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Stationary Concepts for Experimental 2x2 Games

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Abstract: Four stationary concepts for completely mixed 2x2 games are experimentally compared: Nash equilibrium, quantal response equilibrium, sample-7 equilibrium and impulse balance equilibrium. Experiments on 12 games, 6 constant sum games and 6 non-constant sum games were run with 12 independent subject groups for each constant sum game and 5 independent subject groups for each non-constant sum game. Each independent subject group consisted of 4 payers 1 and four players 2 interacting anonymously over 200 periods with random matching. The games were selected to yield a reasonably wide distribution over the parameter space. The comparison of the four theories shows that the order of performance from best to worst is as follows: impulse balance equilibrium, sample-7 equilibrium, quantal response equilibrium, Nash equilibrium. The new concepts of sample-7 equilibrium and impulse balance equilibrium are explained in the text.

1. Introduction

Experimental evidence suggests that mixed Nash-equilibrium is not a very good predictor of behavior. Thus EREV AND ROTH (1998 p. 853) conclude as their first summary observation that "…in some of the games the equilibrium prediction does very badly". A normal form game is called completely mixed, if it has only one equilibrium point in which every strategy is used with positive probability. 2x2 games of this kind are of special interest. They are the simplest games for which mixed equilibrium is the unequivocal game theoretic prediction, if they are played as non-cooperative one-shot games.

Mixed equilibrium has several interpretations. One interpretation is that of a rational recommendation for a one-shot game. Another interpretation looks at mixed equilibrium as a result of evolutionary or learning processes in a situation of frequently repeated play with two populations of randomly matched opponents. One may speak of mixed equilibrium as a behavioral stationary concept. KEN BINMORE, JOE SWIERZBINSKI and CHRIS PROULX (Economic Journal 2001) argue in their paper that mixed Nash-equilibrium predicts reasonably well for completely mixed constant sum 2x2 games. However it is difficult to judge the goodness of fit, if there is no comparison to other stationary concepts.

In this paper we will present several alternative stationary concepts for 2x2 games, which can be compared with mixed equilibrium and with each other. For this purpose we have performed experiments on 12 completely mixed 2x2 games. Six of them are constant-sum games and the other six are non-constant-sum games. Each of the constant-sum games was run with 12 independent subject groups and each of the other games with 6 independent subject groups. Each independent subject group consisted of four players 1 and four players 2 interacting in fixed roles over 200 periods with random matching.

The stationary concepts compared were:

- Nash Equilibrium
- Quantal Response Equilibrium
- Sample-7 Equilibrium
- Impulse Balance Equilibrium

Quantal response equilibrium (MCKELVEY, PALFREY, THOMAS, 1995) assumes that players give quantal best responses to the behavior of the others. In the exponential form of quantal response equilibrium, considered here, the probabilities are proportional to an exponential with the expected payoff times a parameter in the exponent.

Sample-7 equilibrium is based on the idea that in a stationary situation a player takes a sample of 7 observations of the strategies played on the other side, and then optimizes against this sample. This yields a mixed strategy depending on the probabilities of strategies on the other side. Sample-7 Equilibrium is a mixed strategy combination consistent with this picture. The concept has been developed by one of the authors (R. SELTEN). As far as we know it cannot be found in the literature. Originally the sample size 7 was chosen in view of the famous paper "The Magical Number 7 Give or Take Two" by MILLER (1951). Later we found that 7 actually gives a better fit than other sample sizes.

Impulse balance equilibrium proposed by one of the authors (R. SELTEN) is based on learning direction theory (SELTEN, BUCHTA, 1999). This learning theory is applicable to the repeated choice of the same parameter in learning situations in which the decision maker receives feedback not only about the payoff for the choice taken, but also for the payoffs connected to alternative actions. If a higher parameter would have brought a higher payoff we speak of an upward impulse and if a lower parameter would have yielded a higher payoff we speak of a downward impulse. The decision maker is assumed to have a tendency to move in the direction of the impulse. In (SELTEN, ABBINK and COX, 2001) impulse balance theory, a semi quantitative version of learning direction theory has been proposed. The learning process itself is not modeled, but only the stationary distribution. In the stationary distribution expected upward impulses are equal to expected downward impulses. As in prospect theory (KAHNEMANN & TVERSKY, 1979) losses are counted double in the computation of impulses (formally this involves the computation of a loss impulse).

Impulse balance equilibrium applies the idea of impulse balance theory to 2x2 games. The probability of choosing one of two strategies say strategy A is looked upon as the parameter, which can be adjusted upward or downward. It is assumed that the second lowest payoff in the matrix is an aspiration level determining what is perceived as profit or loss. In impulse balance equilibrium expected upward and downward impulses are equal for each of both players simultaneously.

Following a suggestion of one of the authors (R. SELTEN) impulse balance equilibrium has been successfully applied to special 2x2 and 2x2x2 games in a paper by AVRAHAMI, GÜTH and KAREEV (2001).

Two of the stationary concepts compared in this paper, Nash equilibrium and impulse balance equilibrium, are parameter free. Sample-7 equilibrium involves one parameter namely the number 7 which however has been chosen in view of admittedly quite weak theoretically considerations confirmed by pilot experiments not included into the main sample of this paper. In view of this, one also could think of sample-7 equilibrium as essentially parameter free. Quantal response equilibrium involves one parameter namely the constant multiplier of expected payoff in the exponent. This parameter has to be adjusted to the data. There are no theoretical considerations, not even very weak ones, which could be used in order to determine this parameter in any other way.

The development of parameter free stationary concepts, which do not require any adjustment to the data, seems to be very important for behavioral theory. With the help of such concepts theoretically interesting situations can be mathematically explored without any adaptation of parameters to experimental or empirical data. Therefore in this paper we are mainly interested in parameter free stationary equilibrium concepts.

As we shall see the comparison yields a clear order with respect to the goodness of fit from best to worst: Impulse balance equilibrium, sample-7 equilibrium, quantal response equilibrium, Nash equilibrium.

In chapter 2 we shall present a more detailed description of the four concepts. Chapter 3 will explain the experimental setup and section 4 will describe the results. Chapter 5 concludes with a summary and discussion.

2. The four stationary concepts

2.1. Nash equilibrium

All the experimental 2x2 games in this paper have the structure shown by figure 1. The arrows around the matrix show the direction of best replies. The Parameters a_L, a_R, b_U and b_D are assumed to be non-negative. Games with negative payoffs probably would require special behavioral considerations which we want to avoid in this paper. The parameters c_L and c_R are player 1's payoff differences in favor of U and D, respectively. Similarly d_U and d_D are payoff differences of player 2 for R and L, respectively. All these payoff differences are assumed to be positive. It is clear that a game with this structure is completely mixed in the sense that it has a uniquely determined completely mixed Nash equilibrium.

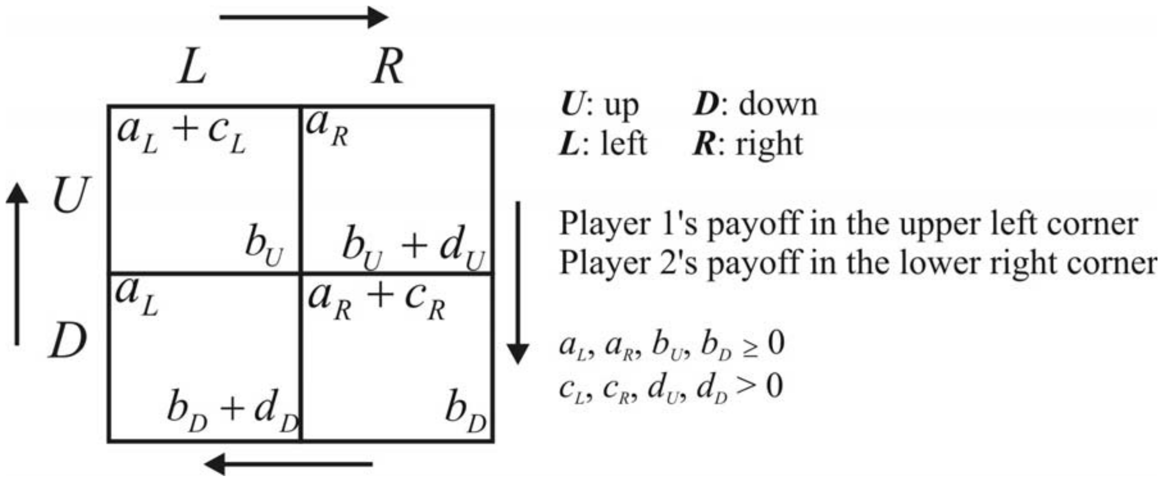

Figure 1: Structure of the experimental 2x2 games.

Let

$$p = (p_U, p_D)$$

be the mixed strategy of player 1 and let

$$q = (q_L, q_R)$$

be the strategy of player 2.

Here p_U is player 1's choice probability for U and q_L is player 2's choice probability for strategy L. In Nash equilibrium the choice probabilities are as follows:

$$p_U = \frac{d_D}{d_U + d_D} \qquad p_D = \frac{d_U}{d_U + d_D}$$

$$q_L = \frac{c_R}{c_L + c_R} \qquad q_R = \frac{c_L}{c_L + c_R}$$

The choice probabilities of a player in Nash equilibrium are independent of his own payoff. They are entirely determined by the payoff differences of the other player. This is a well known counterintuitive property of Nash equilibrium.

2.2. Quantal Response Equilibrium

It is assumed that players choose a "quantal best response" to the strategies of the other player. They make mistakes, taking the mistakes of the other player into account.

Let $E_U(q)$ and $E_D(q)$ be player 1's expected payoff for U and D, resp., against a strategy q of player 2. Similarly $E_L(p)$ and $E_R(p)$ are player 2's expected payoffs for L and R, resp., against a strategy p of player 1.

In quantal response equilibrium the choice probabilities are as follows.

$$p_U = \frac{e^{\lambda E_U(q)}}{e^{\lambda E_U(q)} + e^{\lambda E_D(q)}} \qquad p_D = 1 - p_U$$
$$q_L = \frac{e^{\lambda E_L(p)}}{e^{\lambda E_L(p)} + e^{\lambda E_R(p)}} \qquad q_R = 1 - q_L$$

The formulas for the choice probabilities yield a simultaneous equation system, which determines the choice probabilities as functions of λ. For our data λ=8.84 is the best fitting overall estimate. This value of λ minimizes the sum of squared distances from the actually observed relative choice frequencies. This measure of predictive success will be explained in section 4.2.

The best reply structure of a 2-person game is a pair of mappings (α,β). The mapping α maps the strategies q of player 2 to player 1's set $\alpha(q)$ of pure replies to q and the mapping β maps the mixed strategies p of player 1 to the set β(p) of player 2's pure best replies to p. Nash equilibrium depends only on the best reply structure of the game. However, quantal response equilibria with the same parameter λ can be different for two games with the same best reply structure. If all payoffs of a 2x2 game are multiplied by the same positive factor x the best reply structure remains unchanged, but quantal response equilibrium for a fixed parameter λ does change. The multiplication of all payoffs by x has the same effect as not changing payoffs and replacing λ by $\lambda' = \lambda x$.

Suppose that the payoffs are changed by adding a constant r to all payoffs of player 1 in row R of figure 1 and leaving everything else unchanged. Let $E_U^{'}(q)$ and $E_D^{'}(q)$ be the new payoffs for U and D in the new game obtained in this way. We have

$$E_U^{'}(q) = E_U(q) + q_R r$$
$$E_D^{'}(q) = E_D(q) + q_R r$$

This means that the equation for p_U in the new game can be simplified by dividing numerator and denominator by the common factor $e^{q_R r}$. Therefore the equations for p_U and p_D do not really change in the transition to the new game. The same argument can be applied to the case that a constant is added to player 1's payoff in the column L or players 2's payoff in one of the two rows. We can conclude that such additive changes do not have any effect on the quantal response equilibrium, even if it does not depend on the best reply structure alone.

2.3. Sample-7 Equilibrium

In the stationary state described by p_U, p_D, q_L, and q_R player 1 takes a sample of 7 choices L or R and optimizes against this sample. Player 2 behaves analogously. Sample-7 equilibrium is a special case of **sample-*n* equilibrium** with n = 1,2,.... This concept describes a stationary state of two large populations of players 1 and 2. Every member takes a sample of n past decisions of players on the other side and optimizes against it. The sample-n equilibrium is a stationary state of this system. Here, too, p_U, p_D, p_L and p_R are stationary probabilities of U, D, R and L. Consider two specific players 1 and 2 in both populations. Let k be the number of L's in player 1's sample and let m be the number of D's in player 2's sample. The following can be said about the best replies of the players to their samples

Player 1's best reply to his sample is U for $kc_L > (n-k)c_R$
Player 2's best reply to her sample is L for $md_D > (n-m)d_U$

Let k^* be the lowest integer k such that U is the best reply to a sample in which L appears exactly k times. Similarly let m^* be the lowest number for which L is the best reply to a sample in which D appears exactly m times. In the following we shall assume that we have

$$(*) \qquad k^* c_L \neq (n-k^*) c_R$$

and

$$(**) \qquad m^* d_D \neq (n-m^*) d_U$$

These special cases cannot arise for n=1,...,10 in the experimental games of this paper. If (*) is not satisfied we must have

$$\frac{k^*}{n} = \frac{c_R}{c_L + c_R}$$

and if (**) does not hold the following must be true

$$\frac{m^*}{n} = \frac{d_U}{d_U + d_D}$$

In the games considered here c_L, c_R, d_U and d_D are positive integers such that c_L+c_R and d_U+d_D are always equal to 11. Therefore for n=1,…,10 the payoff differences c_R and d_U are relative prime to n. It follows that (*) and (**) are satisfied for n=1,…,10 by our experimental games.

Conditions (*) and (**) have the consequence that there is exactly one best reply to every sample. In stationary equilibrium the following <u>sample-n equations</u> must be satisfied.

$$p_U = \sum_{k=k^*}^{n} \binom{n}{k} q_L^k (1-q_L)^{n-k}$$

$$q_L = \sum_{m=m^*}^{n} \binom{n}{m} (1-p_U)^m \, p_U^{n-m}$$

It will be shown that a game with the structure shown in figure 1 has exactly one sample-n equilibrium if (*) and (**) are satisfied.

The right hand sides of the sample-n equations can be described as decumulative binomial probabilities. Consider a binomial process with probability x for success and 1-x for failure. We use the following notation

$$b(j,n,x)=\binom{n}{j}x^j(1-x)^{n-j}$$

$$B(h,n,x) \;=\; \sum_{j=h}^{n}\binom{n}{j}x^j(1-x)^{n-j} \;=\; \sum_{j=h}^{n}b(j,n,x)$$

It is clear, that $b(j,n,x)$ is the probability of exactly j successes in a sample of n if x is the probability of success. This is also true for x=0 and j=0 or x=1 and j=n if one adopts the convention 0°=1 Therefore in this paper the expression 0° is always understood as being equal to 1.

$B(h,n,x)$ is the decumulative binomial probability for at least h successes in a sample of n if the success probability is x. We also make use of the following notation:

$$b_x(h,n,x)=\frac{\partial(h,n,x)}{\partial x}$$

$$B_x(h,n,x)=\frac{\partial B(h,n,x)}{\partial x}$$

In order to derive the uniqueness result about sample-n equilibrium we have to show that the following proposition holds

Proposition 1: For n=1,2,… and h=1,…,n the function $B(h,n,x)$ in monotonically increasing in x from $B(h,n,0)$=0 to $B(h,n,1)$=1.

Proof: We have

$$b_x(h,n,x)=\binom{n}{h}\left[hx-(n-h)(1-x)\right]x^{h-1}(1-x)^{n-h-1}$$

In view of

$$hx-(n-h)(1-x)=h-nx$$

this yields

$$b_x(h,n,x)=\binom{n}{h}[h-nx]x^{h-1}(1-x)^{n-h-1}$$

We can conclude that the following is true

$$b_x(h,n,x) \quad \begin{cases} >0 & for\, h<nx \\ =0 & for\, h=nx \\ <0 & for\, h>nx \end{cases} \qquad \text{for } 0<x<1$$

At x=0 or x=1 the derivative $B_x(h,n,x)$ maybe zero even if it is positive or negative for $0<x<1$. Nevertheless one can conclude that $B_x(h,n,x)$ is increasing in x for h<nx and decreasing in x for $h>nx$ from x=0 to x=1.

In view of the definition of $B(h,n,x)$ we have

$$B(h+1,n,x) = b(h+1,n,x) + B(h,n,x)$$

for h=1,…,n-1

and therefore

$$B_x(h+1,n,x) - B_x(h,n,x) = b_x(h,n,x)$$

for h=1,…,n-1

Let h^* be the greatest integer h with $h \leq nx^*$. Obviously the following is true

$$B_x(h+1,n,x) - B_x(h,n,x) \;=\; \begin{cases} >0 & for\, h<h^* \\ =0 & for\, h=h^* \\ <0 & for\, h>h^* \end{cases}$$

It can be seen easily that we have

$$\begin{aligned} &B(1,n,x) = 1-(1-x)^n \\ &B_x(1,n,x) = n(1-x)^{n-1} \\ &B(n,n,x) = x^n \\ &B_x(n,n,x) = nx^{n-1} \end{aligned}$$

The first of these four equations follows by the fact that the probability of no success is $(1\text{-}x)^n$.

For n=1 the right hand side for the equation for $B(1,n,x)$ is x. Therefore the proposition holds for n=1. In the following we assume that n is at least 2.

The formula for $B_x(1,n,x)$ shows that this derivative is positive for $0\leq x<1$. Therefore $B(1,n,x)$ is increasing in x from x=0 to x=1. Similarly the equation for $B_x(n,n,x)$ shows, that $B(n,n,x)$ is increasing in x from zero to one.

The inequality for the difference of $B_x(h+1,n,x)$ and $B_x(h,n,x)$ shows that for given x the derivative $B_x(h,n,x)$ is increasing with h for $h = 1,\ldots,h^*$, non-decreasing from h^* to h^*+1 and then decreasing with h for $h = h^*+1,\ldots,n$. As we have seen $B_x(n,n,x)$ is still positive for $0<x<1$ in spite of the negative difference for $h=h^*+1,\ldots,n$. Therefore $B(h,n,x)$ is increasing for $h = 1,\ldots,n$.

It is an immediate consequence of the definition of $B(h,n,x)$ that

$$B(h,n,0) = 0$$

and

$$B(h,n,1) = 1$$

hold.

If the probability of a success is zero, then the probability of at least h successes is also 0. If the probability of success is 1 then the probability of at least h successes is also 1. This completes the proof of proposition 1.

Proposition 2 is the uniqueness result for sample-n equilibrium.

<u>Proposition 2:</u> Consider a completely mixed 2x2 game and assume that for each of two players the absolute value of a payoff difference divided by the sum of the absolute values of both payoff differences is not equal to an integer multiple of $\frac{1}{n}$. Then for n=2,… this 2x2 game has a uniquely determined sample-n equilibrium.

<u>Proof:</u> the conditions on the payoff differences in the assertion are nothing else than inequalities (*) and (**). The sample-n equations can be expressed as follows

$$\begin{aligned} p_U &= B(k^*, n, q_L) \\ q_L &= B(m^*, n, 1 - p_U) \end{aligned}$$

It follows by proposition 1 that in the first sample-n equation p_U is a monotonically increasing in q_L from p_U=0 for q_L=0 to p_U=1 at p_L=1. Similarly in the second sample-n equation q_L is monotonically decreasing at p_U from q_L=1 for p_U=0 to q_L=0 at p_U=1. It is clear that in a (q_L, p_U)-diagram the two sample-n equations are represented by two curves which must intersect in one and only one point. Therefore the assertion holds for games with the structure shown in figure 1.

In figure 1 the arrows indicating best replies move in a clockwise direction. Of course there are also 2x2-games with the opposite orientation of these arrows. These games can be obtained from the ones shown in figure 1 by an interchange of the two rows. It is clear that therefore the proposition holds in this case, too. This completes the proof of proposition 2.

The sample-n equations for p_U and q_L are determined by the parameters $k,^*$ m^* and n. The parameters k* and m* can have the values k^*=1,…,n and m^*=1,…,n. The values of k^* as well as of m^* cannot have the value 0, since in this case one of the players would have a pure best reply regardless of what the other player does. This cannot be true in a completely mixed 2x2-game.

In view of proposition 2 for each pair (k^*,m^*) the equation system has exactly one solution. Therefore, if (*) and (**) are satisfied, there are only n^2 possible solutions of the equation system independently of the payoff parameters. We shall now explain a relationship between the numbers k^* and m^* and the Nash equilibrium strategies of the game.

Let p_U^N , p_D^N , p_L^N and p_R^N be the choice probabilities in Nash equilibrium for U, D, L, R respectively. In Nash equilibrium player 1's payoffs for U and D are equal. Since the payoff components are a_L and a_R are equal in both rows, this yields

$$q_L^N c_L = q_R^N c_R$$

The number k^* can be described as the smallest integer k with

$$\frac{k}{n} c_L > \left(1 - \frac{k}{n}\right) c_R$$

The comparison of this inequality with the last equation shows that k^* must be the smallest integer k with

$$\frac{k}{n} > q_L^N = \frac{c_R}{c_L + c_R}$$

The number m^* is the smallest number m with

$$\frac{m}{n} d_D > \left(1 - \frac{m}{n}\right) d_U$$

In view of

$$p_D^N d_D = p_U^N d_U$$

we can conclude that m^* is the smallest number m with

$$\frac{m}{n} > 1 - p_U^N = \frac{d_U}{d_U + d_D}$$

The integers k^* and m^* are determined by the choice probabilities q_L^N and p_U^N respectively. Curiously enough the parameters k^* and m^* which determine the behavior of player 1 and player 2, resp., depend on the Nash equilibrium choice probabilities of the other player. Nevertheless a relationship between Nash equilibrium and sample-*n* equilibrium is established in this way.

For the special case of sample-7 equilibrium table 1 shows this connection between the two stationary concepts. The rows and columns correspond to intervals for the Nash equilibrium strategy of player 1 and player 2, respectively. The fields show the values for p_U (above) and q_L (below) in sample-7 equilibrium. The table is based on a clockwise orientation of the arrows for the best replies as in figure 1.

Figure 2 conveys the same information as table 1, but in the form of two column diagrams. It can be seen that the choice probability p_U of the sample-7 equilibrium is not at all constant along a line at which p_U^N and therefore v_U is constant. Along such a line p_U is strongly decreasing with increasing w_L. Similarly in the second diagram q_L strongly increases with v_U and p_U^N on a line with constant q_L^N.

A property of sample-*n* equilibrium connected with these features of figure 2 concerns the effect of an increase of one payoff of one player in one of the fields of the bimatrix.

As we have seen k^* can be described as the smallest integer k with $\frac{k}{n} > q_L^N$ or in other words with:

$$k > \frac{nc_R}{c_L + c_R}$$

Suppose that player 1's payoff at (U,L) is increased and all other payoffs remain constant. This is equivalent to increasing c_L and leaving a_L, a_R, b_U, b_D, c_R, d_U and d_D constant. If the increase is sufficiently great, then $\frac{nc_R}{(c_L + c_R)}$ and therefore k^* is decreased. This means that the curve of the first sample-*n* equation is shifted upwards (except at q_L=0 and q_L=1) whereas the curve for the second sample-*n* equation remains unchanged. Therefore at the new intersection p_U is greater and p_L is smaller than before. We can conclude that a sufficiently big increase of player 1's payoff at (U,L) increases p_U and decreases q_L. Unlike Nash equilibrium sample-*n* equilibrium has the property that a player's own payoffs matter for his choice probability.

It can be seen without difficulty that a ceteris paribus increase of player 1's payoff at (U,R) also increases the sample-7 equilibrium probability p_U, if it is on the one hand big enough to change the equilibrium at all and on the other hand c_R is still positive after the change. Such a change increases a_R and decreases c_R and therefore k^* with the same consequences as above.

An increase of player 1's payoff at a field of the matrix where he plays U increases player 1's sample-n equilibrium probability p_U provided that anything is changed at all and the new game is still in the class described by figure 1. It can be seen without difficulty that this conclusion can be generalized to any ceteris paribus increase of a player *i*'s payoff at a field (x,y) of the bimatrix. Provided that the new game is still in the class described by figure 1, the new, sample-*n* equilibrium, if it is different from the old one, will have a higher p_x if player 1's payoff is increased and a higher p_y if player 2's payoff is increased.

One may say that in sample-*n* equilibrium players are attracted to strategies with high own payoff whereas in Nash equilibrium own payoffs do not have any influence on a players behavior.

v_U \ w_L	**0**	**.143**	**.286**	**.429**	**.571**	**.714**	**.857**
0	.336 .057	.243 .137	.192 .224	.151 .318	.117 .419	.086 .533	.057 .664
.143	.467 .086	.382 .185	.321 .286	.272 .392	.228 .501	.185 .618	.137 .753
.286	.581 .117	.499 .228	.442 .339	.387 .448	.339 .558	.286 .679	.224 .808
.429	.682 .151	.608 .272	.552 .387	.500 .500	.448 .613	.392 .728	.318 .849
.571	.776 .192	.714 .321	.661 .442	.613 .552	.558 .661	.501 .772	.419 .883
.714	.863 .247	.815 .382	.772 .499	.728 .608	.679 .714	.618 .815	.533 .914
.857	.943 .336	.914 .477	.883 .581	.849 .682	.808 .776	.757 .863	.664 .943

v_U **lower boundary of interval containing player 1's Nash equilibrium choice probability** p_U^N

w_L **lower boundary of interval containing player 2's Nash equilibrium choice probability** q_L^N

in the field sample-7 $\boxed{\begin{matrix} p_U \\ q_L \end{matrix}}$

Table 1: Connection between the Nash equilibrium and the sample-7 equilibrium.

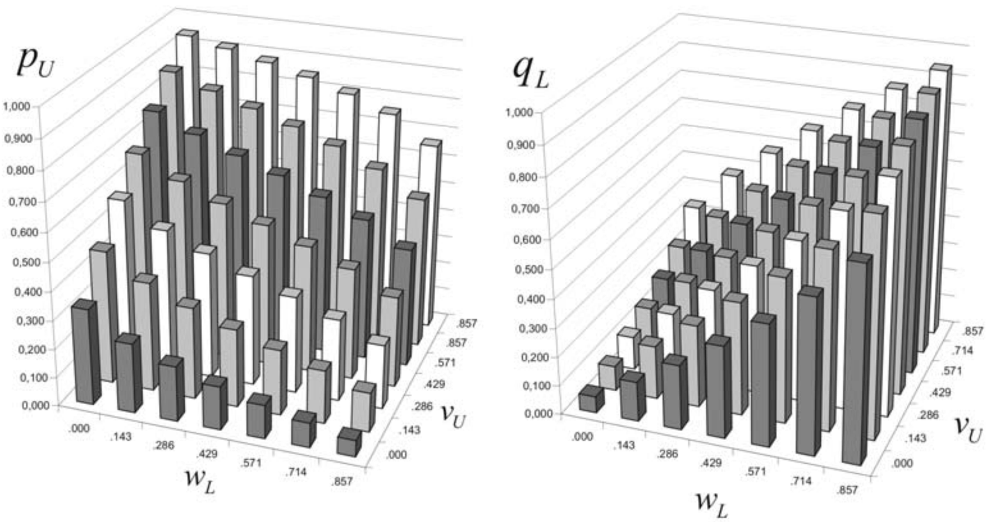

Figure 2: Connection between the Nash equilibrium and the sample-7 equilibrium. The left diagram shows p_U and the right one q_L

2.4. Impulse Balance Equilibrium

As has been already explained in the introduction, impulse balance theory is not applied to the original game, but to a transformed game, in which losses with respect to a natural aspiration level get twice the weight as gains above this level. Figure 1 shows that player 1 receives at least a_R if he chooses U and at least a_L if he chooses D. Player 1 can secure the higher one of both payoffs a_L and a_R by choosing one of his pure strategies.
Therefore the maximum of a_R and a_L is a natural aspiration level for player 1. Similarly the maximum of b_U and b_D is a natural aspiration level for player 2.

Define

$$s_1 = \max(a_L, a_R)$$
$$s_2 = \max(b_U, b_D).$$

We construct the transformed game by leaving player i's payoff below and at s_i unchanged and by reducing the difference of higher payoffs to s_i by the factor ½. Figure 2 shows this <u>impulse</u> <u>balance</u> <u>transformation</u> for the example of the experimental game 3.

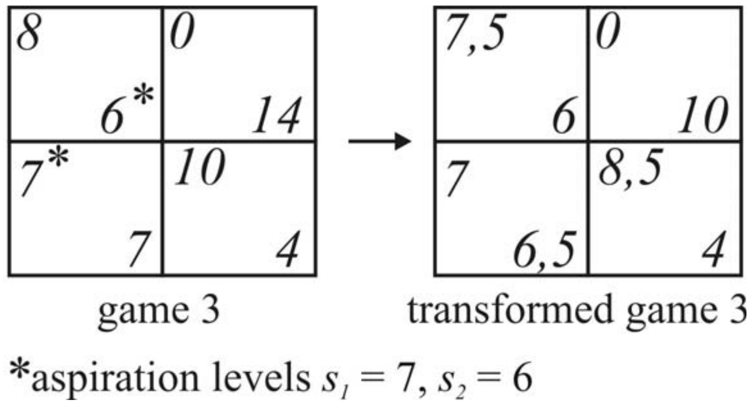

Figure 2: Impulse Balance Transformation of experimental game 3.

The payoff differences in the transformed game corresponding to c_L, c_R, d_U, d_D are denoted by c_L^*, c_R^*, d_U^*, d_D^*. If after a play player *i* could have obtained a higher payoff by the choice of his other strategy, he receives an impulse in the direction of his other strategy. The size of this impulse is the forgone payoff in the transformed game. If for example player 1 chooses *U* and the other player chooses R, then player 1 receives an impulse of c_R^* = 8.5 in the direction of *D*. A player receives no impulse if the payoff for the strategy he did not choose was lower than the one he obtained. Figure 3 shows the impulses obtained to the strategy not chosen, similar to a payoff table.

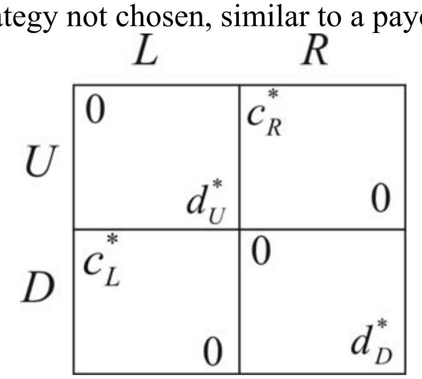

Figure 3: Impulse in the direction of the strategy not chosen.

Impulse balance equilibrium requires that player 1's expected impulse from *U* to *D* is equal to his expected impulse from *D* to *U*. Similarly player 2's expected impulse from L to R must be equal to her expected impulse from *R* to *L*. This yields to following to impulse balance equations:

$$p_U q_R c_R^* = p_D q_L c_L^*$$
$$p_U q_L d_U^* = p_D q_R d_D^*$$

The left hand side of the first impulse balance equation is player 1's expected impulse from *U* to *D* and the right side is player 1's expected impulse from *D* to *U*. If the left hand side is greater than the right hand side then player 1 receives stronger impulses from *R* to *D* and this will decrease q_R and increase q_L. This creates a tendency in the direction of impulse balance. An analogues interpretation can be given to the second impulse balance equation. Of course this is only a heuristic argument, in this paper we do not want to explore the dynamics of impulse balance equilibrium.

The impulse balance equations cannot be satisfied unless p and q are completely mixed. This can be seen as follows. Suppose that p_U=0 holds, then the first impulse balance equation can not be satisfied, unless we have q_L=0. However in this case the second impulse balance equation is not satisfied since the left side is 0 and the right side is d_d^*. Similarly p_D=0 leads to q_R=0 by the fist impulse balance equation. The left side of the second impulse balance equation becomes d_U^* and the right side is 0. We can conclude that p must be completely mixed.

Now suppose that q_L=0 holds. Since p is completely mixed then the left side of the first impulse balance equation is positive and the right side is 0. Since p is completely mixed. Similarly if q_R=0, the left side of the first impulse balance equation is 0 and the right side is positive. It follows that also q is completely mixed.

In order to solve the impulse balance equation system we introduce the following definitions:

$$u=\frac{p_U}{p_D} \qquad v=\frac{q_L}{q_R} \qquad c=\frac{c_L^*}{c_R^*} \qquad d=\frac{d_U^*}{d_D^*}$$

We divide the first impulse balance equation by p_D, q_L and c_R^* and the second impulse balance equation by p_D, q_L *and* d_D^* with the help of the definitions of u, v, c and d impulse balance equation can be rewritten as follows:

$$u=cv$$
$$duv=1$$

Replacing u by cv in the second equation yields

$$v=\frac{1}{\sqrt{cd}}$$

With the help of the first equation we obtain

$$u=\sqrt{\frac{c}{d}}$$

The definition of u together with p_D = 1-p_U yields

$$p_U=\frac{u}{1+u}$$

In the same way we can conclude that

$$q_L = \frac{v}{1+v}$$

holds.

Together with the formulas for u and v this leads to the following result:

$$p_U = \frac{\sqrt{c}}{\sqrt{c}+\sqrt{d}}\,, \qquad p_D = \frac{\sqrt{d}}{\sqrt{c}+\sqrt{d}}$$
$$p_L = \frac{1}{1+\sqrt{cd}}\,, \qquad p_R = \frac{\sqrt{cd}}{1+\sqrt{cd}}\,.$$

It can be seen that the choice probabilities of a player in impulse balance equilibrium are not independent of his payoff. Suppose that c_L is increased whereas all other parameters of the 2x2 game in figure 1 remain constant. Then c_L^* will also be increased, whereas c_R^*, d_U^* and d_D^* remain constant. This means that d will remains unchanged and c is increased. It can be seen that the increase of c results in an increase of p_U and in a decrease of q_L. This shows that unlike in Nash equilibrium a player's strategy is influenced by his own payoff.

We have seen that an increase of player 1's payoff for (U,L) increases his choice probability of U in impulse balance equilibrium. The same is true, if his payoff for (U,R) is increased. In this case c_R is decreased and therefore also c_R^* regardless of whether s_1 is a_L or a_R. We can conclude that in impulse balance equilibrium the increase of a payoff of player 1 in a row increases his choice probability for this row. Of course an analogues statement holds for player 2.

3. Experimental Design

3.1. Procedure

The experimental data were obtained in 54 sessions with 16 subjects each and 864 altogether. The subjects were students of the University of Bonn, mainly majoring in economics or law. The experiments were run in the Bonn laboratory of experimental economics. The computer program was based on the toolbox RatImage developed by Abbink and Sadrieh (1995). Only one game was played in each session.

At the beginning of a session oral and written instructions were given to the subjects. The written instructions (in German) are shown in appendix B. The subjects were informed about the game matrix including the payoffs of both players. They were told that they would interact with randomly changing opponents and always be in the same player role over 200 periods. Actually in each session there were two independent subject groups with four participants in the role of player 1 and four participants in the role of player 2. The players played against randomly chosen opponents but only within their independent group. They were not informed about the fact that there are two groups. We did not lie to them but conveyed the impression that they interact with 15 other players.

After the instruction the participants were sitting in separate cubicals and made their decisions by mouse click. The decisions in a play were made without any information about the choices of the other players. After each of the 200 plays they received feedback about the other player's choice and their payoff, the period number and their cumulative payoff. No limit was imposed on the decision time. The subjects were not permitted to take notes of any kind about their playing experience. They were also not permitted to talk to each other during the experiment and they had no opportunity to see the screens of other participants. After each experiment, participants had to fill in a questionnaire, shown in appendix C.

Each participants received 5 € and in addition to this a money payoff proportional to their game payoff accumulated over the 200 periods. The exchange rate was 1.6 €-Cent per payoff point. An experimental session took 1.5 to 2 hours and the average earning of a subject was about 24 € including the show up fee.

In some sessions a digit span test DAVIS (1931), DELLA SALA ET AL. (1999) preceded the game playing. This test is designed to measure the short time memory size. However we shall make no use of the data collected by this test in this paper. Therefore the details of the digit span test will not be explained here.

3.2. Experimental Games

Figure 4 shows the twelve games used in our Experiment. The constant sum games are shown on the left side of the figure 4 and the non-constant sum games on the right side of figure 4. The non-constant sum game right next to a constant sum game in the figure 4 has the same best reply structure. We say that the two games form a pair. The non-constant sum game in a pair is derived from the constant sum game in this pair by adding the same constant to player 1's payoff in the column for *R* and 2's payoff in the row for *U*. It is clear that this does not change the best reply structure.

Nash equilibrium and sample-7 equilibrium depend only on the best reply structure and therefore yield the same predictions for both games in a pair. In section 2.2 it has been explained that adding a constant to all payoffs of player 1 in a specific column or to all payoffs of player 2 in a row does not change the quantal response equilibrium, even if this concept does not depend only on the best reply structure. Therefore quantal response equilibrium, too, yields the same prediction for the two games in a pair.

The determination of impulse balance equilibrium involves a transition from the original game to the transformed game. The impulse balance equilibrium depends on the best reply structure of the transformed game which is generally different from that of the original game. Therefore this concept yields different predictions for the games in a pair.

Constant Sum Games

Game 1

10, 10	0, 18
9, 9	10, 8

Game 2

9, 4	0, 13
6, 7	8, 5

Game 3

8, 6	0, 14
7, 7	10, 4

Game 4

7, 4	0, 11
5, 6	9, 2

Game 5

7, 2	0, 9
4, 5	8, 1

Game 6

7, 1	1, 7
3, 5	8, 0

Non-Constant Sum Games

Game 7

10, 12	4, 22
9, 9	14, 8

Game 8

9, 7	3, 16
6, 7	11, 5

Game 9

8, 9	3, 17
7, 7	13, 4

Game 10

7, 6	2, 13
5, 6	11, 2

Game 11

7, 4	2, 11
4, 5	10, 1

Game 12

7, 3	3, 9
3, 5	10, 0

L: left R: right
U: up D: down

Player 1's payoff is shown in the upper left corner
Player 2's payoff is shown in the lower right corner

Figure 4: Experimentally investigated games.

In the selection of the experimental games we have been guided by several considerations explained in the following. Two pilot experiments were run with the games shown in figure 5. Game A is similar to the game played by OCHS (1995) and also by GOEREE, HOLT, CHARLES and PALFREY (2000). In the questionnaires the subjects who had played game A often reported attempts to cooperate.

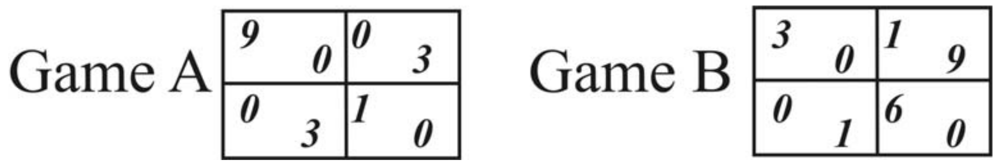

Figure 5: Structure of the pilot experiments.

Even if these attempts failed they may have had an influence on the observed relative frequencies. Therefore we decided to explore constant sum games extensively. Constant sum games offer no cooperation opportunities. We wanted to contrast them with similar non-constant sum games offering some scope for cooperation.

The concepts of sample-7 equilibrium and impulse balance equilibrium have been developed on the basis of the pilot experiments with games A and B. Therefore the experimental results obtained with these games are not included in the comparison of the four theories.

The selection of the constant sum games was guided by the idea, that on the one hand a reasonably wide distribution over the parameter space should be achieved, and on the other hand the number of games should be small enough to permit a sufficiently large number of independent subject groups in every case.

The games explored here have 8 payoffs but the best reply structure is characterized by two parameters. The Nash equilibrium choice probabilities p_U^N and q_L^N will serve as these two parameters in the following figures. Figure 6 show the six Nash equilibria for the experimental games.

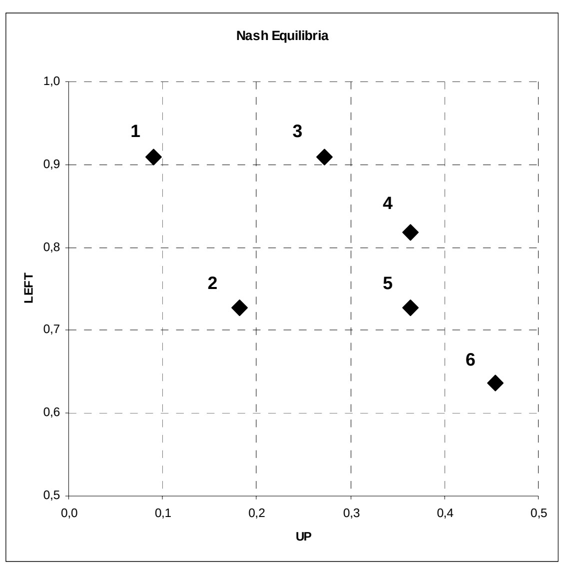

Figure 6: Nash equilibria of the games 1-6.

In all six cases p_U^N is between 0 and .5 and q_L^N is between .5 and 1. Therefore only this part of the parameter space is shown in figure 6. The best reply structure remains essentially unchanged if the rows or columns or the role of both players are exchanged. Such transformations yield all the points in figure 7.

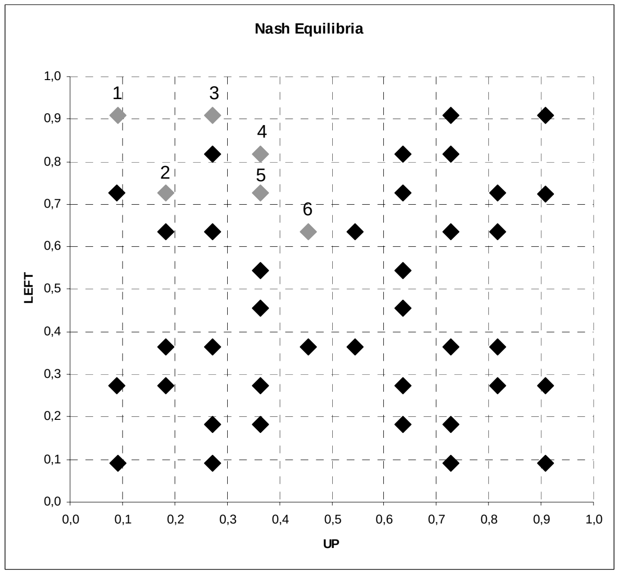

Figure 7: Permutations of rows, columns, or player roles transform the 6 experimental games into 44 games with the Nash equilibria shown in the figure.

It can be seen, that the six games together with their automorphic transformations are widely distributed over the parameter space. However we intentionally underrepresented cases in which one of the choice probabilities was near to .5. In our sample of 6 only game 6 has this property. In the middle of the parameter space, where both parameters are .5, every reasonable theory predicts equal probabilities for all strategies. The greater the distance from the midpoint is, the more the stationary concepts compared in this paper differ with respect to their predictions.

Since constant sum games are more basic we have run experiments with 12 independent subject groups for each of the 6 constant sum games but only 6 independent subject groups for each of the non-constant sum games.

4. Experimental Results

4.1. Predicted and Observed Relative Frequencies

We begin our descriptions of the results obtained by a number of figures showing the predictions of the four stationary concepts together with the observed overall relative frequencies for each of the experimental games. The numerical values are shown in table 2.

		Nash Equilibrium	Quantal Equilibrium	Sample-7 Equilibrium	Impulse Equilibrium	Observed Average 12 Observations
Game 1	***U***	.091	.057	.089	.057	**.079**
	L	.909	.664	.650	.664	**.690**
Game 2	***U***	.182	.185	.172	.185	**.217**
	L	.727	.619	.491	.619	**.527**
Game 3	***U***	.273	.137	.161	.137	**.163**
	L	.909	.753	.765	.753	**.793**
Game 4	***U***	.364	.286	.259	.286	**.286**
	L	.818	.679	.710	.679	**.736**
Game 5	***U***	.364	.286	.297	.286	**.327**
	L	.727	.679	.628	.679	**.664**
Game 6	***U***	.455	.448	.400	.448	**.445**
	L	.636	.613	.600	.613	**.596**
		Nash Equilibrium	**Quantal Equilibrium**	**Sample-7 Equilibrium**	**Impulse Equilibrium**	**Observed Average 6 Observations**
Game 7	***U***	.091	.057	.104	.057	**.141**
	L	.909	.664	.634	.664	**.564**
Game 8	***U***	.182	.185	.258	.185	**.250**
	L	.727	.619	.561	.619	**.586**
Game 9	***U***	.273	.137	.188	.137	**.254**
	L	.909	.753	.764	.753	**.827**
Game 10	***U***	.364	.286	.304	.286	**.366**
	L	.818	.679	.724	.679	**.699**
Game 11	***U***	.364	.286	.354	.286	**.331**
	L	.727	.679	.646	.679	**.652**
Game 12	***U***	.455	.448	.496	.448	**.439**
	L	.636	.613	.575	.613	**.604**

Table 2: Four stationary concepts together with the observed relative frequencies for each of the experimental games

Figures 8 and 9 show the results for game 1 and game 6 in full parameter space. Figure 8 shows that for game 1 the predictions of Nash Equilibrium and quantal response equilibrium are relatively far from the observed values, whereas the impulse balance equilibrium and the sample-7 equilibrium are quite near to them. Figure 9 shows, that in game 6 the predictions of all 4 concepts are quiet near to the observations. This is due to the fact that game 1 is near to the border of the parameter space, whereas game 6 is near to the middle. As we have pointed out before one can expect little differentiation near to the middle of the parameter space and a greater discrimination near to the border.

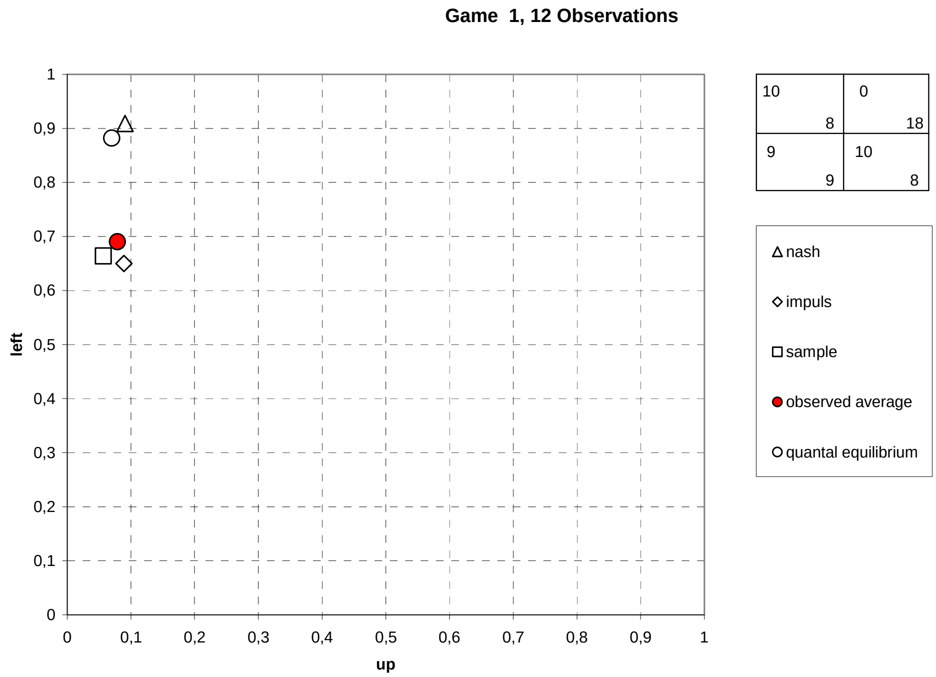

Figure 8: Visualization of the theoretical equilibria and the observed average (Game 1).

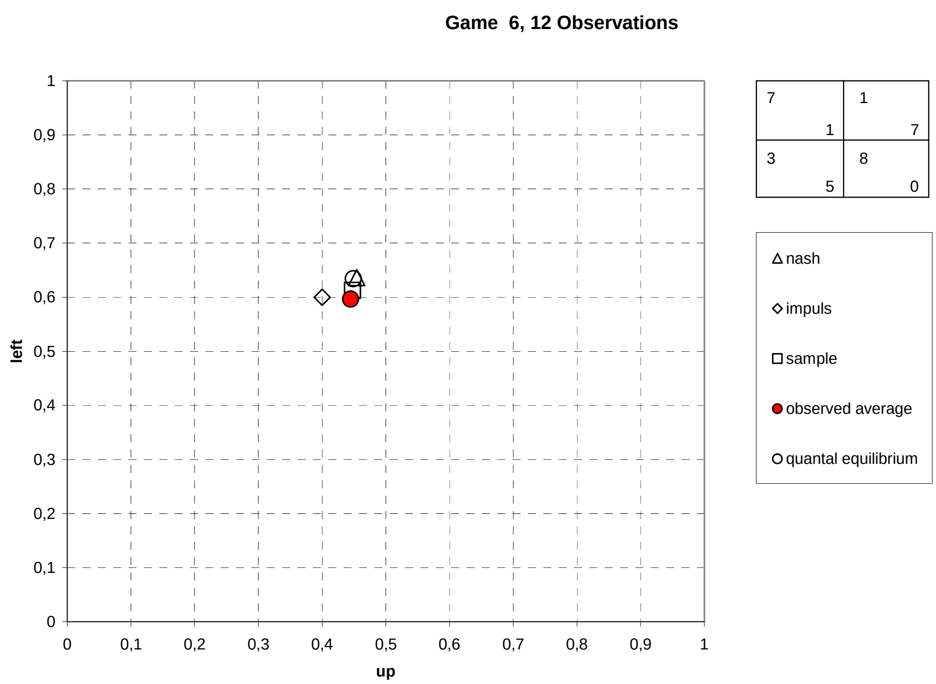

Figure 9: Visualization of the theoretical equilibria and the observed average (Game 6).

In figures 10 and 11 we show cutouts of the whole parameter space with predictions and observed averages for all 12 games. Apart from the fact that the Nash equilibrium of game 2 is nearer to (.5,.5) than that of game 3, the games 1-6 are the farther from the middle of the parameter space the lower is their order in the numbering. One can see that the discrimination between the concepts tends to be worse for games nearer to the middle of the parameter space. For games 1 to 5 the impulse balance equilibrium and sample-7 equilibrium are nearer to the observed values than the other two concepts, but for game 6, both concepts have a similar distance to the observed average as Nash equilibrium and quantal response equilibrium. Due to the nearness to the middle random fluctuations seem to play a greater role for game 6.

The predictions of impulse balance equilibrium and sample-7 equilibrium tend to be near to each other. Therefore random fluctuations make the comparisons between these two concepts difficult. The cutouts for the games 1 to 6 show that in games 2, 3 and 4 impulse balance equilibrium is nearer to the observed values, whereas in games 1, 5 and 6 sample-7 equilibrium is nearer to them.

The cutouts for games 7 to 12 show a similar picture as those for games 1 to 6 as far as the discrimination between the four concepts near the border is concerned, but there are differences in other respects. In games 7, 8, 10 and 11 impulse balance equilibrium is nearest to the observed values, but surprisingly not only in game 12 but also in game 9 Nash equilibrium and quantal response equilibrium are nearer to them.

As has been explained in 3.2, impulse balance equilibrium yields different predictions for the two games in a pair, since the basic idea of impulse balance is applied to the transformed game and not to the original one. The other three concepts yield the same predictions for the two games in a pair.

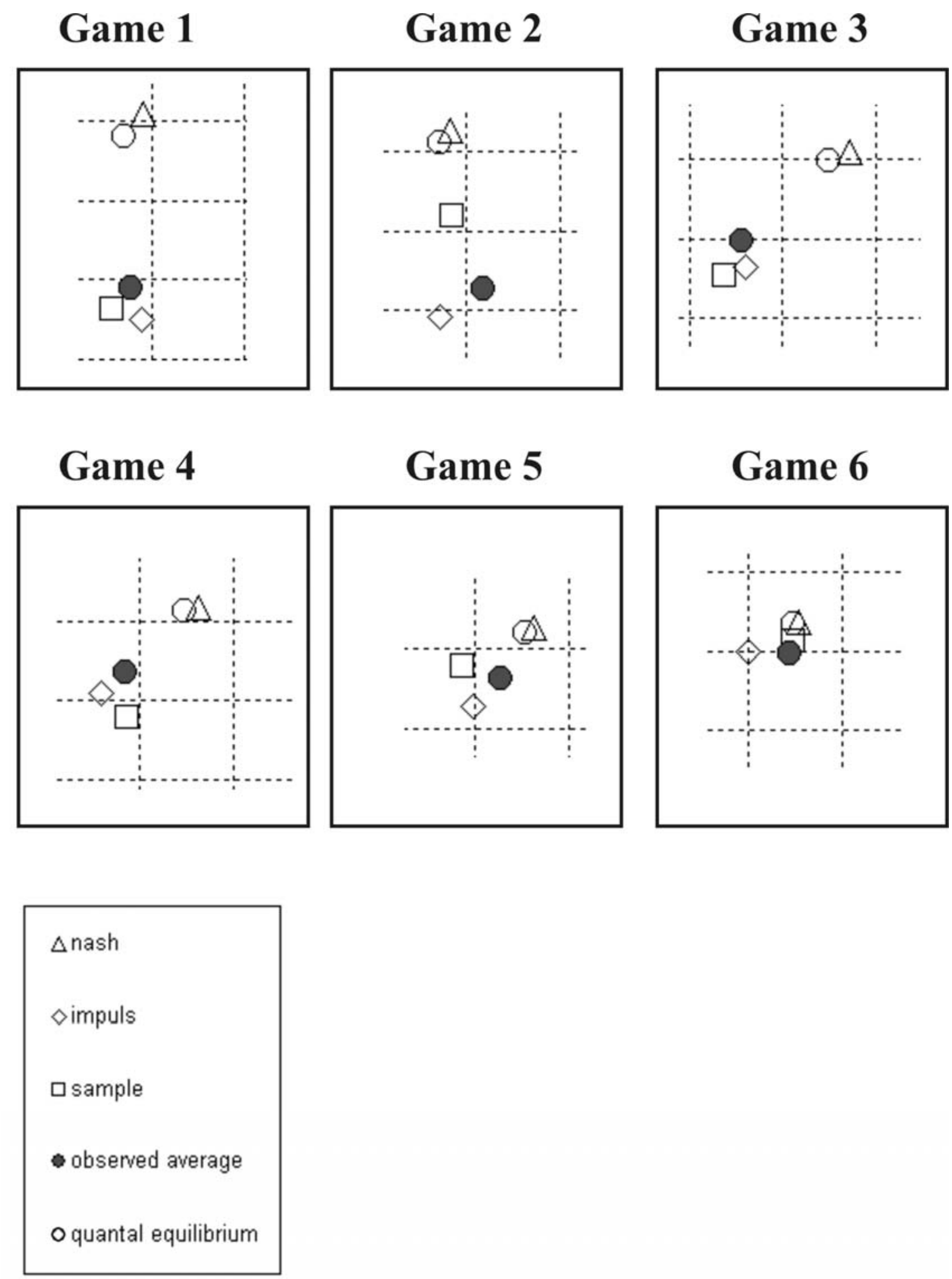

Figure 10: Visualization of the theoretical equilibria and the observed average in the constant sum games.

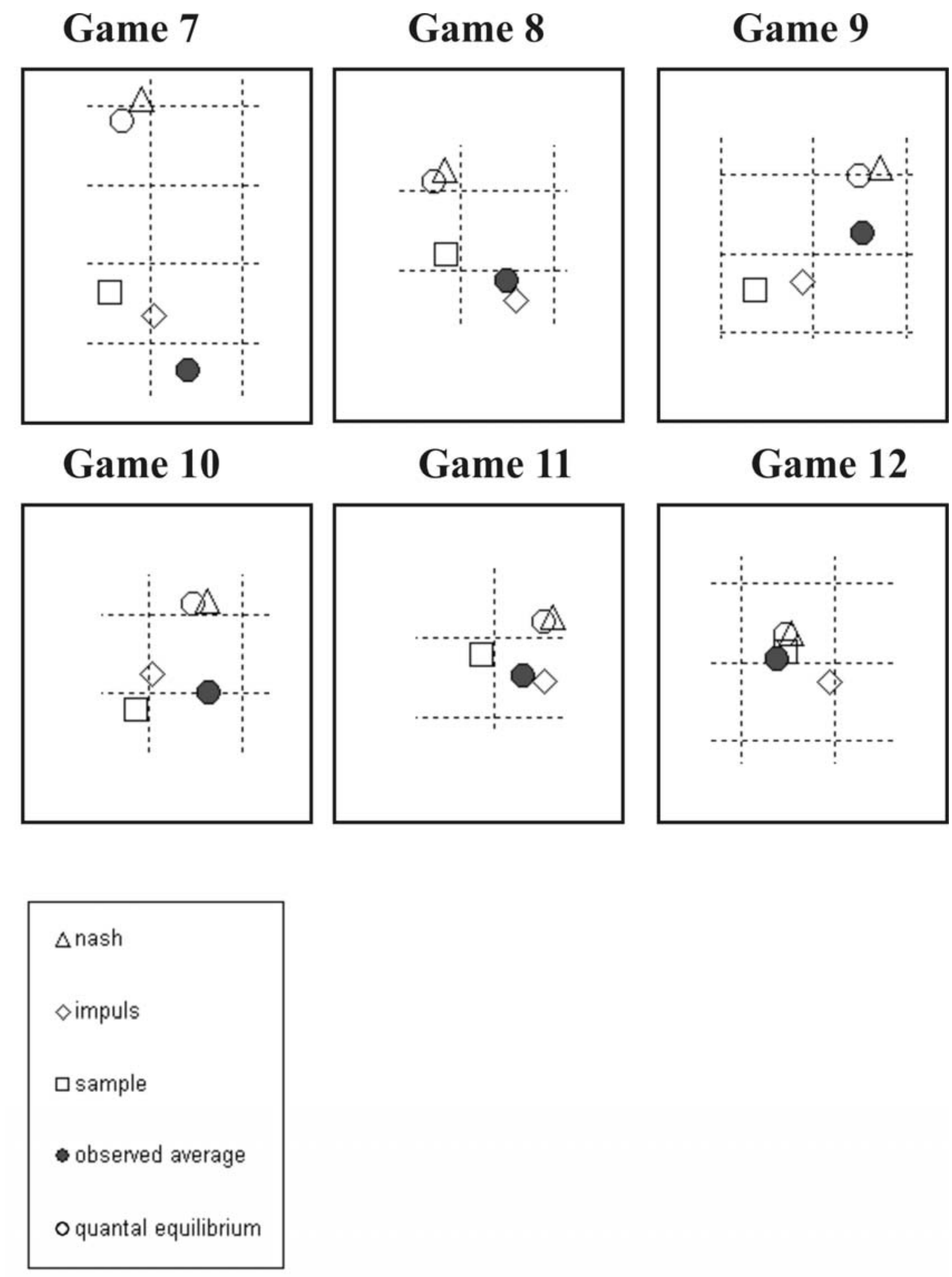

Figure 11: Visualization of the theoretical equilibria and the observed average in the non-constant sum games.

4.2. The Measure of Predictive Success

We look at the four theories compared in this paper as predictions of the relative frequencies of playing of U and L in an independent subject group playing one of the games 1 to 12. We do not want to assert that a player uses the same mixed strategy in all 200 periods of a session and we also do not assume that all players in the same role always behave in the same way. Presumably the players are engaged in complex learning processes which differ from person to person. Nevertheless such behaviour may result in frequencies of U and L which can be predicted reasonable well by stationary concepts. It is important to know how well observed relative frequencies can be explained without going into the details of stochastic learning models.

For a theory predicting a point in an Euclidian space the squared distance of theoretical and observed values is a reasonable measure of predictive success, in the sense that the predicted success is the greater the smaller this distance is. In the following we want to explain how this measure is applied to our data. Each Game i with i=1,…,12 has been played by s independent subject groups with s=12 for i= 1,…,6 and s=6 for i=7,…,12
We use the index j with j=1,…,s for the subject groups. Let and f_{iUj} and f_{iLj} be the relative frequencies of U and L in the j-th independent subject group playing game i. Consider a prediction p_U and p_L for these relative frequencies then

$$Q_{ij} = (f_{iUj}-p_U)^2+(f_{iLj}-p_L)^2$$

is the squared distance of the j-th observation for game i from the prediction for game i. The **mean squared distance** for the data of this game i from (p_U,p_L) is as follows

$$Q_i = \frac{1}{s}\sum_{j=1}^{s} Q_{ij}$$

We shall look at the overall predicted success but also at the predicted success of the constant sum games 1 to 6 and the non-constant sum games 7 to 12 separately. Define:

$$Q_C = \frac{1}{6}\sum_{i=1}^{6} Q_i\,, \qquad Q_N = \frac{1}{6}\sum_{i=7}^{12} Q_i\,, \qquad Q = \frac{1}{12}\sum_{i=1}^{12} Q_i$$

The indices C and N stand for constant sum and non-constant sum games. The **mean squared distances** Q_C, Q_N and Q will be the basis of our comparison of the four theories.

For every game i let f_{iU} and f_{iL} be the mean values of f_{iUj} and f_{iLj} with j=1,…,s:

$$f_{iU} = \frac{1}{s}\sum_{i=1}^{s} f_{iUj} \qquad \text{for } i=1,\ldots 12$$

$$f_{iL} = \frac{1}{s}\sum_{i=1}^{s} f_{iLj} \qquad \text{for } i=1,\ldots 12$$

The expression

$$S_i = \frac{1}{s}\sum_{j=1}^{n}(f_{iUj} - f_{iU})^2 + (f_{iLj} - f_{iL})^2 \qquad \text{for } i=1,\ldots 12$$

is the **sampling variance** of game *i* and

$$T_i = (f_{iU} - p_U)^2 + (f_{iL} - p_L)^2 \qquad \text{for } i=1,\ldots 12$$

is the **theory specific component** of the mean squared distance. The mean squared distance for a game can be split into these two components:

$$Q_i = S_i + T_i \qquad \text{for } i=1,\ldots 12$$

Define

$$S_C = \frac{1}{6}\sum_{i=1}^{6} S_i\,, \qquad S_N = \frac{1}{6}\sum_{i=7}^{12} S_i\,, \qquad S = \frac{1}{12}\sum_{i=1}^{12} S_i$$

$$T_C = \frac{1}{6}\sum_{i=1}^{6} T_i\,, \qquad T_N = \frac{1}{6}\sum_{i=7}^{12} T_i\,, \qquad T = \frac{1}{12}\sum_{i=1}^{12} T_i$$

The mean squared distances Q_C, Q_N and Q can also be split into two components

$$Q_C = S_C + T_C, \qquad Q_N = S_N + T_N, \qquad Q = S + T$$

Since the mean sampling variances S_C, S_N and S do not depend on the theory under consideration it does not really matter whether the comparison of theories is based on Q_C, Q_N and Q or alternatively on T_C, T_N and T. However, the mean squared distances Q_C, Q_N and Q are more natural measures of predictive success. A high sampling variance limits the accuracy of prediction even if the theory specific component is very small. Therefore the mean squared distance of the individual observations from the theory is more adequate as a measure of predictive success.

For no theory the mean squared distance Q can be smaller than S. The sampling variance S is an unavoidable part of Q.

4.3. Comparison of the Sample Sizes

Originally sample-7 equilibrium had been considered as a theory to be compared with the data, since the sample size 7 finds some admittedly weak support in the psychological literature (Miller 1951). The sample size 7 seems to be connected to the average capacity of short time memory. However, it is not really clear, whether this is relevant for the behavior in our experiments. Therefore another sample size could yield a better fit for our data.

In order to check this we compared the predictive success for sample-*n* equilibria with different sample sizes.

Figure 12 shows the overall mean squared distances *Q* for the sample-*n* equilibria with *n*=2,…,10. It can be seen immediately that the average squared distance is smallest for *n*=7. This means that sample-7 equilibrium has the best fit to the data. Therefore we do not have to add other sample-*n* equilibria to the four theories compared in this paper.

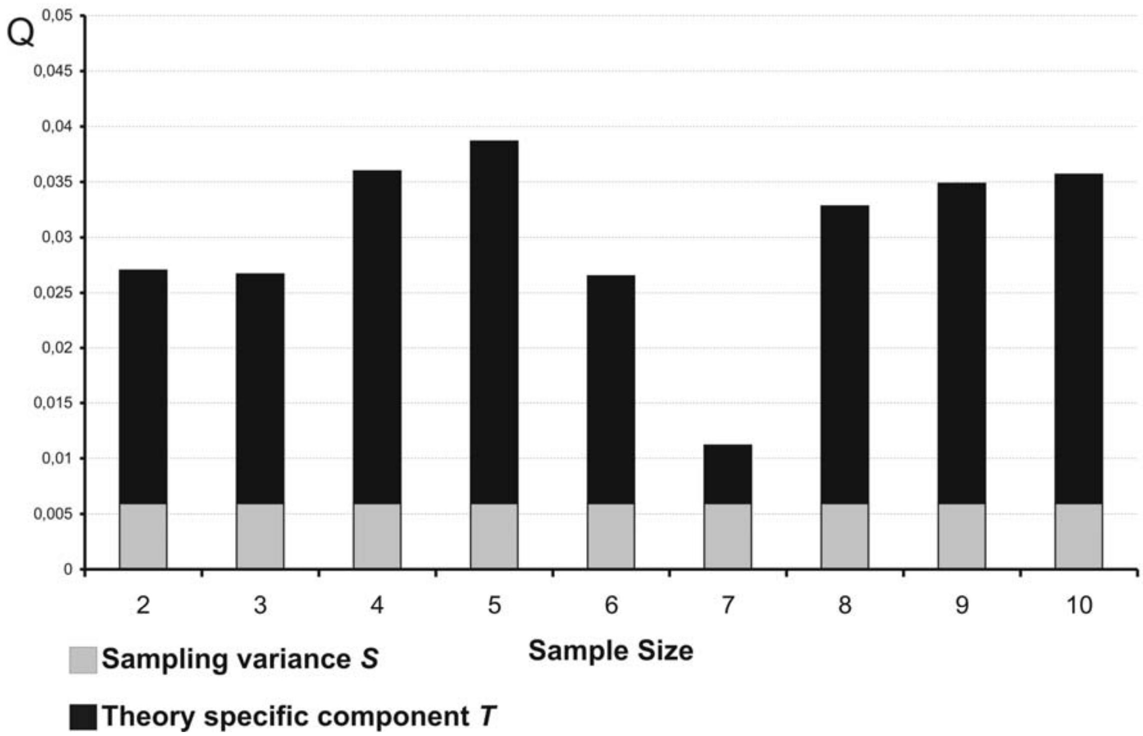

Figure 12: Overall mean squared distances *Q* for the sample-*n* equilibria

The figure also shows the mean sampling variance in grey It can be seen that for the sample size 7 the mean squared distance *Q* is much nearer to its unavoidable part *S* than for all other sample sizes.

4.4. Original versus Transformed Games

The basic idea of impulse balance is applied to the transformed game rather than the original one. This idea could also be applied directly to the original game. As we shall see later, the application to the transformed game yields a better fit to the data. This was already true for the pilot study on games A and B. We therefore decided to test impulse balance theory in the form described in section 2. However, it is of interest to examine the question how the direct application compares to the concept of impulse balance equilibrium proposed here.

It could be the case that not only the predictive power of impulse balance equilibrium but also that of other concepts is increased by applying them to the transformed game rather than to the original one.

We shall examine this question for Nash equilibrium and sample-7 equilibrium. Contrary to Nash equilibrium and quantal response equilibrium, sample-7 equilibrium fits the data quite well. It is therefore of special interest to explore whether a better fit could be obtained by applying it to the transformed game rather the original one. If in this way one obtained a better fitting version of sample-7 equilibrium, then this version should be compared with the other three theories.

We did not examine, what happens, if quantal response equilibrium is applied to the transformed game rather the original one. In the cutouts of figures 10 and 11 quantal response equilibrium is always very near to Nash equilibrium and it can be expected that this would not change in an application to the transformed game.

Figure 13 shows the overall mean squared distances for Nash equilibrium, sample-7 equilibrium and impulse balance equilibrium applied directly to the original game or to the transformed game. It can be seen that only impulse balance theory profits from being applied to the transformed game whereas Nash equilibrium and sample-7 equilibrium do not gain by being modified in this way.

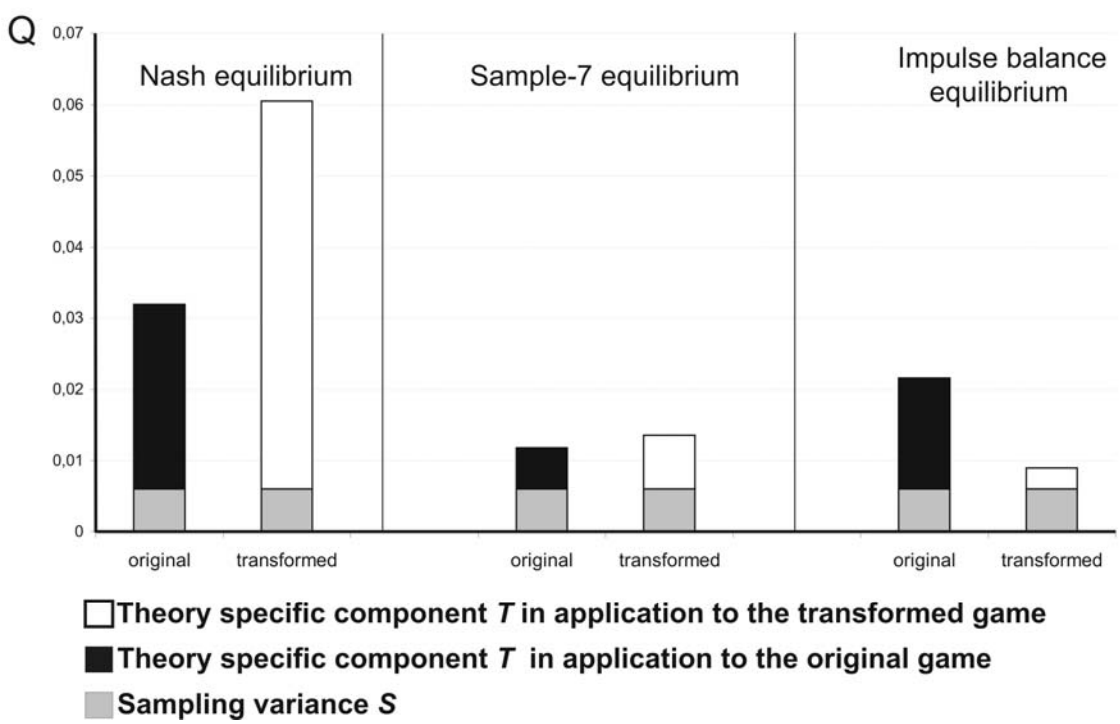

Figure 13: Advantages and disadvantages of applying a concept to the transformed game rather the original one.

The figure also shows the decomposition of the mean squared distance Q into the sampling variance S (grey) and the theory specific component T (black and white resp.). The difference between the theory specific components between the applications to the original game and the transformed one are even more dramatic in the case of impulse balance equilibrium, if one looks at the theory specific components instead of the mean squared distance.

In view of figure 6 it seems to be justified not to add the modifications of Nash equilibrium and sample-7 to the list of the four theories which are the main focus in this paper.

As we shall see in the next section, impulse balance equilibrium is a significantly better predictor for our data than the other theories. Figure 14 shows that this success is not mainly due to the use of the transformed game. Otherwise the predictive success of other concepts should be improved as well if they are applied to the transformed game rather than the original one. This is not the case.

4.5. Comparison of the Four Theories

Table 3 shows the mean squared distances of the four theories for the twelve games separately. It also contains the sampling variance for each game.

	Nash equilibrium	**Quantal response equilibrium**	**Sample-7 equilibrium**	**Impulse balance equilibrium**	**Sampling variance**
Game 1	.0572	.0460	.0103	.0108	.00909
Game 2	.0483	.0428	.0164	.0102	.00693
Game 3	.0321	.0250	.0087	.0073	.00523
Game 4	.0169	.0137	.0072	.0054	.00403
Game 5	.0149	.0136	.0115	.0117	.00953
Game 6	.0042	.0039	.0027	.0045	.00246
Game 7	.1237	.1082	.0189	.0081	.00178
Game 8	.0298	.0269	.0106	.0060	.00531
Game 9	.0212	.0192	.0332	.0224	.01409
Game 10	.0208	.0196	.0134	.0111	.00665
Game 11	.0098	.0084	.0059	.0036	.00307
Game 12	.0045	.0042	.0033	.0073	.00317

Table 3: Squared distances of the four theories.

Figure 14 shows the overall mean squared distances of the four theories compared in this paper. It can be seen that there is a clear order of success: Impulse balance equilibrium, sample-7 equilibrium, quantal response equilibrium and Nash equilibrium. The figure also shows the sampling variance *Q* in grey and the theory specific components in black.

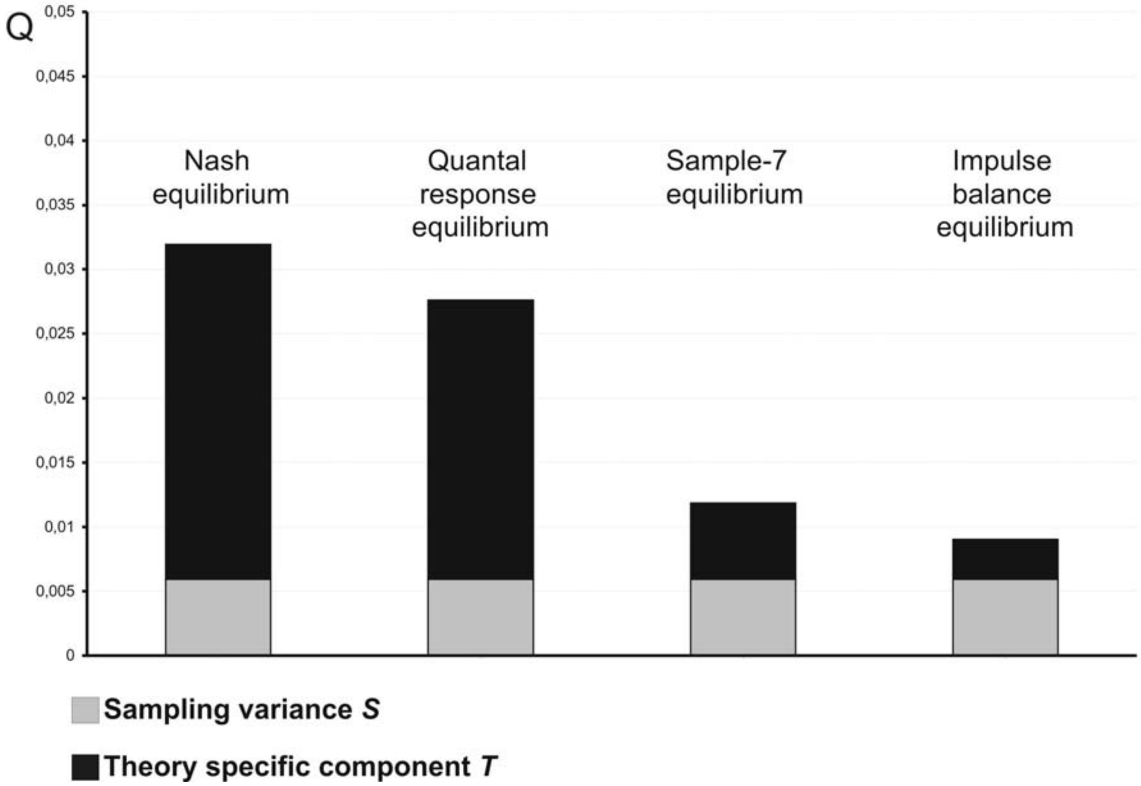

Figure 14: Overall mean squared distances of the four stationary concepts compared to the observed average.

4.6. Changes over time

The question arises whether the order of predictive success of the four theories remains stable over time. It could be the case that the superiority of impulse balance theory is an initial effect which becomes weaker and is finally reversed in the long run. Of course we can investigate this question only within the span of the 200 periods played in our experiments. For this purpose we compared the first hundred periods with the second hundred periods. Figure 15 shows the mean squared distances decomposed into sampling variance (grey) and the theory specific components (black and white resp.) for periods 1-100 (left) and 101-200 (right) for the four theories compared in this paper. It can be seen that in the second half of the experiments the order of predictive success is the same one as in the first half. Interestingly, for each of the four theories the predictive success in the second half of the experiments is better than in the first half. The sampling variance however is greater in the second half, than in the first half.

A two tailed matched pairs Wilcoxon signed rank test applied to the sampling variances for the first half and the second half in the twelve games shows no significant difference. Therefore we interpret the difference between the sampling variances in figure 15 as due to a random effect. The improvement of all four theories in the second half of the experiment indicates a movement of the observed relative frequencies nearer to the convex hull of the theoretical probability vectors. The relative frequencies for the first and the second half of game 4 are both inside the convex hull but for the other 11 games the relative frequencies for the first half are outside the convex hull. In the second half they are either inside (4 games) or still outside but nearer to the convex hull (7 games).

It is clear that the order of predictive success of the four theories is not due to peculiarities of behavior in early periods. Within the 200 periods one does not observe any reversal of this order. The data suggest convergence to a stationary distribution near to the impulse balance equilibrium. However this convergence is not due to a decrease of the sampling variance which is even slightly higher in the second half, than in the first half of the experiments. Of course one cannot assert anything about what will happen in the course of, say, 500 periods but our results do not convey the impression that a change of the order of the predictive success must be expected.

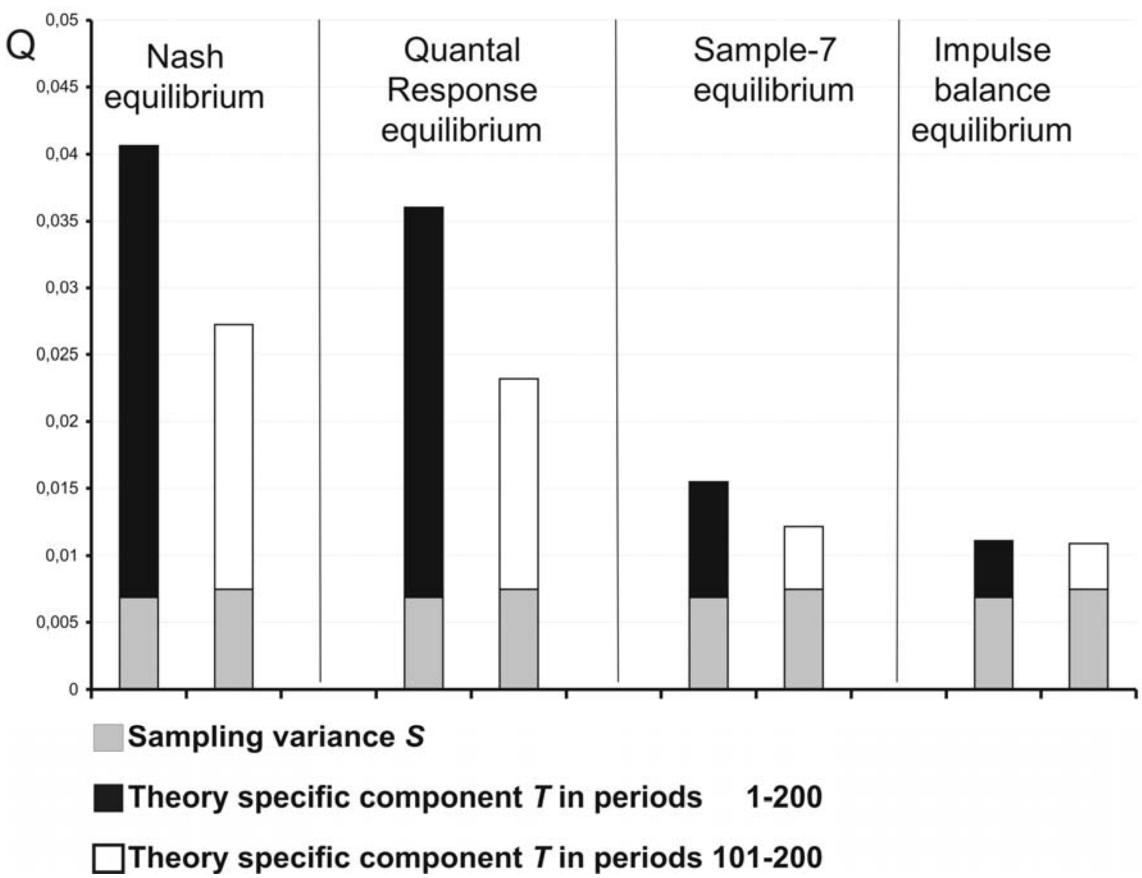

Figure 15: Comparison of predictive success in the first half and second half of the experiments.

4.7. Significance of the comparisons of predictive success

In section 4.1. we have pointed out that the discrimination between the four concepts tends to be the worse the nearer the games are to the middle of the parameter space. Therefore we cannot expect significant results for the twelve or six observations for each of the games separately. It is more reasonable to apply a test to all constant sum games together and to do the same for all non-constant sum games together.

In order to compare the performance of two theories in the six constant sum games we apply the Wilcoxon matched pairs signed rank test to the squared distances of the theoretical values from the observed relative frequencies for the 72 independent subject groups.

In the application of this test differences of the squared distances are computed for each of the 72 observations and then ranked from 1 to 72 according to their absolute value. Smaller absolute values receive a lower rank. The test statistic is the sum of the ranks in favor of first theory, in the sense that the squared distance for the first theory is lower than that for the second theory. This means that higher differences count more than lower ones, since they are less likely to be disturbed by random fluctuations. Therefore the fact, that games near the middle of the parameter space discriminate less among the theories, is automatically taken into account by the Wilcoxon matched pairs signed rank test.

Table 4 shows the two tailed significances in favor of the row concept.

	Sample-7 Equilibrium	**Quantal Equilibrium**	**Nash Equilibrium**
Impulse Balance Equilibrium	5% 10%	1% 1%	1% 1%
Sample-7 Equilibrium		1% 1%	1% 1%
Quantal Response Equilibrium			1% 1%

Above: games 1-6, constant-sum games (72 observations)
Below: games 7-12, non-constant sum games (36 observations)

Table 4: Significances of overall comparisons in favor of row concepts, two tailed matched pairs Wilcoxon signed rank test.

A clear order with respect to the goodness of fit emerges: Impulse balance equilibrium, sample-7 equilibrium, quantal response equilibrium and Nash equilibrium. The goodness of fit decreases in this order.

Impulse balance equilibrium is significantly more successful than the sample-7 equilibrium, but the difference is less pronounced than that for the other two stationary concepts.

5. Summary and Discussion

Four stationary concepts for completely mixed 2x2 games have been compared in this paper. For this purpose experiments on 12 games have been run, 6 constant sum games with 12 independent subject groups each and 6 non-constant sum games with 6 independent subject groups each.

The games were selected in such a way, that the constant sum games were reasonably well distributed over the parameter space. Each non-constant sum game had the same best reply structure as an associated constant sum game.

Each subject group consisted of 8 participants, four playing on one side and four on the other. Each subject group played only one game over 200 periods with random matching.

The literature reports about similar experiments with 2x2 games (GOEREE, HOLT, and PALFREY (2000); BINMORE, SWIERZBINSKI, AND PROULX. (2001); GOEREE, HOLT and PALFREY (2000); OCHS (1995)). Usually the number of periods played is much lower and more than one game has been played by the same subjects in one session. Thus in the Experiments by GOEREE, HOLT, and PALFREY (2000) the number of periods was 40. Several games were played one after the other in the experiments by BINMORE, SWIERZBINSKI, AND PROULX. (2001). We wanted a greater number of periods because it is doubtful whether a stationary state can be reached within only relatively few periods. Play must be long enough to wash out initial effects. If several games are played one after the other transfer of experience may occur from earlier to later games. Moreover data from different games played by the same subject are not statistically independent from each other. In our experiment each subject participated in only one independent subject group. This is necessary for an appropriate application of statistical tests.

Our results show that impulse balance theory has a grater predictive success than the other three stationary concepts, sample-7 equilibrium, quantal response equilibrium and Nash equilibrium. This is true for the constant sum games and the non-constant sum games examined separately.

Instead of sample-7 equilibrium one could also consider sample-*n* equilibria with other sample sizes *n*. The case *n*=7 is suggested by the finding that 7 seems to be near to the average number of items which can be kept in short-term memory (MILLER 1951). It has been shown that sample size 7 fits our data much better than other sample sizes between 2 and 10.

It is of great importance that even for completely mixed constant sum 2x2 games Nash equilibrium fails in comparison to other concepts.

In this paper we concentrated on games played repeatedly with random matching by two populations. The literature reports also experiments on 2x2 games played repeatedly by the same two opponents. Behavior in such games may very well be different from that in games played by populations. If two subjects play the same 2-person zero-sum game hundred times against each other, they will be concerned about not being predictable. This may drive them nearer to maximin strategies. The experimental investigation by O'NEILL (1987) and an empirical paper by WALKER & WOODERS (2005) on "Minimax Play at Wimbledon" suggests that this may be the case.

In our experiments quantal response equilibrium performs significantly better than Nash equilibrium. Quantal response equilibrium was applied with the same free parameter estimated from the data for all games. This parameter was quite high, probably because it has to accommodate relatively many games with a diverse structure. This is maybe the reason, why the predictions of quantal response equilibrium are on the one hand very near to those of Nash equilibrium theory and on the other hand nevertheless significantly better. Of course the addition of a parameter which can be adjusted to the data is always likely to improve performance.

In the same way as Nash equilibrium and quantal response equilibrium, sample-7 equilibrium is still a concept based on best replies, even if these are not best replies to the equilibrium strategies of the others, but to a random sample of strategies on the other side. We have also shown that the sample size 7 fits the data better than other sample sizes between 2 and 10.

Impulse balance equilibrium is very different from the three other concepts since it is not based on best replies. It cannot be considered to be a modification of Nash equilibrium. Impulse balance is different from optimization even in one person decision problems (SELTEN, ABBINK AND COX (2001), OCKENFELS AND SELTEN (2005)). Moreover impulse balance equilibrium is applied to a transformed game. The transformation is based on the idea that losses relative to a natural reference point (the second lowest payoff) count double.

Impulse balance theory could also be applied to the original game but the application to the transformed game improves its performance. If Nash equilibrium or sample-7 equilibrium is applied to the transformed game rather the original one, the performance of these concepts becomes worse. The transformation is an important part of impulse balance theory but it is not the only reason for its success.

It is not easy to understand why the predictions of impulse balance equilibrium and sample-7 equilibrium are not very far apart, in spite of the fact, that they are based on very different principles. This is maybe peculiar to our sample. It would be desirable to devise experiments which permit a better discrimination between the two theories.

In this paper we look at stationary concepts without any discussion of learning processes. The comparison of our data with learning processes will be the subject matter of a later paper. As far as movement over time is concerned we looked only at differences between periods 1-100 and 101-200. We have seen that the order of predictive success of the four theories does not change from the first half to the second half of the experiments. The sampling variance is slightly increased but all four theories are more successful in the second half than in the first half. The mean frequencies of individual observations seem to move nearer to the theoretical predictions, even if within a game the variance of the relative frequencies in independent subject groups does not change significantly. One cannot know whether the stationary distribution is reached within the 200 periods but the evidence conveys the impression that one comes near to it.

Stationary concepts are of great importance especially if they do not depend on parameters which have to be adjusted to the data. Impulse balance theory does not involve any such parameters and can be used in theoretical investigations just like Nash equilibrium. It is possible to generalize impulse balance theory to general games in normal form. It would certainly be desirable to gain experiences with games with more than two strategies or more than two players.

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Appendix A

Observation	Game 1		Game 2		Game 3		Game 4		Game 5		Game 6	
	U	*L*	*U*	*L*	*U*	*L*	*U*	*L*	*U*	*L*	*U*	*L*
1	.104	.716	.255	.583	.218	.836	.291	.748	.154	.873	.453	.604
2	.079	.640	.175	.510	.154	.716	.230	.818	.378	.690	.439	.621
3	.091	.794	.156	.431	.210	.778	.320	.714	.358	.676	.430	.591
4	.109	.688	.210	.616	.217	.844	.245	.748	.276	.648	.398	.604
5	.085	.571	.240	.409	.154	.700	.318	.684	.341	.635	.444	.619
6	.059	.730	.151	.601	.232	.785	.360	.718	.320	.659	.389	.654
7	.184	.575	.286	.591	.081	.856	.283	.723	.295	.689	.463	.574
8	.044	.770	.195	.580	.170	.795	.284	.661	.329	.659	.421	.544
9	.048	.750	.225	.563	.093	.723	.371	.750	.353	.561	.438	.626
10	.056	.755	.229	.448	.133	.873	.249	.805	.328	.651	.535	.594
11	.034	.524	.206	.551	.164	.829	.266	.741	.366	.583	.428	.560
12	.056	.768	.275	.441	.130	.778	.213	.720	.431	.640	.505	.566
Agg 12 Observations	**.079**	**.690**	**.217**	**.527**	**.163**	**.793**	**.286**	**.736**	**.327**	**.664**	**.445**	**.596**

Observation	Game 7		Game 8		Game 9		Game 10		Game 11		Game 12	
	U	*L*	*U*	*L*	*U*	*L*	*U*	*L*	*U*	*L*	*U*	*L*
1	.151	.531	.199	.571	.164	.744	.451	.745	.274	.645	.441	.653
2	.103	.563	.180	.665	.105	.793	.416	.711	.289	.659	.414	.653
3	.176	.596	.246	.529	.188	.839	.299	.634	.336	.688	.431	.559
4	.178	.575	.341	.610	.299	.869	.365	.729	.410	.631	.463	.568
5	.090	.530	.314	.585	.355	.844	.416	.713	.378	.678	.458	.664
6	.149	.586	.220	.559	.413	.874	.246	.665	.301	.611	.428	.529
Agg 6 Observations	**.141**	**.564**	**.250**	**.586**	**.254**	**.827**	**.366**	**.699**	**.331**	**.652**	**.439**	**.604**

Appendix B: Written instructions

Merkblatt zum Matrixexperiment

An diesem Experiment nehmen 16 Personen teil. Jeder Teilnehmer ist entweder ein Spieler 1 oder ein Spieler 2. Diese Rolle behalten Sie über die ganze Dauer des Experimentes bei.
Das Spiel erstreckt sich über 200 Runden.
In jeder Runde spielt jeder Spieler 1 mit einem Spieler 2. Die 8 Spielerpaare werden in jeder Runde neu zufällig zusammengestellt.

Auf dem Bildschirm sehen sie eine Matrix mit vier Feldern.

In jeder Runde haben sie die Möglichkeit zwischen Zeile A oder Zeile B zu wählen.
Ihre eigene Auszahlung ist auf dem Bildschirm umrandet dargestellt. Ihre Auszahlung hängt von ihrer eigenen Wahl und der Wahl des anderen Spielers ab. Nachdem Sie diese Wahl getroffen haben, färbt sich Ihre gewählte Zeile rot. Nach der Wahl des anderen färbt sich das Feld gelb, in dem der Betrag steht, der ihnen ausgezahlt wird.

	A	**B**
A	**10** **8**	**0** **18**
B	**9** **9**	**10** **8**

Es gibt zwei Gruppen von Spielern. In jeder Gruppe hat jeder Spieler dieselbe Matrix, aber die Matrizen sind für beide Gruppen verschieden. Sie spielen immer mit einem Spieler aus der anderen Gruppe.
In jeder Runde werden 8 Spielerpaare zufällig zusammengestellt. Ihnen wird also in jeder Runde ein neuer Mitspieler zugelost. Ihre Mitspieler haben immer dieselbe Matrix.

Nach jeder Runde wird Ihnen mitgeteilt welche Auszahlung sie in der letzten Runde erhielten. Der Umrechnungskurs für ihre Auszahlung wird Ihnen auf dem Bildschirm bekannt gegeben.

Appendix C: Questionnaire

Fragebogen zum Matrixexperiment

Terminalnummer/Kartennummer:

Studienfach: .... Semester: ....

Beruf: ....

Geschlecht: ☐ männlich ☐ weiblich Alter: ....

Beschreiben Sie kurz die Gründe für Ihre Entscheidungen: ....
....
.
....
....
..
Hat sich Ihr Entscheidungsverhalten im Laufe des Experiments verändert? Wenn ja, wie?
....
.
....
....
....
...
Kommentar zum Experiment: ....
....
....
....
...

Vielen Dank für Ihre Mitarbeit.

Appendix D: Screenshot of Game 1