BONN ECON DISCUSSION PAPERS

Discussion Paper 3/2002

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by

Matthias Kräkel

January 2002

Bonn Graduate School of Economics
Department of Economics
University of Bonn
Adenauerallee 24 - 42
D-53113 Bonn

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U-type versus J-type Tournaments as Alternative Solutions to the Unverifiability Problem*

Matthias Kräkel, *University of Bonn***

Abstract

This paper discusses the properties of stylized U.S. ("U-type") and Japanese tournaments ("J-type"), which can both solve the unverifiability problem of labor contracts. Under a zero-profit condition, both tournament types will yield first-best efforts if workers are homogeneous and risk neutral. This result will no longer hold for J-type tournaments if the employer has all the bargaining power. However, if workers are risk averse or one worker has a lead a J-type tournament may dominate a U-type tournament.

Keywords incentives, personnel policy, tournaments, unverifiability

Journal of Economic Literature Classification Numbers J31, J33

* I would like to thank Silke Becker, Christian Grund, Peter-J. Jost and Gunter Steiner for helpful comments.

** Matthias Kräkel, Department of Economics, BWL II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, phone: +49-228-739211, fax: +49-228-739210, email: m.kraekel@uni-bonn.de.

1 Introduction

A major problem of labor contracts arises from the fact that often workers' efforts, their human capital investments, or their outputs cannot be verified by a third party. These workers' inputs or outputs, respectively, are usually called unverifiable or non-contractible, because explicit labor contracts cannot be made contingent on them. Therefore, labor contracts are often incomplete giving room for opportunistic behavior. Consider, for example, the case of unverifiable firm-specific human capital, which is sketched by Kanemoto and McLeod (1989, p. 386). If the worker is paid before he has made his investment, this worker will have no incentive to invest in his human capital afterwards. If, the other way round, the employer promises to compensate the worker after having made the investment, later on the employer can renege on the original agreement and does not pay the promised wage.[1] When the worker anticipates the employer's opportunistic behavior, he will underinvest in his human capital. In total, following Kanemoto and MacLeod we can speak of a double-sided moral hazard problem.

Malcomson (1984, 1986) has offered a general solution to the unverifiability problem. He shows that tournament compensation schemes will be contractible, even if the workers' labor inputs or outputs are unverifiable. Tournaments compensate workers according to the ordinal ranks of their inputs or outputs, respectively, where the tournament prizes are specified in advance, before the tournament has started. This last point is decisive,

[1] Here, there is an hold-up problem, where the employer captures the quasi-rent which is generated by the worker's specific investment.

because the fixed tournament prizes are usually contractible so that the employer cannot save labor costs opportunistically by understating the workers' results. By anticipating this, workers have incentives to exert effort or to invest in human capital to attain the winner prize in the tournament.

In the evolutionary process of labor market institutions, two different types of tournaments have evolved as alternative solutions to the unverifiability problem (see Kanemoto and MacLeod 1989, pp. 386-388; 1992, pp. 144-147). The first type can be called *U-type tournament,* because they are typical of U.S. firms (see, e.g., Baker, Gibbs, and Holmström 1994a, 1994b). Here, the tournament prizes are wages that are attached to jobs along a firm's hierarchy. These wages are rising with the hierarchy level, where the jobs are located. On each level of the hierarchy, workers compete in tournaments against each other to win a promotion to the next level. Thus, the winner prize consists of the wage increase by being promoted to the next level. Since the hierarchical wage structure of a firm is usually verifiable, such U-type tournaments prevent employers from opportunistic behavior and, thereby, workers from underinvestment in effort or human capital, respectively.

The second type of tournament can be found in Japanese firms and will be called *J-type tournament.* The central component of this kind of tournament is an aggregate wage bill which is the outcome of a bargaining process between the employer and the local union.[2] For example, this wage bill can be the sum of the bonus payments which are biannually given to their workers by

[2] It is typical for the Japanese labor market that all workers of a single firm are represented by a single union which acts as a "voice mechanism".

large Japanese firms.[3] When the employer has agreed upon the aggregate wage bill, he can no longer save labor costs by opportunistically rating his workers' results, because the wage bill is verifiable. The wage bill is then shared among the workers by using a tournament. In Japanese firms, there are no strictly separated jobs which are precisely defined by special sets of tasks that are delegated to single jobs. In other words, there are no definite demarcation lines between "jobs" in Japan. Therefore, wages cannot be attached to jobs as in U-type tournaments. They have to be attached to persons. For this purpose, each Japanese worker takes place in a rating or assessment process (called "satei"), in which the workers are subjectively judged by their supervisors (e.g., Itoh 1991; Endo 1994). There is a kind of tournament between the workers, because the more merit points a worker has made the larger is his individual share in the aggregate wage bill. For example, if there are two workers, A and B, which are rated by a supervisor who declares that A has performed three times as good as B, then A will receive 75% of the wage bill and B only 25%.[4]

This paper offers an analytical comparison between U-type and J-type tournaments to show under which conditions a U-type tournament will dominate a J-type tournament and vice versa.[5] Both tournament types are based

[3] Such bonus payments are of great importance to Japanese workers, because bonuses can make up 18-22% of a worker's yearly income; see Kanemoto and MacLeod (1992, p. 145). Other authors even speak of nearly 30%; see Itoh (1991, pp. 348-350); Ito (1992, pp. 231-239).

[4] Of course, the Japanese rating and compensation system is much more complex, but this paper only focuses on the differences between the two stylized types of tournaments.

[5] Note that only two stylized types of tournaments are discussed in the paper, which

on the same general idea to solve the unverifiability problem – the employer commits himself by fixing an aggregate wage structure or an aggregate wage bill in advance. Nevertheless, the following results will show that despite the same general idea the two tournament types will have specific advantages and disadvantages: If workers are homogeneous and risk neutral either tournament type will lead to first-best efforts given that the employer faces a zero-profit condition. If, however, the employer has all the bargaining power U-type tournaments will again achieve first-best efforts which will not hold for J-type tournaments in general. With heterogeneous workers neither tournament will usually yield first-best outcomes. But labor costs may be lower for the employer in the U-type tournament. On the other hand, a J-type tournament may dominate a U-type one if workers are risk averse or if one of the workers has a lead.

Before starting with the basic model it is important to emphasize that J-type tournaments entail some analytical difficulties. For this tournament type, we have to compute the expected quotient of a worker's output and total output of both workers. Unfortunately, it is not unusual that this expected value does not exist (see Mood, Graybill and Boes 1974, p. 181). For this reason, the standard Lazear-Rosen framework using a production technology that is linear in effort and exogenous noise has not been adopted in this paper. Instead, a two-point distribution (i.e., each worker's output is

does not allow a direct comparison between real U.S. and Japanese labor practices. For example, U-type tournaments can also be found in some Japanese firms when workers compete against each other in promotion tournaments along the vertical hierarchy (see Itoh 1991).

either high or low) is used assuming that a worker's effort is identical with his probability of realizing the high output. By this, we can calculate expected outcomes without any problem. As an alternative, one could strictly focus on the unverifiability problem and neglect any uncertainty. This setting would be appropriate for mimicking situations where a worker's output is mainly determined by effort instead of luck. This alternative modelling is used in a companion paper (see Kräkel 2001). However, neglecting any uncertainty will yield the same problem as in the Lazear-Rosen framework if there is not sufficient noise (i.e., if the density of the composed random variable is not flat enough): In U-type tournaments, pure-strategy equilibria do not exist. Of course, mixed-strategy equilibria make U-type tournaments less attractive from an employer's viewpoint (e.g., U-type tournaments no longer yield first-best outcomes). Nevertheless, U-type tournaments may still dominate J-type ones, when the number of participants is not too large.

The paper is organized as follows. In Section 2, the basic model is introduced. As a benchmark result it is shown that both tournament types lead to first-best efforts when workers are risk neutral, homogeneous and have all the bargaining power. Section 3 considers the opposite case where the employer has all the bargaining power. Section 4 deals with heterogeneous and Section 5 with risk averse workers. Section 6 discusses the implications of one worker having a lead in the tournament. Further aspects that are important when comparing U-type and J-type tournaments are verbally discussed in Section 7. Section 8 concludes.

2 The Basic Model

A (U-type or J-type) firm employs two risk neutral workers, A and B. The employer is assumed to be risk neutral, too. Each worker i ($i = A, B$) chooses an effort $e_i \in (0, 1)$ which is unobservable by the employer (hidden action).[6] The output (in monetary terms) of each worker i, π_i, can take on one of two possible values: $\pi_i \in \{\pi_l, \pi_h\}$ with $\pi_l < \pi_h$. The high output π_h is realized with probability e_i and the low output π_l with probability $1 - e_i$ ($i = A, B$). Thus, by exerting effort the workers can influence the probability of a high output. Obviously, each effort choice $e_i \in (0, 1)$ leads to a different probability distribution or lottery $(\pi_h, e_i; \pi_l, 1 - e_i)$, where $(\pi_h, e''; \pi_l, 1 - e'')$ dominates $(\pi_h, e'; \pi_l, 1 - e')$ within the meaning of first-order stochastic dominance when $e'' > e'$. It is assumed that the π_i are stochastically independent and unverifiable, but observable by all parties. Each worker's expected utility can be characterized by his expected wage payment minus his disutility of effort which is (in monetary terms) described by the cost function $c(e_i)$ with $c(0) = 0$, $c'(e_i) > 0$, and $c''(e_i) > 0$. As in Lazear and Rosen (1981) it is assumed that the workers have all the bargaining power and the employer has to maximize the workers' expected utilities for a given zero-profit condition.

As a reference solution the *first-best effort* can be calculated. Here, it is the effort level which maximizes $\pi_h e_i + \pi_l (1 - e_i) - c(e_i)$. From the first-order condition we obtain

$$e^{FB} = c'^{-1}(\pi_h - \pi_l) \qquad (i = A, B) \tag{1}$$

[6] Alternatively, e_i can be interpreted as human capital investment. The standardization of the variable e_i is necessary, because subsequently it is used as a probability.

as first-best effort, where $c'^{-1}(\cdot)$ is the inverse of the marginal cost function $c'(\cdot)$. This inverse $c'^{-1}(\cdot)$ is an increasing function because of the convexity of $c(e_i)$.

Now, we can derive the equilibrium outcomes of the two tournament types in the given basic model. Either tournament can be described by a two-stage game. On the first stage, the owner chooses a winner and a loser prize (in the U-type tournament), or an aggregate wage bill (in the J-type tournament). On the second stage, the two workers compete against each other by choosing their effort variables.

In the *U-type tournament* the employer decides about a loser prize $w_2 \geq 0$ and a winner prize $w_1 \geq w_2$, before the competition between the two workers starts. On the second stage, each worker i $(i = A, B;\ j \neq i)$ exerts effort e_i to maximize his expected utility

$$\begin{aligned} EU_i(e_i) &= w_1 e_i (1-e_j) + \frac{w_1+w_2}{2}\left(e_i e_j + (1-e_i)(1-e_j)\right) \\ &\quad + w_2 (1-e_i) e_j - c(e_i). \end{aligned} \tag{2}$$

With probability $e_i(1-e_j)$ worker i becomes the winner of the tournament and receives the winner prize w_1. He gets the loser prize w_2 with probability $(1-e_i)e_j$. If the two workers produce identical outputs, the winner of the tournament will be randomly chosen by the employer using a fair coin.[7] This event happens with probability $e_i e_j + (1-e_i)(1-e_j)$. The first-order

[7] As an alternative, total payment $w_1 + w_2$ is shared equally between A and B. If w_1 and w_2 correspond to certain jobs, then the employer will use job rotation to share the payment equally.

condition for optimal effort yields[8]

$$e_i^U = c'^{-1}\left(\frac{w_1 - w_2}{2}\right) \qquad (i = A, B)\,. \tag{3}$$

Eq. (3) shows that the optimal effort e_i^U will be large, if the winner prize is high, the loser prize w_2 (as a kind of fall-back position) is low, and the marginal cost function $c'(\cdot)$ has a flat shape. e_i^U can be interpreted as worker $i's$ reaction function to the employer's choice of w_1 and w_2.

On the first stage, the employer chooses w_1 and w_2 to maximize workers' expected utilities $EU_i(e_i)$ subject to the incentive compatibility constraint (3) and the zero-profit condition

$$\begin{aligned} w_1 + w_2 &= 2\left(\pi_h e_i + \pi_l (1 - e_i)\right) \quad \Leftrightarrow \\ \frac{w_1 + w_2}{2} &= \pi_l + (\pi_h - \pi_l)e_i. \end{aligned} \tag{4}$$

Condition (4) requires that total labor costs equal total expected output. Since workers behave symmetrically on the second stage (i.e., $e_A^U = e_B^U =: e^U$), it suffices to consider only one of the workers on the first stage. Because of symmetry the employer wants to maximize

$$EU_i(e_i) = \frac{w_1 + w_2}{2} - c(e_i) \tag{5}$$

subject to (3) and (4). Substituting (4) into (5) gives

$$EU_i\left(e^U\right) = \pi_l + (\pi_h - \pi_l)\, e^U - c\left(e^U\right). \tag{6}$$

with e^U being described by Eq. (3). The employer's first-order conditions are

$$\frac{\partial EU_i\left(e^U\right)}{\partial w_1} = (\pi_h - \pi_l)\frac{e^{U\prime}}{2} - c'\left(e^U\right)\frac{e^{U\prime}}{2} = 0 \tag{7}$$

[8] The second-order condition holds.

$$\frac{\partial EU_i\left(e^U\right)}{\partial w_2} = -\left(\pi_h - \pi_l\right)\frac{e^{U'}}{2} + c'\left(e^U\right)\frac{e^{U'}}{2} = 0 \tag{8}$$

with $e^{U'} := dc'^{-1}(x)/dx$. Eqs. (7) and (8) show that first-best effort is achieved in the U-type tournament, i.e. $e^U = c'^{-1}(\pi_h - \pi_l) = e^{FB}$.

In the *J-type tournament*, the employer chooses an aggregate wage bill[9] $w \geq 0$. On the second stage, worker i ($i = A, B$; $j \neq i$) maximizes his expected utility

$$\begin{aligned} EU_i(e_i) &= \frac{\pi_h}{\pi_l + \pi_h} we_i(1 - e_j) + \frac{w}{2}(e_i e_j + (1 - e_i)(1 - e_j)) \\ &\quad + \frac{\pi_l}{\pi_l + \pi_h} w(1 - e_i)e_j - c(e_i). \end{aligned} \tag{9}$$

Hence, worker i receives the fraction $\pi_i/(\pi_i + \pi_j)$ of the aggregate wage bill w in each of the four events (π_h, π_l), (π_l, π_h), (π_l, π_l), and (π_h, π_h). The first-order condition leads to[10]

$$e_i^J = c'^{-1}\left(\frac{w[\pi_h - \pi_l]}{2[\pi_h + \pi_l]}\right) \qquad (i = A, B). \tag{10}$$

On the first stage, the employer chooses w for maximizing $EU_i(e_i)$ given by Eq. (9) subject to the workers' incentive constraint (10) and the zero-profit condition

$$\frac{w}{2} = \pi_l + (\pi_h - \pi_l)e_i. \tag{11}$$

Because of the symmetric outcome of the second-stage subgame (i.e. $e_A^J =$

[9] The bargaining process between the employer and the local union is neglected here to make both kinds of tournaments comparable. In addition, local unions also exist in the United States. Thus, in many cases the hierarchical wage structure (w_1, w_2) is a bargaining outcome, too.

[10] Again, the second-order condition holds.

$e_B^J =: e^J$) the employer's objective function can be written as

$$EU_i\left(e^J\right) = \frac{w}{2} - c\left(e^J\right) \overset{(11)}{=} \pi_l + (\pi_h - \pi_l)\, e^J - c\left(e^J\right) \tag{12}$$

with e^J given by Eq. (10). From the first-order condition we obtain

$$\begin{aligned} \frac{\partial EU_i\left(e^J\right)}{\partial w} &= (\pi_h - \pi_l) \cdot e^{J'} \cdot \frac{\pi_h - \pi_l}{2\left(\pi_h + \pi_l\right)} - c'\left(e^J\right) \cdot e^{J'} \cdot \frac{\pi_h - \pi_l}{2\left(\pi_h + \pi_l\right)} = 0 \\ &\Leftrightarrow \quad (\pi_h - \pi_l) = c'\left(e^J\right) \\ &\Leftrightarrow \quad c'^{-1}(\pi_h - \pi_l) = e^J \overset{(10)}{=} c'^{-1}\left(\frac{w\left[\pi_h - \pi_l\right]}{2\left[\pi_h + \pi_l\right]}\right) \end{aligned} \tag{13}$$

with $e^{J'} := dc'^{-1}(x)/dx$. According to Eq. (13), the employer chooses the aggregate wage bill $w = 2(\pi_h + \pi_l)$ that leads to first-best effort $e^J = c'^{-1}(\pi_h - \pi_l) = e^{FB}$.

The previous findings can be summarized in the following proposition:[11]

Proposition 1 *If the workers are homogeneous and risk neutral, and have all the bargaining power, then* $e_i^U = e_i^J = e^{FB}$ $(i = A, B)$.

According to the benchmark result of Proposition 1, both tournament types yield first-best outcomes under the given conditions. It can easily be shown that the result will also hold for $n > 2$ workers in a symmetric equilibrium (see the Appendix).[12] Because of risk neutrality there is no trade-off between incentives and risk sharing, i.e. the employer can always set appropriate incentives without a risk premium. Homogeneity among the two workers ensures symmetric equilibria in the tournament subgame on the

[11] For the outcome of the U-type tournament see also Lazear and Rosen (1981), pp. 844-846.

[12] Hence, tournament size does not play the same decisive role as in Kräkel (2001).

second stage. Thereby, choosing a uniform wage structure (w_1, w_2) or w for both workers cannot lead to distorted incentives for one of the workers in the tournament. Altogether, Proposition 1 may explain why both tournament types can be observed in practice at the same time. Unfortunately, the given conditions – homogeneity, risk neutrality, zero-profit condition – can hardly be found in practice. Hence, these three conditions will be dropped in the following sections to compare the U-type and the J-type tournament in a more realistic setting.

3 Reversed Bargaining Power

In contrast to Section 2, now it is assumed that the employer has all the bargaining power. In other words, the zero-profit condition is replaced by the standard principal-agent assumption that each worker has a reservation value $\bar{v}$ and the employer wants to maximize expected profits net off labor costs $w_1 + w_2$ or w, respectively. The solution to the employer's maximization problem has to meet two restrictions – the workers' incentive constraint given by Eq. (3) or Eq. (10), respectively, and the participation constraint which can be written as $(w_1 + w_2)/2 - c(e_i) \geq \bar{v}$ or $w/2 - c(e_i) \geq \bar{v}$ because of the symmetric tournament outcome, which does not depend on the distribution of bargaining power. The following result can be obtained:

Proposition 2 *If the workers are homogeneous and risk neutral, and the employer has all the bargaining power, then* $e_i^U = e^{FB}$ $(i = A, B)$*, but in general* $e_i^J \neq e^{FB}$ $(i = A, B)$*.*

Proof. See the Appendix. ■

The findings of Proposition 2 are surprising: Despite homogeneity and risk neutrality the J-type tournament does usually not lead to first-best efforts any longer when the employer instead of the workers has all the bargaining power. However, the U-type tournament again yields first-best efforts.

Now we can discuss what causes the difference between the U-type and the J-type result. At first sight, one can speculate that the sharing rule $\pi_i / (\pi_i + \pi_j)$ leads away from first-best effort. For the J-type tournament there does not exist a trade-off between incentives and risk sharing, too. But since π_i is both part of the numerator and part of the denominator, the sharing rule generates another trade-off: exerting more effort raises the expected value of the numerator as well as the expected value of the denominator. Eq. (A19) in the Appendix , however, shows that this new trade-off is not the major problem: theoretically, the employer can induce first-best incentives by choosing $w = 2(\pi_h + \pi_l)$.

The difference between the U-type and the J-type result can be better explained by the number of the employer's personnel policy variables. In the U-type tournament the employer has two policy variables, w_1 and w_2, and two restrictions, the incentive and the participation constraint. By appropriately choosing w_1 and w_2 the employer can meet both restrictions when maximizing his expected surplus. A J-type employer also has two restrictions, but he has only one policy variable − the total wage bill w. In general, w cannot meet both restrictions at the same time when expected surplus is maximized. Proposition 1 shows that this problem vanishes under a zero-profit condition, which can be directly substituted into the employer's objective function. By this, the employer effectively has only to meet one restriction – the incentive constraint. In this maximization problem, the employer is able to achieve the first-best solution even with only one policy variable w.

4 Heterogeneous Workers

In this section, it is assumed that workers have different cost functions $c_A(e_A) = c(e_A)$ and $c_B(e_B) = k \cdot c(e_B)$ where $c(\cdot)$ is convex and $k > 0$ with $k \neq 1$. Depending on whether $k < 1$ or $k > 1$, worker B has a cost advantage or a cost disadvantage compared to worker A, respectively. Now, the first-best efforts are given by

$$e_A^{FB} = c'^{-1}(\pi_h - \pi_l) \qquad \text{and} \qquad e_B^{FB} = c'^{-1}\left(\frac{\pi_h - \pi_l}{k}\right). \tag{14}$$

The following results can be derived:

Proposition 3 *Let the workers be heterogeneous and risk neutral:*
(i) If the workers have all the bargaining power, then $e_A^U \neq e_A^{FB} \neq e_A^J$ *and* $e_B^U \neq e_B^{FB} \neq e_B^J$.
(ii) If the employer has all the bargaining power, both participation constraints may be binding in the U-type and in the J-type tournament. But then $e_A^U \neq e_A^{FB} \neq e_A^J$ *and* $e_B^U \neq e_B^{FB} \neq e_B^J$ *in general. In the U-type tournament at least one participation constraint is binding, which cannot be guaranteed in the J-type tournament.*

Proof. See the Appendix. ■

The proposition shows that heterogeneity prevents the employer from achieving first-best outcomes in either tournament type in general. Even in the U-type tournament first-best efforts cannot be guaranteed any longer, when workers are heterogeneous. The intuition for this finding is that in both tournament types the employer has to choose a uniform wage structure for both workers. But since heterogeneous workers react differently to the same wage structure this will typically lead to distorted incentives.[13]

When the workers have all the bargaining power, the employer is forced to maximize the expected utilities of both workers with the same wage policy (w_1, w_2) or w, respectively. But this is impossible, because from the incentive constraints we know that the two workers choose different efforts given a uniform wage policy, since $k \neq 1$. In the U-type tournament, the employer still has two policy variables w_1 and w_2, whereas the J-type employer can only decide about the total wage bill w. Nevertheless, this comparative advantage for the U-type employer does not apply here, because the reaction functions $e_A = e_A(w_1, w_2)$ and $e_B = e_B(w_1, w_2)$ only depend on the prize spread $\Delta w := w_1 - w_2$. In other words, when the workers have all the bargaining power both kinds of tournaments become rather similar − the employer must solely care for the workers' incentives and these incentives can only be controlled by a single policy variable Δw or w, respectively.

[13] Of course, if the employer is able to identify the two types of workers, he can solve his incentive problem by introducing a handicap system or by organizing two separate tournaments for A-type and B-type workers, respectively (see Lazear and Rosen 1981, pp. 861-863).

When the employer has all the bargaining power, in the U-type tournament the employer's maximization problem is constrained by four restrictions, but he still has only two policy variables. In general, binding participation constraints for both workers and first-best efforts do not hold at the same time. The possibility of first-best efforts cannot be completely excluded. But then first-best implementation is costly for the employer in general: one participation constraint will not be binding, which means that the employer will have relatively high labor costs, because one worker gets more than his reservation value. Compared to the case of homogeneous workers, now there will be a welfare loss due to heterogeneity. This loss is completely borne by the employer, since each worker at least receives his reservation value. Similar results hold for the J-type tournament. But Proposition 3 shows that U-type tournaments may still lead to better results than J-type tournaments. When first-best effort is implemented without two binding participation constraints, in the U-type tournament there is one binding participation constraint, whereas in the J-type tournament there may be no binding participation constraints. This result also seems to be intuitively plausible. Obviously, in the U-type tournament at least one participation constraint must be binding, because otherwise the employer can still lower w_1 and w_2 to save labor costs. When decreasing w_1 and w_2 by the same amount this cost saving policy will not result in distorted incentives, because e_i^U (and thereby $c_i\left(e_i^U\right)$) only depends on the prize spread Δw. In the J-type tournament, the employer's only policy variable is w. Decreasing w to make one of the participation constraints binding will automatically decrease the workers efforts, too. In other words, also the case of heterogeneous workers indicates that the U-type tournament may be more advantageous compared to the J-type tournament because of a greater number of policy variables.

5 Risk Averse Workers

Now, we drop the risk neutrality assumption of the basic model which seems to be crucial for obtaining first-best efforts in either tournament type under homogeneity and a zero-profit condition. To focus on the impact of risk aversion homogeneous workers and a zero-profit condition are assumed in this section, too. It is assumed that each worker i has a concave utility function which gives him utility $u(w_i - c(e_i))$ when receiving a wage w_i in any tournament type. To derive explicit solutions for the equilibrium efforts the cost function is assumed to be quadratic: $c(e_i) = 0.5ce_i^2$ with $c > 0$.[14] For simplicity, the analysis is restricted to the most plausible case of symmetric equilibria.

It is well-known that introducing risk aversion into tournaments implies some analytical difficulties. Hence, as in Lazear and Rosen (1981, pp. 852-853) and McLaughlin (1988, pp. 228-231) first-order and second-order Taylor series expansions are used to derive approximate results. Let, again, $\Delta\pi = \pi_h - \pi_l$ and $\Delta w = w_1 - w_2$, and let $r = -u''/u'$ denote the Arrow-Pratt coefficient of absolute risk aversion. The following results can be obtained:[15]

[14] This assumption is not crucial. Without it equilibrium efforts can be described implicitly, see Lazear and Rosen (1981), McLaughlin (1988).

[15] To be precise, in the U-type tournament $r = -u''((w_1 + w_2)/2 - c(e^U))/\ u'((w_1 +$

Proposition 4 *In a symmetric subgame perfect equilibrium of a U-type tournament, the employer chooses*

$$\Delta w = \frac{2\Delta\pi}{1+rc} \tag{15}$$

on the first stage, and the workers

$$e^U = \frac{\Delta\pi}{c\,(1+rc)} \tag{16}$$

on the second stage.[16] *In a symmetric subgame perfect equilibrium of a J-type tournament, equilibrium strategies* w *and* e^J *are described by*

$$ce^J + \frac{\Delta\pi c w r}{\pi_l + \pi_h}\left(e^J - \left[e^J\right]^2\right) + \frac{\Delta\pi^2 w^2 r}{4\left(\pi_l+\pi_h\right)^2}\left(1-2e^J\right) = \Delta\pi \tag{17}$$

$$\text{and} \qquad e^J = \frac{\Delta\pi w}{2c\left(\pi_l+\pi_h\right)}. \tag{18}$$

If $r \to \infty$*, then* $\Delta w = e^U = 0$*, but* $w = \frac{3}{2}c\frac{\pi_l+\pi_h}{\Delta\pi}$ *and* $e^J = \frac{3}{4}$.

Proof. See the Appendix ■

According to Proposition 4 neither the U-type nor the J-type tournament will lead to first-best effort $e^{FB} = \Delta\pi/c$ if workers are risk averse. From Eq. (16) this is obvious for the U-type tournament. Since $r > 0$ we have $e^U < e^{FB}$. In the J-type tournament, $w = 2(\pi_l + \pi_h)$ would induce first-best efforts according to Eq. (18). But inserting $w = 2(\pi_l+\pi_h)$ into Eq. (17) and solving for e^J shows that the coefficient of risk aversion r does not cancel out.

$w_2)/2 - c(e^U))$ and in the J-type tournament $r = -u''(w/2 - c(e^J))/u'(w/2 - c(e^J))$. Following McLaughlin (1988), fn. 4, we do not differentiate between these two expressions in detail to focus on the major differences between the two tournament types.

[16]Note that with quadratic costs we have to assume $\Delta\pi < c$ so that $e^{FB} = \Delta\pi/c$ can be interpreted as probability. This assumption also ensures that $e^U < 1$.

The intuition for this results is the same as in the basic principal-agent model with hidden action: Risk aversion causes a trade-off between incentives and risk sharing and, therefore, in equilibrium the principal prefers not to induce full incentives.

More interestingly, Proposition 4 shows that the J-type tournament will dominate the U-type tournament (i.e., $e^J > e^U = 0$) if workers are sufficiently risk averse.[17] This result is also intuitively plausible. In the U-type tournament, there is always considerable risk for the workers since only two outcomes are possible corresponding to quite different utility levels – a worker becomes the winner of the tournament and receives $u(w_1 - c(e^U))$ or he loses and only gets $u(w_2 - c(e^U)) << u(w_1 - c(e^U))$.[18] Since in the symmetric equilibrium the probability mass is equally distributed between these two events, a U-type tournament represents a very risky lottery for the workers. On the other hand, in the J-type tournament the workers' wages (i.e., their shares in the wage bill w) directly depend on the magnitude of the realized outputs. Hence, this tournament type has strong parallels to a piece rate system and from Lazear and Rosen (1881) we already know that piece rates are relatively advantageous when workers are risk averse since the mass of the income distribution is concentrated near the mean. In case of a J-type tournament this means that for each worker the most likely outcome is to realize a utility level $u(w/2 - c(e^J))$. In the simple model considered here, only three outcomes are possible: A worker realizes $u(\pi_h/(\pi_l+\pi_h)\cdot w - c(e^J))$

[17] Note that $e^J = 3/4$ is not an extreme small value since $e_i \in (0,1)$.

[18] From the zero-profit condition we know that equilibrium prizes are $w_1 = \pi_l + \left[\Delta\pi^2 + \Delta\pi c\right]/\left[c\left(1 + rc\right)\right] >> w_2 = \pi_l + \left[\Delta\pi^2 - \Delta\pi c\right]/\left[c\left(1 + rc\right)\right]$.

or $u(\pi_l/(\pi_l + \pi_h) \cdot w - c(e^J))$ each with probability $e^J - \left[e^J\right]^2$, or he ends up with $u(w/2 - c(e^J))$ with probability $2\left[e^J\right]^2 + 1 - 2e^J$. It can easily be checked, that $2\left[e^J\right]^2 + 1 - 2e^J > e^J - \left[e^J\right]^2$ for all possible values of e^J.

6 Worker with a Lead

As is known in the literature, tournaments may suffer from intermediate information (see, e.g. McLaughlin 1988, p. 249; Prendergast and Topel 1993a, p. 362): In practice, tournaments are not one-shot games but reach over several periods. If during this time workers get intermediate information about who is leading and who has been left behind incentives may immediately break down. In this section, it will be analyzed which type of tournament deals better with this problem.

For analytical tractability some simplifying assumptions have to be introduced. To derive explicit solutions for the equilibrium efforts, again the cost function is assumed to be quadratic: $c(e_i) = ce_i^2$ with $c > 0$.[19] Both workers are homogeneous and risk neutral, and the employer faces a zero-profit condition. It is assumed that the employer has chosen a first-best wage structure $\Delta w = w_1 - w_2 = 2(\pi_h - \pi_l) = 2\Delta\pi$ or $w = 2(\pi_l + \pi_h)$, respectively, before the tournament starts. We will then look at a random shock that exogenously happens at the beginning of the tournament: Worker j ($j = A$ or $j = B$) gets a lead $\lambda > 0$, i.e. his output already amounts to λ before both workers choose their efforts and realize π_i and π_j. At last, for analytical tractability

[19] Again, assuming $\Delta\pi < c$ guarantees equilibrium efforts that lie between zero and one.

let $\pi_l = 0$ and $\pi_h = \pi > 0$. Therefore, $\Delta w = w = 2\pi$. This model variant leads to the following Proposition.[20]

Proposition 5 *If $\lambda > \pi$, then $e_i^J > e_i^U = 0$ $(i = A, B)$.*
If $\lambda < \pi$, then $e_i^J > e_i^U$ given that c and λ are sufficiently small, otherwise $e_i^J < e_i^U$ $(i = A, B)$.

Proof. See the Appendix ■

The intuition for the first part of. Proposition 5 is as follows: If $\lambda > \pi$, in the U-type tournament worker j will have such a large lead that worker i will have no chance to catch up with him. Thus, the best the two workers can do is to choose zero efforts for minimizing their effort costs. On the other hand, in the J-type tournament the workers' wages $\pi_i/(\pi_i + \pi_j) \cdot w$ $(i = A, B)$ directly depend on the concrete magnitude of their realized outputs. By this, each worker will always have an incentive to choose a positive effort irrespective of whether one of them has a lead or not.

However, the second part of Proposition 5 shows that J-type tournaments do not always work better than U-type ones when workers have a lead. Given $\lambda < \pi$, J-type tournaments will only remain more favorable than U-type ones if the cost parameter c and the lead λ are not too large. In the U-type tournament, due to the ordinal ranking incentives are independent of the specific value of λ given $\lambda < \pi$. But this is not true for the J-type tournament. Here, equilibrium efforts will be decreasing in λ if c and λ are relatively high (see the Appendix). In this situation, incentives are rather low

[20] Without the simplifying assumptions $\pi_l = 0$ and $\pi_h = \pi$, also two cases have to be distinguished: $\lambda > \Delta\pi$ and $\lambda < \Delta\pi$.

in the J-type tournament, because effort is very costly and workers become discouraged by the high lead. Hence, if the lead is not prohibitively high (i.e., if we have $\lambda < \pi$), then a U-type tournament may lead to better work incentives because of its winner-take-all character.

7 Further Discussion

There are further aspects which are important when comparing U-type and J-type tournaments, but which have not been discussed formally in the previous sections. First, J-type tournaments are accompanied by a subjective performance evaluation ("satei"). Here, a supervisor subjectively collect performance information about the individual workers which is typically unverifiable. But because such subjective assessments leave room for favoritism (Prendergast and Topel 1993b), influence activities (Milgrom and Roberts 1988) and hidden gaming (Laffont 1988, 1990) J-type tournaments seem to have an additional disadvantage compared to U-type tournaments. However, at first sight this argument does not hold. In practice, U-type tournaments often need subjective performance evaluation, too, when supervisors are asked to give recommendations for promotions. In this context, the same problems as in the J-type tournament may also arise in the U-type one.

Nevertheless, there are good reasons why attaching wages to jobs and promoting workers to a different job in a U-type tournament can mitigate problems due to subjective assessments which does not hold for a J-type tournament, since it is not accompanied by job promotion. For example, we can think of a three-level hierarchy with the employer on the highest level, a supervisor on the middle level, and two workers on the lowest level. If the supervisor has to recommend one of the two workers for a job promotion, he might play a hidden game with them. The supervisor could announce to recommend the worker who is willing to pay the highest bribe. This worker need not be the one with the highest output or with the highest talent. For simplicity, we can assume symmetric uncertainty about the workers' talents (i.e., at the beginning of the game neither the employer nor the two workers know the exact talents) and that the Monotone Likelihood Ratio Property (MLRP) holds so that a worker's realized output in the tournament and expected talent can be treated as similar from the employer's viewpoint when deciding about promotion. Now, the employer can decide to tie the supervisor's compensation to the future performance of the promoted worker, to the future performance of the department where the worker is promoted, or even to firm's performance (see also Prendergast and Topel 1993a, p. 360). By this, hidden gaming will not work. Ex post, the supervisor will always recommend the worker with the highest output who is also the one with the highest expected talent because of the MLRP. Anticipating this, no worker is willing to pay any bribe to the supervisor in the previous hidden game. Moreover, work incentives are restored because each worker wants to be promoted and can be sure of the supervisor's honesty when making recommendations.

Attaching wages to jobs in a U-type (or promotion) tournament also mitigates problems due to favoritism (Prendergast and Topel 1993b, pp. 38-44). Let us again assume the above sketched model with symmetric talent uncertainty and MLRP. If wages are attached to jobs the supervisor can only do his most preferred worker a favor by recommending him for promotion to the better paid job. But if the employer ties the supervisor's pay to firm performance and attaches the highest wages to key jobs which need the most talented workers to guarantee a high firm output, the supervisor will realize high opportunity costs from favoritism. This lessens the probability of favoritism. By anticipating that the supervisor will honestly recommend the most talented worker (with a high probability), the workers will choose high efforts in the initial tournament because of the wage increase following a promotion and because of the MLRP.

A similar point is made by Fairbum and Malcomson (1994) who also discuss a model with symmetric talent uncertainty and MLRP. They emphasize that in any tournament that is not combined with a job promotion the employer ex post will be indifferent whom to declare the winner of the tournament. Hence, the winner prize will be auctioned among the workers who use bribes as bids. By this, the worker with the highest bribe becomes the winner of the tournament, and anticipating this no worker is willing to exert more than minimal effort. However, the situation is completely different when the winner of the tournament is promoted to an important and demanding job on a higher hierarchy level which needs a worker with high talent. Due to the MLRP assumption the employer promotes the worker with the best performance in the tournament. Again, by anticipating the employer's behavior workers' incentives in the U-type tournament are restored. To summarize, there exist several situations in practice where U-type tournaments, which combine prizes with job assignments, work better than J-type tournaments when subjective performance evaluation is needed. As such unverifiable performance measurement has been the key assumption and the starting point of this paper we have an important additional advantage of U-type tournaments compared to J-type ones.[21]

A completely different aspect is the filtering of common noise which is one of the central motives for using tournaments (see Lazear and Rosen 1981, pp. 856-857; Holmström 1982; Green and Stokey 1983). We can assume, for example, a linear Lazear-Rosen like production technology so that the output of worker i ($i = A, B$) is described by $\pi_i = e_i + \varepsilon_i + \eta$. Again e_i denotes effort, ε_i represents individual and η common noise. Of course, in the U-type tournament the common noise η is filtered out: Worker i's winning probability is described by prob$\{\pi_i > \pi_j\}$. This probability can be simplified to prob$\{\varepsilon_j - \varepsilon_i < e_i - e_j\}$ which does not depend on η. However, J-type tournaments do not have this desirable property. Since worker i's wage in the J-type tournament is given by the fraction $\pi_i(\pi_i + \pi_j)$ of the aggregate wage bill w, common noise η is not filtered out. Thus, if filtering of common noise is an important objective of an employer he will prefer U-type to J-type tournaments.

At last, it has to be pointed out that there are some parallels between J-type tournaments and piece rate or bonus systems, since wages in J-type

[21] However, when introducing a supervisor in the Fairburn-Malcomson model there may be inefficiently many or inefficiently few promotions given that workers are risk averse, see Fairburn and Malcomson (2001).

tournaments do not only depend on rank but are directly influenced by a worker's output level. This common feature of piece rates and J-type tournaments implies that J-type tournaments also have similar advantages (e.g., in connection with risk aversion; see Section 5) and disadvantages (e.g., no filtering of common noise) as piece rate systems. In addition, J-type tournaments can even come closer to piece rates by modifying the given wage structure. Especially, we can think of a wage bill $w(e_i, e_j)$ that is no longer exogenous in the tournament subgame. For example, this would mimic the stylized fact that negotiated bonus payments in Japanese firms also depend on firm performance. Note that in the simple model considered in this paper firm performance is described by $\pi_i+\pi_j$. Hence, we can assume that the fraction $w = \alpha \cdot (\pi_i + \pi_j)$ $(0 < \alpha < 1)$ is paid as aggregate wage bill in the J-type tournament. Substituting this expression into worker i's wage $w_i = \frac{\pi_i}{\pi_i+\pi_j} w$ leads to $w_i = \alpha \cdot \pi_i$. In other words, in this variant the J-type tournament is identical with a piece rate system. But it is important to stress that in all model variants with $w(e_i, e_j)$ the central property of tournaments – contractibility despite the existence of unverifiable efforts and outputs of workers – is lost. In case of $w(e_i, e_j)$ and unverifiable performance the employer will always be able to save labor costs by claiming that the workers' performance has been low.

8 Conclusions

In this paper, U-type and J-type tournaments have been compared to analyze under which conditions U-type tournaments dominate J-type ones and vice versa. The findings show that both tournament types will yield first-best efforts if workers are homogeneous and risk neutral and have all the bargaining power (i.e., the employer faces a zero-profit condition). With heterogenous workers, neither tournament type leads to first-best efforts in general. The analytical results and the discussion also show that U-type tournaments will be dominant, if the employer has all the bargaining power, if there are workers with medium-sized leads, if problems due to subjective performance evaluation are severe, and if filtering of common noise is important. On the other hand, J-type tournaments will dominate U-type tournaments, if workers are substantially risk averse and if there are workers with small or large leads. Altogether, the analysis also helps to explain why these two tournament types can be both observed in practice and why they have evolved in the evolution of labor market institutions.

Appendix

First-best outcomes in tournaments with $n > 2$ workers:

As indicated in the text the analysis is restricted to symmetric equilibria which are the most plausible ones in case of homogeneity. Let e_i again denote worker i's effort and $e_j = e$ the uniform effort of each other worker $j \neq i$. Worker i's expected utility in the U-type tournament can then be written (using a binomial distribution) as

$$\begin{aligned}
EU_i(e_i) &= w_2 + \Delta w e_i \binom{n-1}{0} e^0 (1-e)^{n-1} \\
&\quad + \frac{\Delta w}{2} e_i \binom{n-1}{1} e (1-e)^{n-2} \\
&\quad + \frac{\Delta w}{3} e_i \binom{n-1}{2} e^2 (1-e)^{n-3} \\
&\quad + \cdots \\
&\quad + \frac{\Delta w}{n-1} e_i \binom{n-1}{n-2} e^{n-2} (1-e) \\
&\quad + \frac{\Delta w}{n} e_i \binom{n-1}{n-1} e^{n-1} \\
&\quad + \frac{\Delta w}{n} (1-e_i)(1-e)^{n-1} - c(e_i) \\
&= w_2 + e_i \sum_{j=0}^{n-1} \frac{\Delta w}{j+1} \binom{n-1}{j} e^j (1-e)^{n-1-j} \\
&\quad + \frac{\Delta w}{n} (1-e_i)(1-e)^{n-1} - c(e_i)
\end{aligned}$$

with $\Delta w = w_1 - w_2$.The first-order condition $\partial EU_i / \partial e_i = 0$ together with the symmetry condition $e_i = e_j = e$ gives

$$\sum_{j=0}^{n-1} \left[\frac{\Delta w}{j+1} \binom{n-1}{j} e^j (1-e)^{n-1-j} \right] - \frac{\Delta w}{n} (1-e)^{n-1} = c'(e). \qquad \text{(A1)}$$

Shortly, the incentive constraint (A1) yields $e^U = e^U(\Delta w)$ for a given n. Worker i's expected utility can be written as

$$EU_i\left(e^U\right) = w_2 + \frac{\Delta w}{n} - c\left(e^U\right) \tag{A2}$$

and the employer's zero-profit condition as

$$w_2 + \frac{\Delta w}{n} = \pi_l + \Delta\pi e^U \tag{A3}$$

with $\Delta\pi = \pi_h - \pi_l$. Inserting $e^U = e^U(\Delta w)$ and (A3) into (A2) we obtain the employer's unconstrained objective function:

$$EU_i\left(e^U(\Delta w)\right) = \pi_l + \Delta\pi e^U(\Delta w) - c\left(e^U(\Delta w)\right). \tag{A4}$$

From the first-order condition

$$\frac{\partial EU_i}{\partial \Delta w} = \Delta\pi \frac{\partial e^U}{\partial \Delta w} - c'\left(e^U\right)\frac{\partial e^U}{\partial \Delta w} = 0 \tag{A5}$$

it immediately follows that $e^U = c'^{-1}(\Delta\pi) = e^{FB.}$

In the J-type tournament, the workers' incentive constraint can be characterized as

$$e^J = e^J(w) = \arg\max_{e_i}\left\{E\left[\frac{\pi_i}{\left(\sum_{j\neq i}\pi_j\right)+\pi_i}\right]w - c(e_i)\right\} \tag{A6}$$

where $E[\cdot]$ denotes the expectation operator. Since due to symmetry $EU_i\left(e^J\right) = \frac{w}{n} - c\left(e^J\right)$, and the zero-profit condition is given by $\frac{w}{n} = \pi_l + \Delta\pi e^J$, the first-order condition

$$\frac{\partial EU_i}{\partial w} = \Delta\pi\frac{\partial e^J}{\partial w} - c'\left(e^J\right)\frac{\partial e^J}{\partial w} = 0$$

again leads to first-best effort.

Proof of Proposition 2:

Because of symmetry, it suffices to consider only one of the workers. In the U-type tournament, the employer's maximization problem can be described by the following Lagrangian:

$$\begin{aligned} L &= \pi_h e_i + \pi_l (1 - e_i) - \frac{w_1 + w_2}{2} + \lambda_1 \cdot \left[\frac{w_1 - w_2}{2} - c'(e_i) \right] \\ &+ \lambda_2 \cdot \left[\frac{w_1 + w_2}{2} - c(e_i) - \bar{v} \right] \end{aligned} \tag{A7}$$

with λ_1 and λ_2 as multipliers. The optimality conditions for e_i, w_1, and w_2 are:

$$\pi_h - \pi_l - \lambda_1 c''(e_i) - \lambda_2 c'(e_i) = 0 \tag{A8}$$

$$-\frac{1}{2} + \frac{1}{2}\lambda_1 + \frac{1}{2}\lambda_2 = 0 \tag{A9}$$

$$-\frac{1}{2} - \frac{1}{2}\lambda_1 + \frac{1}{2}\lambda_2 = 0 \tag{A10}$$

$$\frac{w_1 - w_2}{2} = c'(e_i) \tag{A11}$$

$$\frac{w_1 + w_2}{2} - c(e_i) \geq \bar{v} \tag{A12}$$

$$\lambda_2 \geq 0 \tag{A13}$$

$$\lambda_2 \cdot \left[\frac{w_1 + w_2}{2} - c(e_i) - \bar{v} \right] = 0. \tag{A14}$$

From Eqs. (A9) and (A10) we obtain $\lambda_2 = 1$ (i.e., the participation constraint is binding) and $\lambda_1 = 0$. Thus, Eq. (A8) gives:[22]

$$e_i^U = c'^{-1}(\pi_h - \pi_l) \overset{(1)}{=} e^{FB}. \tag{A15}$$

[22] The optimal tournament prizes are $w_1 = \bar{v} + c(e^{FB}) + c'(e^{FB})$ and $w_2 = \bar{v} + c(e^{FB}) - c'(e^{FB})$. Hence, we have a kind of "winner-takes-it-all lottery", where ex ante each worker wins with probability $\frac{1}{2}$ and has an expected utility that equals his reservation value $\bar{v}$. Ex post, however, the winner (loser) gets more (less) than his reservation value $\bar{v}$. This generates strong incentives in the tournament.

For the J-type tournament, a Lagrangian can be constructed analogously:

$$\begin{aligned} L &= \pi_h e_i + \pi_l (1 - e_i) - \frac{w}{2} + \lambda_1 \cdot \left[\frac{w (\pi_h - \pi_l)}{2 (\pi_h + \pi_l)} - c' (e_i) \right] \\ & +\lambda_2 \cdot \left[\frac{w}{2} - c (e_i) - \bar{v} \right]. \end{aligned} \tag{A16}$$

The optimality conditions for e_i and w are:

$$\pi_h - \pi_l - \lambda_1 c'' (e_i) - \lambda_2 c' (e_i) = 0 \tag{A17}$$

$$-\frac{1}{2} + \frac{(\pi_h - \pi_l) \lambda_1}{(\pi_h + \pi_l) 2} + \frac{1}{2} \lambda_2 = 0 \tag{A18}$$

$$\frac{w (\pi_h - \pi_l)}{2 (\pi_h + \pi_l)} = c' (e_i) \tag{A19}$$

$$\frac{w}{2} - c (e_i) \geq \bar{v} \tag{A20}$$

$$\lambda_2 \geq 0 \tag{A21}$$

$$\lambda_2 \cdot \left[\frac{w}{2} - c (e_i) - \bar{v} \right] = 0. \tag{A22}$$

Conditions (A17)−(A22) show that, in general, the J-type tournament will not lead to the first-best effort. To generate first-best effort in the J-type tournament the multipliers have to be $\lambda_1 = 0$ and $\lambda_2 = 1$ in Eq. (A17). In addition, this makes condition (A18) hold. In (A19) the aggregate wage bill has to be $w = 2 (\pi_h + \pi_l)$. Condition (A20) has to hold with equality because of $\lambda_2 = 1$. But since $\bar{v}$ and w are certain numbers, this last requirement is usually not met. Hence, in general $e_i^J \neq e^{FB}$ $(i = A, B)$.

Proof of Proposition 3:

(i) Let $\Delta\pi = \pi_h - \pi_l$. In the *U-type tournament*, the zero-profit condition is given by

$$w_1 + w_2 = \Delta\pi (e_A + e_B) + 2\pi_l \tag{A23}$$

and the incentive constraints by

$$e_A = c'^{-1}\left(\frac{\Delta w}{2}\right) \quad \text{and} \quad e_B = c'^{-1}\left(\frac{\Delta w}{2k}\right) \tag{A24}$$

with $\Delta w = w_1 - w_2$. The employer maximizes $EU_i(e_i)$ according to Eq. (2) (with $c_i(e_i)$ instead of $c(e_i)$) for $i = A$ and $i = B$ subject to (A23) and (A24). Substituting (A23) into (2) gives

$$\begin{aligned} EU_i(e_i) &= (\Delta\pi(e_i + e_j) + 2\pi_l - w_2)\, e_i(1 - e_j) \\ &\quad + \frac{\Delta\pi(e_i + e_j) + 2\pi_l}{2}(2e_ie_j + 1 - e_j - e_i) \\ &\quad + w_2(1 - e_i)\, e_j - c_i(e_i) \end{aligned} \tag{A25}$$

$(i, j = A, B;\ i \neq j)$. The first-order conditions $\frac{\partial EU_i}{\partial w_1} = \frac{\partial EU_i}{\partial w_2} = 0$ $(i = A, B)$ subject to (A24) after some rearranging yield $e_A = e_B$, which by using (A24) requires $k = 1$. But this contradicts the heterogeneity assumption $k \neq 1$. Hence, the employer cannot maximize the expected utilities of both workers at the same time and achieve the first-best solution.

In the *J-type tournament*, the employer's zero-profit condition is described by (A23) with w instead of $w_1 + w_2$. The incentive constraints are given by

$$e_A = c'^{-1}\left(\frac{w\Delta\pi}{2(\pi_h + \pi_l)}\right) \quad \text{and} \quad e_B = c'^{-1}\left(\frac{w\Delta\pi}{2k(\pi_h + \pi_l)}\right) \tag{A26}$$

Because of the zero-profit condition the employer has to maximize

$$EU_i(e_i) = (\Delta\pi(e_i + e_j) + 2\pi_l)\frac{\Delta\pi(e_i - e_j) + \pi_l + \pi_h}{2(\pi_h + \pi_l)} - c_i(e_i) \tag{A27}$$

$(i, j = A, B;\ i \neq j)$ with respect to w subject to (A26). It can easily be shown that the employer will not be able to maximize both $EU_A(e_A)$ and

$EU_B(e_B)$ with the same aggregate wage bill w if $k \neq 1$. Hence, there is no first-best solution in the J-type tournament, too.

(ii) In the *U-type tournament* the Lagrangian for the employer's maximization problem is:

$$\begin{aligned} L &= \pi_h e_A + \pi_l (1 - e_A) + \pi_h e_B + \pi_l (1 - e_B) - w_1 - w_2 \\ &+ \lambda_1 \cdot \left[\frac{\Delta w}{2} - c'(e_A) \right] \\ &+ \lambda_2 \cdot \left[\frac{w_1 + w_2}{2} + \frac{\Delta w (e_A - e_B)}{2} - c(e_A) - \bar{v} \right] \\ &+ \lambda_3 \cdot \left[\frac{\Delta w}{2k} - c'(e_B) \right] \\ &+ \lambda_4 \cdot \left[\frac{w_1 + w_2}{2} + \frac{\Delta w (e_B - e_A)}{2} - kc(e_B) - \bar{v} \right]. \end{aligned} \tag{A28}$$

The first line in (A28) corresponds to the employer's expected surplus. The second and the fourth line describe the incentive constraints for the two workers. The third and the fifth line characterize the workers' participation constraints, which follow from Eq. (2) and from $e_A \neq e_B$ in general. In the optimum the following conditions must hold:

$$\Delta\pi - \lambda_1 c''(e_A) + \frac{\Delta w \lambda_2}{2} - \lambda_2 c'(e_A) - \frac{\Delta w \lambda_4}{2} = 0 \tag{A29}$$

$$\Delta\pi - \frac{\Delta w \lambda_2}{2} - \lambda_3 c''(e_B) + \frac{\Delta w \lambda_4}{2} - \lambda_4 k c'(e_B) = 0 \tag{A30}$$

$$-1 + \frac{\lambda_1 + \lambda_2}{2} + \frac{\lambda_2 (e_A - e_B)}{2} + \frac{\lambda_3}{2k} + \frac{\lambda_4}{2} + \frac{\lambda_4 (e_B - e_A)}{2} = 0 \tag{A31}$$

$$-1 + \frac{-\lambda_1 + \lambda_2}{2} - \frac{\lambda_2 (e_A - e_B)}{2} - \frac{\lambda_3}{2k} + \frac{\lambda_4}{2} - \frac{\lambda_4 (e_B - e_A)}{2} = 0 \tag{A32}$$

$$\frac{\Delta w}{2} = c_i'(e_i) \tag{A33}$$

$$\frac{(w_1 + w_2)}{2} + \frac{\Delta w (e_i - e_j)}{2} - c_i(e_i) \geq \bar{v} \tag{A34}$$

with $i, j = A, B$, $i \neq j$, and $\lambda_2, \lambda_4 \geq 0$. Eqs. (A31) and (A32) together yield $\lambda_2 + \lambda_4 = 2$, which implies that at least one participation constraint must be binding. Thus, we have to consider two different cases: First, let us assume that both participation constraints are binding, i.e. (A34) holds with equality. To implement first-best efforts e_A^{FB} and e_B^{FB}, the employer has to choose $\Delta w = 2\Delta\pi$ in (A33). Altogether, first-best efforts and binding participation constraints will lead to

$$\begin{aligned} \Delta\pi\left(e_A^{FB} - e_B^{FB}\right) - c\left(e_A^{FB}\right) &= \bar{v} - \frac{w_1 + w_2}{2} \\ &= -\Delta\pi\left(e_A^{FB} - e_B^{FB}\right) - kc\left(e_B^{FB}\right). \end{aligned} \tag{A35}$$

In general, the three expressions in (A35) will not be identical.[23]

Secondly, when only one of the participation constraints is binding, first-best efforts may be consistent with the optimality conditions (A29)$-$(A34). But then one of the workers gets more than his reservation value, i.e. the employer can only implement the first-best effort at high costs.

[23] For example, in case of quadratic costs $c(e_i) = 0.5\, e_i^2$ the first and the third expression in (A35) require $k = 1$, which contradicts the heterogeneity condition $k \neq 1$.

The Lagrangian in the *J-type tournament* is:

$$
\begin{aligned}
L &= \pi_h e_A + \pi_l (1 - e_A) + \pi_h e_B + \pi_l (1 - e_B) - w \\
&\quad + \lambda_1 \cdot \left[\frac{w}{2} \frac{\Delta\pi}{\pi_h + \pi_l} - c'(e_A) \right] \\
&\quad + \lambda_2 \cdot \left[\frac{w}{2} \frac{\Delta\pi (e_A - e_B) + \pi_h + \pi_l}{\pi_h + \pi_l} - c(e_A) - \bar{v} \right] \\
&\quad + \lambda_3 \cdot \left[\frac{w}{2} \frac{\Delta\pi}{\pi_h + \pi_l} - kc'(e_B) \right] \\
&\quad + \lambda_4 \cdot \left[\frac{w}{2} \frac{\Delta\pi (e_B - e_A) + \pi_h + \pi_l}{\pi_h + \pi_l} - kc(e_B) - \bar{v} \right]. \qquad \text{(A36)}
\end{aligned}
$$

Analogous optimality conditions as in the U-type tournament can be used to distinguish between two cases: First, if both participation constraints bind and the employer wants to implement first-best efforts (i.e., $w = 2(\pi_h + \pi_l)$), the participation constraints will lead to:

$$
\begin{aligned}
\Delta\pi \left(e_A^{FB} - e_B^{FB} \right) - c\left(e_A^{FB} \right) &= \bar{v} - \pi_h - \pi_l \qquad \text{(A37)} \\
&= -\Delta\pi \left(e_A^{FB} - e_B^{FB} \right) - kc\left(e_B^{FB} \right).
\end{aligned}
$$

Again, these three expressions are not identical in general.

Secondly, if at least one participation constraint does not bind, first-best efforts may be implementable. But now, it is even possible that both constraints do not bind, because $\lambda_2 = \lambda_4 = 0$ may be consistent with the optimality conditions.

Proof of Proposition 4:

In the U-type case, worker i's expected utility can be written as

$$\begin{aligned} EU_i(e_i) &= u(w_1 - c(e_i))\, e_i (1-e_j) + u(w_1 - c(e_i)) \frac{e_i e_j + (1-e_i)(1-e_j)}{2} \\ &\quad + u(w_2 - c(e_i)) \frac{e_i e_j + (1-e_i)(1-e_j)}{2} + u(w_2 - c(e_i))(1-e_i) e_j \\ &= \frac{1}{2} u(w_1 - c(e_i))(e_i - e_j + 1) \\ &\quad + \frac{1}{2} u(w_2 - c(e_i))(e_j - e_i + 1). \end{aligned} \tag{A38}$$

The workers' first-order condition $\partial EU_i / \partial e_i = 0$ yields after inserting the symmetry condition $e_i = e_j = e^U$:

$$c'\left(e^U\right) = \frac{u\left(w_1 - c\left(e^U\right)\right) - u\left(w_2 - c\left(e^U\right)\right)}{u'\left(w_1 - c\left(e^U\right)\right) + u'\left(w_2 - c\left(e^U\right)\right)}. \tag{A39}$$

Following McLaughlin (1988, p.228) we define $y_t = w_t - c\left(e^U\right)$ $(t = 1, 2)$ and $\bar{y} = (y_1 + y_2)/2 = \bar{w} - c\left(e^U\right)$ with $\bar{w} = (w_1 + w_2)/2$. Second-order Taylor series expansion of the numerator of (A39) gives

$$\begin{aligned} u(y_1) - u(y_2) &\approx u(\bar{y}) + (y_1 - \bar{y}) u'(\bar{y}) + \frac{1}{2}(y_1 - \bar{y})^2 u''(\bar{y}) \\ &\quad - u(\bar{y}) - (y_2 - \bar{y}) u'(\bar{y}) - \frac{1}{2}(y_2 - \bar{y})^2 u''(\bar{y}) \\ &= \Delta w u'(\bar{y}), \end{aligned} \tag{A40}$$

since $y_1 - \bar{y} = -(y_2 - \bar{y}) = \Delta w / 2$. Using a first-order expansion to approximate the denominator of (A39) yields

$$\begin{aligned} u'(y_1) + u'(y_2) &\approx u'(\bar{y}) + (y_1 - \bar{y}) u''(\bar{y}) + u'(\bar{y}) + (y_2 - \bar{y}) u''(\bar{y}) \\ &= 2u'(\bar{y}). \end{aligned} \tag{A41}$$

Hence, the workers' incentive constraint (A39) can be approximately written as

$$c'\left(e^U\right) = \frac{\Delta w}{2}. \tag{A42}$$

Applying analogous second-order expansion to the expected utilities $EU_i(e_i) \overset{(A38)}{=} \frac{1}{2}u(y_1) + \frac{1}{2}u(y_2)$ leads to

$$EU\left(e^U\right) \approx u(\bar{y}) + \frac{\Delta w^2}{8}u''(\bar{y}).$$

The employer has to maximize (A43) subject to (A42) and the zero-profit condition $\bar{w} = \pi_l + \Delta\pi e^U$. Using the quadratic cost function, (A42) can be written as $e^U = \Delta w/(2c)$. Substituting this and the zero-profit condition into (A43) yields the following unconstrained objective function for the employer:

$$EU(\Delta w) = u\left(\pi_l + \Delta\pi\frac{\Delta w}{2c} - \frac{\Delta w^2}{8c}\right) + \frac{\Delta w^2}{8}u''\left(\pi_l + \Delta\pi\frac{\Delta w}{2c} - \frac{\Delta w^2}{8c}\right).$$

Neglecting terms of order three (as in McLaughlin 1988, p. 230) the first-order condition gives

$$\begin{aligned} \frac{\partial EU}{\partial \Delta w} &= \left(\frac{\Delta\pi}{2c} - \frac{\Delta w}{4c}\right)u'(\bar{y}) + \frac{\Delta w}{4}u''(\bar{y}) = 0 \\ &\iff \Delta w = \frac{2\Delta\pi}{1+rc} \end{aligned} \tag{A45}$$

with $r = -u''(\bar{y})/u'(\bar{y})$ as coefficient of absolute risk aversion. Inserting (A45) in $e^U = \Delta w/(2c)$ we obtain Eq. (16).

In the J-type tournament, the expected utility of worker i is

$$\begin{aligned} EU_i(e_i) &= u\left(\frac{\pi_h}{\pi_l+\pi_h}w - c(e_i)\right)e_i(1-e_j) \\ &\quad + u\left(\frac{w}{2} - c(e_i)\right)(e_ie_j + (1-e_i)(1-e_j)) \\ &\quad + u\left(\frac{\pi_l}{\pi_l+\pi_h}w - c(e_i)\right). \end{aligned} \tag{A46}$$

From the first-order condition $\partial EU_i/\partial e_i = 0$ together with the symmetry condition $e_i = e_j = e^J$ we obtain

$$c'(e^J) = \frac{u(y_h)(1-e^J) + u(y_m)(2e^J - 1) - u(y_l)e^J}{u'(y_h)\left(e^J - [e^J]^2\right) + u'(y_m)\left(2\left[e^J\right]^2 + 1 - 2e^J\right) + u'(y_l)\left(e^J - [e^J]^2\right)} \tag{A47}$$

with $y_h := \left(\frac{\pi_h}{\pi_l+\pi_h} w - c\left(e^J\right)\right)$, $y_l := \left(\frac{\pi_l}{\pi_l+\pi_h} w - c\left(e^J\right)\right)$, and $y_m := (y_h + y_l)/2$ $= w/2 - c\left(e^J\right)$. Using first-order Taylor series approximation, the numerator of (A47) can be written as

$$\begin{aligned} &[u(y_m) + (y_h - y_m)u'(y_m)]\,(1-e^J) + u(y_m)(2e^J - 1) \\ &\quad - [u(y_m) + (y_l - y_m)u'(y_m)]\,e^J = \frac{\Delta\pi w}{2\left(\pi_l + \pi_h\right)} u'\left(y_m\right). \end{aligned} \tag{A48}$$

The same approximation method for the denominator gives

$$\begin{aligned} &[u'(y_m) + (y_h - y_m)u''(y_m)]\left(e^J - \left[e^J\right]^2\right) + u'(y_m)\left(2\left[e^J\right]^2 + 1 - 2e^J\right) \\ &\quad + [u'(y_m) + (y_l - y_m)u''(y_m)]\left(e^J - \left[e^J\right]^2\right) = u'\left(y_m\right) \end{aligned} \tag{A49}$$

so that the incentive constraint (A47) can be simplified to Eq. (18) because of the quadratic cost function. Using the symmetry condition $e_i = e_j = e^J$, and y_h, y_m and y_l in (A46) the workers' expected utility in equilibrium can be approximated by a second-order Taylor series expansion around y_m:

$$EU(e^J) = u(y_m) + \frac{\Delta\pi^2 w^2}{4\left(\pi_l + \pi_h\right)^2} u''\left(y_m\right)\left(e^J - \left[e^J\right]^2\right). \tag{A50}$$

Inserting the zero-profit condition $\frac{w}{2} = \pi_l + \Delta\pi e^J(w)$ into (A50) with $e^J(w)$ being defined by Eq. (18) gives

$$\begin{aligned} EU(e^J) &= u\left(\pi_l + \Delta\pi e^J(w) - \frac{c}{2}\left[e^J(w)\right]^2\right) \\ &\quad + \frac{\Delta\pi^2 w^2}{4\left(\pi_l + \pi_h\right)^2} u''\left(\pi_l + \Delta\pi e^J(w) - \frac{c}{2}\left[e^J(w)\right]^2\right)\left(e^J(w) - \left[e^J(w)\right]^2\right). \end{aligned} \tag{A51}$$

When again neglecting terms of order three the first-order condition

$$\frac{\partial EU}{\partial e^J} = \left(\Delta\pi - ce^J\right)\frac{de^J}{dw}u'(y_m) + \frac{\Delta\pi^2 w}{2\left(\pi_l+\pi_h\right)^2}u''(y_m)\left(e^J - \left[e^J\right]^2\right) \quad \text{(A52)}$$
$$+\frac{\Delta\pi^2 w^2}{4\left(\pi_l+\pi_h\right)^2}u''\left(y_m\right)\left(1-2e^J\right)\frac{de^J}{dw} = 0$$

after some rearranging leads to Eq. (17) using the fact that $de^J/dw = \Delta\pi/\left(2c\left(\pi_l+\pi_h\right)\right)$ and $r := -u''(y_m)/u'(y_m)$.

Now the last part of Proposition 4 can easily be checked. From (15) and (16) we immediately have $\lim_{r\longrightarrow\infty}\Delta w = \lim_{r\longrightarrow\infty} e^U = 0$. Dividing both sides of (17) by r and calculating $\lim_{r\rightarrow\infty}$ yields

$$2c\left(e^J - \left[e^J\right]^2\right) + \frac{\Delta\pi w}{2\left(\pi_l+\pi_h\right)}\left(1-2e^J\right) = 0. \quad \text{(A53)}$$

Together with Eq. (18) we get the last result of Proposition 4.

Proof of Proposition 5:

In the U-type tournament with $\lambda < \pi$ the workers' objective functions are given by

$$\begin{aligned} EU_i\left(e_i\right) &= w_1 e_i\left(1-e_j\right) + w_2\left[1-e_i\left(1-e_j\right)\right] - c\left(e_i\right) \\ EU_j\left(e_j\right) &= w_2\left(1-e_j\right)e_i + w_1\left[1-\left(1-e_j\right)e_i\right] - c(e_j). \end{aligned}$$

Using the quadratic cost function and $\Delta w = 2\pi$ the first-order conditions yield

$$\begin{aligned} e_i^U &= \frac{c\Delta w}{c^2+\Delta w^2} = \frac{2c\pi}{c^2+4\pi^2} \qquad \text{and} \\ e_j^U &= \frac{\Delta w^2}{c^2+\Delta w^2} = \frac{4\pi^2}{c^2+4\pi^2}\,. \end{aligned}$$

In the J-type tournament, irrespective of the concrete value of λ, workers' expected utilities are

$$EU_i(e_i) = \frac{\pi}{\pi+\lambda} we_i(1-e_j) + \frac{\pi}{2\pi+\lambda} we_i e_j - c(e_i)$$

$$\begin{aligned} EU_j(e_j) &= w\left[e_j(1-e_i) + (1-e_j)(1-e_i)\right] + \frac{\pi+\lambda}{2\pi+\lambda} we_i e_j \\ &\quad + \frac{\lambda}{\pi+\lambda} w(1-e_j)e_i - c(e_j). \end{aligned}$$

The first-order conditions together with the quadratic cost function and $w = 2\pi$ can be used to find

$$e_i^J = \frac{2\pi^2 c(\pi+\lambda)(2\pi+\lambda)^2}{4\pi^4(\pi^2+c^2) + c^2\lambda(3\pi+\lambda)\left(4\pi^2+3\pi\lambda+\lambda^2\right)}$$

$$e_j^J = \frac{4(2\pi+\lambda)\pi^5}{4\pi^4(\pi^2+c^2) + c^2\lambda(3\pi+\lambda)\left(4\pi^2+3\pi\lambda+\lambda^2\right)}.$$

Hence, $e_i^U < e_i^J$ and $e_j^U < e_j^J$ if

$$c^2 < \frac{4\pi^3\left(3\pi^3+8\pi^2\lambda+5\pi\lambda^2+\lambda^3\right)}{\lambda(\pi+\lambda)(2\pi+\lambda)^2}$$

$$\text{and} \qquad c^2 < \frac{4\pi^5(\pi+\lambda)}{(2\pi+\lambda)\left(\pi^3+5\pi^2\lambda+4\pi\lambda^2+\lambda^3\right)}$$

which is true for sufficiently small c and also for sufficiently small λ since the right-hand sides of the two inequalities are both strictly decreasing in λ.

In addition, we have

$$\frac{\partial e_i^J}{\partial\lambda} = -2\pi^2 c(2\pi+\lambda)\frac{(38\lambda^2\pi^3+8\pi\lambda^4+28\lambda\pi^4+25\lambda^3\pi^2+8\pi^5+\lambda^5)c^2-(16\pi+12\lambda)\pi^6}{\left(4\pi^6+4\pi^4c^2+12c^2\lambda\pi^3+13c^2\lambda^2\pi^2+6c^2\lambda^3\pi+c^2\lambda^4\right)^2}$$

which is negative for sufficiently large values of λ and c, and

$$\frac{\partial e_j^J}{\partial\lambda} = -4\pi^5\frac{(-4\pi^6+20\pi^4c^2)+52c^2\lambda\pi^3+49c^2\lambda^2\pi^2+20c^2\lambda^3\pi+3c^2\lambda^4}{\left(4\pi^6+4\pi^4c^2+12c^2\lambda\pi^3+13c^2\lambda^2\pi^2+6c^2\lambda^3\pi+c^2\lambda^4\right)^2} < 0$$

since we have to assume $\pi < c$ to ensure $e^{FB} = \Delta\pi/c \in (0,1)$.

References

Baker, G.P., Gibbs, M., and Holmström, B. (1994a). The internal economics of the firm: evidence from personnel data. *Quarterly Journal of Economics* 109: 881-919.

Baker, G.P., Gibbs, M., and Holmström, B. (1994b). The wage policy of a firm. *Quarterly Journal of Economics* 109: 921-955.

Endo, K. (1994). *Satei* (personal assessment) and interworker competition in japanese firms. *Industrial Relations* 33: 70-82.

Fairburn, J.A. and Malcomson, J.M. (1994). Rewarding performance by promotion to a *different* job. *European Economic Review* 38: 683-690.

Fairburn, J.A. and Malcomson, J.M. (2001). Performance, promotion, and the Peter Principle. *Review of Economic Studies* 68: 45-66.

Green, J.R. and Stokey, N.L. (1983). A comparison of tournaments and contracts. *Journal of Political Economy* 91: 349-364.

Holmström, B. (1982). Moral hazard in teams. *Bell Journal of Economics* 13: 324-340.

Ito, T. (1992). *The Japanese Economy.* Cambridge (Mass.), London.

Itoh, H. (1991). Japanese human resource management from the viewpoint of incentive theory. *Ricerche Economiche* 45: 345-376.

Kanemoto, Y., and MacLeod, W.B. (1989). Optimal labor contracts with non-contractible human capital, *Journal of the Japanese and International Economies* 3: 385-402.

Kanemoto, Y., and MacLeod, W.B. (1992). Firm reputation and self-enforcing labor contracts, *Journal of the Japanese and International Economies* 6: 144-162.

Kräkel, M. (2001). U-type versus J-type tournaments. Bonn Graduate School of Economics, University of Bonn, Discussion Paper No. 34/2001.

Laffont, J.-J. (1988). Hidden gaming in hierarchies: Facts and models. *Economic Record* 64: 295-306.

Laffont, J.-J. (1990). Analysis of hidden gaming in a three-level hierarchy. *Journal of Law, Economics, and Organization* 6: 301-324.

Lazear, E.P., and Rosen, S. (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy* 89: 841-864.

Malcomson, J.M. (1984). Work incentives, hierarchy, and internal labor markets. *Journal of Political Economy* 92: 486-507.

Malcomson, J.M. (1986). Rank-order contracts for a principal with many agents. *Review of Economic Studies* 53: 807-817.

McLaughlin, K.J. (1988). Aspects of tournament models. *Research in Labor Economics* 9: 225-256.

Milgrom, P. and Roberts, J, (1988). An economic approach to influence activities in organizations. *American Journal of Sociology*, Supplement 94: S154-S179.

Mood, A.M., Graybill, F.A., and Boes, D.C. (1974). Introduction to the theory of statistics. third edition (Singapore).

Prendergast, C. and Topel, R. (1993a). Discretion and bias in performance evaluation. *European Economic Review* 37: 355-365.

Prendergast, C. and Topel, R. (1993b). Favoritism in organizations. NBER Working Paper No. 4427.