how to vote on propositions like the one "to amend the Controlled Substances Act to add gamma hydroxybutyric acid and ketamine to the schedules of control substances" (an arbitrary example picked from the US House agenda Nov. 1999).

a certain policy decision, whereas voters, in general, lack this knowledge. Some authors suggest that many voters would vote differently if they were better informed (Bartels [3]).

In our model, we take the extreme stance that no voter knows how a policy decision affects himself while assembly members know this with certainty.[2] An assembly, in our model, is a random draw from the electorate and hence, from a constitutional or ex ante perspective, it is a device which estimates the majority preference of the electorate. The way this device works is simply by informing the assembly members on their own preferences. Knowing her own policy preference, each assembly member votes accordingly in the assembly. Thus, the decision made by the assembly (via majority voting) reflects – albeit to a limited degree – the decision the electorate would have made in a referendum if all voters had been fully informed. Assembly members bring about costs for the society which consist of their remuneration and of their office cost. We assume that these are the costs of getting the assembly member informed and that these costs are shared equally among the electorate.

We show that under these assumptions a rational electorate will decide to install an assembly, irrespective of the distribution of preferences in the electorate, as long as enough is at stake in the policy decisions compared to the office costs. This result relies on some version of the median-voter theorem. After deriving some intuitive comparative static results, we use this model to explain data on the empirical relation between electorate and assembly size. The single parameter that has to be estimated for testing our model is the cost per representative. So we needed samples where we could expect this cost to be similar and where nevertheless the assemblies had real decision power. Hence we looked for groups of electorates and assemblies forming federations or federation-like systems. The data we used are from Switzerland, Germany, and the European Union.

The paper is organized in the following way. In Section 2, we provide a brief sketch

[2] In the sequel, we will refer by the term "assembly" to a body to which a decision is delegated like parliaments, councils, or standing committees.

of the related literature. This is followed by the model and some comparative static results in Section 3. In Section 4, we provide the exact relationship between assembly and population size by assuming that all majority shares are equally likely. In Section 5, we use the result derived in Section 4 to explain data on the empirical relation between electorate and assembly size. In Section 6, we conclude our analysis. Some proves are relegated in an appendix in Section 7.

2 Related Literature

To the best of our knowledge, the first to address the issue of justifying representative government, in a somewhat formal way, were Buchanan and Tullock [6]. They discuss, in a short section (in Chapter 15), the optimal ratio of assembly members to the electorate (or the optimal degree of representation). They introduce the concepts of external and decision-making costs of representation. The former is the expected cost to a single voter of decisions against his preference and the latter is the cost of finding a majority agreement in the assembly. Assuming both functions to be positive for all assembly sizes and the first one decreasing and the second one increasing, they define the optimal degree of representation as the assembly size which minimizes the sum of both cost functions. Although there is no reference to information asymmetry and no further foundation or specification of the cost functions, they suggest comparative static results similar to ours. They show that the optimal degree of representation is increasing in electorate size at a less than proportional rate. This result is justified by their assumption that the external cost is 'nearly' unaffected by an increase in the size of the electorate.

Since Buchanan and Tullock [6], the issue has not been properly addressed.[3] Closest

[3] Yet, there seems to be a growing literature in political economy on representation. This literature deals with different questions and from a different perspective. Cf. Austen-Smith/Banks [2], Feddersen/Pesendorfer [7], Besley/Coate [4], and Osborne/Slivinski [10].

to our approach is the paper by Tanguiane [11]. Although his paper suggests a way out of Arrow's paradox, there are similarities in the way the democratic process is perceived. Tanguiane presents measures of representativeness of a single politician's preferences for the community's preference profile and extends this measure to cabinets and councils. This "capability of individuals to represent the social preference" is measured by the weight of a coalition and by the probability of a choice situation. Policy choice is perceived as the repeated selection between binary alternatives. Given a probability measure on the set of all possible binary policy decisions of the community, a single decision maker is the more representative for the community the higher the measure of those instances in which he represents a majority ("majority representativeness"). For a council, Tanguiane takes the majority opinion of the council on each binary decision and compares it with the majority opinion of the community. He defines the representativeness of a randomly chosen council with k voters and the maximal value of a council's representativeness. The former variable increases in council size. Tanguiane derives a lower bound for this variable depending on the probability of the society being divided into almost equal coalitions.

In the same spirit as in Tanguiane's model, we describe the election of a council (assembly) by a random draw from the electorate. Similarly, we will be concerned with the probability measure of those instances where the assembly majority is at odds with the electorate majority. Finally, increasing "representativeness" of the assembly in assembly size is a property we obtain in our framework, too. But in our model, we explicitly take the information problem into account, hence we need not refer to the notion of representativeness. Instead, we take a constitutional viewpoint. Unlike Tanguiane, we identify conditions under which a rational electorate would install a representative democracy at a constitutional stage. Moreover, we derive a relationship for the optimal assembly size as a function of the electorate size which we use to explain real world assembly sizes.

3 The Model

The electorate consists of N voters, where N is an odd integer. Each voter $i \in \{1, .., N\}$ knows that the community will face, at time $t = 1$, a policy choice from two alternatives denoted by $\{A, B\}$. She also knows that she will have strict preferences over $\{A, B\}$, that is, either she will prefer A to B or she will prefer B to A but not both. Moreover, she will not find out which is her preferred alternative till the policy is implemented at time $t = 2$. When drawing up the constitution at time $t = 0$, the voter can either favor delegation to an informed assembly of size $Z \leq N$ or can favor policy choice by referenda ($Z = 0$). To avoid ties in the assembly, only odd numbers can be chosen. Define the set of all possible assembly sizes (other than $Z = 0$) by the index set $I = \{1, 3, .., N\}$ which is simply all odd integers up to N. The constitutional decision is made by majority rule. If a single assembly size Z is undefeated in pairwise contest, then it is institutionalized.

Assembly members get the necessary information for knowing how they will be affected by a policy decision, while voters lack this information. Each assembly member brings about a fixed cost of amount $c > 0$ for getting information. This cost will be shared equally among the electorate. That is, the cost to any voter i of having an informed assembly of size Z is given by $\frac{cZ}{N}$. We normalize utilities such that voter i's utility is $u_i > 0$ if her preferred alternative is implemented and her utility is $-u_i$ otherwise.[4] The two main assumptions of our model are the following.

Assumption 1 $\forall i$, $0 < u_i < c$.

This assumption states that no single voter can afford to acquire the necessary in-

[4] It might seem odd that the voter has an estimation of this utility loss at hand while not having a clue about the expected utility from the single alternatives. But many situations are like this: think of two policy measures which might affect general unemployment in different ways (e.g. joining the monetary union or not). Although there might be (Knightian) uncertainty about the consequences on employment of these measures, the individual voter can have a clear estimate of what losing her job would cost her.

formation on her own. To calculate the expected advantage of delegation to voter i, she has to know the distribution of true preferences over $\{A, B\}$. The term "true preference" refers to the alternative which the voter would choose if she was fully informed. The next assumption is on the distribution of true preferences.

Assumption 2 $\forall i,\ \Pr\{A \succ_i B\} = \Pr\{B \succ_i A\}$.

This assumption states that ex ante both preferences are equally likely. Let $f(x) = \Pr\{\text{number of } A\text{-preferrer} = Nx\}$ denote the marginal distribution of A-preferrers. To calculate the ex ante expected advantage of delegation, only $f(x)$ of the joint distribution of preference profiles is needed. To illustrate this point we provide the marginal distribution of A-preferrers for the case of Binomial Distribution in the following example.

Example 1 *Nature chooses with equal probability between two complementary binomial distributions. In situation (i), each voter has the preference* $\{A \succ_i B\}$ *with* $\Pr\{A \succ_i B\} = \alpha \geq \frac{1}{2}$, *in situation (ii),* $\Pr\{A \succ_i B\} = 1 - \alpha$. *Using ex ante symmetry,* $\Pr\{$*number of* A*-preferrer* $= Nx\}$ *is given by* $f_\alpha(x) = \binom{N}{Nx}\left(\frac{1}{2}\alpha^{Nx}(1-\alpha)^{N(1-x)} + \frac{1}{2}\alpha^{N(1-x)}(1-\alpha)^{Nx}\right)$.

For our model we do *not* assume any specific distribution like the binomial distribution. We simply assume that there is some joint distribution that satisfies ex ante symmetry (Assumption 2).

An elected assembly is perceived as a random draw from the electorate. Each assembly member gets the information about her true preference and votes, in the assembly, accordingly. For a given true preference profile in the electorate, the preferences of assembly members over the alternatives is hypergeometrically distributed. Since for each voter both preferences are equally likely (ex ante), we focus on the question of majority and minority instead of conditioning on preferences. That is, at $t = 0$ each voter i has to worry only about (i) the probability that her true preference will be a majority preference $\Pr(\{i \in maj\})$ and (ii) the probability that the assembly decides against the majority in the electorate $\Pr(\{ass \neq maj\})$. The voter will incur the loss $-u_i$ (a) if she is in the majority and the assembly is against the majority and (b) if she is in the minority and the assembly follows the majority. In the opposite cases, she will gain u_i. Therefore, the expected benefit from delegating the decision to an informed assembly of size Z is given by $V(N, Z)\ u_i = [\Pr\{(\text{a})\} + \Pr\{(\text{b})\}]\,(-u_i) + [1 - \Pr\{(\text{a})\} - \Pr\{(\text{b})\}]\,u_i$. After simplifying $V(N, Z)$ we get the expected benefit to be

$$V(N, Z)u_i = [1 - 2\Pr(\{i \in maj\} \cap \{ass \neq maj\}) - 2\Pr(\{i \notin maj\} \cap \{ass = maj\})]u_i$$

Thus, voter i's preferred Z is either given by

$$Z_i(c, N) \in \arg\max_{Z \in I} \left[U_i(c, N, Z) = V(N, Z)\ u_i - \frac{cZ}{N} \right] \tag{1}$$

or by $Z_i = 0$, the latter being the case if and only if $U_i(c, N, Z_i(c, N)) < 0$. We refer to the maximization problem (1) by the term *interior maximization* (since this considers the maximizer in the set I) and to the one including $Z = 0$ by the term *global maximization* (the maximizer in the set $I \cup \{0\}$). Defining the incremental utility of voter i, for an assembly of size Z, by $\Delta U_i(c, N, Z) \equiv U_i(c, N, Z + 2) - U_i(c, N, Z)$, we can write the optimality condition for the interior maximization problem (1) as

$$u_i\ \Delta V(N, Z) - 2\frac{c}{N} \leq 0 < u_i\ \Delta V(N, Z - 2) - 2\frac{c}{N} \tag{2}$$

where $\Delta V(N, Z) = V(N, Z + 2) - V(N, Z)$.

Before we can discuss the properties of the interior maximization problem, we need one additional assumption. Assume that no voter ever wants more than a quarter of the population in the assembly and hence no assembly size bigger than this is ever suggested by anyone in the constitutional debate.

Assumption 3 $\forall i,\ Z_i < \frac{N}{4}$

Given that our rationale for the rationality of government by delegation is along the lines of division of labor between governing and non-governing members of society, this assumption is rather mild. For the empirical data, the highest assembly to electorate ratio we encounter is 1/300.

Remark 1 *Instead of an a priori bound on the assembly size, one could introduce an assumption on the distribution $f(x)$. Only for distributions $f(x)$ for which the bulk of probability mass is concentrated around $x = \frac{1}{2}$* and *with the possibility that $Z_i > N/4$, the following lemma is not applicable. For $f(x)$-functions derived from the binomial distribution of Example 1 (for any α) or the $f(x)$ of equal probability which will be used later, we can dispense with the above assumption.*[5] *Although we would not need the assumption for the $f(x)$ of equal probability, we can use this case to judge the restrictiveness of the assumption. In this case the assumption would be equivalent to assuming $\frac{u_i}{c} < \frac{N(N+8)}{4(N+2)}$ for all i. Clearly, this would be no restriction at all in light of Assumption 1.*[6]

Lemma 1 *For all $0 < Z < \frac{N}{4}$, either $V(N,Z)$ is independent of Z or $V(N,Z)$ is increasing and has decreasing first differences in Z. Thus, the interior maximizer $Z_i(c,N)$ of $U_i(c,N,Z)$ is generically unique, i.e. only for $c \in \mathbb{R}_+$ of measure zero there are two values Z_i and $Z_i + 2$ in the* $\arg\max$ *of the interior maximization problem (1).*

Proof. See Appendix A.

From the first statement of this Lemma it follows that for all u_i, there is a $Z_i(c,N)$ such that $U_i(c,N,Z)$ is increasing (decreasing) in Z for $0 < Z < Z_i(c,N)$ ($Z > Z_i(c,N) > 0$).

[5] The latter can be seen from Lemma 4 whose proof does not need the assumption.

[6] The inequality can be easily derived by solving $Z_i = \frac{N}{4}$ with Z_i taken from eq. (5).

Thus, for generic costs c, the voters preferences over $\hat{I}$ have a unique peak at $Z_i(c, N)$. However, voters preferences need not be single peaked over $\hat{I} \cup \{0\}$ since we have not ruled out $U_i(c, N, 1) < 0 = U_i(c, N, 0)$. But still the constitutional decision is unambiguous.

Proposition 1 *Order the individual u_i's such that $u_{(1)} \geq u_{(2)} \geq ... \geq u_{(N)}$. Then under Assumptions 1 -3 and for generic $c \in \mathbb{R}_+$ there is an assembly size $\hat{Z}$ which cannot be defeated by majority voting by any other alternative. Moreover, an assembly of size $\hat{Z} > 0$ will be set up if and only if*

$$U_{(\frac{N-1}{2})}(c, N, Z_{(\frac{N-1}{2})}) > 0 \tag{3}$$

■

Proof. From Lemma 1 it not only follows that preferences of voters are single peaked on $\hat{I} \equiv \{Z \in I | \ 0 < Z < N/4\}$, but, since voters utility functions differ only by the factor u_i by which the function V is weighted, we can readily draw two further conclusions. First, the ranking of individuals by their most preferred assembly size must correspond to the ranking by their u_i's. Second, the individuals preferences are "nested" in the sense that $\forall i, j \in \{1, ..., N\}$, such that $i < j$ we have $U_{(i)}(c, N, Z) < U_{(j)}(c, N, Z) \ \forall Z \in \hat{I}$.
Since preferences are single peaked on $\hat{I}$, the median voter theorem tells us that if people vote only on this restricted set of alternatives, then the alternative $Z_{(\frac{N+1}{2})}(c, N)$ would be a Condorcet winner. Now, when we also take into consideration the "referendum" option Z_0, there are two possibilities. Either the preferences of the voters are such that $Z_{(\frac{N+1}{2})}$ would defeat Z_0 in pairwise comparison, in which case we are done, or the opposite is true, that is, more than half of the voters prefer Z_0 over $Z_{(\frac{N+1}{2})}$. In this latter case Z_0 must be a Condorcet winner. This can be seen as follows. Let k be the voter with the largest index among those who vote for Z_0 (in the pairwise comparison with $Z_{(\frac{N+1}{2})}$). Clearly, since Z_0 is the winning alternative, we must have $k \geq \frac{N+1}{2}$. Also, by nestedness it follows that for all $i \leq k$, $U_{(i)}(c, N, Z) < 0 \ \forall Z \in \hat{I}$. Hence, Z_0 would defeat any alternative from $\hat{I}$ with at least the same majority as it defeats $Z_{(\frac{N+1}{2})}$. ■

We now provide comparative static results on the the interrelation between the parameters u_i, c and the optimal assembly size $\hat{Z}$. From the interior maximization problem (1), the following Lemma follows immediately.

Lemma 2 *For each voter i, the assembly size Z_i solving (1) is non-increasing in c, non-decreasing in u_i and constant for a proportional increase in c and u_i.*

Proposition 2 *The assembly size $\hat{Z}$ which results from majority voting at the constitutional stage under the assumptions of Proposition 1 is non-decreasing in c, non-increasing in any u_i and constant for a proportional increase in c and all u_i.*

Proof. This follows from the preceding lemma together with the cited proposition. ■

To sum up: if the costs of getting a single member informed is not prohibitively high compared to the median voter's u_i, there will be an assembly set up on the constitutional level. In such a situation, it is perfectly rational to delegate the decision to a better informed assembly. Thus, the institution of a representative assembly can be justified in individualistic terms. Moreover, if delegating is more costly per delegate, we expect the optimal number of delegates to decrease. Similarly, if more is at stake for the single voter (higher u_i), the additional benefit from each added delegate is higher. Thus, Z_i goes up.

4 Equally likely majority shares

It would be very interesting to show a consistent relation between N and $\hat{Z}$. But this is not possible without further assumptions since there is no clear-cut way to parametrize the distribution $f(x)$ for different N and since $\hat{Z}$ depends delicately on this distribution. With the strong distributional assumption on $f(x)$, which we will introduce in the next paragraph, the interrelation between N and $\hat{Z}$ is determined. Hence, in this section we drop Assumption 3 and instead assume the following:

Assumption 4 *$f(x) = \frac{1}{N+1}$ for all $Nx \in \{0, 1, ..N\}$*

Assumption 4 implies that all majority shares are equally likely.With this additional assumption, the first differences of the $V(N, Z)$ function, that is $\Delta V(N, Z) = V(N, Z+2) - V(N, Z)$, can be simplified and we find a closed-form solution for the interior maximization problem (1).

Lemma 3 *Under Assumptions 2 and 4,*

$$\Delta V(N, Z) = \frac{(N+2)}{2N\,(Z+2)\,(Z+4)} \tag{4}$$

Proof. See Appendix A. ■

Lemma 4 *Under Assumption 4, Z_i solves the interior maximization problem (1) if and only if*

$$Z_i = 2\left\lfloor \sqrt{\left(\frac{1}{4} + \frac{Nu_i}{16c} + \frac{u_i}{8c}\right)} \right\rfloor - 1 \tag{5}$$

where $\lfloor x \rfloor := \max\{n \in \mathbb{N} |\; n \leq x\}$.

Proof. From the optimality condition (2) we get $\Delta V(N, Z) < \frac{2c}{Nu_i} \leq \Delta V(N, Z-2)$. By substituting the simplified form of $\Delta V(N, Z)$ from equation(4) in this inequality we get $\frac{1}{2c}\sqrt{(4c^2 + 2cNu_i + 2cu_i)} - 3 \leq Z_i < \frac{1}{2c}\sqrt{(4c^2 + 2cNu_i + 2cu_i)} - 1$. The last inequality implies (5). ■

Proposition 3 *Under Assumptions 1, 2 and 4, the assembly size $\hat{Z}(c, N)$ which is chosen at the constitutional stage is non-decreasing in N. Moreover, if $\hat{Z}(c, N) > 0$ for some N, then the assembly size $\hat{Z}(c, N)$ tends to infinity for $N \to \infty$.*

Proof. We see from Lemma 4, that for all voters i, $\hat{Z}_i(c, N)$ is non-decreasing in N and tends to infinity as $N \to \infty$. Then, from Proposition 1, it is obvious that this also holds for the $\hat{Z}(c, N)$ chosen by the society. ■

5 Explaining Real World Assembly Sizes

The main reason for developing a simple model in the last section was to explain why delegation of political power to representatives can be rational. This simple model can now be used to judge whether real world assemblies can be rationalized in this way. We continue to assume that the marginal distribution $f(x)$ satisfies Assumption 4. Since it is not possible to estimate the utility values u_i from the data, we assume identical utility values for our estimations.

Assumption 5 $\forall i,\ u_i \equiv u(=1)$.

With this additional assumption, Proposition 1 and Lemma 4 together imply the following relationship between electorate and optimal assembly size:

$$\hat{Z}(c, N) = 2\left\lfloor \sqrt{\left(\frac{1}{4} + \frac{N}{16c} + \frac{1}{8c}\right)} \right\rfloor - 1$$

This can now be fitted to our data. We used data (Z_j, N_j) for some federal systems, because within a federal system one might expect the information cost c to be similar. This parameter c has been estimated by non-linear least square methods for each data set. We fitted the equation

$$\hat{Z} = 2\sqrt{\left(\frac{1}{4} + \frac{N}{16c} + \frac{1}{8c}\right)} - 1 + \varepsilon$$

because the non-linear least square method involves Taylor-series approximations and the floor-function $\lfloor\rfloor$ is not differentiable. The fitted equation is plotted together with the original data in the following figures. We present the data for (a) all countries of the European community (Figure 1) and (b) the parliaments of the "Länder" (states) in Germany (Figure 2), and (c) for the Swiss Cantons (Figure 3).

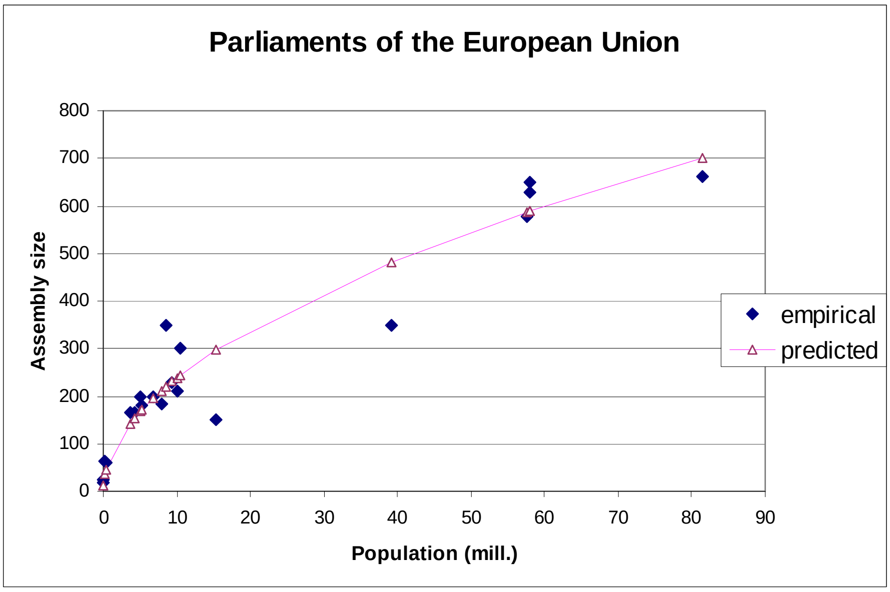

Figure 1: Empirical and predicted $\hat{Z}(c, N)$ for the parliaments of the European Union.

In order to judge the fit of our model, we compare it with two more standard statistical models. These are

$$Z = a + bN + \varepsilon \tag{*}$$

$$Z = a + bN + cN^2 + \varepsilon \tag{**}$$

We estimated the parameters (a, b, c) by OLS for both models and compared the fit with the one of our model $\hat{Z}(c, N)$. Since our model is more parsimonious (one parameter fitted instead of two or three), one cannot expect a better fit of our model. Still, with respect to the used measures SSR (sum of squared residuals) and ρ (correlation between predicted and true assembly size), our model outperforms models (*) and (**) in the EU data set,

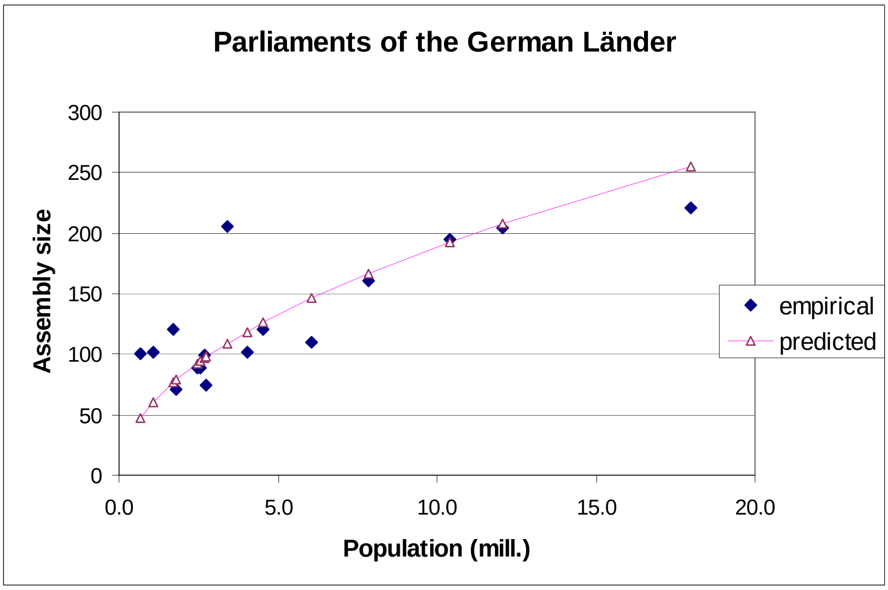

Figure 2: Empirical and predicted $\hat{Z}(c, N)$ for the parliaments of the German States ("Länder").

and compares quite well to the other models in the other data sets, too (cf. Table 1).

Table 1

		model $\hat{Z}$	model (*)	model (**)
EU	SSR	68,907	99,609	84,406
	ρ	0.96	0.94	0.95
Switzerland	SSR	18,790	18,003	10,047
	ρ	0.89	0.82	0.90
German	SSR	18,930	13,928	13,615
	ρ	0.79	0.80	0.81

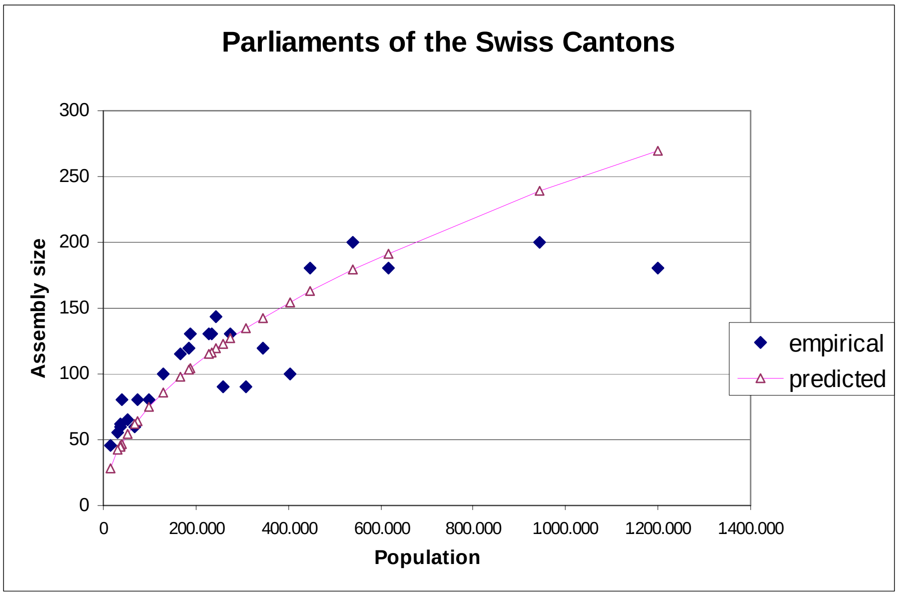

Figure 3: Empirical and predicted $\hat{Z}(c, N)$ for the parliaments of the Swiss Cantons.

We see that a good part of the empirical relation between population and assembly size can be explained by our model fitting a single parameter. But this exercise has to be interpreted with caution. In addition to the stringent assumptions $1, 2$ and 5, we imposed the supposition that the cost c is the same in the relevant set of jurisdictions. So if we observe a departure from our predicted relation, this can either be due to (i) different costs of information, (ii) missing rationality in the design of the institutions or (iii) the limitation of our model.

6 Conclusion

In this paper we suggest a political economy explanation for the existence of representative assemblies. The focus is on the delegation perspective. Electing an assembly like a parliament or committee is an act of delegating decisions to better informed decision makers. Assuming extreme ignorance on the voter's side and full information for the delegate, we showed that an electorate of rational voters would decide to install a representative system (if the cost of informing a delegate was not insurmountable). In line with our intuition, the model yielded a bigger assembly for smaller information costs and for voters having a bigger stake in the decision. In order to keep the model consistent, we assumed an objective probability model generating the random preferences for the voters. Moreover, voters beliefs about the distribution of preferences were assumed to be in line with the objective generation of randomness. Interestingly, one could open up the model at this point: voters can also have different beliefs about the way their preferences are generated and distributed, as long as these beliefs conform with Assumption 2 and 3. This implies that a society need not agree about the distribution of relevant preferences in order to accept the fact that a constitutional design of representative democracy is in line with their own interest. We compared the prediction of the model with a sample of real world assemblies. Fitting a single parameter for the cost of delegation to our data, we found that our model can explain a considerable part of the relation between population and assembly size.

The model can now be extended to include more features of reality. In particular, one would like to model different degrees of information both within the electorate and within the assembly. Furthermore, a two-level extension can take into account the fact that parliament members delegate many decisions to committees and vote in line with their party. These are the few relevant extensions that can be taken up for future research.

A Appendix

Proof of Lemma 1 To prove the Lemma, we first simplify the term $V(N,Z)$ and its first difference. We then prove that $V(N,Z)$ is either independent of Z or $V(N,Z)$ is strictly increasing at a decreasing rate in assembly size Z for all odd integers Z strictly less than $\lfloor \frac{N}{4} \rfloor$. This is done in three steps. Finally, we prove the last statement of the Lemma. We start by simplifying the term $V(N,Z)$ and it's first difference in the next few paragraphs.

Let x represent the fraction of the population preferring alternative A to B and let $f(x)$ represent the marginal distribution of A preferrers in an electorate of size N. Therefore, the probability that the majority consists of a share of y voters is given by

$$g(y) \equiv \Pr(max\{x, 1-x\}) = \begin{cases} f(y) + f(1-y) & \text{if } y \in \left\{\frac{N+1}{2N}, \ldots, 1\right\} \\ 0 & \text{otherwise.} \end{cases}$$

Let $h(k; N, Ny, Z)$ represent the event that, in a random selection of Z voters (odd in number) from the population of size N, exactly k voters are selected from the majority group of size Ny. Therefore, the number of ways in which the event $h(k; N, Ny, Z)$ can occur is $\binom{Ny}{k}\binom{N(1-y)}{Z-k}$ and hence $\Pr(h(k; N, Ny, Z)) = \frac{\binom{Ny}{k}\binom{N(1-y)}{Z-k}}{\binom{N}{Z}}$. Thus, h is hypergeometrically distributed. Let $H(k; N, Ny, Z)$ represent the cumulative event that, in a random selection of Z voters from the population of size N, *at most* k voters are selected from the majority group of size Ny, that is $H(k; N, Ny, Z) = \bigcup_{l=max[0,Z-N(1-y)]}^{k} h(l; N, Ny, Z)$. Since $[h(l; N, Ny, Z)]_{l=max[0,Z-N(1-y)]}^{k}$ are mutually exclusive (or disjoint), $\Pr(H(k; N, Ny, Z)) = \sum_{l=max[0,Z-N(1-y)]}^{k} \Pr(h(l; N, Ny, Z)) = \sum_{l=max[0,Z-N(1-y)]}^{k} \frac{\binom{Ny}{l}\binom{N(1-y)}{Z-l}}{\binom{N}{Z}}$.

Conditional on y, the events $\{ass \neq maj\}$ and $\{i \in maj\}$ are independent since they can be perceived as two independent draws from a given urn with Ny balls of one color and $N(1-y)$ balls of the other color, where $\{ass \neq maj\}$ refers to a draw of Z balls (odd in number) such that at most $\frac{Z-1}{2}$ balls are from the set of Ny balls of the first color and $\{i \in maj\}$ refers to the draw of a single ball of the first color. Thus, the conditional probability of these two events are $\Pr(\{ass \neq maj\} \mid y) = \Pr\left(H(\frac{Z-1}{2}; N, Ny, Z)\right)$ and $\Pr(\{i \in maj\} \mid y) = y$. Using these two conditional probabilities we now simplify the term $V(N, Z)$.

$$\begin{aligned}
&V(N,Z)\\
&= \sum_{y=\frac{N+1}{2N}}^{1} g(y)[1 - 2\Pr(\{i \in maj\} \mid y)\Pr(\{ass \neq maj\} \mid y) - 2\Pr(\{i \notin maj\} \mid y)\Pr(\{ass = maj\} \mid y)]\\
&= \sum_{y=\frac{N+1}{2N}}^{1} g(y)[1 - 2y\Pr(\{ass \neq maj\} \mid y) - 2(1-y)(1 - \Pr(\{ass \neq maj\} \mid y))]\\
&= 1 - 2\sum_{y=\frac{N+1}{2N}}^{1} g(y)[(1-y) + (2y-1)\Pr\left(H(\tfrac{Z-1}{2}; N, Ny, Z)\right)]\\
&= (2E(y) - 1) - 2\sum_{y=\frac{N+1}{2N}}^{1} g(y)(2y-1)\Pr\left(H(\tfrac{Z-1}{2}; N, Ny, Z)\right).
\end{aligned}$$

Using this simplification and by substituting $2E(y) - 1 = C$ we get

$$V(N,Z) = C - 2\sum_{y=\frac{N+1}{2N}}^{1} g(y)(2y-1)\Pr\left(H\left(\frac{Z-1}{2}; N, Ny, Z\right)\right) \tag{6}$$

Finally, we consider the first difference $\Delta V(N, Z) = V(N, Z+2) - V(N, Z)$. Simplifying the first difference using (6) and then substituting $w(N, Ny, Z) = \Pr\left(H(\frac{Z-1}{2}; N, Ny, Z)\right) - \Pr\left(H(\frac{Z+1}{2}; N, Ny, Z+2)\right)$ we get

$$\Delta V(N,Z) = 2\sum_{y=\frac{N+1}{2N}}^{1} g(y)(2y-1)w(N, Ny, Z) \tag{7}$$

We now prove that the term $V(N, Z)$ is increasing in Z at a decreasing rate for all odd numbers Z strictly less than $[\frac{N}{4}]$ in three steps. In the first two steps we prove that $V(N, Z)$ is strictly increasing in Z for all odd numbers $Z < N$. In the third step we use the restriction that Z is odd and strictly less than $\lfloor \frac{N}{4} \rfloor$ to show that $\Delta V(N, Z)$ is strictly decreasing.

Step 1 : $w(N, y, Z) > 0$ for all $y \in \{\frac{1}{2} + \frac{1}{2N}, \ldots, 1 - \frac{Z+1}{2N}\}$.

Proof: Using the result $\Pr(A) - \Pr(B) = \Pr(A \cap B^c) - \Pr(B \cap A^c)$, we can rewrite the term $w(N, y, Z)$ as $w(N, y, Z) = \Pr\left(H(\frac{Z-1}{2}; N, Ny, Z) \cap H^c(\frac{Z+1}{2}; N, Ny, Z+2)\right) - \Pr\left(H(\frac{Z+1}{2}; N, Ny, Z+2) \cap H^c(\frac{Z-1}{2}; N, Ny, Z)\right)$. Observe, first, that the compound event $H(\frac{Z-1}{2}; N, Ny, Z) \cap H^c(\frac{Z+1}{2}; N, Ny, Z+2)$ is the joint occurrence of (1) *at most* $\frac{Z-1}{2}$ voters from the majority group Ny in a random selection of Z voters from the population of size N and (2) *more than* $\frac{Z+1}{2}$ voters from the majority group Ny in a random selection of $Z+2$ voters from the population of size N. This compound event can occur if and only if (i) in a random selection of Z voters from the population N, exactly $\frac{Z-1}{2}$ voters are selected from the majority group of size Ny (and hence exactly $\frac{Z+1}{2}$ voters are selected from the non-majority group $N(1-y)$), that is $h\left(\frac{Z-1}{2}; N, Ny, Z\right)$ and (ii) given (i), the event that in a random selection of 2 voters from the remaining population $(N - Z)$, both are selected from the remaining $\left(Ny - \frac{Z-1}{2}\right)$ voters of the majority group. Thus, (ii) is the unconditional event $h(2; N-Z, Ny - \frac{Z-1}{2}, 2)$. Therefore, the compound event $H(\frac{Z-1}{2}; N, Ny, Z) \cap H^c(\frac{Z+1}{2}; N, Ny, Z+2)$ is given by the simultaneous occurrence of the mutually independent events $h\left(\frac{Z-1}{2}; N, Ny, Z\right)$ and $h\left(2; N-Z, Ny - \frac{Z-1}{2}, 2\right)$. Therefore, using this observation in the first compound probability term of $w(N, y, Z)$ and then by simplifying it we get

1. $\Pr\left(H(\frac{Z-1}{2}; N, Ny, Z) \cap H^c(\frac{Z+1}{2}; N, Ny, Z+2)\right) = \Pr\left(h(\frac{Z-1}{2}; N, Ny, Z)\right) \frac{\binom{Ny - \frac{Z-1}{2}}{2}}{\binom{N-Z}{2}}.$

Observe, next, that the compound event $H(\frac{Z+1}{2}; N, Ny, Z+2) \cap H^c(\frac{Z-1}{2}; N, Ny, Z)$ is the joint occurrence of $(1')$ *at most* $\frac{Z+1}{2}$ voters from the majority group Ny in a random selection of $Z+2$ voters from the population of size N and $(2')$ *at least* $\frac{Z+1}{2}$ voters from the majority group Ny in a random selection of Z voters from the population of size N. This compound event can occur if and only if (i') in a random selection of Z voters from the population N, exactly $\frac{Z+1}{2}$ voters are selected from the majority group of size Ny (and hence exactly $\frac{Z-1}{2}$ voters are selected from the non-majority group $N(1-y)$), that is $h\left(\frac{Z+1}{2}; N, Ny, Z\right)$ and (ii') given (i'), the event that in a random selection of 2 voters from the remaining population $(N-Z)$, none are selected from the remaining $\left(Ny - \frac{Z+1}{2}\right)$ voters of the majority group. Thus, (ii') is the unconditional event $h(0; N - Z, Ny - \frac{Z+1}{2}, 2)$ and the compound event $H(\frac{Z+1}{2}; N, Ny, Z+2) \cap H^c(\frac{Z-1}{2}; N, Ny, Z)$ is given by the simultaneous occurrence of the mutually independent events $h\left(\frac{Z+1}{2}; N, Ny, Z\right)$ and $h\left(0; N-Z, Ny - \frac{Z+1}{2}, 2\right)$. Using this observation in the second probability term of $w(N, y, Z)$ and then simplifying it we get

2. $\Pr\left(H(\frac{Z+1}{2}; N, Ny, Z+2) \cap H^c(\frac{Z-1}{2}; N, Ny, Z)\right) = \Pr\left(h(\frac{Z+1}{2}; N, Ny, Z)\right) \frac{\binom{N(1-y)-\frac{Z-1}{2}}{2}}{\binom{N-Z}{2}}$.

Finally, using **1** and **2** and simplifying $w(N, y, Z)$ we get

$$w(N, y, Z) = \frac{N(2y-1)\left(Ny - \frac{Z-1}{2}\right)}{(N-Z-1)(N-Z)} \Pr\left(h(\frac{Z-1}{2}; N, Ny, Z)\right) \geq 0$$

The term $\Pr\left(h(\frac{Z-1}{2}; N, Ny, Z)\right)$ is strictly positive if and only if $Ny \geq \frac{Z-1}{2}$ and $N(1-y) \geq \frac{Z+1}{2}$ $\left[\Leftrightarrow y \leq 1 - \frac{Z+1}{2N}\right]$, where the first condition is redundant because $Ny > N(1-y)$. Hence, $w(N, y, Z) > 0$ for all $y \in \{\frac{N+1}{2N}, \ldots, 1 - \frac{Z+1}{2N}\}$.

Step 2 : $V(N, Z)$ is either independent of Z or increasing in Z for all odd integers $Z < \frac{N}{4}$.

Proof: Observe that $\Delta V(N, Z) = V(N, Z+2) - V(N, Z) = 2 \sum_{y=\frac{N+1}{2N}}^{1} g(y)(2y-1)w(N, y, Z)$. Since $w(N, y, Z) = 0$ for all $y \in \{1 - \frac{Z-1}{2N}, \ldots, 1\}$, we get the first difference $\Delta V(N, Z) = 2 \sum_{y=\frac{N+1}{2N}}^{1} g(y)(2y-1)w(N, y, Z) = 2 \sum_{y=\frac{N+1}{2N}}^{1-\frac{Z+1}{2N}} g(y)(2y-1)w(N, y, Z) \geq 0$. The last step follows from Step [1]. Only if $g(y) = 0$ for all $y \in \{\frac{N+1}{2N}, \ldots, 1 - \frac{Z+1}{2N}\}$ then $\Delta V(N, Z) = 0$. In this case, $V(N, Z)$ is independent of Z. In all other cases, there is at least *one* $y \in \{\frac{N+1}{2N}, \ldots, 1 - \frac{Z+1}{2N}\}$ such that $g(y) > 0$ and hence $\Delta V(N, Z) > 0$ (since $w(N, y, Z) > 0$ for this y).

Step 3 : If $V(N,Z)$ is not independent of Z, then $\Delta V(N,Z)$ is decreasing in Z for all odd integers $Z < \frac{N}{4}$.

Proof: To prove this Step, we show that the difference term $\mathcal{P}(N,Z)$, given by $\mathcal{P}(N,Z) = \Delta V(N,Z) - \Delta V(N,Z+2) > 0$. Observe that

$$\mathcal{P}(N,Z) = 2 \sum_{y=\frac{N+1}{2N}}^{1-\frac{Z+1}{2N}} G(N,y) \left\{ \frac{\Pr\left(h(\frac{Z-1}{2}; N-1, Ny-1, Z)\right)}{(N-Z-1)} - \frac{\Pr\left(h(\frac{Z+1}{2}; N-1, Ny-1, Z+2)\right)}{(N-Z-3)} \right\}$$

where $G(N,y) = g(y)Ny(2y-1)^2$. For simplicity, let us substitute $2m+1$ for the assembly size Z. Thus, $\mathcal{P}(N,Z) = \mathcal{P}(N,2m+1) = 2 \sum_{y=\frac{N+1}{2N}}^{1} G(N,y)\mathcal{S}(N,y,2m+1)$ where $\mathcal{S}(N,y,2m+1) = \left\{ \frac{\Pr(h(m;N-1,Ny-1,2m+1))}{(N-2m-2)} - \frac{\Pr(h(m+1;N-1,Ny-1,2m+3))}{(N-2m-4)} \right\}$. Simplifying the term $\mathcal{S}(N,y,2m+1)$, by using the identity $\binom{a}{b+1} = \left(\frac{a-b}{b+1}\right)\binom{a}{b}$, we get $S(N,y,2m+1) = \frac{\Pr(h(m;N-1,Ny-1,2m+1))}{(N-2m-2)}\{1-\mu(N,y,2m+1)\}$ where the expression $\mu(N,y,2m+1) = \frac{2(Ny-m-1)(N(1-y)-m-1)(2m+3)}{(N-2m-3)(N-2m-4)(m+2)}$. Observe that given N and m, $\mu(N,y,2m+1)$ attains it's maximum at $y = \frac{N+1}{2N}$, that is $\mu(N,y,2m+1) \leq \mu(N,\frac{N+1}{2N},2m+1)$ for all $y = \frac{N+1}{2N},\ldots,1$. Also observe that the term $\mu(N,\frac{N+1}{2N},2m+1) = \frac{(N-2m-1))(2m+3)}{(N-2m-4)(2m+4)} < 1$ for all $Z = 2m+1 < \frac{N-9}{4}$. Since we have assumed that the assembly size is strictly less than $\frac{N}{4}$ and since the highest assembly size we used in the expression $\mathcal{P}(N,Z)$ is $Z+4$, it must be the case that $Z+4 < \frac{N}{4}$, that is $Z < \frac{N-16}{4}$ $\left(< \frac{N-9}{4}\right)$. Therefore, we get $\mu(N,\frac{N+1}{2N},2m+1) < 1$ and hence $\mu(N,y,2m+1) < 1$ for all $y = \frac{N+1}{2N},\ldots,1$. This proves that $\mathcal{S}(N,y,2m+1) > 0$ for all $y = \frac{N+1}{2N},\ldots,1-\frac{m+1}{N}$ and $\mathcal{S}(N,y,2m+1) = 0$ for all $y = 1-\frac{m}{N},\ldots,1$. Thus, $\mathcal{P}(N,Z) > 0$ since (i) $G(N,y) \geq 0$ for all $y = \frac{N+1}{2N},\ldots,1$ with at least one strict inequality, (ii) $\mathcal{S}(N,y,Z) > 0$ for all $y = \frac{N+1}{2N},\ldots,1-\frac{Z+1}{2N}$ and (iii) $\mathcal{S}(N,y,Z) = 0$ for all $y = 1-\frac{Z-1}{2N},\ldots,1$.

We finally prove the last statement of the Lemma. Using the results on $V(N,Z)$ it is obvious that the function $U_i(N,Z,u_i) = V(N,Z)u_i - \frac{c}{N}Z$ has non-decreasing first difference for all $u_i > 0$. This implies single-peakedness except for the case when $\Delta U_i(N, Z, u_i) = 0$ has a solution $Z^* \in I$. Obviously, this is only the case for a set of $c \in \mathbf{R}_+$ of measure zero. ■

Proof of Lemma 3 Recall that $\Delta V(N, Z) = 2\sum_{y=\frac{N+1}{2N}}^{1-\frac{Z+1}{2N}} g(y)(2y-1)w(N, y, Z)$ and from Step [1] of Lemma 1 we know that $w(N, y, Z) = \frac{Ny(2y-1)}{(N-Z-1)} \Pr\left(h(\frac{Z-1}{2}; N-1, Ny-1, Z)\right)$. Since each possible proportion $x \in \{0, \frac{1}{N}, \dots, 1\}$, of voters preferring A to B in the population of size N, is assumed to be equally likely ex-ante, it follows that $f(x) = \frac{1}{N+1}$ for all $x \in \{0, \frac{1}{N}, \dots, 1\}$ and hence all majority shares are equally likely, that is

$$g(y) = \begin{cases} \frac{2}{N+1} & \text{if } y \in \left\{\frac{N+1}{2N}, \dots, 1\right\} \\ 0 & \text{otherwise.} \end{cases}$$

By substituting $g(y)$ in the first difference term $\Delta V(N, Z)$ and then simplifying it by replacing $N = 2n+1$ and $Z = 2m+1$ we get $\Delta V(N, Z) = \Delta V(2n+1, 2m+1) = A(n, m)B(n, m)$ where $A(n, m) = \frac{(2n-2m-2)!(2m+1)!4}{m!(m+1)!(2n+2)!(2n+1)}$ and $B(n, m) = \sum_{X=n+1}^{2n-m}(2X - (2n+1))^2 \frac{X!((2n+1)-X)!}{(X-m-1)!((2n+1)-X-m-1)!}$. By replacing $X = n+j$ in the summand of $B(n, m)$ we obtain $B(n, m) = \sum_{j=1}^{n-m}(2j-1)^2 \frac{(n+j)!(n+1-j)!}{(n-m-1+j)!(n-m-j)!}$. To find the value of $B(n, m)$, we first define $T(n, m) = \frac{\Gamma(n+2)\Gamma(n+\frac{5}{2})\Gamma(m+2)\Gamma\left(\frac{1}{2}\right)}{\Gamma(n-m)\Gamma\left(n-m-\frac{1}{2}\right)\Gamma\left(m+\frac{7}{2}\right)}$ and show that $B(n, m) = T(n, m)$ for all $n > m \geq 0$. This is done in the following two steps.

Step 1 : For all $n > m$:

$$B(n+1, m) = (m+1)(2n+2-m)B(n, m-1) + B(n, m) \tag{8}$$

Proof. Define

$$c(j, n, m) := \frac{(n+j)!\,(n-j+1)!}{(n-m-j)!\,(n-m+j-1)!}$$

Using this, we write

$$B(n, m) = \sum_{j=1}^{n-m} (2j-1)^2\, c(j, n, m)$$

and

$$
\begin{aligned}
&B(n+1,m) = \sum_{j=1}^{n+1-m} (2j-1)^2\, c(j,n+1,m) \\
&= \sum_{j=1}^{n+1-m} (2j-1)^2 \tfrac{(n+1+j)(n+2-j)}{(n-m+j)(n-m+1-j)} c(j,n,m) \\
&= \sum_{j=1}^{n-m} (2j-1)^2 \left(\tfrac{(n+1+j)(n+2-j)}{(n-m+j)(n-m+1-j)} - 1\right) c(j,n,m) \\
&\quad + \sum_{j=1}^{n-m} (2j-1)^2\, c(j,n,m) + (2n-2m+1)^2\, c(n-m+1,n+1,m) \\
&= \sum_{j=1}^{n-m} (2j-1)^2 \tfrac{(m+1)(2n+2-m)}{(n-m+j)(n-m+1-j)} c(j,n,m) + B(n,m) \\
&\quad + (2n-2m+1)^2 \left((m+1)\,(2n+2-m)\, c(n-m+1,n,m-1)\right) \\
&= (m+1)\,(2n+2-m) \sum_{j=1}^{n+1-m} (2j-1)^2\, c(j,n,m-1) + B(n,m) \\
&= (m+1)\,(2n+2-m)\, B(n,m-1) + B(n,m)
\end{aligned}
$$

■

Step 2 : For all $n > m, n \in \mathbb{N} \cup \{0\}, m \in \mathbb{N} \cup \{0,-1\}$:[7]

$$B(n,m) = T(n,m) \tag{9}$$

Proof.

It is quite easy to see that (9) holds for $m = n-1$ since $B(n,n-1) = c(1,n,n-1) = \Gamma(n+2)\Gamma(n+1)$ and $T(n,n-1) = \Gamma(n+2)\Gamma(n+1)$ .We now show that (9) holds for

[7] The only properties of the gamma function we will need in what follows are:

1. $\Gamma(x) = (x-1)!$ whenever $x \in \mathbb{N}$. $\Gamma(x)$ is defined for all $x \in \mathbb{R}\backslash\{-1,-2,-3,...\}$.
2. $x\Gamma(x) = \Gamma(x+1)$
3. $\Gamma(\frac{x}{2})\Gamma(\frac{x+1}{2}) = 2^{x-1}\Gamma(x)\Gamma(\frac{1}{2})$ which can be easily checked by induction using (2.).

$m = -1$ and any integer $n > m$.

$$\begin{aligned}
B(n,-1) &= \sum_{j=1}^{n+1} (2j-1)^2 \, \frac{(j+n)!\,(n-j+1)!}{(j+n+1-1)!\,(n-j+1)!} \\
&= \sum_{j=1}^{n+1} (2j-1)^2 = 4\sum_{j=1}^{n+1} j^2 - 4\sum_{j=1}^{n+1} j + (n+1) \\
&= 4\frac{(n+1)(n+2)(2n+3)}{6} - 4\frac{(n+1)(n+2)}{2} + (n+1) \\
&= 1 + \frac{11}{3}n + 4n^2 + \frac{4}{3}n^3 \\
&= \frac{4}{3}(n+\frac{3}{2})\,(n+1)\left(n+\frac{1}{2}\right)
\end{aligned}$$

$$\begin{aligned}
T(n,-1) &= \frac{\Gamma(n+2)\Gamma(n+\frac{5}{2})\Gamma(1)\Gamma(\frac{1}{2})}{\Gamma(n+1)\Gamma(n+\frac{1}{2})\Gamma(\frac{5}{2})} \\
&= \frac{((n+1)\Gamma(n+1))\left(\left(n+\frac{3}{2}\right)\left(n+\frac{1}{2}\right)\Gamma\left(n+\frac{1}{2}\right)\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(n+1\right)\Gamma\left(n+\frac{1}{2}\right)\left(\frac{1}{2}\frac{3}{2}\Gamma\left(\frac{1}{2}\right)\right)} \\
&= \frac{4}{3}(n+\frac{3}{2})\,(n+1)\left(n+\frac{1}{2}\right) = B(n,-1)
\end{aligned}$$

Note that $B(n,n-1)$ and $B(n,-1)$, together with the recurrence relation (8), guarantees that $B(n,m)$ are defined for all $n > m \geq -1$. Moreover, we have proved that $B(n,n-1) = T(n,n-1)$ and $B(n,-1) = T(n,-1)$. Finally, to complete the proof we will show that $T(n,m)$ satisfies the same recurrence relation as $B(n,m)$ (given by (8)). We derive first

$$\begin{aligned}
T(n+1,m) &= \frac{(n+2)\left(n+\frac{5}{2}\right)}{\left(n-m-\frac{1}{2}\right)(n-m)} T(n,m) \\
T(n,m-1) &= \frac{\left(m+\frac{5}{2}\right)}{(m+1)\left(n-m-\frac{1}{2}\right)(n-m)} T(n,m)
\end{aligned}$$

and hence

$$
\begin{aligned}
& (m+1)(2n+2-m)\,T(n,m-1)+T(n,m) \\
= & \left(\frac{(m+1)(2n+2-m)\left(m+\frac{5}{2}\right)}{(m+1)\left(n-m-\frac{1}{2}\right)(n-m)}+1\right)T(n,m) \\
= & \frac{(2n+2-m)\left(m+\frac{5}{2}\right)+\left(n-m-\frac{1}{2}\right)(n-m)}{\left(n-m-\frac{1}{2}\right)(n-m)}T(n,m) \\
= & \frac{(n+2)\left(n+\frac{5}{2}\right)}{\left(n-m-\frac{1}{2}\right)(n-m)}T(n,m) \\
= & T(n+1,m)
\end{aligned}
$$

Hence, we have proved that $B(n,m) = T(n,m)$ for all $n,m \in \mathbb{N}$, $n > m \geq -1$. Therefore, it is obvious that $B(n,m) = T(n,m)$ for all $n,m \in \mathbb{N}$, $n > m \geq 0$.

This gives us the claimed form of $\Delta V(N,Z)$:

$$
\begin{aligned}
& \Delta V(2n+1,2m+1) \\
= & A(n,m)T(n,m) \\
= & 2\frac{\Gamma(2m+2)\,\Gamma(2n-2m-1)}{(2n+1)\,\Gamma^2(m+1)\,(m+1)\,\Gamma(2n+3)}\frac{\Gamma(n+2)\,\Gamma\left(n+\frac{5}{2}\right)\Gamma(m+2)\,\Gamma(\frac{1}{2})}{\Gamma(n-m)\,\Gamma\left(n-m-\frac{1}{2}\right)\Gamma\left(m+\frac{7}{2}\right)} \\
= & \frac{1}{8}\frac{\left(n+\frac{3}{2}\right)}{\left(m+\frac{5}{2}\right)\left(m+\frac{3}{2}\right)\left(n+\frac{1}{2}\right)}
\end{aligned}
$$

Finally, we substitute $m = \frac{Z-1}{2}$, $n = \frac{N-1}{2}$ and get

$$
\Delta V(N,Z) = \frac{1}{2}\frac{(N+2)}{(Z+4)(Z+2)\,N} \tag{10}
$$

B Appendix

1. **Data for the Western European Countries**[8]

State	**Pop. [mill.]**	**Ass.**	**State**	**Pop. [mill.]**	**Ass.**
Monaco	0.03	18	Sweden	8.6	349
Liechtenstein	0.03	25	Portugal	9.4	230
Iceland	0.26	63	Belgium	10	212
Luxembourg	0.39	60	Greece	10.4	300
Ireland	3.6	166	Netherlands	15.3	150
Norway	4.3	165	Spain	39.1	350
Finland	5	200	France	57.7	577
Denmark	5.2	179	U.K.	57.9	650
Switzerland	6.8	200	Italy	58	630
Austria	7.9	183	Germany	81.5	662

[8] Source: Boden [**5**]

2. **Data for Germany**

Bundesland	**Population [1000]**	**Assembly**
Bremen	663	100
Saarland	1,072	102
Hamburg	1,705	121
Mecklenburg-Vorpommern	1,789	71
Thüringen	2,449	88
Brandenburg	2,601	88
Sachsen-Anhalt	2,649	99
Schleswig-Holstein	2,777	75
Berlin	3,387	206
Rheinland-Pfalz	4,031	101
Sachsen	4,460	120
Hessen	6,052	110
Niedersachsen	7,899	161
Baden-Württemberg	10,476	195
Bayern	12,155	204
Nordrhein-Westfalen	18,000	221

This data was obtained from the "Statistisches Bundesamt Deutschland" and "Bayerisches Landesamt für Statistik und Datenverarbeitung".[9]

[9] The data can be downloaded from the following sites: *www.statistik-bund.de/jahrbuch/jahrtab1.html* *www.bayern.de/lfstad/BW98/a-z/l.html*

3. **Data for Switzerland**

Kanton	**Population**	**Assembly**	**Kanton**	**Population**	**Assembly**
AI	14, 946	46	TG	227, 285	130
OW	32, 225	55	FR	234, 307	130
UR	35, 487	62	SO	243, 908	144
NW	37, 657	60	BL	258, 602	90
GL	38, 708	80	VS	275, 632	130
AR	53, 737	65	TI	308, 498	90
JU	68, 818	60	LU	345, 357	120
SH	73, 552	80	GE	403, 067	100
ZG	97, 758	80	SG	447, 609	180
SZ	128, 248	100	AG	540, 639	200
NE	165, 649	115	VD	616, 275	180
GR	186, 026	120	BE	943, 427	200
BS	188, 458	130	ZH	1, 198, 569	180

This data was obtained from the Swiss "Bundesamt für Statistik" and the "Institut für Politikwissenschaften Universität Bern" by private communication (KTW_anzahlsitze.xls).

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