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Information in tournaments under limited liability

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Information in tournaments under limited liability [‡]

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Abstract

The problem of designing tournament contracts under limited liability and alternative performance measures is considered. Under risk neutrality, only the best performing agent receives an extra premium if the liability constraint becomes binding. Under risk aversion, more than one prize is awarded. In both situations, performance measures can be ranked if their likelihood ratio distribution functions differ by a mean preserving spread. The latter result is applied to questions of contest design and more general forms of relative performance payment.

Keywords: contest, information, likelihood ratio distribution, tournament

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1 Introduction

Tournaments and contests are widely used for compensation and incentive purposes. To this end, planners apply a variety of performance indicators. In some cases, the performance measure is identical to the organizer's objective, in others, there seems to be only an indirect relation to what is sought to be procured by the tournament. In all cases, however, the incentive effects of potential performance measures are at least implicitly taken into account.

From an economic perspective, incentive effects of performance measurement have been extensively discussed within a standard agency framework. Starting with the pioneering work of Holmström [11], numerous papers have analyzed questions of information efficiency under the assumption of optimal contracts in a moral hazard setting. In doing so, they established several criteria for ranking alternative information systems (see, for example, Gjesdal [6], Holmström [12], Grossman and Hart [9], Amershi and Hughes [1], Kim [13], or Demougin and Fluet [5]).

Only little work, however, has so far been done on information efficiency in a tournament setting where contracts are exogenously restricted to the order of the agents' performance. Under such a restriction, the criteria derived in the standard setting are not naturally valid. As Holmström ([11], p. 86) states, "if, for administrative reasons, one has restricted attention *a priori* to a limited class of contracts (e.g., linear price functions or instruction-like step functions), then informativeness may not be sufficient for improvements within this class". In general, the same objection applies to the other criteria. Due to the widespread use of tournaments, it is therefore worthwhile reasoning whether the criteria derived in the standard agency setting also apply under the specific restrictions given by a tournament contract.

The classical agency literature discusses the use of information in tournaments mainly with regard to optimal contracts. Adapting his sufficient statistics results derived from the standard agency setting, Holmström ([12], proposition 7) proves that relative performance evaluation will be valuable if and only if the agent's outputs are stochastically dependent. Similarly, Green and Stokey ([8], proposition 1) show in a more specific setting that individual contracts dominate tournaments whenever the agents' outputs admit only idiosyncratic risk. Conversely, if there is common uncertainty, tournaments will dominate individual contracts when the common shock becomes diffuse (ibid., proposition 2). Mookherjee ([20], proposition 4) applies Holmström's [11] informativeness result to show that a tournament contract will be optimal if an agent's rank in output is statistically sufficient for all available information with respect to his action choice.

In all of these results, informativeness criteria are applied to distinguish between different types of contracts. We shall return to this important question when applying our general results. At first, however, we look for criteria to rank information systems in a setting where contracts are exogenously restricted to rank orders. To distinguish the analysis from the previous research on *optimal* contracts, we deliberately confine ourselves to analyzing situations where the agents' performances are stochastically independent. According to Holmström's [12] result mentioned above, a tournament contract will not be optimal in this setting, and an application of the general informativeness results will thus not be valid. We show that nonetheless, the main criteria also apply to the tournament setting. Thereafter, we use these criteria to distinguish between different types of relative performance evaluation.

The remainder of this paper is organized as follows: In section 2, the analytical framework is presented. Section 3 describes the main characteristics of an information system with respect to the principal's optimization problem. Sections 4 and 5 present information efficiency results and some application to the comparison of alternative contract types. Concluding remarks are given in section 6.

2 Model

Consider a single-period agency setting in which a risk-neutral principal hires a number of agents $i = 1, \ldots, n \quad (n \geq 2)$ to perform identical tasks. The agents who decide to participate provide a productive input $a_i \in A = [\underline{a}, \overline{a}] \subset \mathbb{R}$ not observed by the principal. The resulting outputs $x_i \in X \subseteq \mathbb{R}$ which accrue to the principal are independent identically distributed random variables. Their probability distribution function $F(x_i; a_i)$ is parameterized by the agent's action choice. Increases in a_i are assumed to shift the distribution function to the right in the sense of first-order stochastic dominance. Thus, ceteris paribus the principal will prefer higher effort levels.

All agents have identical preferences. These can be described by utility functions which are additively separable in monetary income w_i and effort a_i, such as

$$U_i(w_i, a_i) = u(w_i) - d(a_i),$$

where $u(w^i)$ denotes the agent's utility on monetary income and $d(a_i)$ denotes the agent's disutility of action a_i. We assume that the agents are effort-averse and weakly risk-averse[1], i.e. $u' > 0, u'' \leq 0$, $d' > 0$ and $d'' > 0$.

Before hiring the agents, the principal chooses an information system k from a set $\mathcal{K}$ of feasible information systems. Information system k consists of signals $y_1^k, \ldots, y_n^k \in Y^k = \left[\underline{y}^k, \bar{y}^k\right] \subseteq \mathbb{R}$ which become observable to the principal and the agents without cost after the action choices have been taken. The signals ${}_i^k$ are independent identically distributed random variables with distribution function $G^k(y_i^k; a_i)$ and probability density function $g^k(y_i^k; a_i)$ which depend on the respective agent's effort. The latter is assumed to be twice differentiable in a_i, and its support is independent of a_i. Signal y_i^k can be regarded as a performance measure for agent i. In the simplest case, performance is measured by output ($y_i = x_i$). More generally, y_i^k may be an index of all available information on the agent's action. We assume that higher effort can be inferred from

[1]Results are presented separately for risk-neutral and strictly risk averse agents.

a higher performance score, i.e. the monotone likelihood ratio property (MLRP) is assumed to hold for any information service k.

Given the information system, the principal designs an ordinal payment scheme which determines agent i's compensation according to his rank r_i in the order of the observed signals. Let w^j denote the compensation stipulated for the jth-lowest rank j within the order of performance measures. In the case of multiple ranking agents the respective prize is awarded via randomization[2]. Under these assumptions, agent i's expected utility from action choices $\mathbf{a} = (a_1, \ldots, a_n)$ can be written as

$$\mathrm{E}\left[U_i(\mathbf{a})\right] = \sum_{j=1}^{n} u(w^j) p_{ij}^k(\mathbf{a}) - d(a_i),$$

where $p_{ij}^k(\mathbf{a}) = \mathrm{Prob}\{r_i = j \mid (a_1, \ldots, a_n)\}$ denotes the probability that agent i will achieve rank j in the tournament [3].

The principal wants all agents to participate. Hence, their expected utilities have to reach a certain reservation level U^R which is assumed to be identical for all agents. Furthermore, we assume that agents are of restricted wealth, and thus compensation has to exceed a liability level w^{min} for each agent.

3 The principal's problem and properties of information systems

The principal seeks to maximize his expected profit net of wage payments. His problem is to select compensations $\mathbf{w} = (w^1, \ldots, w^n)$ from a set $\mathcal{W}^n \subset \mathbb{R}^n$ of feasible compensations such that the agents choose actions $\hat{a}_i$ which maximize his expected net profit.

[2] However, this will occur with zero probability because the density functions g^k have no mass points.

[3] According to this diction, rank n denotes the highest outcome.

Due to the principal's risk neutrality, this problem can be split up, first considering the least cost way of achieving a given action profile and then turning to the question of which actions to implement. To our purpose of comparing performance measures, the interesting part is the first. Therefore, similar to Kim's [13] analysis of the standard agency model, we will focus on the question of what type of information system k implements a particular effort profile $\mathbf{a}$ at the lowest cost. In doing so, we restrict the analysis to symmetric Nash equilibria of the tournament game. Consequently, all agents choose the same action $\hat{a}$, and each agent's probability of winning is $1/n$. The principal's cost minimization problem for the symmetric equilibrium given information systems k is given by

$$\min_{\mathbf{w}} \quad \sum_{j=1}^{n} w^j \tag{1}$$

$$\text{s.t.} \quad \frac{1}{n}\sum_{j=1}^{n} u(w^j) - d(\hat{a}) \geq U^R \tag{2}$$

$$\hat{a} \in \arg\max_{a_i} \left\{ \sum_{j=1}^{n} u(w^j) p_{ij}^k(a_i, \hat{\mathbf{a}}_{-i}) - d(a_i) \right\} \tag{3}$$

$$w^j \geq w^{min} \quad \forall j. \tag{4}$$

The participation constraints (2) guarantee that all agents accept the contract. The Nash - incentive constraints (3) ensure that given his opponents equilibrium strategies $\hat{\mathbf{a}}_{-i}$, the desired action a_i is in agent i's own best interest.

In this optimization problem, information system k is characterized by a vector $\mathbf{p}_i^k = (p_{i1}^k, \ldots, p_{in}^k)$ of ranking probabilities. In order to compare information systems with regard to their cost of inducing a certain action a, we are interested in properties of these probabilities, which in more detail can be written as[4]

$$p_{ij}^k(a_i, \hat{\mathbf{a}}_{-i}) = \frac{1}{n} \int_{Y^k} \frac{g^k(y; a_i)}{g^k(y; \hat{a})} g_{j:n}^k(y; \hat{a}) dy, \tag{5}$$

[4]See Green and Stokey [8], p. 355.

where $g^k_{j:n}(y;a)$ denotes the density of the $(j:n)$-order statistic under distribution G^k. For $a_i = \hat{a}$, the integral in (5) is 1, and $p^k_{ij} = 1/n$. Differing from $\hat{a}$, the agent varies his ranking probabilities. The way in which these changes work at $\hat{a}$ mainly determines the incentive effects of information system k. Similar to the standard agency setting, further insight into the quality of a performance measure can be gained under the first-order approach. As in the standard model, it is valid under the additional assumption that $G^k(y^k;a_i)$ is convex in a (convexity of the distribution function condition, CDFC)[5]. The incentive constraints (3) can then be replaced by first-order conditions

$$\sum_{j=1}^{n} u(w^j)\frac{\partial}{\partial a_i}p^k_{ij}(\hat{\mathbf{a}}) - d'(\hat{a}) = 0 \tag{6}$$

governed by the marginal probabilities

$$\frac{\partial}{\partial a_i}p^k_{ij}(\hat{\mathbf{a}}) = \frac{1}{n}\int\limits_{Y^k}\frac{g^k_a(y;\hat{a})}{g^k(y;\hat{a})}g^k_{j:n}(y;\hat{a})dy. \tag{7}$$

The integral in (7) is the expected value of the score function $\frac{g_a}{g}$ for the $(j:n)$ - order statistic of performance scores. By MLRP, this function, which for simplicity is often referred to as the likelihood ratio, is increasing in y^k. Therefore, the agents' action choices $\hat{a}$ in the symmetric equilibrium are determined by

$$\frac{1}{n}\sum_{j=1}^{n} u(w^j)\mathrm{E}\left[lr^{k,\hat{a}}_{j:n}\right] = d'(\hat{a}), \tag{8}$$

where $lr^{k,\hat{a}}_{j:n}$ denotes the $(j:n)$ - order statistic of likelihood ratios derived from g^k at point $\hat{a}$. Note that $\sum_{j=1}^{n}\mathrm{E}\left[lr^{k,\hat{a}}_{j:n}\right] = n\mathrm{E}\left[lr^{k,\hat{a}}\right] = 0$ by the assumption of non-moving supports[6]. Therefore, the incentive effects of some prizes $w^1,\ldots,w^l$ $(l<n)$ will be negative, whereas those of the remaining ones will be positive.

In the following sections, we will exploit further properties of moments and distributions of order statistics in order to compare different information services. Section

[5]A proof is available from the author upon request.

[6]For the relation of order statistics, see Arnold et al. 1992, p. 110.

4 derives results for the risk neutral agency, and section 5 presents the findings for risk-averse agents. Both sections contain applications to questions of contest design and more general forms of relative performance payment.

4 risk neutral agents

4.1 Optimal reward structure

If the agents are risk-neutral, the principal's problem simplifies to

$$\min_{w^1,\ldots,w^n} \quad \sum_{j=1}^{n} w^j \tag{9}$$

$$\text{s.t.} \quad \frac{1}{n}\sum_{j=1}^{n} w^j - d(a) \geq U^R \tag{10}$$

$$\frac{1}{n}\sum_{j=1}^{n} w^j \mathrm{E}\left[lr_{j:n}^{k,a}\right] = d'(a) \tag{11}$$

$$w^j \geq w^{min} \quad \forall j. \tag{12}$$

Similar to Lazear's and Rosen's [15] analysis of a tournament with two agents, the first-best solution can be achieved under *any* informative performance measure as long as the liability constraints (12) are not binding. Starting from equal prizes for all ranks, the principal just has to increase the prize differentials $w^j - w^{j-1}$ for arbitrary ranks j with positive value of $\mathrm{E}\left[lr_{j:n}^{k,a}\right]$ until the Nash incentive constraints (11) are fulfilled. By adjustment of w^1, the participation constraint can be fulfilled with equality. Implementation is without additional cost because of the agent's risk neutrality. The resulting total compensation cost is $n\left(d(a)+U^R\right)$.

Under limited liability, however, this procedure, in general, will not be feasible. For low levels of U^R and $\mathrm{E}\left[lr_{j:n}^{k,a}\right]$, the liability constraints will become binding in the optimal solution. As a consequence, it will matter to which ranks the prize differentials are allocated. Due to the agents' risk neutrality, however, the optimal prize structure is apparently simple:

Proposition 1 *If the agents' liability constraint is binding under information system k, the cost-minimizing tournament only awards a prize to the best performing agent.*

Proof The proof is in the appendix.

Essentially, the proof of proposition 1 shows that compensation cost can be lowered by shifting compensation from lower ranks to the the highest rank of the tournament. The economics of the result are similar to those in the standard agency setting as derived by Demougin and Fluet [4]. If the agents are risk neutral, income smoothing only matters with regard to the minimum wage. Incentives, however, are least costly provided by rewarding only those results the probability of which reacts most sensitive to changes in the agent's effort. In the contest setting, due to the MLRP this is the top rank $r_{n:n}$.

The proposition also complies with results of Moldovanu and Sela [18] who find that in a symmetric equilibrium of privately informed contestants, a total premium is most effectively allocated to only the winner of the contest[7]. Similar to the moral hazard setting analyzed here, the result is driven by the fact that a single prize provides the strongest incentives for risk neutral contestants. However, since under private pre-decision information different types of agents choose different effort levels in equilibrium, Moldovanu's and Sela's result requires linear or concave cost functions for which variations in effort have no cost increasing effects. In the present setting, convexity of the cost function is not an issue since all agents choose identical actions.

[7]The same result is derived by Glazer and Hassin [7] in a related framework, but under more restrictive assumptions.

4.2 Information efficiency

Given the structure of the optimal contract, a comparison of alternative information systems is straightforward. Whenever the agents' liability constraints are binding, the optimal reward scheme takes the form $\mathbf{w} = (w^{min}, \ldots, w^{min}, w^n)$. In this scheme, w^n has to be chosen to fulfil the agents' Nash incentive compatibility constraint (11), which takes the form

$$\frac{1}{n}(w^n - w^{min})\mathrm{E}[lr^{k,a}_{n:n}] = d'(\hat{a}). \tag{13}$$

According to (13), the necessary wage spread to induce action $\hat{a}$ is given by

$$(w^n - w^{min}) = \frac{nd'(\hat{a})}{\mathrm{E}[lr^{k,a}_{n:n}]}.$$

As a consequence, the total compensation cost under information system k in a symmetric equilibrium of n contestants with action choices $\hat{a}$ can be written as

$$C^k_n(\hat{a}) = n \cdot \max\left\{ d(\hat{a}) + U^R, w^{min} + \frac{d'(\hat{a})}{\mathrm{E}[lr^{k,a}_{n:n}]} \right\}. \tag{14}$$

By inspection of (14), it is obvious that an information systems cost impact is solely determined by $\mathrm{E}[lr^{k,a}_{n:n}]$:

Proposition 2 *In the symmetric equilibrium $\hat{a}$ of the tournament under information system k, total compensation cost is the lower, the higher $E[lr^{k,\hat{a}}_{n:n}]$.*

Proof Obvious from (14). C^k_n is decreasing in $\mathrm{E}[lr^{k,a}_{n:n}]$. □

Given the prominent role of the likelihood ratio in (14), proposition 2, when related to the literature on informativeness criteria, provides a direct reference to Kim's [13] criterion of a mean preserving spread of likelihood ratio distribution functions. Kim [13] proves that in a standard agency setting with one risk-averse agent, an action $\hat{a}$ can be induced under a signal y^l at a lower cost than under another signal y^m if the distribution function of the likelihood ratio $\frac{g^l_a}{g^l}$ under signal y^l differs from that under signal y^m by a mean preserving spread (MPS)[8]. Since due to the assumption of a non-moving support the expected likelihood ratio is zero for all information systems, the MPS relation reduces to second order stochastic dominance. In order to exploit this property, Kim essentially shows that the compensation cost is a concave function of likelihood ratios, the expectation of which is lower under second order stochastic dominance[9]. A related convexity argument can be applied here to establish the mean preserving spread criterion as a device to rank information systems in the tournament setting:

Proposition 3 *In the symmetric equilibrium $\hat{a}$ of the tournament, total compensation cost under information system y^l is lower than that under information system y^m if the distribution function of the likelihood ratio $lr^{l,\hat{a}} = \frac{g_a^l(y^l;\hat{a})}{g^l(y^l;\hat{a})}$ under signal y^l differs from that under signal y^m by a mean preserving spread.*

Proof [10] The proof is in the appendix.

The proof of proposition 3 makes use of the fact that the well known consequences of second order stochastic dominance between univariate distributions extend to the product distribution of i.i.d. random variables. By the convexity of the maximum operator, it is thus obvious that a mean preserving spread relation yields a unique order of highest order statistics. Therefore, like in the standard agency setting, information systems can be compared by the distributions of their likelihood ratios. Similarly, the mean preserving spread property only provides a local criterion for a specific action $\hat{a}$. To make general predictions for arbitrary levels of a, the relation must hold for *all* $a \in (\underline{a}, \overline{a}]$. This, again, has been proven by Kim ([13], proposition 4) to follow from

[8]For the definitions of a mean preserving spread see Rothschild and Stiglitz [21].

[9]The result can also be carried forward to a standard agency model with a risk neutral agent who is of limited wealth.

[10]A related proof for risk-averse agents can be found in Budde and Gaffke [3].

the criterion of Blackwell informativeness (the opposite is not true). From this, the following conclusion is obvious:

Corrollary 1 *In* any *symmetric equilibrium of the tournament, total compensation cost under information system y^l is lower than that under information system y^m if y^l is Blackwell sufficient for y^m with respect to a.*

Proof The claim follows from proposition 3 by the relation of Blackwell sufficiency and the mean preserving spread criterion.

Although more information systems will be comparable by the mean preserving spread criterion, Blackwell sufficiency is useful for at least two reasons. First, it is a global criterion which does not focus on a particular effort level $\hat{a}$. Therefore, information systems can be compared by it without specifying which action is sought to be induced. Second, and for the application even more important, the criterion refers to the signal distributions instead of the distributions of likelihood ratios. Usually, this will make its use much easier. In the following subsection, we apply Corollary 1 to rank different types of relative performance payment.

4.3 Application

4.3.1 Alternative forms of relative performance evaluation

Perhaps the most important property of tournament contracts is the fact that the total compensation paid to all n agents is constant. Malcomson [16] uses this property to propose tournaments as a general device to overcome the unverifiability problem, i.e. tournaments can be used for compensation even if the applied performance measures are not verifiable and the principal could misreport these measures in order to cut wages. Yet, this is impossible under a tournament contract as long as contracts and payments are observable.

Tournaments, however, are not the only compensation form to fulfil the desired property of a constant total wage payment. In particular, Japanese firms make extensive use of a special kind of relative performance payment in which a constant bonus W is distributed to workers of a group according to their relative outputs. Agent i's wage in accordance with outputs $x_1, \ldots, x_n$ is given by

$$w_i = w_0 + \frac{x_i}{\sum_{j=1}^{n} x_j} W. \tag{15}$$

Due to its similarity to a tournament, this type of compensation has also been referred to as a *J-Type tournament* after to its Japanese origin as opposed to *U-Type tournaments* of the form described in section 2, which are predominantly applied in the US (Kräkel [14]).

With regard to the general question of compensation cost analyzed here, the two types of compensation contracts can be compared by application of the criteria derived in the previous section:

Proposition 4 *In the symmetric equilibrium $\hat{a}$ of the tournament game, total compensation cost in a U-type tournament is lower than that in a J-type tournament.*

Proof The proof is in the appendix.

The proof of proposition 4 makes use of the fact that the bonus portion in (15) is identical to a contest success function. This contest success function, in turn, is known in a two-player contest to be identical to the winning probability under exponentially distributed outputs (see Hirshleifer and Riley [10], p. 380n.). The proof generalizes this property by assuming an n-player tournament and shows that risk-neutral agents assess a J-Type tournament equal to a U-Type tournament with an additional randomization. This randomization, however, weakens the incentives of the contest, leading to a higher compensation cost.

4.3.2 Contest design

Moldovanu and Sela [19] analyze (amongst others) the question of whether a contest should be split into several sub-contests in a situation of private pre-decision information. They prove that for linear or convex cost functions, the grand contest generates a higher expected output than any contest divided into subgroups of equal size (ibid., Theorem 1). Adapting this question to the present moral hazard situation, we find the following result:

Proposition 5 *Total compensation cost to induce a certain action $\hat{a}$ in a symmetric equilibrium of risk neutral contestants is lower under a grand contest of n agents than under any split contest of subgroups with $n_1 \in \{2, \ldots n-2\}$ and $n_2 = n - n_1$ agents.*

Proof Average compensation cost per agent in each of the subgroups is

$$c_{n_i}(\hat{a}) = \frac{C_{n_i}(\hat{a})}{n_i} = \max\left\{ d(\hat{a}) + U^R, w^{min} + \frac{d'(\hat{a})}{\mathrm{E}[lr^{k,a}_{n_i:n_i}]} \right\}, \quad i = 1, 2. \tag{16}$$

Since $\mathrm{E}[lr^{k,a}_{n_i:n_i}] < \mathrm{E}[lr^{k,a}_{n:n}]$ for $n_i < n$, average compensation cost is higher in each subgroup, from which the claim follows by the fact that $n_1 + n_2 = n$. □

The result is derived from the fact that the average cost (16) is decreasing in the number of contestants[11]. Due to the agent's risk neutrality, the fact that each agent's probability of winning decreases does not result in an additional cost. Due to the MLRP, however, compensation reacts most sensitively to changes in the agents' effort if they compete in a grand contest.

[11]This is in line with proposition 2 in Moldovanu and Sela [19].

5 Risk averse agents

5.1 Optimal reward structure

If the competing agents are risk averse, the proposed extreme prize schedule in which only the best performing agent receives an extra payment will no longer be optimal. This can be illustrated by the following counterexample:

Example Consider a group of $n = 3$ risk-averse agents competing in a contest with prize structure $\mathbf{w} = (w^1, w^2, w^3)$. Prizes are allocated according to signals $y_i \in \mathbb{R}^+$ which follow the same family of probability distributions described by cumulative distribution functions $G(y_i \mid a_i) = 1 - \exp(y_i/a_i)$. Thus, the agent's performance measures are exponentially distributed with mean a_i. Furthermore, let the agents' preferences be described by identical utility functions $U_i(w_i, a_i) = \sqrt{w_i} - a_i^2$, and let their reservation utilities be $U^R = 0$. Prizes have to be nonnegative. Assuming a symmetric equilibrium of the contest game, the principal wants to implement an equilibrium effort $\hat{a} = 1$ for all agents. His cost minimization problem described in (1) - (4) then becomes

$$\min_{w^1,w^2,w^3} \quad w^1 + w^2 + w^3 \tag{17}$$

$$\text{s.t.} \quad \frac{1}{3}\left[\sqrt{w^1} + \sqrt{w^2} + \sqrt{w^3}\right] - 1 \geq 0 \tag{18}$$

$$-\frac{2}{3}\sqrt{w^1} + -\frac{1}{6}\sqrt{w^2} + \frac{5}{6}\sqrt{w^3} = 2 \tag{19}$$

$$w^j \geq 0 \quad j = 1, 2, 3. \tag{20}$$

The coefficients in (19) are the expected values of the likelihood ratio order statistics under the exponential distribution with mean 1. The cost minimizing prize structure is given by $w^1 = 0, w^2 = (6/7)^2$ and $w^3 = (18/7)^2$. Obviously, it assigns positive prizes to more than just the top ranking position. □

Similar to the situation analyzed in proposition 1, the agent's liability constraint is binding in the example. However, the contract proposed there would impose too much risk on the agents. Therefore, incentives have to be provided also by w^2. This is less effective than solely rewarding the best performing agent, but under risk aversion also less costly.

5.2 Information efficiency

Given the counterexample, the ranking criteria derived in the previous section cannot directly be translated to the model with risk averse-agents because they build on the extreme contract of proposition 1. Under a more general prize structure, the compensation will not only depend on the value of $\mathrm{E}\left[lr_{n:n}^{k,\hat{a}}\right]$ as in (14), but in general on the expectations of *all* likelihood ratio order statistics. Yet, since $\mathrm{E}\left[\mathrm{E}\left[lr_{n:n}^{k,\hat{a}}\right]\right] = 0$ for all k, the relation of order statistics used in propositions 2 and 3 cannot hold for *all* ranks. However, if the distribution function of the likelihood ratio $lr^{l,\hat{a}} = \frac{g_a^l(y^l;\hat{a})}{g^l(y^l;\hat{a})}$ under signal y^l differs from that under signal y^m by a mean preserving spread, the same should hold for the likelihood ratio distribution functions of the ranks achieved in a contest under these measures. Intuitively, this results in prizes which are less dispersed, which in turn yields lower compensation cost due to the agents' risk aversion.

To prove this intuition, we first give a condition of less dispersed prizes under which total compensation cost is reduced (lemma 1). Subsequently, we prove that this condition is fulfilled under the mean preserving spread criterion (proposition 6).

Lemma 1 *Let* $\mathbf{w} = (w^1, \ldots, w^n)$ *and* $\mathbf{v} = (v^1, \ldots, v^n)$ *be incentive compatible prize schedules fulfilling restrictions (2), (4) and (8) in the symmetric equilibrium of the tournament. If the utility spreads resulting from that prizes under a concave utility* *function u are higher under* $\mathbf{w}$*, then total compensation cost is less under* $\mathbf{v}$*, i.e.*

$$u(w^j) - u(w^{j-1}) \geq u(v^j) - u(v^{j-1}) \textit{ for } j = 2, \ldots, n \tag{21}$$

$$\Rightarrow \sum_{j=1}^{n} w^j \geq \sum_{j=1}^{n} v^j. \tag{22}$$

Proof The proof is in the appendix.

The lemma intuitively follows from the agents' risk aversion and limited liability. Under an optimal prize structure, either the agents' participation constraint or their liability constraint will be binding. If the participation constraint is binding under both schedules, the higher utility spreads under schedule $\mathbf{w}$ produce a mean preserving spread relation of the distribution functions of utilities. The claim then follows from the agents' risk aversion. If on the other hand the liability constraint is binding, the higher utility spreads under $\mathbf{w}$ result in prizes which are higher for each rank. In this case, the claim is even more obvious.

The lemma can be used to compare different information structures. For this purpose, it is convenient to write the agents' expected utility in a way which refers to utility spreads:

$$\mathrm{E}U_i(\mathbf{a}) = u(w^1) + \sum_{j=2}^{n} \left[u(w^j) - u(w^{j-1})\right] P_{ij}^k(\mathbf{a}) - d(a_i). \tag{23}$$

The term $P_{ij}^k(\mathbf{a}) = \mathrm{Prob}\{r_i > j-1\}$ denotes the probability that agent i achieves *at least* rank j in the tournament under information system y^k. Given his opponents' effort $\hat{a}$ in the symmetric equilibrium, this probability is given by

$$P_{ij}^k(a_i, \mathbf{a}_{-i}) = \int_{Y^k} (1 - G^k(y^k; a_i)) g_{j-1:n-1}(y^k; \hat{a}) dy^k. \tag{24}$$

Substituting (24) in (23), the agent's first-order condition becomes

$$\frac{\partial}{\partial a_i} \mathrm{E}U_i(\mathbf{a}) = \sum_{j=2}^{n} \left(\left[u(w^j) - u(w^{j-1})\right] \int_{Y^k} -G_a^k(y^k; a_i) g_{j-1:n-1}(y^k; \hat{a}) dy^k \right) - d'(a_i). \tag{25}$$

This expression can be used to prove that the mean preserving spread criterion also applies to the setting with risk averse agents. For this purpose, we make use of a recent finding by Demougin and Fluet [5] who prove that Kim's mean preserving spread criterion is equivalent to their so-called integral condition, which is defined for the transformed signals $z^l = G^l(y^l;\hat{a})$ and $z^m = G^m(y^m;\hat{a})$. Due to the assumption of non-moving supports, z^k is as informative as y^k since G^k is strictly monotonic and an optimal contract can be based on z^k as well as on y^k. Denote by $H^l(z^l,a)$ and $H^m(z^m,a)$ the cumulative distribution functions of that signals, given a. The integral condition is fulfilled if

$$-H^l_a(z \mid \hat{a}) \geq -H^m_a(z \mid \hat{a}) \quad \forall z \in [0,1] \tag{26}$$

and is identical to the fact that the distribution function of $lr^{l,\hat{a}}$ differs from that of $lr^{m,\hat{a}}$ by a mean preserving spread (see [5], proposition 3). The main advantage of the criterion is that in contrast to the mean preserving spread relation, it allows for a simple and intuitive comparison of information structures in the standard agency setting (see [5], proposition 1). Similarly, the criterion can be applied in the contest setting to prove the following result:

Proposition 6 *In the symmetric equilibrium $\hat{a}$ of the tournament of risk-averse agents, total compensation cost under information system y^l is lower than that under information system y^m if the distribution function of the likelihood ratio $lr^{l,\hat{a}}$ under signal y^l differs from that under signal y^m by a mean preserving spread.*

Proof Let $\mathbf{w} = (w^1, \ldots, w_n)$ denote the optimal prize structure under information system y^m or z^m respectively[12], fulfilling the agents incentive compatibility constraint

$$\frac{\partial}{\partial a_i}\mathrm{E}U_i(\mathbf{a}) = \sum_{j=2}^{n}\left(\left[u(w^j)-u(w^{j-1})\right]\int_0^1 -H_a^m(z^m;a_i)h_{j-1:n-1}^m(z^m;\hat{a})dz^m\right)-d'(a_i). \tag{27}$$

By comparing this to the incentive constraint

$$\frac{\partial}{\partial a_i}\mathrm{E}U_i(\mathbf{a}) = \sum_{j=2}^{n}\left(\left[u(v^j)-u(v^{j-1})\right]\int_0^1 -H_a^l(z^l;a_i)h_{j-1:n-1}^l(z^l;\hat{a})dz^l\right)-d'(a_i) \tag{28}$$

under information system z_l derived from y^l and the respective prize schedule $\mathbf{v} = (v_1,\ldots,v_n)$, the integral condition can be applied:

Since z^l and z^m are values of cumulative distribution functions, they follow a uniform distribution on $[0,1]$. Thus, $h_{j-1:n-1}^l = h_{j-1:n-1}^m$ for all j. From this, the integral in (27) is smaller than that in (28) for each j, provided that (26) is fulfilled. Therefore, there exists a prize schedule $\mathbf{v}$ such that (28) is fulfilled and $u(w^j)-u(w^{j-1}) \geq u(v^j)-u(v^{j-1})$ for $j=2,\ldots,n$. The claim then follows from Lemma 1. □

The intuition of the result is readily carried forward from the arguments in Demougin and Fluet [5]. Relating the integral condition to their previous findings on bonus type contracts in the risk-neutral agency (see Demougin and Fluet [4]), they argue that under risk aversion, a signal is preferred in an optimal contract if it is also preferred under any bonus contract (see Demougin and Fluet [5], 490). The latter is obviously fulfilled under the integral condition.

We also make use of this fact and show that if a signal is preferred under any bonus contract, it is also preferred in a tournament. From a single agent's perspective, a tournament in this regard can best be described as a series of bonus contracts with randomized aspiration levels. These levels are given by the performances of the agent's

[12] Optimal prizes are identical under y^m and z^m because of the monotonicity of the distribution function.

rivals in the tournament. If a signal is more sensitive with respect to the agent's action for any possible value of these levels, it is also more sensitive in expected terms.

5.3 Application

Similar to the analysis of the tournament with risk neutral agents, the information efficiency results can be applied to compare different types of tournaments. In doing so, we again refer to the analysis of Moldovanu and Sela [19] of contest architecture. Our aim is to reinforce their result on the efficiency of the grand contest in the moral hazard setting analyzed here. Different to our proof in section 4, however, we cannot simply compare functions of total compensation cost as in (14) because now the compensation cost depends on the agent's risk attitude. To derive the desired result, we therefore at first prove that average compensation cost is decreasing in the number of agents (proposition 7), and then turn to the question of whether to split the contest or not (proposition 8).

Proposition 7 *Average compensation cost to induce a certain action $\hat{a}$ in a symmetric equilibrium of risk averse agents is decreasing in the number of contestants.*

Proof The proof is in the appendix.

The proof of proposition 7 makes use of the fact that the principal's optimization problem (1)–(4) is similar to the one of a standard single agent model in which the agent's performance is measured by his rank among $n-1$ agents choosing the equilibrium action $\hat{a}$. The proof shows that this signal becomes more informative in the sense of the mean preserving spread criterion when the number n of competitors increases. At first glance, this seems counterintuitive because each of the contestants adds noise to the performance measure. At the same time, however, the number of ranks increases, thereby enriching the principal's opportunities to calibrate the contract. As Malcomson [17] shows, for an infinite number of competitors this results in the equivalence of a rank order contract and a piece rate contract.

The result can directly be applied to answer the initial question:

Proposition 8 *Total compensation cost to induce a certain action $\hat{a}$ in a symmetric equilibrium of risk averse contestants is lower under a grand contest of n players than under any split contest of subgroups with $n_1 \in \{2, \ldots n-2\}$ and $n_2 = n - n_1$ players.*

Proof Obvious because average compensation cost is higher in both sub-contests, compared to the grand contest, which follows from proposition 7.

The reasoning behind proposition 8 is similar to the one of the preceding proposition 7. Although the grand contest determines an agent's compensation based on the noisiest information, it dominates all other architectures because it allows for the most precise stipulation of prizes. Since in general the tradeoff of these two effects is not obvious, the main contribution of the two propositions is to prove that the latter effect always dominates the former. At the same time, the difference to a model without exogenous restriction to a rank order tournament is highlighted. Without the restriction, each agent would receive a payment which is based only on his individual performance, because outputs are assumed to be independent. Since any contract based on y_i^k can be written, any information on another agent's output only adds noise to the compensation. Therefore, in that sense the result contrasts Holmström' s [11] informativeness result.

6 Conclusion

This paper analyzed whether the informativeness criteria derived for information systems in a standard agency setting of moral hazard, where the principal chooses an optimal contract in the second best solution, also apply to a tournament setting where the contract is exogenously restricted to be rank-dependent. As a main result, Kim's [13] mean preserving spread criterion was approved to be capable of ranking performance measures in the symmetric equilibrium of the tournament game. As a consequence, Blackwell sufficiency also applies. Although in view of these parallels transferability seems to be obvious, it is not trivial. The key feature connecting the two settings is that the MPS relation of likelihood ratios carries forward from the original signals to the ranks in the contest. Only from this, the result from second best contracts also holds in the constrained model.

Various applications of the result are possible. We used it to compare different types of contracts. The key idea is to attribute the comparison of contracts to that of different information systems using the same type of contract. While the present paper focussed on the comparison of specific contracts, the procedure could also be applied to more general questions of contract design. In particular, it may be used to identify conditions under which tournaments are optimal agreements with regard to a special class of contracts. One such class could be given by contracts which distribute a constant sum of payments among a group of agents. This class is of particular interest with respect to unverifiable or subjective performance information, as mentioned in subsection 4.3. Therefore, the furnished results may be a device to prove the optimality of tournaments as a solution to the so-called *unverifiability problem*.

A Proofs

Proof of proposition 1: Suppose not. Let $\mathbf{w} = (w^1, \ldots, w^n)$ denote the respective compensation schedule, with $w^j \geq w^{min}$ for all j and $w^j > w^{min}$ for at least one $j \in \{1, \ldots, n-1\}$. We show that this contract can be improved by one of the type described in the proposition.

To that purpose, consider the wage structure $\mathbf{v}' = (v^1, \ldots, v^n)$ with

$$v^j = w^{min} \quad \text{for } j = 1, \ldots, n-1$$

and

$$v^n = w^n + \sum_{j=1}^{n-1} (w^j - w^{min}) \frac{\mathrm{E}[lr^{k,\hat{a}}_{j:n}]}{\mathrm{E}[lr^{k,\hat{a}}_{n:n}]}.$$

According to (11), the incentive effects of $\mathbf{v}$ are identical to those of $\mathbf{w}$:

$$\begin{aligned} & \sum_{j=1}^{n} v^j \mathrm{E}\left[lr^{k,\hat{a}}_{j:n}\right] \\ = \; & \sum_{j=1}^{n-1} w^{min} \mathrm{E}\left[lr^{k,\hat{a}}_{j:n}\right] + \mathrm{E}[lr^{k,\hat{a}}_{n:n}] \left[w^n + \sum_{j=1}^{n-1} (w^j - w^{min}) \frac{\mathrm{E}[lr^{k,\hat{a}}_{j:n}]}{\mathrm{E}[lr^{k,\hat{a}}_{n:n}]} \right]. \\ = \; & \sum_{j=1}^{n} w^j \mathrm{E}[lr^{k,\hat{a}}_{j:n}]. \end{aligned}$$

The total wage payment, however, is lower under $\mathbf{v}$:

$$\begin{aligned} \sum_{j=1}^{n} v^j &= (n-1) w^{min} + w^n + \sum_{j=1}^{n-1} (w^j - w^{min}) \frac{\mathrm{E}[lr^{k,\hat{a}}_{j:n}]}{\mathrm{E}[lr^{k,\hat{a}}_{n:n}]} \\ &< (n-1) w^{min} + w^n + \sum_{j=1}^{n-1} (w^j - w^{min}) \\ &= \sum_{j=1}^{n} w^j. \end{aligned}$$

The inequality follows from the fact that $\mathrm{E}[lr^{k,\hat{a}}_{j:n}] < \mathrm{E}[lr^{k,\hat{a}}_{n:n}]$ for all $j < n$ by MLRP. □

Proof of proposition 3: Denote by $L^{k,\hat{a}}$ the distribution function of the likelihood ratio $lr^{k,\hat{a}}$, $k = l, m$. If $L^{l,\hat{a}}$ differs from $L^{m,\hat{a}}$ by a mean preserving spread, it is said to be *larger than* $L^{m,\hat{a}}$ *in the convex order*, which means that

$$\mathrm{E}_{L^{l,\hat{a}}}[\phi] \geq \mathrm{E}_{L^{m,\hat{a}}}[\phi]$$

for any convex function $\phi : \mathbb{R} \to \mathbb{R}$, provided the expectation exists[13]. The same holds for the joint distributions

$$M^{l,\hat{a}}(z_1, \ldots, z_n) = \prod_{i=1}^{n} L^{l,\hat{a}}(z_i) \quad \text{and} \quad M^{m,\hat{a}}(z_1, \ldots, z_n) = \prod_{i=1}^{n} L^{m,\hat{a}}(z_i)$$

of independent identically distributed random variables $z_i \in \mathbb{R}$ which are distributed according to L^l_a and L^m_a, respectively. The expectation of any convex function $\psi : \mathbb{R}^n \to \mathbb{R}$ is higher under distribution $M^{l,\hat{a}}$ (This follows from Theorem 5.A.3. in Shaked & Shantikumar [22].). The relation also applies to the convex function $\psi(z_1, \ldots, z_n) = \max_{i=1,\ldots,n} z_i$. Thus,

$$\mathrm{E}[lr^{l,\hat{a}}_{n:n}] = \mathrm{E}_{M^{l,\hat{a}}} \left\{ \max_{i=1,\ldots,n} z_i \right\} \geq \mathrm{E}_{M^{m,\hat{a}}} \left\{ \max_{i=1,\ldots,n} z_i \right\} = \mathrm{E}[lr^{m,\hat{a}}_{n:n}], \tag{29}$$

which establishes the proposed relation due to proposition 2. □

Proof of proposition 4: Consider performance measures $y_i \in [-\infty, 0]$ with cumulative distribution function $G(y_i \mid x_i) = \exp(x_i y_i)$ and probability density function $g(y_i \mid x_i) = x_i \exp(x_i y_i)$ parameterized by the output x_i. Suppose that the signals y_i are used in a U-Type tournament of the form derived in proposition 1, and only the best performing agent receives a prize. Given $\mathbf{x} = (x_1, \ldots, x_n)$, agent $i's$ probability of winning

[13] See Shaked & Shanthikumar [22], p. 55, for a definition of convex orders, and Scarsini [23], p. 357, for the (explicit) relation of convex orders and mean preserving spreads.

this prize is

$$\begin{aligned}
\text{Prob}\left[y_i = \max\{y_j\}_{j=1,\dots,n} \mid \mathbf{x}\right] &= \int\limits_{-\infty}^{0} g(y \mid x_i) \prod_{\substack{j=1 \\ j\neq i}}^{n} G(y \mid x_j)\, dy \\
&= \int\limits_{-\infty}^{0} x_i \exp(x_i y) \prod_{\substack{j=1 \\ j\neq i}}^{n} \exp(x_j y)\, dy \\
&= \int\limits_{-\infty}^{0} x_i \exp\left(y \sum_{i=1}^{n} x_j \right) dy \\
&= \frac{x_i}{\sum_{j=1}^{n} x_j}.
\end{aligned}$$

Taking into account the stochastic nature of the outputs x_j, agent j's (ex ante) expected utility is given by

$$\text{E}U_i = \int\limits_{X} \left[w^{min} + \frac{x_i}{\sum_{j=1}^{n} x_j} w^n \right] \prod_{j=1}^{n} f(x_j; a_j) dx_1 \dots dx_n - d(a_i).$$

This is identical to his utility in a J-type tournament in which the shared bonus W is equal to the winner prize w^n and the base salary w_0 is given by w^{min}. Therefore, a comparison of compensation cost in a U-type to that in a J-type tournament is equivalent to a comparison of the costs in U-type tournaments under performance measures x_i and y_i.

Given the previous results, however, the latter is straightforward. Since y_i depends on a_i only via x_i, its probability density function, given a_i, can be written as

$$g(y_i \mid a_i) = \int\limits_{X} g(y_i \mid x) f(x \mid a_i) dx.$$

Since the function $g(y_i \mid x_i)$ meets the requirements of a Markov kernel, x_i is Blackwell sufficient for y_i. From this, the claim immediately follows by corollary 1. □

Proof of lemma 1 The proof analyzes the possible cases regarding the agents' liability constraints.

1. The agents' liability constraint is not binding under **v** and **w**.

   In this case, the participation constraints is binding and $\mathrm{E}[u(w)] = \mathrm{E}[u(v)]$. Denote by F^w and F^v the cumulative distribution functions of one agent's utilities resulting from **w** and **v** in the symmetric equilibrium. From the relation of utility spreads (21), it follows that

   (a) $u(w^n) \geq u(v^n)$ (obvious).

   (b) $u(w^1) \leq u(v^1)$ (obvious).

   (c) The distribution functions F^v and F^w only cross once, i.e. $\exists \hat{u}$ such that

   $$F^w(u) \begin{cases} \leq F^v(u) & \forall \quad u < \hat{u} \\ \geq F^v(u) & \forall \quad u > \hat{u}, \end{cases}$$

   because the jumps in the cumulative distribution functions are $p_{ij} = 1/n$ for each rank j.

   Since $\mathrm{E}[u(w)] = \mathrm{E}[u(v)]$, it must hold that

   $$\int_{-\infty}^{U} F^w(u)du = \int_{-\infty}^{U} F^v(u)du$$

   for all U such that $F^v(U) = F^v(U) = 1$. From this and 1a–1c above it follows that

   $$\int_{-\infty}^{U} F^w(u)du \geq \int_{-\infty}^{U} F^v(u)du \tag{30}$$

   for all $U \in \mathbb{R}$. Therefore, F^w and F^v differ by a mean preserving spread (cf. Rothschild and Stiglitz [21], p. 230f.), and the expectation of each convex function is lower under F^v. Since due to the agent's risk aversion the inverse utility function is convex, expected compensation of a single agent (and thus total compensation of all agents) is lower under **v**.

2. The agents' liability constraint is binding under both **w** and **v**.

   Then, $v^1 = w^1$. From this and (21), it follows that $w^j \geq v^j \quad \forall j$, and therefore $\sum_{j=1}^{n} w^j \geq \sum_{j=1}^{n} v^j$.

3. The agents' liability constraint is binding under **w** and not binding under **v**.

   In this case, the participation constraints will be binding under **v**, but not necessarily under **w**, and $\mathrm{E}[u(w)] \geq \mathrm{E}[u(v)]$. From 1 above, it immediately follows that $\sum_{j=1}^{n} w^j \geq \sum_{j=1}^{n} v^j$.

4. The agents' liability constraint is binding under **v** and not binding under **w**.

   In this case, the participation constraint is binding under **w** and $w^1 \geq v^1$. From this and (21), it follows that $\mathrm{E}[u(v)] \leq \mathrm{E}[u(w)] = U^R$, a contradiction. $\square$

Proof of proposition 7 In the symmetric equilibrium of n risk neutral contestants, each player's compensation is based on his rank $r_{in}^{\hat{a},k} \in \{1,\ldots,n\}$, and all ranks are equally likely. In what follows, it is shown that the likelihood ratio distribution function of $r_{in}^{\hat{a},k}$ is a mean preserving spread of that of $r_{i,n-1}^{\hat{a},k}$, the rank in a contest of $n-1$ participants. The claim then follows from Kim's (1995) results in the standard agency setting.

From (7), the likelihood ratio $\frac{\frac{\partial}{\partial a_i} p_{ij}^k(\hat{\mathbf{a}})}{p_{ij}^k(\hat{\mathbf{a}})}$ is given by $\mathrm{E}[lr_{j:n}^{k,\hat{a}}]$. Yet, from a triangle rule in order statistics (see Arnold et al., Theorem 5.3.1), expectations of order statistics are related as follows:

$$j\mathrm{E}[lr_{j+1:n}^{k,\hat{a}}] + (n-j)\mathrm{E}[lr_{j:n}^{k,\hat{a}}] = n\mathrm{E}[lr_{j:n-1}^{k,\hat{a}}]. \tag{31}$$

This can be exploited to construct the likelihood ratio distribution function of $r_{i,n}^k$ from that of $r_{i,n-1}^k$ by a sequence of mean preserving spreads s_j, $j = 1,\ldots,n-1$, where s_j

is defined as follows:

$$s_j = \begin{cases} \frac{j}{n}\frac{1}{n-1} & \text{for } \mathrm{E}[lr^{k,\hat{a}}_{j:n}] \\ -\frac{1}{n-1} & \text{for } \mathrm{E}[lr^{k,\hat{a}}_{j:n-1}] \\ \frac{n-j}{n}\frac{1}{n-1} & \text{for } \mathrm{E}[lr^{k,\hat{a}}_{j+1:n}]. \end{cases}$$

Thus, s_j distributes the probability mass $1/(n-1)$ of $\mathrm{E}[lr^{k,\hat{a}}_{j:n-1}]$ to $\mathrm{E}[lr^{k,\hat{a}}_{j:n}]$ and $\mathrm{E}[lr^{k,\hat{a}}_{j+1:n}]$. It is a spread (which defers probability mass to the tails of a distribution) because $\mathrm{E}[lr^{k,\hat{a}}_{j:n}] \leq \mathrm{E}[lr^{k,\hat{a}}_{j:n-1}] \leq \mathrm{E}[lr^{k,\hat{a}}_{j+1:n}]$, and it is mean preserving because

$$\begin{aligned} \mathrm{E}[s_j] &= \frac{j}{n}\frac{1}{n-1}\mathrm{E}[lr^{k,\hat{a}}_{j:n}] - \frac{1}{n-1}\mathrm{E}[lr^{k,\hat{a}}_{j:n-1}] + \frac{n-j}{n}\frac{1}{n-1}\mathrm{E}[lr^{k,\hat{a}}_{j+1:n}] \\ &= \frac{1}{n}\frac{1}{n-1}\left(j\mathrm{E}[lr^{k,\hat{a}}_{j:n}] - n\mathrm{E}[lr^{k,\hat{a}}_{j:n-1}] + (n-j)\mathrm{E}[lr^{k,\hat{a}}_{j+1:n}]\right) = 0 \end{aligned}$$

by the triangle rule (31) and the fact that $\mathrm{E}\left[\mathrm{E}[lr^{k,\hat{a}}_{j:n-1}]\right] = \mathrm{E}[lr^{k,a}] = 0$. The resulting probabilities

$$p\left(\mathrm{E}[lr^{k,\hat{a}}_{j:n}]\right) = \begin{cases} \frac{n-1}{n}\frac{1}{n-1} = \frac{1}{n} & \text{for } j = 1 \\ \frac{i}{n}\frac{1}{n-1} + \frac{n-(i+1)}{n}\frac{1}{n-1} = \frac{1}{n} & \text{for } j = 1, \ldots, n-1 \\ \frac{n-1}{n}\frac{1}{n-1} = \frac{1}{n} & \text{for } j = n \end{cases}$$

are those in the contest of n agents. □

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