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Performance of Procrastinators: On the Value of Deadlines*

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Earlier work has shown that procrastination can be explained by quasi-hyperbolic discounting. We present a model of effort choice over time that shifts the focus away from completion to performance on a single task. We show that quasi-hyperbolic discounting is detrimental for performance. More intrestingly, we find that being aware of the own self-control problems not necessarily increases performance. Extending this framework to a multi-task model, we show that deadlines help an agent to structure his workload more efficiently, which in turn leads to better performance. Moreover, being restricted by deadlines increases a quasi-hyperbolic discounter's well-being. Thus, we give a theoretical underpinning for recent empirical evidence and numerous casual observations.

JEL classification: A12, D11

Keywords: Effort Choice; Deadlines; (Quasi-) Hyperbolic Discounting; Naiveté; Present-Biased Preferences; Sophistication

1 Introduction

Life is pervaded by situations where people have a certain span of time to work on a task, and the final reward depends on how much devotion they put into their work: students studying for the final of a class they take or writing their thesis, employees working on a long-term project, etc. Next to the final deadline, these tasks often have additional interim deadlines: mandatory problem sets maybe a prerequisite to pass a class; students meet in regular intervals with their thesis advisor to report on their progress; employees have to hold several presentations at different stages over the course of the whole project. A rational decision maker with time-consistent preferences would not welcome such restrictions on his choice set. But when people

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impulsively procrastinate, such interim deadlines can be reasonable.[1] Earlier research has shown that procrastination on the completion of a task can be explained by hyperbolic discounting. This paper analyzes the behavior of hyperbolic discounters in a model of effort choice over time that shifts the focus away from completion to performance. We show that interim deadlines are a useful commitment device for a hyperbolic discounter to increase his "long-run utility". Moreover - and more interestingly - interim deadlines are performance-enhancing. Thus, implementing interim deadlines not only is in the interest of the hyperbolic discounter himself, but there is also scope for the employer of such an agent to benefit from doing so, even in the absence of any cost of delay. Therefore, our paper gives a theoretical underpinning for the frequent observation of interim deadlines.

We start out from a model where an individual has a given number of periods to work on a single task. In each period this person can invest costly effort into this task. Effort is modeled as a continuous decision variable. In the final period the individual receives a reward which depends on the total amount of effort he has invested. Since serious procrastination can hardly be explained by exponential discounting with a reasonable discount factor, we adopt the assumption that the agent discounts (quasi-)hyperbollically, which gives rise to time-inconsistent preferences.[2] We compare the performance of three types of persons. Next to the benchmark of a time-consistent individual without self-control problems, we consider two types of hyperbolic discounters: naive persons who are totally unaware of these problems on the one hand, and sophisticated persons who are fully aware of these problems on the other hand. Mainly, we ask three questions regarding procrastination, performance and deadlines: First, is procrastination detrimental for performance? Second, does sophistication increase an individual's performance and overall well-being? Third, do interim deadlines enhance performance, and if so, how? The answer to the first question unambigously is yes, procrastination hampers performance. With regard to the second question, we find that sophistication may actually hurt an individual, even in an environment with immediate costs and delayed rewards. In order to provide an intuition for why this may be the case, we identify and discuss the effects that drive the differences in the behavior of sophisticated and naive agents. As it turns out, it is exactly the awareness of conflicting intra-personal preferences that possibly makes a sohisticate take undesirable actions today in order to strategically manipulate the behavior of his future selves. This finding is in contrast to earlier work on hyperbolic discounting which has shown that when costs are immediate and rewards are delayed, awareness of self-control problems will never hurt

[1] We do not claim that procrastination issues are the only explanation for observing interim deadlines. Other explanations may be preferences for risk diversification or motives for information aquisition.

[2] See O'Donoghue and Rabin (2005) for some illustrative numerical examples.

an individual.[3] In order to answer the final question, we augment the basic model by introducing a second task. Two different regimes are compared: a regime with interim deadline and a regime without interim deadline. If no interim deadline is imposed, the agent can work on both tasks up to the final period, whereas under an interim deadline he has only half the time to perform on the first task, and the whole span of time to work on the second task. We show that being exposed to a deadline is beneficial for time-inconsistent agents. Interim deadlines help hyperbolic discounters to structure their workload and to allocate their effort more efficiently, leading to an overall better performance, which in turn improves long-run utility.

Our paper draws on two different strands of literature on time-inconsistent preferences. First, the literature on time-inconsistent procrastination, initiated by Akerlof (1991), and secondly the literature on time-inconsistent consumption-saving decisions, first studied by Laibson (1996). Earlier work on procrastination assumes that the decision that an individual has to make is *when* to do a task. In general, these papers are interested in the effects of awareness on behavior. O'Donoghue and Rabin (1999b) consider a setting where a single task has to be performed exactly once over a certain span of time. Each period, a person faces the binary decision whether to complete the task or not. They find that being sophisticated with regard to self-control problems leads to an earlier completion of the task. When costs are immediate and rewards are delayed, this in turn implies that sophistication never hurts a person. In O'Donoghue and Rabin (2001b) and O'Donoghue and Rabin (2007), these results are shown to carry over to situations where an individual has to choose which task to perform from a menu of mutually exclusive tasks or where a person engages in long-term projects.[4] In the literature on time-inconsistent consumption-saving decisions, which was carried on by Laibson (1997, 1998), Laibson et al. (1998), Angeletos et al. (2001), and Diamond and Kőszegi (2003), an individual has to decide each period anew how much to consume and how much to save, a continuous decision variable. Here, most researchers assume sophisticated beliefs.[5] The analysis of sophisticated hyperbolic discounters and continuous action spaces is fairly complicated. All the above contributions circumvent the arising analytical problems by assuming that the agent's instantaneous utility function for consumption is of the constant-relative-risk-aversion (CRRA) type. Borrowing the essential framework from this literature, in particular the assumption of a CRRA-utility function and sophisticated beliefs, Fischer (1999) analyzes procrastination issues, showing

[3] See, for example, O'Donoghue and Rabin (1999b, 2001b, 2007).

[4] O'Donoghue and Rabin (2007) assume that a project requires two periods to be completed, one in which it is started, and a second period in which it is finished. The decision the agent has to take each period, however, remains a binary one.

[5] Diamond and Kőszegi (2001) briefly discuss the behavior of naive agents without comparing sophisticates and naifs.

that sophisticated persons choose a decreasing leisure profile over time. To the best of our knowledge, our paper is the first that compares the behavior of naive and sophisticated individuals in a continuous action space framework. We consider differently aware persons who, over a certain span of time, have to decide each period how much effort to spend on a task, where effort is modeled as a continuous decision variable.

Moreover, we analyze the value of interim deadlines as commitment technology. O'Donoghue and Rabin (1999c) analyze optimal incentive schemes when a principal, who faces a cost of delay, hires a time-inconsistent agent, who faces a stochastic task cost, to perform a single task once. They find that under certain circumstances it is optimal to implement a deadline scheme, that is, to fix a date beyond which procrastination is severely punished. While this kind of deadline in a sense compares to the final deadline in our model, our main interest is in the impact of interim deadlines. That interim deadlines may be a valuable commitment mechanism for hyperbolic discounters is conjectured in O'Donoghue and Rabin (2005). We show that this indeed is the case, and moreover we lay open the beneficial effect of interim deadlines. Laibson (1997) considers illiquid assets as a commitment device. In our context, this idea would translate into an individual having today the possibility to commit his tommorow-self not to postpone a certain amount of work to the day after tomorrow. This clearly is different from the kind of commitment embedded in the interim deadlines that we consider.

The rest of the paper is structured as follows: In Section 2 we present the basic single-task model, and briefly review the concept of (quasi-) hyperbolic discounting and the notions of naiveté and sophistication. This model is analyzed in Section 3. In Section 4 we identify the effects driving the differences in behavior of differently aware agents and discuss the impact of awareness on performance and overall satisfaction. Section 5 extends the basic model to allow for a meaningful analysis of the effect of deadlines on performance. The final section concludes. All proofs are deferred to the appendix.

2 The Model

An agent has to perform a task, e.g. writing a term paper. He has two periods to work on that task in the sense that in each period $t \in \{1, 2\}$ the agent chooses an effort level $e_t \geq 0$ which he invests in the task. If the agent invests some positive effort in period t then in the same period an effort cost $c(e_t)$ arises. This cost function is assumed to be time-invariant. The agent is rewarded for the task in period 3. This delayed reward, which is assumed to be a function of total effort invested, is denoted by $g(\sum_{t=1}^{2} e_t)$.

Assumption 1 *It is assumed that the cost function and that the reward function satisfy the following properties:* $\forall x > 0$,

$$c'(x) > 0, \quad c''(x) > 0, \quad c(0) = 0, \quad c'(0) = 0$$
$$g'(x) > 0, \quad g''(x) < 0, \quad g(0) = 0, \quad g'(0) > 0$$

To motivate the above functional assumptions, once again consider the example of the student who has to write a term paper.[6] The effort is the time he spends on writing the paper. Thus, the costs of effort are the opportunity costs of not enjoying leisure time. Making the standard assumption of decreasing marginal utility of leisure time is equivalent to assuming a convex cost function. The reward function is the expected grade of the term paper. The expected grade increases when the student spends more time on writing the paper. Typically, by investing somewhat more effort the probability to receive a C instead of a D increases significantly, whereas the increase in effort necessary to receive an A instead of a B is much higher.

Within this framework, we study the behavior of individuals with time-inconsistent preferences due to hyperbolic discounting.[7] In particular, we assume that a person's intertemporal preferences from the perspective of period t are given by

$$U_t(u_t, u_{t+1}, ..., u_T) = u_t + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u_\tau,$$

where u_t denotes that person's instantaneous utility in period t. This functional form, which often is referred to as quasi-hyperbolic discounting, captures the essence of hyperbolic discounting.[8] While $\delta \in (0, 1]$ represents a time-consistent discount factor, $\beta \in (0, 1]$ introduces a time-inconsistent preference for immediate gratification and represents a person's self-control problem: for $\beta < 1$, at any given moment the person has an extra bias for the present over the future.[9] In order to focus on the effects that arise from the present bias embodied in the agent's preferences,

[6]We focus on a three-period model, the shortest possible time horizon that actually generates quasi-hyperbolic discounting effects. For longer time horizons the analysis becomes very quickly very complicated.

[7]Hyperbolic discounting refers to a person discounting events in the near future at a higher discount rate than events in the distant future. For an overview of empirical studies that provide evidence of hyperbolic discounting, see Frederick et al. (2002).

[8]Throughout this paper, we use the terms "present-biased preferences", and "(quasi-)hyperbolic discounting" interchangeably.

[9]While originally introduced by Phelps and Pollak (1968) to study intergenerational altruism, these present-biased preferences have been "rediscovered" by Laibson (1996, 1997) to study intra-personal, time-inconsistent decision problems. Besides procrastination and consumption-saving decisions, present-biased preferences have been applied to a broad range of contexts of economic interest, for example contract design (DellaVigna and Malmendier (2004, 2006)), industrial organization (Nocke and Peitz (2003), Sarafidis (2005)), bargaining (Akin (forthcoming)), information acquisition (Carrillo and Mariotti (2000), Benabou and Tirole (2000)), and labor economics (DellaVigna and Paserman (2005)).

we abstract from time-consistent exponential discounting, that is, formally we set $\delta = 1$.

An individual is modeled as a composite of autonomous intertemporal selves. These selves are labeled according to their respective periods of control over the effort decision. During its period of control, self t observes all past effort choices. The current self cannot commit future selves to a particular path of effort decisions. Within this framework, we study three types of agents: time-consistent agents (TC) as a benchmark, and two types of hyperbolic discounters, naifs (N) and sophisticates (S).[10] A naif is completely unaware of future self-control problems and hence wrongly predicts his future behavior: He believes that his future self's preferences will be identical to his current self's, not realizing that as the date of action gets closer his tastes will have changed. A sophisticate, in contrast, is fully aware of his future self-control problems and therefore correctly predicts how he will behave in the future. The first-period intertemporal utility of an agent of type $i \in \{TC, N, S\}$ is given by $U_1^i = -c(e_1) - \beta c(e_2) + \beta g\left(e_1 + e_2\right)$. Accordingly, given first-period effort $\hat{e}_1$, the second-period intertemporal utility takes the form $U_2^i = -c(e_2) + \beta g\left(\hat{e}_1 + e_2\right)$. The parameter $\beta \in (0, 1)$ measures the degree of present bias. For a time-consistent agent we have $\beta = 1$.

Following the literature on present-biased preferences, we assume that agents follow *perception-perfect strategies*, that is, strategies such that in all periods a person chooses the optimal action given her current preferences and her perception of future behavior. In each period, time-consistent and naive agents are just choosing an optimal effort path. While a time-consistent agent will always follow the effort path chosen in the first period, a naif, in contrast, will often revise his chosen effort path as his preferences change over time. Sophisticates, on the other hand, in a sense play a game against their future selves. Their behavior therefore incorporates reactions to behavior by their future selves that they cannot directly control as well as attempts to strategically manipulate the behavior of their future selves.

3 The Analysis

In this section, we solve the model for the three types of agents: time-consistent individuals, naifs and sophisticates. Hyperbolic discounters have a preference for immediate gratification. As was shown, for instance in O'Donoghue and Rabin (1999b), due to this present bias hyperbolic discounters are prone to procrastinate working on unpleasant tasks. Therefore, in our model with continuous effort choice over several periods, one should expect both naifs and sophisticates to procrastinate

[10] The two extreme assumptions about awarness, naiveté and sophistication, already have been discussed by Strotz (1956) and Pollak (1968).

in the sense of an increasing effort profile over time. Moreover, compared to a time-consistent agent, both types of hyperbolic discounters perceive immediate effort costs as higher relative to future effort costs and future rewards. Hence, one should expect both types of hyperbolic discounters to exert less effort in total than a time-consistent agent. We begin the analysis with the benchmark case of an agent without self-control problems.

The Time-Consistent Agent Since the preferences of a time-consistent agent do not change over time, his intertemporal decision problem boils down to maximizing lifetime utility, U_1^{TC}, by choosing both first- and second-period effort levels simultaneously. From the corresponding first-order conditions we immediately obtain that a TC chooses the same effort level in both periods. This optimal effort level, e^{TC}, is characterized by

$$c'(e^{TC}) = g'(e^{TC} + e^{TC}) \ . \tag{1}$$

Hence, a TC prefers to smooth effort in the sense that in each period he invests the same effort level in the task.[11] This is intuitively plausible: With the cost of effort being a convex function, a time-consistent agent can improve on any uneven allocation of effort over time by keeping total effort - and thus the final reward - constant, but shifting effort from the high-effort period to the low-effort period, thereby reducing total effort costs.

The Naive Agent A naive agent is unaware that his preferences will change over time. In the first period he believes that his second-period self will have the same preferences, that is, he believes he will stick to the plan he chooses now. When the second period finally rolls around, however, a naif's preferences will have changed.

Definition 1 *A perception-perfect strategy for a naive agent is given by* $(e_1^N, e_2^N(\hat{e}_1))$ *such that (i)* $(e_1^N, e_2^{TC}) \in \arg\max_{(e_1,e_2)} U_1^N(e_1, e_2)$*, and (ii)* $\forall\, \hat{e}_1 \geq 0$*,* $e_2^N(\hat{e}_1) \in \arg\max_{e_2} U_2^N(\hat{e}_1, e_2)$*. Let* $e_2^N = e_2^N(e_1^N)$

In the first period a naive agent maximizes U_1^N with respect to e_1 and e_2.[12] The actual first-period effort, e_1^N, and the planned second-period effort, e_2^{TC}, are characterized by the following conditions:

$$g'(e_1^N + e_2^{TC}) = c'(e_2^{TC}) \tag{2}$$

$$\beta g'(e_1^N + e_2^{TC}) = c'(e_1^N) \ . \tag{3}$$

[11] This clearly is an artifact of our choice to abstract from time-consistent discounting. With $\delta < 1$, a time-consistent agent would choose an increasing effort path, as was shown by Fischer (2001).

[12] Equivalently, we could solve for the behavior of a time-consistent agent in period 2 for a given first-period effort, $e_2^{TC}(e_1)$. Then, wrongly believing himself to behave time-consistently in the future, in period 1 a naive agent maximizes U_1^N with respect to e_1 subject to $e_2 = e_2^{TC}(e_1)$. We will actually make use of this procedure in the appendix.

Since there is no decision to be made after period 2, beliefs about own future behavior play no further role in determining the second-period effort. Hence, in the second period a naive person maximizes U_2^N with respect to e_2. The corresponding first-order condition which caracterizes the second-period effort, e_2^N, is given by

$$\beta g'(e_1^N + e_2^N) = c'(e_2^N) \; . \tag{4}$$

From equations (1)-(4) the following result is readily obtained.

Proposition 1 *(i) A naive agent invests more effort in period 2 than in period 1, i.e.,* $e_1^N < e_2^N$*. (ii) The total effort a naive agent invests is lower than the total effort of a time-consistent person, i.e.,* $e_1^N + e_2^N < 2e^{TC}$*. (iii) A naive agent is overly optimistic when predicting his future-self's willingness to work, i.e.,* $e_2^N < e_2^{TC}$*.*

Parts (i) and (ii) of Proposition 1 state that the two intuitive conjectures made above hold true for naive hyperbolic discounters. According to part (i), a naive agent procrastinates in the beginning and tries to catch up in the end. Part (ii) compares the behavior of a naif and a time-consistent agent. The present bias leads to higher perceived costs for a naif, which makes him exhibit lower overall effort than a time-consistent agent. Moreover, part (iii) says that a naive agent overestimates his own capabilities. Believing that he will behave time-consistently in the future, a naive agent makes ambitious plans today, that he does not follow through tomorrow.

The Sophisticated Agent In contrast to a naif, a sophisticate is fully aware that his preferences will change. Therefore, correctly predicting his own future behavior, a sophisticate plays a game against his future self, which can be solved per backwards induction.

Definition 2 *A perception-perfect strategy for a sophisticated agent is given by* $(e_1^S, e_2^S(\hat{e}_1))$ *such that (i)* $\forall\, \hat{e}_1 \geq 0$*,* $e_2^S(\hat{e}_1) \in \arg\max_{e_2} U_2^S(\hat{e}_1, e_2)$*, and (ii)* $e_1^S \in \arg\max_{e_1} U_1^S(e_1, e_2^S(e_1))$*. Let* $e_2^S = e_2^S(e_1^S)$*.*

For a given first period effort level $\hat{e}_1$, in period 2 a sophisticate maximizes U_2^S with respect to e_2. The second-period effort obviously is a function of the first-period effort, $e_2^S(\hat{e}_1)$, and satisfies the corresponding first-order condition,

$$\beta g'(\hat{e}_1 + e_2^S(\hat{e}_1)) = c'(e_2^S(\hat{e}_1)) \; . \tag{5}$$

Differentiating (5) with respect to e_1 yields

$$\frac{de_2^S(e_1)}{de_1} = -\frac{\beta g''(e_1 + e_2^S(e_1))}{\beta g''(e_1 + e_2^S(e_1)) - c''(e_2^S(e_1))} \in (-1, 0) \; .$$

The above derivative describes how a second-period sophisticate reacts to a change in the first-period effort. A higher first-period effort reduces the second-period effort.

Due to the strict convexity of the cost function, however, the absolut value of this reduction is lower than the increase in effort in the first period. In the first period the sophisticate maximizes U_1^S with respect to e_1 subject to $e_2 = e_2^S(e_1)$. In the appendix we show that the effort level that globally maximizes U_1^S, e_1^S, is characterized by the corresponding first-order condition.[13] This first-order condition is given by

$$-c'(e_1^S) + \beta g'\left(e_1^S + e_2^S(e_1^S)\right) + \frac{de_2^S(e_1^S)}{de_1}\beta\left[g'\left(e_1^S + e_2^S(e_1^S)\right) - c'(e_2^S(e_1^S))\right] = 0. \quad (6)$$

With the bahavior of a sophisticated agent being characterized by (5) and (6), the following result is obtained.

Proposition 2 *(i) A sophisticated agent invests more effort in period 2 than in period 1, i.e.,* $e_1^S < e_2^S$. *(ii) The total effort a sophisticated agent invests is lower than the total effort of a time-consistent person, i.e.,* $e_1^S + e_2^S < 2e^{TC}$.

Except for the fact that a sophisticated agent correctly predicts his own future behavior, his behavior otherwise qualitatively parallels that of a naive agent: First, a sophisticated agent procrastinates working on the task in the sense of an increasing effort profile over time.[14] Secondly, with the present bias increasing the perceived cost of effort, in total a sophisticate works less than a time-consistent agent.[15]

4 Comparison of the Naive and the Sophisticated Agent

Having compared the behavior of both types of hyperbolic discounters with the behavior of a time-consistent agent, now we are interested in how naifs and sophisticates compare to each other. Put differently, what effects does awareness of self-control problems have on performance and overall satisfaction? To answer this question a welfare criterion needs to be defined. Following O'Donoghue and Rabin (1999b, 2005) we use people's long-run preferences.

Definition 3 *A person's long-run preferences are given by* $U_0(e_1, e_2) \equiv -c(e_1) - c(e_2) + g(e_1 + e_2)$.

Long-run preferences reflect a person's preferences when asked from a prior perspective when she has no option to indulge immediate gratification. To formalize this long-run perspective, it is assumed that there is a (fictitiuos) period 0 where a

[13] While there is not necessarily a unique perception-perfect strategy for a sophisticated agent, all perception-perfect effort pairs are characterized by the corresponding first-order conditions. Multiple perception-perfect strategies are a well-known phenomenon for sophisticated hyperbolic discounters, see for instance O'Donoghue and Rabin (2007).

[14] A similar result can be found in Fischer (1999) for log utility functions.

[15] Similar results can be found in the consumption-saving literature for sophisticated present-biased consumers, see for instance Laibson (1996).

person has no decision to make.[16] It turns out that comparing first period efforts is sufficient to answer the question who is better off, naifs or sophisticates.

Lemma 1 *Suppose that* $e_1^i > e_1^j$*, for* $i, j \in \{S, N\}$ *and* $i \neq j$*. Then (i)* $e_2^i < e_2^j$*, (ii)* $e_1^i + e_2^i > e_1^j + e_2^j$*, and (iii)* $U_0^i \geq U_0^j$*.*

The lemma has a clear intuition. Since there is no decision to be made in the future, awareness plays no role in the second period. Hence, for a given effort level from the first period, both types of hyperbolic discounters face the same problem in period 2. Consequently, the type who works more in the first period works less in the second period. Due to the convexity of the cost function, however, the difference in first-period efforts is larger than the difference in second-period efforts. Thus, the type who invests more effort in the first period, in the end also has the overall better performance. The optimal effort levels from a long-run perspective are those chosen by a TC. While for both types of hyperbolic discounters total effort is below this optimal level of total effort, the type who works more in the first period is closer to the optimal total effort. Moreover, this total effort is more evenly - and thus, more efficiently - allocated over the two periods. Therefore, the type of hyperbolic discounter who works more in the first period is better of from a long-run perspective.

An intuitive guess would be that a sophisticate, who is aware of his self-control problems, will exhibit a higher first-period effort - and hence a higher total effort - than a naif. This would also be in line with previous research. For instance, O'Donoghue and Rabin (1999b) show that "when costs are immediate, sophisticates do at least as well as naifs (i.e. $U_0^S \geq U_0^N$)" (p.113).[17] While previous research analyzing the effects of awareness solely focuses on models with discrete action spaces, we analyze a continuous action space model. The following simple example demonstrates that the earlier result that sophisticates are always better off than naifs when costs are immediate does not hold true in general.[18]

Example: Let the cost function be $c(e) = (5/3)(1+z)(1/10)^z e^2$ for $e \leq 1/10$, $c(e) = (1/3)e^{1+z} - 1/3(1/10)^{1+z}(1-z)/2$ for $e \in (1/10, 1)$ and $c(e) = (1/6)(1+z)e^2 + 1/3[1 - (1/10)^{1+z}(1-z)/2 - (1+z)/2]$ for $e \geq 1$. The reward function is given by

[16] Another possibility would be to apply the Pareto criterion, where one outcome is deemed better than another if and only if the person views it as better at all points in time. A discussion of these two welfare criteria for hyperbolic discounters is provided in O'Donoghue and Rabin (2005).

[17] That sophisticates are better off than naifs when costs are immediate is shown in several other papers. O'Donoghue and Rabin (2001b), extend their earlier finding to a setting where a person has to choose which task to perform from a nenu of mutually exclusive tasks. Most recently, considering long-term projects, O'Donoghue and Rabin (2007) have shown that in contrast to sophisticates, naifs may start costly projects but then procrastinate finishing these projects, thus never reaping the reward.

[18] That sophistication may hurt a hyperbolic discounter is well known in the literature for models where costs are delayed and rewards are immediate like models of addiction, see O'Donoghue and Rabin (2001a).

$g(e_1+e_2) = 2(e_1+e_2) - (1/2)(e_1+e_2)^2$ for $e_1+e_2 \leq 2$ and $g(e_1+e_2) = 2$ otherwise. Suppose that $z = .005$ and $\beta = 1/4$.[19] The optimal effort choices of a sophisticate in the perception-perfect equilibrium are $e_1^S = .02602$ and $e_2^S = .63700$. In contrary, a naif chooses $e_1^N = .03718$ and $e_2^N = .62595$ in the perception-perfect equilibrium. In this example, a naif invests more effort in the task than a sophisticate both in the first period and in total. Hence, a naif is better of than a sophisticate from a welfare point of view, i.e., $U_0^S - U_0^N < 0$. Thus, in contrast to earlier findings, awareness of future self-control problems can hurt the agent even in a model of immediate costs and delayed rewards.[20]

As the above discussion suggests, characterizing the impact of awareness is complicated. Identifying the underlying effects that drive the different behavior of naifs and sophisticates, however, allows us to derive sufficient conditions for a sophisticate exhibiting higher first-period effort than a naif.

Pessimism Effect and Incentive Effect Why does sophistication may not help to increase first-period effort and thereby long-run utility? What are the driving forces behind this observation? O'Donoghue and Rabin (1999a, 2001a) carefully identify two effects how awareness of self-control problems can influence an agent's behavior. First, as O'Donoghue and Rabin (1999a) point out, "sophistication about future self-control problems can make a person pessimistic about future behavior" (p.16). Knowing that - from today's perspective - the future self will not behave optimally, may induce a sophisticate to directly respond to his future shortcommings. Reasoning like "I know that I won't work hard tomorrow, so I'll work more today" probably is familiar to everyone. This is what O'Donoghue and Rabin (1999a, 2001a) call the *pessimism effect*. This, however, is only half the story. Sophistication about one's own self-control problems has a second, less direct effect on today's behavior. Knowing about his own future misbehavior also makes a sophisticate aware of the need and the potential to strategically influence his future behavior via his behavior today. This second channel is labeled *incentive effect* by O'Donoghue and Rabin (1999a, 2001a).[21] So the following question is immediately at hand: How are these effects operative in the model presented in this paper?

A sohisticate in period 1 realizes that he will work less in period 2 than is optimal

[19] While the cost function is continuously differentiable, it is not twice continuously differentiable. Thus, the example does not fit perfectly to our Assumption 1.

[20] While this result may be somewhat counterintuitive, there actually is empirical evidence supporting this suggestion. Wong (2006) finds that time-inconsistency is associated with lower class performance irrespective of awareness. Effects of time-inconsistency on class performance, however, are smaller in magnitude and less statistically significant under naiveté than under sophistication.

[21] The pessimism effect and the incentive effect represent a decomposition of the "sophistication effect" identified by O'Donoghue and Rabin (1999b).

from today's perspective. He directly responds to his future shortcomings by working more today. Thus, due to the pessimism effect a sophisticate tends to work more in period 1 than a naif.[22] The incentive effect, however, in tendency leads to a lower first-period effort. The first-period self of a sophisticate would like to see his future self invest more effort in the task than he actually does. Since the second-period self increases effort when first-period effort is reduced, the first-period self can create incentives for his future self to work more by working less today. Formally, adding and subtracting $\beta g'(e_1 + e_2^{TC}(e_1))$ from dU_1^S/de_1 yields the following formulation of the marginal utility of a sophisticate in period 1:

$$\frac{dU_1^S}{de_1} = \beta g'(e_1 + e_2^{TC}(e_1)) - c'(e_1) + \underbrace{\beta\left[g'(e_1 + e_2^S(e_1)) - g'(e_1 + e_2^{TC}(e_1))\right]}_{PE} + \underbrace{(1-\beta)(de_2^S/de_1)c'\left(e_2^S(e_1)\right)}_{IE},$$

where $e_2^{TC}(e_1)$ is the effort a TC chooses in period 2 for a given first period effort. Note that the first term equals zero for $e_1 = e_1^N$. The second term, PE, is positive and reflects the pessimism effect. The agent knows that his future self chooses $e_2^S(e_1)$ instead of $e_2^{TC}(e_1)$, which would be optimal from today's perspective. The third term, IE, is negative and characterizes the impact of the incentive effect.[23] Given that U_1^S is a quasi-concave function in e_1, then a sophisticate chooses higher effort levels than a naif if the incentive effect does not outweigh the pessimism effect.

At first glance, the two effects seem to be weighted by the present bias parameter β. For a low degree of present bias the pessimism effect seems to be more important than the incentive effect. The agent cares more about a high reward than delegating work to his future self, and thus works harder today. On the other hand, for a high degree of present bias the incentive effect seems to be more important. The agent's perceived cost in the second period is remarkably lower than his cost today. Thus, the agent prefers to create incentives for his future self to work harder by working less today.[24] When having a closer look at the problem, however, it turns out that things are more complicated. When the present bias is low ($\beta \to 1$) then e_2^S is close

[22] O'Donoghue and Rabin (1999a, 2001a) use the term pessimism effect in models of addictive goods and present-biased preferences. In addictive good models, where rewards are immediate and costs are delayed, the pessimism effect can hurt the agent. In our context, the pessimism effect helps the sophisticate to achieve a better performance than a naif. Thus, in the model of this paper the term pessimism effect is a little bit misleading. Here, it would be more suitable to call this effect "realism effect".

[23] To be precise, it is not possible to completely disentangle the two effects, because the incentive effect is only operative if the pessimism effect is operative.

[24] And indeed, this is what happens in our example: For a high degree of present-biasedness, $\beta = 1/4$, sophistication hurts the agent because it makes him work less in the first period than under naiveté. For a low degree of present bias, on the other hand, for instance if $\beta = 3/4$, a sophisticate works more than a naif, and hence is better off . A similar finding is obtained by Gruber and Kőszegi (2001) who analyze the behavior of sophisticates in a model of addictive goods.

to e_2^{TC} and there is not much pessimism involved. When the present bias is extreme ($\beta \to 0$) then $de_2^S/de_1 \to 0$ and the agent cannot set incentives for his future self effectively.

With pessimism effect and incentive effect moving in opposite directions, it is complicated to obtain general results concerning the comparison of naive and sophisticated behavior. Nevertheless, using the insights gained from the above discussion we can characterize sufficient conditions for the cost and reward function such that sophisticated agents are better off than naive ones.

Lemma 2 *Suppose that $c'''(\cdot) \leq 0$ and $g'''(\cdot) \leq 0$. Then a sophisticated agent chooses a strictly higher effort in the first period than a naive agent, i.e., $e_1^S > e_1^N$.*

In the proof of the above lemma we compile sufficient conditions such that the incentive effect never outweighs the pessimism effect. So Lemma 2 states a very intuitive result: given the pessimism effect outweighs the incentive effect, then sophisticates choose higher first-period efforts than naifs.

Proposition 3 *Suppose that $c'''(\cdot) \leq 0$ and $g'''(\cdot) \leq 0$. Then the long-run utility of a sophisticated agent is at least as great as the long-run utility of a naive agent, i.e., $U_0^S \geq U_0^N$. Moreover, the performance of a sophisticated agent is strictly higher than the performance of a naive agent, i.e., $e_1^S + e_2^S > e_1^N + e_2^N$.*

5 Deadlines

In daily life deadlines are an often encountered phenomenon. As an example consider the "good-standing rules" of the Bonn Graduate School of Economics: after a year of coursework, a first paper has to be completed at the end of the second year, a second paper at the end of the third year, and a third paper at the end of the fourth year. A rational decision-maker with time-consistent preferences would not welcome constraints on his choices. But if people impulsively procrastinate, and if they are also aware of their procrastination problems, deadlines can be strategic and reasonable. Perhaps the best empirical demonstration is the study of Ariely and Wertenbroch (2002), which we will discuss in more detail at the end of this section. In this section we ask if and how the behavior of a present-biased agent is affected by the existence of deadlines. Our main finding is that deadlines help an individual to structure his workload more efficiently, which decreases effort costs and in turn improves performance.

A Multi-Task Model To tackle this question we have to modify the simple framework introduced above. While we stick to the case of two periods, we now assume that there are two independent tasks to be undertaken by the agent, task A and task B. We consider two regimes: *deadline* and *no deadline*. When the agent faces no (interim) deadline he is completely free in his decision how to divide his effort on tasks and over time. More precisely, the agent can work in both periods on both tasks. When there is an (interim) deadline, however, the agent can invest effort in task A only in period 1, whereas he can work on task B in both periods.[25] The reward for a task depends on the total effort invested in that task up to its deadline.[26] Effort costs for a particular period are determined by the sum of efforts invested in both tasks in that period. Formally, let e_{it} denote the effort invested in task $i \in \{A, B\}$ in period $t \in \{1, 2\}$. Moreover, let $e_t = e_{At} + e_{Bt}$ be the total effort that the agent exhibits in period t, and $e_i = e_{i1} + e_{i2}$ be the total effort invested in task i. The reward for task $i \in \{A, B\}$ then is given by $g_i(e_{i1} + e_{i2})$, and the total effort cost in period $t \in \{1, 2\}$ is $c(e_{At} + e_{Bt})$. We assume that the grade function is the same for both tasks, that is, $g_A(\cdot) = g_B(\cdot) = g(\cdot)$. Moreover, we keep the functional assumptions imposed in Section 3. In all that follows, the double-superscript refers to the regime that the agent faces: D for a situation with a deadline, and ND for a situation without a deadline.

The Time-Consistent Agent As a benchmark, consider a time-consistent agent who faces no deadline. In the above language, the intertemporal utility of this agent in period 1 is given by

$$U_1^{TC^{ND}} = -c(e_{A1} + e_{B1}) - c(e_{A2} + e_{B2}) + g(e_{A1} + e_{A2}) + g(e_{B1} + e_{B2}).$$

Choosing $e_{A1}, e_{A2}, e_{B1}, e_{B2}$ in order to maximize this expression yields

$$c'(e_1^{TC^{ND}}) = c'(e_2^{TC^{ND}}) = g'(e_A^{TC^{ND}}) = g'(e_B^{TC^{ND}}). \tag{7}$$

It follows immediately that a time-consistent agent equates effort over tasks and smoothes effort over time, that is, $e_A = e_B$ and $e_1 = e_2$. Put differently, when $2e^{TC^{ND}}$ denotes the overall effort that a time-consistent agent invests over the two periods, then he invests $e^{TC^{ND}}$ in the first period and $e^{TC^{ND}}$ in the second period. Moreover, $e^{TC^{ND}}$ is spent on task A and $e^{TC^{ND}}$ is spent on task B. Note, however, that a time-consistent agent does not care about how he splits up his per period effort between the two tasks as long as he invests evenly in both tasks. This implies that being subject to a deadline does not help a time-consistent agent. When investment

[25] In order to obtain a comparison of the two regimes in terms of the effort level chosen, we introduce a second task which allows us to consider a regime-independent reward scheme. With only one task, the reward under the regime without deadlines would have to be function of total effort only, whereas the reward under the regime of deadlines would have to be a function of both first-period effort and total effort, making a comparison infeasible.

[26] Our model also compasses another kind of deadline where task B is handed out after the deadline for task A, as it is typically the case for students' homework assignments. Formally, $e_{B1} = 0$ a priori. Since - and now we are jumping ahead - the agent optimally chooses $e_{B1} = 0$ anyway, this does not impose any additional restrictions and results do not chnage.

in task A is possible only in period 1, for a desired overall effort level $2e^{TC^{ND}}$ the time-consistent agent still can choose $e_A^{TC^D} = e_1^{TC^D} = e^{TC^{ND}}$ and $e_B^{TC^D} = e_2^{TC^D} = e^{TC^{ND}}$.

The Sophisticated Agent First consider a sophisticate who faces no deadline. Having two periods of time to work on two tasks is similar to having two periods of time two work on one task. The only additional question is how to divide the total effort on the two tasks. The reward function is identical for both tasks, thus it is optimal to invest half of the total effort in each task. From the single-task exercise we know that a sophisticate has a tendency to work more in period 2 than in period 1. By always working harder in the second period the agent can achieve effort smoothing over tasks in the second period irrespectively of the proportion of first period effort spend on a specific task. This observation allows us to focus on the agent's effort choice over time. With effort being spread out evenly among the two tasks, the optimal second-period effort as a function of first-period effort, $e_2^{S^{ND}}(\hat{e}_1)$, is charachterized by

$$c'(e_2^{S^{ND}}(\hat{e}_1)) = \beta g'((1/2)(\hat{e}_1 + e_2^{S^{ND}}(\hat{e}_1))). \tag{8}$$

The effort level choosen by a sophisticate in the first period is determined by the following first-order condition,[27]

$$\beta g'((1/2)(e_1 + e_2^{S^{ND}}(e_1))) - c'(e_1) + \frac{de_2^{S^{ND}}(e_1)}{de_1}\beta\left[g'((1/2)(e_1 + e_2^{S^{ND}}(e_1))) - c'(e_2^{S^{ND}}(e_1))\right] \overset{!}{=} 0\ . \tag{9}$$

Note that the two first-order conditions are very similar to those obtained in the single task case. Recapitulatory, when not facing a deadline, a sophisticated agent equates effort over tasks like a time-consistent agent, but does not achieve effort-smoothing over time, i.e. $e_1^{S^{ND}} < e_2^{S^{ND}}$ and $e_A = e_B = (1/2)(e_1^{S^{ND}} + e_2^{S^{ND}})$, where $e_2^{S^{ND}} = e_2^{S^{ND}}(e_1^{S^{ND}})$.

Next, consider a situation where a sophisticated agent faces a deadline in the sense described above: task A is due at the end of the first period, while task B is due at the end of the second period. Put differently, the agent can invest effort in task A only in period 1, whereas he can work for task B in both periods. Formally, $e_{A2} = 0$, $e_A = e_{A1}$ and $e_{B2} = e_2$. For given effort levels $\hat{e}_A$ and $\hat{e}_{B1}$, in the second period the agent's utility is given by

$$U_2^{S^D} = -c(e_{B2}) + \beta g(\hat{e}_A) + \beta g(\hat{e}_{B1} + e_{B2})\ .$$

[27] The first-order approach is valid according to the same reasoning as in the single-task case.

The optimal second-period effort invested in task B as a function of the first-period-effort invested in task B, $e_{B2}^{S^D}(\hat{e}_{B1})$, satisfies

$$c'(e_{B2}^{S^D}(\hat{e}_{B1})) = \beta g'(\hat{e}_{B1} + e_{B2}^{S^D}(\hat{e}_{B1})) \; . \tag{10}$$

Differentiation of (10) yields

$$\frac{de_{B2}^{S^D}(e_{B1})}{de_{B1}} = -\frac{\beta g''(e_{B1} + e_{B2}^{S^D}(e_{B1}))}{\beta g''(e_{B1} + e_{B2}^{S^D}(e_{B1})) - c''(e_{B2}^{S^D}(e_{B1}))} \in (-1, 0) \; .$$

Correctly predicting his own future behavior, in period 1 a sophisticated agent chooses e_A and e_{B1} in order to maximize his intertemporal utility,

$$U_1^{S^D} = -c(e_A + e_{B1}) - \beta c(e_{B2}^{S^D}(e_{B1})) + \beta g(e_A) + \beta g(e_{B1} + e_{B2}^{S^D}(e_{B1})) \; .$$

This utility maximization problem, however, does not have an interior solution.[28] When facing a deadline, a sophisticated agent considers it optimal to work exclusively on task A in the first period, that is, $e_{B1}^{S^D} = 0$. Intuitively, the single-task case and the no-deadline case suggest that a present-biased agent will work harder in the second period. Hence, under a deadline, there is a tendency to invest more effort in task B anyway. But then investing in task B in the first period is not optimal, because due to decreasing marginal rewards the agent can benefit from shifting first-period effort from task B to task A. While intuitively plausibel, the formal proof of this statement is somewhat elaborate and therefore deferred to the appendix. The effort levels which are chosen strictly positive, $e_A^{S^D}$ and $e_{B2}^{S^D}$, are characterized as follows:

$$c'(e_A^{S^D}) = \beta g'(e_A^{S^D}) \tag{11}$$
$$c'(e_{B2}^{S^D}) = \beta g'(e_{B2}^{S^D}) \tag{12}$$

From (11) and (12) it follows immediately that $e_A^{S^D} = e_{B2}^{S^D}$. To sum up: When facing a deadline, a sophisticated agent smoothes effort over time and equates effort over tasks. Moreover, he does not invest in task B in period 1. Let e^{S^D} denote the effort level that is chosen under a regime of deadlines in each period and per task. Formally we have $e_1^{S^D} = e_A^{S^D} = e^{S^D}$ and $e_B^{S^D} = e_2^{S^D} = e^{S^D}$.

After all, we are interested in whether deadlines are helpful to overcome self-control problems and thereby to improve performance and the agent's satisfaction. The following proposition compares the behavior and well-being of a sophisticate under both regimes, deadlines and no deadlines.

[28] With interior solution we refer to a pair of first-period effort choices (e_A, e_{B1}) with $0 < e_A, e_{B1} < \infty$.

Proposition 4 *When facing a deadline, a sophisticated agent chooses a higher effort level in the first period and a higher total effort level than under a regime without a deadline, i.e.,* $e_1^{SND} < e^{SD}$ *and* $e_1^{SND} + e_2^{SND} < 2e^{SD}$*. Moreover, the sophisticated agent is strictly better off from a long-run perspective when facing a deadline, i.e.,* $U_0^{SD} > U_0^{SND}$*.*

The above proposition has a clear intuition: a deadline helps a sophisticate to better structure his work on the two tasks. He has to complete task A in the first period and therefore he cannot procrastinate finishing task A as he does without a deadline. Thus, the deadline helps the sophisticate to combat procrastination and thereby effort is allocated more efficiently over the two periods. This more efficient allocation reduces effort cost, which in turn leads to a higher overall effort and a better performance. The optimal total effort level from a long-run perspective is the one chosen by a TC. Furthermore, for any total effort level the optimal allocation is investing equal amounts in both tasks and exhibiting the same amount of effort in each period. Irrespectively of the regime, deadline or no deadline, the total effort a sophisticate invests in the tasks is below the optimal total effort of a TC. With a deadline, however, the level of total effort a sophisticate chooses is closer to a TC's total effort. Moreover, this more desirable level of total effort is more evenly allocated over the two periods. For this reason a sophisticate is better off when being constrained by a deadline.[29]

The Naive Agent Since the analysis for the naive agent is completely analogous to the one of the sophisticated agent for the regime with a deadline and to the single-task case for the regime without a deadline, we defer the formal analysis to the appendix. Here we briefly state the main results and then move on to a discussion of our findings.

When not facing a deadline, a naive agent equates efforts over tasks, but chooses a higher effort level in the second period, that is, $e_1^{NND} < e_2^{NND}$. When being subject to a deadline, a naive agent also equates effort effort over tasks, but - in contrast - smoothes effort over time. In particular, the first-period effort is spent exclusively on task A and the second-period effort is spent exclusively on task B. Formally, $e_1^{ND} = e_A^{ND} = e^{ND}$ and $e_B^{ND} = e_2^{ND} = e^{ND}$. As a consequence, under a deadline a naive agent achieves a more desirable allocation of his effort, which in turn leads to a higher level of total effort under deadlines. Hence, with the same reasoning as above, a deadline also makes a naive agent better off.

Proposition 5 *When facing a deadline, a naive agent chooses a higher effort level in the first period and a higher total effort level than under a regime without a*

[29]That restrictions on the choice set may help to reduce procrastination is also shown by O'Donoghue and Rabin (2001).

deadline, i.e., $e_1^{NND} < e^{ND}$ *and* $e_1^{NND} + e_2^{NND} < 2e^{ND}$. *Moreover, from a long-run perspective, being subject to a deadline makes a naive agent strictly better off, i.e.,* $U_0^{ND} > U_0^{NND}$.

One question is immediately at hand: Which type of hyperbolic discounter benefits more from being exposed to an interim deadline? As it turns out, under a deadline sophisticates and naifs choose the same allocation of effort, that is, $e^{SD} = e^{ND}$.[30] Thus, with long-run utility being the same for both types of hyperbolic discounters when facing a deadline, we just have to compare long-run utilities when there are no deadlines in order to answer the question of interest. With effort being evenly distributed over tasks no matter what, the situation without an interim deadline is comparable to the single-task case. Hence, from our earlier findings we know that in general it is undetermided which type of hyperbolic discounter benefits more from being exposed to deadlines. When $c'''(\cdot) \leq 0$ and $g'''(\cdot) \leq 0$, however, a naive agent will benefit at least as much from the imposition of a deadline as a sophisticated agent.

Discussion We have shown so far that simple deadlines can help people with self-control problems to improve their performance. The reason is that being exposed to deadlines allows people to allocate their effort more efficiently, which in turn leads to a higher amount of total effort and an overall better performance. Our findings are highly in line with the empirical observations of Ariely and Wertenbroch (2002). They demonstrate the value and effectiveness of deadlines for improving task performance in two different studies both conducted at MIT. In one study participants were "native English speakers [who were given the task to] proofread papers of other students to evaluate writing skills". Participants were randomly assigned to one of three conditions: evenly-spaced deadlines, end-deadline, or self-imposed deadlines.[31] In each condition a participant had to read three texts and payment was contingent on the quality of the proofreading with a penalty for each day of delay.[32] The number of errors correctly detected was highest in the evenly-spaced-deadlines condition, followed by the self-imposed-deadlines condition, with the lowest performance in the end-deadline condition. Moreover, particpants were asked to estimate how much time they had spent on each of the three texts. Participants in the evenly-spaced-deadlines condition spent the highest amount of time on each text, followed by the participants of the self-imposed-deadlines condition, while

[30] This result, which is an artefact of our model where the agent faces as many deadlines and tasks as periods, is formally established in the proof of Proposition 5.

[31] While the evenly-spaced deadlines condition is comparable to our deadline regime, our regime of no deadlines corresponds to the end-deadline condition.

[32] By setting their deadlines as late as possible, the participants would have the most time to work on the texts and the highest flexibility in arranging their workload.

participants of the end-deadline condition have invested the least amount of time. Ariely and Wertenbroch (2002) summarize these observations as follows: "[T]he results show that when deadline constraints increased, performace improved [and] time spend on the task increased" (p.223). These observations are predicted by our theoretical analysis of agents with self-control problems: a deadline increases total effort, which in turn improves performance. In the other study professionals participating in an executive-education course at MIT had the task to write three short papers. Participants were randomly assigned to one of two treatments: no-choice or free-choice. In the no-choice treatment deadlines were fixed and evenly spaced, in the free-choice treatment participants were free to choose the deadlines. In both treatments deadlines were binding and there was a penalty for late submission.[33] The main finding is that the grade in the no-choice treatment is significantly higher than the grade in the free-choice treatment. This observation also is in line with the theoretical results obtained in this paper.

The focus of the latter study is on self-imposed deadlines and inefficiencies arising due to suboptimal spacing of these deadlines. Even though we do not endogenize the timing of deadlines, our model also captures this result - in a highly stylized way. Let ΔU_0^S denote the long-run utility gain of a sophisticated agent from being exposed to a deadline. Formally, $\Delta U_0^S \equiv U_0^{S^D} - U_0^{S^{ND}}$. Analogously define $\Delta U_1^S \equiv U_1^{S^D} - U_1^{S^{ND}}$ to be the utility gain of a sophisticated agent from being exposed to a deadline as perceived from the beginning of the first period. Correctly predicting his future behavior, a sophisticate will always welcome being subject to a deadline in (fictitious) period zero. When asked in period 1, however, a sophisticate is not very enthusiastic about facing a deadline. Formally, $\Delta U_1^S < 0 < \Delta U_0^S$.[34] In period zero, a naive agent considers a deadline neither helpful nor harmful, that is, $\Delta U_0^N = 0$. In period 1, on the other hand, a naive agent considers a deadline an undesirable restriction. Formally we have $\Delta U_1^N < 0$. Thus, while both types of time-inconsistent agents may be willing to accept a deadline long before the task is to be performed, this will not be the case when the task is immediately at hand. Hence, when interpreting "suboptimal spacing of tasks" as not setting deadlines at all, asking present-biased agents too late whether they are willing to accept deadlines or to voluntarily impose deadlines on themselves may lead to agents rejecting this opportunity. Moreover, this finding illustrates what O'Donoghue and Rabin (2005) point out to be general principles when considering "incentives and present bias". Present-biased individuals

[33] Besides giving the students the most time to work on the papers and the highest flexibility in arranging their workload, by setting their deadlines as late as possible they would also have the opportunity to learn the most about the topic before submitting the papers. Students also had an incentive to set submission dates late because the penalty would be applied only to late submissions and not to early ones. Finally, students who wanted to submit assignments early could privately plan to do so without precommitting to the instructor.

[34] This result is readily established by a simple revealed-preference argument.

are sensitive to exactly how decisions are made - e.g. choosing in advance vs. in the moment. When all consequences of a decision are sufficiently far in the future, however, present bias is not a problem and it may be possible to induce better behavior when people are given the opportunity to make decisions now about future behavior.

6 Conclusion

Empirical evidence suggests that people have self-control problems, in particular a tendency to procrastinate unpleasant tasks. Former research has shown that this procrastinative behavior can be explained by (quasi-)hyperbolic discounting. The focus of this paper is not on procrastination itself, but on the effects of (quasi-) hyperbolic discounting and awareness of the arising self-control problems on performance. We present a simple model in which an agent has two periods to work on a specific task. His performance depends on the total effort invested. We find that self-control problems reduce performance. Moreover, sophistication about one's own self-control problems not necessarily leads to better performance than naiveté.

In a next step, in a slightly augmented version of the basic model, we analyze the value and effectiveness of interim deadlines as commitment device. In line with recent empirical evidence we find that interim deadlines improve performance when individuals impulsively procrastinate. This improvement of performance, which makes a present-biased agent better off from a welfare point of view, is based on a more favorable allocation of effort. The restrictions imposed by deadlines help an agent to better structure his workload, which in turn leads to lower effort costs and an overall higher effort level. These results are of interest not only because they provide a theoretical underpinning of recent empirical work, but also because they explain many types of deadlines encountered in daily life. To get back to one of the examples that we have mentioned so far: Deadlines implemented by the "good-standing" rules of graduate schools make grad students work focused on each of their papers, finishig a paper thoroughly before starting another one, thereby improving chances to write high-quality papers. Without these deadlines, grad students cannot commit themselves to work in their last year in school exclusively on the their final paper. Instead, they possibly will end up spending effort on - perhaps unfinished - older papers, resulting in a bunch of low-quality papers that are finished in a hurry and written sloppy.

The model of this paper is simple in the sense that we consider the shortest possible time horizon that actually generates quasi-hyperbolic discounting effects. Without imposing further assumptions on cost and reward functions, analyzing a longer time horizon in a continuous action space framework, in particular the analysis of the behavior of sophisticated individuals, becomes very complicated very quickly. In the literature the arising complications are sidestepped by assuming instantaneous utility functions of the CRRA type. Facing the trade off between the analysis of a longer time horizon on the one hand, and less restrictive functional assumptions on the other hand, we opted for the latter. We think, however, that the main insights are to be obtained in our model. Moreover, we refrain from considering partial naiveté. The behavior of a partially naive person will be somewhere between the two extremes that we have analyzed, sophistication and naiveté. With both extreme types of hyperbolic discounters, naifs and sophisticates, benefiting from the presence of interim deadlines, we feel that this result should carry over to the case of partially naive individuals.

A Appendix

Proof of Proposition 1: As mentioned in Footnote 12, in order to establish the proposition, we follow a different but nevertheless equivalent way than proposed in the paper. In period 1, a naive agent believes that he is time-consistent in period 2. Thus, we first analyze what effort a TC chooses in period 2, given an arbitrary effort level of the first period, $\hat{e}_1$. This effort choice, which maximizes $U_2^{TC} = -c(e_2) + g(\hat{e}_1 + e_2)$, obviously is a function of the first-period effort. Thus, $e_2^{TC}(\hat{e}_1)$ is characterized by the corresponding first-order condition,

$$g'(\hat{e}_1 + e_2^{TC}(\hat{e}_1)) = c'(e_2^{TC}(\hat{e}_1)) \ . \tag{A.1}$$

Differentiating (A.1) with respect to e_1 yields $de_2^{TC}(e_1)/de_1 \in (-1, 0)$. With $U_1^N = -c(e_1) - \beta c(e_2^{TC}(e_1)) + \beta g(e_1 + e_2^{TC}(e_1))$ being a strictly concave function of e_1, the effort level that a naive agent invests in the first period, e_1^N, is implicitly characterized by the following first-order condition:

$$\beta g'\left(e_1^N + e_2^{TC}(e_1^N)\right) = c'(e_1^N) \ . \tag{A.2}$$

The actual problem of a naive agent in period 2 is to maximize $U_2^N = -c(e_2) + \beta g(e_1^N + e_2)$ over his second-period effort choice. The optimal second-period effort, e_2^N, satisfies

$$\beta g'\left(e_1^N + e_2^N\right) = c'(e_2^N) \ . \tag{A.3}$$

Comparison of (A.1)-(A.3) allows to establish the proposition. We prove each part of the proposition in turn.

(iii) Comparison of (A.1) and (A.3) immediately yields $e_2^N < e_2^{TC}(e_1^N) = e_2^{TC}$.

(i) Suppose, in contradiction, that $e_1^N \geq e_2^N$. Then $c'(e_1^N) \geq c'(e_2^N)$, which in turn implies $\beta g'(e_1^N + e_2^{TC}(e_1^N)) \geq \beta g'(e_1^N + e_2^N)$. But since $e_2^N < e_2^{TC}(e_1^N)$ and $g''(\cdot) < 0$ we have $\beta g'(e_1^N + e_2^{TC}(e_1^N)) < \beta g'(e_1^N + e_2^N)$, a contradiction.

(ii) From our considerations of the TC we know that $g'(\hat{e}_1 + e_2^{TC}(\hat{e}_1)) = c'(e_2^{TC}(\hat{e}_1))$ for all $\hat{e}_1$. Hence, $c'(e^{TC}) = c'(e_2^{TC}(e^{TC})) = g'(e^{TC} + e_2^{TC}(e^{TC})) > \beta g'(e^{TC} + e_2^{TC}(e^{TC}))$. For e_1^N we must have $c'(e_1^N) = \beta g'(e_1^N + e_2^{TC}(e_1^N))$. Since $de_2^{TC}/de_1 \in (-1, 0)$, $g''(\cdot) < 0$ and $c''(\cdot) > 0$, we immediately obtain that $e_1^N < e^{TC}$. Now it immediately follows that $e_1^N + e_2^N < e_1^N + e_2^{TC}(e_1^N) < e^{TC} + e_2^{TC}(e^{TC}) = 2e^{TC}$, where the first inequality holds by (i) and the second inequality holds because $e_1^N < e^{TC}$ and $de_2^{TC}(e_1)/de_1 \in (-1, 0)$.

This concludes the proof. ■

Proof of Proposition 2: First we prove that the effort choice in the first period of a sophisticated agent is characterized by a first-order condition. We can rule out corner solutions to be optimal: With $c(e) \longrightarrow \infty$ as $e \longrightarrow \infty$, $e_1 = \infty$ is not a candidate for the agent's first-period effort. Next we show that $e_1 = 0$ also is not optimal. The derivative of U_1^S with respect to e_1 can be rewritten as follows:

$$\frac{dU_1^S}{de_1} = \left[\frac{de_2^S(e_1)}{de_1}(1-\beta) + 1\right] c'(e_2^S(e_1)) - c'(e_1),$$

where we used twice the fact that $\beta g'\left(e_1 + e_2^S(e_1)\right) = c'(e_2^S(e_1))$. Since $e_2^S(0) > 0$ and $de_2^S(e_1)/de_1 \in (-1, 0)$, we have $dU_1^S/de_1|_{e_1=0} > 0$. Note that U_1^S is a differentiable and hence continuous function, which establishes the desired result.

Next, we prove each part of the proposition in turn.

(i) From (5) and (6) it follows immediately that $\beta g'\left(e_1^S + e_2^S(e_1^S)\right) - c'(e_1^S) > 0$, which in turn implies that $c'(e_2^S(e_1^S)) = \beta g'(e_1^S + e_2^S(e_1^S)) > c'(e_1^S)$. Thus, $e_2^S(e_1^S) > e_1^S$.

(ii) Suppose, in contradiction, that $e_1^S + e_2^S(e_1^S) \geq 2e^{TC}$. We know that $\beta g'(e_1^S + e_2^S(e_1^S)) - c'(e_1^S) > 0 = g'(e^{TC} + e^{TC}) - c'(e^{TC})$. With $g''(\cdot) < 0$ and $c''(\cdot) > 0$, $e_1^S + e_2^S(e_1^S) \geq 2e^{TC}$ immediately implies $e_1^S < e^{TC}$. Furthermore, $\beta g'(e_1^S + e_2^S(e_1^S)) - c'(e_2^S(e_1^S)) = 0 = g'(e^{TC} + e^{TC}) - c'(e^{TC})$, which under the above functional assumptions implies that $c'(e_2^S(e_1^S)) < c'(e^{TC})$. But this means that $e_2^S(e_1^S) < e^{TC}$, which leads to a contradiction to the assumption that $e_1^S + e_2^S(e_1^S) \geq 2e^{TC}$.

This concludes the proof. ■

Proof of Lemma 1: For a given first-period effort e_1, both the naive agent and the sophisticated agent face the same maximization problem in period 2. This allows us to write $e_2^N = e_2^S(e_1^N)$. For $i, j \in \{S, N\}$ and $i \neq j$, together with $de_2^S(e_1)/de_1 \in (-1, 0)$, this observation immediately yields that $e_1^i > e_1^j$ implies $e_2^i = e_2^S(e_1^i) < e_2^S(e_1^j) = e_2^j$ and $e_1^i + e_2^i > e_1^j + e_2^j$. It remains to show that $e_1^i > e_1^j$ implies $U_0^i = -c(e_1^i) - c(e_2^S(e_1^i)) + g(e_1^i + e_2^S(e_1^i)) \geq -c(e_1^j) - c(e_2^S(e_1^j)) + g(e_1^j + e_2^S(e_1^j)) = U_0^j$. Define $H(e_1) \equiv -c(e_1) - c(e_2^S(e_1)) + g(e_1 + e_2^S(e_1))$. In order to establish the desired result, it suffices to show that

$$\frac{dH(e_1)}{de_1} = g'(e_1 + e_2^S(e_1)) - c'(e_1) + \frac{de_2^S(e_1)}{de_1}\left[g'(e_1 + e_2^S(e_1)) - c'(e_2^S(e_1))\right] > 0$$

for all $e_1 \in [0, e_1^i]$. Since, by Propositions 1 and 2, $e_1^i < e_2^i = e_2^S(e_1^i)$ for $i \in \{S, N\}$, and moreover $de_2^S(e_1)/de_1 < 0$, we have $e_1 < e_2^S(e_1)$ for all $e_1 < e_1^i$. This in turn implies $g'(e_1 + e_2^S(e_1)) - c'(e_1) > g'(e_1 + e_2^S(e_1)) - c'(e_2^S(e_1)) > 0$, where the last inequality follows from (5). Together with $de_2^S(e_1)/de_1 \in (-1, 0)$, the desired result follows. ■

Proof of Lemma 2: By the revealed preference argument, for the first-period effort choices of a naive and a sophisticated agent, e_1^N and e_1^S, the following two inequalities have to hold:

$$\begin{aligned} &- c(e_1^N) - \beta c(e_2^{TC}(e_1^N)) + \beta g(e_1^N + e_2^{TC}(e_1^N)) \\ &\qquad\qquad \geq -c(e_1^S) - \beta c(e_2^{TC}(e_1^S)) + \beta g(e_1^S + e_2^{TC}(e_1^S)) \end{aligned}$$

and

$$\begin{aligned} &- c(e_1^S) - \beta c(e_2^S(e_1^S)) + \beta g(e_1^S + e_2^S(e_1^S)) \\ &\qquad\qquad \geq -c(e_1^N) - \beta c(e_2^S(e_1^N)) + \beta g(e_1^N + e_2^S(e_1^N)) \end{aligned}$$

Taken together these two inequalities imply

$$\begin{aligned} &\left[g(e_1^N + e_2^{TC}(e_1^N)) - c(e_2^{TC}(e_1^N))\right] - \left[g(e_1^N + e_2^S(e_1^N)) - c(e_2^S(e_1^N))\right] \\ &\quad \geq \left[g(e_1^S + e_2^{TC}(e_1^S)) - c(e_2^{TC}(e_1^S))\right] - \left[g(e_1^S + e_2^S(e_1^S)) - c(e_2^S(e_1^S))\right] \,. \end{aligned} \tag{A.4}$$

Define $F(e_1) \equiv \left[g(e_1 + e_2^{TC}(e_1)) - c(e_2^{TC}(e_1))\right] - \left[g(e_1 + e_2^S(e_1)) - c(e_2^S(e_1))\right]$. Since both sides of (A.4) have the same structure, a sufficient condition for $e_1^S \geq e_1^N$ to hold is $dF(e_1)/de_1 < 0$. From (A.1) and (5) we know that $g'(e_1 + e_2^{TC}(e_1)) = c'(e_2^{TC}(e_1))$ and $\beta g'(e_1 + e_2^S(e_1)) = c'(e_2^S(e_1))$. Hence,

$$\frac{dF(e_1)}{de_1} = \left[g'(e_1 + e_2^{TC}(e_1)) - g'(e_1 + e_2^S(e_1))\right] - (1 - \beta) g'(e_1 + e_2^S(e_1)) \frac{de_2^S(e_1)}{de_1} \,. \tag{A.5}$$

For $\beta = 0$ we have $dF(e_1)/de_1 = \left[g'(e_1 + e_2^{TC}(e_1)) - g'(e_1 + e_2^S(e_1))\right] < 0$ since $de_2^S(e_1)/de_1 = 0$ in this case. For $\beta = 1$ we have $e_2^{TC}(e_1) = e_2^S(e_1)$ for all e_1, and hence $dF(e_1)/de_1 = 0$. Thus, $\frac{d}{d\beta}(dF(e_1)/de_1) > 0$ is a sufficient condition for $dF(e_1)/de_1 < 0$ for all $\beta \in (0,1)$. Tackling this derivative by brute force yields

$$\begin{aligned}\frac{d}{d\beta}\left[\frac{dF(e_1)}{de_1}\right] &= -g''(\cdot)\frac{de_2^S}{d\beta} \\ &\quad - \left[-g'(\cdot)\frac{de_2^S}{de_1} + (1-\beta)g''(\cdot)\frac{de_2^S}{d\beta}\frac{de_2^S}{de_1} + (1-\beta)g'(\cdot)\frac{d(de_2^S/de_1)}{d\beta}\right] \\ &= (1-\beta)\frac{-2g'(\cdot)g''(\cdot)c''(e_2^S) + \frac{de_2^S}{d\beta}\beta g'(\cdot)\left[g''(\cdot)c'''(e_2^S) - g'''(\cdot)c''(e_2^S)\right]}{\left[\beta g''(\cdot) - c''(e_2^S)\right]^2},\end{aligned}$$

where we made use of the fact that

$$\frac{de_2^S}{d\beta} = -\frac{g'(\cdot)}{\beta g''(\cdot) - c''(e_2^S)}.$$

and

$$\frac{d\{de_2^S/de_1\}}{d\beta} = \frac{c''(e_2^S)[g''(\cdot) + \beta g'''(\cdot)\{de_2^S/d\beta\}] - \beta g''(\cdot)c'''(e_2^S)}{[\beta g''(\cdot) - c''(e_2^S)]^2}.$$

Under the imposed functional assumptions, a sufficient condition for $\frac{d}{d\beta}(dF(e_1)/de_1) > 0$ for all $\beta \in (0,1)$ is $c'''(\cdot) \leq 0$ and $g'''(\cdot) \leq 0$. Together with the above observation that $dF(e_1)/de_1 < 0$ for $\beta = 0$ and $dF(e_1)/de_1 = 0$ for $\beta = 1$, this implies that $dF(e_1)/de_1 < 0$ for all $\beta \in [0,1)$. This allows us to conclude that $e_1^N \leq e_1^S$ when $c'''(\cdot) \leq 0$ and $g'''(\cdot) \leq 0$.

Next, we will show that $e_1^N \neq e_1^S$ for $c'''(\cdot) \leq 0$ and $g'''(\cdot) \leq 0$, which completes the proof. Suppose in contradiction that $e_1^N = e_1^S$. The first-order condition of the utility maximization problem of the first-period sohisticate can be written as follows:

$$\begin{aligned}&\beta g'(e_1^S + e_2^{TC}(e_1^S)) - c'(e_1^S) + \beta\left[g'(e_1^S + e_2^S(e_1^S)) - g'(e_1^S + e_2^{TC}(e_1^S))\right] \\ &\qquad + \frac{de_2^S(e_1)}{de_1}\beta\left[g'(e_1^S + e_2^S(e_1^S)) - c'(e_2^S(e_1^S))\right] = 0.\end{aligned}$$

Setting $e_1^N = e_1^S$ in the above equation yields

$$\left[g'(e_1^N + e_2^S(e_1^N)) - g'(e_1^N + e_2^{TC}(e_1^N))\right] - (1-\beta)g'(e_1^N + e_2^S(e_1^N))\frac{de_2^S(e_1)}{de_1} = 0. \quad \text{(A.6)}$$

Note that the left-hand side of (A.6) is $dF(e_1)/de_1|_{e_1=e_1^N}$. For $c'''(\cdot) \leq 0$ and $g'''(\cdot) \leq 0$, however, we have just shown that $dF(e_1)/de_1 < 0$ for $\beta \in [0,1)$, a contradiction. ■

Proof of Proposition 3: Follows immediately from Lemmas 1 and 2. ■

Proof of Proposition 4: The proof consists of three major parts. First, we formally derive the behavior of a sophisticated agent when facing no deadline. Next we show that when facing a deadline, the utility maximization problem of a sophisticated agent in the first period indeed is solved by a first-period effort pair (e_A, e_{B1}) with $e_A > 0$ and $e_{B1} = 0$. Last, we prove each of the results explicitely stated in the proposition.

PART 1: Consider a sophisticated agent who faces no deadline. In period 2, this agent maximizes his intertemporal utility by choosing e_{A2} and e_{B2}. Let α_2 denote the fraction of the effort chosen in period 2 that this agent invests in task A, that is, $e_{A2} = \alpha_2 e_2$ and $e_{B2} = (1-\alpha_2)e_2$. For given effort levels $\hat{e}_{A1}$ and $\hat{e}_{B1}$ from the first period, the sophisticate's utility in period 2 is

$$U_2^{S^{ND}} = -c(e_2) + \beta g(\hat{e}_{A1} + \alpha_2 e_2) + \beta g(\hat{e}_{B1} + (1-\alpha_2)e_2) \; .$$

The corresponding first-order condition with respect to e_2 yields the usual condition that in the optimum marginal cost equals marginal reward:

$$\frac{\partial U_2^{S^{ND}}}{\partial e_2} \overset{!}{=} 0 \iff c'(e_2) \overset{!}{=} \beta g'(\hat{e}_{A1} + \alpha_2 e_2)\alpha_2 + \beta g'(\hat{e}_{B1} + (1-\alpha_2)e_2)(1-\alpha_2). \quad \text{(A.7)}$$

The first-order condition with respect to α_2, ignoring the constraint that $\alpha_2 \in [0,1]$ for the moment, implies that the agent equates efforts over tasks:

$$\frac{\partial U_2^{S^{ND}}}{\partial \alpha_2} \overset{!}{=} 0 \iff \hat{e}_{A1} + \alpha_2 e_2 \overset{!}{=} \hat{e}_{B1} + (1-\alpha_2)e_2. \quad \text{(A.8)}$$

Solving (A.8) for the optimal allocation of the second-period effort over tasks, α_2^S, reveals

$$\alpha_2^S = \frac{1}{2} + \frac{\hat{e}_{B1} - \hat{e}_{A1}}{2e_2}.$$

Note that $\alpha_2^S \in [0,1]$ for $|\hat{e}_{B1} - \hat{e}_{A1}| \leq e_2^S$. Combining (A.7) and (A.8), and plugging in α_2^S yields that the second-period effort as a function of the first-period effort, $e_2^{S^{ND}}(\hat{e}_1)$, satisfies

$$c'(e_2^{S^{ND}}(\hat{e}_1)) = \beta g'((1/2)(\hat{e}_1 + e_2^{S^{ND}}(\hat{e}_1))). \quad (8)$$

Differentiation of (8) yields

$$\frac{de_2^{S^{ND}}(e_1)}{de_1} = -\frac{\frac{1}{2}\beta g''(\frac{1}{2}(e_1 + e_2^{S^{ND}}(e_1)))}{\frac{1}{2}\beta g''(\frac{1}{2}(e_1 + e_2^{S^{ND}}(e_1))) - c''(e_2^{S^{ND}}(e_1))} \in (-1, 0).$$

In period 1 a sophisticated agent then chooses his effort level in order to maximize the intertemporal utility of his first-period self,

$$U_1^{S^{ND}} = -c(e_1) - \beta c(e_2^{S^{ND}}(e_1)) + 2\beta g((1/2)(e_1 + e_2^{S^{ND}}(e_1))) \; .$$

According to the same reasoning as in the single-task case, the optimal first-period effort is characterized by the following first-order condition:

$$\beta g'((1/2)(e_1 + e_2^{SND}(e_1))) - c'(e_1) + \frac{de_2^{SND}(e_1)}{de_1}\beta\left[g'((1/2)(e_1 + e_2^{SND}(e_1))) - c'(e_2^{SND}(e_1))\right] \stackrel{!}{=} 0\,. \quad (9)$$

From (8) we know that $\beta g'(\frac{1}{2}(\hat{e}_1 + e_2^{SND}(\hat{e}_1))) - c'(e_2^{SND}(\hat{e}_1)) = 0$ for all $\hat{e}_1$, and in particular for $\hat{e}_1 = e_1^{SND}$. Since $de_2^{SND}(e_1)/de_1 < 0$, in combination with (9) this implies that $\beta g'(\frac{1}{2}(e_1^{SND} + e_2^{SND}(e_1^{SND})) - c'(e_1^{SND}) > 0$. Taken together these two observations yield $c'(e_2^{SND}(e_1^{SND})) = \beta g'(\frac{1}{2}(e_1^{SND} + e_2^{SND}(e_1^{SND}))) > c'(e_1^{SND})$. Since $c''(\cdot) > 0$, it follows that when facing no deadline, a sophisticated agent increases effort over time, that is, $e_1^{SND} < e_2^{SND}(e_1^{SND})$. Note that with $e_1^{SND} < e_2^{SND}(e_1^{SND})$ we have $\alpha_2^S \in [0, 1]$.

Part 2: Next, we provide the proof that when facing a deadline, the utility maximization problem of a sophisticated agent in the first period is solved by a first-period effort pair (e_A, e_{B1}) with $e_A > 0$ and $e_{B1} = 0$. To prove this result, we proceed in three steps. First, we show that we cannot have an interior solution $(e_A, e_{B1}) \gg (0, 0)$ with $e_A < \infty$ and $e_{B1} < \infty$. Second, we rule out solutions in which the agent chooses an infinite amount of effort for at least one task, and also the solution that the agent does not exhibit any effort at all in the first period. Third, we show that an effort pair (e_A, e_{B1}) with $e_{B1} > 0 = e_A$ is not a solution.

Step 1: Suppose, in contradiction, that there is an interior solution. This solution then would be characterized by the following first-order conditions:

$$\frac{\partial U_1^{SD}}{\partial e_A} = \beta g'(e_A) - c'(e_A + e_{B1}) \stackrel{!}{=} 0 \quad (A.9)$$

$$\frac{\partial U_1^{SD}}{\partial e_{B2}} = \beta g'(e_{B1} + e_{B2}^{SD}(e_{B1})) - c'(e_A + e_{B1}) + \frac{de_{B2}^{SD}(e_{B1})}{de_{B1}}\beta\left[g'(e_{B1} + e_{B2}^{SD}(e_{B1})) - c'(e_{B2}^{SD}(e_{B1}))\right] \stackrel{!}{=} 0 \quad (A.10)$$

Combining (A.9) and (A.10) yields

$$\beta g'(e_{B1} + e_{B2}^{SD}(e_{B1})) - \beta g'(e_A) = -\frac{de_{B2}^{SD}(e_{B1})}{de_{B1}}\beta\left[g'(e_{B1} + e_{B2}^{SD}(e_{B1})) - c'(e_{B2}^{SD}(e_{B1}))\right] > 0,$$

where the inequality follows from (10). This last inequality implies that $e_{B1} + e_{B2}^{SD}(e_{B1}) < e_A$. From (A.9) it follows that e_A decreases as e_{B1} increases. Comparing (10) and (A.9) yields that for $e_{B1} = 0$ we have $e_A = e_{B2}^{SD}(0)$. Since $d(e_{B1} +$

$e_{B2}^{SD}(e_{B1}))/de_{B1} > 0$ it follows that $e_{B1} + e_{B2}^{SD}(e_{B1}) \geq e_A$ for all $e_{B1} \geq 0$, a contradiction. Hence, the utility maximization problem of a sophisticated agent in the first period cannot have an interior solution.

Step 2: Obviously we can rule out effort choices where the agent invests an infinite high effort in one or both tasks since this would lead to an intertemporal utility of minus infinity. To see that it is not optimal to exert no positive effort at all in the first period, let $\alpha_1 \in [0,1]$ denote the fraction of e_1 which is dedicated to task B, that is, $e_{A1} = (1-\alpha_1)e_1$ and $e_{B1} = \alpha_1 e_1$. For each α_1, by (10) the optimal second-period effort satisfies $\beta g'(\alpha_1 e_1 + e_{B2}^{SD}(\alpha_1 e_1)) = c'(e_{B2}^{SD}(\alpha_1 e_1))$. With this notation, the intertemporal utility in the first period is given by $U_1^{SD} = -c(e_1) - \beta c(e_{B2}^{SD}(\alpha_1 e_1)) + \beta g((1-\alpha_1)e_1) + \beta g(\alpha_1 e_1 + e_{B2}^{SD}(\alpha_1 e_1))$. Differentiating with respect to e_1, taking into account that $\beta g'(\alpha_1 e_1 + e_{B2}^{SD}(\alpha_1 e_1)) = c'(e_{B2}^{SD}(\alpha_1 e_1))$, and rearranging yields

$$\frac{dU_1^{SD}}{de_1} = \beta(1-\alpha_1)g'((1-\alpha_1)e_1) - c'(e_1) + \alpha_1 c'(e_{B2}^{SD}(\alpha_1 e_1))\left[1 + (1-\beta)\frac{de_{B2}^{SD}(e_{B1})}{de_{B1}}\right].$$

Evaluated at $e_1 = 0$ we have $dU_1^{SD}/de_1|_{e_1=0} = \beta(1-\alpha_1)g'(0) + \alpha_1 c'(e_{B2}^{SD}(0))[1 + (1-\beta)(de_{B2}^{SD}(e_{B1})/de_{B1})] > 0$, for all $\alpha_1 \in [0,1]$.

Step 3: We are left with two possible candidates for the corner solution: (i) $e_A = 0$ and $e_{B1} > 0$, or (ii) $e_A > 0$ and $e_{B1} = 0$. To see that (i) can be ruled out, suppose that $e_A = 0$ and $e_{B1} > 0$. For $e_A = 0$ to be optimal it must hold that

$$\beta g'(0) - c'(e_{B1}) \leq 0, \tag{A.11}$$

otherwise it would be optimal to invest some positive effort in task A. Since e_{B1} is assumed to be strictly positive, the following first-order condition has to hold:

$$\beta g'(e_{B1} + e_{B2}^{SD}(e_{B1})) - c'(e_{B1}) + \frac{de_{B2}^{SD}(e_{B1})}{e_{B1}}\beta\left[g'(e_{B1} + e_{B2}^{SD}(e_{B1})) - c'(e_{B2}^{SD}(e_{B1}))\right] = 0.$$

The last term of the left-hand side of the above equation is negative, which implies that $\beta g'(e_{B1} + e_{B2}^{SD}(e_{B1})) - c'(e_{B1}) > 0$. Taken together with (A.11) this yields $\beta g'(e_{B1} + e_{B2}^{SD}(e_{B1})) > g'(0)$. This in turn implies $e_{B1} + e_{B2}^{SD}(e_{B1}) < 0$, which is not possible. This establishes the desired result.

Part 3: Having shown that an effort pair (e_A, e_{B1}) with $e_A > 0$ and $e_{B1} = 0$ solves the utility maximization problem of a sophisticated agent in the first period, we now prove each part of the proposition. First we show that a sophisticate exhibits a higher first-period effort when facing deadlines. Suppose, in contradiction, that $e_1^{SND} \geq e^{SD}$. From (8) and (12) we know, respectively, that $c'(e_2^{SND}(e_1^{SND})) = \beta g'(\frac{1}{2}(e_1^{SND} + e_2^{SND}(e_1^{SND})))$ and $c'(e^{SD}) = \beta g'(\frac{1}{2}(e^{SD} + e^{SD}))$. Since $de_2^{SND}(e_1^{SND})/de_1 \in (-1,0)$, $e_1^{SND} \geq e^{SD}$ implies that $e_2^{SND}(e_1^{SND}) \leq e^{SD}$, which in turn in implies $e_1^{SND} +$

$e_2^{SND}(e_1^{SND}) \geq 2e^{SD}$. From (9), however, we know that $\beta g'(\frac{1}{2}(e_1^{SND} + e_2^{SND}(e_1^{SND}))) - c'(e_1^{SND}) > 0$. Together with (12) this implies that $\beta g'(\frac{1}{2}(e_1^{SND} + e_2^{SND}(e_1^{SND}))) - \beta g'(\frac{1}{2}(e^{SD} + e^{SD})) > c'(e_1^{SND}) - c'(e^{SD}) \geq 0$, where the last inequality holds by our initial assumption that $e_1^{SND} \geq e^{SD}$. With $g'(\cdot)$ strictly decreasing, this last expression implies $e_1^{SND} + e_2^{SND} < 2e^{SD}$, a contradiction. Therefore we must have $e_1^{SND} < e^{SD}$. Together with $e_2^{SND}(e^{SD}) = e^{SD}$, which follows from (8) in combination with (11) or (12), and $de_2^{SND}(e_1)/de_1 \in (-1, 0)$, $e_1^{SND} < e^{SD}$ immediately implies $e_1^{SND} + e_2^{SND}(e_1^{SND}) < 2e^{SD}$. It remains to show that a sophisticate indeed is better off under a deadline from a long-run perspective, i.e., $U_0^{SD} > U_0^{SND}$. Let α and γ denote the allocation of some level of total effort e^{Total} over tasks and time, respectively. Since time-consistent agents and sophisticated agents, both under a deadline and under no deadline, divide effort evenly among tasks, fix $\alpha = \frac{1}{2}$. Long-run utility then is given by $U_0(e^{\text{Total}}, \gamma) = -c(\gamma e^{\text{Total}}) - c((1-\gamma)e^{\text{Total}}) + 2g(\frac{1}{2}e^{\text{Total}})$. Fixing $\gamma = \frac{1}{2}$, it is readily verified that $U_0(e^{\text{Total}}, \frac{1}{2})$ is a strictly concave function of e^{Total} which obtains its maximum for $e^{\text{Total}} = 2e^{TC}$. Hence, with $e_1^{SND} + e_2^{SND}(e_1^{SND}) < 2e^{SD} < 2e^{TCND}$ we have $U_0^{SD} = U_0(2e^{SD}, \frac{1}{2}) > U_0(e_1^{SND} + e_2^{SND}(e_1^{SND}), \frac{1}{2})$. Next, fixing an arbitrary level of total effort $e^{\text{Total}} > 0$, $U_0(e^{\text{Total}}, \gamma)$ is a strictly concave function with its maximum obtained at $\gamma = \frac{1}{2}$. Hence, $U_0^{SND} < U_0(e_1^{SND} + e_2^{SND}(e_1^{SND}), \frac{1}{2})$, which establishes the desired result. ■

Proof of Proposition 5: First consider a naive agent who faces no deadline. Since he predicts his own future behavior to be time-consistent, his first-period belief about his second-period choice of effort is determined by the following maximization problem:

$$\max_{e_2, \alpha \in [0,1]} \; -c(e_2) + g(\hat{e}_{A1} + \alpha e_2) + g(\hat{e}_{B1} + (1-\alpha)e_2).$$

As before, α denotes the fraction of e_2 invested in task A. The corresponding first-order condition with respect to α, ignoring the constraint that $\alpha \in [0, 1]$ for the moment, is given by

$$g'(\hat{e}_{A1} + \alpha e_2)e_2 + g'(\hat{e}_{B1} + (1-\alpha)e_2)(-e_2) \stackrel{!}{=} 0. \tag{A.12}$$

From (A.12) it follows immediately that the naive agent plans to distribute effort evenly over tasks, $e_A = \hat{e}_{A1} + \alpha e_2 = \hat{e}_{B1} + (1-\alpha)e_2 = e_B$. Solving for the optimal allocation of the second-period effort over tasks, α^{TC}, yields

$$\alpha^{TC} = \frac{1}{2} + \frac{\hat{e}_{B1} - \hat{e}_{A1}}{2e_2}.$$

The first-order condition with respect to e_2 is given by

$$-c'(e_2) + \alpha g'(\hat{e}_{A1} + \alpha e_2) + (1-\alpha)g'(\hat{e}_{B1} + (1-\alpha)e_2) \stackrel{!}{=} 0.$$

Taking into account that effort will be split evenly among tasks, this first-order condition can be rewritten as follows

$$g'((1/2)(\hat{e}_1 + e_2^{TC}(\hat{e}_1))) = c'(e_2^{TC}(\hat{e}_1)). \tag{A.13}$$

Let $e_2^{TC} = e_2^{TC}(\hat{e}_1)$. Moreover, note that $\alpha^{TC} \in [0,1]$ for $|\hat{e}_{B1} - \hat{e}_{A1}| \leq e_2^{TC}$.

In the first period a naive agent chooses his effort level in order to maximize his intertemporal utility,

$$U_1^{NND} = -c(e_1) - \beta c(e_2^{TC}(e_1)) + 2\beta g((1/2)(e_1 + e_2^{TC}(e_1))) \; .$$

The optimal first-period effort choice, e_1^{NND}, is characterized by

$$\beta g'((1/2)(e_1^{NND} + e_2^{TC}(e_1^{NND}))) = c'(e_1^{NND}). \tag{A.14}$$

The second-period utility of a naive agent takes the following form:

$$U_2^{NND} = -c(e_2) + \beta g(e_{A1}^{NND} + \alpha e_2) + \beta g(e_{B1}^{NND} + (1-\alpha)e_2) \; ,$$

where $e_1^{NND} = e_{A1}^{NND} + e_{B1}^{NND}$. Ignoring the constraint that $\alpha \in [0,1]$ for the moment, the first-order condition with respect to α,

$$\beta g'(e_{A1}^{NND} + \alpha e_2)e_2 + \beta g'(e_{B1}^{NND} + (1-\alpha)e_2)(-e_2) \overset{!}{=} 0,$$

again implies that effort is split evenly among tasks, $e_A = e_{A1}^{NND} + \alpha e_2 = e_{B1}^{NND} + (1-\alpha)e_2 = e_B$. Solving for the optimal effort allocation over tasks in the second period, α^N, yields

$$\alpha^N = \frac{1}{2} + \frac{e_1^{NND} - e_2^{NND}}{2e_2}.$$

Knowing this, the first-order condition with respect to e_2 can be rewritten as

$$\beta g'((1/2)(e_1^{NND} + e_2^{NND}(e_1^{NND}))) = c'(e_2^{NND}(e_1^{NND})), \tag{A.15}$$

where $e_2^{NND}(e_1^{NND})$ denotes the optimal total effort choice of a naive agent in the second period when there are no deadlines. Let, $e_2^{NND} = e_2^{NND}(e_1^{NND})$. Note that $\alpha^N \in [0,1]$ for $|e_{B1}^{NND} - e_{A1}^{NND}| \leq e_2^{NND}$. Since, by comparison of (A.13) and (A.15), for any $\hat{e}_1$ we have $e_2^{TC}(\hat{e}_1) > e_2^{NND}(\hat{e}_1)$, comparing (A.14) and (A.15) reveals that $e_1^{NND} < e_2^{NND}$, thus α^{TC} and α^N are feasible solutions.

Next, consider the case where a naive agent faces a deadline, formally, $e_{A1} = e_A$, $e_{A2} = 0$, and $e_{B2} = e_2$. The utility of a naive agent in the first period is given by

$$U_1^{ND} = -c(e_A + e_{B1}) - \beta c(e_2^{TC}(e_{B1})) + \beta g(e_A) + \beta g(e_{B1} + e_2^{TC}(e_{B1})) \; ,$$

where $e_2^{TC}(e_{B1})$ is characterized by

$$g'(e_{B1} + e_2^{TC}(e_{B1})) = c'(e_2^{TC}(e_{B1})). \tag{A.16}$$

The first-order conditions of utility maximization take the following form:

$$\frac{\partial U_1^{ND}}{\partial e_A} = \beta g'(e_A) - c'(e_A + e_{B1}) \stackrel{!}{=} 0 \tag{A.17}$$

$$\frac{\partial U_1^{ND}}{\partial e_{B1}} = \beta g'(e_{B1} + e_2^{TC}(e_{B1})) - c'(e_A + e_{B1}) \stackrel{!}{=} 0 \tag{A.18}$$

If the above maximization problem has interior solutions, $e_A > 0$ and $e_{B1} > 0$, then these solutions are characterized by (A.17) and (A.18). When both first-order conditions are satisfied, we have $g'(e_A) = g'(e_{B1} + e_2^{TC}(e_{B1}))$, that is, at an interior solution we must have $e_A = e_{B1} + e_2^{TC}(e_{B1})$. By (A.17), however, it is immediate that e_A is decreasing in e_{B1}. Moreover, comparing (A.16) and (A.17) reveals that $e_2^{TC}(e_{B1}) > e_A$ for $e_{B1} = 0$. Together with $de_2^{TC}(e_{B1})/de_{B1} \in (-1, 0)$, these last two observations imply that $e_A < e_{B1} + e_2^{TC}(e_{B1})$ for all $e_{B1} \geq 0$, a contradiction. Hence, the naive agent's first-period utility maximization problem has a corner solution. Similar reasoning as in the case of the sophisticate allows us to restrict attention to the following two candidates for this corner solution: (i) $e_A^{ND} > 0$ and $e_{B1}^{ND} = 0$ or (ii) $e_A^{ND} = 0$ and $e_{B1}^{ND} > 0$. For (ii) to be the solution to the naive agent's first-period problem, the following conditions have to hold:

$$\beta g'(0) - c'(e_{B1}^{ND}) \leq 0 \tag{A.19}$$

$$\beta g'(e_{B1}^{ND} + e_{B2}^{TC}(e_{B1}^{ND})) - c'(e_{B1}^{ND}) = 0 \tag{A.20}$$

Obviously, for (A.19) and (A.20) to hold simultaneously it is required that $e_{B1}^{ND} + e_2^{TC}(e_{B1}^{ND}) \leq 0$, which can never be the case. Therefore we are left with $e_A^{ND} > 0$ and $e_{B1}^{ND} = 0$, that is, in the first period the agents invests only in task A. This first-period effort is characterized by

$$\beta g'(e_A^{ND}) = c'(e_A^{ND}). \tag{A.21}$$

The second-period utility of a naive agent under a regime with a deadline takes the following form:

$$U_2^{ND} = -c(e_{B2}) + \beta g(e^{ND}) + \beta g(e_{B2}) \ .$$

The optimal second-period effort then satisfies

$$\beta g'(e_{B2}^{ND}) = c'(e_{B2}^{ND}). \tag{A.22}$$

Comparing (A.21) and (A.22) yields $e_A^{ND} = e_{B2}^{ND}$, that is, when facing a deadline a naive agent equates effort over tasks and smoothes effort over time. Let the effort level that is chosen by a naive agent under a regime of deadlines per period and per task be denoted by e^{ND}.

To show that a naive agent chooses a higher effort level in the first period when facing a deadline, suppose, in contradiction, that $e_1^{NND} \geq e^{ND}$. Then $\beta g'(e^{ND}) = c'(e^{ND}) \leq c'(e_1^{NND}) = \beta g'(\frac{1}{2}(e_1^{NND} + e_2^{TC}(e_1^{NND}))$, where the first equality holds by (A.21) and the second equality holds by (A.14). But with $g''(\cdot) < 0$, this implies $e^{ND} \geq \frac{1}{2}(e_1^{NND} + e_2^{TC}(e_1^{NND})) > e_1^{NND}$, a contradiction. Hence we must have $e_1^{NND} < e^{ND}$. Together with $e_2^{NND}(e^{ND}) = e^{ND}$, which follows from (A.15) in combination with (A.21) or (A.22), and $de_2^{NND}(e_1)/de_1 \in (-1, 0)$, $e_1^{NND} < e^{ND}$ immediately implies $e_1^{NND} + e_2^{NND} < 2e^{ND}$. That is, when facing a deadline, a naive agent exhibits a higher total effort level than under regime without a deadline.

To see that $U_0^{ND} > U_0^{NND}$ the same reasoning applies as in the case of the sophisticate. For a formal argument we refer to the proof of Proposition 4. Intuitively, under deadlines a naive chooses a more desirable total effort level than under no deadlines, which moreover is allocated more efficiently over the two periods. ■

References

[1] **Akerlof, G.A. (1991):** Procrastination and Obedience, *American Economic Review*, Vol. 81, 1-19.

[2] **Akin, Z. (forthcoming):** Time Inconsistency and Learning in Bargaining Games, *International Journal of Game Theory.*

[3] **Ariely, D.** and **K. Wertenbroch (2002):** Procrastination, Deadlines, and Performance: Self-Control by Precommitment, *Psychological Science*, Vol. 13, 219-224.

[4] **Bénabou, R.** and **J. Tirole (2002):** Self-Confidence and Personal Motivation, *Quarterly Journal of Economics*, Vol. 117, 871-915.

[5] **Carrillo, J.D.**, and **T. Mariotti (2000):** Strategic Ignorance as Self-Disciplining Device, *Review of Economic Studies*, Vol. 67, 529-544.

[6] **DellaVigna, S.** and **U. Malmendier (2004):** Contract Design and Self-Control: Theory and Evidence, *Quartely Journal of Economics*, Vol. 119, 353-402.

[7] **DellaVigna, S.** and **U. Malmendier (2006):** Paying Not to Go to the Gym, *American Economic Review*, Vol. 96, 694-719.

[8] **DellaVigna, S.** and **M.D. Paserman (2005):** Job Search and Impatience, *Journal of Labor Economics*, Vol. 23, 527-588.

[9] **Diamond, P.** and **B. Kőszegi (2003):** Quasi-Hyperbolic Discounting and Retirement, *Journal of Public Economics*, 87, 1839-1872.

[10] **Fischer, C. (1999):** Read this Paper Even Later: Procrastination with Time-Inconsistent Preferences, *Resources for the Future discussion paper 99-20.*

[11] **Fischer, C. (2001):** Read this Paper Later: Procrastination with Time-Consistent Preferences, *Journal of Economic Behavior and Organization*, 46, 249-269 .

[12] **Frederick, S.**, **G. Loewenstein** and **T. O'Donoghue (2002):** Time Discounting and Time Preference, *Journal of Economic Literature*, Vol. 40, 351-401.

[13] **Gruber, J.** and **B. Kőszegi (2001):** Is Addiction Rational? Theory and Evidence, *Quarterly Journal of Economics*, Vol. 116, 1261-1303.

[14] **Laibson, D. (1996):** Hyperbolic Discount Functions, Undersaving, and Savings Policy, *NBER Working Paper Series*, No.5635, Cambridge, MA.

[15] **Laibson, D. (1997):** Golden Eggs and Hyperbolic Discounting, *Quarterly Journal of Economics*, Vol. 112, 443-477.

[16] **Nocke, V.** and **M. Peitz (2003):** Hyperbolic Discounting and Secondary Markets, *Games and Economic Behavior*, Vol. 44, 77-97.

[17] **O'Donoghue, T.** and **M. Rabin (1999a):** Addiction and Self-Control, in *Addiction: Entry and Exits*, J. Elster, editor, Russel Sage Foundation.

[18] **O'Donoghue, T.** and **M. Rabin (1999b):** Doing It Now Or Later, *American Economic Review*, Vol. 89, 103-124.

[19] **O'Donoghue, T.** and **M. Rabin (1999c):** Incentives for Procrastinators, *Quarterly Journal of Economics*, Vol. 114, 769-816.

[20] **O'Donoghue, T.** and **M. Rabin (2001a):** Addiction and Present-Biased Preferences, *working paper*, University of California, Berkeley.

[21] **O'Donoghue, T.** and **M. Rabin (2001b):** Choice and Procrastination, *Quarterly Journal of Economics*, Vol. 116, 121-160.

[22] **O'Donoghue, T.** and **M. Rabin (2005):** Incentives and Self-Control, *working paper*, University of California, Berkeley.

[23] **O'Donoghue, T.** and **M. Rabin (2007):** Procrastination on Long-Term Projects, *Journal of Economic Behavior and Organization*, doi: 10.1016/j.jebo.2006.05.005.

[24] **Phelps, E.S.** and **R.A. Pollak (1968):** On Second-Best National Saving and Game-Equilibrium Growth, *Review of Economic Studies*, Vol. 35, 185-199.

[25] **Pollak, R.A. (1968):** Consistent Planning, *Review of Economic Studies*, Vol. 35, 201-208.

[26] **Strotz, R.H. (1956):** Myopia and Inconsistency in Dynamic Utility Maximization, *Review of Economic Studies*, Vol. 23, 156-180.

[27] **Wong, W.-K. (2006):** How Much Time-Inconsistency Is There and Does It Matter? Evidence on Self-Awareness, Size, and Effects, *working paper*, National University of Singapore.