BONN ECON DISCUSSION PAPERS

Discussion Paper 2/2002

# Strategic Delegation and Mergers in Oligopolistic Contests

by

**Matthias Kräkel, Dirk Sliwka**

January 2002

Bonn Graduate School of Economics
Department of Economics
University of Bonn
Adenauerallee 24 - 42
D-53113 Bonn

The Bonn Graduate School of Economics is
sponsored by the

Deutsche Post World Net

MAIL EXPRESS LOGISTICS FINANCE

# Strategic Delegation and Mergers in Oligopolistic Contests*

Matthias Kräkel and Dirk Sliwka, *University of Bonn***

**Abstract**

In this paper, we combine the strategic delegation approach of Fershtman-Judd-Sklivas with contests. The results show that besides a symmetric equilibrium there also exist asymmetric equilibria in which one owner induces pure sales maximization to his manager so that all the other firms drop out of the market. If merging is allowed on an initial stage, the resulting merged subgame perfect equilibria show that there is strictly more merging under contest than under Cournot competition. We also compare our findings with the previous results on contest models with delegation and find that the outcomes for the Fershtman-Judd-Sklivas incentive scheme clearly differ. Especially, in our model we have a prisoner's-dilemma like situation where delegation is individually rational for each owner, but all owners are worse off compared to non-delegation.

JEL classification: L1, M2.

* Financial support by the Deutsche Forschungsgemeinschaft (DFG) (project no. KR 2077/2-1) is gratefully acknowledged.

** Matthias Kräkel (corresponding author) and Dirk Sliwka, Department of Economics, BWL II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany, e-mail: m.kraekel@uni-bonn.de, phone: +49-228-739211, fax: +49-228-739210.

# 1 Introduction

In large firms, especially in public corporations, there is a separation of ownership and control. Managerial theories of the firm and agency theory has emphasized that this separation leads to inefficiencies due to asymmetric information and differing objectives of managers and owners (e.g., Williamson 1964, Jensen and Meckling 1976, Fama and Jensen 1983). On the other hand, a growing literature on strategic delegation has highlighted that owners will profit from delegating decisions to managers if delegation serves as a self-commitment device (Fershtman 1985, Vickers 1985). For example, when firms compete against each other, owners may wish their managers to act more aggressively by putting a positive weight on sales in the managerial incentive contracts. Here, delegation to a manager may be beneficial for an owner as without delegation the owner would maximize profits and the other competitors would expect that the owner will do so.

Strategic delegation games usually consist of two stages – the first stage where managerial compensation is chosen by the owners, and the second stage where the managers compete in an oligopolistic market against each other. The previous literature on strategic delegation has focused on Cournot and Bertrand competition on the second stage (e.g., Fershtman and Judd 1987, Sklivas 1987). But, in practice, many competitive situations are much better described by oligopolistic contests. For example, firms have to spend resources in advance to compete for a highly profitable order from a public institution or from a private corporation. Or, firms invest resources for advertising to obtain large market shares (Schmalensee 1976, 1992)[1]. In addition, firms often compete in R&D contests against each other (e.g., Loury 1979). There are also a lot of litigation contests for brand names or patent rights between firms (e.g., Wärneryd 2000). As a further example, we can

[1] E.g., according to Scherer (2000) advertising expenditures are very high in the pharmaceutical industry.

think of oligopolistic competition in new markets, which often look like a contest. In such markets, it is important for firms to implement a new technical standard as a first-mover to realize network externalities (Besen and Farrell 1994). Successful competitors (like Microsoft, for example) can be labeled contest winners, whereas less successful firms can be described as contest losers. Either example characterizes a highly competitive situation where each firm has sunk expenditures irrespective of the outcome of the competition. In these cases, oligopolistic competition is far better modelled by a contest than by Cournot or Bertrand competition.

The aim of this paper is threefold. First, we want to combine the Fershtman-Judd-Sklivas approach with contest competition and ask how the optimal incentives for managers will look like in this alternative form of oligopoly. In the models of Fershtman and Judd (1987) and Sklivas (1987) the owners choose a linear combination of profits and sales as incentive scheme for their managers. In case of Cournot competition each owner puts a positive weight on sales, whereas in the Bertrand model owners prefer negative weights on sales. We will show that there are parallels but also strong differences to Cournot and Bertrand competition. On the one hand, like in the Cournot model a symmetric equilibrium exists in which the owners put a positive weight on sales. On the other hand, in contrast to both Cournot and Bertrand competition, despite a completely symmetric market structure asymmetric (preemptive) equilibria exist in which one owner induces pure sales maximization to his manager so that all the other firms drop out of the market. As oligopolistic contests define a rather strong form of competition there will be also strong incentives for owners to limit competition by merging. Therefore, we will also discuss merging in contests with strategic delegation. Interestingly, mergers have already been discussed for the Cournot model with strategic delegation (Ziss 2001, Gonzalez-Maestre and Lopez-Cunat 2001) on the one hand, and for the contest model without strategic delegation (Huck et al. 2001) on the other hand. In Section 4 we will contrast our results with the findings of these papers

Secondly, the model can be used to explain real market behavior. In the last years, the markets for gas and electricity and the telecommunication market have become deregulated in Germany and other European countries. As a consequence, a couple of private firms enter into these markets at the same time. They use a lot of promotional expenditures to aggressively fight for market shares. This competitive situation can be best described by a contest model. Our results show that delegation in contests leads to a symmetric equilibrium in which all owners choose a sales-oriented compensation for their managers to make them highly aggressive (see Proposition 3). By this, all managers spend immense resources to become the contest winner. In addition, the theoretical results show that aggressive market behavior is highest for a small number of competitors (see Eq. (12) of Proposition 3). This fits quite well with the stylized facts for the deregulated markets which are only entered by few large competitors. The symmetric equilibrium result may also be used to explain the excessive spending of ressources by the so-called dotcom firms in the last years. In this case, network externalities become so important that managers are given strong incentives mainly to care for sales than for profits.

Thirdly, we want to contrast our findings with the previous models on delegation in contests to demonstrate the impact of the Fershtman-Judd-Sklivas incentive scheme on contest competition. As in the Fershtman-Judd-Sklivas framework we assume that, on the first stage of the game, managers are given a linear incentive scheme conditional on profits and sales. On the second stage, the managers compete in a logit-form contest against each other.[2]

[2] We adopt the logit-form contest because it fits best within the industrial organization context. In addition, all the above cited literature on oligopolistic contests also deal with the logit-form.

Delegation in contests has been discussed before by Baik and Kim (1997), Konrad et al. (1999), Schoonbeek (2000), and Wärneryd (2000). However, none of the these models discuss the Fershtman-Judd-Sklivas compensation scheme. Baik and Kim (1997) consider a logit-form contest where players can delegate their decisions to an agent who is compensated according to his ability and the contest prize. In contrast to our model, incentive parameters are exogenously given and the principals differ in their preferences concerning the winner prize. The findings of Baik and Kim show that in case of bilateral delegation less resources are spent compared to the non-delegation case. Interestingly, our results for the symmetric equilibrium show that the Fershtman-Judd-Sklivas incentive scheme will result in higher resource choices in case of delegation.

Konrad et al. (1999) discuss delegation in an all-pay auction. Here, the agent has to pay an up-front fee to the principal prior to the contest and gets a fixed remuneration in case of winning where the remuneration is derived endogenously. Konrad et al. show that both principals gain from delegation, and that delegation contracts are asymmetric despite the symmetric structure of the game. However, our findings for the logit-form contest demonstrate that the owners' profits are lower when delegating decisions to managers compared to the non-delegation case. In Schoonbeek (2000) only one player can delegate his decisions to an agent who receives a fraction of the prize in case of winning the logit-form contest. The fraction is optimally chosen by the risk averse principal. In equilibrium, the more risk averse the principal the higher is the chosen fraction because the principal wants to minimize the risk of not obtaining the winner prize. Wärneryd (2000) considers delegation in a logit-form contest where the agent receives an endogenously chosen payment when winning the contest. The equilibrium of the game is symmetric and both principals strictly gain from delegation.

The paper is organized as follows. In the next section, the basic model is described. In Section 3, we solve the basic two-stage game. Section 4 deals with merging in contests with strategic delegation. Section 5 concludes.

## 2 The Basic Model

We consider a two-stage model with $n$ ($n \geq 2$) firms where each firm is characterized by its owner and its manager. All players are risk neutral. On the first stage (compensation stage), each owner $i$ has to decide about an incentive scheme for his manager $i$ ($i = 1, \ldots, n$). Following Fershtman and Judd (1987) and Sklivas (1987), it is assumed that owner $i$ chooses a linear combination of profits ($\Pi_i$) and sales ($S_i$) as incentive scheme:

$$O_i = \alpha_i \Pi_i + (1 - \alpha_i) S_i \tag{1}$$

with $\alpha_i \geq 0$. As in Fershtman and Judd (1987) the manager's total compensation package is given by $A_i + B_i O_i$ ($B_i > 0$) where $A_i$ and $B_i$ are chosen by owner $i$ so that the manager's compensation just equals his reservation value.[3]Hence, in the following we only have to care about the managers' incentives. It is assumed that incentive contracts can exclusively be written on the basis of $\Pi_i$ and $S_i$, but not on other variables that determine profits and sales.[4] According to Eq. (1), $\alpha_i < 1$ ($\alpha_i > 1$) means that owner $i$ puts a positive (negative) weight on sales whereas $\alpha_i = 1$ induces pure profit

[3]Note that the given incentive scheme characterizes all possible contracts that are linear in profits and sales. Let, e.g., $O_i' = x_i \Pi_i + y_i S_i$. Hence, $A_i + B_i O_i' = A_i + B_i x_i \Pi_i + B_i y_i S_i$. By choosing $x_i = \alpha_i / B_i$ and $y_i = (1 - \alpha_i) / B_i$ we obtain Eq. (1).

[4]For example, we can assume that owner $i$ only observes the realizations of $\Pi_i + \varepsilon_i$ and $S_i + \gamma_i$ with $\varepsilon_i$ and $\gamma_i$ being random variables with $E[\varepsilon_i] = E[\gamma_i] = 0$. As we will see later, profits and sales directly depend on the resources spent by the managers. Hence, the last assumption excludes incentive contracts that directly depend on the managers' resource decisions.

maximization. Each owner $i$ wants to maximize profits $\Pi_i$.[5]

On the second stage (contest stage), each manager $i$ observes all the chosen incentive schemes.[6] After that the managers compete for market shares by spending resources $\mu_i \geq 0$ of firm $i$ given in monetary terms (e.g., promotion expenditures). As in Schmalensee (1976, 1992), for example, this competition is modelled as a logit-form contest in which $S > 0$ denotes the total market volume of sales (i.e., the market size), and

$$s_i = \frac{\mu_i}{\mu_i + \sum\limits_{j \neq i} \mu_j} \tag{2}$$

firm $i$'s share in $S$ depending on the resources spent by all firms.[7] The more resources manager $i$ spends relative to the other managers the higher will be firm $i$'s market share. Eq. (2) assumes that $\mu_i + \sum_{j \neq i} \mu_j > 0$. Otherwise, let $s_i = 1/n$. Altogether, $S_i = s_i \cdot S$ describes firm $i$'s sales and $\Pi_i = S_i - \mu_i$ firm $i$'s profits. Substituting for $S_i$ and $\Pi_i$ in Eq. (1) we can rewrite the incentive scheme for manager $i$ as

$$O_i = \frac{\mu_i}{\mu_i + \sum\limits_{j \neq i} \mu_j} S - \alpha_i \mu_i. \tag{3}$$

Therefore, by choosing his incentive variable $\alpha_i$ owner $i$ directly influences his manager's cost function on the contest stage. If owner $i$ chooses a low (high) value for $\alpha_i$ he will make the use of the firm's resources cheap (expensive) for his manager which results in a more (less) aggressive behavior of manager $i$ in the contest. In the next section, we will solve this basic two-stage model

[5] Remember that each manager always receives his reservation value. Therefore, owner $i$ maximizes the surplus of firm $i$.

[6] Otherwise, there would be serious problems of delegation working as a self-commitment device. See Katz (1991) and Bagwell (1995), but also Bonanno (1992) and Kalai and Fershtman (1997).

[7] Here, $S$ corresponds to the winner prize of the contest and $s_i$ to the winning probability of firm $i$. We assume that the power parameter equals one to exclude scale effects when discussing mergers in Section 4.

to compare the optimal incentive scheme to the findings of Fershtman-Judd-Sklivas. Where possible we will also discuss parallels to the previous literature on delegation in contests.

# 3 Delegation in Oligopolistic Contests

We start by considering the game played among the managers on the contest stage given their respective incentive schemes (3). Manager $i$'s best reply to the resource expenditures of the other managers is then described by the solution of

$$\max_{\mu_i} \frac{\mu_i}{\sum\limits_j \mu_j} S - \alpha_i \mu_i \quad \text{s.t. } \mu_i \geq 0. \tag{4}$$

Note that the second stage corresponds to an asymmetric contest between $n$ players with possibly differing marginal costs of resource expenditures as for instance partially analyzed in Hillman and Riley (1989). We extend their analysis by not only characterizing the set of managers with positive contributions but explicitly computing those contributions.

The Kuhn-Tucker conditions of program (4) are

$$S \sum_{j \neq i} \mu_j \Big/ \left( \sum_j \mu_j \right)^2 - \alpha_i = 0 \text{ if } \mu_i > 0, \tag{5}$$

$$S \sum_{j \neq i} \mu_j \Big/ \left( \sum_j \mu_j \right)^2 - \alpha_i \leq 0 \text{ if } \mu_i = 0. \tag{6}$$

For all managers choosing strictly positive values for $\mu_i$ we therefore must have that in equilibrium the ratio $\alpha_i / \sum_{j \neq i} \mu_j$ must be the same for all managers and equal to $S / \left( \sum_j \mu_j \right)^2$. For all other managers this ratio has to be larger or equal than $S / \left( \sum_j \mu_j \right)^2$. We solve this (in)equality for $\mu_i$ to obtain a manager's reaction function:

$$\mu_i = \max\left\{\sqrt{\frac{S}{\alpha_i}\sum_{j\neq i}\mu_j} - \sum_{j\neq i}\mu_j, 0\right\}.$$

To find an equilibrium we must solve this system of $n$ equalities. A solution is given in the following result:

**Proposition 1** *If all incentive variables $\alpha_i$ are strictly positive, there exists a Nash equilibrium on the contest stage with the following properties. There is a subset $H \subseteq \{1,..,n\}$ of all managers who spend positive resource levels. The managers contained in the subset are those that have the $m = \#H$ lowest $\alpha_i$ values. The resource expenditure of each manager in the subset is*

$$\mu_i = (m-1)\,S\frac{\left(\sum_{j\in H\backslash\{i\}}\alpha_j\right) - (m-2)\,\alpha_i}{\left(\sum_{j\in H}\alpha_j\right)^2}. \tag{7}$$

*The managers with the $n-m$ highest $\alpha_i$ do not spend any resources. For the $\alpha_i$ of the managers contributing positive amounts the following condition holds:*

$$\sum_{j\in H\backslash\{i\}}\alpha_j > (m-2)\,\alpha_i. \tag{8}$$

*The managers with the two lowest $\alpha_i$ always contribute a strictly positive amount.*

**Proof:** See the Appendix. ■

The results of Proposition 1 show that only the most aggressive managers (i.e., the ones with the lowest $\alpha_i$) will choose positive contributions in the contest whereas the other managers drop out of the market by spending zero ressources. According to Eq. (7), the owners can directly influence the behavior of their managers but also the behavior of the other firms' managers. Especially, by choosing a low value for the incentive variable on the compensation stage owner $i$ makes his manager more aggressive in the subsequent contest.

Proposition 1 generalizes a result by Hillman and Riley (1989) who have only given condition (8) without solving for the equilibrium values of $\mu_i$. Note that this result implies a simple algorithm by which a second-stage equilibrium can be computed. To do this one simply has to sort the incentive variables $\alpha_i$. Without loss of generality take $\alpha_1 < \alpha_2 < ... < \alpha_n$. We know that the two managers with the lowest $\alpha_i$ will always choose positive resources. Hence, we start off with the third, checking condition (8) for $i = 3$ and $H = \{1, 2\}$. If it is met we will continue with $i = 4$ and check condition (8) for $H = \{1, 2, 3\}$ and so on, until the condition is violated.

In Proposition 1 we have analyzed equilibria in the contest when all owners have chosen strictly positive incentive variables on the compensation stage. It remains to look at situations in which one or more managers face compensation schemes with a zero incentive variable:

**Proposition 2** *There is no Nash equilibrium on the contest stage when for at least two managers the respective incentive variables $\alpha_i$ are zero. If exactly one manager $i$ has a compensation scheme with $\alpha_i = 0$, there is a continuum of Nash equilibria, in which this manager $i$ chooses a preemptive quantity $\mu_i \geq \frac{S}{\alpha_k}, \forall k \neq i$, and all other managers $j \neq i$ choose $\mu_j = 0$.*

**Proof:** To see that no Nash equilibrium will exist if there are two managers $i$ and $j$ with $\alpha_i, \alpha_j = 0$ just note that given for any (pure or mixed) strategy of manager $i$ manager $j$ can always be better off by "overbidding" since the use of ressources is costless for him.

If only one manager $i$ has $\alpha_i = 0$ he is indifferent between any quantity $\mu_i$ given that all other managers stay out and choose $\mu_j = 0$. On the other hand, for a given quantity $\mu_i$ by manager $i$ and $\mu_k = 0$ for all managers

$k \notin \{i, j\}$ we know from (6) that for a manager $j$ it is optimal to choose $\mu_j = 0$ iff

$$S \frac{\mu_i}{(\mu_i)^2} - \alpha_j \leq 0 \Leftrightarrow \frac{S}{\mu_i} \leq \alpha_j.$$

This condition holds for all $j \neq i$ iff $\mu_i \geq \frac{S}{\alpha_k}, \forall k \neq i$. ■

Hence, a manager with a very aggressive compensation schedule such that he is only rewarded for sales but not at all punished for expenditures will in equilibrium spend at least some resources to preempt any other manager from entering the contest.

Now we can go back to the first stage of the game where the owners simultaneously choose the incentive schemes for their respective managers. First, we restrict the analysis to searching only for symmetric equilibria on this stage. To characterize the best response by an owner $i$ we first have to compute his profits for given incentive variables set by his competitors. As we look for a symmetric equilibrium, we have $\alpha_j$ is equal to some $\alpha$ for all $j \neq i$. We have to check whether there is some $\alpha$ for which indeed choosing $\alpha_i = \alpha$ is a best response of owner $i$. Note that we can conclude from condition (8) that the manager of firm $i$ will only spend a strictly positive amount of resources if

$$\alpha_i < \frac{(n-1)\,\alpha}{n-2}.$$

This condition is clearly met in a symmetric equilibrium. If this condition is not met the owner makes zero profits. If it is met, however, we know from (7) in Proposition 1 that his manager spends the following amount of resources given that $\alpha$ and $\alpha_i$ are strictly positive:

$$\mu_i = \frac{(n-1)\,S\left((n-1)\,\alpha - (n-2)\,\alpha_i\right)}{\left((n-1)\,\alpha + \alpha_i\right)^2}. \tag{9}$$

We can again apply (7) to get for $j \neq i$ that

$$\mu_j = \mu = \frac{(n-1)\,S\alpha_i}{\left((n-1)\,\alpha + \alpha_i\right)^2}. \tag{10}$$

Using this we derive the following result:

**Proposition 3** *There is a symmetric subgame perfect equilibrium such that, on the first stage, each owner chooses an incentive variable*

$$\alpha_i^* = \frac{n^2 - 2n + 2}{n(n-1)}. \tag{11}$$

*On the second stage, each manager spends resources*

$$\mu_i^* = \frac{(n-1)^2}{n(n^2 - 2n + 2)} S. \tag{12}$$

*Each owner's equilibrium profits are given by*

$$\Pi_i^* = \frac{S}{n(n^2 - 2n + 2)}. \tag{13}$$

**Proof:** See the Appendix. ■

Proposition 3 offers some parallels to the findings of Fershtman and Judd (1987) and Sklivas (1987) for Cournot competition on the second stage. In analogy to Cournot competition, strategic delegation in oligopolistic contests also leads to a symmetric equilibrium where each owner usually puts a positive weight on sales (i.e., $\alpha_i^* < 0$) to make his manager more aggressive. In addition, like in Theorem 4 of Fershtman and Judd (1987) $\alpha_i^* \rightarrow 1$ as $n$ approaches infinity. Hence, similar to the result of Fershtman and Judd for $n \rightarrow \infty$ we have a perfectly competitive market where each owner chooses pure profit maximization as incentive scheme and realizes zero profits (see (13)). Note that $\alpha_i^* = 1$ is equivalent to the case of an entrepreneurial firm which is managed by the owner himself. In other words, there are not any strategic advantages from delegation in this situation. On the contrary, each owner will prefer not to delegate decisions to a manager to save labor costs since each manager will receive his reservation value as compensation when being hired. But, in oligopolistic contests according to Eq. (11) owners will also choose pure profit maximization in case of a duopoly (i.e., $n = 2$) which does not hold for the case of Cournot competition (see Eq. (17) in Fershtman and Judd 1987, p. 936).

As mentioned in the introduction, the results of Proposition 3 fit well with the aggressive behavior of firms that have entered in recently deregulated markets, like the gas and electricity market and the market for telecommunication in Germany and other European countries. There, a few large corporations use immense expenditures for advertising to obtain large market shares relative to each other. This very strong competition can be best characterized as an oligopolistic contest. Interestingly, our results show that managerial firms act very aggressively in oligopolistic contests and the more aggressively the less competitors participate in the contest (see Eq. (12)). Analogously, there are also only few large managerial firms in the deregulated market case. According to Proposition 1, the most aggressive firms drive the other ones out of the market. Presumably, this motive also holds for the aggressive behavior in the deregulated markets. But note that our model can only partly contribute to the discussion of deregulated markets, since in the telecommunication case, for example, firms do not only compete by advertising but also by prices which can be better discussed within the Bertrand model. However, price competition can be mimicked by the contest model, because lowering product prices is equivalent to invest in future market shares by increasing $\mu_i$.

Moreover, we can also compare the results of Proposition 3 to the findings of previous papers that have discussed delegation in contests without using the Fershtman-Judd-Sklivas incentive scheme. In the logit-form contest model of Baik and Kim (1997) players exert less resources in the case of delegation than in the non-delegation case. However, this result clearly differs from the results of Proposition 3. Without delegation (i.e., $\alpha_i = \alpha = 1$ in (9) and (10)), each firm will choose

$$\hat{\mu}_i = (n-1)\,\frac{S}{n^2} \tag{14}$$

in oligopolistic contests, but this is strictly less than the resources chosen by the managers in case of delegation: $\hat{\mu}_i - \mu_i^* \overset{(12)}{=} -\frac{(n-2)(n-1)S}{n^2(n^2-2n+2)} \leq 0$. This result is also intuitively plausible, as in the context of industrial organization owners want their managers to become more aggressive and not more passive.

The findings of both the all-pay auction model of Konrad et al. (1999) and the logit-form contest model of Wärneryd (2000) show that the owners or principals will strictly gain from delegating. Again, this sharply contrasts with the results of our model using the Fershtman-Judd-Sklivas incentive scheme.[8] Without delegation the profits of each owner can be computed by inserting $\hat{\mu}_i, \forall i$, according to (14) into (20):

$$\hat{\Pi}_i = \frac{S}{n^2}. \tag{15}$$

Subtracting the profits in case of delegation, which are described by Eq. (13), yields $\hat{\Pi}_i - \Pi_i^* = \frac{n^2-3n+2}{n^2(n^2-2n+2)} S \geq 0$. Hence, in our model the $n$ owners are collectively worse off by delegating decisions to managers. Nevertheless, the delegation decision is individually rational (given that the manager's reservation value is not too large): If all owners but $i$ do not delegate decisions (i.e., $\alpha_k = 1$, $\forall k \neq i$) owner $i$'s profits will be

$$\Pi_i \overset{(22)}{=} S\alpha_i \frac{n-1-(n-2)\alpha_i}{(\alpha_i + n - 1)^2}.$$

The first-order condition $\partial \Pi_i / \partial \alpha_i = 0$ leads to

$$\alpha_i = \frac{n-1}{2n-3} < 1 \quad \text{for } n > 2.$$

Therefore, with the exception of the duopoly case owner $i$ will hire a manager and put a positive weight on sales as a best response if all other owners do not delegate. Altogether, we have a kind of prisoner's-dilemma situation here. For each owner delegating is strictly dominant (given that $n > 2$ and the manager's reservation value is not too large), but all owners together will

[8] This also contrasts with the results for the Cournot model (see Fershtman and Judd 1987, p. 932).

be better off if each one does not delegate. Interestingly, in the model of Wärneryd (2000) there is just the opposite prisoner's-dilemma situation.[9]

In Proposition 2 we have seen that if a single owner has decided to make his manager very aggressive by conditioning his compensation only on sales and setting the incentive variable $\alpha_i$ equal to zero, there will be equilibria on the contest stage where this manager drives all other firms out of the market by supplying at least a certain amount of resources. Now, it is interesting to consider whether it is possible that in a subgame perfect equilibrium one owner will indeed on the previous stage be able to set $\alpha_i$ equal to zero and achieve preemption by its management. It is important to note that such an equilibrium requires that the incentive variables set by those firms which are driven out of the market on the second stage must neither be too high nor too low. If they are too high the preempting owner can profitably deviate by forgoing complete preemption and choosing a strictly positive incentive variable. By doing this he may gain a sufficiently large share of $S$ without making his manager spend too much resources, and this of course will only be possible if the competing managers are sufficiently passive, i.e. their incentive variables are sufficiently large. If, however, the incentive variables of those firms are too small and, hence, the competing managers are relatively aggressive, we know from Proposition 2 that preemption requires relatively large resource expenditures by the preempting manager, which in turn might make preemption to expensive for the owner. The following result shows that such equilibria nonetheless exist:

**Proposition 4** *Preemptive subgame perfect equilibria exist, in which a single owner $i$ chooses $\alpha_i = 0$. All other owners choose $\alpha_k = \frac{n+1}{n}$. At stage 2, manager $i$ spends resources $\mu_i = \frac{nS}{n+1}$, whereas all managers $k \neq i$ choose*

[9] See Wärneryd (2000), pp. 152-153. However, in the two-principal all-pay auction model of Konrad et al. (1999) with endogenous delegation decisions we have two asymmetric equilibria in which one principal delegates and the other not (p. 11).

$\mu_k = 0$.

**Proof:** See the Appendix. ■

The result of Proposition 4 is surprising. Although the structure of the two-stage game is completely symmetric, asymmetric equilibria exist in which one owner $i$ chooses pure sales maximization as incentive scheme for his manager which makes all the other firms $k \neq i$ drop out of the contest. In this case, a sales maximizing manager is rational from owner $i$'s point of view as the owner becomes a monopolist in the market. Note that all the other owners cannot gain by choosing an equally aggressive manager. If these owners also choose $\alpha_k = 0$ resources will be costless for all the managers on the contest stage. This would result into complete escalation where each manager chooses infinitely many resources and each owner makes infinitely large losses. Hence, dropping out of the market will be the best response of the other owners $k \neq i$ to owner $i$ choosing $\alpha_i = 0$.

This finding clearly differs from the results of Fershtman and Judd (1987) and Sklivas (1987) for both Cournot and Bertrand competition. There, only symmetric equilibria exist where either all owners choose a positive or a negative weight for sales. The finding also differs from the results of the previous models on delegation in contests. Baik and Kim (1997) discuss an asymmetric game and, therefore, find asymmetric equilibria. Konrad et al. (1999) also find asymmetric equilibria for a certain parameter range, but no preemptive equilibria. Wärneryd (2000) exclusively finds symmetric equilibria for his symmetric logit-form contest model.

# 4 Mergers in Oligopolistic Contests

In this section we will only consider the symmetric equilibrium of Proposition 3. Given a completely symmetric game with homogeneous owners and homogeneous managers a symmetric outcome of the game appears to be the most plausible one. Now, we introduce an additional stage of the game on which the owners decide about merging. The timing of the game is as follows: On the first stage, the $n$ owners decide about merging. Here, merging means that the acquired firms are shut down. This assumption is not critical at all in this model since we have excluded any scale effects and costs are linear. Hence, the only effect of merging is weakened competition which would result in higher profits for the raider.[10] As in the basic model, now on stage 2 the remaining owners choose incentive schemes for their managers, and on stage 3 the managers spend resources in the oligopolistic contest given the observed incentive schemes of all owners.

Contests are a rather strong form of competition. Therefore, we can presume that owners have considerable interests in limiting competition by merging with other firms. This aspect can be called *competition effect*. On the other hand, merging is not costless for the raider as he has to pay the owner of the target firm his foregone profits (*cost effect*). Thus, we have a strict trade-off between competition and cost effect: The more firms an owner buys the less competitive is the market but the higher are the payments to the owners of the firms that are taken over. In addition, we have to notice that merging generates positive externalities because each remaining firm, not only the raider himself, profits from merging. Interestingly, there are three recent papers that also deal with mergers in oligopoly in a related setting. Ziss (2001) and Gonzalez-Maestre and Lopez-Cunat (2001) consider merging in a Cournot model with strategic delegation, whereas Huck et al. (2001) discuss mergers in oligopolistic contests without strategic delegation. We will see that merging in contests with strategic delegation will lead to different results compared to the three mentioned papers.

As a reference case we compare the equilibrium results in the model with strategic delegation to one where decisions are directly taken by the owners.

[10] Note that the symmetric equilibrium profits given by Eq. (13) are decreasing in $n$.

Let $\Pi_r(x)$ denote the symmetric equilibrium profits from the basic two-stage game when $x$ firms compete under the regime $r\epsilon\{D, ND\}$ where $D$ stands for the case of strategic delegation and $ND$ denotes the non-delegation case. Hence, if $r = D$ the profit function $\Pi_r(x)$ will be described by Eq. (13) and for $r = ND$ by Eq. (15). For example, without merging under regime $r$ each owner receives profits $\Pi_r(n)$.

Similar to Gonzalez-Maestre and Lopez-Cunat (2001) and Ziss (2001) we proceed by first examining what kind of mergers are profitable before analyzing equilibria. A merger will be profitable if the profit of the merged entity exceeds the sum of the individual firms' profits without the merger. Alternatively, one could think of a single firm acquring $t \leq n-1$ target firms by paying their owners the foregone profits. A merger will be *profitable* for an owner if[11]

$$\begin{aligned} \Pi_r(n-t) - t\Pi_r(n) &> \Pi_r(n) \Leftrightarrow \\ \Pi_r(n-t) &> (t+1)\Pi_r(n). \end{aligned} \tag{16}$$

Here, $\Pi_r(n-t)$ describes the raider's ex-post profits, $\Pi_r(n)$ his ex-ante profits before merging, and $t\Pi_r(n)$ the payments to the owners of the target firms. As an alternative, one can think of a non-cooperative game where exactly one owner gets the possibility to make a simultaneous take-it-or-leave-it offer to a given number of $t$ other firms before the delegation and competition stage. If all accept the merger will go along. If a single firm rejects the offer the whole merger fails. In this case, there is a unique subgame perfect equilibrium in which the $t$ firms are acquired if and only if this condition holds.

The following Proposition compares the profitableness of merging under both regimes:

[11] For this condition see also inequality (11) in Ziss (2001), p. 478, and Gonzalez-Maestre and Lopez-Cunat (2001), p. 1267.

**Proposition 5** *(i) Under regime $r = D$, if $n \leq 5$ all mergers will be profitable. If, however, $n > 5$ there will be a cut-off value $\hat{t}_D$ so that acquiring $t \geq \hat{t}_D$ firms will be profitable whereas the acquisition of $t < \hat{t}_D$ firms is not profitable.*
*(ii) Under regime $r = ND$, if $n \leq 3$ all mergers are profitable. If, however, $n > 3$ there will be a cut-off value $\hat{t}_{ND}$ so that acquiring $t \geq \hat{t}_{ND}$ firms will be profitable whereas the acquisition of $t < \hat{t}_{ND}$ firms is not profitable. The set of profitable mergers is larger in the delegation regime (i.e., $\hat{t}_D \leq \hat{t}_{ND}$).*
*(iii) In both cases, monopolization by acquiring all firms yields the highest profits.*

**Proof:** See the Appendix. ■

Proposition 5 shows that monopolization through merging is always profitable under either regime. The results also demonstrate that there is more merging under the delegation than under the non-delegation regime.

A key intuition to understand Proposition 5 is that there are increasing returns to scale *with respect to the number of acquired firms $t$*. On the one hand, the expenditures for acquiring $t$ firms (cost effect) is linearly increasing in $t$ as all acquired firms are bought for the same price. On the other hand, returns to the mergers rise in $t$ at an increasing rate as the profits are split among less firms and in addition competition is weakened when more and more firms are bought. This is illustrated in Figure 1 which shows the costs of acquiring $t$ firms (dotted line) and the profits (solid line) under the delegation regime ($S = 1$, $n = 10$).

Therefore it becomes clear that if any mergers are profitable then large mergers are. Note furthermore that a firm which acquires less than $n - 1$ firms delivers a public good to all other market participants. The profits of all other remaining firms are identical with the acquiring firm's profits. Hence, this firm has to bear all costs but has to share the entire gains. As the proposition shows, for a small number of acquired firms the costs are too

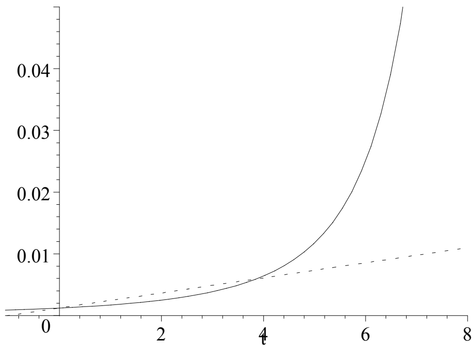


Figure 1: Competition versus cost effect.

high relative to the gains.

When the market is quite large, it only pays for an owner to buy a lot of firms to fully exploit the competition effect. Note that even the cost effect works in the same direction: As $n$ becomes large, the price for buying another firm given by Eq. (13) becomes small, i.e. for a very competitive situation the foregone profits of the target firms become negligible.

Next, we can compare the results of Proposition 5 to the findings of the previous literature. Huck et al. (2001) only consider the regime $r = ND$. Whereas they show that without scale effects acquiring $n-1$ firms is profitable, we have shown the stronger result that without delegation there is a larger set of profitable mergers even though the public good problem exists.[12] Gonzalez-Maestre and Lopez-Cunat (2001) discuss merging in Cournot competition with delegation using a Fershtman-Judd-Sklivas incentive scheme. They show that – similar to our findings – given regime $r = D$ there exists a critical fraction of merged firms so that merging of more (less) firms will be profitable (unprofitable).[13] Also similar to our model there is more merging

[12] Compare Proposition 5 in Huck et al. (2001). If scale effects are sufficiently strong all mergers will be profitable in their model.

[13] See Proposition 1 in Gonzalez-Maestre and Lopez-Cunat (2001), p. 1267.

under the delegation than under the non-delegation regime in the Cournot model.[14]

If one firm has the possibility to acquire all other firms in the beginning, it can monopolize the market. As we have shown in claim (iii) of Proposition 5 this is indeed the most profitable strategy in a game, where one firm can make simultaneous offers to all others. But from Proposition 4, we know that there is another way in which monopolization can occur in equilibrium: If an owner offers his manager a very aggressive incentive scheme, it may be possible to preempt all other market participants. It is interesting to know whether monopolization by preemption or by merger leads to higher profits:[15]

**Corollary 1** *Under regime $r = D$, monopolization by merging is more profitable than monopolization by preemption.*

**Proof:** According to Proposition 4, profits from monopolization by preemption are $S - \frac{nS}{n+1} = \frac{S}{n+1}$. By merging with $n-1$ other firms an owner's payoffs are given by

$$S - (n-1)\frac{S}{n(n^2-2n+2)} = S\frac{n^3-2n^2+n+1}{n(n^2-2n+2)}.$$

These payoffs will be higher than the payoffs from preemption, if

$$\begin{aligned} S\frac{n^3-2n^2+n+1}{n(n^2-2n+2)} &> \frac{S}{n+1} \\ S\frac{-2n^3+n^2+n^4+1}{(n+1)\,n(n^2-2n+2)} &> 0, \end{aligned}$$

which holds for all $n \geq 2$. ■

At first sight, the finding of the corollary seems to be somewhat surprising. As the number of payments to the owners of the target firms increases in $n$

[14] See Gonzalez-Maestre and Lopez-Cunat (2001) and Ziss (2001).

[15] More formally, this can be modelled by allowing one owner ex-ante to choose among two game forms: one in which the owner can buy all other firms in the above specified way before the game proceeds and another one where he can choose an incentive scheme as a Stackelberg leader without merging.

whereas preemption seems to work against 100 and 1,000 firms in the same way we would expect that at least for large $n$ monopolization by merging might be too costly. But a closer look shows that especially for large $n$ (i.e., for large markets) merging is much more attractive than preemption. If $n$ increases, the number of payments for the target firms will also increase in case of merging, but the magnitude of each payment will decrease much faster (see Eq. (13)). In addition, preemption is not the same for 100 and 1,000 competing firms. Proposition 4 shows that the preemptive amount of resources strictly increases in the number $n-1$ of preempted firms (i.e., $\partial\mu_i/\partial n = S/(n+1)^2 > 0$). Therefore, not merging with the whole market but preemption will be too costly if markets are quite large.

Up to now merging has been exogenously given or resulted from the acquisition proposal by a single firm. In the next step, we can look for endogenous mergers as equilibrium outcomes when all firms can simultaneously bid for others. Following Kamien and Zang (1990) and Gonzalez-Maestre and Lopez-Cunat (2001), we model the merger game in the following way: On a first stage, each owner $j$ chooses a vector of bids $B^j = \left(B_1^j, B_2^j, .., B_n^j\right)$ for all firms including a bid $B_j^j$ for his own firm which defines the owner's reservation price. Then, each firm is allocated to the highest bidder. In case of a tie among a buyer and a seller, the buyer acquires the firm. If there is a tie between two buyers the buyer with the lower index gets the firm. After the bidding process the game proceeds as above with a potentially lower number of firms. In the delegation regime, each owner of a remaining firm chooses a compensation scheme for his manager and in both regimes the contest takes place among the remaining firms. Note that all remaining firms are technologically identical as we assumed constant returns to scale.

As has been first pointed out by Kamien and Zang (1990) it is not clear that all profitable mergers will take place. Owners might prefer to free-ride on the merger decisions of other firms since all firms that are still active after the merging stage of the game will profit from merging. As we will see below, the acquisition price for a firm will be higher than $\Pi_r(n)$ as the outside option of a target firm is now raised by the free-rider effect.

We proceed by characterizing subgame perfect equilibria of the game for both regimes. Again following the previous literature we will speak of a *merged subgame perfect equilibrium* (or *merged SPE*) if at least one owner has acquired at least one firm ex post. Following Kamien and Zang and the subsequent literature on mergers in oligopoly models[16] we first give a necessary condition that must hold for any owner who has bought at least one firm in equilibrium. Consider any merged SPE. Let $m$ denote the number of active firms after the merging stage. Take any owner who posseses at least 2 firms in equilibrium. Let $q$ be the number of firms this owner possesses in equilibrium (i.e., he has bought $t = q - 1$ firms). In this case the total price such an owner is willing to pay for the acquired firms cannot exceed the difference between his profits $\Pi_r(m)$ with the merger and his profits when he does not acquire the $q - 1$ firms which is given by $\Pi_r(m + q - 1)$.

On the other hand, note that each owner of an acquired firm has the option, not to accept an offer by raising his ask price to a sufficiently high level. In that case, the number of active firms would be $m + 1$. Hence, each bid for an acquired firm must be greater or equal than $\Pi_r(m + 1)$. Then for an owner having $q \geq 2$ firms in equilibrium the following necessary condition must hold for a merged SPE:

$$\begin{aligned} \Pi_r(m) - \Pi_r(m + q - 1) &\geq (q - 1)\Pi_r(m + 1) \Leftrightarrow \\ \Pi_r(m) - (q - 1)\Pi_r(m + 1) &\geq \Pi_r(m + q - 1) \qquad (17) \end{aligned}$$

with $r \epsilon \{D, ND\}$. Note that the difference to condition (16) for a profitable merger refers to the payment to each selling owner. In a merged SPE, this

[16] See Kamien and Zang (1990), p. 486; Gonzalez-Maestre and Lopez-Cunat (2001), p. 1269; Ziss (2001), p. 481.

payment will be higher as any owner of an acquired firm anticipates that if he does not accept the bid for his own firm, still other acquisitions will take place increasing his profits above the profits without any merger.

Condition (17) helps to reduce the number of possible equilibrium outcomes significantly. In a second step, a vector of bids is constructed for each potential outcome such that indeed such an outcome is sustained in a subgame perfect equilibrium. The following results can be derived:

**Proposition 6** *(i) Under regime $r = D$, if $n \geq 10$, there will be no merged SPE. If, $n \leq 9$, the merged SPE are the following:*
*a monopoly if $n \epsilon \{2, 3, 4\}$*
*a duopoly if $n \epsilon \{4, 5, 6, 7, 8\}$*
*a three-firm oligopoly if $n \epsilon \{6, 7, 8, 9\}$*
*a four-firm oligopoly if $n = 8$.*
*(ii) Under regime $r = ND$, if $n \geq 5$, there will be no merged SPE. If, $n \leq 4$, the merged SPE are the following:*
*a monopoly if $n \epsilon \{2, 3, 4\}$*
*a duopoly if $n = \{3, 4\}$.*
*(iii) In case of monopoly payoffs under $r = D$ and $r = ND$ are identical.*

**Proof:** See the Appendix. ■

Comparing Proposition 6 to the results of Proposition 5 shows that not all profitable mergers are part of a merged SPE. Especially, for the non-delegation case there are only two types of equilibria. The fact that there is less merging under the conditions of a simultaneous-bidding equilibrium for both regimes can be explained by the strictly higher payments for firms. Here, each buyer has to pay $\Pi_r(m + 1)$ instead of $\Pi_r(n)$ which makes merging less attractive. But again, under equilibrium conditions there is more merging under the delegation than under the non-delegation regime. This can be explained by the fact that for a given number of competitors profits under the delegation regime are always lower than under the non-delegation regime. Hence, the payments that have to be made to the selling owners are also strictly lower under delegation than under non-delegation (i.e., $\Pi_D(m+1) < \Pi_{ND}(m+1)$) which strongly favors merging in connection with delegation.[17]

We can also compare our results with the findings for the Cournot model. According to Proposition 2 in Ziss (2001, p. 481) in the Cournot model delegation cannot reduce the set of candidate merged SPE. This also holds for our contest model. In Proposition 2, Gonzalez-Maestre and Lopez-Cunat (2001, p. 1270) explicitly derive the merged SPE for the case of delegation in Cournot competition. Comparing their results to the results of our Proposition 6 under $r = D$ shows that there is more merging in the contest model. This result fits well with the previous findings that contests describe a more competitive situation than the Cournot model. Therefore, the incentives to limit competition by merging will be greater in the contest than in the Cournot model. Gonzalez-Maestre and Lopez-Cunat (2001, p. 1271) also consider the non-delegation regime for Cournot competition. Similar, to the contest case delegation increases the set of merged SPE for Cournot competition, too. But under the given non-delegation regime there will be again more merging in the contest than in the Cournot model.

The last proposition has shown that there are multiple merged SPE for a given number of initial firms. For example, an initial number of eight firms may lead to a duopoly, a three-firm oligopoly, or a four-firm oligopoly. However, the number of SPE can be restricted to the most plausible ones by using the dominance criterion. In analogy to Gonzalez-Maestre and Lopez-Cunat (2001, p. 1271) we will use the concept of *undominated SPE* (*USPE*)

[17]Note that this argument does not apply in the same way to the case of profitable mergers. This is due to the fact that the difference between profits under $r = ND$ and $r = D$ is very high for low numbers of competitors but strictly decreases in this number. Hence, the difference $\Pi_{ND}(m+1) - \Pi_D(m+1)$ is more decisive than $\Pi_{ND}(n) - \Pi_D(n)$.

to restrict our attention to those merged SPE, for which – given a certain number of initial firms – there is no other SPE with payoffs that are greater or equal for all owners and strictly greater for at least one owner.[18] We obtain the following result:

**Proposition 7** *For $r = D$, the set of merged USPE ist the following:*
*If $n = 9$ the three-firm oligopoly will be the unique USPE.*
*If $n = 8$ or $n = 5$ the duopoly will be the unique USPE.*
*If $n = 7$ or $n = 6$ the three-firm oligopoly and the duopoly will be USPE.*
*For $n = 2, 3, 4$ monopoly is the unique USPE.*

**Proof:** See the Appendix. ■

The proposition shows that especially merged SPE in form of small oligopolies are merged USPE. Due to the competition effect these oligopolies are the most attractive ones for the owners. The largest market form that can be sustained as an USPE is a three-firm oligopoly. However, for $n = 9$ a three-firm oligopoly is the unique type of merged USPE because it is the only SPE type. For $n = 7$ and $n = 6$ initial firms, both buyers and sellers earn more in the duopoly case than in the case of a three-firm oligopoly. But in the latter one there are also owners that have neither bought nor sold any firm. These owners free-ride on the merging activities of the other firms and have very high payoffs which are higher than the payoffs of certain buyers in the duopoly case.

# 5 Conclusion

Many oligopolistic markets can be best described by a contest model. This holds for oligopolies in which firms mainly compete by promotion expenditures, for R & D races, for new markets with network externalities, for

[18]Note that contrary to Gonzalez-Maestre and Lopez-Cunat (2001) our definition of USPE is based on weak dominance.

recently deregulated markets, and other market situations that are highly competitive. Our results show that strategic delegation in oligopolistic contests leads to different outcomes than delegation in Cournot or Bertrand competition. Especially, although the market structure is completely symmetric there are also asymmetric equilibria in the contest model where one owner uses a very aggressive incentive scheme for his manager to preempt all the other firms.

In a next step, we allow firms to limit competition by merging prior to the contest. The results for the merged subgame perfect equilibria show that there is strictly more merging under contest than under Cournot competition. This can be explained by the fact that contests characterize a very strong form of oligopolistic competition. Hence, firms will also have strong incentives to limit competition by merging.

The comparison of our results with the findings of previous contest models with delegation show that the Fershtman-Judd-Sklivas incentive scheme used in our model leads to opposite results compared to alternative compensation forms which are used in the previous contest models. For example, we find a prisoner's dilemma-like situation where each owner prefers delegation to non-delegation irrespective of the decisions of the other owners (given that employing a manager is not too costly), but all owners together are worse off in case of delegation compared to non-delegation. However, in other contest models with delegation there is just the reverse kind of prisoner's dilemma situation.

**Appendix**

*Proof of Proposition 1*:

To show that Proposition 1 indeed describes an equilibrium, we first check that for a manager choosing a strictly positive $\mu_i$ (and thus being an element of $H$), (7) satisfies condition (5) given that all other managers $j \in H\backslash\{i\}$ who choose strictly positive values of $\mu_j$ do this according to (7). Inserting (7) yields

$$S\frac{\sum_{k\in H\backslash\{i\}}\left((m-1)\,S\frac{\sum_{j\in H\backslash\{k\}}\alpha_j-(m-2)\,\alpha_k}{\left(\sum_{j\in H}\alpha_j\right)^2}\right)}{\left(\sum_{k\varepsilon H}\left((m-1)\,S\frac{\sum_{j\in H\backslash\{k\}}\alpha_j-(m-2)\,\alpha_k}{\left(\sum_{j\in H}\alpha_j\right)^2}\right)\right)^2}-\alpha_i=0 \qquad (18)$$

But this can be rearranged:

$$\begin{aligned}
&\sum_{k\in H\backslash\{i\}}\left(\frac{\left(\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\,\alpha_k}{\left(\sum_{j\in H}\alpha_j\right)^2}\right)\\
&=\ \alpha_i\,(m-1)\left(\sum_{k\in H}\left(\frac{\left(\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\,\alpha_k}{\left(\sum_{j\in H}\alpha_j\right)^2}\right)\right)^2 \Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
&\left(\sum_{j\in H}\alpha_j\right)^2\sum_{k\in H\backslash\{i\}}\left(\left(\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\,\alpha_k\right)\\
&=\ \alpha_i\,(m-1)\left(\sum_{k\in H}\left(\left(\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\,\alpha_k\right)\right)^2 \Leftrightarrow
\end{aligned}$$

$$\left(\sum_{j\in H}\alpha_j\right)^2\left(\left(\sum_{k\in H\backslash\{i\}}\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\left(\sum_{k\in H\backslash\{i\}}\alpha_k\right)\right)$$
$$= \alpha_i(m-1)\left(\left(\sum_{k\in H}\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\left(\sum_{k\in H}\alpha_k\right)\right)^2 \Leftrightarrow$$

$$\left(\sum_{j\in H}\alpha_j\right)^2\left(\left(\sum_{k\in H}\sum_{j\in H\backslash\{k\}}\alpha_j\right)-\left(\sum_{j\epsilon H\backslash\{i\}}\alpha_j\right)-(m-2)\left(\sum_{k\in H\backslash\{i\}}\alpha_k\right)\right)$$
$$= \alpha_i(m-1)\left(\left(\sum_{k\in H}\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\left(\sum_{k\in H}\alpha_k\right)\right)^2 \Leftrightarrow$$

$$\left(\sum_{j\in H}\alpha_j\right)^2\left(\left(\sum_{k\in H}\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-1)\left(\sum_{k\in H\backslash\{i\}}\alpha_k\right)\right)$$
$$= \alpha_i(m-1)\left(\left(\sum_{k\in H}\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\left(\sum_{k\in H}\alpha_k\right)\right)^2$$

and we obtain:

$$\left(\sum_{j\varepsilon H}\alpha_j\right)^2\left(\left(\sum_{k\varepsilon H}\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-1)\left(\sum_{k\in H\backslash\{i\}}\alpha_k\right)\right)$$
$$= \alpha_i(m-1)\left(\left(\sum_{k\varepsilon H}\sum_{j\in H\backslash\{k\}}\alpha_j\right)-(m-2)\left(\sum_{k\varepsilon H}\alpha_k\right)\right)^2.$$

Note that

$$\begin{aligned}\left(\sum_{k\varepsilon H}\sum_{j\in H\backslash\{k\}}\alpha_j\right) &= \alpha_2+\alpha_3+\alpha_4+....+\alpha_m \\ &\quad +\alpha_1+\alpha_3+\alpha_4+..+\alpha_m \\ &\quad +\alpha_1+\alpha_2+\alpha_4+...+\alpha_m \\ &\quad +... \\ &\quad +\alpha_1+\alpha_2+\alpha_3+..+\alpha_{m-1} \\ &= (m-1)\left(\sum_{j\varepsilon H}\alpha_j\right).\end{aligned}$$

Using this we get

$$\left(\sum_{j\varepsilon H}\alpha_j\right)^2\left((m-1)\sum_{j\varepsilon H}\alpha_j-(m-1)\sum_{k\in H\backslash\{i\}}\alpha_k\right)$$
$$=\alpha_i\,(m-1)\left((m-1)\sum_{j\varepsilon H}\alpha_j-(m-2)\sum_{j\varepsilon H}\alpha_j\right)^2,$$

which by rearranging yields that finally (18) is equivalent to

$$(m-1)\sum_{j\varepsilon H}\alpha_j-(m-1)\sum_{k\in H\backslash\{i\}}\alpha_k=\alpha_i\,(m-1)$$

which is clearly always the case. To obtain the condition for positive resource expenditures just check that for $\mu_i>0$ we must have

$$(m-1)\,S\frac{\sum_{j\in H\backslash\{i\}}\alpha_j-(m-2)\,\alpha_i}{\left(\sum_{j\varepsilon H}\alpha_j\right)^2}>0 \qquad (19)$$

and this is equivalent to the given condition $\sum_{j\neq i}\alpha_j>(m-2)\,\alpha_i$. Now consider a different manager $k$ with $\alpha_k<\alpha_i$. For this manager the left-hand side is larger (as $\alpha_k$ in the sum is replaced by the larger $\alpha_i$) and the right-hand side smaller. Hence, manager $k$ will also choose a strictly positive value

$\mu_k$. To see that in equilibrium at least the two managers with the two lowest $\alpha_i$ will spend positive amounts, simply note that if the manager with the second lowest $\alpha_i$ does not choose a positive $\mu_i$ the manager with the lowest $\alpha_i$ can always reduce his own resource expenditure and be better off. Hence, there is no equilibrium with only one manager spending a positive amount. There is also no equilibrium where $\mu_i = 0$, $\forall i$, as in that case one manager can deviate and gain $S$ with an arbitrarily small value of $\mu_i$.

*Proof of Proposition 3*:

Owner $i$'s profits are given by

$$\Pi_i = S\frac{\mu_i}{\sum_j \mu_j} - \mu_i = S\frac{\mu_i}{(n-1)\mu + \mu_i} - \mu_i. \tag{20}$$

By inserting (9) and (10) we get

$$S\frac{S(n-1)(\alpha(n-1) - \alpha_i(n-2))}{S(n-1)(\alpha(n-1) - \alpha_i(n-2)) + (n-1)\alpha_i S(n-1)} - \frac{S(n-1)(\alpha(n-1) - \alpha_i(n-2))}{(\alpha_i + \alpha(n-1))^2}. \tag{21}$$

This can be simplified to obtain

$$\frac{S((\alpha-1)(n-1) + \alpha_i)(\alpha(n-1) - \alpha_i(n-2))}{(\alpha_i + \alpha(n-1))^2} \tag{22}$$

This function is continuous for strictly positive values of $\alpha$ and $\alpha_i$. Taking the first derivative and solving for $\alpha_i$ yields a unique candidate for an internal global maximum:

$$\alpha_i = \frac{(n-1)\alpha(n-(n-1)\alpha)}{\alpha(n-1) + n - 2}. \tag{23}$$

It can easily be checked that the first derivative of (22) is strictly positive for $\alpha_i$ smaller than (23) and strictly negative for larger $\alpha_i$. Hence, it must be a global maximum. We still have to check that owner $i$ has no incentive to deviate and choose $\alpha_i = 0$ as in this case the profits are no longer given by (22). From Proposition 2 we know that in this case there is a continuum of

Nash equilibria in the following contest subgame such that $\mu_i$ is sufficiently large. To sustain the symmetric equilibrium we consider here, we simply have to pick a subgame perfect continuation on the contest stage with $\mu_i$ large enough to make this deviation unprofitable for owner $i$.

Finally, to obtain the symmetric equilibrium we must have that $\alpha_i = \alpha$ which gives us

$$\alpha = \frac{n^2 - 2n + 2}{n(n-1)}.$$

The resource expenditures in equilibrium can be computed by inserting this into (9):

$$\mu = \frac{(n-1)S((n-1)\alpha - (n-2)\alpha)}{((n-1)\alpha + \alpha)^2} = \frac{(n-1)^2}{n(n^2 - 2n + 2)}S.$$

By inserting $\mu$ into (20) we finally obtain the owners' profits

$$\Pi_i = S\frac{\mu}{n\mu} - \mu = \frac{S}{n(n^2 - 2n + 2)}.$$

*Proof of Proposition 4*:

From Proposition 2 we know that indeed no manager has an incentive to deviate on stage 2. On stage 1, first check that an owner $k$ whose manager will spend no resources at all on stage 2 cannot gain by choosing a different value for $\alpha_k$. Such an owner is indifferent between all strictly positive values for $\alpha_k$ as his firm is always driven out of the market. Furthermore, choosing $\alpha_k = 0$ would lead to infinite losses (see Proposition 2).

It remains to check whether owner $i$ can profitably deviate by choosing a positive value for $\alpha_i$. With $\alpha_i = 0$ owner $i$ receives $\Pi_i = S - \mu_i$. As we only want to show the existence of preemptive equilibria, it suffices to give an example for the strategies of all other owners such that indeed preemption by choosing $\alpha_i = 0$ is the best response of owner $i$. For simplicity we look for an equilibrium in which all other owners $k \neq i$ behave symmetrically and choose a given value $\alpha_k > 0$. As we have seen in Proposition 2 there will be a continuum of Nash equilibria on the second stage if $\alpha_i = 0$. A preemptive equilibrium should be easiest to sustain when the Nash equilibrium played on the contest stage is the best possible for the preempting owner. That is the case when $\mu_i = \frac{S}{\alpha_k}$. Owner $i$'s profits are given by $S - \mu_i = S\left(1 - \frac{1}{\alpha_k}\right)$ in that case. If we are able to find a strictly positive value for $\alpha_k$ such that owner $i$ has no incentive to deviate by choosing a strictly positive value for $\alpha_i$, we will have a subgame perfect equilibrium. When choosing a positive value for $\alpha_i$ we know from Eq. (23) in the proof of Proposition 3 that owner $i$'s best internal response to $\alpha_k$ is given by

$$\alpha_i = \frac{(n-1)\,\alpha_k\,(n-(n-1)\,\alpha_k)}{\alpha_k\,(n-1)+n-2}.$$

Owner $i$'s profits in that case can be computed by inserting this value into Eq. (22) and simplifying which yields

$$\frac{1}{4}S\frac{(\alpha_k\,(n-1)-n+2)^2}{\alpha_k\,(n-1)}.$$

We have to check, whether there exists a strictly positive value for $\alpha_k$ such that this expression is not greater than the highest profits with preemption, $S\left(1 - \frac{1}{\alpha_k}\right)$, or

$$\begin{aligned} \frac{1}{4}S\frac{(\alpha_k\,(n-1)-n+2)^2}{\alpha_k\,(n-1)} &\leq S\left(1-\frac{1}{\alpha_k}\right) \Leftrightarrow \\ (-n+\alpha_k n-\alpha_k)^2 &\leq 0. \end{aligned}$$

This is true if and only if the term in brackets has value 0, which is equivalent to $\alpha_k = \frac{n}{n-1}$. Hence, we have indeed found a value for $\alpha_k$ such that preemption can be sustained in equilibrium.

*Proof of Proposition 5*:

(i) First, we consider regime $r = D$. Substituting for $\Pi_D(n)$ according to

(13) in (16) gives

$$\frac{S}{((n-t)^2-2(n-t)+2)(n-t)}-(t+1)\frac{S}{(n^2-2n+2)n} > 0 \Leftrightarrow \quad (24)$$

$$-St\frac{n^3-3tn^2+3nt^2-t^3-5n^2+7nt-3t^2+6n-4t-2}{((t-(n-1))^2+1)(n-t)(n^2-2n+2)n} > 0.$$

Since the denominator is positive, the inequality can be simplified to

$$\Lambda(t) := n^3-3tn^2+3nt^2-t^3-5n^2+7nt-3t^2+6n-4t-2<0.$$

Note that the polynomial $\Lambda(t)$ has exactly one real-valued root:

$$\hat{t}_D = \frac{1}{6}\Omega(n) - 6\frac{\frac{1}{3}-\frac{1}{3}n}{\Omega(n)}+n-1 \quad \text{with}$$

$$\Omega(n) = \sqrt[3]{\left(-108n^2+108n+12\sqrt{(12-36n+117n^2-174n^3+81n^4)}\right)}.$$

We have $\Lambda(t)>0$ for $t<\hat{t}_D$, and $\Lambda(t)<0$ for $t>\hat{t}_D$. It can also be checked that $0<\hat{t}_D<1$ for $n=5$, and $1<\hat{t}_D<2$ for $n=6$. The higher $n$ the larger will be the root $\hat{t}_D$. Altogether, for $n\leq 5$ we have $\Lambda(t)<0$, $\forall t\geq 1$, i.e. merging is always profitable for an owner no matter how much firms are taken over. If, on the other hand, $n>5$ then merging with $t<\hat{t}_D$ other firms will not be profitable whereas an owner will gain from merging with $t>\hat{t}_D$ other firms.

(ii) Now, we consider regime $r=ND$. From condition (16) together with (15) we obtain:

$$\frac{S}{(n-t)^2} > (t+1)\frac{S}{n^2} \Leftrightarrow -n^2+2nt-t^2+2n-t>0 \quad (25)$$

The graph of the left-hand side is a parabola open to the bottom. Hence, it is positive for values between both roots. Thus, the inequality holds for

$$n-\frac{1}{2}-\frac{1}{2}\sqrt{4n+1}<t<n-\frac{1}{2}+\frac{1}{2}\sqrt{4n+1}.$$

Check that the upper root is larger than $n$, and the lower root is smaller than $n$, strictly increasing in $n$ and larger than 1 if and only if $n\geq 2+\sqrt{2}$ (i.e.

$n \geq 4$). Hence, for $n \leq 3$ all mergers are profitable, whereas for $n > 3$ there is a cut-off value $\hat{t}_{ND}$ such that at least $\hat{t}_{ND}$ firms have to be acquired for a profitable merger.

To show that there is more merging with delegation we have to check that (25) implies (24). Replacing $k = n - t$ (25) is equivalent to

$$\frac{S}{k^2} > (n-k+1)\frac{S}{n^2} \Leftrightarrow \frac{S}{k\,(k^2-2k+2)} > \frac{k}{n}(n-k+1)\frac{S}{n\,(k^2-2k+2)}.$$

Hence, we have to show that

$$\begin{aligned} & \frac{k}{n}(n-k+1)\frac{S}{n\,(k^2-2k+2)} \geq (n-k+1)\frac{S}{(n^2-2n+2)\,n} \\ \Leftrightarrow\ & kn\,(n-k) \geq 2\,(n-k)\,. \end{aligned}$$

But the latter is true for $n \geq 2$.

(iii) We will show that monopolization is indeed in both cases the most profitable strategy. We start by examining regime $r = D$. Monopolization will yield higher profits than buying less firms such that $k > 1$ firms remain active if revenue $S$ in case of monopolization less the price of buying $n-1$ firms exceeds the revenue with $k$ active firms less the price of buying $n-k$ firms:

$$\begin{aligned} S-(n-1)\frac{S}{(n^2-2n+2)\,n} &> \frac{S}{(k^2-2k+2)\,k}-(n-k)\frac{S}{(n^2-2n+2)\,n} \\ &\Leftrightarrow\ 1 > \frac{1}{\left((k-1)^2+1\right)k}+(k-1)\frac{1}{\left((n-1)^2+1\right)n} \end{aligned}$$

We know that for $k > 1$ indeed

$$\begin{aligned} 1 &> \frac{k}{\left((k-1)^2+1\right)k} = (k-1)\frac{1}{\left((k-1)^2+1\right)k}+\frac{1}{\left((k-1)^2+1\right)k} \\ &> (k-1)\frac{1}{\left((n-1)^2+1\right)n}+\frac{1}{\left((k-1)^2+1\right)k}. \end{aligned}$$

Hence, monopolization is always the most profitable merger strategy. The proof for $r = ND$ proceeds analogously. Monopolization will yield the highest profits if

$$S-(n-1)\frac{S}{n^2} > \frac{S}{k^2}-(n-k)\frac{S}{n^2} \Leftrightarrow 1 > \frac{1}{k^2}+(k-1)\frac{1}{n^2}.$$

We know that for $k > 1$ indeed

$$1 > \frac{1}{k} = \frac{1}{k^2} + (k-1)\,\frac{1}{k^2} > \frac{1}{k^2} + (k-1)\,\frac{1}{n^2}.$$

*Proof of Proposition 6*:

(ii) The proof starts with the simpler case of regime $r = ND$. Using Eq. (15) the necessary condition (17) can be written as:

$$\begin{aligned}
& \frac{S}{m^2} - (q-1)\frac{S}{(m+1)^2} \geq \frac{S}{(m+q-1)^2} \\
\Leftrightarrow \quad & -S\frac{(q-1)\left(m^2q^2 - q - 3m^2q + 2m^3q - 2mq - 2m^2 + m^4 + 1 - 4m^3\right)}{m^2\left(m+1\right)^2\left(m+q-1\right)^2} \geq 0 \\
\Leftrightarrow \quad & \Psi(q) := m^2q^2 - q - 3m^2q + 2m^3q - 2mq - 2m^2 + m^4 + 1 - 4m^3 \leq 0. \quad (26)
\end{aligned}$$

First, we consider the necessary condition (26) for the monopoly case $m = 1$ which gives $q^2 - 4q - 4 \leq 0$. This inequality is met by integers $q \leq 4$. Next, note that

$$\begin{aligned}
\frac{d\Psi(q)}{dq} &= 2m^2q - 1 - 3m^2 + 2m^3 - 2m \\
&= (2q-3)m^2 + (m^2-1)2m - 1 > 0 \quad \text{for all } q \geq 2 \text{ and } m > 1.
\end{aligned}$$

Hence, if (26) does not hold for $q = 3$ it will also not hold for any $q > 3$. Inserting $q = 3$ into (26) yields $m^4 + 2m^3 - 2m^2 - 6m - 2 \leq 0$, which does not hold for integers $m > 1$. Inserting $q = 2$ into (26) gives $m^4 - 4m^2 - 4m - 1 \leq 0$, which only holds for $m \leq 2$. Altogether, we have two candidates for a merged SPE: a duopoly with $m = 2$ and $q \leq 2$, and a monopoly with $m = 1$ and $q \leq 4$. Note that there can be no merged SPE in a market with more than 4 initial firms.

In the next step, we have to check whether there exist equilibrium bids of the owners on the first stage of the game that make the derived duopoly and monopoly an outcome of a subgame perfect equilibrium. Here, we follow the formal structure of Gonzalez-Maestre and Lopez-Cunat (2001), pp. 1276-1277, without repeating all the details. But the main idea is the following: After the merging stage, there are three subsets of owners – $N_0$ consisting of those owners who have zero firms (the sellers), $N_1$ consisting of those owners who possess exactly one firm (owners that have not participated in merging), $N_2$ consisting of those owners who have at least two firms (the buyers). In the bidding process of stage 1, a $N_0$-owner demands a price equal to $\Pi_r(m+1)$ for his firm and makes negative bids for all other firms, a $N_1$-owner demands a price equal to $\infty$ for his firm and makes negative bids for all the other firms, and a $N_2$-owner demands a price equal to $\infty$ for his firm, offers a price equal to $\Pi_r(m+1)$ to the firms he want to buy and makes negative bids for the firms he does not want to buy. Most of the arguments given by Gonzalez-Maestre and Lopez-Cunat are independent of the type of competition that follows on the third stage and, therefore, also hold for our contest model. But we have to check whether a $N_2$-owner wants to buy less than the predicted number of $q-1$ firms.[19]

Now, we come back to our two candidate equilibria above. In the monopoly case ($m = 1$), an owner possesses $q = n \leq 4$ firms ex post. We have to check whether the monopolist has an incentive to deviate and buy less than $q-1$ firms. For $n = 4$, the owner's payoffs in case of monopolization are $S - 3\Pi_{ND}(2) \overset{(15)}{=} S/4$. He will gain from buying two firms instead of three if $\Pi_{ND}(2) - 2\Pi_{ND}(2) > S/4$, which is not true; he will buy one instead of three firms if $\Pi_{ND}(3) - \Pi_{ND}(2) = (S/9) - (S/4) > S/4$, which is not true either. For $n = 3$, the owner's payoffs in case of monopolization are $S - 2\Pi_{ND}(2) = S/2$. Again, he will not deviate and buy only one instead two firms, because then his profits will be $\Pi_{ND}(2) - \Pi_{ND}(2) = 0$. Note that buying zero instead of $q-1$ firms is already precluded by the necessary condition. This also applies

[19] This part of the proof in Gonzalez-Maestre and Lopez-Cunat (2001), p. 1277, directly refers to the Cournot model and, therefore, does not apply for our contest model.

to the duopoly case ($m = 2$) with $q = 2$ so that there is no deviating by buying zero instead of one firm. To summarize, the two candidate market structures do indeed appear as subgame perfect equilibria

(i) Next, regime $r = D$ is considered. Inserting $\Pi_D(\cdot)$ according to Eq. (13) and, for brevity, using the number of bought firms $t = q - 1$ instead of $q$ the necessary condition (17) can be written as:

$$\begin{aligned}\frac{S}{m\,(m^2-2m+2)} - \frac{tS}{(m+1)\,((m+1)^2-2(m+1)+2)} &\geq \\ \frac{S}{(m+t)\,((m+t)^2-2(m+t)+2)} &\Leftrightarrow\end{aligned}$$

$$\begin{aligned}&-St\frac{(m^3-2m^2+2m)t^3+(9m^2-1-9m^3-5m+3m^4)t^2}{m\,(m^2-2m+2)\,(m+1)\,(m^2+1)\,(m+t)\left((m-1)^2+(t-1)^2+2mt\right)}\\ &-St\frac{(2-13m^4+15m^3-13m^2+3m+3m^5)t}{m\,(m^2-2m+2)\,(m+1)\,(m^2+1)\,(m+t)\left((m-1)^2+(t-1)^2+2mt\right)}\\ &-St\frac{2m-2-9m^3+3m^2-7m^5+9m^4+m^6}{m\,(m^2-2m+2)\,(m+1)\,(m^2+1)\,(m+t)\left((m-1)^2+(t-1)^2+2mt\right)} \geq 0 \Leftrightarrow\end{aligned}$$

$$\begin{aligned}(m^3-2m^2+2m)t^3+(9m^2-1-9m^3-5m+3m^4)t^2&\\ +(2-13m^4+15m^3-13m^2+3m+3m^5)t&\\ -2+2m-9m^3+3m^2-7m^5+9m^4+m^6 &\leq 0.\end{aligned} \tag{27}$$

First, we check for a monopoly described by $m = 1$ and $t = n - 1$. Inserting into (27) gives $n^3 - 6n^2 + 6n - 4 \leq 0$. It can easily be checked that this inequality only holds for integers $n \leq 4$. Hence, we have the same candidate outcome as in the non-delegation case above. Again, we have to check whether the monopolist has an incentive to buy less than $n - 1$ firms. For $n = 4$, the monopolist's payoffs are $S - 3\Pi_D(2) \overset{(13)}{=} S/4$. He will gain from buying two instead of three firms if $\Pi_D(2) - 2\Pi_D(2) > S/4$, which is not true; he will buy one instead of three firms if $\Pi_D(3) - \Pi_D(2) = (S/15) - (S/4) > S/4$, which is not true either. For $n = 3$, monopolizing yields $S - 2\Pi_D(2) = S/2$.

If the owner buys only one firm his profits will be $\Pi_D(2) - \Pi_D(2) = 0 < S/2$. Altogether, we have the same merged SPE as under regime $r = ND$.

It remains to look for further equilibria by checking condition (27) for $m > 1$. If $m = 2$, condition (27) will simplify to $-74 - 36t + t^2 + 4t^3 \leq 0$ which holds for integers $t \leq 3$. For $m = 3$, (27) can be rewritten as $-455 - 25t + 65t^2 + 15t^3 \leq 0$ which is met by integers $t \leq 2$. If $m = 4$, condition (27) will be $-1290 + 510t + 315t^2 + 40t^3 \leq 0$ which only holds for the integer $t = 1$. For all other integers $m \geq 5$ condition (27) cannot hold. Hence, besides the monopoly we have three further candidate equilibria that meet the necessary condition for a merged SPE:

$$\begin{aligned} m = 2 \quad &\text{with} \quad t \leq 3 \text{ and therefore } 4 \leq n \leq 8, \\ m = 3 \quad &\text{with} \quad t \leq 2 \text{ and therefore } 6 \leq n \leq 9, \\ m = 4 \quad &\text{with} \quad \ \ t = 1 \text{ and therefore } n = 8. \end{aligned} \tag{28}$$

For any of these cases to be a merged SPE $N_2$-owners must not be interested to buy $\tau < t$ instead of $t$ firms. This condition is met if

$$\Pi_D(m + t - \tau) - \tau\Pi_D(m + 1) - [\Pi_D(m) - t\Pi_D(m + 1)] \leq 0 \Leftrightarrow$$
$$\Delta(m, t, \tau) := \Pi_D(m + t - \tau) - \Pi_D(m) + (t - \tau)\Pi_D(m + 1) \leq 0.$$

Inserting for all the relevant cases given by (28) and computing the values for $\Delta(m, t, \tau)$ by using (13) yields: $\Delta(2, 3, 2) = -\frac{7}{60}S$, $\Delta(2, 3, 1) = -\frac{11}{120}S$, $\Delta(2, 2, 1) = -\frac{7}{60}S$, and $\Delta(3, 2, 1) = -\frac{1}{60}S$. Again, buying zero instead of $t$ firms is already precluded by the necessary condition. To sum up, the candidate equilibria described by (28) are indeed merged SPE.

(iii) The proofs for (i) and (ii) have shown that for $n = 4$ monopolist's payoffs will be $S/4$ ex post, and for $n = 3$ they will be $S/2$ under either regime. For $n = 2$ and $r = D$, we have $S - \Pi_D(2) \overset{(13)}{=} \frac{3}{4}S$. For $n = 2$ and $r = ND$, we also obtain $\frac{3}{4}S$.

*Proof of Proposition 7*:

For $n = 9$, the three-firm oligopoly is the only merged SPE.

For $n = 8$, we have three candidate USPE – the duopoly, the three-firm oligopoly, and the four-firm oligopoly. Let $u_{+t}$ denote the payoffs of an owner that has bought $t$ other firms, $u_-$ the payoffs of an owner who has sold his firm, and $u_0$ the payoffs of an owner that is neither a buyer nor a seller. For the duopoly case and $n = 8$, we only have buyers and sellers. Their respective payoffs are $u_{+3} = \Pi_D(2) - 3\Pi_D(3) = 0.05S$ and $u_- = \Pi_D(3) = 0.066667S$. In the case of a three-firm oligopoly and $n = 8$, two owners have bought two other firms and one owner has bought one other firm. The respective payoffs are $u_{+2} = \Pi_D(3) - 2\Pi_D(4) = 0.016667S$, $u_{+1} = \Pi_D(3) - \Pi_D(4) = 0.041667S$, and $u_- = \Pi_D(4) = 0.025S$. In the case of a four-firm oligopoly and $n = 8$, each owner has bought one other firm: $u_+ = \Pi_D(4) - \Pi_D(5) = 0.013235S$ and $u_- = \Pi_D(5) = 0.011765S$. The comparison shows that the duopoly is the unique USPE.

Now, we consider the situation with $n = 7$ initial firms. The duopoly and the three-firm oligopoly are the only candidate USPE. In the duopoly case, five firms have been acquired; hence, one duopolist has bought three firms ($u_{+3} = 0.05S$) and the other one two firms ($u_{+2} = \Pi_D(2) - 2\Pi_D(3) = 0.11667S$). The payoffs of the selling owner are again $u_- = \Pi_D(3) = 0.066667S$. In the case of a three-firm oligopoly, four firms have been acquired – either two owners have bought two firms and the third owner zero firms, or two owners have bought one firm and one owner two firms. The respective payoffs are $u_{+2} = 0.016667S$, $u_{+1} = 0.041667S$, $u_0 = 0.066667S$, $u_- = 0.025S$. Comparing these values (especially, $u_{+3} = 0.05S$ and $u_0 = 0.066667S$) shows that both candidates are merged USPE.

For $n = 6$, again the two possible merged SPE are a duopoly and a three-firm oligopoly. In the first case, both owners have bought two other firms, or one owner has bought three firms and the other owner only one firm. In case of a three-firm oligopoly, each owner has acquired one firm, or one owner has bought two, the next owner one and the last owner zero firms. Similar to the case of seven initial firms both merged SPE are USPE.

For $n = 5$, the duopoly is the only merged SPE and, therefore, the unique merged USPE.

For $n = 4$, now a monopoly is also a candidate USPE. The respective payoffs are $u_{+3} = S - 3\Pi_D(2) = 0.25S$ and $u_- = \Pi_D(2) = 0.25S$. The only alternative equilibrium outcome is a duopoly, where one owner has acquired two and the other owner zero firms, or both owners have bought one firm. The respective payoffs show that the duopoly is (weakly) dominated by the monopoly.

For $n = 3$ and $n = 2$, the monopoly is the only equilibrium outcome and, hence, the unique merged USPE.

## References


Bagwell, K., 1995, "Commitment and Observability in Games", *Games and Economic Behavior*, 8, 271–280.

Baik, K.H. and I.-G. Kim, 1997, "Delegation in Contests", *European Journal of Political Economy*, 13, 281–298.

Besen, S.M. and J. Farrell, (1994, "Choosing How to Compete: Strategies and Tactics in Standardization", *Journal of Economic Perspectives*, 8:2, 117–131.

Bonanno, G., 1992, "Deterrence, Observability and Awareness", *Economic Notes*, 21, 307–315.

Fama, E.F. and M.C. Jensen, 1983, "Separation of Ownership and Control", *Journal of Law and Economics*, 26, 301–325.

Fershtman, C., 1985, "Managerial Incentives as a Strategic Variable in Duopolistic Environment", *International Journal of Industrial Organization*, 3, 245–253.

Fershtman, C. and K.L. Judd, 1987, "Equilibrium Incentives in Oligopoly", *American Economic Review*, 77, 927–940.

Fershtman, C. and E. Kalai, 1997, "Unobserved Delegation", *International Economic Review*, 38, 763–774.

Gonzalez-Maestre, M. and J. Lopez-Cunat, 2001, "Delegation and Mergers in Oligopoly", *International Journal of Industrial Organization*, 19, 1263–1279.

Hillman, A.L. and J.G. Riley, 1989, "Politically Contestable Rents and Transfers", *Economics and Politics*, 1, 17–39.

Huck, S., Konrad, K.A. and W. Müller, 2001, "Merger and Collusion in Contests", Discussion Paper FS IV 01-04, *Wissenschaftszentrum Berlin.*

Jensen, M.C. and W.H. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure", *Journal of Financial Economics,* 3, 305−360.

Kamien, M.I. and I. Zang, 1990, "The Limits of Monopolization Through Acquisition", *Quarterly Journal of Economics*, 105, 465−499.

Katz, M.L., 1991, "Game-Playing Agents: Unobservable Contracts as Precommitments", *Rand Journal of Economics*, 22, 307−328.

Konrad, K.A., Peters, W. and K. Wärneryd, 2000, Delegation in First-Price All-Pay Auctions, *SSE/EFI Working Paper Series in Economics and Finance*, no. 316, Stockholm School of Economics.

Loury, G.C., 1979, "Market Structure and Innovation", *Quarterly Journal of Economics*, 94, 395−410.

Scherer, F.M., 2000, "The Pharmaceutical Industry", in Culyer, A.J. and Newhouse, J.P. (Eds.): *Handbook of Health Economics*, Vol. 1B, 1297−1336.

Schmalensee, R., 1976, "A Model of Promotional Competition in Oligopoly", *Review of Economic Studies*, 43, 493−507.

Schmalensee, R., 1992, "Sunk Costs and Market Structure: A Review Article", *Journal of Industrial Economics*, 40, 125−134.

Schoonbeek, L., 2000, "A Delegated Agent in a Winner-Take-All Contest", University of Groningen, mimeo.

Sklivas, S.D., 1987, "The Strategic Choice of Managerial Incentives", *Rand Journal of Economics*, 18, 452−458.

Vickers, J., 1985, "Delegation and the Theory of the Firm", *Economic Journal, Supplement*, 95, 138−147.

Wärneryd, K., 2000, "In Defense of Lawyers: Moral Hazard as an Aid to Cooperation", *Games and Economic Behavior*, 33, 145−158.

Williamson, O.E., 1964, *The Economics of Discretionary Behavior: Managerial Objectives in a Theory of the Firm*, London.

Ziss, S., 2001, "Horizontal Mergers and Delegation", *International Journal of Industrial Organization*, 19, 471−492.