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## **The Two-Person Harvard Game: An Experimental Analysis**

by

**Jose Apesteguia**

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# The Two-Person Harvard Game: An Experimental Analysis\*

Jose Apesteguia

Laboratorium für experimentelle Wirtschaftsforschung, University of Bonn

Adenauerallee 24-42, D-53113 Bonn, Germany

apesteguia@wiwi.uni-bonn.de

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## Abstract

Two subjects have to repeatedly choose between two alternatives,  $A$  and  $B$ , where payoffs of an  $A$  or  $B$ -choice depend on the choices made by both players in a number of previous choices. Locally, alternative  $A$  gives always more payoff than alternative  $B$ . However, in terms of overall payoffs exclusive choice of  $B$  is a better strategy. The equilibrium predicted by the theory of melioration is to exclusively play  $A$ , while the Nash equilibrium is to almost exclusively play  $B$ . The predictive values of such equilibria are analyzed under three different informational conditions. Special attention is paid to the learning processes exhibited by players.

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# The Two-Person Harvard Game: An Experimental Analysis

## 1. - Introduction

The two-person Harvard game consists of two players that have to repeatedly choose between two alternatives, *A* and *B*, where the outcome of an *A* or *B*-choice depends on the number of *A*-choices made by both players in the last nine periods. For any strategy of the opponent, a pattern of individual behavior focused on local payoffs is in direct contradiction with a pattern of individual behavior focused on overall payoffs, the former involving a substantial loss in payoffs with respect to the latter. More precisely, for any strategy of the opponent, locally an *A*-choice gives always more value than a *B*-choice. However, if *A* is chosen in all the nine previous choices, an *A*-choice gives less value than a *B*-choice when *B* is chosen in all nine previous choices. Furthermore, every *A*-choice reduces the opponent's payoffs in the current and future periods.

The Harvard game was originally formulated in the framework of one decision-maker<sup>1</sup> (see Herrnstein, 1991; Herrnstein and Prelec, 1991; Herrnstein *et al.* 1993; Kudadjie-Gyamfi and Rachlin, 1996; and Warry *et al.* 1999). The extension developed in this paper to a game theoretic environment is a new one. It will be argued below that the two-person Harvard game represents the interdependent environment in the consumption of a good with addictive characteristics, or the exploitation problem of a common-pool resource.

The interesting feature of the two-person Harvard game is that situates the predictions of the unique *Nash equilibrium* and of what will be called here the *Stable Matching Equilibrium in Games* at two extremes. The *Stable Matching Equilibrium in Games* is the equilibrium prediction developed here based on the psychological theory of individual decision-making known as melioration (see Herrnstein, 1991, 1997; Herrnstein and Prelec, 1988, 1991, 1992). The theory of melioration states that individuals at each period select that alternative that provided more value in the previous period. It will be shown that this pattern of behavior applied to the two-person Harvard game predicts that both players exclusively choose

alternative  $A$ . In contrast to this, the Nash equilibrium predicts that both players almost<sup>2</sup> exclusively choose alternative  $B$ .

Therefore, while the Stable Matching Equilibrium in Games predicts maximal exploitation of the common-pool resource, or maximal consumption of the alternative with addictive characteristics, the Nash equilibrium calls for almost complete avoidance of the addictive alternative, or for almost complete protection of the common-pool resource.

In this experimental analysis the two-person Harvard game is analyzed under three different informational characterizations. In the first one players are informed neither of the payoff structure, nor of their opponent's past choices. In the second treatment, players have information on their opponent's behavior, and in the final one, players are given some qualitative information on the payoff structure. Therefore, the predictive value of the Nash equilibrium and the Stable Matching Equilibrium in Games are analyzed under three different informational conditions. Furthermore, it is the aim of this paper to study the individual learning processes followed by players in the course of the game. A method to analyze the "quality" of individual learning will be proposed. Finally, it will be developed a classification of players on the basis of the dynamic-adjustment processes exhibited.

The organization of the paper is as follows. Section 2 formally introduces the two-person Harvard game and derives the Nash equilibrium and the Stable Matching Equilibrium in Games. Section 3 describes the experimental procedure. In Section 4 the experimental results are reported, and Section 5 concludes the paper.

## 2. - The Two-Person Harvard Game

There are two players,  $i=1, 2$ , that have to repeatedly choose between two alternatives,  $A$  and  $B$ . Let  $x_i$  denote the number of  $A$ -choices made by player  $i$  in the last ten choices (including the current one),  $x_i \in X_i = \{0, 1, \dots, 10\}$ . Functions  $v_i^A(x)$  and  $v_i^B(x)$  represent the  $i$ -th payoffs obtained from a single choice of  $A$  and  $B$  when  $x$ , where

$$\begin{aligned} v_i^A(x_i, x_j) &= a - bx_i - cx_j, \\ v_i^B(x_i, x_j) &= d - bx_i - cx_j, \end{aligned} \quad i, j = 1, 2, i \neq j.$$

The parameterization followed in this paper is  $(a, b, c, d) = (16, 0.8, 0.2, 10)$ . Figure 1 represents the  $i$ -th payoff functions when the number of  $A$ -choices made by the opponent is zero and ten.<sup>3</sup> The  $i$ -th payoff functions for any other value of  $x_j$  in  $X_j$  are in between those represented in the figure. Note that, for all  $x_j$  in  $X_j$ , the payoff function of  $A$  is above the payoff function of

*B*. Therefore, for all  $x = (x_1, x_2) \in X = X_1 \times X_2$ , locally a single *A*-choice gives more value than a single *B*-choice. However, note also that  $v_i^A(10, x_j) < v_i^B(0, x_j)$  holds for all  $x_j$  in  $X_j$ . That is, for any strategy of the opponent, if *A* is chosen in all nine previous choices, an *A*-choice gives less value than a *B*-choice, when *B* is chosen in all nine previous choices.

To interpret the two-person Harvard game, consider that alternative *A* represents an addictive alternative, or a bad habit. That is, irrespective of the individual's own strategy and that of the opponent, alternative *A* always gives a higher local payoff than alternative *B*, but, at the same time, it reduces the individual's own payoffs derived from both alternatives in the current and future periods. Therefore, the consequences of choosing *A* once are suffered in the next rounds. With a *B*-choice, however, the individual receives a lower local payoff, but contributes to a larger overall payoff. Furthermore, whenever an individual selects the alternative with addictive characteristics (alternative *A*) this reduces the opponent's payoffs in the current and future periods. One enjoys *A* and *B* to a lesser extent whenever the person with whom one is interacting chooses the alternative with addictive characteristics.

Another interpretation of the two-person Harvard game is in the vein of a common-pool resource (CPR) game. A CPR is a resource for which returns are subtractable and the level of appropriation is difficult to limit. Typical examples of CPRs are fisheries, forests, groundwater basins, and irrigation systems. It is often argued that CPRs are subject to the so-called tragedy of the commons (see Hardin, 1968), that predicts that the pursuit of self interest by individuals leads to overexploitation of the CPR.<sup>4</sup> Hence, in the two-person Harvard game, alternative *A* represents a common-pool resource; the more one and the opponent harvest the CPR, the less payoff one will gain from choosing the CPR. On the other hand, *B* represents an alternative market with increasing payoffs, but negatively affected by the CPR exploitation. Thus, locally, the CPR alternative is always attractive, however, as it will be seen below, players are better off protecting the CPR, and hence investing almost always in the alternative market.

## 2.1.- Nash Equilibrium

It is easy to show that there is a unique Nash equilibrium that happens to be symmetric and in dominant strategies. For any player  $i = 1, 2$ , all  $x$  in  $X$ , and any period provided that there are at least nine more periods to play, one's local payoff differential between an *A* and a *B*-choice is  $(a-d)>0$ . However, in overall terms, since an *A*-choice increases  $x_i$  in one unit for the next ten choices (including the current one), an *A*-choice gives a total of  $a-10b$ . Hence, it can be

seen that since  $a-10b < d$ ,  $B$  is a dominant *best-reply* for any period, provided there are at least nine more periods to play. By applying a similar analysis for the last nine periods of the game, it is concluded that the best-reply is to play  $A$ , instead of  $B$ , in the last seven periods. So, the Nash equilibrium calls for both players always to play  $B$ , except in the last seven periods when  $A$  must be chosen. Using the parameters above presented and considering 120 periods, the Nash equilibrium gives an individual total payoff of 1194. Note, however, that the Nash equilibrium is not Pareto optimal. If, instead of playing  $A$  in the last seven periods, both players play  $A$  in the last six periods having played  $B$  in the rest, each player will obtain a total payoff of 1195.

It is important to note that the three experimental treatments studied in this paper are with incomplete information, and therefore, players cannot derive the Nash equilibrium from an *a priori* analysis. Note, however, that by trying different simple choice patterns, players may realize that, although an  $A$ -choice is locally more attractive than a  $B$ -choice, repeated  $A$ -choices imply that  $A$ -payoffs tend to decrease, while repeated  $B$ -choices imply that  $B$ -payoffs tend to increase, the latter being more attractive in terms of payoffs. This would lead to playing best-reply, and hence to Nash equilibrium.

## **2.2.- Stable Matching Equilibrium in Games**

Melioration is a simple myopic dynamic-adjustment theory of individual decision-making where the decision-maker has to repeatedly choose one alternative from a set of alternatives. The theory of melioration states that individuals consider the value obtained from a single choice of each alternative and then shift choice to alternatives that provide a higher value. Obviously, such a dynamic-adjustment theory may imply a significant loss in terms of overall payoffs.

The extension of melioration developed here to a game theoretic context such as the two-person Harvard game is straightforward. It follows the dynamic-adjustment process above described taking the strategy of the opponent as given. This describes a reply function that will be called here the melioration-reply function. Hence, since for all  $x \in X$  a single  $A$ -choice always gives a higher local payoff than a single  $B$ -choice, exclusive choice of  $A$  is a dominant melioration-reply. Therefore, exclusive choice of  $A$  on the part of both players is the equilibrium predicted by melioration. Drawing from the terminology used in the original characterization of melioration (see Herrnstein and Prelec, 1992), this is called here the *Stable Matching Equilibrium in Games* (SMEG).

It is of relevance to note that, according to the parameters used, the Stable Matching Equilibrium in Games gives 745 units of payoff. Hence, with respect to the Stable Matching Equilibrium in Games, the Nash equilibrium brings about a 60% gain in efficiency.

Therefore, while melioration predicts maximal exploitation in the CPR, or maximal consumption of the addictive alternative, the Nash equilibrium calls for almost complete avoidance of the addictive alternative, or for almost complete protection of the CPR. Recall, however, that the exploitation of the CPR (or the consumption of the addictive alternative) as it is referred to by the Nash equilibrium, is Pareto inefficient.

### **3. - Experimental Procedure**

The present experimental study was conducted in the fall semester of 2000 at the Public University of Navarra. Sixty volunteer undergraduate students, primarily from economics, took part. The experiment was divided into six sessions with ten participants each. In each session participants were randomly divided into five independent pairs.

Three treatments were run, each one comprising ten independent two-person Harvard games. In Treatment I, no information was provided as to the nature of the payoff structure. Players had to figure out the payoff contingencies throughout their relatively long exposure to the game. The hypothesis here is that players will focus on local payoffs, and hence that melioration will drive behavior towards the Stable Matching Equilibrium in Games. The instructions for Treatment I were as follows (original instructions in Spanish).

Welcome to today's experiment!

In this room there are ten participants who will be put into pairs. The pairs will be fixed throughout the experiment. You will not know who the other member of your pair is. In this sense, the experiment is totally anonymous.

The experiment comprises a sequence of 120 periods.

The aim in this experiment is to accumulate as many points as possible. The way to accumulate points is to choose between two alternatives in each period: either *A*, or *B*. The other member of your pair will also choose between *A* and *B*.

At the end of the experiment you will be privately paid in cash according to the total points that you have accumulated. Every 100 points equals 180 pesetas.

Thank you very much for your participation!

Hence, although players knew that they were playing with an opponent, they did not know with whom, nor did they have any feedback from the opponent. Participants remained in the same pair for all 120 periods. The computer screen, that was presented and explained to subjects, had two buttons, one for alternative *A* and one for alternative *B*, situated at the



bottom of the screen, and two counters at the top of the screen, one showing the number of the period being played, and the other showing the total points accumulated through time. In each session the role of *A* and *B*-choices was interchanged across participants. This was to avoid any bias regarding the labeling or screen positioning of the buttons.

In Treatment II players knew at each decision period the previous choice of their opponent. It can be hypothesized that when information on the previous choice of the opponent is available, imitation of the opponent's behavior may arise, thus resulting in players trying out different choice patterns, a behavior which may prove more successful in terms of payoffs. Hence, it is hypothesized that behavior in Treatment II will tend to shift toward Nash equilibrium. Instructions for Treatment II were as for Treatment I, except that this extra information was announced.

In Treatment III the only difference with Treatment I was the provision of some qualitative information on the nature of the payoff structure. Drawing from Herrnstein *et al.* (1993) and Kudadjie-Gyamfi and Rachlin (1996), the following was included in the instructions of those players taking part in Treatment III.

The points that you score in each period depend on your decision in that period and in previous periods, as well as on the decisions of the other member of your pair in that period and previous periods. If you choose *A* repeatedly, the number of points that you and the other member of your pair will receive from both alternatives decreases. On the other hand, if you choose *B* repeatedly, the number of points that you and the other member of your pair will receive from both alternatives increases.

The hypothesis here is that when this qualitative information on the nature of the payoff structure is available, behavior will be more in accordance with the overall analysis of the game, and therefore tend to Nash equilibrium.

Common to all treatments was the fact that no communication was allowed. On average, a session, including the instructions phase, took one hour and fifteen minutes. Participants were privately paid in cash directly after completing the 120 periods. Average earnings were about 9.3 euros.<sup>5</sup>

#### **4. - Experimental Results**

The data analysis is structured in three subsections. The first analyzes the predictive value of the Nash equilibrium and the Stable Matching Equilibrium in Games. Also, by contrasting data between treatments, an analysis is obtained of the effect on behavior of the three different levels of information studied here. In Section 4.2 the learning processes exhibited by players are studied. Finally, Section 4.3 provides a classification of players on the basis of the

individual behavior displayed in the final third of the experiment. The decision rule adopted in all the statistical tests that follow is in terms of a significance level of the 5%.

#### 4.1. – Nash Equilibrium and Stable Matching Equilibrium in Games

Table I shows the proportions of *A*-choices at the individual and game level over the 120 periods, the first third, the middle third, and the final third of the experiment. Note that the average proportion of *A*-choices in the final third of the experiment is close to 90% in Treatments I and II, while in Treatment III it approaches 70%. Figure 2 plots the cumulative relative frequency distributions of *A*-choices for the three treatments.

**Observation 1.** In Treatments I and II the proportion of *A*-choices is significantly higher than the proportion of *B*-choices, while in Treatment III it cannot be rejected that there is no difference in the proportion of *A* and *B*-choices.

In each treatment the proportions of *A*-choices at the game level in the final third of the experiment are taken. Analysis of the data at game level rather than at individual level permits the use of independent observations (ten independent observations per treatment). Thus, except when explicitly stated to the contrary, data is always considered at the game level. Analysis of the final third of the experiment is justified in terms of the interest in studying behavior after the learning phase of the game. It will be shown below that individual behavior in the final third of the experiment shows a remarkably stable pattern. The Wilcoxon signed-ranks test is used to test whether, in each treatment, it cannot be rejected that there is no difference in the proportion of *A* and *B*-choices (for this, and the tests to be applied below, see Siegel and Castellan, 1988). Hence, the null hypothesis of no difference in the proportion of *A* and *B*-choices is rejected in Treatments I and II at significance levels below 0.1% ( $T = 0$  in both cases), in favor of the alternative hypothesis of a higher proportion of *A*-choices. The null hypothesis, however, cannot be rejected in Treatment III ( $T = 12$ ,  $P$ -value equal to 6.54%).

Consider Figures 3, 4, and 5, where individuals are grouped together on the basis of the proportion of *A*-choices exhibited in the final third of the experiment. There it can be seen that, while in Treatments I and II a great majority of players shows behavior consistent with the melioration-reply function (more than 60% of the players chose *A* at least 90% of the time), Treatment III shows a higher dispersion across individual behavior. However, even in

Treatment III, 70% of players chose *A* at least 70% of the time. In the whole experiment only one player shows a proportion of *A*-choices of less than 0.1. Observed choice is now contrasted across treatments.

**Observation 2.** For each of the three thirds of the experiment, while Treatment III shows a lower proportion of *A*-choices than the other two treatments, there is no difference between Treatments I and II.

The Jonckheere test for ordered alternatives is used to test the null hypothesis of no difference in the proportion of *A*-choices among the three treatments, against the alternative hypothesis of an equal or higher proportion of *A*-choices in Treatment I, as compared with Treatment II, and in Treatment II as compared with Treatment III. For each of the three blocks of data, the null hypothesis of no difference is rejected at significance levels below 0.01% ( $J^* = 3.27, 4.506, \text{ and } 3.213$  for the first, middle, and final third, respectively). When the null hypothesis is rejected, the Jonckheere test guarantees that at least one of the inequalities is strict. It does not, however, say which. A multiple comparison analysis between treatments is therefore applied. Table II shows that, for each of the three thirds of the experiment, while it cannot be rejected that Treatments I and II show no difference in the proportion of *A*-choices, Treatment III shows a lower proportion of *A*-choices than the other two treatments.

Therefore, with regard to the hypothesis on imitation, it can be seen that the provision of information on the past choice of the opponent does not induce a significant change in behavior. Imitation can only be successful when one of the two players checks the consequences of repeated choice of *B*, and the other player imitates that behavior, realizing that such a choice pattern is better in terms of overall payoffs. However, as it will be shown in Section 4.2, in the vast majority of cases players do not systematically try different patterns of choice, and therefore, more successful behavior in terms of overall payoffs cannot be imitated.

Regarding the hypothesis that the provision of qualitative information on the payoff structure would induce behavior consistent with best-reply, the following considerations are in order. Note that the proportion of *A*-choices in Treatment III is lower than in the other two treatments throughout all the experiment (Observation 2). Also, note that Treatment III shows a higher dispersion on the types of individual behavior (Figure 5). However, it can be seen that on aggregate, behavior is not closer to the Nash equilibrium than to the Stable Matching Equilibrium in Games (Observation 1, Table I, and Figure 2). Therefore, although the

provision of explicit information on the payoff structure clearly lowers the proportion of A-choices, it is not sufficient to promote the type of individual behavior consistent with best-reply. Interestingly, Section 4.3 will show that Treatment III gives rise to a new, different type of behavior from those considered so far.

## 4.2. - Learning

Consider Table I, Figure 2, and Table III where the Spearman rank-order correlation coefficients are calculated at the individual and treatment level. The Spearman rank-order correlation coefficient is a non-parametric measure of association that is used here to relate the proportion of A-choices with time. For the calculus of the coefficients, data are divided in ten consecutive blocks of twelve periods.

**Observation 3.** In each one of the three treatments individuals display a tendency to learn to play melioration-reply.

In Table I it is seen that in the majority of cases the proportion of A-choices tends to increase. Also, in Figure 2 it can be appreciated that for each of the three treatments, the cumulative relative frequency distributions of A-choices show an increasing rate. Further to this, note that Table III shows that only one negative Spearman rank-order correlation coefficient is significant at the 5% level, while more than the 50% of the coefficients are significant positive coefficients (one-sided). Furthermore, at the aggregate, the three treatments show highly positive correlation coefficients. All this shows a tendency to learn to play according to melioration-reply rather than according to best-reply.

The learning processes used by players in the experiment can be further analyzed by studying the number of *runs* exhibited by players. A run is a succession of choices of the same type, either A or B, followed and preceded by a choice of the opposite type, or by no choice. It is hypothesized that the number of runs tends to decline through time. This would indicate that players, after having tried different choice patterns, tend to adopt a stable pattern of choice. Table IV shows the average number of runs per game for the entire experiment, the first third, the middle third, and the final third.<sup>6</sup>

**Observation 4.** In each of the three treatments, players tend to adopt a stable pattern of behavior.

For each of the three treatments, the Page test, a non-parametric procedure for dependent samples, is used to test the null hypothesis of no difference in the number of runs in the three thirds of the experiment, against the alternative hypothesis of a decreasing number of runs. The null hypothesis is then rejected in each of the three treatments at significance levels below 1% ( $L = 137, 136.5$ , and  $132$ , respectively). It can, therefore, be argued that players in the three treatments tend to adopt a stable pattern of behavior. The three treatments are now contrasted in terms of the number of runs to see whether the informational conditions analyzed in this paper have an impact on the individual learning processes.

**Observation 5.** In each of the three thirds, Treatment III shows a higher number of runs than the other two treatments. Furthermore, it cannot be rejected that Treatments I and II show no difference in the number of runs in the three blocks of data.

For each of the three blocks of data, the Jonckheere test is applied to test the null hypothesis of no difference in the number of runs across the three treatments, against the alternative hypothesis of an equal or lower number of runs in Treatment I, than in Treatment II, and in Treatment II, than in Treatment III. Then, the null hypothesis of no difference is rejected for each of the three blocks of data at significance levels below 0.001% ( $J^* = 3.821, 4.62$ , and  $3.935$ , for the first third, the middle third, and the final third, respectively). Table V provides the results of the multiple comparison analysis. There it is shown that, for each of the three blocks of data, Treatment III shows a higher number of runs than the other two treatments, while it cannot be rejected that there is no difference between Treatments I and II in the number of runs in each of the three thirds. Therefore, the provision of information on the payoff structure has a clear effect on the number of runs exhibited by players, and hence on the individual learning processes followed.

Consider now the length of the runs (the number of periods that constitute the runs) of the less frequently chosen alternative (alternative  $B$  in the vast majority of cases). This gives an idea of the “quality” of the learning process. If, when choosing the less frequently chosen alternative, short runs are extensively used, the learning of the payoff structure of the two-person Harvard game is seriously limited. Such a pattern of choice makes difficult to realize that repeated  $B$ -choice is best in terms of overall payoffs. Figures 6, 7, and 8 show the frequencies of these runs in terms of their length for the first, middle, and final third of the

experiment. The lengths of runs are between one period and six periods. Runs longer than 6 periods have not been observed.

**Observation 6.** In the three thirds of each of the three treatments there are more one-period runs than of any other length. Also, in the three treatments, the number of runs of any length tends to decline.

Figures 6, 7, and 8 clearly show what is stated in Observation 6.

**Observation 7.** While Treatment III shows a higher number of one and two-period runs than the other two treatments, it cannot be rejected that there is no difference in the number of runs longer than two periods among the three treatments.

Table VI shows the results of the application of the Jonckheere test when, for each of the three thirds, the three treatments are contrasted on the basis of the number of runs of each length.<sup>7</sup> It can be seen that, for each of the three thirds, the null hypothesis of no difference among the three treatments in the number of one and two-period runs is rejected at significance levels below 5%, in favor of an equal or higher number of one and two-period runs in Treatment III than in Treatment II, and in Treatment II than in Treatment I. The null hypothesis of no difference cannot be rejected in the case of three-period runs. Table VII reports the results of the multiple comparison analysis for one and two-period runs.

Runs of the less frequently chosen alternative longer than two periods are rare in the three treatments. This holds even in the first third of the experiment. As it was previously argued, this shows a poor searching strategy on the payoff contingencies, that, of course, hinders the learning of best-reply. Moreover, the fact that Treatment III differs from the other two treatments only in the number of one and two-period runs of the less frequently chosen alternative, shows that the provision of information explicitly stressing the importance of repeated *B*-choice has only a superficial impact on behavior.

#### **4.3. - Classification of Players**

What follows is a proposed classification of players by the type of behavior exhibited. This must be taken as a suggested classification. Criteria other than the one followed here could also be applied.

For each player an analysis will be made of the final third of data, when some stabilization in behavior has been reached. Players are classified on the basis of the decision rule that most closely approaches their observed behavior. The decision rules considered are the melioration-reply function, the best-reply function, and the alternating decision rule. The first two are already known. The former calls for repeated selection of alternative *A*, while the latter calls for repeated selection of alternative *B*, except, of course, in the last 7 periods where *A* must be chosen. So, if a player chooses alternative *A* (alternatively *B*) 90% of the time or more, that player will be classified as following strict melioration-reply (strict best-reply). If a player chooses alternative *A* (*B*) between 75% and 90% of the time, that player will be classified as following melioration-reply (best-reply).<sup>8</sup>

The alternating decision rule calls for a stable pattern of alternation between *A* and *B*-choices. In the post-experimental debriefing some players reflected the alternating decision rule. The argument of these players was that alternating between *A* and *B*-choices enabled them to maintain a reasonable level of payoffs on alternative *A*. *B*-choices, therefore, were somewhat like investments on which the payoffs were attached to the *A*-alternative. The specific version of the alternating decision rule varies across players. Some show behavior that closely approaches a symmetric pattern of alternation (a number of *A*-choices followed by the same number of *B*-choices, where the number of consecutive equal choices was primarily one or two), while others show asymmetry in the number of consecutive choices of the same sign. Of course, the same player may show different patterns of alternation over time.

**Observation 8.** On aggregate, the majority of players (more than 75%), can be classified as exhibiting behavior consistent with melioration-reply, while only one player out of sixty can be classified as exhibiting behavior consistent with best-reply. Treatments I and II show a similar classification of players. It is in Treatment III where a significant change occurs; the percentage of players classified as exhibiting behavior consistent with melioration-reply falls from 90% to 45%, and the percentage of players classified as exhibiting behavior consistent with the alternating decision rule increases from practically zero to 50%.

Table VIII shows the classification of players per treatment. As in previous sections, Treatments I and II show significantly similar results that clearly favor melioration. In these treatments the great majority of subjects (90%) is classified as exhibiting behavior consistent with melioration-reply. The provision of qualitative information on the payoff structure is not sufficient for players to learn to play best-reply. No player in Treatment III can be classified

as exhibiting behavior consistent with best-reply. However, 50% of the subjects in Treatment III, those classified as exhibiting behavior consistent with the alternating decision rule, seem to show some awareness of the payoff contingencies, but they are still too interested in the relatively high local payoffs of alternative A. It must also be mentioned that in the entire experiment only one player (in Treatment I) showed behavior consistent with the best-reply function.

## **5. - Concluding Remarks**

The experimental results obtained favor the Stable Matching Equilibrium in Games, rather than the Nash equilibrium. The mean proportions of A-choices in the final third of the experiment in Treatments I and II are close to 90%, while in Treatment III a proportion of almost 70% is observed. With no information on the payoff structure (Treatments I and II), 90% of the players were classified as exhibiting behavior consistent with melioration. The provision of qualitative information on the payoff structure (Treatment III) lowers this percentage to 45%. It must also be noted that, with the provision of such information, a new type of decision-making rule emerges, the alternating decision rule, held up by the fact that 50% of the players in Treatment III are classified as exhibiting behavior consistent with it. Players following the alternating decision rule show some awareness of the payoff contingencies of the game in the sense that they choose *B* intermittently in order to prevent excessive reduction in A-payoffs. Only one player out of 60 apparently learnt to play best-reply. It is remarkable that this player took part in Treatment I, where no information on the payoff structure was provided. The analysis of the learning processes exhibited by players also show some interesting results. First, it emerges that as the experiment proceeds individuals display a tendency to learn to play according to melioration, rather than according to best-reply. That is, not only does time fail to enhance intuition on the game, but it leads players to show even clearer meliorating behavior. Furthermore, the “quality” of the individual learning processes has been studied. It has been argued that the length in periods of the runs of the less frequently chosen alternative serves as an indicator of how hard players try to learn about the contingencies of the game. Hence, since the runs of the less frequently chosen alternative are mostly short, the learning processes exhibited by players are “poor” in the sense that they do not provide the chance to learn to play best-reply.

More research on the Harvard game could be conducted. In particular it would be of interest to see the effect of increasing the degree of information provided on the payoff structure. It



has been shown in this experiment that explicitly describing the essential features of the payoff structure is not sufficient to induce behavior consistent with best-reply. It may be hypothesized that the provision, for example, of tables relating different patterns of choice with the corresponding payoffs would favor best-reply.

Also, the hypothesis that imitation may shift behavior in the direction of Nash equilibrium could be further analyzed. One possibility would be to give players information on a number of previous choices of the opponent, paired with the corresponding payoffs. The interest here would be to see whether the provision of this information is sufficient to promote searching strategies for choice patterns that provide the opportunity of learning best-reply.

## 6.- References

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FIGURE 1  
THE TWO-PERSON HARVARD GAME

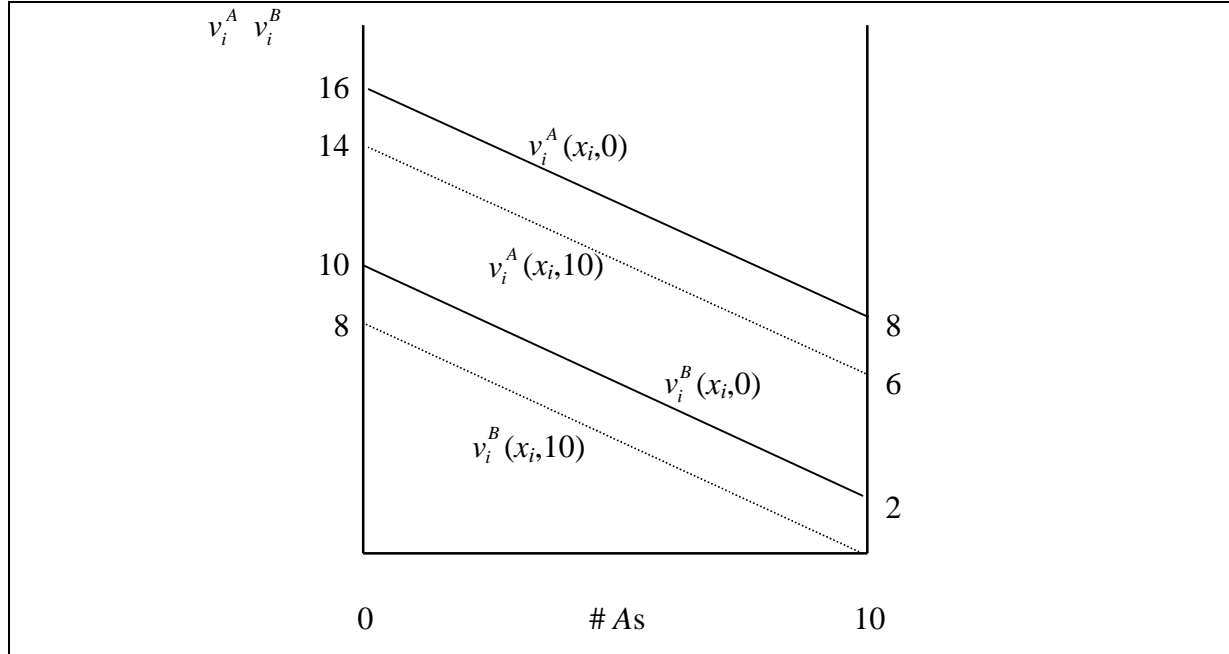


FIGURE 2  
CUMMULATIVE RELATIVE FREQUENCY DISTRIBUTIONS

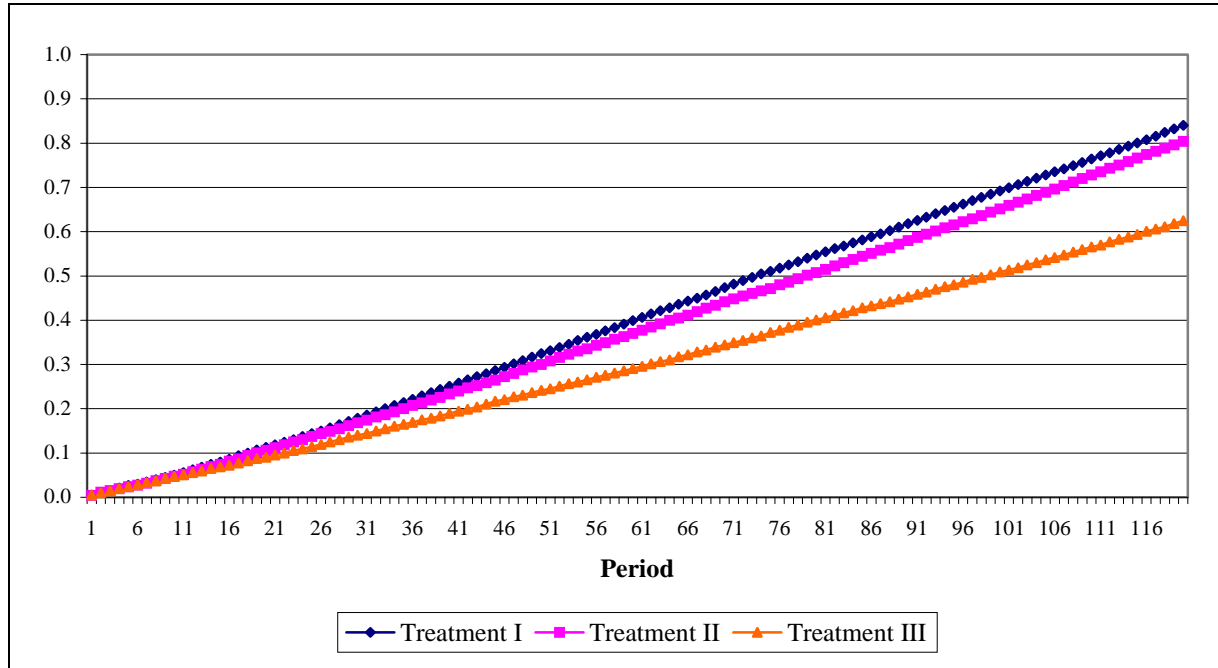


FIGURE 3  
FREQUENCY OF PLAYERS ACCORDING TO THE PROPORTION OF A-CHOICES IN  
THE FINAL THIRD OF THE EXPERIMENT IN TREATMENT I

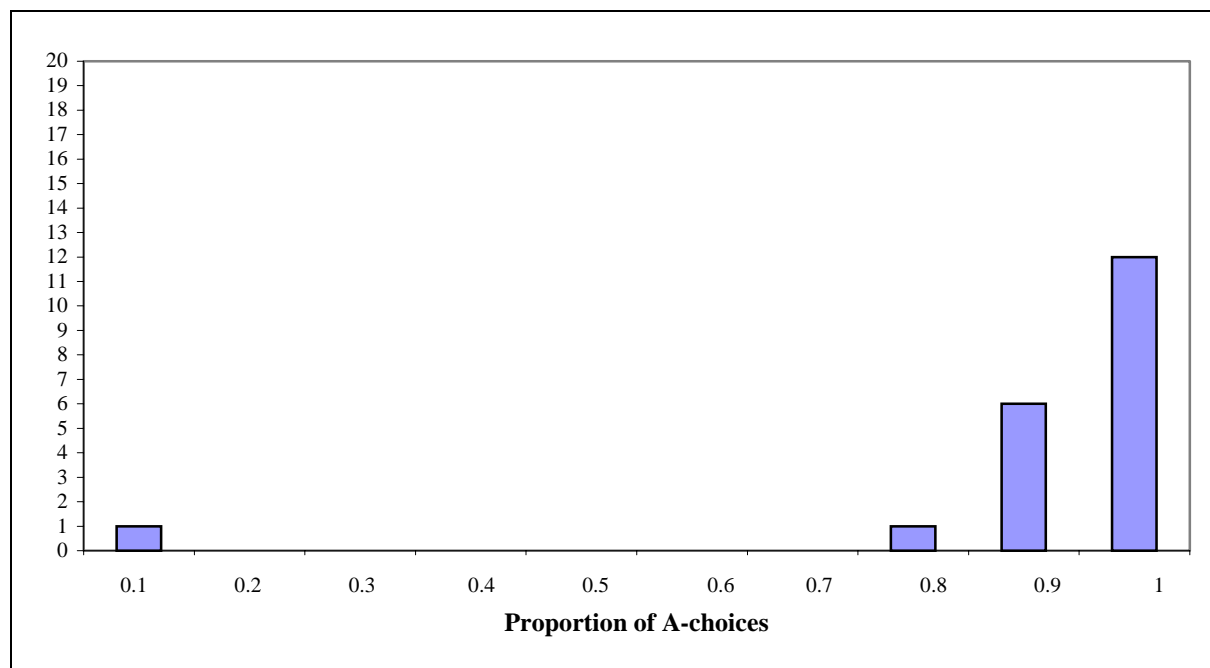
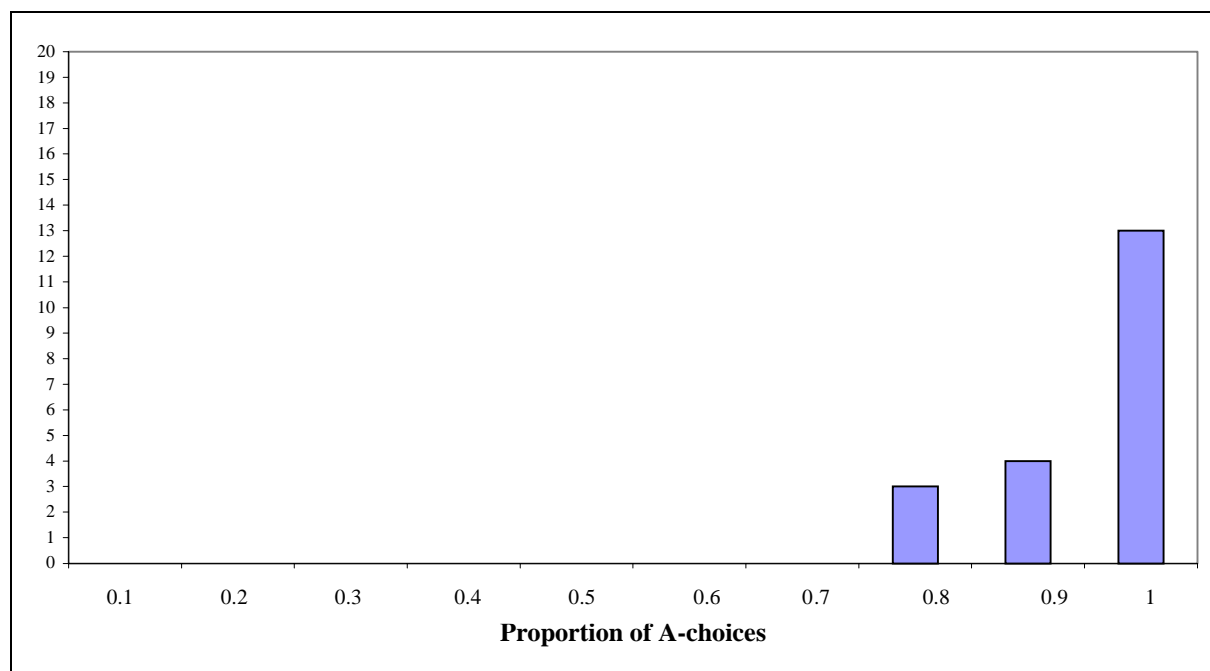
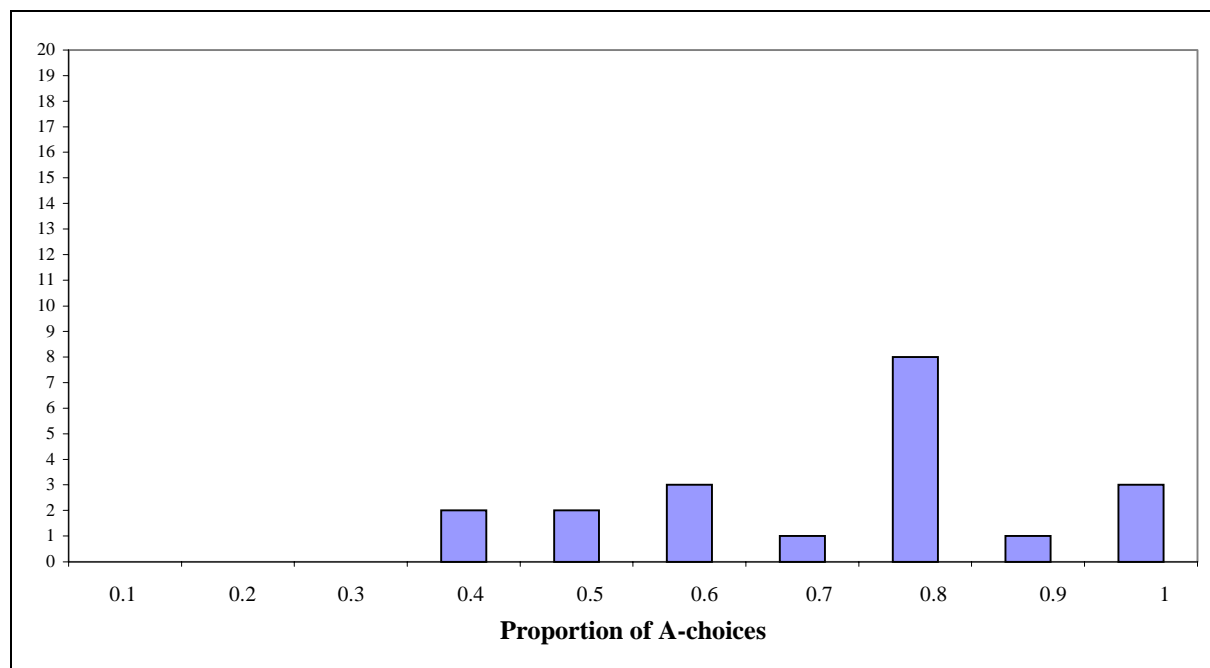


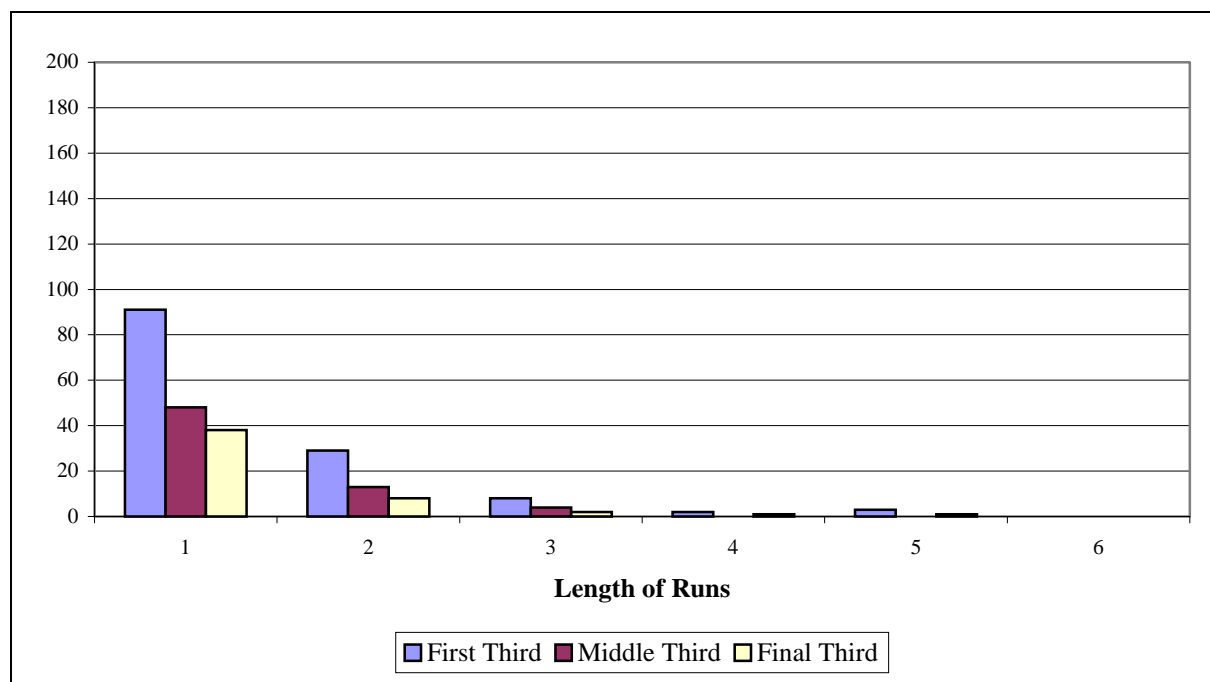
FIGURE 4  
FREQUENCY OF PLAYERS ACCORDING TO THE PROPORTION OF A-CHOICES IN  
THE FINAL THIRD OF THE EXPERIMENT IN TREATMENT II



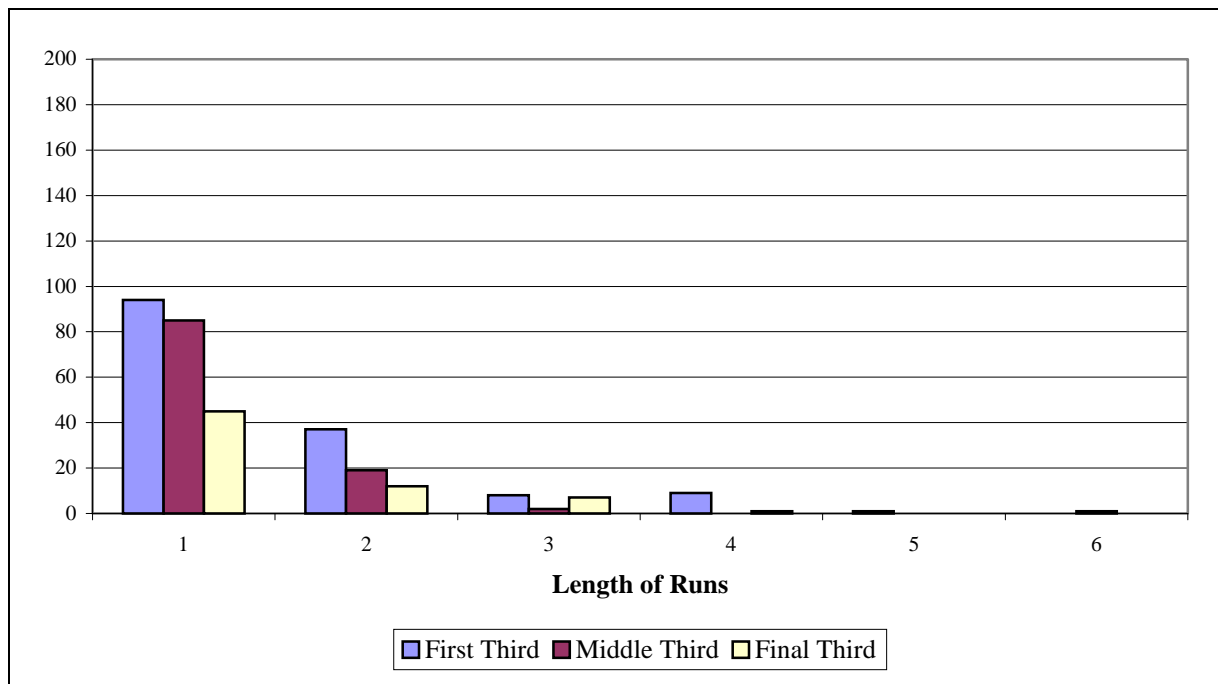
**FIGURE 5**  
**FREQUENCY OF PLAYERS ACCORDING TO THE PROPORTION OF A-CHOICES IN**  
**THE FINAL THIRD OF THE EXPERIMENT IN TREATMENT III**



**FIGURE 6**  
**FREQUENCY OF RUNS OF THE LESS FREQUENTLY CHOSEN ALTERNATIVE BY**  
**LENGTH IN TREATMENT I**



**FIGURE 7**  
**FREQUENCY OF RUNS OF THE LESS FREQUENTLY CHOSEN ALTERNATIVE BY**  
**LENGTH IN TREATMENT II**



**FIGURE 8**  
**FREQUENCY OF RUNS OF THE LESS FREQUENTLY CHOSEN ALTERNATIVE BY**  
**LENGTH IN TREATMENT III**

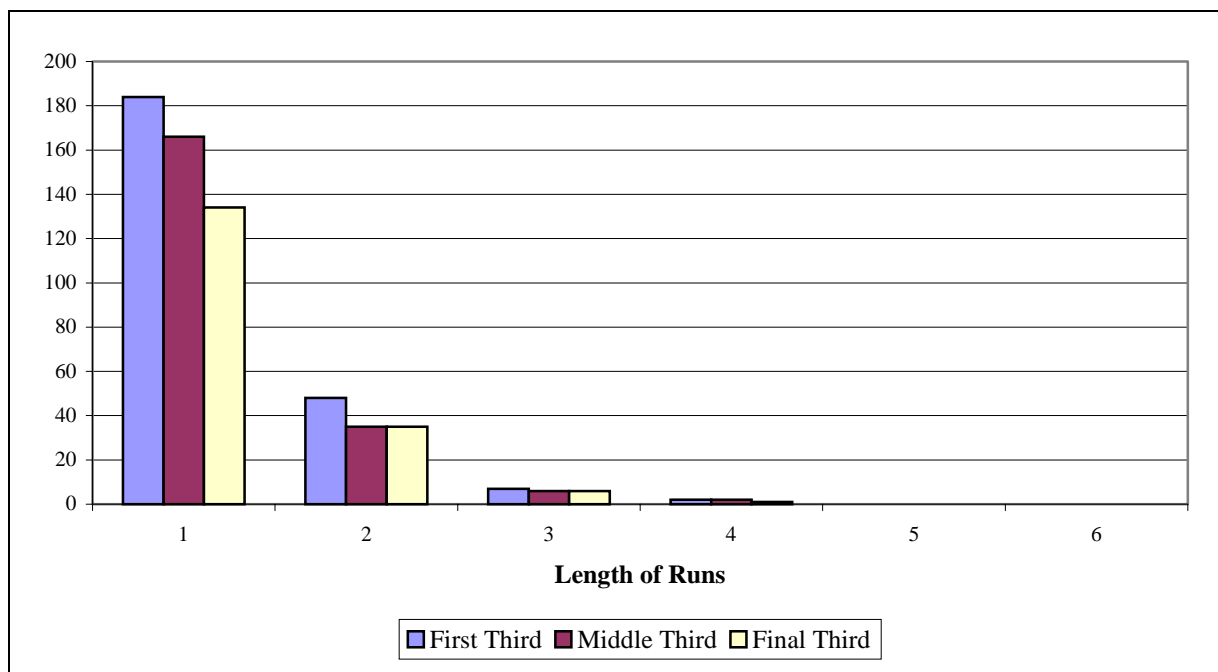


TABLE I  
PROPORTIONS OF A-CHOICES

A) AT THE INDIVIDUAL LEVEL

Game: Player	Treatment I				Treatment II				Treatment III			
	All Data	First Third	Middle Third	Final Third	All Data	First Third	Middle Third	Final Third	All Data	First Third	Middle Third	Final Third
1:1	0.74	0.63	0.70	0.90	0.83	0.78	0.80	0.90	0.74	0.70	0.75	0.78
1:2	0.82	0.78	0.90	0.78	0.84	0.78	0.85	0.90	0.65	0.50	0.75	0.70
2:1	0.63	0.50	0.58	0.83	0.82	0.75	0.75	0.95	0.74	0.70	0.75	0.78
2:2	0.95	0.90	0.98	0.98	0.79	0.58	0.88	0.93	0.68	0.60	0.70	0.75
3:1	0.86	0.78	0.85	0.95	0.83	0.80	0.88	0.83	0.54	0.60	0.48	0.55
3:2	0.96	0.90	1.00	0.98	0.80	0.63	0.85	0.93	0.82	0.75	0.80	0.90
4:1	0.83	0.70	0.90	0.88	0.65	0.50	0.65	0.80	0.60	0.50	0.50	0.80
4:2	0.83	0.73	0.95	0.80	0.78	0.55	0.80	0.98	0.51	0.53	0.50	0.50
5:1	0.82	0.73	0.85	0.88	0.90	0.80	0.95	0.95	0.38	0.35	0.38	0.43
5:2	0.81	0.68	0.88	0.88	0.88	0.85	0.85	0.93	0.41	0.40	0.48	0.35
6:1	0.94	0.88	0.95	1.00	0.80	0.60	0.88	0.93	0.64	0.53	0.63	0.78
6:2	0.96	0.93	0.98	0.98	0.72	0.58	0.85	0.73	0.68	0.63	0.68	0.73
7:1	0.83	0.60	0.90	0.98	0.78	0.85	0.80	0.70	0.79	0.58	0.83	0.98
7:2	0.80	0.60	0.88	0.93	0.74	0.75	0.75	0.73	0.56	0.60	0.55	0.53
8:1	0.88	0.88	0.85	0.90	0.88	0.73	0.93	0.98	0.74	0.78	0.78	0.68
8:2	0.87	0.78	0.95	0.88	0.93	0.95	0.90	0.95	0.38	0.40	0.38	0.38
9:1	0.88	0.73	0.98	0.95	0.92	0.90	0.88	0.98	0.80	0.58	0.88	0.95
9:2	0.89	0.78	0.90	1.00	0.81	0.55	0.88	1.00	0.70	0.58	0.75	0.78
10:1	0.98	0.93	1.00	1.00	0.79	0.68	0.83	0.88	0.69	0.68	0.68	0.73
10:2	0.56	0.68	0.85	0.15	0.61	0.43	0.55	0.85	0.43	0.43	0.45	0.43
Aggregated Proportions of A-choices by Treatment												
	0.84	0.75	0.89	0.88	0.80	0.70	0.82	0.89	0.62	0.57	0.63	0.67

B) AT THE GAME LEVEL

Game	1	2	3	4	5	6	7	8	9	10
Treatment I										
All Periods	0.78	0.79	0.91	0.83	0.81	0.95	0.81	0.87	0.89	0.77
First Third	0.70	0.70	0.84	0.71	0.70	0.90	0.60	0.83	0.75	0.80
Middle Third	0.80	0.78	0.93	0.93	0.86	0.96	0.89	0.90	0.94	0.93
Final Third	0.84	0.90	0.96	0.84	0.88	0.99	0.95	0.89	0.98	0.58
Treatment II										
All Periods	0.83	0.80	0.82	0.71	0.89	0.76	0.76	0.90	0.86	0.70
First Third	0.78	0.66	0.71	0.53	0.83	0.59	0.80	0.84	0.73	0.55
Middle Third	0.83	0.81	0.86	0.73	0.90	0.86	0.78	0.91	0.88	0.69
Final Third	0.90	0.94	0.88	0.89	0.94	0.83	0.71	0.96	0.99	0.86
Treatment III										
All Periods	0.65	0.68	0.82	0.51	0.41	0.68	0.56	0.38	0.70	0.43
First Third	0.50	0.60	0.75	0.53	0.40	0.63	0.60	0.40	0.58	0.43
Middle Third	0.75	0.70	0.80	0.50	0.48	0.68	0.55	0.38	0.75	0.45
Final Third	0.70	0.75	0.90	0.50	0.35	0.73	0.53	0.38	0.78	0.43

TABLE II  
MULTIPLE COMPARISONS BETWEEN TREATMENTS ACCORDING TO THE PROPORTIONS OF A-CHOICES\*

	First Third	Middle Third	Final Third
Treatments I and II	3,10 <sup>a</sup>	6,25 <sup>a</sup>	0,4 <sup>a</sup>
Treatments I and III	12,52 <sup>b</sup>	16,70 <sup>b</sup>	12,5 <sup>b</sup>
Treatments II and III	9,42 <sup>b</sup>	10,45 <sup>b</sup>	12,1 <sup>b</sup>

\* Critical Value, 8.378.

<sup>a</sup> The null hypothesis of no difference in the proportion of A-choices cannot be rejected.

<sup>b</sup> The null hypothesis is rejected in favor of a lower proportion of A-choices in Treatment III.

TABLE III  
SPEARMAN RANK-ORDER CORRELATION COEFFICIENTS

Player	Treatment I	Treatment II	Treatment III
Game 1-Player 1	0.84*	0.23	0.30
Game 1-Player 2	0.17	0.48	0.52
Game 2-Player 1	0.70*	0.72*	0.44
Game 2-Player 2	0.44	0.87*	0.77*
Game 3-Player 1	0.81*	-0.11	0.03
Game 3-Player 2	0.62*	0.95*	0.71*
Game 4-Player 1	0.44	0.87*	0.70*
Game 4-Player 2	0.45	0.98*	-0.13
Game 5-Player 1	0.28	0.55	0.34
Game 5-Player 2	0.65*	0.21	-0.39
Game 6-Player 1	0.62*	0.93*	0.86*
Game 6-Player 2	0.38	0.32	0.31
Game 7-Player 1	0.94*	-0.35	0.90*
Game 7-Player 2	0.88*	-0.03	-0.65*
Game 8-Player 1	0.15	0.91*	-0.45
Game 8-Player 2	0.30	0.09	-0.26
Game 9-Player 1	0.69*	0.66*	0.91*
Game 9-Player 2	0.81*	0.93*	0.92*
Game 10-Player 1	0.70*	0.82*	0.68*
Game 10-Player 2	-0.43	0.95*	0.47
Aggregated Coefficients by Treatment			
	0.71*	0.98*	0.95*

\* Significant at the 5% level (one-sided).

TABLE IV  
AVERAGE NUMBER OF RUNS PER GAME

Game	1	2	3	4	5	6	7	8	9	10	Total
Treatment I											
All Periods	32.5	35.5	20	30.5	37.5	12	29	20	23	14	25.4
First Third	15.5	17.5	12	17	18	8	16	9	17	10	14
Middle Third	12	11.5	6	5.5	9.5	4	9.5	7	5	2.5	7.25
Final Third	7	8	4	9	11	2	5	6	3	3.5	5.85
Treatment II											
All Periods	25	31	33	46.5	20.5	33.5	39.5	19	26.5	51.5	32.6
First Third	12	16.5	16	21.5	10.5	17	12.5	9	17.5	22.5	15.5
Middle Third	10.5	10	10	18.5	8	9.5	12.5	7	8	20.5	11.45
Final Third	4.5	6	9	8	4	8.5	16.5	4	2	9.5	7.2
Treatment III											
All Periods	52	56.5	60.5	91.5	77.5	54	63	60.5	56	65	63.65
First Third	23	21.5	23.5	35	21.5	20.5	26	20	32	22.5	24.55
Middle Third	14.5	19	22	33	30	20	21	20	14.5	21.5	21.55
Final Third	15	17	16.5	23.5	26.5	15	16.5	21.5	10	21.5	18.3



TABLE V  
MULTIPLE COMPARISONS BETWEEN TREATMENTS ACCORDING TO THE NUMBER OF RUNS\*

Treatments	First Third	Middle Third	Final Third
Treatments I and II	2.55 <sup>a</sup>	6.4 <sup>a</sup>	2.3 <sup>a</sup>
Treatments I and III	15.15 <sup>b</sup>	17.15 <sup>b</sup>	15.4 <sup>b</sup>
Treatments II and III	12.59 <sup>b</sup>	10.75 <sup>b</sup>	13.1 <sup>b</sup>

\* Critical Value, 8.378.

<sup>a</sup> The null hypothesis of no difference in the number of runs cannot be rejected.

<sup>b</sup> The null hypothesis is rejected in favor of a higher number of runs in Treatment III.

TABLE VI  
TEST OF NO DIFFERENCE IN THE NUMBER OF RUNS OF THE LESS FREQUENTLY CHOSEN  
ALTERNATIVE BY LENGTH ACROSS THE THREE TREATMENTS (JONCKHEERE TEST)\*

	First Third		Middle Third		Final Third	
	<i>J</i> *	Approx. <i>P</i> -value	<i>J</i> *	Approx. <i>P</i> -value	<i>J</i> *	Approx. <i>P</i> -value
One Period Runs	3.232 <sup>b</sup>	0.07%	4.544 <sup>b</sup>	0.003%	3.783 <sup>b</sup>	0.011%
Two-Period Runs	1.882 <sup>b</sup>	3%	2.414 <sup>b</sup>	0.8%	3.042 <sup>b</sup>	0.12%
Three Period Runs	-0.19 <sup>a</sup>	42.47%	0.456 <sup>a</sup>	32.64%	0.57 <sup>a</sup>	28.43%

\* Critical Value, 1.645.

<sup>a</sup> The null hypothesis of no difference in the number of runs among the three treatments cannot be rejected.

<sup>b</sup> The null hypothesis is rejected in favor of an equal or higher number of runs in Treatment III, than in Treatment II, and in Treatment II, than in Treatment I.

TABLE VII  
MULTIPLE COMPARISONS BETWEEN TREATMENTS ACCORDING TO THE NUMBER OF RUNS OF  
THE LESS FREQUENTLY CHOSEN ALTERNATIVE BY LENGTH\*

	One-Period Runs			Two-Period Runs		
	First Third	Middle Third	Final Third	First Third	Middle Third	Final Third
Treatments I and II	0.7 <sup>a</sup>	7.2 <sup>a</sup>	1.15 <sup>a</sup>	3.3 <sup>a</sup>	2.85 <sup>a</sup>	2.6 <sup>a</sup>
Treatments I and III	12.8 <sup>b</sup>	16.95 <sup>b</sup>	14.75 <sup>b</sup>	7.5 <sup>a</sup>	9.6 <sup>b</sup>	11.8 <sup>b</sup>
Treatments II and III	12.1 <sup>b</sup>	9.75 <sup>b</sup>	13.6 <sup>b</sup>	5.8 <sup>a</sup>	6.75 <sup>a</sup>	9.2 <sup>b</sup>

\* Critical Value, 8.378

<sup>a</sup> The null hypothesis of no difference in the number of runs cannot be rejected.

<sup>b</sup> The null hypothesis is rejected in favor of a higher number of runs in Treatment III.

TABLE VIII  
CLASSIFICATION OF PLAYERS

	Treatment I	Treatment II	Treatment III	Total
Strict Melioration	13 (65%)	13 (65%)	3 (15%)	29 (48.3%)
Melioration	6 (30%)	4 (20%)	6 (30%)	10 (26.7%)
Strict best-reply	1 (5%)	-	-	1 (1.7%)
best-reply	-	-	-	-
Alternating	-	1 (5%)	10 (50%)	11 (18.3%)
Other	-	2 (10%)	1 (5%)	7 (5%)
Total	20	20	20	60

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<sup>1</sup> The term “Harvard game” was coined by Rachlin and Laibson in Herrnstein (1997, p. 189).

<sup>2</sup> The “almost” means that at the very end of the game it pays to make a few *A*-choices. This issue will be studied below, when the Nash equilibrium is derived.

<sup>3</sup> The representation in Figure 1 takes  $X_i$  as the continuum between zero and ten.

<sup>4</sup> However, for evidence on the contrary see Ostrom *et al.* (1994).

<sup>5</sup> The experimental data will be made available upon request.

<sup>6</sup> Note that the sum of the runs of the three blocks of data does not typically equal the number of runs observed in all the 120 periods taken together. This is because dividing data in blocks may divide a run into two, and hence it is counted in two blocks.

<sup>7</sup> Since the number of runs of four periods or more is practically concentrated around zero, the Jonckheere test is not applicable. However, this is a sign of the similitude among the three treatments on these types of runs.

<sup>8</sup> In the case of best-reply, those *A*-choices observed in the last 7 periods will not be counted in the 10% (strict best-reply) or 25% (best-reply) limit of admissible *A*-choices.