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by

Manisha Chakrabarty

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Bonn Graduate School of Economics
Department of Economics
University of Bonn
Adenauerallee 24 - 42
D-53113 Bonn

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The Law Of Aggregate Demand : Empirical Evidence From India Using Nonparametric Direct Average Derivative Estimation Procedure*

Manisha Chakrabarty†

Key Words: Aggregation, Heterogeneity, Income effect, Law of demand, Average derivative estimator, Nonparametric regression.

JEL Classification Number: D12, C12, C14.

Abstract

This paper attempts to provide empirical evidence of the positive definiteness of the mean income effect matrix, a sufficient condition for market demand to satisfy the *law of demand* derived by Härdle, Hildenbrand and Jerison [HHJ(1991)]. Increasing heterogeneity in spending of populations of households leads to this sufficient condition which is falsifiable from cross-section data. Based on this framework we use the National Sample Survey (NSS) 50-th round data (1993-1994) for the rural sector of Maharashtra to examine the empirical viability of this condition. Due to a restrictive assumption on the density function and several other limitations of the indirect method we use the nonparametric direct average derivative estimation procedure [Stoker (1993)], unlike the indirect method used in the HHJ paper. It is shown that the income effect matrix is, indeed, positive definite. The required heterogeneity condition is also well supported in this data where one can not expect too much variation in spending patterns of population given the source of data, i.e., rural sector of a developing economy.

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†Correspondance to : Dr. Manisha Chakrabarty
Wirtschaftstheoretische Abteilung II
University Of Bonn, Lennestrasse. 37, D 53113, Bonn, Germany
Tel: +49-228-737993, e-mail: manisha.chakrabarty@wiwi.uni-bonn.de

1 Introduction

We know that pure theory of individual consumer behaviour does not lead to useful structure of the market demand such as the *law of demand.* This property requires that the Jacobian matrix of price derivatives is negative definite which is not ensured by the utility maximizing behaviour of individuals. There is a straightforward way to derive this *law of demand* by imposing the homotheticity of utility function; this restrictive assumption, however, is not well supported by empirical evidence [for a detailed discussion of aggregate demand see Mas-Collel, Whinston and Green (1995)].

In the paper by Härdle, Hildenbrand and Jerison (1991) [from now on we define them as HHJ] it is shown that the *law of demand* holds at the aggregate level without assuming it to hold at the individual level. Hence it is not necessary to impose specific condition on the individual preferences. Thus their approach is to explain some of the structures found in the aggregate data with only minimum reference to the behaviour of individuals. They have considered the statistical distribution of preferences and income and the condition to satisfy the *law of demand* follows from aggregation of heterogeneous household's demand. This alternative approach has rightfully been described as *the macroscopic microeconomic approach* by Aruka (2000) because it takes into account the population of households as a whole. The novelty of their approach can be traced to several points, (i) this model does not consider any behavioural assumption and structural modelling, (ii) it takes into account the heterogeneity of preferences, i.e., without any identical or homothetic preferences assumption and (iii) the required distributional property can be tested from the most commonly available data source, i.e., cross-section survey data.

The *law of demand* is equivalent to negative definiteness of the Jacobian matrix of price derivatives of mean (aggregate) demand. This Jacobian matrix can be decomposed into a mean of individual Slutsky substitution matrices and a mean of income effect matrices. Then assuming negative semidefiniteness of the mean substitution matrix[1], the sufficient condition for the *law of demand* to hold is positive definiteness of the mean income effect matrix. This does not follow from the theory of individual consumption behaviour. HHJ have shown that this condition can be derived from the assumption of increasing dispersion of the distribution of demands. Hence, if no sufficiently large number of consumers violates the law of demand for the same commodity bundle then the *law of demand* will hold in the aggregate.

[1]This would follow from the utility maximisation, more generally, from the weaker condition, i.e., Weak Axiom of Revealed Preference (WARP). Yet this does not imply a sufficiently strong negative substitution matrix to offset the income effect whose property can not be derived from the theory.

We use the nonparametric direct average derivative estimation (DADE) procedure to estimate the mean income-effect matrix and also the variance-covarince matrix of consumption expenditure from cross-section data where each element of the matrix is obtained as an average derivative of the mean response function. Though this direct method is asymptotically equivalent to the indirect method used in HHJ's paper [Stoker (1991)], in the small sample case, the indirect method is subject to more bias and is unstable. We use the NSS 50-th round (1993-1994) cross-section data for the rural sector of Maharashtra, India, to examine the empirical viability of this condition which has been tested empirically in the developed economy of UK and France. Quite contrary to developed economies this rural sector data of the developing economy has a very limited scope to have sufficient heterogeneity in the individual consumers' demand curves. This is due to the fact that in such an economy consumers' consumption basket mainly consists of inferior goods. Here consumers don't get much scope to reallocate their expenditure patterns in favour of better quality goods belonging to the same commodity group to overcome the negative income effect due to nonavailability of such goods. Additionally, the income level is also not sufficiently large to establish this increasing heterogeneity on average.

In the following section we describe briefly the theory of HHJ and the estimation procedure; section 3 describes the data used in our analysis; section 4 analyses the results obtained and finally, conclusions are drawn in section 5.

2 Theoretical & Empirical Background

Each household h's demand function is described by the function

$$f_h : (p, x_h) \rightarrow f_h(p, x_h) \in \mathcal{R}^l_+$$

where $p \in \mathcal{R}^l_{++}$ denotes the price vector of the l commodities and x_h is the total expenditure of household h. The Individual demand function is assumed to satisfy the budget identity and homogeneity condition. Market demand is, therefore, defined by

$$F(p) = \frac{1}{\#H} \sum_{h \in H} f_h(p, x_h) \tag{2.1}$$

The *law of demand* is defined by the monotonicity of F

$$(p - q) \cdot (F(p) - F(q)) < 0 \tag{2.2}$$

for any two different price vectors p and q. It is known that F is monotone if the Jacobian matrix

$$\frac{\partial F(p)}{\partial p} = (\frac{\partial F_k(p)}{\partial p_j})_{j,k=1,\dots l}$$

is negative definite for every $p \in \mathcal{R}^l_{++}$. If the individual demand function f_h is differentiable then the Jacobian matrix of price derivatives can be decomposed into the Slutsky substitution matrix and the income effect matrix, such as,

$$\partial_p f_h(p, x_h) = S f_h(p, x_h) - \partial_x f_h(p, x_h) f_h(p, x_h)^T \tag{2.3}$$

T denotes the transpose, the matrix $Sf_h(p, x_h)$ is called the Slutsky substitution matrix and the second element is called the matrix of income effects. Then the Jacobian matrix of the mean demand function F can be written as

$$\begin{aligned} \partial_p F(p) &= \frac{1}{\#H} \sum_{h \in H} S f_h(p, x_h) - \frac{1}{\#H} \sum_{h \in H} \partial_x f_h(p, x_h) f_h(p, x_h)^T \\ &= \quad \overline{S}(p) \qquad\qquad\qquad - \quad \overline{M}(p) \end{aligned} \tag{2.4}$$

Now assuming that all individual demand functions satisfy the Weak Axiom of Reveleaded Preference (WARP) implies that the individual substitution matrix $Sf_h(p, x_h)$ is negative semidefinite and hence the same property holds for the mean of the individual substitution matrices. Even if some households violate the WARP slightly, their effect on the mean Slutsky matrix $\overline{S}$ can be counterbalanced by other households who satisfy the axiom. A sufficient condition for the market *law of demand* is, therefore, the positive definiteness of the mean income effect matrix $\overline{M}(p)$. This property does not follow from the rationality of individual behaviour. To establish the desired property of $\overline{M}(p)$, HHJ proceed as follows: For convenience consider a large population with different demand functions and different levels of income. Then one can consider labeling of the demand functions by an index $\alpha \in \mathcal{A}$, where the index set $\mathcal{A}$ may be a finite set with $f^\alpha(p, x) \neq f^{\alpha'}(p, x)$ if $\alpha \neq \alpha'$. Similarly one can treat income as a continuous variable so that each household h is described by a pair $(x_h, \alpha_h) \in \mathcal{R}_+ \times \mathcal{A}$, i.e., by household's budget x_h and demand function f^{α_h}. Then the market (mean) demand function F is defined as

$$F(p) = \int_{\mathcal{R}_+ \times \mathcal{A}} f^\alpha(p, x) d\mu \tag{2.5}$$

where μ is the joint density of x and α on $\mathcal{R}_+ \times \mathcal{A}$. The mean income effect matrix $\overline{M}(p)$ involves $\partial_x f_h(p, x)$ [equation 2.4] which is unobservable, hence a matrix A, which can be estimated from cross-section data, is defined as

$$A = \int_{\mathcal{R}_+} (\partial_x \overline{G(x)}) \rho(x) dx \tag{2.6}$$

with $\overline{G(x)}$ is the matrix with elements $\overline{g_{jk}} = \int_{\mathcal{A}} f^\alpha_j(x).f^\alpha_k(x) d\mu|x$. $\mu|x$ denotes the conditional distribution of α given the budget level x and ρ denotes the price-independent marginal density of income. This matrix A can be estimated from cross-section data since each element of this matrix is an average derivative of the regression function $x \to \int g_{jk}(x, \alpha) d\mu | x$.

The symmetrized matrix $M = \overline{M} + \overline{M}^T$ is given by

$$M = \int_{\mathcal{R}_+} [\int_{\mathcal{A}} \partial_x G(x, \alpha) d\mu | x] \rho(x) dx$$

with $G(x, \alpha) = (g_{jk}(x, \alpha)) = (f_j^\alpha(x) \cdot f_k^\alpha(x))_{j,k=1,..,l}$. One obtaines $M = A$ if for every x

$$\int_{\mathcal{A}} \partial_x G(x, \alpha) d\mu | x = \partial_x \int_{\mathcal{A}} G(x, \alpha) d\mu | x \tag{2.7}$$

i.e., the conditional mean of the derivatives of the product of demand functions is equal to the derivative of the conditional mean of the product functions. HHJ described this condition as *metonymy.*.

The implicit assumption is that if consumers at one income level were all given a little more income, the resulting distribution of their demands would resemble the distribution of consumers that already have this slightly higher income. A sufficient condition for a *metonymic* population is that the distribution of all other relevant determinants of demand are locally independent of the income level. Hence it might well happen that this *metonymy* condition is not satisfied for the whole population, but after appropriate stratification of the population of households into subpopulation with more homogeneous characteristics which seem to affect demand, this *metonymy* condition is satisfied. *Metonymy* in each subpopulation s implies that $M_s = A_s$. Since $\mu = \sum p_s \mu_s$, $M = \sum p_s M_s = \sum p_s A_s$, where p_s is the fraction of population in the subgroup s. Therefore, the *metonymy* of the entire population can be tested empirically with the null hypothesis

$$H_0 : A = \sum p_s A_s \tag{2.8}$$

Now from equation (2.6) we have seen that $\overline{G(x)}$, the average value of squared demands $f(x)$ across all households with total expenditure x, measures the heterogeneity of households' demands for a given level of total expenditure. A large value of it implies substantial diversity of demand across individuals with the same level of total expenditure x. The *Law of demand* requires $E(\overline{G'(x)}) > 0$, implying that as x increases the spread or heterogeneity of demand increases on an average. Increasing dispersion is the required distributional property which leads to the *law of demand.* This has been empirically tested for a collection of consumption goods using UK and French data by Hildenbrand and Kneip (1993). Lewbel (1994) gives a nice view of this *increasing spread* in the one-commodity case. One possible explanation may be that poorer households must devote a larger fraction of their total expenditure on necessities. Therefore they have less variety in the allocation across goods, while the rich households with a higher level of x have much scope for varying their consumption expenditure. Whether this condition of increasing variation in consumption expenditure even with a limited range of x holds or not is one of the major thrusts in this paper.

This special form of heteroscedasticity has been tested in terms of the covariance matrix of demands, in which the diagonal elements indicate the variance of the demands for a particular good by x households. The jk-th component of the conditional covariance matrix $C(x)$ can be written as

$$\int_{\mathcal{A}} f_j^{\alpha}(x) f_k^{\alpha}(x) d\mu | x - \overline{f_j(x)}\;\overline{f_k(x)} \tag{2.9}$$

where $\overline{f_j(x)} = \int_{\mathcal{A}} f_j^{\alpha}(x) d\mu | x$ is the Engel curve for the j-th good. The average derivative of the conditional covariance matrix $C(x)$ is given by

$$V = \int \partial_x C(x) \rho(x) dx \tag{2.10}$$

The average increasing dispersion of demands with the increase in the size of the budget x implies positive semidefiniteness of the matrix V with strict positive diagonal components.

From the above expressions 2.6 and 2.10 it is well understood that the estimation technique should be based on the average derivative method. We use the nonparametric average derivative estimation methodology [Stoker, 1991]. We divide each household's demand by the mean budget. The prices are normalized to 1, quite reasonable in a single period cross-section study. Therefore the demand for a good by a particular household is household's expenditure on the good divided by the mean budget for the whole population [see HHJ]. Hence in this model the regressor is X_h, defined as $X_h = \frac{x_h}{\overline{x}}$ and the regressand is the joint product of the demnd functions $Y_h = \frac{f_j^{\alpha h}(x_h) f_k^{\alpha h}(x_h)}{\overline{x}^2}$. The regression function, therefore, is $m(x) = E(Y_h | X_h = x)$ and the average derivative is defined as $\delta = E(m'(x))$ with $m' = \frac{\partial m}{\partial x} = \int m'(x) \rho(x) dx$.

In this paper we use the direct average derivative estimator (DADE), in which we take the average of estimated values of the derivative $\widehat{m'(x)}$. Though this direct estimators and the indirect estimator used in HHJ paper are asymptotically equivalent, there are some limitations of the indirect method in the small sample case. The indirect estimator is based on the assumption that the density function $\rho(x)$ vanishes on the boundary which makes the estimator more unstable. If one takes a cut-off point or a bandwidth such that a sufficient number of observations belongs to the tail-region, then the basic assumption will not be satisfied and it may also create oversmoothing in the intermediate region. Then one should consider the case of variable bandwidth, which is yet to be resolved. Besides, the indirect method is subject to more bias as it's expectation differs from the true average derivative by the boundary terms, which vanishes only in the limit and which is also controlled by the choice of cut-off point. Above all, this vanishing boundary condition is not satisfied in the subpopulations if the subpopulation is of high-density region and in this case the indirect estimator is not a consistent estimator of the true average derivative.

The DADE estimator of δ is defined as

$$\hat{\delta} = \frac{1}{N} \sum_{h=1}^{N} \widehat{m'(x_h)}, \quad h = 1,,,,N \tag{2.11}$$

While N is the number of households. We use the Nadaraya-Watson Kernel Regression estimator of the regression function $m(x)$ as

$$\widehat{m(x)} = \frac{\widehat{D(x)}}{\widehat{\rho(x)}}$$

where

$$\widehat{D(x)} = \frac{1}{Nb} \sum_{h=1}^{N} K(\frac{x - X_h}{b}) Y_h$$

and the Rosenblatt-Parzen kernel estimator $\widehat{\rho(x)}$ of the density estimator is

$$\widehat{\rho(x)} = \frac{1}{Nb} \sum_{h=1}^{N} K(\frac{x - X_h}{b})$$

Here b is the bandwidth and we have used the quartic kernel of the form

$$K(u) = \frac{15}{16}(1 - u^2)^2 \ \ for |u| \leq 1$$

We explore quite a high range of values of bandwidth values, ranging from $.25 * std\ dev(X_h)\ to\ 2 * std\ dev(X_h)$ and we choose that value of b where the matrix components and eigenvalues are most stable.

To test for the positive definiteness of the income effect matrix and the V matrix we use the bootstrap test [Härdle and Hart (1992)] concerning the smallest eigenvalue of the income-effect matrix. To define a small sample distribution of the minimum eigenvalue for the income effect matrix we consider a bootstrap sample (X_h^*, Y_h^*) drawn at random with replacement from the original sample and we compute A^* for each of the sample. Then we calculate the minimum eigen value λ^* and estimate the upper and lower confidence bounds c_{up} and c_{low} to conduct a test of the hypothesis that $H_0 : \lambda = 0$ against the alternative that $H_1 : \lambda > 0$ at the 95 % confidence level. We consider the bootstrap distribution of $\hat{\lambda}^* - \hat{\lambda}$ with the 95 % confidence interval $[-L_1^*, L_2^*]$. Therefore, the confidence interval for λ is $(c_{low} = \hat{\lambda} - L_2^*,\ c_{up} = \hat{\lambda} + L_1^*)$ because the bootstrap distribution of $\sqrt{N}(\hat{\lambda}^* - \hat{\lambda})$ is asymptotically close to $\sqrt{N}(\hat{\lambda} - \lambda)$ [Härdle and Hart (1992)]. A bootstrap test then is conducted in the following way [Hildenbrand (1994), Hildenbrand & Kneip (1993)]:

1. Reject the hypothesis that the matrix is not postive definite if $c_{low} > 0$, i.e., if the entire confidence interval for the smallest eigenvalue is positive.
2. Reject the hypothesis of positive semidefiniteness if $c_{up} < 0$, i.e., the entire confidence interval for the minimum eigenvalue is negative[2].

Now to test for *metonymy* we have used the test procedure suggested by HHJ. If $\widehat{\xi_1}$ is the vector of elements (here we have tested only for the diagonal elements of the mean income-effect matrix) of $\hat{A}$ and $\widehat{\xi_2}$ is the vector of diagonal elements of the weighted average of the $\hat{A}_s$ matrices, then the test for the difference between these elements in $\widehat{\xi_1}$ and $\widehat{\xi_2}$ is formulated as

$$W = N(\widehat{\xi_1} - \widehat{\xi_2})^T \widehat{\Sigma^{-1}} (\widehat{\xi_1} - \widehat{\xi_2}) \tag{2.12}$$

which follows chi-square with $(no\ of\ elements - 1)$ degrees of freedom, T indicates the transpose. The variance-covariance matrix of differences is estimated as

$$\widehat{\Sigma} = N^{-1} \sum_{h=1}^{N} (\widehat{r_h^1} - \widehat{r_h^2})(\widehat{r_h^1} - \widehat{r_h^2})^T$$

where $\widehat{r_h} = [\widehat{m'(X_h)} - (Y_h - \widehat{m(X_h)})]\frac{\hat{\rho}'}{\hat{\rho}}$. This is done for the stratified (indicated by $\widehat{r_h^2}$) and also for the unstratified $(\widehat{r_h^1})$ case.

3 Data

The data used is the National Sample Survey (NSS) 50-th round data for the period 1993-1994. We have considered only the rural sector of Maharashtra. The data consists of 4393 households. To avoid the very low density at rather large extreme values of total expenditure we exclude certain observations as outliers which are in the range $Q(75) + 3IQR < total\ \ expenditure < Q(25) - 3IQR$, where $Q(75)$ and

[2]Here we would like to mention that in all bootstrap samples we use the same value for bandwidth b as were used in constructing $\hat{A}$, as suggested by Härdle & Hart (1992).

$Q(25)$ denote the 3rd and the 1st quartile and IQR represents the inter-quartile-range of total expenditure. Hence our final data consists of 4301 observations.

We have considered four commodities, these are *food, beverages, intoxicants* and *fuel-light*[3]. We form subgroups to test the condition of *metonymy* for the whole population. This condition is satisfied if the income effect matrix of the whole population is equal to the weighted average of the subgroups' income-effect matrices. If it is not equal then the whole population or some subgroups must violate the condition of *metonymy.*

The characteristics we consider to form subgroups are two-folds - occupation status of the head of the household and household composition. The occupation types we consider are the following: nonagricultural self-employed households (SENA), agricultural labourer households (AL), other labourer households (OL), agricultural self-employed households (SEA), and other households (OTHERS).

Regarding the household composition we consider the following ten subgroups: single adult (1A), two adults (2A), more than two adults ($\geq 3A$), single adult with children (1A+$\geq 1C$), two adults with one child (2A+1C), two adults with 2 children (2A+2C), two adults with more than 2 children (2A $+\geq 3C$), more than two adults with 1 child ($\geq 3A+1C$), with 2 children ($\geq 3A+2C$), with more than two children ($\geq 3A+\geq 3C$).

The descriptive statistics of all commodity expenditures and of total expenditure for the whole sample and for subgroups are presented in the appendix.

4 Results

Table 4.1 presents the components of the $\hat{A}$ matrix multiplied by 100. The diagonal components depict the joint product of own demand and off-diagonal terms depict the cross-product. The largest value for *food* is self-explanatory given the fact that in the rural sector the highest expenditure share is on this commodity. The minimum eigenvalue is, indeed, positive.

Though in section 2 we have described how we have chosen the bandwidth parameter b, nevertheless we have tried with a range of b values around that chosen b value. But the final conclusion of the positive definiteness of the $\hat{A}$ matrix has never changed. Now to test the hypothesis of positive definiteness as described in section 2, we construct the bootstrap distribution of the minimum eigenvalue of this $\hat{A}$ matrix. We could not reject the hypothesis of positive definiteness at the 95%

[3]The detailed description of the commodities included in the groups is described in the appendix. Other commodities like services, rent, transport can not be taken into consideration because very few households are reported for this consumption categories in the survey.

Food	Beverage	Intoxicants	Fuel & light
55.64	4.85	2.27	6.38
	.83	.23	.58
		.25	.26
			.87
Minimum Eigen value: $\lambda_{min} \times 100 = .13$			

Table 4.1: The Income effect Matrix $\hat{A} \times 100$ for the whole sample

confidence level because $c_{low} = .102 > 0$.

To support the *law of demand* we further explore the covariance matrix to examine the special form of heteroscedasticity, i.e., increase in variances of demands as the level of x increases. Table 4.2 depicts the V matrix and table 4.3 presents the diagonal components of $C(x)$, i.e., the variances of the demands for budget levels at .5, 1, 1.5 and 2 times the mean budget. From table 4.3 we clearly see that for all goods the variance increases with an increase in x, though the increase is at the highest level for *food*. For *fuel & light* a small change seems plausible as there is not much scope to change it's expenditure level except some reallocations within the group, which is possible only at a very limited level in the rural sector. In case of *intoxicants*, there are lots of zero consumption and people do underreport or misreport the level of expenditure which may explain the observed values. The variances against level of total expenditure for all commodities are also depicted in figure 4.1 which clearly shows that on average variances increase with the level of total expenditure. Also from table 4.2 we see that the V matrix is positive semidefinite with all diagonal components positive. The bootstrap test procedure as described in section 2 in terms of the minimum eigenvalue of this matrix indicates that we cannot reject the hypothesis of positive semidefiniteness as $c_{low} = .124 > 0$. Therefore, even in a rural economy with highest expenditure share on *food*, it is shown that on average the heterogeneity of demands increases with the increase in total expenditure.

The smallest eigenvalues for all subpopulations are described in table 4.4. For all subpopulations, based on either occupation status or household composition, minimum eigenvalues turn out to be positive. The very small eigenvalues only occur in small subpopulations. The highest value of λ for the AL group is obvious if we see appendix for the descriptive statistics. In this group the average values of expenditure shares on the commodity groups are highest implying larger values of the elements of the $\hat{A}$ matrix[4].

[4] We have also explored with the indirect average derivative procedure and we observe that for

Food	Beverage	Intoxicants	Fuel & light
3.66	.01	.03	.21
	.38	.02	.01
		.16	-.001
			.15
Minimum Eigen value: $\lambda_{min} \times 100 = .14$			

Table 4.2: The Average Derivative of The Conditional Covariance Matrix, $\hat{V}$, For The Whole Sample.

commodity	.5	1	1.5	2
Food	.005	.017	.043	.081
Beverages	.001	.003	.010	.010
Intoxicants	.0004	.001	.002	.002
Fuel & light	.0004	.001	.002	.003

Table 4.3: The Diagonal Elements Of The Conditional Covariance Matrix For .5, 1, 1.5 & 2 Times The Mean Budget.

<table>
<tr><th colspan="10">Occupation Strata</th></tr>
<tr><td colspan="2">SENA</td><td colspan="2">AL</td><td colspan="2">OL</td><td colspan="2">SEA</td><td colspan="2">OTHERS</td></tr>
<tr><td colspan="10">$\lambda_{min} \times 100$</td></tr>
<tr><td colspan="2">.09</td><td colspan="2">.12</td><td colspan="2">.10</td><td colspan="2">.10</td><td colspan="2">.10</td></tr>
<tr><th colspan="10">Household Composition strata</th></tr>
<tr><td>1A</td><td>2A</td><td>$\geq 3A$</td><td>$1A+ \geq 1C$</td><td>2A+1C</td><td>2A+2C</td><td>$2A+ \geq 3C$</td><td>$\geq 3A + 1C$</td><td>$\geq 3A + 2C$</td><td>$\geq 3A+ \geq 3C$</td></tr>
<tr><td colspan="10">$\lambda_{min} \times 100$</td></tr>
<tr><td>.09</td><td>.17</td><td>.11</td><td>.02</td><td>.01</td><td>.03</td><td>.08</td><td>.10</td><td>.11</td><td>.16</td></tr>
</table>

Table 4.4: Minimum Eigen Values of $\hat{A}$ For Different Subsamples

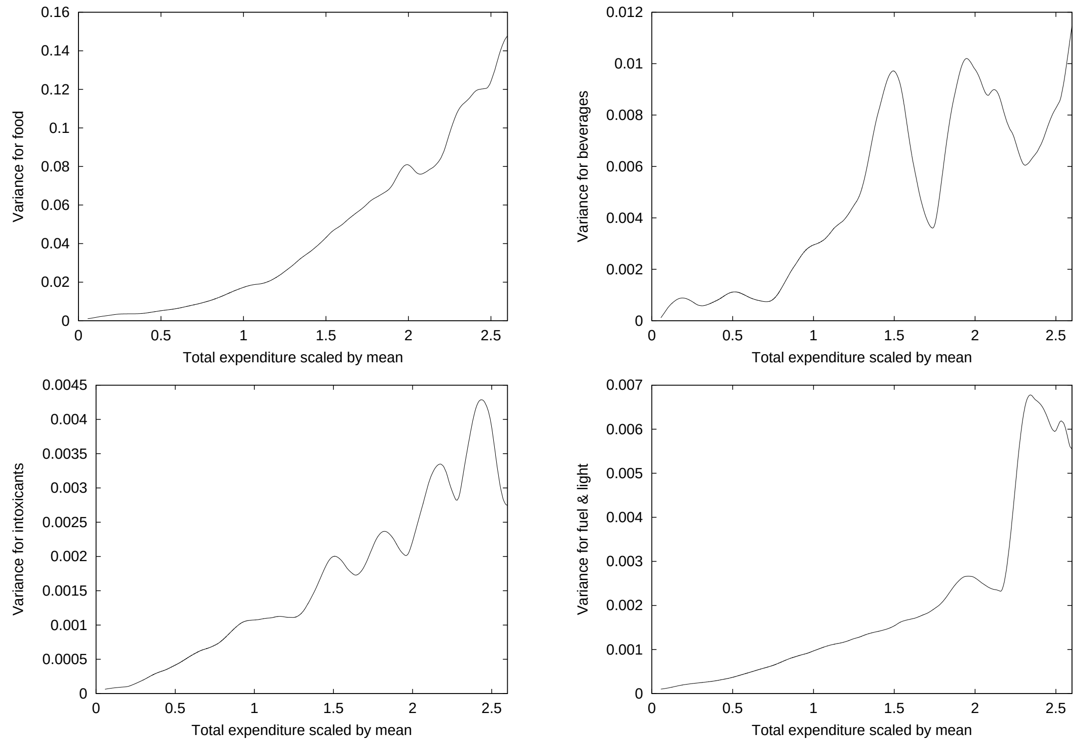

Figure 4.1: Plot of variances

The chi-square value of the test statistics for the equality of the weighted average of the subpopulation matrices and $\hat{A}$ is .514 for the occupation stratification and .640 for household composition stratification. Hence the hypothesis that the matrices are equal cannot be rejected and the *metonymy* condition is satisfied.

5 Conclusion

The main thrust of this paper is to provide empirical evidence of Hildenbrand's (1994) claim that heterogeneity of consumers' demands results in downward sloping uncompensated demand curves in the aggregate, more generally, in the *law of demand*. To justify this we use the direct nonparametric average derivative estimation methodology, different from Härdle, Hildenbrand and Jerison (1991) to avoid certain limitations in their methodology. We concentrate on the rural sector data of a

the whole sample with more than 4000 observations, the $\hat{A}$ matrix and $\hat{\lambda}$ values do not differ very much between these two estimation procedures. Yet for the subsamples, especially with very few observations, the indirect method always gives a high $\hat{\lambda}$ value, indicating a bias towards accepting positive definiteness.

developing economy in which not much variation in demands can be expected. However, we have sucessfully established empirical evidence which supports his claim that *rationality of individual households plays only a minor role.* But there exist certain limitations in this study which need further exploration. First of all, if one takes into consideration disposable income instead of total expenditure, then the acceptance of negative semidefiniteness of the Slutsky matrix would create difficulties and needs certain behavioural assumptions. Also the incorporation of demographic characteristics into the demand function makes it more difficult to accept the condition of negative semidefiniteness because demographic factors correlate with income. Above all, the extension of the analysis in case of leisure goods can pose a challenge to this conclusion because of the upward sloping demand curves for leisure by some individuals.

References

- Aruka, Y : Family Expenditure Data in Japan and The Law of Demand: Macroscopic Microeconomic View ; mimeo, Chuo University, Tokyo, Japan, 2000.
- Härdle, W and Hart, J: A Bootstrap Test for Positive Definiteness of Income Effect Matrices ; Economic Theory, Vol-8, 276-290, 1992.
- Härdle, W, Hildenbrand, W and Jerison, M: Empirical Evidence on the Law of Demand ; Econometrica, Vol-59, No-6, 1525-1549, 1991.
- Hildenbrand, W : Market Demand; Princeton, Princeton Univ. Press, 1994.
- Hildenbrand, W and Kneip, A: Family Expenditure Data, Heteroscedasticity and the Law of Demand ; Ricerche Economiche, Vol-47, 137-165, 1993.
- Lewbel, A: An Examination of Werner Hildenbrand's Market Demand ; Journal of Economic Literature, Vol-32, 1832-1841, 1994.
- Mas-Colell, A, Whinston, M and Green, J.R: Microeconomic Theory; Oxford, Oxford University Press, 1995.
- Stoker, T. M: Equivalence of Direct, Indirect and Slope Estimators of Average Derivatives ; Barnett, W.A, Powel, J.L and Tauchen, G. eds. Nonparametric and Semiparametric Methods in Econometrics and Statistics, Cambridge, CUP, 99-124, 1991.

Appendix

- **Food: Food includes cereals & cereals substitutes, Pulses, milk and milk products, edible oil, meat-egg & fish, all vegetables, fruits, sugar, salt and spices.**
- **Beverages: It includes tea, coffee, cold beverages, fruit juice, chocolate, confectionery, cake-pastry, jam, pickles, sauce, processed food.**
- **Intoxicants: The commodities in this group are pan and it's ingredients, tobacco, liquor, ganja, beer, other drugs etc.**
- **Fuel & light: It includes coke, electricity, kerosene, coal gas, coal, charcoal, other oil used for lighting, candle, other fuel and light.**

Sample type	Number of observations	Food expenditure	Beverages expenditure	Intoxicants expenditure	Fuel & light expenditure	Total expenditure
WHOLE	4301	757.23 (444.23)	64.47 (91.03)	33.21 (49.07)	101.77 (60.74)	1343.84 (841.49)
SENA	336	862.87 (447.54)	80.88 (100.01)	35.24 (53.94)	114.80 (62.27)	1576.75 (896.94)
AL	1256	535.53 (264.28)	43.31 (76.31)	30.36 (41.53)	80.46 (41.66)	918.37 (523.99)
OL	286	701.50 (374.12)	72.09 (101.96)	35.46 (54.90)	99.53 (48.72)	1279.98 (673.20)
SEA	1285	924.21 (487.62)	65.16 (67.28)	37.65 (53.29)	115.67 (69.25)	1595.66 (881.28)
OTHERS	1138	796.16 (468.53)	80.31 (115.64)	30.16 (48.29)	106.31 (63.87)	1476.34 (915.87)
1A	186	224.70 (127.27)	59.13 (147.41)	10.74 (23.26)	45.09 (23.43)	486.42 (376.58)
2A	370	414.02 (209.47)	36.21 (62.60)	24.97 (41.80)	70.01 (35.71)	786.65 (520.79)
$\geq 3A$	689	788.89 (393.22)	68.07 (115.01)	37.32 (49.39)	109.03 (61.23)	1395.31 (795.49)
$1A+ \geq 1C$	115	473.76 (250.56)	36.44 (45.51)	8.82 (16.26)	75.67 (47.20)	820.75 (488.21)
2A+1C	240	538.76 (239.53)	52.76 (69.19)	20.68 (27.87)	83.55 (42.80)	1039.57 (664.82)
2A+2C	343	617.97 (279.95)	55.66 (76.65)	26.68 (36.45)	87.53 (41.65)	1117.46 (605.13)
$2A+ \geq 3C$	589	680.33 (283.65)	59.63 (82.11)	34.13 (46.49)	91.95 (43.15)	1205.41 (625.38)
$\geq 3A + 1C$	549	858.87 (402.38)	65.77 (72.09)	32.86 (49.30)	112.67 (56.25)	1511.35 (821.55)
$\geq 3A + 2C$	525	938.78 (443.01)	77.53 (89.22)	41.26 (54.77)	118.49 (68.28)	1626.50 (826.52)
$\geq 3A+ \geq 3C$	695	1095.49 (542.20)	82.73 (90.93)	44.73 (62.39)	132.07 (75.66)	1901.63 (970.36)

Table A.1: Descriptive Statistics. The terms in the parenthesis indicate standerd deviations.