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Optimal Trading Mechanisms for an Informed Seller

by

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Optimal Trading Mechanisms for an Informed Seller

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Abstract

We consider the situation where the owner of some good wants to sell the good to one of several potential buyers. We assume that the owner possesses private information about the buyers' valuations of the good, and analyze this model as an informed principal mechanism design model. In an undominated perfect Bayesian equilibrium of the model the seller gives the object to the person who values the object most, and receives a transfer payment from each potential buyer such that all ex-ante expected rents are extracted from the buyers.

JEL-Classification: C72, D82

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1 Introduction

Consider the situation where the owner of some good is to design some mechanism for selling the good to one of several potential buyers. Assuming that the potential buyers have private information about their valuations of the good this leads us to the problem of optimal auction design. Though optimal auction design has received a lot of attention in the economic literature during the last two decades, the situation where the seller has some private information about the buyers' valuations of the object to be sold does not seem to have been considered so far. But under some circumstances the seller clearly does possess such private information.

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For example, consider the following variation of the classic Akerlof model (c.f. Akerlof (1970)), where the owner of a used car wants to sell this car: Assume that there are two potential buyers, an academic and a mechanic. Moreover, assume that the car seems to be in good shape to both the academic and the mechanic, but there is a possibility that the car has some damage which has to be fixed soon. Whether or not the car has such a damage is private information of the owner. If the car is undamaged, then it is worth €9.000 to the mechanic and €10.000 to the academic (the academic might rely more on a car). If the car turns out to be damaged, the repair will cost the academic €3.000, whereas the mechanic can do it himself for €1.000. If the state of the car was common knowledge, the owner could sell it for either €10.000 or €8.000. But given that this state is private information of the owner, such a solution is unlikely as the owner would always have an incentive to claim that the car is undamaged. So the question arises what kind of trading mechanisms the informed owner can implement in such a situation.

If some seller possessing private information wants to find the best mechanism for selling her object, then this situation amounts to a mechanism design problem for an informed principal. Milgrom and Weber (1982) have compared ex-ante expected seller revenues for a few well-known auctions in a model where the seller has some private information which is *affiliated* with the bidders' valuations of the object. Such a comparison can be carried out without taking into account that the choice of a mechanism by the principal may reveal some of her private information. Furthermore, Milgrom and Weber assume that the informed principal does not misrepresent her private information. In our optimal mechanism design approach, on the other hand, we have to allow for the possibility that the choice of a mechanism reveals some private information as well as for the possibility that the seller may misrepresent her private information. More precisely, we shall assume that the seller is to select a trading mechanism within the following *contract proposal game*: First, the principal (seller) obtains some private information about the buyers' valuations of the object to be sold, and then selects some trading mechanism or contract (a game form). Next the agents (buyers) update their prior beliefs about the principal's type based on any information revealed by the choice of mechanism, and decide whether or not to accept this mechanism ("voluntary participation"). Finally, if the mechanism is accepted, then the implied game is played by the principal and the agents. Maskin and Tirole (1990, 1992) present some non-cooperative solution concepts for general contract proposal games.[1] We shall use for our analysis an extension of the latter model to an arbitrary number of agents and to a

[1]The first to analyze an informed principal mechanism design model was Myerson (1983), but most of his solution concepts incorporate ideas of cooperative game theory.

more general information structure provided by Tisljar (2002).[2]
The paper is organized as follows: In Section 2 we present the common-value mechanism design model for an informed seller to be analyzed. Moreover, we review the solution concept to be utilized for this model. In Section 3 we solve our mechanism design model, and Section 4 concludes with a short summary and with some comments on the connection of our optimal trading design model to optimal auction design models.

2 The Model

Suppose the owner of an object – the seller or principal (indexed by $i = 0$) – wants to sell this object to one of n potential buyers (the agents, indexed by $i = 1, \ldots, n$). The monetary value of the object for agent i is given by $v_i \in \mathbb{R}$, and the monetary value of the object for the seller is $v_0 \in \mathbb{R}$. We assume that $v_i = v_i(t)$ depends on some $t \in T$ for all $i = 1, \ldots, n$ (where $T \subseteq \mathbb{R}$ is some countable set),[3] and that t is private information of the seller. All agents consider t to be some random variable with prior probabilities given by $\pi^t \in [0, 1]$ ($t \in T$) such that $\sum_{t \in T} \pi^t = 1$. Though the buyers' valuations are assumed to be private information of the seller, we assume that v_0 is common knowledge.[4]
Let $\alpha_i = 1$ if the object is sold to agent i, and $\alpha_i = 0$ otherwise ($i = 1, \ldots, n$).[5] Moreover, let $p_i \in \mathbb{R}$ be the transfer paid by agent i to the seller. Then the utilities derived from $(\alpha, p) = (\alpha_1, \ldots, \alpha_n, p_1, \ldots, p_n)$ (with $\alpha_1 + \ldots + \alpha_n \leq 1$) are given by

$$\begin{aligned} u_0((\alpha, p), t) &= \sum_{j=1}^{n} p_j + \left(1 - \sum_{j=1}^{n} \alpha_j\right) v_0, && \text{and} \\ u_i((\alpha, p), t) &= -p_i + \alpha_i v_i(t) && (i = 1, \ldots, n). \end{aligned}$$

Let us assume that

$$\sup_{t \in T,\ i=1,\ldots,n} |v_i(t)| < \infty,$$

that all possible transfer payments are bounded by some arbitrary number greater than $\sup_{t \in T,\ i=1,\ldots,n} |v_i(t)| + |v_0|$, and that all players are expected utility maximizers. Furthermore, assume that the seller and the buyers play the following contract proposal game:

[2] Maskin and Tirole (1992) assume for their model that there is one agent only and that the type set of the principal is finite.

[3] Allowing $T \subseteq \mathbb{R}$ to be some arbitrary Borel set would not change our analysis, but then we would have to make use of certain measurability assumptions (see Tisljar (2002)).

[4] Although this assumption may seem rather artificial, it is not unusual in the economic literature. In standard auction design models, for example, it is usually assumed that the seller's valuation of the object to be sold is common knowledge.

[5] The restriction to $\alpha_i \in \{0, 1\}$ (rather than $\alpha_i \in [0, 1]$) is not essential, i.e. we could also consider the sale of some divisible good.

1. Nature draws the seller's type $t \in T$; t is revealed to the seller, but all buyers only know the probability measure according to which t is distributed.

2. The seller chooses some game form (a contract) specifying some action spaces for the seller and for all buyers, and for each combination of actions some (α, p), i.e. some transfer payments to be made to the seller as well as some decision as to who is going to receive the seller's object.

3. The n buyers update their prior beliefs about the seller's type based on any information revealed by the selection of the game form, and then simultaneously decide whether or not to play the associated game.

4. If all agents have accepted, then the game associated with the game form proposed by the seller is played. Otherwise the seller keeps her object and no transfer payments are made.

Each pure-strategy equilibrium of the contract proposal game has to specify for each $t \in T$ some game form to be selected by the seller, as well as some outcome for the implied game (if the game form is accepted; otherwise the seller and the buyers settle with the status quo $(\alpha, p) = (0, 0)$). Hence, each pure-strategy equilibrium implements some allocation rule

$$(\alpha, p) : T \to \{(\alpha_1, \ldots, \alpha_n) \in \{0, 1\}^n \mid \alpha_1 + \ldots + \alpha_n \leq 1\} \times \mathbb{R}^n.$$

The following definition concerning allocation rules will turn out helpful for the characterization of equilibria of the contract proposal game:

Definition 1 An allocation rule (α, p) is

- *incentive-compatible* (IC), if for all $t, s \in T$

$$\sum_{i=1}^{n} p_i(t) + \left(1 - \sum_{i=1}^{n} \alpha_i(t)\right) v_0 \geq \sum_{i=1}^{n} p_i(s) + \left(1 - \sum_{i=1}^{n} \alpha_i(s)\right) v_0 \ ,$$

- *individual rational for the seller* (IRS), if

$$\sum_{i=1}^{n} p_i(t) + \left(1 - \sum_{i=1}^{n} \alpha_i(t)\right) v_0 \geq v_0 \quad \forall\, t \in T,$$

- *(ex-ante) individual rational for agent i* (IRi), if

$$\sum_{t \in T} \pi^t \alpha_i(t) v_i(t) \geq \sum_{t \in T} \pi^t p_i(t) \ (i = 1, \ldots, n), \text{ and}$$

- *not strictly dominated for the seller at $\tau \in T$* (NDτ), if there does not exist any incentive-compatible allocation rule $(\widetilde{\alpha}, \widetilde{p})$ such that

$$\widetilde{\alpha}_i(t) v_i(t) > \widetilde{p}_i(t) \ \forall \ t \in T, \ i \in \{1, \ldots, n\}$$

and

$$\sum_{i=1}^{n} \widetilde{p}_i(\tau) + \left(1 - \sum_{i=1}^{n} \widetilde{\alpha}_i(\tau)\right) v_0 > \sum_{i=1}^{n} p_i(\tau) + \left(1 - \sum_{i=1}^{n} \alpha_i(\tau)\right) v_0.$$

The definitions of incentive-compatibility and (ex-ante) individual rationality are standard. Strict dominance for the seller is defined in terms of the seller's utility, only; as far as the buyers are concerned, some incentive-compatible allocation rule which strictly dominates as defined above is guaranteed to be strictly preferred to the status quo, but not necessarily strictly preferred to the dominated allocation rule (for all $t \in T$).[6]
Tisljar (2002) demonstrates that all pure-strategy perfect Bayesian equilibria (PBE) of the contract proposal game implement some allocation rule which is incentive-compatible, individual rational for the seller, ex-ante individual rational for each agent, and not strictly dominated for the seller at any $t \in T$.[7] These conditions are due to a revelation and inscrutability principle for the informed principal mechanism design model. The revelation part of the theorem says that the seller can restrict herself to selecting some direct revelation mechanism, i.e. some incentive-compatible allocation rule. The inscrutability part implies that in a PBE all types of principal may select the *same* incentive-compatible allocation rule. Hence, the choice of this allocation rule does not reveal any of the principal's private information, and therefore the allocation rule has to be ex-ante individual rational for the agents to be accepted. Moreover, the allocation rule has to be individual rational and not strictly dominated for the seller at any $t \in T$, otherwise at least one type of principal would have an incentive to deviate from the equilibrium to the status quo or to some dominating allocation rule, respectively.[8]

[6] Being not strictly dominated for the seller at any $t \in T$ is closely related to being a *Rothschild-Stiglitz-Wilson* allocation rule in the model of Maskin and Tirole (1992): Both conditions ensure that the allocation rule under consideration yields the principal at least as much utility as she could achieve by selecting any other allocation rule which is guaranteed to be accepted by all agents regardless of how the agents update their prior beliefs.

[7] To derive these necessary conditions Tisljar restricts his analysis to equilibria where the mechanism selected by the principal is accepted by all agents (for all $t \in T$). Since the principal can always secure herself the status quo by selecting a mechanism which implements $\alpha = p = 0$ irrespective of the players' actions (and hence the principal need not resort to mechanisms which are rejected), the restriction to equilibria "with acceptance" is not severe.

[8] Though Tisljar (2002) proves that all pure-strategy perfect Bayesian equilibrium allocation rules ("with acceptance") have to be (IC), (IRS), (IRi) for all i, and (NDt) for

Furthermore, it is reasonable to assume that any equilibrium allocation rule is *undominated* as defined below:

Definition 2 An incentive-compatible allocation rule (α, p) is *undominated* if there does not exist any allocation rule $(\widetilde{\alpha}, \widetilde{p})$ which is (IC), (IRS), (IRi) for all $i = 1, \ldots, n$, (NDt) for all $t \in T$, and such that

$$\sum_{j=1}^{n} \widetilde{p}_j(t) + \left(1 - \sum_{j=1}^{n} \widetilde{\alpha}_j(t)\right) v_0 > \sum_{j=1}^{n} p_j(t) + \left(1 - \sum_{j=1}^{n} \alpha_j(t)\right) v_0 \ \forall\ t \in T.$$

To see why, suppose that (α, p) is some allocation rule which is expected to be implemented in an equilibrium of the contract proposal game, and assume that (α, p) is dominated by some other equilibrium candidate $(\widetilde{\alpha}, \widetilde{p})$ as defined above. Now the seller could select $(\widetilde{\alpha}, \widetilde{p})$ instead of (α, p) and point out to the agents that they should not infer anything from this deviation from equilibrium play, as *all* types of seller strictly prefer $(\widetilde{\alpha}, \widetilde{p})$ over (α, p). If the agents follow this argument and do not update their prior beliefs, then they should accept $(\widetilde{\alpha}, \widetilde{p})$ as this yields them a non-negative expected utility (since $(\widetilde{\alpha}, \widetilde{p})$ is (IRi) for all i). But then (α, p) is an unlikely outcome of the contract proposal game, because the seller can do better by announcing $(\widetilde{\alpha}, \widetilde{p})$.[9]

3 Undominated Perfect Bayesian Equilibria

In the previous section we have identified five conditions for an undominated pure-strategy perfect Bayesian equilibrium allocation rule for the contract proposal game. In this section we demonstrate that there is an essentially unique allocation rule satisfying all these conditions. To do so, we first prove the following lemma.

Lemma 1 *Each allocation rule which is incentive-compatible and ex-ante individual rational for the buyers yields the seller a utility of at most*

$$\sum_{s \in T} \pi^s \max_{j=1,\ldots,n} \{v_j(s), v_0\} = \sum_{s \in T} \pi^s \max\{v_0, \max_{j=1,\ldots,n} v_j(s)\},$$

irrespective of the seller's type.

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all t, he shows that these conditions are also sufficient for a perfect Bayesian equilibrium only for the case that mechanisms are restricted exogenously to incentive-compatible allocation rules. But if the set of feasible mechanisms is more general, then any allocation rule satisfying the necessary conditions above is implementable at least by some Bayesian equilibrium of the contract proposal game (though not necessarily by a *perfect* Bayesian equilibrium). Therefore, in the remainder of the paper we shall assume that any allocation rule satisfying (IC), (IRS), (IRi) for all i, and (NDt) for all t can be implemented by the seller.

[9] This reasoning follows an argument given in Myerson (1983).

Proof: Consider some incentive-compatible allocation rule (α, p). Then

$$\sum_{i=1}^{n} p_i(t) + \left(1 - \sum_{i=1}^{n} \alpha_i(t)\right) v_0 \geq \sum_{i=1}^{n} p_i(\tilde{t}) + \left(1 - \sum_{i=1}^{n} \alpha_i(\tilde{t})\right) v_0$$

for all $t,\ \tilde{t} \in T$, i.e. there exists some $u^* \in \mathbb{R}$ such that

$$\sum_{i=1}^{n} p_i(t) + \left(1 - \sum_{i=1}^{n} \alpha_i(t)\right) v_0 = u^*$$

for all $t \in T$.
Moreover, if (α, p) is individual rational for the agents then (α, p) satisfies

$$\sum_{t \in T} \pi^t \alpha_i(t) v_i(t) \geq \sum_{t \in T} \pi^t p_i(t)$$

for all $i = 1, \ldots, n$. Hence

$$\begin{aligned}
u^* &= \sum_{t \in T} \pi^t u^* \\
&= \sum_{t \in T} \pi^t \left\{ \sum_{i=1}^{n} p_i(t) + \left(1 - \sum_{i=1}^{n} \alpha_i(t)\right) v_0 \right\} \\
&= \sum_{i=1}^{n} \sum_{t \in T} \pi^t p_i(t) + \sum_{t \in T} \pi^t v_0 - v_0 \sum_{t \in T} \left(\pi^t \sum_{i=1}^{n} \alpha_i(t) \right) \\
&\leq \sum_{i=1}^{n} \sum_{t \in T} \pi^t \alpha_i(t) v_i(t) + \sum_{t \in T} \pi^t v_0 - v_0 \sum_{t \in T} \left(\pi^t \sum_{i=1}^{n} \alpha_i(t) \right) \\
&= \sum_{t \in T} \left(\pi^t \underbrace{\left\{ \sum_{i=1}^{n} \alpha_i(t)(v_i(t) - v_0) + v_0 \right\}}_{\leq\ \max_{i=1,\ldots,n}\{v_i(t), v_0\}} \right) \\
&\leq \sum_{t \in T} \left(\pi^t \max_{i=1,\ldots,n} \{v_i(t), v_0\} \right),
\end{aligned}$$

since $\alpha_1(t) + \ldots + \alpha_n(t) \leq 1$ for all $t \in T$. *Q.E.D.*

Lemma 1 provides some upper bound on the utility the seller can obtain in the contract proposal game. Next we shall show that this upper bound can actually be achieved by the seller. To do so, let $i^*(t) \in \{1, \ldots, n\}$ for all $t \in T$ be such that

$$v_{i^*(t)}(t) = \max_{i=1,\ldots,n} v_i(t)$$

for all $t \in T$, and consider the following allocation rule:

$$\alpha_i^*(t) := \begin{cases} 1 & : \quad i = i^*(t),\ v_i(t) \geq v_0 \\ 0 & : \quad i \neq i^*(t) \text{ or } v_i(t) < v_0 \end{cases} \qquad (t \in T,\ i = 1, \ldots, n), \quad \text{and}$$

$$p_i^*(t) := \begin{cases} \nu_i - \frac{v_0}{n}(1 - \Pi) & : \quad \sum_{j=1}^n \alpha_j^*(t) = 0 \\ \nu_i + \frac{v_0}{n}\Pi & : \quad \sum_{j=1}^n \alpha_j^*(t) = 1 \end{cases} \qquad (i = 1, \ldots, n),$$

where

$$\Pi := \sum_{t \in T,\ \sum_{j=1}^n \alpha_j^*(t) = 0} \pi^t$$

and

$$\nu_i := \sum_{s \in T} \pi^s \alpha_i^*(s) v_i(s) \qquad\qquad (i = 1, \ldots, n).$$

So (α^*, p^*) specifies that the seller's object is given to some buyer who values it most, unless the seller herself values the object even more (in which case the seller keeps the object). Each buyer has to pay an amount to the seller which only depends on whether or not the seller keeps the object herself (but not on which buyer gets the object in case the seller does not keep it). This yields the seller a utility of

$$\begin{aligned}
&\sum_{i=1}^n p_i^*(t) + \left(1 - \sum_{i=1}^n \alpha_i^*(t)\right) v_0 \\
= &\begin{cases} \sum_{i=1}^n \nu_i - v_0(1 - \Pi) + v_0 & : \quad \sum_{j=1}^n \alpha_j^*(t) = 0 \\ \sum_{i=1}^n \nu_i + v_0\Pi & : \quad \sum_{j=1}^n \alpha_j^*(t) = 1 \end{cases} \\
= &\sum_{i=1}^n \sum_{s \in T} \pi^s \alpha_i^*(s) v_i(s) + v_0 \Pi \\
= &\sum_{s \in T,\ \sum_{j=1}^n \alpha_j^*(s) = 1} \left(\pi^s \sum_{i=1}^n \alpha_i^*(s) v_i(s)\right) + \sum_{s \in T,\ \sum_{j=1}^n \alpha_j^*(s) = 0} \pi^s v_0 \\
= &\sum_{s \in T} \left(\pi^s \max_{i=1,\ldots,n} \{v_i(s), v_0\}\right)
\end{aligned}$$

for all $t \in T$, and buyer i's ex-ante expected utility from (α^*, p^*) is

$$\begin{aligned}
&\sum_{t \in T} \pi^t \left(-p_i^*(t) + \alpha_i^*(t) v_i(t)\right) \\
= &-\nu_i + \frac{v_0}{n}(1 - \Pi)\Pi - \frac{v_0}{n}\Pi(1 - \Pi) + \sum_{t \in T} \pi^t \alpha_i^*(t) v_i(t) \\
= &-\sum_{s \in T} \pi^s \alpha_i^*(s) v_i(s) + \sum_{t \in T} \pi^t \alpha_i^*(t) v_i(t) \\
= &\ 0.
\end{aligned}$$

The following theorem shows that (α^*, p^*) satisfies all necessary conditions for being a pure-strategy perfect Bayesian equilibrium allocation rule.

Theorem 1 *The allocation rule (α^*, p^*) is (IC), (IRS), (IRi) for all i, and (NDt) for all $t \in T$.*

Proof: Since (α^*, p^*) is obviously (IC), (IRS), and (IRi) for all i, it remains to show that (α^*, p^*) is not strictly dominated for the seller at any $t \in T$. Suppose to the contrary that there exists some $\tau \in T$ and some incentive-compatible allocation rule $(\widetilde{\alpha}, \widetilde{p})$ such that

$$\widetilde{\alpha}_i(t)v_i(t) > \widetilde{p}_i(t) \ \forall \ t \in T, \ i \in \{1, \dots, n\}$$

and

$$\sum_{i=1}^{n} \widetilde{p}_i(\tau) + \left(1 - \sum_{i=1}^{n} \widetilde{\alpha}_i(\tau)\right) v_0 > \sum_{s \in T} \pi^s \max_{i=1,\dots,n} \{v_i(s), v_0\}.$$

Clearly, $(\widetilde{\alpha}, \widetilde{p})$ is ex-ante individual rational for all buyers, and therefore by Lemma 1

$$\sum_{i=1}^{n} \widetilde{p}_i(\tau) + \left(1 - \sum_{i=1}^{n} \widetilde{\alpha}_i(\tau)\right) v_0 \leq \sum_{s \in T} \pi^s \max_{i=1,\dots,n} \{v_i(s), v_0\},$$

a contradiction. *Q.E.D.*

With the help of Lemma 1 it easily follows that (α^*, p^*) is the essentially unique undominated pure-strategy equilibrium allocation rule ((α^*, p^*) is unique as far as the seller's utility is concerned).

Corollary 1 *(α^*, p^*) is an undominated allocation rule. Moreover, any undominated pure-strategy perfect Bayesian equilibrium allocation rule yields the seller the same utility as (α^*, p^*).*

Proof: Since any incentive-compatible allocation rule which is ex-ante individual rational for all agents yields the seller at most

$$\sum_{s \in T} \left(\pi^s \max_{j=1,\dots,n} \{v_j(s), v_0\} \right)$$

(c.f. Lemma 1), (α^*, p^*) is undominated.
Furthermore, consider some allocation rule which is (IC), (IRS), (IRi) for all i, and (NDt) for all $t \in T$. By Lemma 1 this allocation rule does not yield the seller more than $\sum_{s \in T} (\pi^s \max_{j=1,\dots,n}\{v_j(s), v_0\})$. If the allocation rule yields the seller less than $\sum_{s \in T} (\pi^s \max_{j=1,\dots,n}\{v_j(s), v_0\})$, then it is dominated by (α^*, p^*). *Q.E.D.*

Note that (α^*, p^*) is efficient. That the buyers do not receive any expected rents should not be surprising, given that they do not have any private information nor any bargaining power. What may be surprising, however, is the fact that the seller's utility is only $E[\max_{i=1,\ldots,n}\{v_i, v_0\}]$, even if the seller has some type t^* such that

$$\max_{i=1,\ldots,n}\{v_i(t^*), v_0\} > E[\max_{i=1,\ldots,n}\{v_i, v_0\}].$$

Under complete information a seller with such a type would clearly receive a utility greater than $E[\max_{i=1,\ldots,n}\{v_i, v_0\}]$. The reason why (α^*, p^*) can be supported as an equilibrium is the fact that it is impossible for the seller to credibly reveal her private information (even if her type is indeed above average), and this would be necessary to receive a utility of more than $E[\max_{i=1,\ldots,n}\{v_i, v_0\}]$. In technical terms, the allocation rule the seller can implement has to be incentive-compatible, and since the seller's valuation of her object is assumed to be common knowledge, each incentive-compatible allocation rule yields the seller the same utility irrespective of her type. So a seller of above average type is worse off due to the fact that the buyers' valuations are her private information, whereas a seller of below average type benefits from the presence of private information.
Finally, let us review the Akerlof model described in the introduction. If we assume that the car as such is worthless for the seller (she might have lost her driving licence) and if we apply the solution (α^*, p^*) of the contract proposal game to this model, then this yields the following outcome: If the car is undamaged, then it is given to the academic (who values it at €10.000), otherwise it is given to the mechanic (who has a valuation of €8.000 for the damaged car). This is the same outcome as in the common knowledge scenario. However, if the type of the car is private information of the seller, then both the academic and the mechanic have to pay – the academic €10.000 times the prior probability that the car is undamaged, the mechanic €8.000 times the prior probability that the car is damaged (for this result it is obviously essential that the buyers are not risk-averse).

4 Conclusion

This paper considers an optimal trading design model for an informed seller. We have shown that in an undominated pure-strategy perfect Bayesian equilibrium of the implied contract proposal game the seller gives the object to the person who values the object most (unless the seller values the object even more, in which case she keeps it), and receives a transfer payment from each potential buyer such that all ex-ante expected rents are extracted from the buyers.
Though the optimal trading design model is interesting in its own right, it can also be considered as a first step in analyzing optimal auction design

models with two-sided incomplete information. In optimal auction design models it is usually assumed that the bidders possess some private information about their valuations of the object to be sold, whereas the seller does not have any such private information. Though this assumption may be appropriate for some auction design settings, and it may serve as a useful first approximation to the true information structure in other situations, in general we might expect that the seller as well as a bidder possesses some private information about a bidder's valuation. If, for example, some piece of art is to be sold, some auction house is typically in a better position than any of the bidders to estimate the resale value of the item. Therefore, the standard auction design literature considers only one polar case of the more general information structure, whereas our optimal trading design model analyzes the other polar case (where only the seller possesses some private information). An obvious next step is to consider an auction model with two-sided incomplete information.

REFERENCES

AKERLOF, G. (1970): "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics,* 84, 488–500.

MASKIN, E., AND J. TIROLE (1990): "The Principal-Agent Relationship with an Informed Principal: The Case of Private Values," *Econometrica,* 58, 379–409.

——— (1992): "The Principal-Agent Relationship with an Informed Principal, II: Common Values," *Econometrica,* 60, 1–42.

MILGROM, P.R., AND R.J. WEBER (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica,* 50, 1089–1122.

MYERSON, R.B. (1983): "Mechanism Design by an Informed Principal," *Econometrica,* 51, 1767–1797.

TISLJAR, R. (2002): "Mechanism Design by an Informed Principal – Pure-Strategy Equilibria for a Common Value Model," Bonn Graduate School of Economics, Universität Bonn, Discussion Paper No. 21/2002.