Bonn Econ Discussion Papers

Discussion Paper 3/2006

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February 2006

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Organization of R&D With Two Agents and Principal[*]

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Abstract

In order to deliver an innovation principals employ competing agents in some circumstances, while employing research team in other circumstances. This paper compares various structures of R&D to provide a rational behind this observation. It is assumed, that the principal can employ either one agent, two competing agents or two agents, cooperating in a team. Which of the available structures will be chosen by principal, depends on value of prize in stake, technological benefits of team production and team structure. Due to the positive effect on incentives, competing agents always generate larger profit to the principal, than a single agent. Further, they often perform better than the team, even when the latter has significant technological benefits. However, the performance of the team may be improved, if it is organized as a hierarchy with the team leader (who is responsible for allocation of resources) and his subordinate. The paper provides conditions on parameters, which determine whether the principal should employ a team or competing agents for performing R&D.

Keywords: moral hazard, hierarchy, team production, competition, organization of R&D
JEL Classification: O31, L23, C72

1 Introduction

There is a large body of literature which employs the principal-agent approach to the analysis of financing of innovation. This literature largely assumes a structure

[*]For many valuable discussions and suggestions I am grateful to Avner Shaked, Eugen Kovac and Patrick Schmitz.

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of a research unit to be exogenous and investigates the interaction of a principal with either a single agent or multiple agents forming a team or competing with each other. However, the decision of the principal on how many agents to employ and how to structure their interaction is not explored sufficiently. Yet, such decision, should be key for the principal, since it inevitably influences both the technological capabilities of the research unit and the incentives of the agents.

Anecdotal evidence suggest, that there are circumstances, when principals (i.e. grant agencies, venture capitalists, government agencies) decide to employ competing agents. For example, McKinsey, a consultancy, describes a case of a company in the chemical industry which was seeking ways to change the old chemical production processes to more competitive synthesis routes. The company used competing research teams to attack the problem.[1] Similarly, Vulcan Inc., which is a multi-division corporation, owned by former Microsoft co-founder Paul Allen, has contracted three competing agencies for the project Halo, aimed at the development of the problem-solving software.[2] On the other hand it is not unusual for a financier to grant a financial support to a consortium of co-operating independent teams, rather than to each of these team separately. Numerous examples include research grants by National Science Foundation of the US or by National Institute of Health. The purpose of this paper is to contribute to a theory, which rationalizes a decision of the principal to employ competing agents at some instances, while to contract a non-competing team or a single agent at other instances.

A theoretical justification for why it can be profitable for the principal to finance competing agents rather than a single agent is provided in Levitt (1995). Che and Yoo (2001) analyze the attractiveness of the team production versus stand-alone production in the repeated setting. Hemmer (1995) shows, that if there are synergies from performing two tasks, then assigning a team to the subsequent tasks results in higher product quality than assigning separate agents to each of those tasks. Goldfain and Kovac (2005) compare benefits from employing competing agents, rather than a single agent, in a dynamic framework with multiple stages of R&D.

The present paper contributes to the literature on financing of innovation by investigating, when it is in principal's interests to assign competing agents to the same task and when he prefers that agents cooperate in a team. The team may have significant technological benefits (synergy effects). However, in the presence of moral hazard the ability of team to exploit these synergies efficiently may depend on the team's structure. On the other hand, in the moral hazard framework competition acts as an important incentive device and may prove to be a superior organizational structure, comparing with team production, despite

[1]The case study is available at `http://www.mckinsey.com/clientservice/chemicals`

[2]The information about the project can be found at `http://www.projecthalo.com`

the lower efficiency of R&D and duplication of research costs.

To support this intuition, I analyze four alternative structures of R&D: stand-alone agent, competing agents, team with equal partners and hierarchical team. These structures are compared in a unified framework, where research is financed by a principal, but the agents have a discretion to decide whether to allocate money into the project or to divert them for private consumption.

It is shown, that due to the positive effect which competition has on incentives, the principal always prefers to employ competing agents, rather than a single agent. The paper is therefore focused on comparing team production (where team may be organized as a hierarchy or as equal partners team) with competing agents. A team, where agents are treated identically by the principal, suffers from the free-riding problem, and therefore often performs worse, than competing agents. I conclude, that such team will be employed, only if technological benefits of team production are high, or if a prize in stake is not too large.

The existing literature on team production suggests several mechanisms, which allow to alleviate the free-riding problem in a one-shot model. The general idea behind these mechanisms is, that agents are required to monitor each other and submit the report to the principal, based on their observations (Miller 1997, Ma 1988, Marx and Squintani 2002). However, these mechanisms are difficult to implement in the environment where agents feel guilty about spying on their team mates. Therefore, I suggest an alternative mechanism, which under certain conditions allows to increase the attractiveness of team production. Namely, the principal may decrease the moral hazard in team by assigning one of the agents to the principal position (position of the team leader) above the other agent (subordinate).

If the team leader can observe effort of an agent on the lower level of hierarchy, then such hierarchical structure gives the team leader power to punish his subordinate, if the latter shirks. This arrangement unambiguously improves incentives of the subordinate, while also resulting in the efficient allocation of investment recourses. Hence, the hierarchical team structure significantly improves performance of the team and increases range of parameters where the principal chooses to employ agents, cooperating in a team.

On the other hand, if the team leader cannot observe and verify the effort of his team mate, then the hierarchical team structure may improve incentives of the former, although the incentives of the latter remain unaltered. I show that in this situation there exists a range of parameters, where the team leader over-invests in R&D, comparing with his best response reaction (i.e. comparing with his investment choice in equal-partners team). For this range of parameters the hierarchical team may become a superior arrangement comparing with alternative team structure.

However, for the unobservable effort of the subordinate the hierarchical team always leads to suboptimal allocation of investments and to the loss of efficiency in terms of success probability. This efficiency loss increases with investment resources, allocated to the agents. Hence, for sufficiently large value of the prize in stake (implying large value of investment resources) the principal will never organize a research team as a hierarchy, if the team leader is not able to monitor his subordinate.

Aside from the literature on financing of innovation, this paper is related to several other streams of literature. First, it contributes to the literature on team incentives by analyzing alternative structures of team production and drawing implications for the optimal contract and the investment decisions of the agents. Second, by addressing benefits and disadvantages of team production comparing to the competitive setup, this paper is close in spirit to the literature on formation of joint ventures.

In the literature on formation of joint ventures there are only few papers which explicitly take a principal-agent approach. The most relevant among them is Fabrizi and Lippert (2003). The authors investigate, how the presence of moral hazard influences decisions of two firms, each of which has one agent, to conduct a joint project. The authors focus in their analysis on welfare effect of joint venture and on the market strategies of the firms. Unlike their work, the present paper studies influence of different organizational structures on financing decisions of the principal, incentives of the agents and their investment decisions.

There is a stream of papers in the literature on team incentives, which similarly to my paper investigate the internal structure of a team and its implementation for productivity (Macho-Stadler and Perez-Castrillo 1993, Itoh 1991, Itoh 1993). This literature, however, has very different focus from the present paper: it is focused on the analysis of the effects which mutual help, cooperation and reciprocity within the team have on incentives. The existing literature on internal structure of a team usually compares a group of individuals, where each is concentrated in his own task with a team, where agents help each other. This is not the issue in the present paper. Here the cooperation between the agents, in case when they form a team, is simplified and boils down to the specialization in different tasks, which is the source of synergy effects in the model. The paper contributes to the literature by investigating a different aspect of team production, namely, whether and when a team should be organized as a hierarchy.

The structure of this paper is the following. The basic framework of the model is described in Section 2. The setup with a single agent, competing agents and a team with equal partners is discussed through Sections 3 to 5. Section 6 is devoted to the comparison of team setup with competing agents setup. In Section 7, I investigate the alternative team structure, where one of the agents has a principal position over his peer, and draw the implications of such team structure for the incentives of the agent an profit of the principal. Finally, the conclusions are summarized in Section 8. Proofs and figures can be found in Appendix.

2 Basic framework of the model

The basic structure of this model is following. There are two identical risk-neutral agents (entrepreneurs), who have an idea how to solve a particular problem. The problem (which I will henceforth call a project), if solved successfully, yields a prize of size R. For example, if the project is to find a cure against a deceases, then R may represent a discounted stream of all future payoff, generated by sales of this cure. Moreover, I assume that in case of competition between agents, if one of them successfully completes the project, the solution will be patented, so that the follower does not earn anything.

It is assumed that the agents have no wealth and the necessary funds for research and development are provided by a principal (venture capitalist, grant agency or a firm which has subcontracted research and development to the agents) in exchange for a share in the project. Although finances are provided by the principal, allocation decisions are made by agents. They can either invest funds or divert them for private uses. The principal is not able to observe the allocation decision. All he can observe is a success or a failure of the project.

The principal is risk-neutral and maximizes his expected payoff from the project. I assume that the principal has all bargaining power, which means that after paying the agents their contractual payoffs, he retains all the residual surplus. The principal offers agents a contract, which specifies the share of each agent in case of success and the size of investment.[3] Moreover, the principal can decide whether to allow the agents to compete for the patent or to order them to form a research team and to join their research efforts. In the former case the winning agent patents his invention and shares the prize with principal. In the latter case, the prize is shared between the team and the principal.

The success of the project is stochastic and depends on efforts of both agents as well as on the structure of the research (team or competition). It is assumed, that the effort of agents in this model is equivalent to their monetary investments into R&D. For illustration, consider the case with one agent. Let the principal transfer amount c to this agent. The agent allocated amount $x \leq c$ to the research and consumes $c - x$. I assume, that the probability of success is given by:

$$p(x) = 1 - e^{-x}.$$

To justify this particular functional form, I make a realistic assumption, that the project is successful only if the agent finishes his research before time $T = 1$

[3] It will become clear later, than in case of hierarchical team the principal offers a contract only to the team leader.

elapses[4]. I further assume, that the probability of success is $p(x) = P(t_x < 1)$, where t_x is an exponentially distributer random variable with mean $\frac{1}{x}$. Note, that from the relationship between the exponential and the Poisson distribution, it follows, that x is an average number of successes in unit of time, while $\frac{1}{x}$ is the expected time, until the first success is made. It is clear therefore, that the higher is investment x, the shorter is expected time, which elapses before a success occurs.[5] The interpretation of x as a parameter of exponential distribution is not essential for the model. Another way to justify the functional form of $p(x)$ is to assume, that by investing x an agent generates a random variable t_x, which is uniformly distributed on the interval $[0, e^x]$. This variable can be interpreted as a distance covered between the initial stage of R&D and its final stage. Assume, that the project is successful, if the random variable takes the value $t_x \geq 1$. The probability of this event is $p(x) = 1 - e^{-x}$.

The research and development is modelled as a one-shot game. After the contract is signed the agents make one-time decision on how much to invest and the probability of success is a function of this investment.

3 Single agent

Let me first consider the most simple case when only one agent is employed. The game has two stages: in the first stage the principal offers a contract, where he determines amount of investment funds c and the share of the agent β^S. In the second stage the agent (given the terms of a contract) allocates $x \leq c$ into R&D and consumes $c - x$. In case of success the agent receives a share β^S of R. I will denote the profit of the agent Π_1^S, where S stands for "single". Here I also use the index of agent $i = 1$ to avoid confusion in further discussion, when the second agent will be introduced The game is solved backwards, starting from the agent's problem:

$$\max_{x \in [0,c]} \Pi_1^S = R\beta^S(1 - e^{-x}) + c - x. \tag{1}$$

Note, that agent's profit consists of two parts. First, he enjoys a reward $R\beta^S$ in case of success (which happens with probability $1-e^{-x}$). Second, he also consumes part of funds at his discretion, so that $c - x \geq 0$.[6] The solution to the problem is

[4]The assumption of limited financing horizon justified, since it is common for the venture capital firms or grant agencies to set the time limits within which the research must be completed. For theoretical justification see Goldfain and Kovac (2005).

[5]I also make a technical (normalization) assumption, that investment of x monetary units translates to a probability $1 - e^{-x}$.

[6]To avoid confusion, note, that β^S denotes a percent share of the prize, while $R\beta^S$ is a monetary share.

the following:

1. $x = 0$, if $R\beta^S \leq 1$,
2. $x \in (0, c)$: is such that $R\beta^S = e^x$, if $1 \leq R\beta^S \leq e^c$,
3. $x = c$, if $R\beta^S \geq e^c$.

The principal chooses the terms of the contract, namely amount of investment c and the share of an agent β^S, taking into account the solution of Problem 1. Note that if the principal chooses $R\beta^S \leq 1$, the agent will consume all funds, which leaves a principal with a negative profit $\Pi_P^S = -c$. He can do better by not investing in the project at all. Hence, if the principal decides to invest in the project, he will never choose $R\beta^S \leq 1$. So, we can limit our attention to the investigation of the strategies, which dominate $R\beta^S \leq 1$:

$$
\begin{aligned}
\max_{c,\beta} \quad & \Pi_P^S = R(1-\beta^S)(1-e^{-x}) - c \\
s.t. \quad & (ICS)\ R\beta^S \geq e^x, \\
& (RCS)\ x \leq c, \\
& (CSS)\ (R\beta^S - e^x)(x - c) = 0.
\end{aligned}
$$

The incentive compatibility constraint (ICS) ensures, that the agent invests in R&D. According to resource constraint (RCS) he can only invest as much as c. Finally, according to complimentary slackness condition (CSS) at least one of the two other constrains should be binding, as follows from the equilibrium conditions above. If the incentive constraint does not bind, the agent invests all available funds, so that RCS binds. If the recourse constraint does not bind, then the incentive constraint will necessarily be binding.

Both the incentive compatibility constraint and the recourse constraint will bind in the optimum. There is a following intuition behind this result. Assume, that the (RCS) does not bind, so that $x^* < c$, where x^* is the equilibrium choice of the agent. If the principal marginally decreases c, the agent's investment does not change (the probability of success stays unaltered), but the investment cost declines, so that the profit of the principal increases. Hence, in optimal solution the principal always chooses c so, that the (RCS) binds. The same intuition justifies why the incentive compatibility constraint should be binding. Indeed, assume that the constraint does not bind, so that $R\beta^S > e^x$. Then the principal can decrease a share of the agent (hence, increase his own share) without altering the probability of success. So, in optimum the principal will choose such β^S, that (ICS) constraint binds.

The explicit solution to the principal problem is derived in the proof to Proposition 1, where I also formally prove, that both (ICS) and (RCS) constrains bind in optimum.

Proposition 1. *Assume that the principal employs one agent and let $R > 2$. Then in SPNE the following statements hold:*

1. *Values of c and β^S are such, that the agent invests all funds which he receives from the principal, i.e. $x = c$.*

2. *The optimal amount of investment is $c = \ln \frac{1}{2}(-1 + \sqrt{1 + 4R})$, if $R > 2$ and $c = 0$, if $R \leq 2$*

3. *The reward of the agent is $R\beta^S = e^c$.*

Both the equilibrium amount of investment c and the reward of the agent $R\beta^S$ increase in the value of R. This is the essence of the tradeoff which the principal faces. He is willing to increase his investment, if the project promises a lucrative payoff. However, in order to ensure that the agent does not divert funds to the private consumption, the principal has to balance the incentive constraint of the latter by promising him a larger share of the prize.

Note further, that the $c = 0$ as $R = 2$. Since the equilibrium investment expenditures of the principal increase in R, then for any $R \leq 2$ the principal will not employ a stand-alone agent in equilibrium.

4 Competing agents

In a setting with competing agents, the prize is shared between the winning agent and the principal. After the terms of a contract (i.e., the share of each agent in case of success and the amount of investments) are announced, the agents decide which part of funds they allocate to R&D and which part they consume.

Let the principal transfer amount c to the first agent and amount d to the second agent. Let $x \leq c$ be the funds which the first agent allocates to the project and $c - x \geq 0$ are the funds that he diverts to the private consumption. Likewise, I define y and $d - y$. Second agent wins the prize, if he successfully completes the project at time t_y, such that $t_y \leq t_x$ and $t_y \leq 1$, where t_x is a time, when first agent completes his project. Hence, the probability that second agent succeeds is:

$$P(t_y \leq t_x \ \wedge \ t_y \leq 1) = P(t_y \leq 1 \leq t_x) + P(t_y \leq t_x < 1) =$$
$$= e^{-x}(1 - e^{-y}) + \int_0^1 \int_0^t xe^{-xt} ye^{-yu} du dt = \left(1 - \frac{x}{x+y}\right)\left(1 - e^{-(x+y)}\right).$$

Then, the expected payoff of the second agent is Π_2^C, where "C" stands for "competition" is :

$$\Pi_2^C(x, y) = R\beta_2^C \frac{y}{x+y}\left(1 - e^{-(x+y)}\right) + d - y,$$

where $R\beta_2^C$ is the share which the second agent receives according to a contract. Analogically, the expected payoff of the first agent is:

$$\Pi_1^C(x,y) = R\beta_1^C \frac{x}{x+y}\left(1 - e^{-(x+y)}\right) + c - x,$$

where $R\beta_1^C$ is the share which the first agent receives according to a contract. Note, that conditional on the fact, that at least one agent succeeds, the probability that first agent succeeds $\frac{x}{x+y}$ and the probability that the second agent succeeds is $\frac{y}{x+y}$. This result is consistent with the finding of the literature on contests and patent races (Tullock 1980, Dixit 1987, Loury 1979). Note further, that for $y = 0$, the profit Π_1^C reduces to the profit of an agent in stand-alone situation.

In equilibrium, each agent plays his best response to the rival's strategy by choosing amount of investment x or, respectively, y, taking $R\beta_2^C$, $R\beta_1^C$, c and d as given. Let us consider the best response correspondence for the first agent. Taking the derivative of Π_1^C I receive:

$$\frac{\partial \Pi_1^C}{\partial x} = R\beta_1^C\left(\frac{1 - e^{-(x+y)}}{(x+y)^2}y + \frac{x}{x+y}e^{-(x+y)}\right) - 1.$$

Hence, the best response of the first agent to the investment choice of the second agent is

1. $x = 0$, if $R\beta_1^C \leq \frac{y}{1-e^{-y}}$,
2. $x \in (0,c)$ such that $R\beta_1^C = \frac{(x+y)^2}{(1-e^{-(x+y)})y + xe^{-(x+y)}(x+y)}$, if $\frac{y}{1-e^{-y}} \leq R\beta_1^C \leq \frac{(c+y)^2}{(1-e^{-(c+y)})y + ce^{-(c+y)}(c+y)}$
3. $x = c$, if $R\beta_1^C \geq \frac{(c+y)^2}{(1-e^{-(c+y)})y + ce^{-(c+y)}(c+y)}$.

The best response of the second agent can be derived similarly. Depending on the parameters, there are seven potential equilibria in the last stage of the game: $(0,0)$, $(x,0)$, $(0,y)$, (x,y), (c,y), (x,d), (c,d). The conditions for each of those equilibria to occur, are described in Table 1 in Appendix.

The problem of the principal is to choose the terms of the contract so that the residual expected payoff (gross payoff net of agents compensation) is maximized. I first derive the optimal contract for each possible equilibrium and then choose the one, which delivers the principal the highest profit. If the equilibrium investment decision is $(x,y) = (0,0)$, then the optimal solution is not to employ the agents and hence the profit of the principal is zero. The equilibrium decision such, that

$x \in (0,c]$, $y=0$ or $x=0$, $y \in (0,d]$ is equivalent to at most one agent being employed. Solution of the problem in this case is described in previous section. Finally, if neither x nor y are zero in equilibrium, then the problem of principal is to maximize his profit subject to incentive compatibility constraints, which are described by equilibrium conditions (see Table 1). The principal receives his share of the prize if at least one of the agents wins, which happens with probability $1-(1-p(x))(1-p(y)) = (1-e^{-(x+y)})$. In equilibrium the principal is going to treat the agents symmetrically, so that $\beta_1^C = \beta_2^C$ and $c = d$. Further, the optimal contract will be such, that the agents find it just incentive compatible to allocate all recourses which they receive to R&D. In other words, they will receive exactly a share which makes them to invest $x = y = c$ into the project. The problem of the principal in the reduced form (i.e. with binding constrains and symmetric agents) can be written as:

$$\max_{\beta^C,\ c} \quad \Pi_P^C = R(1-\beta^C)(1-e^{-2x}) - 2c$$
$$s.t. \quad R\beta^C := R\beta_1^C = R\beta_2^C = \tfrac{e^{2x}4x^2}{2x^2-x+e^{2x}x}\ ,$$
$$x = c.$$

The solution to this problem leads to the optimal contract and is formalized in Proposition 2.

Proposition 2. *Let competing agents be employed. Assume that $R > 2$ and that the agents are identical. Then for the optimal contract following statements hold:*

1. *The optimal choice of $\beta_1^C, \beta_2^C, c, d$ leads to unique SPNE such that the agents allocate all the funds into R&D: $x = c$, $y = d$.*

2. *In equilibrium the agents are treated symmetrically: $\beta_1^C = \beta_2^C$, $c = d$.*

3. *Equilibrium level of investment c increases in R and is given by*

$$R = \frac{e^{2c}\left[4c(e^{2c}-1) + 3(e^{2c}-1)^2 + c^2(4+8e^{2c})\right]}{(e^{2c}+2c-1)^2}$$

4. *The rewards of the agents are given by* $R\beta_1^C = R\beta_2^C := R\beta^C = \dfrac{4c}{1-e^{-2c}+2ce^{-2c}}$.

According to Proposition 2, if the principal employs competing agents, he will suggest a contract which leads to unique SPNE with $(x,y) = (c,c)$. From the relationship between R and c it follows, that $R \to 2$ as $c \to 0$ (one can see this by applying the L'Hospital rule twice). Hence, for $R \leq 2$ the competing agents will not be employed.

The optimal contract for the competing agents is developed under the assumption, that only the winner of the patent race receives a reward. It is easy to see that any contract, where the follower also receives some reward, will be strictly worse from the principal's point of view. Indeed, for each amount of investment the optimal contract will be such, that agents invest all received funds in R&D. Hence, the probability of success will be the same in a setting, where the follower receives some reward, as in the setting where he receives nothing. However, in the former case the incentives of the agents to invest are significantly weaker, which increases the rent they can extract from the principal. Hence, in more general contract, where an agent earns a reward $R\beta^L$ if he wins the patent race, and a reward $R\beta^F$, if he looses this race[7], the principal will optimally choose $R\beta^F = 0$.

The last step which is left to be done is to decide whether the principal indeed prefers to employ competing agents instead of a single agent.

Corollary 1. *Assume, $R > 2$. Then the principal always prefers to employ competing agents, rather than a single agent.*

The competition has a twofold effect in this model. It increases probability of success and disciplines agents by decreasing the rent which they can extract from the principal. These effects soften the principal's tradeoff. Comparing to the set-up with a single agent, he can afford increasing the investment level by larger amount, while increasing the reward to the agents only by smaller share. As is shown in the proof to the proposition above, in equilibrium the principal will invest more if he employs competing agent, than if he employs a single agent. However, the share, which he pays to each agent is smaller in the former case, then in the latter. This result conforms to the intuition in Goldfain and Kovac (2005), where the authors show (in different modelling framework) that competition is always beneficial for the principal, when the agents are identical.

5 Team production

An alternative organization of R&D, which the principal might use, is a research team. In the team the agents join their efforts in order to complete the project. If the project is successful, the prize is divided according to the contract between the principal and the team. I assume, that the principal observes neither individual nor the joint contribution of the agents to research. Hence, the agents receive their reward only if the team succeeds. The problem inherited in the team production is free-riding. If the team wins the prize, each agent receives his share, no matter how much he has invested in the research. Hence, the agents face a tradeoff

[7] "L" and "F" stay for the "Leader" and the "Follower" respectively.

between increasing a joint probability of success by investing and increasing their own payoff by consuming the funds.

The team, however, may have technological benefits compared to other organizational structures of the research unit. These technological benefits from cooperation (further referred to as *synergy effects*) allow the team to generate a higher success probability for fixed amount of investments, than each agent in stand-alone situation can achieve. I model the team production by assuming that the joint investments influence the probability of success in the following way:

$$p(x, y) = 1 - e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}.$$

In the Appendix, I provide a formal model which justifies the choice of the function $(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}$ for modelling of the synergy effects in team production. It is shown that this specification reflects a situation, where probability of success depends on two skills or production factors, and each of two agents is "talented" in different skill (so, that it is relatively "cheap" for him). In this case the joint production (i.e. the probability of success) is maximized, when each agent specializes in his cheapest factor.[8] The parameter $\alpha \in [0, 1)$, that is assumed to be a common knowledge, characterizes a degree of complementarity between skills of team members, which is the source of synergies in this model. If $\alpha = 0$, then there are no technological benefits from employing a team. It succeeds with the probability $p = 1 - e^{-(x+y)}$, which is the same as if a single agent invests $(x + y)$. Moreover, this probability exactly equals the probability that at least one of the competing agents succeeds, as derived in Section 4.[9] If $\alpha > 0$, i.e. there are synergies in team production, then for the same amount of investment x and y the team succeeds with higher probability, than a single agent, who invests the same amount (or, for that matter, than at least one of two independent agents).

5.1 Optimal contract for team

This section is devoted to the solution of principal's problem in case when the team is employed. I assume, that the principal signs two separate contracts with both agents, in which he determines investment funds allocated to each agent and a share of each agent in case of success. As before, the game is solved backwards starting from the last stage, where the agents in team choose x and y, given the terms of contract.

[8] Specialization is one of the sources of synergy effects. Another possible source is a better organization of work, which allows to decrease the duplication of effort. See Lippert (2005) for a model, relevant in this case.

[9] This is a result of a property of the exponential distribution: if probability of success as function of investments is $p(x) = 1-e^{-x}$, then $p(x+y) = 1-(1-p(x))(1-p(y)) = 1-e^{-(x+y)}$.

I assume, that the agents choose their efforts simultaneously. Hence, each member of the team maximizes his own profit, by playing the best response to the investment decision of his team-mate, taking the terms of contract as given. The decision problem of the first and second agent respectively are:

$$\max_{x\in[0,c]} \quad \Pi_1^T = R\beta_1^T(1 - e^{-(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-a}}}) + c - x,$$

$$\max_{y\in[0,d]} \quad \Pi_2^T = R\beta_2^T(1 - e^{-(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-a}}}) + d - y.$$

Consider the problem of the first agent. The first derivative of the payoff function is the following:

$$\frac{\partial \Pi_1^T}{\partial x} = R\beta_1^T(x^{1-\alpha} + y^{1-\alpha})^{\frac{\alpha}{1-\alpha}} x^{-\alpha} e^{-(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-a}}} - 1.$$

Let me define $k := \dfrac{c^\alpha e^{(c^{1-\alpha}+y^{1-a})^{\frac{1}{1-\alpha}}}}{(c^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}$. Then, the best response of the first agent (i.e., optimal choice of x) to any y of the second agent is:

1. $x = 0$, if $R\beta_1^T \leq 1$ and $y = 0$,
2. $x \in (0, c)$ such that $R\beta_1^T = \dfrac{x^\alpha e^{(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-\alpha}}}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}$, if $k \geq R\beta_1^T \geq 1$
3. $x = c$ if $R\beta_1^T \geq k$.

Similarly, I can define the best response for the second agent. As in the case of competition, depending on values of $R\beta_1^T$, $R\beta_2^T$, c and d, different equilibria can occur. The equilibrium conditions and resulting choice of (x, y) are summarized in Table 2 in Appendix. As above, I will derive the optimal contract for each possible equilibrium and then will choose the contract, which maximizes the payoff of the principal. If $(x, y) = (0, 0)$ is played in equilibrium, then the principal prefers not to employ the team, which yields him a zero profit. Otherwise, the principal maximizes his profit subject to incentive compatibility constraints, given by equilibrium conditions, described in Table 2. There are two considerations to be taken into account. First, the principal will give the agents exactly the amount of money, which they are willing to invest (given their reward in case of success and synergy effects). Hence, it must be the case, that $x = c$ and $y = d$. The intuition behind this result is analogical to the one, discussed in Section 3 and in proof to the Proposition 1. Further, notice that for fixed investment $(c+d)$ the probability of success is maximized, if the agents invest equal amounts to R&D. Hence, the principal will offer symmetric contracts to both agents, so that $x = y = c = d$ (this result is proved in the proposition below). Therefore, we can write the reduced form of the principal's problem:[10]

$$\max_{\beta^T,c,x} \quad \Pi_P^T = R(1-2\beta^T)(1-e^{-2^{\frac{1}{1-\alpha}}x}) - 2c \tag{2}$$

$$s.t. \qquad R\beta^T := R\beta_1^T = R\beta_2^T = \frac{e^{2^{\frac{1}{1-\alpha}}x}}{2^{\frac{\alpha}{1-\alpha}}},$$
$$x = c.$$

The solution to the optimal choice model is formulated in the following proposition.

Proposition 3. *The optimal contract for a team has the following features:*

- *If $R > 1 + 2^{\frac{1}{1-\alpha}}$, then the optimal choice of $\beta_1^T, \beta_2^T, c, d$ leads to unique equilibrium, where agents allocate all the funds into R&D, so that $x = c$, $y = d$.*
- *If $3 \cdot 2^{\frac{\alpha}{\alpha-1}} < R \leq 1 + 2^{\frac{1}{1-\alpha}}$ then there are two SPNE: $(0,0)$ and (c,c).*
- *If $R \leq 3 \cdot 2^{\frac{\alpha}{\alpha-1}}$ then the equilibrium value of investment expenditures is $c = 0$.*
- *The agents are treated symmetrically: $\beta_1^T = \beta_2^T$, $c = d$.*
- *In equilibrium (c,c) optimal c is given by $R = 2^{\frac{\alpha}{\alpha-1}}e^{2^{\frac{1}{1-\alpha}}c}(1+2e^{2^{\frac{1}{1-\alpha}}c})$, so that: $c^* = 2^{\frac{1}{\alpha-1}}\ln\frac{1}{4}\left(-1+\sqrt{1+2^{\frac{1}{1-\alpha}}4R}\right)$.*
- *The reward of the agents is : $R\beta_1^T = R\beta_2^T = \dfrac{e^{2^{\frac{1}{1-\alpha}}c}}{2^{\frac{\alpha}{1-\alpha}}}$.*

The most surprising result of the Proposition 3 is that teams with very high synergy effects (α close to 1) might end up investing nothing to the research and development. From the proposition it follows, that the higher is the synergy effect, the higher should be the price in stake to ensure full-investment equilibrium. Actually, for α close to 1, the prize should be infinitely large to ensure unique (c,c) equilibrium. For smaller prizes the agents can as well choose equilibrium $(0,0)$. The intuition for this is that for high synergy effects it is sufficient to invest a small amount in order to have a success with high probability. Hence, the principal will allocate relatively small c to project and consequently will promise low shares to the agents. If one of the team peers decides to divert funds, the other has very

[10] The general form is given in the proof to Proposition 3

low probability of winning the prize alone, and the share is not large enough to justify the effort.

However, the equilibrium $(0,0)$ is Pareto-dominated by equilibrium (c,c). Indeed using the Proposition 3, the expected profit of each agent in the full investment equilibrium is $\Pi(c,c) = 2^{\frac{\alpha}{\alpha-1}}(e^{2^{\frac{1}{1-\alpha}}c} - 1)$. In equilibrium $(0,0)$ each agent earns $\Pi(0,0) = c$. It is then straightforward, that $\Pi(c,c) > \Pi(0,0)$ for any $\alpha \in (0,1]$ and $c > 0$.

Many game-theorists, the most prominent among which are Harsanyi and Selten (1992), consider the Pareto-dominant equilibrium to be a natural focal point in a game where equilibria are Pareto-ranked.[11] The experimental literature, on the other hand, shows, that in this type of games (called coordination games), the Pareto-dominant Nash equilibrium is not always the unique outcome. However, according to Cooper, DeJong, Forsythe and Ross (1990) the coordination failure is likely to happen when decisions of the agents are influenced by presence of a cooperative dominated strategy, which (provided that the agents are able to cooperate) gives both of them larger payoff, than the Pareto-dominant equilibrium. Their results suggest, that otherwise the agents are likely to choose the Pareto-dominant equilibrium.

As I show below, in the present model collusion between agents would lead to the full-investment decision, which is also Pareto-dominant equilibrium in the simultaneous move game. Hence, based on the argument of Harsanyi and Selten (1992) and the experimental results of Cooper et al. (1990) I will assume, that in the simultaneous move game the agents are able to choose equilibrium (c,c) over equilibrium $(0,0)$.[12]

Let me further notice, that the contract described in Proposition 3 is collusion-proof in a sense, that if the agents collude, they will choose to invest all funds in their discretion. Indeed, assume that the agents collude on maximizing joint profit, which then has to be shared between them. In this case each agent takes into account a positive externality which his choice of investment has on the joint profit. An agent's equilibrium investment decision therefore, will be at least as large, as in the case of self-profit maximization.[13] Moreover, as I have shown

[11] According to Harsanyi and Selten (1992) p.221-223, among two equilibria U and V, equilibrium U dominates equilibrium V if it results in strictly higher payoff for both players. The authors take a point of view, that "there is no risk involved in a situation, where expectations can be coordinated by common payoff interests of the relevant players".

[12] Alternatively, to eliminate equilibrium $(0,0)$ I can assume that there are mechanisms, which prevent the agents from choosing zero investment simultaneously. For example, I can assume that there are tiny fixed costs of running a research laboratory, so that each player has to invest at least $\varepsilon > 0$ into R&D.

[13] Kandel and Lazear (1992) show more formally, that maximization of a joint surplus in a team leads to the higher effort level, than the effort level, resulting from individual profit

above, equilibrium (c, c) generates larger profit, than equilibrium $(0, 0)$. This means, that given contract in place, the colluding agents would prefer to coordinate on equilibrium (c, c). Hence, given the optimal contract, equilibrium investment decision of the agents does not depend on whether they collude or not.

6 Team versus competition

In this model the problem of the principal is analyzed in two stages. In the second stage he chooses optimal contract, taken the structure of research department (team or competing agents), synergy effect and size of the prize as given. In the first stage the principal chooses such structure of research department, which maximizes his payoff, taking α and R as given. The optimal contract for team and competition was derived in previous sections. Here I will discuss when the principal prefers to employ a team and when he prefers to employ competing agents.

The compensation in a setting with competing agents reminds the relative performance evaluation schemes (RPE). In the literature on contract theory, this scheme is used to penalize the member of a team, who performs worse than his pears. This feature is also present in my model: if the agents compete, each of them is rewarded only if he has better result than his rival (which also means that he wins the prize and patents the invention). On the other hand, the compensation scheme in a team rewards each entrepreneur if the whole team performs well; this is so-called joint performance evaluation (JPE). The insights from the optimal contract literature suggest that in a one-shot game the optimal payment scheme for teams is RPE (Holmstrom 1982, Che and Yoo 2001). Intuitively, this conclusion should also hold if we compare the competing agents and team without synergy effects. However, when the synergy effects present the team could potentially become an attractive arrangement, if the increase in success probability due to synergies is high enough.

In order to compare competitive setting and team, I will first consider the benchmark case without moral hazard. Note, if all actions of agents are observable and verifiable, then the principal can write a contract specifying the level of investment, which agents should allocate into the project. Then their reward is zero for any organizational structure of research unit, and the whole payoff from the projects is retained by the principal. In absence of synergy effect ($\alpha = 0$), the profit of principal is the same regardless whether he employs a team, or competing agent:

$$\Pi_P^T = \Pi_P^C = R(1 - e^{-(c^*+d^*)}) - (c^* + d^*),$$

maximization.

where $(c^*, d^*) \in \text{argmax}_{c,d}\ R(1 - e^{-(c+d)}) - (c + d)$.

It is clear, that if there are synergy effects ($\alpha > 0$), then profit of the principal is larger if he employs team, than if he employs competing agents, since for any c and d the following inequality holds:

$$\Pi_P^T = R(1 - e^{-(c^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}}) - (c + d) > \Pi_P^C = R(1 - e^{-(c+d)}) - (c + d).$$

The picture, however, changes significantly in the presence of moral hazard. According to Propositions 2 and 3 the optimal share of agents and investments of the principal are such that in equilibrium both agents find it incentive compatible to invest all funds their receive. Hence, the principal has two mechanisms how to induce the most efficient investment decision: size of funds and size of share. For given prize R and synergy effect α, the larger is the amount of funds which the agents receive, the larger is the amount which they can potentially divert from investing. On the other hand, the larger is their share, the more prone are the agents to invest as much as they can into R&D. Therefore, the principal always faces a tradeoff between increasing his investment (hence potential probability of success) and increasing the share of the agents in order to balance the incentive constraint.

Let us first consider a case, where $\alpha = 0$. It is intuitive, that, for a fixed R, the share of a prize, paid to a team should be larger, than the share of a prize paid to one of the competing agents. On the contrary, the funds which the principal invests in the optimum should be smaller in the former case. This is the natural result of free-riding, inherited in JPE scheme. Consequently, the profit of the principal should be larger, if he employs competing agents, than if he employs a team without synergy effect. This intuition is confirmed by Corollary 2.

Corollary 2. *Assume, there are no synergies effect in team, i.e. $\alpha = 0$. Then the following statements hold:*

1. *The principal invests less funds in R&D if he employs a team, than if he employs competing agents.*
2. *A share of the prize which is to be paid to the agents in case of success is larger if the team is employed.*
3. *The expected profit of the principal is smaller if he employs a team, than if he employs competing agents.*

It is surprising, that the conclusion of the above proposition does not change much for $\alpha > 0$. Figure 1 shows combinations of α and R, such that the principal is indifferent between team and competition (the graph corresponds to the equation $\Pi_P^T(\alpha, R) - \Pi_P^C(R) = 0$). In the region above the line, team is more beneficial arrangement, than competition and below the line the latter is preferred to the former. Recall, that I restrict the analysis of the team behavior to the full-investment equilibrium (c, c).

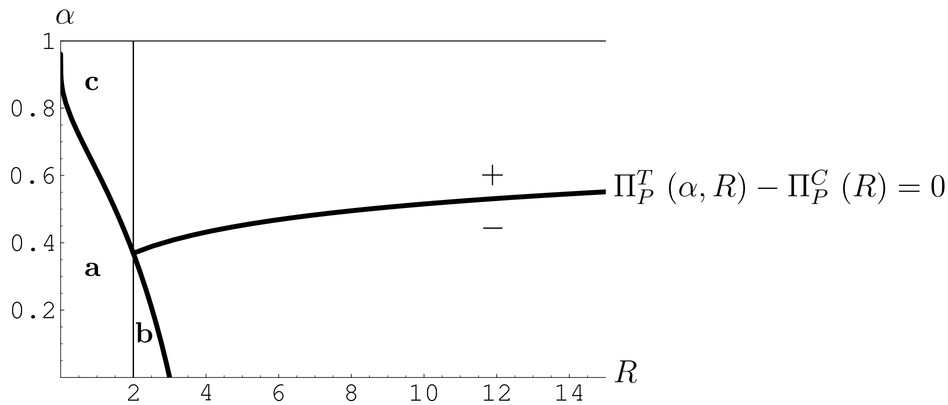

Figure 1: Competing agents versus team.

Competing agents will never be employed, if $R \leq 2$ (on the Figure 1 the corresponding regions are labelled **a** and **c**). On the other hand, the team will never be employed, if $R \leq 3 \cdot 2^{\frac{\alpha}{1-\alpha}}$ (the region corresponds to the union of **a** and **b** on the picture). Let me define $k := \frac{\log 3 - \log 2}{\log 3} \approx 0.369$. If $\alpha > k$, then $3 \cdot 2^{\frac{\alpha}{1-\alpha}} < 2$ and for all $3 \cdot 2^{\frac{\alpha}{1-\alpha}} < R < 2$ the principal can employ team, but not competing agents or a single agent (region **c**). The principal will not finance a project, if $3 \cdot 2^{\frac{\alpha}{1-\alpha}} > R$ (region **a**). If $\alpha \leq k$, then $3 \cdot 2^{\frac{\alpha}{1-\alpha}} \geq 2$ and the principal will employ competing agents (but not a team) for all values of R, such that $2 < R < 3 \cdot 2^{\frac{\alpha}{1-\alpha}}$ (the corresponding region is **b**). When the parameters R and α are such that both the team and competing agents generate a positive profit, the principal chooses an arrangement, which maximizes his own surplus from the project.

Consider now $R > 2$, so that financing of competing agents is feasible. As becomes clear from the picture, the team is never a preferred arrangement, if the synergy effects are moderate ($\alpha \leq 0.369$). But even for significant synergy effects, team is not always better than competition, despite of the technological advantages. For the fixed α, as R increases, the competition becomes more attractive. The reason is that as the value of prize in stake is high, the principal is willing to increase his investment in order to reach higher success probability. But this also means that he has to promise a higher share to the agents in order to balance their incentive compatibility constraint. If the synergy effects are high in the team, the agents can generate a high probability of success by investing only small amount of resources. Hence, it is very tempting for them to divert fraction of funds and therefore the incentive compatible share must be too large, comparing to a structure with competing agents. This leads us to somewhat surprising conclusion. The model suggests, that we should observe the research teams, financed by a third party (venture capitalist, grant agency, etc.), only if the synergy effects of team production are significant or if the prize is not too large. For large prize and moderate synergy effect we should most of the time observe the principals, financing competing agents.

This result is quite counterintuitive. No doubt, there are numerous cases in the practice of R&D financing, when the principal finances competing projects in order to choose then the best output. Nevertheless, it is much more common to see a principal financing a research team, then competing research units. In the following sections I investigate alternative structure, which can under certain conditions increase the attractiveness of a team to the principal.

7 Team with a hierarchical structure

There are several mechanisms, which can reduce the moral hazard within a team and hence increase the surplus of the principal. Important feature of the team work is that agents can often observe the action of their peers, even when the principal cannot. Che and Yoo (2001) show that repeated interaction with a team increases the attractiveness of the team to a principal. In the repeated setting, if one of the team members is observed to shirk during some period of interaction, he is punished by the penalty strategy of his peer, which generates the worst sustainable payoff for each member. This "reputation effect" deters agents from shirking and allows the principal to increase his payoff from a project.

In my model, however, I concentrate on one-time interaction between the principal and the agents. Namely, the interaction will be terminated, as soon as the project succeeds or as soon as the maximal financing horizon elapses. An example of such relationship could be an interaction between a venture capitalist and a firm. It will most of the time be terminated as soon as a firm is taking to IPO or merged with other company.

In the static setting the ability of the team members to monitor each other can still be useful, if it creates a potential for peer pressure. There is a considerable literature on how the peer pressure improves the incentives of team members. This literature usually assumes, that team members are able to commit to punish the free riders, because they derive positive utility from punishing. Further, the punishment is often considered to be non-pecuniary, such as "mental or physical harassment" (Kandel and Lazear 1992, Baron and Gjerde 1997).

There are also papers, which consider the effect of the monetary punishment on the performance of the team. In this case the punishment is executed by the principal, but only if he has verifiable evidence that an agent shirks. This evidence he obtains from reports of the team mates of the agent. Marx and Squintani (2002) show that by including a requirement to monitor and to report on his peer in a contract of each team member, the principal can reach the first-best outcome. However, while such "spying" can be justified in some environments, it can be totally inappropriate in the environments, where people feel guilty for spying on their team members.

I will assume that in my model due to prohibitively large moral costs it is not possible to execute a peer pressure or to write a contract between the principal and agents, which requires agents to monitor and to report on their peers. In such environment an agent can impose a punishment on his team mate only if he has a principal position, i.e., a position of the manager of the research department, the head of the research team, etc. By organizing the research team as a hierarchy, the original principal can utilize ability of agents to observe the effort of their peers by giving an agent on the higher hierarchy level a competence to punish his shirking team mates on the lower level of the hierarchy. Intuitively, that should decrease the moral hazard on behalf of the agents on the lower level of hierarchy.

Hence, in the next sections I will investigate an alternative structure of a team, which is further referred to as a hierarchical team. In this setting, only one agent interacts directly with the principal. He also has a discretion to decide, whether to employ the second agent and which contract offer to him. In fact, he acts as a principal with respect to the second agent. However, whether the team should be organized as a hierarchy, or whether the team members should be employed on equal conditions, is a decision to be made by the original principal, based on his profit maximization problem. I will consider two cases: the case where the first agent (the team leader) can, without incurring any costs, observe and verify the effort of the second agent (subordinate), and the case where this is not possible.

The timing of the game, in case when the principal opts for a team with hierarchy, is the following:

1. The principal signs a contract $(R\beta_1, c)$ with the team leader (A_1).
2. A_1 decides whether to stay alone or to employ the second agent (A_2).
3. If A_2 is employed, then he and A_1 sign a contract $(R\beta_1\beta_2, d \leq c)$.
4. A_1 chooses level of investment $x \leq c - d$.
5. A_2 chooses level of investment $y \leq d$.
6. The outcome is realized and the payoffs are distributed

In the following discussion I will distinguish between case, where stages 4 and 5 are simultaneous and a case, when they are sequential.

7.1 No moral hazard on behalf of agent A_2

Let me first consider a situation, where the team leader can, without incurring any costs, perfectly observe and verify the investment decision of the subordinate. This is likely to be the case, since, unlike the principal, the team leader understands the nature of the project and is working knee to knee with his employee. Hence, the contract between two can be written upon the observable and verifiable effort (i.e., investment level) of A_2. Since A_1 can perfectly observe the effort of his team mate, he can impose a prohibitively large punishment, if the latter diverts part of funds for own consumption.[14] In this case, the second agent does not earn any rent ($R\beta_1\beta_2 = 0$) and will invest $y = d$ into the project. Note, that since A_2 always invests all funds in his discretion, it is not essential, whether the investment decision is made sequentially or simultaneously.

Agent A_1 is obviously better off by employing A_2, than pursuing the project alone, while in the former case he can gain from synergy effects and does not have to pay any rent for this. The problem of A_1 has the following form (in Π_1^H, the superscript "H" stands for "hierarchy"):

$$\begin{aligned} \max_{x,d} \quad & \Pi_1^H = R\beta_1(1 - e^{-(x^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}}) + c - x - d \\ s.t. \quad & x + d \leq c \end{aligned}$$

In the optimal solution to this problem $x = d$, because for any $x + d \leq c$, the probability of success is maximized if $x = d$. Hence, $R\beta_1 = \frac{e^{2^{\frac{1}{1-\alpha}}x}}{2^{\frac{\alpha}{1-\alpha}}}$. The principal will choose the level of investment c and the share $R\beta_1$ so that all funds are invested, which implies, that $x = d = \frac{c}{2}$. This arrangement is obviously more beneficial for the principal, than the team with equal partners. In the latter case, both agents sign a contract with the principal, and since the principal does not observe their effort, he has to pay a rent to both agents. If, however, A_1 observes the effort of A_2, the moral hazard on behalf of the latter agent is eliminated and the principal has to provide incentives only to one agent. The problem of the principal, therefore, is following:

$$\begin{aligned} \max_{\beta,c} \quad & \Pi_P^H = R(1-\beta)(1 - e^{-2^{\frac{1}{1-\alpha}}x}) - 2c \\ s.t. \quad & R\beta^T = \frac{e^{2^{\frac{1}{1-\alpha}}x}}{2^{\frac{\alpha}{1-\alpha}}}, \\ & x = c. \end{aligned}$$

[14] I assume, that there is a mechanism that I do not model or investigate in details, which allows a team leader to punish his subordinate, if he observes that the latter shirks.

From the maximization problem it follows, that the optimal value c is given by equation

$$R = 2^{\frac{\alpha}{\alpha-1}} e^{2^{\frac{1}{1-\alpha}} c}(1 + e^{2^{\frac{1}{1-\alpha}} c}).$$

The threshold value of R, starting from which the project will be financed, is $\tilde{R} = 2 \cdot 2^{\frac{\alpha}{\alpha-1}}$. Note, that since $\tilde{R} < 2$ for any $\alpha \in [0, 1)$, it is never the case, that the principal decides to invest $c > 0$ if he employs competing agents, but invests zero if he employs a team.[15] Note also, that since $\tilde{R} < \hat{R}$ for any $\alpha \in [0, 1)$, the principal will finance the team with hierarchy more often (i.e. for larger range of parameters), than the team with equal partners.[16]

Comparing the optimal contracts for two alternative team structures[17], it is easy to see, that for any given investment level c the principal pays a half share of the prize to the agents in case of hierarchical team, than in case of equal-partners team. At the same time, for given value of total investment expenditures the probability of success is the same. Indeed, for both alternative team structures the optimal contract leads to the full investment equilibrium, where resources are allocated in most efficient manner between the agents (i.e. $x = y$). The final conclusion is, that if the principal can ensure, that agents observe each other effort, he will prefer to organize his research team as a hierarchy. In this hierarchy he interacts directly only with the first agent, who execute a monitoring function over the second agent. Such structure allows to decrease the moral hazard problem and increases attractiveness of a team.

7.2 Non-observable effort of A_2

If the effort of the second agent is not observable, the problem becomes significantly more complicated. Agent A_1 now has three decisions to made: whether to employ A_2 or to develop the project alone, the design of the optimal contract (provided, that A_2 was employed) and own investment decision. The game is solved by backward induction. The problem of the second agent is to choose the optimal level of investment y, such that $y \leq d$:

$$\max_{y \in [0,d]} \Pi_2^H = R\beta_1\beta_2(1 - e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}) + d - y.$$

The derivative of the profit function is

$$\frac{\partial \Pi_2^H}{\partial y} = R\beta_1\beta_2 y^{-\alpha}(x^{1-\alpha} + y^{1-\alpha})^{\frac{\alpha}{1-\alpha}} e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} - 1.$$

[15] Recall, that this was not the case for team with equal partners

[16] $\hat{R} = 3 \cdot 2^{\frac{\alpha}{\alpha-1}}$ is a threshold value for team with equal partners, see Section 5.

[17] The optimal contract for equal partners team is developed in Section 5.

I have extensively discussed on several occasions, why the principal will always choose such a share and investment, that the agents find it just incentive compatible to invest all funds which they receive into $R\&D$ (see for example the proof of Proposition 1 and the discussion in Section 3). The same argument applies to the case when the first agent acts as a principal to the second agent. A_1 will choose such combination (β_2, d), that for any given x the second agent will find it just incentive compatible to allocate all funds, which he receives, into R&D. Hence, the first order condition $\frac{\partial \Pi_2^H}{\partial y} = 0$ will be satisfied at the boundary $y = d$ of the set $[0, d]$. For the ease of notation I will define $t := (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}}$. Then, I can re-write the first order condition of the second agent in the following form:

$$R\beta_1\beta_2 = \frac{y^\alpha}{t^\alpha} e^t. \tag{3}$$

There are two remarks, which arise from the investigation of the agents' problem.

Remark 1. If A_1 decides to employ A_2, he will never choose zero level of investment. When the second agent is employed, he has to be paid a share β_2 of $R\beta_1$ in case of success. Suppose, A_1 does not invest anything in the project. Then, the probability of success is determined by the investments of the second agent: $p = 1 - e^{-y}$. However, if A_1 decides not to employ A_2, he can achieve the same success probability by investing $x = y$ and he does not have to pay a share to the second agent. Hence, if A_1 employs A_2 he will invest $x > 0$ into the project. Note, that this is a special case of the following result: A_1 will always invest at least as much as A_2, so that $x \geq y$. Indeed, the probability of success $p = (1 - e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}})$ is symmetric in x and y. On the other hand, a share which has to be paid to A_2 increases in y. Hence, if A_1 wants to implement the total amount of investment $x + y$, he is better of choosing such contract and level of investment, that $x \geq y$.

Remark 2. If the agents do not observe each other effort, then with respect to their investment decision, they play a simultaneous move game. Note, that any outcome of that game the principal can replicate, by contracting the agents himself, so that agent A_1 will be offered a contract $(R\beta_1(1-\beta_2),\ c-d)$ and agent A_2 will be offered a contract $(R\beta_1\beta_2, d)$. Hence, if in the hierarchical team neither of the agents observe an effort of his partner, the principal can do at least as well by employing a team with equal partners.

The hierarchical team, however, may give the team leader (i.e. agent A_1) a possibility to make his effort observable his subordinate. Most obviously, he can complete part of the project and present results to his employee. Alternatively, the team leader can commit to invest certain amount of effort by committing part of investment resources to the project (buying computers, financing the research laboratory, etc.) and making that decision known to his team mate. Interestingly, if it is at all possible for the team leader to make his effort observable to his team pear, either by performing part of job, or by committing money, he prefers to do that. Indeed, that allows the team leader to play sequential move game with respect to investment decision, rather than simultaneous move game. And in sequential game he can reach at least as large profit, as in the simultaneous move game.[18] The question now remains, whether the principal would ever prefer to employ a hierarchical team, if he understands, that this structure allows the team leader to make his effort observable to the second agent.

Assuming, that the choice of investment is sequential, it is possible to construct a problem of the team leader (agent A_1):

$$\begin{aligned} \max_{\{d,\beta_2,x\}} \quad & \Pi_1^H = R\beta_1(1-\beta_2)(1-e^{-t}) + c - (d+x) \\ s.t. \quad & t = (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}} \\ & R\beta_1\beta_2 = \tfrac{y^\alpha}{t^\alpha} e^t, y = d \\ & 0 \le d \le t, \; x \ge 0 \\ & d + x = d + (t^{1-\alpha} - d^{1-\alpha})^{\frac{1}{1-\alpha}} \le c. \end{aligned} \tag{4}$$

This problem has a complicated first-order condition and is not tractable for all value of α. However, investigation of the first-order conditions, allows to make a number of propositions about the investment decisions of the agents.

Case 1: $\alpha = 0$

If there are no synergy effects in team production, then the probability of success is $p(x, y) = 1 - e^{-(x+y)}$, where $x + y \le c$. In this case it is obvious, that A_1 will never employ A_2. In stand-alone situation he can implement the same probability of success and he does not have to pay a share to his employee.

As for the principal in this situation, he is better off by employing a single agent A_1, than a team with equal partners. Indeed, the probability of success is the same in both settings, but there is double moral hazard in the latter case. However, according to Corollary 1, the principal receives even higher profit by employing competing agents.

Case 2: $\alpha > 0$.

In the presence of technological benefits of team production, agent A_1 faces a complicated tradeoff. He can employ agent A_2 and allow him to allocate part of

[18] If x^* is the optimal investment choice of the team leader in the sequential move game, than for any (x, y) holds $\Pi_1(x^*, y(x^*)) \ge \Pi_1(x, y)$, where $\Pi_1(x, y)$ is profit of the team leader.

investment funds into the project, which will increase the probability of success due to synergy effect. However, he also has to promise A_2 a share of the prize, large enough to deter the latter from consuming the investment funds. Hence, by employing the second agent, A_1 suffers twice: he has to give away part of the funds, which he otherwise could consume himself, and he has to give up part of the reward in case of success. It is therefore intuitive, that A_1 should be reluctant to employ A_2 and if he does employ him, then it is reasonable to expect, that the latter will be allocated only a small part of total investment funds.

In the solution to A_1's maximization problem (4), the derivatives with respect to d and t become:

$$\begin{aligned}\frac{\partial \Pi_1^H}{\partial d} &= -1 - \alpha d^{\alpha-1}(e^t - 1)t^{-\alpha} + d^{-\alpha}(t^{1-\alpha} - d^{1-\alpha})^{\frac{\alpha}{1-\alpha}}, \\ \frac{\partial \Pi_1^H}{\partial t} &= e^{-t}t^{-(1+\alpha)}\left(\alpha d^\alpha e^t(e^t - 1) - t\left(-R\beta_1 t^\alpha + e^t(d^\alpha e^t + (t^{1-\alpha} - d^{1-\alpha})^{\frac{\alpha}{1-\alpha}})\right)\right).\end{aligned}$$

If the derivative $\frac{\partial \Pi_1^H}{\partial d}$ is decreasing in d, then agent A_1 will optimally decide not to employ the second agent. On the other hand, if he chooses to cooperate with the second agent, then the derivative must necessarily increase for some d.

Lemma 1. *If A_1 employs A_2 in equilibrium, then the optimal solution to the maximization problem (4) will be reached in the interior of the feasible set, so that $\frac{\partial \Pi_1^H}{\partial d} = 0$ and $\frac{\partial \Pi_1^H}{\partial t} = 0$.*

Proposition 4. *For the team with hierarchical structure the following statements hold:*

1. *Agent A_1 will not employ agent A_2, whenever $\alpha \leq \bar{\alpha}$, where $\bar{\alpha} \approx 0.432$ solves the equation $1 - (1-\alpha)^{\frac{\alpha}{2\alpha-1}} + (1 + (1-\alpha)^{\frac{1-\alpha}{2\alpha-1}})\alpha = 0$.*

2. *If A_1 employs A_2, then the allocation of investment resources between agents is suboptimal, i.e. $d < x$.*

The proof of the proposition is given in Appendix A. Let me first notice, that the hierarchical team is an inefficient arrangement. Due to the suboptimal allocation of resources between the agents, it generates a smaller probability of success for given c, then the alternative team structure. In Appendix, I illustrate the allocation of funds in case of team with hierarchy for $\alpha = \frac{2}{3}$ (see Figure 4)[19] This example shows, that A_1 is willing to transfer the second agent only small

[19] Using Lemma 1 above, it is straightforward to derive $R\beta_1$ as function of t. This relationship can be used then to recover equilibrium x and d. Complete solution for $\alpha = \frac{2}{3}$ is provided in the proof of Corollary 3 in Appendix.

fraction of the investment recourses, which he obtains from the principal. In other words, A_1 makes only a minor use of the synergy effects, compared with optimal allocation $x = d$.[20] The unwillingness of A_1 to share is the source of inefficiency in the hierarchical team. If the principal employs hierarchical team, then in order to achieve the same probability of success as in team with equal partners, he has to invest more in the former case, which increases his investment costs.

This inefficiency of hierarchical team has also an implication for the decision of agent A_1 to employ the second agent. The first statement of the Proposition 4 suggests, that when the synergy effects are moderate, then from the point of view of agent A_1 the contribution of A_2 to the project is not sufficient to justify the costs, connected with his employment. Note, however, that when the principal himself employs both agents, he allocates the investment resources in most efficient manner, so that agents invest equal amounts. In this case, due to the optimal use of synergy effect, the participation of the second agent in the project has more sound effect on the probability of success. A_1 never employs the second agent for $\alpha \leq \bar{\alpha}$, where $\bar{\alpha} \approx 0.432$. However, the original principal would have financed a team with equal partners for the same synergy effect, if the value of price in stake were large enough to justify investment costs.[21] Moreover, there exist a set of (α, R) with $\alpha \leq \bar{\alpha}$, such that the principal would actually prefer to employ a team, rather than competing agents.[22]

The continuity of profit function Π_1^H also ensures, that the first agent might end up not employing agent A_2 even for the synergy effects $\alpha > \bar{\alpha}$. To illustrate this, I show in the Appendix, that for $\alpha = 0.5$, the first agent will prefer to stay alone, if the value of prize in stake R is larger, than 14.490. It is quite intuitive, that for moderate synergy effects agent A_1 is reluctant to employ agent A_2, if the prize in stake increases. Indeed, the principal is willing to invest more in the project, if the prize in stake is large. He also has to balance the incentive constraint by offering a larger reward to the agents. Agent A_1 has both investment resources and the reward at his discretion. This amount of money at hand makes his tradeoff between increasing the probability of success by employing the second agent and increasing own utility by consuming the funds, ever more complicated. Hence, for R large enough he may prefer to undertake the project alone.

The bottom line from the Proposition 4 is, that for "large" set of parameters (α, R), the hierarchical team structure is not feasible to the principal in a sense, that in equilibrium the first agent will not employ the second agent. If agent A_1 does employ agent A_2, then the allocation of resources between them is suboptimal, which leads to the loss of efficiency in terms of success probability.

[20]This allocation is optimal from the efficiency point of view, because it maximizes the probability of success for given amount of investment resources.

[21]This follows from Proposition 3.

[22]See Figure 1 in Section 6.

Consider now such (α, R), that the hierarchical team is feasible for the principal. Which team structure serves him better, depends on combination of parameters α and R. However, intuition suggests, that the outcome of the hierarchical team may be better than the outcome of the equal partners team, only if it leads to the investment decision, which the principal is not able to replicate in the corresponding simultaneous move game. In other words, if for fixed $(R\beta_1(1-\beta_2), R\beta_1\beta_2)$ an equilibrium in the hierarchical team is situated to the right from an equilibrium in the team with equal partners, than there is a possibility, that hierarchical team generates larger profit to the principal.[23]

More formally, assume that the optimal contract in the case of hierarchical team requires, that the agents A_1 and A_2 are paid $(R\beta_1(1-\beta_2), R\beta_1\beta_2)$ respectively. Let (x_h, y_h) be the equilibrium investment of agents A_1 and A_2 in the hierarchical team. Similarly, let (x_t, y_t) be the equilibrium investments of agents in the equal partners team with shares fixed to $(R\beta_1(1-\beta_2), R\beta_1\beta_2)$. Then, if $x_h \leq x_t$, the principal can replicate the outcome of the hierarchical team by signing a contract with the agents, where A_1 is allocated an amount $c = x_h$ and A_2 is allocated $d = y_h$, while the shares are $(R\beta_1(1-\beta_2), R\beta_1\beta_2)$. If this is the case, then the hierarchical team does not have any advantages from the principal's point of view.

Consider now the situation, where $x_h > x_t$ (see Figure 5 in Appendix for illustration). In this case the team leader overinvests in the project, comparing with his best response function. In other words, given the corresponding investment choice of the agent A_2, the team leader (agent A_1) would prefer to execute lower level of effort. However, given the sequential structure of the investment, he cannot change his investment decision. Moreover, the principal cannot replicate the outcome of this game by signing a contract with each agent separately. Indeed, in the equilibrium of the corresponding simultaneous move game, $x_t < x_h$. Naturally, in team with equal partners, the principal can implicate the probability of success p_h, which corresponds to investment level (x_h, y_h). Since the principal always allocates resources in the most efficient manner (which is not the case in the hierarchical team), he can achieve the probability p_h by incurring smaller costs. However, the principal also has to adjust the rewards of agents appropriately. In particulary, the reward of the agent A_2 must be increased, since efficiency requires that he is allocated larger amount of resources, than he receives in team with hierarchy. Let the sum of these adjusted shares be $R\beta^T$. If the principal employs hierarchical team, he has to pay a reward of $R\beta_1$. If $R\beta^T > R\beta_1$ and the difference in the investment costs is not too large, than the principal may prefer to employ a hierarchical team, rather than a team with equal partners. The corollary below proves this intuition.

[23] The illustration of such case is provided in Appendix, see Figure 5.

Corollary 3. *There exists an open set of parameters (α, R), such that the hierarchical team is feasible to the principal, and he prefers to employ a hierarchical team, rather than team with equal partners.*

In Appendix, I provide a proof of this corollary by a mean of example. I show, that for $\alpha = \frac{2}{3}$ the principal will finance a hierarchical team for $\alpha \in (0.734, 0.75]$, while for this range of parameters he will set $c = 0$ in case of team with equal partners. For $\alpha \in (0.75, 0.895)$ both teams are feasible, but profit of the principal is larger under hierarchical team. By continuity, this result also holds in the open set around $\alpha = \frac{2}{3}$.

The choice between hierarchical team and team with equal partners depends on the balance between efficiency of R&D and power of incentives. For small values of R the principal might prefer to finance a hierarchical team, if the team leader is prepared to overinvest in the sequential equilibrium, compared with his best response reaction. Indeed, for low values of R the principal is only willing to invest little resources in the project. This implies, that probability of success is small, even when resources are allocated efficiently. Hence, if agent A_1 has incentives to invest intensively in the hierarchical team and the increase in efficiency due to the optimal allocation of resources is not significant, then the principal might prefer the latter structure over the team with equal partners. However, intuition and the examples in Appendix suggest, that the range of parameters, where hierarchical team performs better is small. As the prize in stake increases, so does the equilibrium investment expenditures of the principal. The efficiency becomes now an important issue, because increase in the success probability due to the optimal allocation of funds becomes more significant. It is obvious, that the principal will never employ the hierarchial team, if he has to give up a larger share of the prize, than with the alternative team structure. But also when the opposite is the case, he will prefer to employ the team with equal partners, when the efficiency gain is large enough to justify the reward of the agents. In my examples, the principal will always prefer the team with equal partners, if $\alpha = 0.5$. If $\alpha = \frac{2}{3}$, he will prefer a team with equal partners over the hierarchical team for $R > 0.895$.

8 Conclusion

In this paper I investigate four different organizational structures of the research and development in the framework where the financing decisions (made by a principal) and allocation decisions (made by agents) are separated. The allocation decision is not observable to the principal, which creates a moral hazard problem. The common implication for the contract between the principal and agents in all four cases is, that when the principal decides to increase amount of finances, allocated to the project, he also has to increase a reward of the agents, which they obtain in case of success. Otherwise, they tend to consume part of funds, instead of allocating them into R&D.

However, different structures have different effect on the incentives, which leads to decrease or increase in the rent, allocated to the agents. Competition between agents proves to be a strong incentive device. Despite the duplication of research costs, it leads to a significant decrease in agents' rents, so that the principal will always prefers employing competing agents, rather than a single agent.

Comparing with competition, the team production may have large synergy effects, which result in higher probability of success for the same amount of funds, allocated to the R&D. However, team production suffers from the free-riding problem, which increases incentive compatible reward of the agents, comparing with the competitive structure. The model therefore predicts, that the principal will prefer to use team structure, only if the synergy effects are significant or the prize is not too large. For moderate synergy effects and large value of the prize, the principal will rather employ competing agents.

If, however, one of the agents is able to observe the effort of the other agent, then the principal can improve performance of the team if he assigns the former agent to a principal position (position of a team leader). This agent acts as a second principal to his team pear. He determines the optimal contract (share of the prize and the investment resources), which the latter receives, and can impose a prohibitively large punishment for if he detects shirking. Since the team leader is able to perfectly observe effort of the other agent, the contract between them will be written upon observable effort and the moral hazard on behalf of the second agent will be eliminated.

On the other hand, if the agents cannot observe each other efforts, then (from the principal's point of view) the hierarchical team does not have any benefits, comparing to the case where the principal contracts the agents himself. However, hierarchical team structure may provide the team leader an opportunity to make his effort observable to another agent, either by completing part of the project, or by committing resources to it. If this is the case, then this team structure may still prove to be superior to the equal-partners team, if it results in equilibrium, which the principal cannot replicate by contracting the agents himself. It follows from the analysis, that this will be the case, only if the team leader is willing to overinvest in equilibrium, compared to his best response reaction.

Although sometimes the hierarchical team structure gives the team leader powerful incentives to invest in the project, it also leads to a suboptimal allocation of funds and a significant loss of efficiency in terms of success probability. The team leader is reluctant to involve another agent into the project. Although the joint probability of success could be increased by allocating more funds to the second agent[24], he also has to be rewarded for his effort by receiving a larger share in case of success. On the other hand, the team leader can increase his own utility by consuming these funds, and in addition he could enjoy the larger share of a prize. Hence, he will only distribute a minor part of funds to the second agent. Therefore, for fixed amount of investment expenditures, the hierarchical team leads to much lower success probability than a team, with equal partners.

Due to the low efficiency of a hierarchical team, the team leader will not employ the second agent for large set of parameters (for example, when $\alpha < 0.43$). For (α, R), which belong to this set, the hierarchical team is not feasible to the principal. On the other hand, for the same set of parameters, the team with equal partners team may perform even better than competition, due to the efficient allocation of investment resources. The model also suggests, that even when both alternative team structures are feasible to the principal, he should most of the time structure his research team as a team with equal partners. The reason is that as the prize increases, the principal is willing to allocate more resources into the project. This implies, that increase in probability of success due to the optimal allocation of these resources becomes significant and overweighs any positive effect, which the hierarchical team might have on the incentives of the leader.

An implication of the model is, that in environments, where the team leader has difficulties monitoring his team mates, one should expect to observe team leader executing disproportional large effort. On the contrary, other members of the team should only perform a minor work.

Another implication of this model for the practice of R&D financing is that the financiers (e.g. grant agencies, venture capitalists, firms, subcontracting R&D) should encourage the information exchange and transparency within the research team. Even if this transparency does not increase ability of the financiers to observe efforts, they can benefit from it indirectly by using the hierarchical team structure. On the other hand, if due to working conditions or very different job tasks the agents are not able to observe or understand amount of effort, imputed by their team pears, the financiers should most of the time organize their research unit as a team with equal partners, rather than hierarchy. Especially when the team members cannot observe each other effort, competition may be a superior organizational structure, if the synergy effects in team are moderate and the prize in stake is large enough.

[24]For each amount of money, which the first agent is ready to allocate to the project, the probability of success is maximized, if both agents invest half of that amount.

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A Mathematical appendix

Proof of Proposition 1

As a first step, I will show that the recourse constraint RCS binds in optimum. Assume, that c is large enough, so that RCS does not bind. Denote $f(x) = R(1-\beta(x))(1-e^{-x})$, where $R\beta(x) \geq e^{x}$ according to ICS. The problem of the principal in this case is to maximize expected profit, subject to incentive constraint:

$$\begin{aligned} \max_{x \in [0,c]} \quad & f(x) - c \\ s.t. \quad & ICS. \end{aligned}$$

I will denote the solution to this problem as $\hat{x}$. Note, that the profit of the principal decreases in c. Hence, optimal c is such, that $c = \hat{x}$ and the profit of the principal is $f(\hat{x}) - \hat{x}$.

Assume now, that for any x the principal chooses c so, that the RCS is also binding. Then the problem of the principal is:

$$\begin{aligned} \max_{x \in [0,c]} \quad & f(x) - c \\ s.t. \quad & ICS, \\ & c = x. \end{aligned}$$

Denoting the solution to this problem as x^{*}, I can write the expected profit of the principal as $f(x^{*}) - x^{*}$. Note, that since $x^{*} = \text{argmax} f(x) - x$, for any $\hat{x}$ holds the following inequality: $f(x^{*}) - x^{*} \geq f(\hat{x}) - \hat{x}$. Hence, the principal will always choose c such, that the recourse constraint is binding.

Further, assume, that (ICS) is not binding, so that $R\beta > e^{c}$. This however, cannot be an optimum, because for *any* $R\beta > e^{c}$ the agent will choose the level of investment $x = c$, so that the probability of success does not change. However, the principal can increase his profit by decreasing $R\beta$. Hence, in optimum the principal will choose such β and c, that $R\beta = e^{c}$, i.e. (ICS) constraint will be binding.

With all constraint being binding the Problem (2) looks as follows:

$$\begin{aligned} \max_{c,\beta} \quad & \Pi_P^S = R(1-\beta)(1-e^{-x}) + c - x \\ s.t. \quad & (ICS)\ R\beta = e^{x}, \\ & (RCS)\ x = c, \\ & (CSS)\ (R\beta - e^{x})(x - c) = 0. \end{aligned}$$

The solution to this problem is $c = \ln \frac{1}{2}(-1+\sqrt{1+4R})$. For any $R > 2$, c is positive.

Proof of Proposition 2

The problem of the principal in the general form is

$$\begin{aligned}
\max_{\beta_1^C,\ \beta_2^C,\ c,\ d} \quad & \Pi_P^C = R(1 - \tfrac{x}{x+y}\beta_1^C - \tfrac{y}{x+y}\beta_2^C)(1-e^{-(x+y)}) - (c+d) \\
s.t. \quad & (ICC_1)\ R\beta_1^C \geq \frac{e^{x+y}(x+y)^2}{x^2 - y + e^{x+y}y + xy}\ , \\
& (ICC_2)\ R\beta_2^C \geq \frac{e^{x+y}(x+y)^2}{y^2 - x + e^{x+y}x + xy}\ , \\
& (RCC_1)\ x \leq c, \\
& (RCC_2)\ y \leq d, \\
& (CSC_1) \left(R\beta_1^C - \frac{e^{x+y}(x+y)^2}{x^2 - y + e^{x+y}y + xy}\right)(c-x) = 0, \\
& (CSC_2) \left(R\beta_2^C - \frac{e^{x+y}(x+y)^2}{y^2 - x + e^{x+y}x + xy}\right)(d-y) = 0.
\end{aligned}$$

According to incentive compatibility constraints ICC_1 and ICC_2 the agents choose their investment x and y to maximize their expected utility. On the other hand, since all available funds come from the principal, the agents cannot invest more then c, respectively d, which is described by recourse constraints RCC_1 and RCC_2. Finally, complementary slackness conditions CSC_1 and CSC_2 ensures that at least one of constrains ICC_1, RCC_1 and ICC_2, RCC_2 is binding. If RCC_1 does not bind, than ICC_1 must necessarily be binding. On the other hand, if ICC_1 does not bind, then RCC_1 must be binding according to equilibrium conditions (the same holds for ICC_2 and RCC_2).

I will develop a proof in several steps.

Step 1.

All constrains will be binding, which follows from the same argument as in the proof to Proposition 1.

Step 2.

With all constrains being binding, we can re-write the principal problem in the following form:

$$\max_{x,y} \left(R - \frac{e^{x+y}(x+y)^2}{x^2 - y + e^{x+y}y + xy} - \frac{e^{x+y}(x+y)^2}{y^2 - x + e^{x+y}x + xy}\right)(1 - e^{-(x+y)}) - (x+y).$$

I take the first-order condition w.r.t. x and y and expressing R, so that I receive the system of equation[25]:

$$R = F_1(x,y), R = F_2(x,y).$$

Subtracting second equation from the first one I receive the following expression:

$$\frac{e^{x+y}(e^{x+y}-1)^2(x-y)(x+y)^3(-1+e^{x+y}-(x+y))(e^{x+y}+x+y-1)}{(y^2+x(y+e^{x+y}-1))^2((-1+e^{x+y})y+x(x+y))^2} = 0.$$

The above equality holds, if the following conditions are satisfied:

1. $(y^2+x(y+e^{x+y}-1))^2((-1+e^{x+y})y+x(x+y))^2 \neq 0$,
2. $e^{x+y}(e^{x+y}-1)(x-y)(x+y)^3(-1+e^{x+y}-(x+y))^2(e^{x+y}+x+y-1) = 0$

From the first condition it follows, that $x \neq -y$. The second part holds true if at least one of the following conditions is satisfied:

1. $x = y$,
2. $x = -y$
3. $-1 + e^{x+y} + x + y = 0$,
4. $-1 + e^{x+y} - (x+y) = 0$.

Condition 2 is ruled out based on result that $x \neq -y$. For simplicity I will denote $x + y := a$. Condition 4 holds iff $a = 0$, implying $x = -y$, which we have ruled out. Indeed, denoting, it is easy to see that the function $f(a) = e^a - a - 1$ reaches it's unique minimum at 0, so that $f(0) = 0$. As for condition 3, the function $g(a) = e^a + a - 1$ is strictly increasing in a, and there is unique solution of $g(a) = 0$, namely $a = 0$, which is impossible due to result $x \neq -y$. Hence, it necessarily must be, that condition 1 holds, i.e. $x = y$. This automatically implies, that $R\beta_1 = R\beta_2$ and $c = d = x = y$. Hence the agents are offered a symmetric contract.

Step 3.

[25]The equations are straightforward to receive, but too cumbersome to present them in the paper

I can now re-write a problem of the principal in the following form:

$$\max_{\beta^C,\ c} \quad \Pi_P^C = R(1-\beta^C)(1-e^{-2x}) - 2c$$

$$s.t \qquad R\beta^C = \frac{e^{2x}4x^2}{2x^2 - x + e^{2x}x},$$

$$x = c.$$

The solution to the problem is given by

$$R = \frac{e^{2c}\left(4c(e^{2c}-1) + 3(e^{2c}-1)^2 + c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2} \tag{5}$$

To see, that R is increasing in c, let me re-write the Equation (5) in the following form:

$$R = e^{2c} + \frac{8e^{4c}c^2}{(e^{2c}+2c-1)^2}.$$

It is straightforward to verify, that both function on the right-hand side of the equation increase in c.

Step 4.

Recall, that I assume, $R > 2$. Now let us to verify, that optimal contract $c = d$, $R\beta_1^C = R\beta_2^C := R\beta^C = \dfrac{4c}{1-e^{-2c}+2ce^{-2c}}$, where c is given by Equation (5) leads to unique SPNE. Equilibria $(x,0)$,$(0,y)$,(c,y),(x,d) are ruled out because the agents are treated symmetrically, so that $x = y$ in SPNE.

There are two possibilities left: $(0,0)$ and (c,c). Recall (see Table 1), that equilibrium $(0,0)$ will be played iff $R\beta^C \leq 1$. However, $c > 0$ for any $R > 2$. And $c > 0$ implies, that $R\beta^C > 1$. To see this, note, that $R\beta_C$ is increasing in c and $R\beta^C \to 1$ as $c \to 0$. Hence for any $R > 2$, $(0,0)$ will not be played in equilibrium. Therefore the optimal contract, described in Proposition 2 leads to unique SPNE (c,c).

Proof of Corollary 1

For clarity of notations I will denote as t the amount of finance which a single agent receive and c the amount of finance which each of competing agents receives. The amount of finance and shares of agents are determined from the following expressions (see Proposition 1 and Proposition 2):

$$R = e^t(1+e^t), \qquad R\beta^S = e^t \tag{6}$$

$$R = \frac{e^{2c}\left(4c(e^{2c}-1) + 3(e^{2c}-1)^2 + c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2}, \quad R\beta^C = \frac{4ce^{2c}}{e^{2c}+2c-1} \tag{7}$$

I will first prove that according to the optimal contract the principal invests less if he employs a single entrepreneur, than if he employs competing agents, i.e. $2c > t$. Assume on the contrary that $2c \leq t$. I will show that, contrary to (6) and (7) this implies the following relation:

$$e^t(1+e^t) > \frac{e^{2c}\left(4c(e^{2c}-1)+3(e^{2c}-1)^2+c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2} \tag{8}$$

Note, that it is enough to prove, that (8) holds for $t = 2c$. Plugging $t = 2c$ into (8) and rearranging the terms I obtain the following inequality:

$$(1+e^{2c})(e^{2c}+2c-1)^2 > 4c(e^{2c}-1)+3(e^{2c}-1)^2+c^2(4+8e^{2c}).$$

I will denote the function on the left-hand side as $L(c) = (1+e^{2c})(e^{2c}+2c-1)^2$ and the function on the right-hand side as $R(c) = (4c(e^{2c}-1)+3(e^{2c}-1)^2+c^2(4+8e^{2c}))$. Note, that $L(0) = R(0) = 0$. I will prove now that for any $c > 0$, $L'(c) > R'(c)$, which implies that for any $c > 0$, $L(c) > R(c)$. Taking the respective derivatives, I obtain:

$$\begin{aligned} L'(c) &= 2(-1+2c+e^{2c})(2+(3+2c)e^{2c}+3e^{4c}), \\ R'(c) &= 4(-1-2c^{2c}+4c^2e^{2c}+3e^{4c}+c(2+6e^{2c})). \end{aligned}$$

Both these derivatives are positive for any $c > 0$. Subtracting $R'(c)$ from $L'(c)$ I obtain the following function:

$$f(c) = 2e^{2c}(-4c^2+8c(-1+e^{2c})+3(-1+e^{2c})^2).$$

Since $f(0) = 0$, it is enough to show that function monotonically increases for $c > 0$ in order to prove that for any $c > 0$, $f(c) > 0$. The first derivative of the function $f(c)$ is positive, since

$$-2-e^{2c}+3e^{4c}+c(4e^{2c}-2) > 0.$$

for any $c > 0$. Hence, I have proved that if $2c \leq t$, than the following holds:

$$e^t(1+e^t) \geq e^{2c}(1+e^{2c}) > \frac{e^{2c}\left(4c(e^{2c}-1)+3(e^{2c}-1)^2+c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2}.$$

This, however, contradicts the conditions (6) and (7), which should be satisfied simultaneously for given R. Hence, it must be, that $2c > t$.

Next, I will show, that conditions (6) and (7) imply that $R\beta^C < R\beta^S$. Suppose on the contrary, $R\beta^C \geq R\beta^S$. Then it is enough to show, that for $R\beta^C = R\beta^S$, the following inequality holds (contrary to Equations 6 and 7):

$$e^t(1+e^t) < \frac{e^{2c}\left(4c(e^{2c}-1)+3(e^{2c}-1)^2+c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2}.$$

If $R\beta^C = R\beta^S$, then $e^t = \dfrac{4ce^{2c}}{e^{2c}+2c-1}$. Substituting this to the expression for R, given by (6) I obtain the following expression:

$$R = \frac{4ce^{2c}}{e^{2c}+2c-1}\left(1+\frac{4ce^{2c}}{e^{2c}+2c-1}\right).$$

On the other hand, according to (7),

$$R = \frac{e^{2c}\left(4c(e^{2c}-1)+3(e^{2c}-1)^2+c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2}.$$

However, it is not possible that these two equalities are satisfied simultaneously. Indeed, I will show, that for any $c>0$ holds the inequality:

$$\begin{aligned}\frac{4ce^{2c}}{e^{2c}+2c-1}\left(1+\frac{4ce^{2c}}{e^2c+2c-1}\right) \quad &< \quad \frac{e^{2c}\left(4c(e^{2c}-1)+3(e^{2c}-1)^2+c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2} \Leftrightarrow \\ &\Leftrightarrow \quad 4c^2+8c^2e^{2c} < 3e^{4c}-6e^{2c}+3\end{aligned}$$

To prove that the last inequality holds for any $c>0$, I will take the derivatives of the function $l(c) = 4c^2+8c^2e^{2c}$ and $r(c) = 3e^{4c}-6e^{2c}+3$ and compare them (notice, that $l(0) = r(0) = 0$). The derivatives are $l'(c) = 4(2c+4ce^{2c}+4c^2e^{2c})$ and $r'(c) = 4(3e^{4c}-3e^{2c})$. Comparing $l'(c)$ and $r'(c)$ I establish the following:

$$l'(c) < r'(c) \Leftrightarrow 2c < e^{2c}(3e^{2c}-4c^2+3+4c). \tag{9}$$

Obviously, $2c < e^{2c}$ for any $c>0$. Also, $3e^{2c} > 4c^2$ for any $c>0$. Hence, $l'(c) < r'(c)$. This in turn implies, that

$$e^t(1+e^t) \le \frac{4ce^{2c}}{e^{2c}+2c-1}\left(1+\frac{4ce^{2c}}{e^{2c}+2c-1}\right) < \frac{e^{2c}\left(4c(e^{2c}-1)+3(e^{2c}-1)^2+c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2},$$

which contradicts (6) and (7). Therefore, I conclude, that it must be that $R\beta^C < R\beta^S$.

I have proved, that the principal invests more in case of competing agents ($2c > t$), so that the probability of success is higher than in single-agent case. On the other hand, the share of a prize which would be paid in case of success is smaller if competing agents are employed ($\beta^C < \beta^S$). According to Propositions 1 and 2, the profit of the principal if he employs competing agents, respectively single agent is:

$$\begin{aligned}\Pi_P^C &= R(1-\beta^C)(1-e^{-2c})-2c = (R-\frac{4ce^{2c}}{e^{2c}+2c-1})(1-e^{-2c})-2c;\\ \Pi_P^S &= R(1-\beta^S)(1-e^{-t})-t = (R-e^t)(1-e^{-t})-t.\end{aligned}$$

First, let me notice, that $\Pi_P^S(t)$ is increasing in t. Indeed, the derivative of the function is $Re^{-t} - e^t$ which is positive, if $R > e^{2t}$. According to optimal contract, $R = e^t(1 + e^t)$, which indeed implies $R > e^{2t}$. Hence, I can write the inequality:

$$(R - e^{2c})(1 - e^{-2c}) - 2c > (R - e^t)(1 - e^{-t}) - t.$$

Further, notice, that $e^{2c} > e^t > \dfrac{4ce^{2c}}{e^{2c} + 2c - 1}$. Together with inequality above, that implies

$$\left(R - \frac{4ce^{2c}}{e^{2c} + 2c - 1}\right)(1 - e^{-2c}) - 2c > (R - e^{2c})(1 - e^{-2c}) - 2c >$$
$$> (R - e^t)(1 - e^{-t}) - t.$$

Hence, $\Pi_P^C > \Pi_P^S$.

Production function and probability of success in team

For consistency of presentation I will start the discussion with the case of one agent. Assume, that in order to produce an output the agent needs to acquire two skills or production factors, such as capital, which I will call "K" and labor "L". He allocates the available budget x in order to acquire these skills. The cost of skill K is $p > 1$ and the cost of skill L is normalized to 1. Assume further, that the production function has a CES form $f(K, L) = (K^{1-\alpha} + L^{1-\alpha})^{\frac{1}{1-\alpha}}$, where $0 \leq \alpha < 1$ is a degree of complementarity between K and L. Hence, the agent faces a following maximization problem:

$$\max_{K,L} \quad f(K, L) = (K^{1-\alpha} + L^{1-\alpha})^{\frac{1}{1-\alpha}}$$
$$s.t. \qquad pK + L = x.$$

The optimal solution to this problem is

$$(K^*, L^*) = \left(\frac{xp^{-\frac{1}{\alpha}}}{1 + p^{\frac{\alpha-1}{\alpha}}}, \frac{x}{1 + p^{\frac{\alpha-1}{\alpha}}}\right)$$

and the output, which the agent produces in optimum is $f(x) = x \cdot (1 + p^{\frac{\alpha-1}{\alpha}})^{\frac{\alpha}{1-\alpha}}$. This output converges to x, as p goes to infinity. Therefore, the agent who disposes a budget x and chooses his factors of production optimally, will produce the output of value x, if the price of one factor is infinitely large. This output determines the probability of success $p(x)$ in a way, which I have already discussed earlier:

$$p(x) = 1 - e^{-x}.$$

The output x can be interpreted as a "knowledge" or productivity, which the agent acquires while investing in factors. This productivity determines the parameter of Poisson distribution so, that the average number of success in unit of time is x and the expected time before first success is $1/x$.

Consider now the problem of two agents who jointly maximize the team output. Let one of them have a budget x and another y. One agent invests in K_1 and L_1, where cost of K_1 is $p > 1$ and cost of L_1 is 1 and the other agent invests in K_2 and L_2, with costs 1 and $p > 1$ respectively. The maximization problem therefore is the following:

$$\begin{aligned} \max_{\{K_1,K_2,L_1,L_2\}} \quad & f(\cdot) = \left((K_1+K_2)^{1-\alpha} + (L_1+L_2)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}; \\ s.t. \quad & pK_1 + L_1 = x, \\ & K_2 + pL_2 = y. \end{aligned} \tag{10}$$

The complete solution to this maximization problem is developed below. Here I will discuss the result and the intuition behind it. The problem does not have an interior solution, and depending on values of parameters p and α, the optimal choice of the production factors is the following:

$$[(K_1, L_1), (K_2, L_2)] = \left[(0, x); \left(\frac{y+px}{1+p^{\frac{\alpha-1}{\alpha}}}, \frac{yp^{-\frac{1}{\alpha}} - x}{1+p^{\frac{\alpha-1}{\alpha}}}\right)\right], \text{ if } \frac{x}{y} \leq p^{-\frac{1}{\alpha}},$$

$$[(K_1, L_1), (K_2, L_2)] = [(0, x); (y, 0)], \text{ if } p^{-\frac{1}{\alpha}} < \frac{x}{y} < p^{\frac{1}{\alpha}},$$

$$[(K_1, L_1), (K_2, L_2)] = \left[\left(\frac{xp^{-\frac{1}{\alpha}} - y}{1+p^{\frac{\alpha-1}{\alpha}}}, \frac{x+yp}{1+p^{\frac{\alpha-1}{\alpha}}}\right); (y, 0)\right], \text{ if } \frac{x}{y} \geq p^{\frac{1}{\alpha}}.$$

This optimal solution has the following property. If the budgets of two agents are not very different, so that $p^{-\frac{1}{\alpha}} < \frac{x}{y} < p^{\frac{1}{\alpha}}$, then the agents specialize and each invests in his cheapest factor of production. If, however, one of the agents has too small budget, so that $\frac{x}{y} \leq p^{-\frac{1}{\alpha}}$ or $\frac{x}{y} \geq p^{\frac{1}{\alpha}}$, then he specializes in his the cheapest factor, while his team mate invests in both factors. Then the output at optimal allocation has the following form:

$$f(x, y) = \begin{cases} (y+px)\left(1+p^{\frac{\alpha-1}{\alpha}}\right) & \text{if } \frac{x}{y} \leq p^{-\frac{1}{\alpha}}, \\ \left(x^{1-\alpha} + y^{1-\alpha}\right)^{\frac{1}{1-\alpha}} & \text{if } \frac{x}{y} \in \left(p^{-\frac{1}{\alpha}}, p^{\frac{1}{\alpha}}\right), \\ (x+yp)\left(1+p^{\frac{\alpha-1}{\alpha}}\right) & \text{if } \frac{x}{y} \geq p^{\frac{1}{\alpha}}. \end{cases} \tag{11}$$

If $p \to \infty$, then both agents specialize in their cheapest factor of production and the output $f(x, y)$ is

$$f(x, y) = (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}}.$$

If we interpret this output as a parameter of the Poisson distribution, then the probability that team reaches a success before time $T = 1$ elapses is:

$$p(x, y) = 1 - e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}.$$

Hence, the specification of synergy effect in team, which I have chosen, reflects a case when each of two agents is "talented" in different skill or production factor, and joint production is maximized, when each agent specializes in his cheapest factor. Note, that if both factors are equally expensive, so that price of K and L can be normalized to 1, then no specialization occurs and optimal production has a form $f(x, y) = x + y$ so that probability of success is

$$p(x, y) = 1 - e^{-(x+y)}.$$

Solution to the maximization problem (10)

From the set up of problem (10) and the fact, that the amount of factors is always nonnegative, it is clear that the solution should satisfy the following constraint:

$$L_1 \in [0, x], K_2 \in [0, y].$$

Given this constraints, it is easy to show that there is no interior solution to the problem. Indeed, solving the system of the first-order conditions one obtains the following result:

$$(K_1^*, L_1^*) = \left(-\frac{xp + y}{(1-p)^2}, x + \frac{p^2x + py}{(1-p)^2}\right),$$
$$(K_2^*, L_2^*) = \left(y + \frac{px + yp^2}{(1-p)^2}, -\frac{x + yp}{(1-p^2)}\right).$$

Obviously, this can not be a solution, since $K_1 < 0$ and $L_2 < 0$. Therefore let us consider the boundary solutions. There are eight possible allocations:

1. $L_1 = 0, K_2 \in (0, y)$,
2. $L_1 = 0, K_2 \in (0, y)$,
3. $L_1 = x, K_2 = y$,

4. $L_1 = 0, K_2 = y$,

5. $L_1 = x, K_2 = 0$,

6. $L_1 = 0, K_2 = 0$,

7. $L_1 \in (0, x), K_2 = y$,

8. $L_1 \in (0, x), K_2 = 0$.

Allocation 1 and 7, 2 and 8, 4 and 5 are symmetric and therefore I will develop the complete solution only for the allocations 1,2,3,6 and 7. Recall again problem (10):

$$\begin{aligned} \max_{\{K_1,K_2,L_1,L_2\}} \quad & f(\cdot) = ((K_1+K_2)^{1-\alpha} + (L_1+L_2)^{1-\alpha})^{\frac{1}{1-\alpha}}; \\ s.t. \quad & pK_1 + L_1 = x, \\ & K_2 + pL_2 = y. \end{aligned}$$

Allocation 6 is: $[(K_1, L_1), (K_2, L_2)] = [(\frac{x}{p}, 0), (0, \frac{y}{p})]$. The output resulting from this allocation of factors is

$$f_6(x, y) = \frac{1}{p}(x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}}.$$

Allocation 3 is $[(K_1, L_1), (K_2, L_2)] = [(0, x), (y, 0)]$ and the corresponding output is

$$f_3(x, y) = (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}}.$$

Obviously, for any $p > 1$, $f_6 < f_3$, so that the Allocation 6 cannot be optimal solution to the problem (10).

Allocation 1 leads to the following maximization problem:

$$\begin{aligned} \max_{\{K_1,K_2,L_1,L_2\}} \quad & f(\cdot) = A((K_1+K_2)^{1-\alpha} + (L_1+L_2)^{1-\alpha})^{\frac{1}{1-\alpha}}; \\ s.t. \quad & K_1 = 0, L_1 = x, \\ & K_2 + pL_2 = y, \\ & K_1, K_2, L_1, L_2 \geq 0. \end{aligned}$$

We are interested in interior solution to this problem, because otherwise the problem is analogical either to Allocation 3 or to Allocation 5. The interior solution exists if $x \leq yp^{-\frac{1}{\alpha}}$ and has the following form:

$$[(K_1, L_1), (K_2, L_2)] = [(0, x), \left(\frac{yp^{-\frac{1}{\alpha}} - x}{1 + p^{\frac{\alpha-1}{\alpha}}}, \frac{y + px}{1 + p^{\frac{\alpha-1}{\alpha}}} \right)],$$

and the corresponding production is $f_1(x,y) = (y+px)(1+p^{\frac{\alpha-1}{\alpha}})^{\frac{\alpha}{1-\alpha}}$.

Allocation 2, leads to the maximization problem

$$\begin{array}{rc} \max\limits_{\{K_1,K_2,L_1,L_2\}} & f(\cdot) = ((K_1+K_2)^{1-\alpha} + (L_1+L_2)^{1-\alpha})^{\frac{1}{1-\alpha}}; \\ s.t. & K_1 = \frac{x}{p}, L_1 = 0, \\ & K_2 + pL_2 = y, \\ & K_1, K_2, L_1, L_2 \geq 0. \end{array}$$

The existence of interior solution requires $x \leq yp^{\frac{1}{\alpha}}$ and the solution is the following:

$$\begin{aligned} [(K_1,L_1),(K_2,L_2)] &= [\left(\frac{x}{p},0\right), \left(\frac{y-xp^{-\frac{1}{\alpha}}}{1+p^{\frac{\alpha-1}{\alpha}}}, \frac{p^{-\frac{1}{\alpha}}(\frac{x}{p}+y)}{1+p^{\frac{\alpha-1}{\alpha}}}\right)] \\ f_2(x,y) &= \left(\frac{x}{p}+y\right)(1+p^{\frac{\alpha-1}{\alpha}})^{\frac{\alpha}{1-\alpha}} \ . \end{aligned}$$

Finally, *Allocation 4*: $[(K_1,L_1),(K_2,L_2)] = [(\frac{x}{p},0),(y,0)]$ results in output

$$f_4(x,y) = \left(\frac{x}{p}+y\right).$$

So, to determine the optimal solution we need to compare the value of functions $f_1(x,y)$, $f_2(x,y)$, $f_3(x,y)$ and $f_4(x,y)$. This comparison is relatively straightforward and therefore I only sketch it without going into details. There are several cases to be considered:

1. $y \leq xp^{\frac{1}{\alpha}}$. If this is the case, then for any $p > 1$ the following obviously holds:

$$\begin{aligned} f_4(x,y) = \left(\frac{x}{p}+y\right) &\leq f_2(x,y) = \left(\frac{x}{p}+y\right)(1+p^{\frac{\alpha-1}{\alpha}})^{\frac{\alpha}{1-\alpha}}, \\ f_2(x,y) = \left(\frac{x}{p}+y\right)(1+p^{\frac{\alpha-1}{\alpha}})^{\frac{\alpha}{1-\alpha}} &\leq f_1(x,y) = (y+px)\left(1+p^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

   It is less obvious, that $f_1(x,y) > f_3(x,y)$. To prove this, I will re-write the inequality in the following form:

$$f_1(x,y) > f_3(x,y) \Leftrightarrow (t+p)^{1-\alpha}\left(1+p^{\frac{\alpha-1}{\alpha}}\right)^{\alpha} > 1 + t^{1-\alpha}, \qquad (12)$$

   where $t = \frac{y}{x}$. Notice, that if $t = p^{\frac{1}{\alpha}}$, then the left-hand side of the above inequality, which I will call $L(t)$ is equal to the right-hand side, which I will call $R(t)$, namely $L(t) = R(t) = 1 + p^{\frac{1-\alpha}{\alpha}}$.

Since both $L(t)$ and $R(t)$ are increasing functions of t, it is enough to show, that $L'(t) > R'(t)$ in order to prove that inequality (12) is true. Taking the respective derivatives, one can see, that

$$L'(t) > R'(t) \Leftrightarrow (t+p)^{-\alpha}\left(1+p^{\frac{\alpha-1}{\alpha}}\right)^{\alpha} > t^{-\alpha} \Leftrightarrow \frac{t}{t+p} > \frac{p^{\frac{1-\alpha}{\alpha}}}{1+p^{\frac{1-\alpha}{\alpha}}}$$

where the last inequality always holds for $\frac{y}{x} = t > p^{\frac{1}{\alpha}}$.

2. $xp^{-\frac{1}{\alpha}} \leq y \leq xp^{\frac{1}{\alpha}}$. If this is the case, then there are three outputs to be compared: $f_3(x,y)$, $f_4(x,y)$ and $f_2(x,y)$. The analysis in analogical to previous case and the conclusion is $f_3(x,y) > f_2(x,y) > f_4(x,y)$.

3. $y \leq xp^{-\frac{1}{\alpha}}$. Using the similar approach, as in (1), it is possible to show, that $f_4(x,y) < f_3(x,y)$.

The analysis of the residual allocations $5, 7$ and 8 is symmetric. Hence, among all possible allocations, which satisfy the constrains of problem (10) the allocation which maximize the joint production are:

1. Allocation 1, if $\frac{x}{y} \leq p^{-\frac{1}{\alpha}}$,
2. Allocation 3, if $p^{-\frac{1}{\alpha}} < \frac{x}{y} < p^{\frac{1}{\alpha}}$,
3. Allocation 7, if $\frac{x}{y} \geq p^{\frac{1}{\alpha}}$.

The resulting output therefore is the following:

$$f(x,y) = \begin{cases} (y+px)\left(1+p^{\frac{\alpha-1}{\alpha}}\right) & \text{if } \frac{x}{y} \leq p^{-\frac{1}{\alpha}}, \\ \left(x^{1-\alpha}+y^{1-\alpha}\right)^{\frac{1}{1-\alpha}} & \text{if } \frac{x}{y} \in \left(p^{-\frac{1}{\alpha}}, p^{\frac{1}{\alpha}}\right), \\ (x+yp)\left(1+p^{\frac{\alpha-1}{\alpha}}\right) & \text{if } \frac{x}{y} \geq p^{\frac{1}{\alpha}}. \end{cases} \tag{13}$$

Proof of Proposition 3

The problem of the principal in general form is:

$$\max_{\beta_1^T,\, \beta_2^T,\, c,\, d} \quad \Pi_P^T = R(1-\beta_1^T-\beta_2^T)(1-e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}) - (c+d)$$

$$s.t. \qquad (ICT_1)\ R\beta_1^T \geq \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}x^{\alpha}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}},$$

$$
\begin{aligned}
&(ICT_2)\ R\beta_2^T \geq \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}y^{\alpha}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}\ ,\\
&(RCT_1)\ x \leq c,\\
&(RCT_2)\ y \leq d,\\
&(CST_1)\ \left(R\beta_1^T - \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}x^{\alpha}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}\right)(x-c)=0,\\
&(CST_2)\ \left(R\beta_2^T - \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}y^{\alpha}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}\right)(y-d)=0.
\end{aligned}
$$

As it was the case with competition, at least one of the constrains (ICT_1, RCT_1) and (ICT_2, RCT_2) will be binding. If the incentive constraint is not binding, then an agent will invest all available funds, i.e. resource constrain will be binding. On the other hand, if the recourse constrain is not binding, then equilibrium conditions imply, that incentive constraint is binding.

The proof is developed in several steps.

Step 1. All constrains for the principal problem are binding. The argument is analogical to the proof of Proposition 1.

Step 2. With all constrains being binding, we can re-write the problem of the principal in the following form:

$$
\max_{x,y} \Pi_P^T = R\left(1 - \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}x^{\alpha}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}} - \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}y^{\alpha}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}\right)(1-e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-a}}})-(x+y)
$$

Taking the first-order condition w.r.t x and y and expressing R, I receive the system of equation $R = F_1(x,y), R = F_2(x,y)$.[26] In the optimal solution therefore, $F_1(x,y) = F_2(x,y)$, which can be shown to be equivalent to:

$$
xy(x^{\alpha}-y^{\alpha}) = \alpha\left(e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}-1\right)\left(x^{1-\alpha}+y^{1-\alpha}\right)^{\frac{\alpha}{1-\alpha}}xy(y^{2\alpha-1}-x^{2\alpha-1}).
$$

The above equation obviously holds iff $x = y$. This in turn implies, that $c = d$ and $\beta_1^T = \beta_2^T$.

[26]The first-order conditions are straightforward, but are too cumbersome, and therefore are not presented here.

Step 3. Taking into account the results of two previous steps, I can re-write the problem of the principal in the following form:

$$\max_{\beta^T,c,x} \quad \Pi_P^T = R(1-2\beta^T)(1-e^{-2^{\frac{1}{1-\alpha}}x}) - 2c \tag{14}$$
$$s.t. \qquad R\beta^T = \frac{e^{2^{\frac{1}{1-\alpha}}x}}{2^{\frac{\alpha}{1-\alpha}}},$$
$$x = c.$$

Solving this problem, I receive the optimal amount of investments c:

$$R = 2^{\frac{\alpha}{\alpha-1}} e^{2^{\frac{1}{1-\alpha}}c}(1+2e^{2^{\frac{1}{1-\alpha}}c}), \text{ hence} \tag{15}$$
$$c = 2^{\frac{1}{\alpha-1}} \ln \frac{1}{4}\left(-1+2^{\frac{1}{1-\alpha}}\sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R}\right). \tag{16}$$

Step 4. According to (16), c is monotonically increasing in R. Let us determine the threshold $\hat{R}$, such that $c > 0$ if $R > \hat{R}$. Solving Equation (16) for $c = 0$, I obtain the value of $\hat{R} = 3 \cdot 2^{\frac{\alpha}{\alpha-1}}$. So, for $R \leq 3 \cdot 2^{\frac{\alpha}{1-\alpha}}$ the team will not be employed and for $R > 3 \cdot 2^{\frac{\alpha}{1-\alpha}}$ the team will be employed. In the latter case the profit of the principal is positive. To see this, I will first prove, that Π_P^T is increasing in R, if $R \geq \hat{R}$. I substitute back expression for optimal c into the Π_P^T:

$$\Pi_P^T = \frac{1}{4}\left(4R - 8\sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R} + 2^{3+\frac{1}{\alpha-1}}(2+\ln 4) - \right.$$
$$\left. -2^{3+\frac{1}{\alpha-1}} \ln\left(-1+2^{\frac{1}{\alpha-1}}\sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R}\right)\right).$$

The derivative w.r.t. R, $\dfrac{-5\cdot 2^{\frac{1}{\alpha-1}}+\sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R}}{-2^{\frac{1}{\alpha-1}}+\sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R}}$, is positive if $R > \hat{R}$. Since $\Pi_P^T(\hat{R}) = 0$, for any $R > \hat{R}$, profit of the principal is positive..

Step 5. I have proved that the agents are offered a symmetric contract. If the incentive constraint is satisfied, they allocate all fund to *R&D*. This rules out all equilibria expect $(0,0)$ and (c,c), where optimal c was derived at step 3.

According to the equilibrium conditions (see Table 2), an equilibrium $(0,0)$ will be played if $R\beta_1^T = R\beta_2^T \leq 1$. From the incentives constraint (see maximisation problem 14), $R\beta_1^T = R\beta_2^T = \dfrac{e^{2^{\frac{1}{1-\alpha}}x}}{2^{\frac{\alpha}{1-\alpha}}}$. Hence, $(0,0)$ is not an equilibrium, if

$$\frac{e^{2^{\frac{1}{1-\alpha}}x}}{2^{\frac{\alpha}{1-\alpha}}} > 1 \iff c > 2^{\frac{1}{\alpha-1}} \ln 2^{\frac{\alpha}{1-\alpha}} \iff R > 1 + 2^{\frac{1}{1-\alpha}}.$$

From the incentive constraint it also follows, that $R\beta_1^T \leq 1$ iff $c \leq 2^{\frac{1}{\alpha-1}} \ln 2^{\frac{\alpha}{1-\alpha}}$. Using the expression for c (Equation 16), I obtain the following condition:

$$c \leq 2^{\frac{1}{\alpha-1}} \ln 2^{\frac{\alpha}{1-\alpha}} \qquad \text{iff} \qquad R \leq 1 + 2^{\frac{1}{1-\alpha}}.$$

Hence, there are two equilibria (c, c) and $(0, 0)$, if $3 \cdot 2^{\frac{\alpha}{1-\alpha}} < R \leq 1 + 2^{\frac{1}{1-\alpha}}$. Otherwise, there is a single equilibrium (c, c).

Proof of Corollary 2

The proof is completely analogical to the proof of Corollary 1 and will be therefore abandoned.

Proof of Lemma 1

Recall, that the domain of d and t is such, that $d \in [0, t]$ and $t \in [d, ((c - d)^{1-\alpha} + d^{1-\alpha})^{\frac{1}{1-\alpha}}]$. Let me first consider the derivative $\frac{\partial \Pi_1^H}{\partial d} = -1 - \alpha d^{\alpha-1}(e^t - 1)t^{-\alpha} + d^{-\alpha}(t^{1-\alpha} - d^{1-\alpha})^{\frac{\alpha}{1-\alpha}}$. Agent A_1 employing agent A_2 implies, that this derivative is increasing in $d = 0$, so that the optimal d is positive. Further, for $d = t$ the derivative $\frac{\partial \Pi_1^H}{\partial d}$ takes the value $\frac{\partial \Pi_1^H}{\partial d} = -1 - \frac{\alpha}{t}(e^t - 1) < 0$. Hence, $d = t$ cannot be the optimal solution. Therefore, the optimum should be reached in the interior of the interval $[0, t]$, so that $\frac{\partial \Pi_1^H}{\partial d} = 0$.

Consider now the derivative $\frac{\partial \Pi_1^H}{\partial t}$. I have showed in the Proof of Proposition 1, that it is optimal for the principal to provide such incentives, that agents invest all funds in their discretion into the R&D. Hence, the principal will choose such c, that the optimal choice of x and d satisfies $c = d + x = d + (t^{1-\alpha} - d^{1-\alpha})^{\frac{1}{1-\alpha}}$, which is equivalent to $t = ((c - d)^{1-\alpha} + d^{1-\alpha})^{\frac{1}{1-\alpha}}$. But, for this t to be an equilibrium choice of the first agent, it must be, that $\frac{\partial \Pi_1^H}{\partial t} \geq 0$, or, equivalently

$$R\beta_1 \geq e^t t^{-(1+\alpha)} \left(-\alpha d^\alpha (e^t - 1) + t(d^\alpha e^t + (t^{1-\alpha} - d^{1-\alpha})^{\frac{\alpha}{1-\alpha}}) \right).$$

Since $t = ((c-d)^{1-\alpha} + d^{1-\alpha})^{\frac{1}{1-\alpha}}$ for any β_1, which satisfies the inequality above, the principal will choose β_1, such that the inequality above is just satisfied, so that $\frac{\partial \Pi_1^H}{\partial t} = 0$.

Proof of Proposition 4.

Consider the first statement of the proposition. Agent A_1 will not employ A_2, if derivative $\frac{\partial \Pi_1^H}{\partial d} < 0$ for any d. This condition is satisfied, if

$$-1 - \alpha d^{\alpha-1}(e^t - 1)t^{-\alpha} + \left(\left(\frac{t}{d} \right)^{1-\alpha} - 1 \right)^{\frac{\alpha}{1-\alpha}} < 0.$$

Taking into account, that $e^t - 1 > t$, the following inequality holds:

$$-1 - \alpha d^{\alpha-1}(e^t - 1)t^{-\alpha} + \left(\left(\frac{t}{d}\right)^{1-\alpha} - 1\right)^{\frac{\alpha}{1-\alpha}} < \tag{17}$$

$$< -1 - \alpha d^{-1+\alpha} t^{1-\alpha} + \left(\left(\frac{t}{d}\right)^{1-\alpha} - 1\right)^{\frac{\alpha}{1-\alpha}} \tag{18}$$

Let me substitute $z = \left(\frac{t}{d}\right)^{1-\alpha}$. Note, that $z > 1$ since $d < t$. Then, I can re-write the right-hand side of the inequality (18), as:

$$-1 - \alpha d^{-1+\alpha} t^{1-\alpha} + \left(\left(\frac{t}{d}\right)^{1-\alpha} - 1\right)^{\frac{\alpha}{1-\alpha}} = -1 - \alpha z + (z-1)^{\frac{\alpha}{\alpha-1}}.$$

If $f(z) = 1 + \alpha z - (z-1)^{\frac{\alpha}{\alpha-1}}$ is positive for some α, then the right-hand side of inequality (18) is negative, which implies $\frac{\partial \Pi_1^H}{\partial d} < 0$. The function $f(z)$ reaches its minimum in $z^* = 1 + (1-\alpha)^{\frac{1-\alpha}{2\alpha-1}}$. The value of function in $z = z^*$ is $f(z^*) = 1 - (1-\alpha)^{\frac{\alpha}{2\alpha-1}} + (1 + (1-\alpha))^{\frac{1-\alpha}{2\alpha-1}}$. This value is positive, if $\alpha < \bar{\alpha}$, where $\bar{\alpha} \approx 0.4316$. Hence, for all $\alpha \leq \bar{\alpha}$, $f(z) \geq 0$. This implies that the derivative $\frac{\partial \Pi_1^H}{\partial d}$ is strictly negative, so that agent A_1 will never employ agent A_2.

The second statement of the proposition follows directly from the Lemma 1. If A_1 employs A_2, then the first order condition with respect to d is $\frac{\partial \Pi_1^H}{\partial d} = 0$. Substituting $x = (t^{1-\alpha} - d^{1-\alpha})^{\frac{\alpha}{1-\alpha}}$ I receive the equivalent condition:

$$x^\alpha = d^\alpha + \alpha d^{2\alpha-1}(e^t - 1)t^{-\alpha}.$$

Given $t > 0$, this condition directly implies $x > d$.

Solution for the hierarchical team, $\alpha = 0.5$

Let $\alpha = 0.5$. Then, the derivatives of Π_1^H with respect to d and t are the following:

$$\frac{\partial \Pi_1^H}{\partial d} = \frac{1 - e^t - 4\sqrt{dt} + 2t}{2\sqrt{dt}} \tag{19}$$

$$\frac{\partial \Pi_1^H}{\partial t} = \frac{e^{-t}}{2t^{\frac{3}{2}}}\left(2(R\beta_1 - e^t)t^{\frac{3}{2}} - \sqrt{d}(e^t - 1)(2t - 1)\right). \tag{20}$$

Notice, that In the solution to A_1's maximization problem $d = 0$ if and only if $1 - e^t + 2t \leq 0$.

Sufficiency:

If $1 - e^t + 2t \leq 0$, then $\frac{\partial \Pi_1^H}{\partial d} < 0$ for any $d \in [0, t]$. Hence, optimal solution is $d = 0$.

Necessity:

If $d = 0$ is the optimal solution, then it must be the case, that $\frac{\partial \Pi_1^H}{\partial d} < 0$. Recall, that

$$\frac{\partial \Pi_1^H}{\partial d} = \frac{1 - e^t - 2t}{2\sqrt{dt}} - 2.$$

Hence, for $d = 0$, $\frac{\partial \Pi_1^H}{\partial d} \to -\infty$, if $1 - e^t + 2t \leq 0$ and $\frac{\partial \Pi_1^H}{\partial d} \to +\infty$ if $1 - e^t + 2t > 0$. Therefore, for $d = 0$ to be an optimal solution it is necessarily must be, that $1 - e^t + 2t \leq 0$.

Let me denote $\hat{t}$ the solution of the equation $1 - e^t + 2t = 0$. Then, inequality $1 - e^t + 2t \leq 0$ is equivalent to $t \geq \hat{t}$. It is possible to estimate, that $\hat{t} \approx 1.25643$. Hence, for any $t \geq \hat{t}$, the first agent will optimally choose $d = 0$ and will not employ the second agent. Let us find out, which value of $R\beta_1$ corresponds to this threshold value.

According to Lemma 1, for $t > \hat{t}$, we can limit our attention to the interior solution of A_1's problem, so that $\frac{\partial \Pi_1^H}{\partial d} = 0$ and $\frac{\partial \Pi_1^H}{\partial t} = 0$. From these first-order conditions it is possible to express $R\beta_1$:

$$d = \frac{(1 - e^t + 2t)^2}{16t}, \tag{21}$$

$$R\beta_1 = e^t \left(1 + \frac{(e^t - 1)\sqrt{d}(2t - 1)}{2t^{\frac{3}{2}}}\right). \tag{22}$$

The equations above can be used to find the value of $R\beta_1$, corresponding to threshold value $\hat{t}$: $R\hat{\beta}_1 \approx 3.51286$. For any $R\beta_1 \geq R\hat{\beta}_1$ the first agent will not employ the second agent. To find the value of R, which corresponds to the $R\hat{\beta}_1$, one need to solve the problem of the principal.

$$\begin{aligned} \max_{\{c, \beta_1\}} \quad & \Pi_P^H = R(1 - \beta_1)(1 - e^{-t}) - c \\ s.t. \quad & c = x + d = (t^{1-\alpha} - d^{1-\alpha})^{\frac{1}{1-\alpha}} + d \\ & d \text{ is given by (21)} \\ & R\beta_1 \text{ is given by (22).} \end{aligned}$$

The first order condition to this problem is given by:

$$R = -\frac{1}{8t^3}\Big(e^t(-2+t-4t^3+e^t(6-7t+4t^2)+e^{3t}(2-5t+6t^2)- \\ -e^{2t}(6-11t+10t^2+8t^3))\Big). \qquad (23)$$

Plotting the graph for $R(t)$, where $t \in [0, \hat{t}]$ it is possible to verify, that R increases in t. Using Equation (23), I can now estimate the threshold $\hat{R} = R(\hat{t})$. For the values $R > \hat{R}$ the first agent will prefer not to employ agent A_2. By plugging the value of $\hat{t}$ into the function $f(t)$, it is possible to estimate, that $R(\hat{t}) \approx 14.4903$. Hence, if $R \geq 14.4903$ and $\alpha = 0.5$, the team with hierarchy is not feasible to the principal. Continuity implies, that the same conclusion holds for the open set around $\alpha = 0.5$.

If $R < R(\hat{t})$, then Equation (23) can be used to numerically evaluate the profit of the principal and the parameters of the contract in the situation when the hierarchical team is employed. The fact, that $t = ((c-d)^{1-\alpha} + d^{1-\alpha})^{\frac{1}{1-\alpha}}$ and $d < c$ implies, that $t = 0$ iff $c = 0$. Hence, we can use Equation (23) to find R, such that $c(R) = 0$. By applying the L'Hospital rule three times in a raw, it is possible to establish, that as $R \to 1.5$ as $c \to 0$. Hence, for $R \leq 1.5$ the principal will choose zero level of investments in equilibrium, otherwise he will choose $c > 0$.

To compare the hierarchical team with the team with equal partners, notice, that from the Proposition 3 it follows, that the latter will be financed for $R > 1.5$. Hence, we have to compare profit of the principal, investment expenditures and reward of the agent for two alternative team structures for $R \in (1.5, 14.4903)$. From the numerical comparison (see Figure 2) it follows, that the principal is better off by employing the team with equal partners.

Proof of Corollary 3

The proof is done by a mean of example. Consider $\alpha = \frac{2}{3}$. Using Lemma 1, the first order conditions of the A_1's problem are:

$$d = \frac{27t^4}{8(3t+e^t-1)^3}. \qquad (24)$$

$$R\beta_1 = \frac{e^t}{3t^{\frac{5}{3}}}\left(-6d^{\frac{1}{3}}t^{\frac{4}{3}} + 3t^{\frac{5}{3}} + d^{\frac{2}{3}}(2+3t+e^t(3t-2))\right). \qquad (25)$$

The problem of the principal is

$$\max_{\{c,\beta_1\}} \quad \Pi_P^H = R(1-\beta_1)(1-e^{-t}) - c$$

$$s.t. \quad c = x + d = (t^{1-\alpha} - d^{1-\alpha})^{\frac{1}{1-\alpha}} + d$$

d is given by (24)

$R\beta_1$ is given by (25).

The first order condition results in the following equation:

$$R = \frac{1}{4(3t + e^t - 1)^3} e^t \Big(14 + 4e^t - 27t^2 + 27t^3 + e^{3t}(54t - 26) + \\ + e^t(-46 + 59t - 9t^2) + 18e^{2t}(3 - 6t + 2t^2 + 3t^3) \Big). \tag{26}$$

It is tedious, but relatively straightforward to verify, that R increases in t.

As I have shown above (see example for $\alpha = 0.5$), $c = 0$ iff $t = 0$. Applying L'Hospital rule to the Equation (26), one can establish, that $R \to \frac{47}{64} \approx 0.73$ as $t \to 0$. Since R is increasing in t, the hierarchical team will be allocated the positive amount of investment, if $R > \frac{47}{64} \approx 0.73$. For all $R \in [0.73, 0.75]$, the team with equal partners will not be financed (this follows from the Proposition 3). The profit functions Π_P^H and Π_P^T are increasing in R, which implies, that in the neighborhood of $R = 0.75$ it must be true, that $\Pi_P^H > \Pi_P^T$. From the numerical computations it follows, that $\Pi_P^H = \Pi_P^T$ if $R \approx 0.8952$. Hence, the principal will employ the hierarchical team, if $R \in [0.73, 0.8952]$. For $R > 0.8952$ he will prefer team with equal partners. On Figure 3, I illustrate the profit of the principal under both alternative team structures for $R > 0.8952$.

B Appendix: Tables and Figures

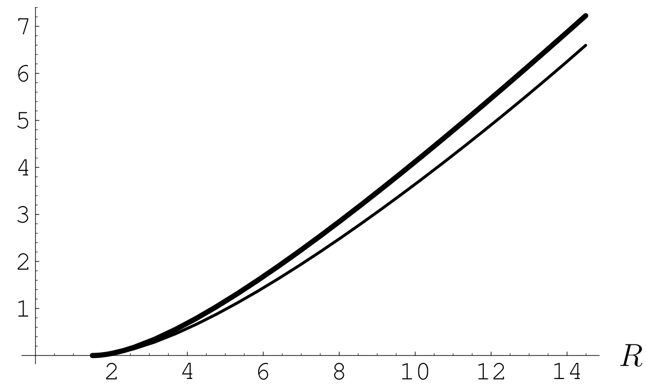

Figure 2: Profit of the principal ($\alpha = \frac{1}{2}, R \in (1.5, 14.4903)$): equal-partner team (thick line) vs. team with hierarchy (thin line).

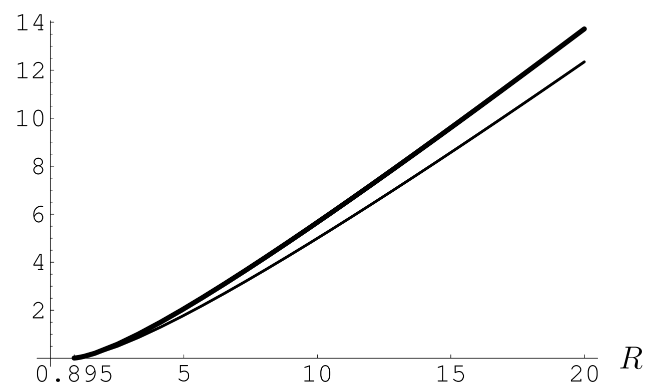

Figure 3: Profit of the principal ($\alpha = \frac{2}{3}, R \geq 0.8952$): equal-partner team (thick line) vs. team with hierarchy (thin line).

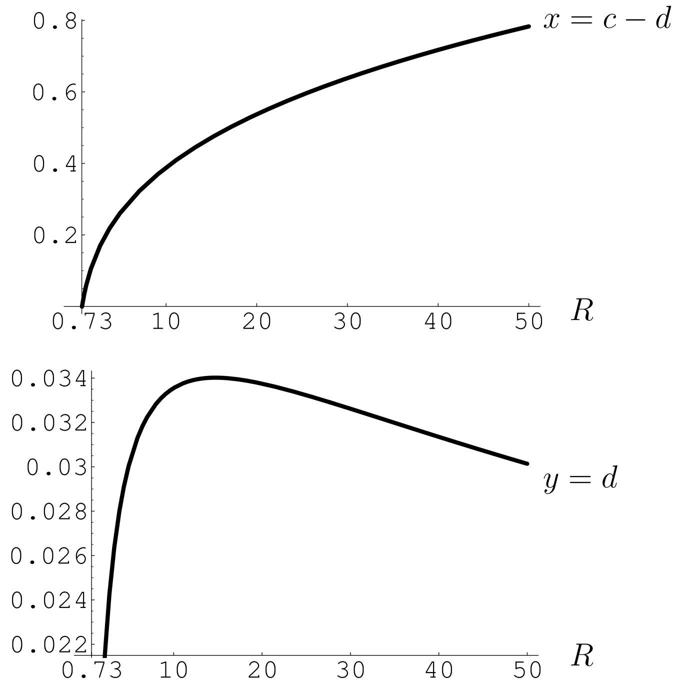

Figure 4: Equilibrium investment level of agents A_1 and A_2 in team with hierarchy: $\alpha = \frac{2}{3}$

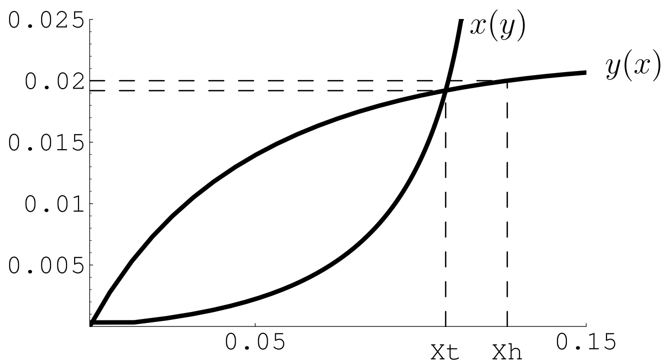

Figure 5: Illustration of a case, where the team leader overinvests in the hierarchical team: $\alpha = \frac{2}{3}, R = 2$. The reaction functions of the agents A_1 (the team leader) and A_2 are $x(y)$ and $y(x)$ respectively.

	$R\beta_1^C$	$R\beta_2^C$
$(0,0)$	$R\beta_1^C \leq 1$	$R\beta_2^C \leq 1$
$(x,0)$	$R\beta_1^C \geq e^x$	$R\beta_2 \frac{1-e^{-x}}{x} \leq 1$
$(0,y)$	$R\beta_1 \frac{1-e^{-y}}{y} \leq 1$	$R\beta_2^C \geq e^y$
(x,y)	$R\beta_1^C = \frac{e^{x+y}(x+y)^2}{x^2-y+e^{x+y}y+xy+y^2}$	$R\beta_2^C = \frac{e^{x+y}(x+y)^2}{y^2-x+e^{x+y}x+xy+y^2}$
(c,y)	$R\beta_1^C \geq \frac{e^{c+y}(c+y)^2}{c^2-y+e^{c+y}y+cy+y^2}$	$R\beta_2^C = \frac{e^{c+y}(c+y)^2}{y^2-c+e^{c+y}c+cy+y^2}$
(x,d)	$R\beta_1^C = \frac{e^{x+d}(x+d)^2}{x^2-d+e^{x+d}x+xd+d^2}$	$R\beta_2^C \geq \frac{e^{x+d}(x+d)^2}{d^2-x+e^{x+d}x+xd+d^2}$
(c,d)	$R\beta_1^C \geq \frac{e^{c+d}(c+d)^2}{c^2-d+e^{c+d}c+cd+d^2}$	$R\beta_2^C \geq \frac{e^{c+d}(c+d)^2}{d^2-c+e^{c+d}c+cd+d^2}$

Table 1: Equilibrium conditions in competition

	$R\beta_1^T$	$R\beta_2^T$
$(0,0)$	$R\beta_1^T \leq 1$	$R\beta_2^T \leq 1$
(x,y)	$R\beta_1^T = \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot x^\alpha}{(x^{1-\alpha}+y^{1-a})^{\frac{\alpha}{1-\alpha}}}$	$R\beta_2^T = \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot y^\alpha}{(x^{1-\alpha}+y^{1-a})^{\frac{\alpha}{1-\alpha}}}$
(c,y)	$R\beta_1^T \geq \frac{e^{(c^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot c^\alpha}{(c^{1-\alpha}+y^{1-a})^{\frac{\alpha}{1-\alpha}}}$	$R\beta_2^T = \frac{e^{(c^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot y^\alpha}{(c^{1-\alpha}+y^{1-a})^{\frac{\alpha}{1-\alpha}}}$
(x,d)	$R\beta_1^T = \frac{e^{(x^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot x^\alpha}{(x^{1-\alpha}+d^{1-a})^{\frac{\alpha}{1-\alpha}}}$	$R\beta_2^T \geq \frac{e^{(x^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot d^\alpha}{(x^{1-\alpha}+d^{1-a})^{\frac{\alpha}{1-\alpha}}}$
(c,d)	$R\beta_1^T \geq \frac{e^{(c^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot c^\alpha}{(c^{1-\alpha}+d^{1-a})^{\frac{\alpha}{1-\alpha}}}$	$R\beta_2^T \geq \frac{e^{(c^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot d^\alpha}{(c^{1-\alpha}+d^{1-a})^{\frac{\alpha}{1-\alpha}}}$

Table 2: Equilibrium conditions in team production