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by

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Product Pricing when Demand Follows a Rule of Thumb*

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We analyze the strategic behavior of firms when demand is determined by a rule of thumb behavior of consumers. We assume consumer dynamics where individual consumers follow simple behavioral decision rules governed by imitation and habit as suggested in consumer research. On this basis, we investigate monopoly and competition between firms, described via an open-loop differential game which in this setting is equivalent to but analytically more convenient than a closed-loop system. We derive a Nash equilibrium and examine the influence of advertising. We show for the monopoly case that a reduction of the space of all price paths in time to the space of time-constant prices is sensible since the latter in general contains Nash equilibria. We prove that the equilibrium price of the weakest active firm tends to marginal cost as the number of (non-identical) firms grows. Our model is consistent with observed market behavior such as product life cycles.

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1. Introduction

In many situations, strategic pricing represents a difficult task for firms, especially if the consumers are not known to strictly follow a given demand function. In reality, however, consumers behave boundedly rationally as observed in numerous psychological and experimental investigations (cf. Conlisk 1996) and appreciated in some areas of industrial organization (cf. Ellison 2006). In the present work, we examine a model which describes how firms shall optimally, i. e. strategically, set their prices or advertising levels when confronted with habitual imitative consumers. Such consumers either imitate popular product choices or form a habit and

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repeatedly purchase the same product. This rule of thumb behavior is psychologically supported (Assael 1984) and acknowledged in the economic literature (e. g. Stigler and Becker 1977, Schlag 1998).

The demand side of a market with habitual imitative consumers has been examined in a companion paper (Matzke and Wirth 2008). There we have shown—using imitation and habit as the only model ingredients—under which conditions a product is feasible (i. e. has sufficient demand to survive lastingly on the market), which types of sales curves can be generated, and how the imitative and habitual parameters influence feasibility and sales evolution. The corresponding model has been stated in form of a population game as defined in Sandholm (2005), using the fact that for a large population size the stochastic process generated by the evolutionary process can be approximated by solutions to ordinary differential equations (Benaïm and Weibull 2003).

For a complete market description, the demand side is here complemented with a supply side model. The demand side consists of the continuous-time consumer population game from Matzke and Wirth (2008). The game of the supply side describes the strategic behavior of firms, which anticipate the consumers' behavior and thus the demand dynamics. With such a description at hand, we can then transfer the concept of welfare into this framework, examine how advertising might influence the results, and show that the model is consistent with observed market patterns such as product life cycles.

As motivated in Matzke and Wirth (2008), in a consumer–seller relationship the strategic variables (e. g. price, quality, output) are determined by the firms. Any influencing parameter on the demand side (such as personal preference or reservation price) is fixed, i. e. exogenous in a market model. Hence, due to their reactive role, consumers naturally do not compete actively with each other or with firms, which prohibits a conventional game theoretic demand side model. In contrast, for the firms a strategic behavior of determining the degrees of freedom can be devised using game theoretic approaches. In particular, we apply a differential game in order to model the firms' behavior.

This paper aims at providing an insight into the strategic response of firms when confronted with demand dynamics generated by the imitative and habitual consumer behavior. The consumer model is in the spirit of Smallwood and Conlisk (1979) as well as von Thadden (1992) in that the consumers are unable or not willing to act strategically and thus act adaptively. Our rational supply side approach differs in methodology. We employ a normal form game in which firms choose a price path at the beginning of the game and gain an according profit. (This could easily be extended to repeated choices of price paths.) A more classic normal form game, which describes a single discrete time step, would not exploit the full richness of the model since the time-continuous demand side calls for at least a continuous-time price path. At the other extreme, a steady price adjustment at all points in time is rather unrealistic since a firm can hardly react continuously without time lag (though, nonetheless, the proposed model will be able to capture even this case). By letting the firms fix a price path at the beginning of the game (e. g. for a certain time period, which is represented by the duration of the game), we strike a balance between both extremes.

We employ a differential game, i. e. a time-continuous game where the state variables (here the consumer subpopulation sizes) follow first-order ordinary differential equations. More specifically, the chosen product prices (control variables) determine the rate of change in consumer subpopulation sizes (state variables), where consumers are grouped into subpopulations according to the product they own. Technically, there is no effect in the opposite direction, which renders the dynamics an open-loop system. However, in a deterministic setting we have equivalence of open and closed loops. This is convenient since closed loops are considered—in contrast to static open loops—as being genuinely strategic since they comprise a feedback in which the control variables are affected by the state variables.

In this context, the following result turns out to be very interesting: The firms' action space generally contains all possible price paths and thus is very complex. However, for a monopoly we will show that a Nash equilibrium often lies in the reduced space of time-constant prices. This justifies that most of the time we return to time-constant price paths and steady state analysis. Nevertheless, we additionally analyze some exemplary cases with general price paths. Moreover, we will see that markets with imitative and habitual consumers behave naturally in that e. g. an increasing number of firms enhances competition and reduces prices. However, perfect competition is generally only achieved in symmetric markets. Finally, advertising is shown to be an effective method to sustain demand, and a welfare definition is suggested.

1.1. Further motivation and related literature

Boundedly rational consumer behavior—as advocated by Ellison (2006), Conlisk (1996), and many others—is often observed in consumer research, in particular imitation of group behavior or habitual purchase (cf. Assael 1984, p. 371ff, 53). Just to mention some exemplary laboratory experiments, Venkatesan (1966) shows that consumers generally conform to group norms. Pingle and Day (1996) summarize experiments which show that boundedly rational behavior such as imitation and habit (which they call "economic choices in reality") represents means in order to get well-performing economic choices in presence of decision costs. Our focus here lies on markets with boundedly rational consumers that follow habitual imitative decision rules as introduced in Matzke and Wirth (2008).

Closely related to the demand dynamic employed here is the model by Smallwood and Conlisk (1979). They consider consumers who buy the same product each period until a breakdown occurs. Then, they choose another product depending on its market share. It is examined how strongly the consumers should rely on product popularities. Despite having been published in 1979 already, there is still social learning literature building on this model, for example Ellison and Fudenberg (1995).

Imitative behavior in general constitutes a well-known and frequently used concept in evolutionary game theory and social learning, compare for instance Schlag (1998), Ellison and Fudenberg (1993), and Banerjee (1992), just to name a few notable papers. Habit, on the other hand, occurs in the habit formation literature (Heaton 1993) as well as implicitly in some industrial organization models (for instance in Smallwood and Conlisk 1979, where habit is implicitly formed as long as no breakdown occurs). Habit may also be interpreted as a special case of learning, since agents learn from past experience (Sobel 2000, p. 257) and positive experience with a good may cause habitual purchase behavior.

Firms are usually more rational than consumers can or aim to be. The reason lies in the large number of agents and equipment that are employed in order to avoid costly wrong decisions. The approximation of rational firms seems reasonable, even though some early work in the field of bounded rationality assumes the opposite, i. e. boundedly rational firms (e.g. Rothschild 1947, Cyert and March 1956). However, in line with most of the recent literature, we restrict bounded rationality to the consumers and to assume fully rational firms (Ellison 2006, p. 4). We will stick to this convention, i. e. our firms aim at maximizing their profits given the demand side and the pricing strategies of the competitors which is modeled via a differential game as introduced by Isaacs (1954). In combination with the two previously mentioned simple rule of thumb ingredients, imitation and habit by consumers, the model will be able to generate typical patterns observed in consumer markets such as product life cycles (de Kluyver 1977, Brockhoff 1967, Polli and Cook 1969).

The outline of this paper is as follows. Section 2 recapitulates the consumer model, defining the consumers' behavioral rules and deducing the resulting demand dynamics. Subsequently, section 3 introduces the competition game played by the supply side. The game is first applied to exemplary monopoly or oligopoly settings, after which more general results and conclusions are drawn for the monopoly and oligopoly case. In section 4, an adequate welfare definition is provided, and a possible generation of product life cycles is described. Additionally, a model extension by advertising is suggested. Finally, we conclude in section 5.

2. Model for boundedly rational demand side

In a preceding article, we introduced an approach to model boundedly rational consumers and analyzed the resulting demand dynamics (Matzke and Wirth 2008). Before we model the firms' strategic response to this demand side, let us briefly recapitulate the consumer model. For details we refer to Matzke and Wirth (2008).

2.1. Methodology

The methodology applied builds upon the work of Sandholm (2006) and consists mainly of a population game with a particular choice of a conditional switch rate.

Each consumer owns at most one unit of n possible products. A consumer owning good i is equipped with an independent Poisson alarm clock of rate R_i, i. e. an alarm clock which rings after an exponentially distributed time with expected value R_i^{-1}. Each time the alarm rings (which is associated with a broken product), the consumer switches to product j with switching probability $p_{ij} = \frac{\rho_{ij}}{R_i}$. If the consumer does not own a product yet, the alarm clock signalizes an arising interest in buying a good. Typically, the frequency R_0 of revisions without any good is larger than the rate R_i of possible replacements of good i, since the goods usually survive longer than the consumers without any good are satisfied.

Let us denote $\rho_{ij}(x,t)$ the conditional switch rate from product i to product j at time t and state $x = (x_1, \dots, x_n)$ (where x_i denotes the market share of consumers owning product i). Obviously, $R_i = \sum_{j=0}^{n} \rho_{ij}(x,t)$ (where subscript 0 stands for consumers without any good).

The switching rates and probabilities of course depend on the product prices, and exactly this dependence will later form the instrument via which firms exert an influence on consumer behavior during their competition. However, the price dependence is not necessary to understand the dynamics of the demand side. Hence, the reader may implicitly understand all parameters to depend on the product prices, but this dependence will not be introduced explicitly until the treatment of the supply side.

The previous definitions characterize a population game with all potential consumers as players (Matzke and Wirth 2008). This game uniquely determines a mean dynamic which describes the temporal change of market shares,

$$\dot{x}_i = \sum_{j=0}^{n} x_j \rho_{ji}(x,t) - x_i \sum_{j=0}^{n} \rho_{ij}(x,t), \quad x_i(0) = x_i^0, \quad i = 0, \dots, n. \tag{1}$$

We define the sales of product i, $S_i(t)$, as the number of units of product $i \in \{1, \dots, n\}$ sold at time t,

$$S_i(t) \quad = \quad N \sum_{j=0}^{n} x_j(t) \rho_{ji}(x(t),t), \quad i = 0, \dots, n, \tag{2}$$

where N denotes the number of possible consumers. Using mean dynamic (1), we obtain a relation between sales and consumer subpopulations,

$$\dot{x}_i + x_i R_i = \sum_{j=0}^{n} x_j \rho_{ji} \quad \Leftrightarrow \quad \frac{S_i}{N} = \dot{x}_i + x_i R_i, \quad i = 0, \dots, n. \tag{3}$$

2.2. Consumer dynamics

Now only the switching probabilities $p_{ij}(x,t) = \frac{\rho_{ij}(x,t)}{R_i}$ remain to be specified. Of those people who do not own any product, the fraction of consumers deciding to buy product i is described by p_{0i}. Consumers' choices are sensitive to market shares or popularities of the products (Smallwood and Conlisk 1979): When consumers passively encounter a product, its level of familiarity rises, thus increasing the possibility for this product to be bought. Consumers may also actively imitate others in buying the same good since the popularity of a product might give information about the product's past performance (Ellison and Fudenberg 1993). As discussed in Matzke and Wirth (2008) a linear relation

$$p_{0i} = \varphi_i x_i, \quad i \neq 0, \tag{4}$$

seems to be a good modeling approach. $\varphi_i \in [0,1]$ generally differs from product to product (Assael 1984, p. 432, 414) and can even be time dependent. It constitutes the accumulated influence of product frequency on the consumers' purchase decision via different mechanisms and can be interpreted as an anticipated product quality. Of course, φ_i depends on the good's properties as there are the price, the (expected) quality, the strength of networking and fashion effects for that product etc.

Let us now turn to those people owning product i. Someone who is content with that good tends to buy a new unit of the same good, even though a better product might exist. Assael (1984, p. 53) summarizes several studies on the topic and comes to the conclusion that a form of habit evolves, leading to repeat purchases of a product without further information search or evaluating brand alternatives. Hence we assume a fixed, product-specific percentage of consumers to develop a buying habit so that

$$p_{ii} = s_i \in [0,1], \quad i \neq 0. \tag{5}$$

The fraction of switching consumers $(1 - p_{ii})$ divides up into the fractions p_{ij} of people switching to product $j \neq i$. They behave just like those consumers not yet owning any good, i. e.

$$p_{ij} = (1 - p_{ii})p_{0j} = (1 - s_i)\varphi_j x_j, \quad i \neq 0 \wedge j \neq 0, i. \tag{6}$$

The switching probabilities p_{i0} and p_{00} are now uniquely determined by the constraints $\sum_{j=0}^n p_{ij} = 1$ and $\sum_{j=0}^n p_{0j} = 1$.

For $i = 1, \ldots, n$, the mean dynamic (1) eventually takes the form

$$\begin{aligned} \dot{x}_i &= x_i\left(\varphi_i R_0 - (1-s_i)R_i - \varphi_i \sum_{\substack{j=1\\ j\neq i}}^n \left[R_0 - (1-s_j)R_j\right] x_j - \varphi_i R_0 x_i\right) \\ &= \varphi_i R_0 x_i\left(\Psi_i - x_i - \sum_{\substack{j=1\\ j\neq i}}^n \Phi_j x_j\right), \end{aligned} \tag{7}$$

where $\Psi_i := 1 - \frac{R_i}{R_0}\frac{1-s_i}{\varphi_i}$ and $\Phi_i := 1 - \frac{R_i}{R_0}(1-s_i)$ stand for "quality" and "habit induction" of product i.

All constants R_i, φ_i and s_i may in principle (and will later) be time-dependent so that product modifications or fashion trends can be modeled.

Population games with the switching probabilities as defined above will represent the demand side of our market model. Let us hence define:

Definition 2.1 (Habitual imitative consumers)**.** *Agents who behave according to the above model with switching probabilities* (4) *to* (6) *are called* habitual imitative consumers*. A population game with such agents is called the demand side of a* market with habitual imitative consumers*.*

3. Strategic pricing in a monopoly & oligopoly

After having repeated the framework of the demand side model, let us now examine markets in which firms anticipate the consumers' actions (indeed, companies do try to predict consumer behavior) and set their prices accordingly.

Naturally, the imitation factor φ_i and habit coefficient s_i depend on the good prices, i. e. $\varphi_i = \varphi_i(\xi_1(t), \dots, \xi_n(t))$, $s_i = s_i(\xi_1(t), \dots, \xi_n(t))$, $i = 1, \dots, n$, where the price of good j at time t is denoted $\xi_j(t)$. To keep things simple while staying sufficiently realistic, we shall assume φ_i and s_i to depend on ξ_i only. The consumers see the prices of all goods, and the probability to buy product i (encoded by φ_i and s_i) rises with falling price ξ_i. They behave like many small iron particles which are attracted by different magnets, representing the products. The strength of a magnet relative to its competitors determines the eventual amount of trapped particles, which illustrates the mechanism of competition. Competing firms will seek a compromise between large margins and sufficiently low prices to attract consumers more strongly than their competitors (via high φ_i and s_i).

Recall that the imitation function has the following interpretation: A consumer owning no good or switching product imitates the population of consumers owning good i with probability $\varphi_i(\xi_i)$. Equivalently, the fraction $\varphi_i(\xi_i)$ of the whole population would purchase good i at a price of ξ_i. Obviously, $\varphi_i(\xi_i)$ represents the normalized demand function of product i, or in probabilistic terms, $\varphi_i(\xi_i)$ is the demand distribution of product i. Hence, let us agree upon the following

Condition 3.1. *In a market with habitual imitative consumers, let* $\varphi_i(\vec{\xi})$ *and* $s_i(\vec{\xi})$ *denote the imitation and habit coefficient for product* i*, depending on the vector* $\vec{\xi}=(\xi_1,\dots,\xi_n)$ *of product prices. Then* φ_i *and* s_i *are monotonously decreasing in* ξ_i*.*

In the following we will abbreviate vectors of scalars according to $(\sigma_i)_{i=1,\dots,m} = \vec{\sigma}$. We are now able to describe a normal form competition game of the firms.

Definition 3.2 (Normal form competition game)**.** *The* normal form competition game *in a market with habitual imitative consumers is a normal form game* $G = (n, \mathfrak{S}, \Pi)$ *with*

- *the number of agents* n *being the number of firms, where each firm produces one product and the products are understood to be characterized by the functions* $s_i(\xi_1,\dots,\xi_n)$ *and* $\varphi_i(\xi_1,\dots,\xi_n)$*,*
- *the set* $\mathfrak{S}$ *of all possible strategy combinations with* $\mathfrak{S} \subseteq [\mathbb{R}_+^{\mathbb{R}_+}]^n = \mathbb{R}_+^{\mathbb{R}_+} \times \dots \times \mathbb{R}_+^{\mathbb{R}_+}$ *(a subset of the space of* n*-tuples over maps* $\xi_i : \mathbb{R}_+ \to \mathbb{R}_+$*,* $t \mapsto \xi_i(t)$*, where* $\xi_i(t)$ *denotes the price of good* i *at time* t*),*
- *the utility function* $\Pi : \mathfrak{S} \to \mathbb{R}_+^n$*,* $\Pi_i(\xi_1,\dots,\xi_n) = F\left[(\xi_i(t) - c_i) S_i\left(\vec{s}(\vec{\xi}(t)), \vec{\varphi}(\vec{\xi}(t)), \vec{x}(t)\right)\right]$*, being the firms' profit, where there are no fixed costs,* c_i *denotes the (time-independent) marginal cost of production for good* i*, and* $\vec{S}(\vec{s}(\vec{\xi}(t)), \vec{\varphi}(\vec{\xi}(t)), \vec{x}(t)) \equiv \vec{S}(t, \vec{x}(0))$ *is the sales vector (cf. (2)) belonging to the population game (7) with* ρ *defined by (4) to (6). The operator* $F : \mathbb{R}^{\mathbb{R}_+} \to \mathbb{R}_+$*, which assigns a non-negative real number to each map* $f : \mathbb{R}_+ \to \mathbb{R}$*, may assume different forms.*

The imitation and habit function $\vec{\varphi}(\vec{\xi})$ and $\vec{s}(\vec{\xi})$ may in general be time dependent. The operator F can e. g. constitute the *cumulated discounted profit* over a certain time period $[0, T]$,

$$F_T[\pi(t)] = \int_0^T \exp[-rt]\pi(t)\mathrm{d}t,$$

where $\pi(t)$ represents the firm's profit at time t. For an infinite time horizon this is extended to

$$F_\infty[\pi(t)] = \int_0^\infty \exp[-rt]\pi(t)\mathrm{d}t.$$

For a zero discount rate r, the latter definition is not well-defined. In this case we resort to the *long-term profit rate*,

$$F_\partial[\pi(t)] = \lim_{T\to\infty} \frac{1}{T}\int_0^T \pi(t)\mathrm{d}t = \lim_{t\to\infty} \pi(t),$$

where the last expression only holds for time-invariant prices in the steady state. In this case (which is for example of interest when the firms would like to validate their prices in an equilibrated market), we will also denote F_∂ as the *steady state profit.*

Note that 3.2 in conjunction with (7) defines a differential game as in Isaacs (1954).

Definition 3.2 allows for a time dependent price. The firms set their price paths initially and then strictly follow these. The lack of opportunities for price path revisions implies no disadvantage for the firms, since the consumer model is deterministic and consumer behavior thus predictable. Therefore, even if the firms would be able to change their prices during the game, they would not do so unless the market conditions were changed by an external event. (Such price path revisions could readily be modeled by extending the normal form competition game to a repeated game.) Put differently, assume the optimal pricing strategies to be functions of the state also, i. e. $\vec{\xi} = \vec{\xi}(\vec{x}, t)$, which would correspond to the closed-loop case. Then, the corresponding optimal path in state space, $\vec{x}(t)$, could be computed by solving the mean dynamic (7). Choosing $\hat{\xi}(t) = \vec{\xi}(\vec{x}(t), t)$, we obtain the optimal open-loop control path, which is exactly equivalent to the corresponding closed-loop path. (Note that this would be different for models with non-Markovian strategies and positive time lags between a firm's action and the others' reaction, in which case trigger strategies, i. e. punishment after path deviation of a competitor, might exist, cf. Dockner, Jørgensen, Van Long, and Sorger (2001).)

Obviously, firms do not at each time maximize their current profit, but choose their price paths in order to obtain an optimal overall profit in the long run.

3.1. Monopoly

As a first illustrative example, let us consider the simplest case possible, the monopoly with fixed prices (as in equilibrated or price-restricted markets). The monopoly strategy space thus reduces to the space $\mathfrak{S} = \mathbb{R}_+$ of time-constant price functions $\xi_1(t) = \xi_1$. For given $\varphi_1(\xi_1)$, $s_1(\xi_1)$ the Nash equilibrium with steady state profit now yields an optimum *monopoly price* (corresponding *oligopoly prices* are addressed in the next section).

Example 3.1 (Steady state monopoly price)**.** *Consider the normal form competition game*

$$G = (1, \mathbb{R}_+, F_\partial[\pi(t)]) = \left(1, \mathbb{R}_+, \lim_{t\to\infty} (\xi_1 - c_1) S_1(s_1, \varphi_1(\xi_1), t)\right),$$

with $\mathfrak{S} = \mathbb{R}_+$ representing the set of all constant price functions in $\mathbb{R}_+$. Let us assume a constant habit function s_1 and the generic piecewise affine imitation function $\varphi_1(\xi_1) = \max\left(0, 1 - \frac{\xi_1}{\Xi_1}\right)$ with maximum reservation price $\Xi_1 \geq c_1$. The price in the (unique) Nash-equilibrium is the maximizer of

$$\max_{\xi_1 \in \mathbb{R}_+} \left[\lim_{t\to\infty} (\xi_1 - c_1) S_1(s_1, \varphi_1(\xi_1), t)\right],$$

where S_1 follows from equation (3) *with $x_1(t) = \frac{\Psi_1}{1+\left(\frac{\Psi_1}{x_1(0)}-1\right)\exp[-tR_0\Psi_1\varphi_1]}$ solving the ordinary differential equation* (7)*. The optimal price is given by*

$$\xi_1^* = \Xi_1 - \sqrt{\frac{R_1}{R_0}(1 - s_1)\Xi_1(\Xi_1 - c_1)} \quad \textit{if} \;\; \xi_1^* > c_1.$$

For $\xi_1^* < c_1$ the firm is recommended not to produce. The product therefore is feasible, if and only if $c_1 \leq \Xi_1 \left[1 - \frac{R_1}{R_0}(1 - s_1)\right]$.[1]

Of course, the constant s_1 in the above example is a very crude approximation (though reasonable if a certain range of prices is not exceeded), since then, obviously,

[1] This is just a different representation of the feasibility condition $\Psi_1[\xi_1 = c_1] > 0$ from Matzke and Wirth (2008).

the monopolist could gain infinite profit by suddenly charging infinite prices (which the consumers will pay due to their habit). Hence, when looking at time dependent prices, one should for example set $s_1(\xi_1) = \varphi_1(\xi_1)$.

A product is feasible if it persists in the steady state and the firm does not make any losses when selling the good. The feasibility condition from the above example naturally extends to a result for general $\varphi_1(\xi_1)$ and $s_1(\xi_1)$.

Proposition 3.1. *The single product on the market with habitual imitative consumers is feasible if and only if* $\Psi_1[\xi_1 = c_1] > 0$.

Proof. Due to condition 3.1, φ_1 and s_1 are monotonously decreasing with ξ_1. Hence, Ψ_1 is so as well. Also, Matzke and Wirth (2008) have shown that a single product on a market is feasible if and only if $\Psi_1 > 0$. Hence, if and only if $\Psi_1[\xi_1 = c_1] > 0$, there exist prices ξ_1 for which the firm makes a positive profit. □

The previous example of an equilibrated monopoly market did not exploit the possibility of time-varying prices. Yet, it might very well be that non-constant prices result in a higher profit: A widely observed pricing strategy consists in charging an elevated price most of the time with (more or less regular) intermittent special offers. This strategy probably aims at making people buy the product during the low-price period and thereby inducing a habit for the high-price period. However, for periodic price changes we will show that in a market with habitual imitative consumers a Nash equilibrium is found to lie in the space of time-constant prices, which in many cases justifies to a priori confine ourselves to steady states and constant pricing. To show this, we will proceed in steps and first prove the criticality, later the optimality of a constant price.

Proposition 3.2. *Let us consider a monopoly market with habitual imitative consumers in which the firm has a periodic price path, i. e. in each time period* $[kT, kT+T], k \in \mathbb{N}$, *the same price path* $\xi_1(t) = \xi_1(t+T) = \xi_1(t+2T) = \cdots$ *is pursued. The appropriate normal form competition game reads*

$$G = \left(1, \mathbb{R}_+^{[0,T]}, \frac{1}{T}\int_0^T (\xi_1(t) - c_1) S_1(s_1(\xi_1(t)), \varphi_1(\xi_1(t)), t)\mathrm{d}t\right),$$

assuming that a periodic state, i. e. a state with $x_1(t) = x_1(t+T) = \cdots$, *has been reached. Then, the proposed profit operator indeed yields the average profit per time. (Instead, profit* F_∂ *could equivalently be used.) Then, if the good is feasible, a constant price is a critical value for the monopoly.*

Proof. For simplicity, we abbreviate $R := \frac{R_1}{R_0}$ and skip the index 1 for all other variables. Also, we will introduce the non-dimensional time $\hat{t} = R_0 t$ and $\hat{T} = R_0 T$, where for ease of notation, the hats are dropped in the following.

Let $\xi(t)$ be periodic with period T and assume that after some equilibration time, all other system variables also behave periodically with same period.

The non-dimensionalized ordinary differential equation $\dot{x} = x\varphi(\Psi - x)$ is of Riccati type and thus readily solved for x,

$$x = \frac{\exp\left(\int_0^t \varphi\Psi \mathrm{d}\tau\right)}{\int_0^t \varphi \exp\left(\int_0^\tau \varphi\Psi \mathrm{d}\theta\right) \mathrm{d}\tau + \frac{1}{x(0)}}, \quad x(0) = \frac{\exp\left(\int_0^T \varphi\Psi \mathrm{d}\tau\right) - 1}{\int_0^T \varphi \exp\left(\int_0^\tau \varphi\Psi \mathrm{d}\theta\right) \mathrm{d}\tau},$$

where the expression for $x(0)$ follows from the periodicity condition $x(0) = x(T)$. Using $S(t)/(NR_0) = \dot{x} + Rx = x[R + \varphi(\Psi - x)]$, the normalized long-term profit rate can be expressed as

$$\frac{\Pi}{NR_0} = \frac{1}{T}\int_0^T (\xi - c)x[R + \varphi(\Psi - x)]\mathrm{d}t.$$

Finally, in appendix A.1, a lengthy sequence of non-trivial transformations proves that the Gâteaux derivative of Π with respect to ξ is zero for all test directions ϑ, if for ξ we substitute the constant price ξ^* which is implicity defined by $\xi^* - c = -\frac{\Psi(\xi^*)}{\Psi'(\xi^*)}$, where $\Psi' \equiv \frac{\mathrm{d}\Psi}{\mathrm{d}\xi}$. In other words, the Euler-Lagrange equation for Π is fulfilled for the constant price ξ^*, and hence ξ^* is critical. □

For the constant price to be a Nash equilibrium, the second variation of the long-term profit rate Π with respect to the price is required to be negative definite. To show this, we need the following lemma, whose proof is given in appendix A.2.

Lemma 3.3. *Let $H : \mathbb{R} \to \{0, 1\}$ be the Heaviside function. The following inequality holds for all $\alpha \in \mathbb{R}$ and Lebesgue-integrable functions $\vartheta : [0, T] \to \mathbb{R}$:*

$$\int_0^T \int_0^T \vartheta(t)\vartheta(\tau) \exp[\alpha(\tau - t + TH(t - \tau))]\mathrm{d}\tau\mathrm{d}t \leq \frac{\exp(\alpha T) - 1}{\alpha}\int_0^T \vartheta^2\mathrm{d}t.$$

Under fairly mild conditions on the functions φ and Ψ at the critical point ξ^* we now obtain the optimality result. We will use the same abbreviations as in the previous proof.

Proposition 3.4. *Let the conditions of proposition 3.2 hold. Moreover, assume $R(\varphi\Psi)' \geq \Psi\Psi'\varphi^2$ at the constant critical price ξ^*. Then, for a feasible good, if $2(\Psi'(\xi^*))^2 > \Psi''(\xi^*)\Psi(\xi^*)$, a constant price is a (local) optimum for the monopoly.*

Proof. Only the negative definiteness of the second variation of Π with respect to ξ remains to be shown. Indeed, using

$$\int_0^T \left\langle \frac{\partial^2 x(t)}{\partial \xi^2}, \vartheta, \vartheta \right\rangle \mathrm{d}t = -\frac{2\Psi'\varphi'\Psi}{\exp(\varphi\Psi T) - 1}\int_0^T \vartheta \exp(\varphi\Psi t)dt \int_0^T \frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t$$
$$+ \frac{(\varphi\Psi)'' - \varphi''\Psi}{\varphi}\int_0^T \vartheta^2 dt - 2\Psi'\varphi'\Psi\int_0^T \frac{\vartheta}{\exp(\varphi\Psi t)}\int_0^t \vartheta \exp(\varphi\Psi\tau)\mathrm{d}\tau\mathrm{d}t$$

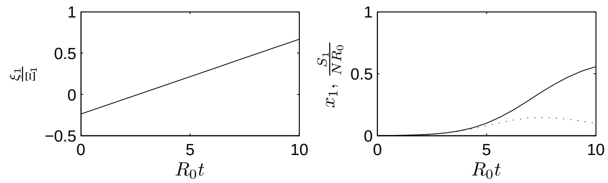

Figure 1: Optimal affine price evolution (left) as well as subpopulation (solid line) and sales (dotted line) evolution (right) for parameter values $R_1 = 0.1R_0$, $T = 10/R_0$, $r = 0$, $c_1 = 0.1/\Xi_1$, and $x_1(0) = 0.001$ (cf. example 3.2).

for any test direction ϑ, appendix A.3 derives

$$\left\langle \frac{\partial^2 \Pi / R_0}{\partial \xi^2}, \vartheta, \vartheta \right\rangle = \frac{N}{T} 2\Psi \left[\frac{R(\varphi\Psi)' - \Psi\Psi'\varphi^2}{\exp(\varphi\Psi T) - 1} \left(\exp(\varphi\Psi T) \int_0^T \vartheta \exp(\varphi\Psi\Theta) \int_\Theta^T \frac{\vartheta}{\exp(\varphi\Psi t)} \mathrm{d}t\mathrm{d}\Theta \right.\right.$$
$$\left.\left. + \int_0^T \vartheta \exp(\varphi\Psi\Theta) \int_0^\Theta \frac{\vartheta}{\exp(\varphi\Psi t)} \mathrm{d}t\mathrm{d}\Theta \right) + \left(\varphi\Psi' - R\frac{\varphi'}{\varphi} - R\frac{\Psi''}{2\Psi'} \right) \int_0^T \vartheta^2 \mathrm{d}t \right].$$

By lemma 3.3 we obtain

$$\begin{aligned} \left\langle \frac{\partial^2 \Pi / R_0}{\partial \xi^2}, \vartheta, \vartheta \right\rangle &\leq \frac{N}{T} 2\Psi \left[\frac{R(\varphi\Psi)' - \Psi\Psi'\varphi^2}{\varphi\Psi} \int_0^T \vartheta^2 \mathrm{d}t + \left(\varphi\Psi' - R\frac{\varphi'}{\varphi} - R\frac{\Psi''}{2\Psi'} \right) \int_0^T \vartheta^2 \mathrm{d}t \right] \\ &= \frac{N}{T} 2R \left(\Psi' - \frac{\Psi''\Psi}{2\Psi'} \right) \int_0^T \vartheta^2 \mathrm{d}t \end{aligned}$$

for $R(\varphi\Psi)' \geq \Psi\varphi^2\Psi'$. Hence, under the assumption $2(\Psi')^2 > \Psi''\Psi$ (and $\Psi' < 0$ and $\Psi \geq 0$ for a feasible product) we have the desired negative definiteness. □

The first condition holds for R small enough, i. e. at least for long-lasting products, the latter condition holds for instance for affine Ψ. Hence it indeed makes sense for some cases to reduce the complex price space $\mathbb{R}_+^{[0,T]}$ by focussing on time-constant prices.

For the issues dealt with so far, the general mathematical system was analytically treatable. For other questions, we have to resort to specific examples in order to obtain a qualitative insight into the characteristics of a market with habitual imitative consumers. Clearly, in such cases it is instructive to consider only very simple forms of s, φ, and especially ξ that just capture the necessary features for the discussed problem at hand. In particular, affine functions (the simplest case possible) are well-suited to study trends (e. g. price trends). The following example is meant to examine the optimal price trend for a good that is sold during a finite time period. It illustrates that proposition 3.4 does not hold for bounded time intervals.

Example 3.2 (Cumulated discounted profit in a monopoly setting)**.** *For simplicity, let us assume* $\varphi_1 = s_1 = 1 - \frac{\xi_1}{\Xi_1}$*, and let us only allow for affine price functions* $\xi_1(\cdot) \in \mathfrak{L}([0,T]) := \{f : [0,T] \to \mathbb{R} \,|\, \exists\, a, b : f(t) = a + bt\}$*. Consider the normal* *form competition game*

$$G = \left(1, \mathfrak{L}([0,T]), \int_0^T \exp[-rt](\xi_1(t) - c_1) S_1\left(s_1(\xi_1(t)), \varphi_1(\xi_1(t)), t\right) \mathrm{d}t\right).$$

For given parameters $R_1, T, r, c_1, x_1(0)$, the optimal price path $\xi_1(t)$ can be found numerically (an analytical solution turns out to be too complex to provide any insight). As a result, for a whole range of realistic parameters we obtain that the product is initially sold below marginal cost, and then the price rises. One example calculation is depicted in figure 1.

Of course, when r is chosen extremely large, this trend is reversed. However, this only happens for values of $r \sim R_0$ to $2R_0$. This would correspond to an interest rate of above 100 % within the time R_0^{-1}, i. e. if on average a consumer thinks of the good only once a year, the interest rate would have to be above 100 % per annum!

From this example, we may conclude that on a market with habitual imitative consumers a beneficial pricing strategy consists in starting at a low price and then increasing the price steadily. It might even be advantageous to initially give away products for free. The underlying idea is to initially strongly increase the market share in order to exploit habitual behavior.

3.2. Oligopoly and polygopoly

In this section we turn to oligopoly and polygopoly markets. As for the monopoly, we will begin with an introductory example and then prove a feasibility result analogous to the result for a single firm. Afterwards, we examine the firms' behavior for an increasing number of competitors.

Example 3.3 (Steady state oligopoly prices)**.** *Consider the situation of example 3.1, this time with n firms. Moreover, we now assume $\varphi_i(\xi_i) = \frac{1}{1+\frac{\xi_i}{\Xi_i}}$ (this choice renders the system analytically solvable and is an approximation to an affine $\varphi_i(\xi_i)$ for low prices). The corresponding normal form competition game reads*

$$G = \left(n, \mathbb{R}_+^n, \lim_{t\to\infty}\left((\xi_i - c_i) S_i(\vec{s}, \vec{\varphi}(\vec{\xi}), t)\right)_{i=1,\dots,n}\right).$$

If all n products are feasible, the steady state Nash equilibrium oligopoly prices ξ_i^* can be computed analytically (cf. appendix A.4),

$$\vec{\xi}^* = \begin{pmatrix} \frac{\Xi_1}{1-\Phi_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{\Xi_n}{1-\Phi_n} \end{pmatrix} \left[\Lambda + \begin{pmatrix} \Lambda_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Lambda_{nn} \end{pmatrix}\right]^{-1} \left[\Lambda \begin{pmatrix} \Phi_1 \\ \vdots \\ \Phi_n \end{pmatrix} + \begin{pmatrix} (1-\Phi_1)\Lambda_{11}\frac{c_1}{\Xi_1} \\ \vdots \\ (1-\Phi_n)\Lambda_{nn}\frac{c_n}{\Xi_n} \end{pmatrix}\right], \quad (8)$$

where Λ is the inverse of matrix A_n, defined as

$$(A_n)_{ij} := \begin{cases} 1, & j = i \\ \Phi_j, & j \neq i \end{cases} .$$

This example is one of the very few cases, where the optimal prices can indeed be calculated analytically. It is not of great importance, but serves to illustrate few general features of oligopoly markets. First of all, we observe that the reservation prices Ξ_i and the marginal costs c_i have a positive effect on ξ_i^*. The reservation price Ξ_i even acts as a kind of proportionality factor on ξ_i^* via the first diagonal matrix in equation (8). Also, due to $1 - \Phi_i = \frac{R_i}{R_0}(1 - s_i)$, the matrix entries and hence the prices tend to infinity as the habit factor s_i approaches one, the value where consumers blindly purchase habitually. A further intuitive fact consists in the shrinking significance of the marginal costs with rising reservation price. Finally, let us note that the different good parameters Φ_i, c_i, Ξ_i affect all prices ξ_j^*, $j = 1, \ldots, n$, and not just the price of that good which they describe.

Let us now turn to the feasibility of goods in an oligopoly.

Proposition 3.5. *Consider an n-product market with habitual imitative consumers on which the products i, $i = 1, \ldots, n-1$, coexist with $0 < \Phi_i < 1$. Then product n is feasible if and only if*

$$\Psi_n[\xi_n = c_n] > \vec{\tilde{x}} \cdot \Phi(\vec{\tilde{\xi}}) = \sum_{i=1}^{n-1} \Phi_i(\tilde{\xi}_i)\tilde{x}_i$$

where $\vec{\tilde{x}}$ is the vector of market shares on the $(n-1)$-goods market (i. e. without product n) in the steady state and $\vec{\tilde{\xi}}$ the corresponding price vector.

Proof. Due to condition 3.1, Ψ_n is monotonously decreasing with ξ_n. Also, Matzke and Wirth (2008) have shown an nth product to be feasible on a market with habitual imitative consumers, if and only if $\Psi_n > \vec{\tilde{x}} \cdot \vec{\Phi}$. Let us assume, $\hat{\xi}$ is such that $\Psi_n[\hat{\xi}] = \vec{\tilde{x}} \cdot \vec{\Phi}(\vec{\tilde{\xi}})$. Then, according to the result just cited, for $\xi_n = \hat{\xi}$ the good does just not exist on the market so that the n-goods market behaves like the $(n-1)$-goods market and firms 1 to $n-1$ choose the prices $\vec{\tilde{\xi}}$. If $\hat{\xi}$ is smaller than c_n, then due to condition 3.1, $\Psi_n[\hat{\xi}]$ is larger than $\Psi_n[\xi_n]$ for $\xi_n \geq c_n$ so that for a profitable price good n still does not persist on the market. If on the other hand $\hat{\xi}$ is larger than c_n, then by decreasing ξ_n a little (to which the other firms react by choosing prices slightly different from $\vec{\tilde{\xi}}$) we obtain a situation in which good n has a non-zero market share and is sold above marginal costs. □

According to Matzke and Wirth (2008), $\Psi_n[\xi_n = c_n]$ represents the hypothetic monopoly market share when the price equals the marginal costs. Hence, intuitively, the above proposition implies that this hypothetic monopoly market share has to be larger than the weighted sum of market shares of products 1 to $n-1$, where the weights $\Phi_i \leq 1$ are the larger the stronger the corresponding goods induce habit.

Next, we shall study market implications from rising numbers of competitors. To start with, let us return to example 3.3 with identical firms.

Proposition 3.6. *In example 3.3, assume a symmetric oligopoly with n identical firms, where each firm optimally chooses the same price $\xi^{*,n}$. Then, for an increasing number of firms the price $\xi^{*,n}$ decreases. In the limit $n \to \infty$, it converges to the marginal cost c.*

Proof. Due to the symmetry of the market, we can skip the indices in equation (8). Also, we readily verify $\Lambda_{ij} = \frac{-\Phi}{(1-\Phi)(1+(n-1)\Phi)}$ for $i \neq j$ and $\Lambda_{ii} = \frac{1+(n-2)\Phi}{(1-\Phi)(1+(n-1)\Phi)}$ so that equation (8) yields

$$\xi^{*,n} = \frac{\Xi\Phi + (n-1)c - (n-2)c(1-\Phi)}{(n-1)-(n-3)(1-\Phi)} \overset{n\to\infty}{\longrightarrow} c.$$

Furthermore,

$$\frac{\xi^{*,n+1}}{\xi^{*,n}} = \frac{n + \left(\frac{\Xi}{c} + \frac{1-\Phi}{\Phi}\right)}{n-1+\underbrace{\left(\frac{\Xi}{c}+\frac{1-\Phi}{\Phi}\right)}_{=:C_1>0}} \cdot \frac{n+\frac{2-3\Phi}{\Phi}}{n+1+\underbrace{\frac{2-3\Phi}{\Phi}}_{=:C_2\geq -1}} = \frac{(n+C_1)(n+C_2)}{\underbrace{(n+C_1)}_{\geq 1}\underbrace{(n+C_2)}_{\geq 0}+C_1-C_2-1}.$$

Also, since for feasible goods, $\xi^{*,n} \geq c$ and $\Psi[\xi^{*,n}] > 0$, we have

$$\begin{aligned} &0 < \Xi\left[\Psi[\xi^{*,n}]\left(1+\frac{\Xi}{c}\right)+\frac{\xi^{*,n}}{c}-1\right] = \frac{\Xi}{c}[\Xi+\xi^{*,n}+(\Psi[\xi^{*,n}]-1)\Xi]+(\Psi[\xi^{*,n}]-1)\Xi \\ &\Leftrightarrow 0 < \frac{\Xi}{c} + \frac{(\Psi[\xi^{*,n}]-1)\Xi}{\Xi+\xi^{*,n}+(\Psi[\xi^{*,n}]-1)\Xi} = C_1 - C_2 - 1, \end{aligned}$$

where in the last step we have used $1-\Phi = (1-\Psi[\xi^{*,n}])\frac{\Xi}{\Xi+\xi^{*,n}}$. Together with the above equation, this yields $\frac{\xi^{*,n+1}}{\xi^{*,n}} < 1$. □

Apparently, competition gets harder the more competitors coexist on the market. In the limit, we obtain perfect competition. This result actually holds for more general symmetric markets, which we will prove step by step. We will first show that steady state Nash equilibrium prices decrease for rising numbers n of firms. Later we will analyze the limit $n \to \infty$.

Lemma 3.7. *Consider a symmetric oligopoly with n identical firms and with habitual imitative consumers. Let $\Phi(\xi)$ and $\Psi(\xi)$ be differentiable. Then, the derivative of the steady state profit rate Π_i^n (of the ith firm in the n-goods market) with respect to the price ξ_i (of the ith product), evaluated at the steady state Nash equilibrium price $\xi^{*,n-1}$ of the $(n-1)$-goods market, is negative,*

$$\left.\frac{\partial \Pi_i^n}{\partial \xi_i}\right|_{\xi_i=\xi_j=\xi^{*,n-1}} < 0.$$

Proof. If all n products coexist on the market, mean dynamic (7) can be written as $A_n\vec{x} = \vec{\Psi}$ and the steady state profit of firm i as

$$\Pi_i^n = NR_i(\xi_i - c_i)x_i = NR_i(\xi_i - c_i)\Lambda_i\vec{\Psi},$$

with matrices A_n and Λ defined as in example 3.3 and Λ_i being the ith row of Λ. Since Ψ_j does not depend on ξ_i for $i \neq j$, we obtain

$$\frac{\partial \Pi_i^n/(NR_i)}{\partial \xi_i} = \sum_{j=1}^n \Lambda_{ij}\Psi_j + (\xi_i - c_i)\Lambda_{ii}\frac{\mathrm{d}\Psi_i}{\mathrm{d}\xi_i} + (\xi_i - c_i)\sum_{j=1}^n \Psi_j \frac{\partial \Lambda_{ij}}{\partial \xi_i}.$$

Due to the market symmetry, we may write $\Psi_i = \Psi$, $\xi_i = \xi$, and $c_i = c$. The derivative $\frac{\partial \Lambda_{ij}}{\partial \xi_i}$ at $\xi_i = \xi_j = \xi$ can equivalently be computed as $\frac{\partial \tilde{\Lambda}_{ij}}{\partial \xi_i}$, where $\tilde{\Lambda}$ is the inverse of matrix

$$(\tilde{A}_n)_{kl} = \begin{cases} 1, & l = k, \\ \Phi, & l \neq k, i, \\ \Phi_i[\xi_i], & l = i, l \neq k. \end{cases}$$

We readily verify $\tilde{\Lambda}_{ij} = \frac{-\Phi}{1+(n-2)\Phi-(n-1)\Phi\Phi_i[\xi_i]}$ for $i \neq j$ and $\tilde{\Lambda}_{ii} = \frac{1+(n-2)\Phi}{1+(n-2)\Phi-(n-1)\Phi\Phi_i[\xi_i]}$ so that

$$\frac{\partial}{\partial \xi_i}\left(\sum_{j=1}^n \Lambda_{ij}\right)\bigg|_{\Phi_i=\Phi} = \frac{\partial}{\partial \Phi_i}\left(\frac{\Phi - 1}{(n-1)\Phi_i\Phi - (n-2)\Phi - 1}\right)\bigg|_{\Phi_i=\Phi}\frac{\mathrm{d}\Phi_i}{\mathrm{d}\xi_i},$$

and $$\frac{\partial \Pi_i^n/(NR_i)}{\partial \xi_i} = \frac{\Psi}{1+(n-1)\Phi}\left[1 + \frac{\xi - c}{1-\Phi}\left(\frac{1+(n-2)\Phi}{\Psi}\frac{\mathrm{d}\Psi}{\mathrm{d}\xi} + \frac{(n-1)\Phi}{1+(n-1)\Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}\xi}\right)\right].$$

We would like to show that this is negative at $\xi_i = \xi_j = \xi^{*,n-1}$, which is the Nash equilibrium price on the $(n-1)$-firms market and hence satisfies $\frac{\partial \Pi_i^{n-1}}{\partial \xi_i}\big|_{\xi_i=\xi_j=\xi^{*,n-1}} = 0$. Solving $\frac{\partial \Pi_i^{n-1}/(NR_i)}{\partial \xi_i}\big|_{\xi_i=\xi_j=\xi^{*,n-1}} = 0$ for $(\xi^{*,n-1} - c)$, we obtain

$$\xi^{*,n-1} - c = -\frac{\Psi(1-\Phi)[1+(n-2)\Phi]}{[1+(n-3)\Phi][1+(n-2)\Phi]\frac{\mathrm{d}\Psi}{\mathrm{d}\xi}\big|_{\xi^{*,n-1}} + (n-2)\Phi\Psi\frac{\mathrm{d}\Phi}{\mathrm{d}\xi}\big|_{\xi^{*,n-1}}}.$$

This can be inserted into $\frac{\partial \Pi_i^n/(NR_i)}{\partial \xi_i}\big|_{\xi_i=\xi_j=\xi^{*,n-1}}$, which yields

$$\frac{\partial \Pi_i^n/(NR_i)}{\partial \xi_i}\bigg|_{\xi_i=\xi_j=\xi^{*,n-1}}$$
$$= \frac{\Psi}{1+(n-1)\Phi}\left[1 - \frac{[1+(n-2)\Phi]^2\frac{\mathrm{d}\Psi}{\mathrm{d}\xi}\big|_{\xi^{*,n-1}} + (n-1)\Phi\Psi\frac{1+(n-2)\Phi}{1+(n-1)\Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}\xi}\big|_{\xi^{*,n-1}}}{[1+(n-3)\Phi][1+(n-2)\Phi]\frac{\mathrm{d}\Psi}{\mathrm{d}\xi}\big|_{\xi^{*,n-1}} + (n-2)\Phi\Psi\frac{\mathrm{d}\Phi}{\mathrm{d}\xi}\big|_{\xi^{*,n-1}}}\right].$$

This is indeed negative, since the fraction in brackets is larger than one: Due to $\frac{\mathrm{d}\Phi}{\mathrm{d}\xi}, \frac{\mathrm{d}\Psi}{\mathrm{d}\xi} < 0$ (follows from condition 3.1), all summands in the numerator and denominator are negative. Furthermore, the first summand of the numerator is smaller (i. e. more negative) than the first summand of the denominator. The same holds for the second summands, if we use that for feasible products $0 < \Psi \le \Phi \le 1$ and hence $\frac{1+(n-2)\Phi}{1+(n-1)\Phi} > \frac{n-1}{n}$. □

The previous lemma now implies the desired result.

Proposition 3.8. *Consider a symmetric oligopoly with n identical firms and with habitual imitative consumers. Let $\Phi(\xi)$ and $\Psi(\xi)$ be differentiable. If $\Phi(\xi)$ and $\Psi(\xi)$ are such that there exists exactly one steady state Nash equilibrium, then the equilibrium price $\xi^{*,n}$ decreases as the number of firms n rises.*

Proof. We show $\xi^{*,n} \le \xi^{*,n-1}$. We may assume

$$\left.\frac{\partial \Pi_i^n}{\partial \xi_i}\right|_{\xi_i=\xi_j=c} \ge 0,$$

since otherwise all firms would choose $\xi_i = c$ as the unique Nash equilibrium price, and $\xi^{*,n-1}$ must have been greater than or equal to marginal cost c so that the proposition would already be proven. Also, due to lemma 3.7,

$$\left.\frac{\partial \Pi_i^n}{\partial \xi_i}\right|_{\xi_i=\xi_j=\xi^{*,n-1}} < 0.$$

Define $f : \mathbb{R} \to \mathbb{R}, \quad \xi \mapsto f(\xi) = \left.\frac{\partial \Pi_i^n}{\partial \xi_i}\right|_{\xi_i=\xi_j=\xi}$. Then f is continuous with $f(\xi^{*,n-1}) < 0$ and $f(c) \ge 0$ so that by Rolle's theorem there exists a price $\xi^{*,n} \in [c, \xi^{*,n-1})$ with $f(\xi^{*,n}) = 0$ at which f changes sign to negative. Hence, $\xi^{*,n} < \xi^{*,n-1}$ is a local maximizer of Π_i^n and thus the unique Nash equilibrium price. □

Hence, despite the consumers' bounded rationality, our model has intuitive competitive features and provides a foundation for perfect competitive equilibrium prices as the number of firms tends to infinity.

Next, we will prepare the second result, which shows that the prices of the weakest products on the market converge against their marginal costs as the number of competitors rises to infinity. This holds for a general polygopoly and directly implies that on the symmetric market all prices converge against marginal costs.

Lemma 3.9. *Consider a polygopoly with n firms and with habitual imitative consumers, where all n products coexist in the steady state Nash equilibrium. For given n, let i_n denote the index of the "weakest" good, i. e. the one with lowest market share $x_{i_n} = \min_{j=1,\dots,n}\{x_j\}$ in the steady state Nash equilibrium. Let $\Phi_{i_n}(\xi_{i_n})$ and $\Psi_{i_n}(\xi_{i_n})$ be differentiable. If there is $\nu > 0$ such that in the steady state Nash equilibrium $\frac{\partial x_{i_n}}{\partial \xi_{i_n}} < -\varepsilon < 0$ for all $n > \nu$, then as the number of firms n tends to infinity, the price of good i_n converges to marginal cost, i. e. $(\xi_{i_n}^{*,n} - c_{i_n}) \to 0$. x_{i_n} shall here be understood as the steady state market share.*

Proof. The steady state profit rate of good i_n is given by $\Pi^n_{i_n} = NR_{i_n}(\xi_{i_n} - c_{i_n})x_{i_n}$ so that

$$\left.\frac{\partial \Pi^n_{i_n}}{\partial \xi_{i_n}}\right|_{\xi_j=\xi_j^{*,n}} = NR_{i_n}\left(x_{i_n}\big|_{\xi_j=\xi_j^{*,n}} + (\xi_{i_n} - c_{i_n}) \left.\frac{\partial x_{i_n}}{\partial \xi_{i_n}}\right|_{\xi_j=\xi_j^{*,n}} \right),$$

where $\xi_j = \xi_j^{*,n}$ indicates evaluation at the steady state Nash equilibrium prices.

For a contradiction, assume there exists $\delta > 0$ such that for all $\mu > \nu$ there is $n(\mu) > \mu$ with $(\xi^{*,n}_{i_n} - c_{i_n}) > \delta$. Hence,

$$\left.\frac{\partial \Pi^{n(\mu)}_{i_{n(\mu)}}}{\partial \xi_{i_{n(\mu)}}}\right|_{\xi_j=\xi_j^{*,n(\mu)}} < NR_{i_{n(\mu)}}\left(x_{i_{n(\mu)}} - \varepsilon\delta\right) \le NR_{i_{n(\mu)}}\left(\frac{1}{n(\mu)} - \varepsilon\delta\right)$$

which is strictly negative for μ large enough. However, this contradicts the Nash equilibrium condition $\left.\frac{\partial \Pi^{n(\mu)}_{i_{n(\mu)}}}{\partial \xi_{i_{n(\mu)}}}\right|_{\xi_j=\xi_j^{*,n(\mu)}} = 0$ so that we obtain $\limsup_{n\to\infty}(\xi^{*,n}_{i_n} - c_{i_n}) \le 0$.

Since trivially, $(\xi^{*,n}_{i_n} - c_{i_n}) \ge 0$, we finally find $\lim_{n\to\infty}(\xi^{*,n}_{i_n} - c_{i_n}) = 0$. □

This lemma is almost what we aimed at, however, it depends on conditions on state variables ($\frac{\partial x_{i_n}}{\partial \xi_{i_n}} < -\varepsilon$) which might not be satisfied. A shrinking market share for rising prices is indeed economically plausible but not necessarily true. Hence, we would like to express all conditions in terms of the control variables Φ_i and Ψ_i, for which we need the following lemma, whose proof is given in appendix A.5.

Lemma 3.10. *Let $0 < \Phi_i < 1$, $i = 1, \dots, n$, and let Λ be the inverse of matrix A_n defined as*

$$(A_n)_{ij} = \begin{cases} 1, & i = j, \\ \Phi_j, & i \ne j, \end{cases} \qquad i, j = 1 \dots n.$$

Then,

$$\Lambda_{ii} \ge 1 \quad \text{and} \quad \Lambda_{ij} \le 0 \quad \text{for all } i, j = 1, \dots, n \text{ with } i \ne j.$$

Now we can prove an estimate for the change of market shares.

Lemma 3.11. *Consider a polygopoly with n firms and with habitual imitative consumers, where all n products coexist in the steady state Nash equilibrium. Let $\Phi_i(\xi_i)$ and $\Psi_i(\xi_i)$ be differentiable. Then*

$$\left.\frac{\partial x_i}{\partial \xi_i}\right|_{\xi_j=\xi_j^{*,n}} \le \left.\frac{\partial \Psi_i}{\partial \xi_i}\right|_{\xi_j=\xi_j^{*,n}}$$

for all $1 \le i \le n$, where x_i is understood as the steady state market share.

Proof. Without loss of generality let $i = 1$. Also, in the following, let all parameters, equations, and derivatives be evaluated at the steady state Nash equilibrium (i. e. at

$\xi_j = \xi_j^{*,n}$). Then, mean dynamic (7) yields $A_n\vec{x} = \vec{\Psi}$, which can be differentiated with respect to ξ_1 to give

$$\frac{\partial A_n}{\partial \xi_1}\vec{x} + A_n\frac{\partial \vec{x}}{\partial \xi_1} = \frac{\partial \vec{\Psi}}{\partial \xi_1} \quad \Leftrightarrow \quad \frac{\partial \vec{x}}{\partial \xi_1} = A_n^{-1}\left(\frac{\partial \vec{\Psi}}{\partial \xi_1} - \frac{\partial A_n}{\partial \xi_1}\vec{x}\right) = A_n^{-1}\begin{pmatrix} \frac{\partial \Psi_1}{\partial \xi_1} \\ -x_1\frac{\partial \Phi_1}{\partial \xi_1} \\ \vdots \\ -x_1\frac{\partial \Phi_1}{\partial \xi_1} \end{pmatrix}.$$

Denoting the inverse of A_n by Λ, the first row of the above equation becomes

$$\frac{\partial x_1}{\partial \xi_1} = \Lambda_{11}\frac{\partial \Psi_1}{\partial \xi_1} - x_1\frac{\partial \Phi_1}{\partial \xi_1}\sum_{j=2}^{n}\Lambda_{1j},$$

which together with the previous lemma and $\frac{\partial \Psi_1}{\partial \xi_1}, \frac{\partial \Phi_1}{\partial \xi_1} \leq 0$ (due to condition 3.1) yields the desired result. □

The previous lemma can be interpreted as follows: The change of steady state market shares x_i is larger than the change of the corresponding "product qualities" Ψ_i, i. e. shares are quite sensitive to quality changes. Lemmata 3.9 and 3.11 can now be combined to yield the following.

Proposition 3.12. *Consider a polygopoly with n firms and habitual imitative consumers, where all n products coexist in the steady state Nash equilibrium. For given n, let i_n denote the index of the "weakest" good, i. e. the one with lowest market share $x_{i_n} = \min_{j=1,\ldots,n}\{x_j\}$. Let $\Phi_{i_n}(\xi_{i_n})$ and $\Psi_{i_n}(\xi_{i_n})$ be differentiable. If there is $\nu > 0$ such that $\frac{\partial \Psi_{i_n}}{\partial \xi_{i_n}}\big|_{\xi_{i_n} = \xi_{i_n}^{*,n}} < -\varepsilon < 0$ for all $n > \nu$, then as the number of firms tends to infinity, the price of good i_n converges to marginal cost, i. e. $(\xi_{i_n}^{*,n} - c_{i_n}) \to 0$.* □

The proof of this result can inductively be repeated for the second "weakest" good, the third "weakest" one, and so on up to the mth "weakest" good, where m is any positive integer.

Corollary 3.13. *Consider a polygopoly with n firms and let the assumptions from proposition 3.12 hold. As the number of firms tends to infinity, the m "weakest" products' prices converge to their marginal costs, where m is any positive integer.* □

As a consequence, since on a symmetric market any good is the "weakest" one, the prices of all goods converge to marginal costs. In general, however, there may be products so superior to the rest of the market that their prices stay away from marginal costs while all other prices converge against marginal costs. The proof of lemma 3.9 can be slightly adapted to show that this may only be true for a finite number of products.

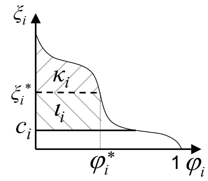

Figure 2: An arbitrary imitation function, plotted as normalized inverse demand function. The consumer surplus equals $CS_i = \frac{\kappa_i}{\varphi_i^*} S_i$, and the producer surplus is given by $PS_i = \frac{\iota_i}{\varphi_i^*} S_i$, where ξ_i^* denotes the current price of product i and $\varphi_i^* = \varphi_i(\xi_i^*)$ is the resulting imitation coefficient.

4. Extensions: Welfare, product life cycle generation, and advertising

To point into possible directions of further research we will briefly discuss three supplements to our model as there are welfare, the generation of product life cycles, and advertising.

4.1. A welfare definition

In the following, we shall propose a suitable welfare definition for our setting to allow for theories of social implications. The producer surplus can be calculated as usual from the Marshallian definition, whereas the consumer surplus has to be obtained differently as a consequence of the non-standard consumer behavior.

Definition 4.1 (Welfare)**.** *Given the imitation function $\varphi_i(\xi_i)$ for good i, the contribution of that good to the producer and consumer surplus at time t can be calculated as*

$$PS_i(t) = (\xi_i(t) - c_i) S_i(t), \tag{9}$$

$$CS_i(t) = \frac{S_i(t)}{\varphi_i(\xi_i(t))} \left(\int_0^{\varphi_i(\xi_i(t))} \left[\varphi_i^{-1}(\varphi) - \xi_i(t) \right] \mathrm{d}\varphi \right). \tag{10}$$

The social welfare takes the form

$$W(t) = \sum_{i=1}^{n} PS_i(t) + CS_i(t). \tag{11}$$

For a single good with an arbitrary imitation function, consumer and producer surplus at a specific time are illustrated in figure 2. Before further motivation, note that the imitation function $\varphi_i(\xi_i)$ can be interpreted as the demand distribution for good i. In other words, for a consumer the probability P to have a reservation price ξ_i^{rp} for good i larger than or equal to some price ξ_i^* is given by the demand distribution, i. e.

$$\varphi_i(\xi_i^*) = \mathrm{P}[\xi_i^{\mathrm{rp}} \geq \xi_i^*].$$

Phrased differently, the imitation function $\varphi_i(\xi_i)$ constitutes the probability distribution of the reservation price ξ_i^{rp} of a set of heterogeneous consumers, and the reservation price ξ_i^{rp} is distributed according to the density $-\frac{\mathrm{d}\varphi_i(\xi_i)}{\mathrm{d}\xi_i}$. (If instead we assume fickle homogeneous consumers, where each individual's reservation price changes from time to time, $\varphi_i(\xi_i)$ can be interpreted as the reservation price probability of a single consumer.) Against this background, the following alternative characterization of the consumer surplus may serve as motivation.

Proposition 4.1. *Let $\varphi_i(\xi_i)\xi_i \to 0$ for $\xi_i \to \infty$, and let ξ_i^* be the current price of product i and $\varphi_i^* = \varphi_i(\xi_i^*)$ the resulting imitation coefficient. Let the consumers' reservation price ξ_i^{rp} for good i have probability density $-\frac{\mathrm{d}\varphi_i(\xi_i)}{\mathrm{d}\xi_i}\big|_{\xi_i^{\mathrm{rp}}} =: -\varphi_i'(\xi_i^{\mathrm{rp}})$.*

(i) Pick one consumer arbitrarily and give her the option to buy either product i or none. The expected value of her utility, $U_i = \max(\xi_i^{rp} - \xi_i^*, 0)$, is then given by

$$\mathrm{E}[U_i] \quad = \quad \int_{\xi_i^*}^{\infty} \varphi_i(\xi_i)\mathrm{d}\xi_i.$$

(For simplicity, we here also allow infinite values of the expected utility.)

(ii) Pick one consumer, who has reservation price $\xi_i^{\mathrm{rp}} \geq \xi_i^*$, i. e. who would buy product i. The expected value of her utility, u_i, is given as

$$\mathrm{E}[u_i] = \frac{\mathrm{E}[U_i]}{\varphi_i^*}.$$

(iii) Assume that those consumers who actually buy product i are uniformly distributed among all potential buyers (i. e. those with $\xi_i^{\mathrm{rp}} \geq \xi_i^*$), then the expected consumer surplus is given by

$$CS_i = \mathrm{E}[u_i]S_i = \frac{\mathrm{E}[U_i]}{\varphi_i^*}S_i.$$

Proof. (i) The consumer's utility U_i is a random variable depending on the reservation price ξ_i^{rp}. Hence,

$$\mathrm{E}[U_i] = \int_0^1 U_i(\xi_i^{\mathrm{rp}})\mathrm{dP}(\xi_i^{\mathrm{rp}}) = \int_0^{\infty} U_i(\xi_i^{\mathrm{rp}})(-\varphi_i'(\xi_i^{\mathrm{rp}}))\mathrm{d}\xi_i^{\mathrm{rp}}$$
$$= -\int_{\xi_i^*}^{\infty} (\xi_i^{\mathrm{rp}} - \xi_i^*)\varphi_i'(\xi_i^{\mathrm{rp}})\mathrm{d}\xi_i^{\mathrm{rp}} = -\left[(\xi_i^{\mathrm{rp}} - \xi_i^*)\varphi_i(\xi_i^{\mathrm{rp}})\right]_{\xi_i^{\mathrm{rp}}=\xi_i^*}^{\infty} + \int_{\xi_i^*}^{\infty} \varphi_i(\xi_i^{\mathrm{rp}})\mathrm{d}\xi_i^{\mathrm{rp}}.$$

The left summand of the last expression equals zero, since $\xi_i^{\mathrm{rp}}\varphi_i(\xi_i^{\mathrm{rp}}) \overset{\xi_i^{\mathrm{rp}}\to\infty}{\longrightarrow} 0$. U_i is a so-called integrable random variable, if and only if $\varphi_i \in L^1([0,\infty))$ (in which case the integral is bounded).

(ii) The consumers with $\xi_i^{\mathrm{rp}} \geq \xi_i^*$ make up $\varphi_i(\xi_i^*)$ of the total population. Among them, ξ_i^{rp} is distributed according to

$$\mathrm{P}[\xi_i^{\mathrm{rp}} \geq \xi] = \begin{cases} 1, & \xi < \xi_i^*, \\ \frac{\varphi_i(\xi_i^{\mathrm{rp}})}{\varphi_i(\xi_i^*)}, & \text{else}, \end{cases}$$

so that analogously to the above, $\mathrm{E}[u_i] = \frac{\mathrm{E}[U_i]}{\varphi_i(\xi_i^*)}$.

(iii) By a change of variables $\varphi = \varphi_i(\xi_i^{\mathrm{rp}})$ and integration by parts we obtain

$$\int_0^{\varphi_i(\xi_i^*)} [\varphi_i^{-1}(\varphi) - \xi_i^*] \mathrm{d}\varphi = -\int_{\xi_i^*}^{\infty} (\xi_i^{\mathrm{rp}} - \xi_i^*) \varphi_i'(\xi_i^{\mathrm{rp}}) \mathrm{d}\xi_i^{\mathrm{rp}} = \int_{\xi_i^*}^{\infty} \varphi_i(\xi_i^{\mathrm{rp}}) \mathrm{d}\xi_i^{\mathrm{rp}} = \mathrm{E}[U_i],$$

which together with the definition of CS_i yields the desired result.

□

Our definition thus resembles the standard approach: CS_i approximates the difference between average reservation price among all buyers and the actual price.

As a brief application, we show that for a symmetric oligopoly, the welfare rises with the number of firms.

Proposition 4.2. *Consider a symmetric oligopoly with n identical firms and with habitual imitative consumers. Let $\varphi(\xi)$ and $s(\xi)$ be differentiable and such that there exists exactly one steady state Nash equilibrium with equilibrium price $\xi^{*,n}$. If the total steady state sales $nS^n := \sum_{i=1}^n S_i(\xi^{*,n})$ increase more strongly in n than $\varphi^{*,n} := \varphi(\xi^{*,n})$, i. e. $\frac{\mathrm{d}(nS^n)}{\mathrm{d}n}/(nS^n) > \frac{\mathrm{d}\varphi^{*,n}}{\mathrm{d}n}/\varphi^{*,n}$, then the welfare increases for a rising number of firms.*

Proof. As before, since all firms are identical we skip the indices. As illustrated in figure 2, the welfare is given by $\frac{nS^n}{\varphi^{*,n}}(\kappa + \iota)$. Define $\Omega^n = \frac{nS^n}{\varphi^{*,n}}$, then

$$\frac{\mathrm{d}\Omega^n}{\mathrm{d}n} = \frac{\frac{\mathrm{d}(nS^n)}{\mathrm{d}n}\varphi^{*,n} - nS^n \frac{\mathrm{d}\varphi^{*,n}}{\mathrm{d}n}}{(\varphi^{*,n})^2},$$

which is larger than 0 by assumption. Also, $\xi^{*,n}$ decreases for a rising number of firms by proposition 3.8, and hence, $\kappa + \iota$ increases due to the monotonicity of $\varphi(\xi)$ (condition 3.1). Altogether, the welfare rises for rising n. □

Apparently, despite the consumers' bounded rationality, we obtain the standard result of an increasing welfare. This implies a certain amount of market efficiency, comparable to a market with rational consumers.

4.2. The generation of product life cycles

In this paragraph, we briefly illustrate how a realistic product life cycle may emerge from our model. As a simple example, consider a consecutive introduction of many

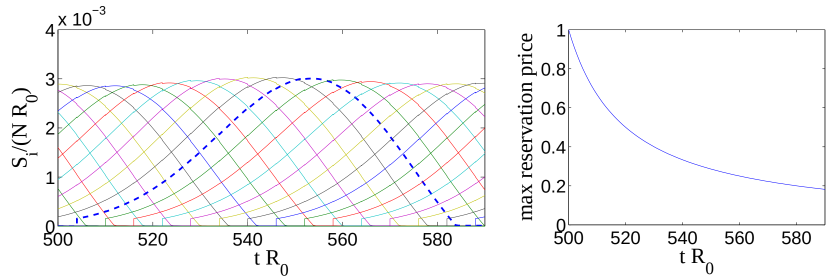

Figure 3: Product life cycles (left) when many products enter the market successively and the maximum reservation price Ξ_i of each product decreases in time (right). In this simulation we chose $\varphi_i(\xi_i) = s_i(\xi_i) = 1 - \frac{\xi_i}{\Xi_i}$ with maximum reservation price $\Xi_i = \frac{\Xi}{1+\alpha R_0(t-t_i)}$, $\alpha = 5 \cdot 10^{-2}$. Furthermore, we have a time interval $T = \frac{6}{R_0}$ between the introduction of products, an alarm clock rate $R_i = 0.025 R_0$ for all goods, an initial subpopulation $x_i(0) = 10^{-3}$, and constant product prices $\xi_i = 0.1\Xi$.

products, all competing with each other. Think for instance of the mobile phone market, where new (innovative) mobile phones frequently enter the market. The maximum reservation price for a product may be assumed highest at its introduction on the market when it still represents the state of the art, and then it decreases in time, as innovation goes on. Hence, also habit and imitation function are highest at the time of product introduction.

The most simple setting is to assume fixed prices ξ_i, simple imitation and habit functions $\varphi_i(\xi_i) = s_i(\xi_i) = 1 - \frac{\xi_i}{\Xi_i}$, new product introductions equally distributed over time, and a simple evolution of the maximum reservation price Ξ_i in time, e. g. $\Xi_i = \frac{\Xi}{1+\alpha R_0(t-t_i)}$, where t_i is the time of introduction of product i. Figure 3 shows the resulting product life cycles of the successively introduced products, obtained from an exemplary simulation.

Notice the classical pattern with a gentle increase of the sales right after product launch, a broad maturity period and a quite steep decline until the product vanishes (cf. for example de Kluyver (1977), Polli and Cook (1969) and others).

4.3. Marketing strategies: Advertisement

In section 2.2, in order to employ specific switching probabilities, we used the mechanism of imitation (4), that is, the probability of buying good i is proportional to the amount x_i of people who already own it. x_i may here be interpreted as the probability that the consumer gets to know the product from other consumers. The multiplicative imitation factor φ_i represents how strongly the consumer is convinced to buy the good when she knows it. However, consumers can also get to know the good via advertisements, which constitute an effective tool for firms to influence the consumers' buying behavior. The probability to see the product's commercial is given by $a_i \in [0,1]$, where a_i depends positively on the advertising budget. The overall probability to become aware of product i (via commercials or other consumers) hence is $a_i + x_i - a_i x_i$ so that (4) and (6) change to

$$p_{0i} = \varphi_i(x_i + a_i - x_i a_i), \quad i \neq 0, \tag{12}$$

$$p_{ij} = (1 - s_i)\varphi_j(x_j + a_j - x_j a_j), \quad i \neq 0 \wedge j \neq 0, i. \tag{13}$$

For the same motivation as in section 2.2, equation (5) remains unchanged,

$$p_{ii} = s_i \in [0,1], \quad i \neq 0. \tag{14}$$

We shall in the following always assume $\varphi_i, s_i > 0$. An interesting question would be whether a non-feasible product can be made feasible by advertising. The following lemma provides an answer for a single good market (where we disregard advertising costs and only examine whether a demand for that good exists).

Proposition 4.3. *The single product on a market with habitual imitative consumers is always feasible if it is advertised, i. e.* $a_1 > 0$*.*

Proof. The mean dynamic for the single good market takes the form

$$\begin{aligned} \dot{x}_1 &= \varphi_1 R_0(x_1 + a_1 - x_1 a_1) + x_1(R_1 s_1 - \varphi_1 R_0(x_1 + a_1 - x_1 a_1)) - R_1 x_1 \\ &= x_1 \varphi_1 R_0[\Psi_1 - 2a_1 - x_1(1 - a_1)] + \varphi_1 R_0 a_1. \end{aligned}$$

In the stationary state $\dot{x}_1 = 0$ we thus obtain

$$x_1 = \begin{cases} \frac{1}{2-\Psi_1}, & a_1 = 1, \\ \frac{\Psi_1 - 2a_1 + \sqrt{(\Psi_1 - 2a_1)^2 + 4a_1(1-a_1)}}{2-2a_1}, & a_1 \neq 1, \end{cases}$$

which is positive for $a_1 > 0$, irrespective of the value of Ψ_1. □

Apparently, commercials help the good to survive on the market. This statement is illustrated in figure 4 where the steady state market share is shown for different advertising levels. For a positive level, the market share is always positive and hence the product feasible.

Note that in proposition 4.3 we only consider the demand side of the market, i. e. we examine whether the product is demanded by consumers in the steady state. We ignore that the firm might not be able to operate in the black because of immense advertising costs.

An analogous result can be shown for an oligopoly.

Proposition 4.4. *On an* n*-product market with habitual imitative consumers, product* i *is always feasible if it is advertised, i. e.* $a_i > 0$ *(unless there is a good* j *with* $\Phi_j = 1$*).*

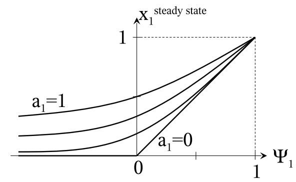

Figure 4: Stable steady state value of the market share x_1 for different advertising levels a_1.

Proof. The mean dynamic reads

$$\dot{x}_i = \varphi_i R_0 x_i \left[\Psi_i - 2a_i - (1-a_i)\left(x_i + \sum_{\substack{j=1\\ j\neq i}}^{n} \Phi_j x_j \right) \right] + \varphi_i R_0 a_i \left(1 - \sum_{\substack{j=1\\ j\neq i}}^{n} \Phi_j x_j \right).$$

For $x_i = 0$ we obtain $\dot{x}_i > 0$. Hence, the system trajectory can never approach $x_i = 0$ so that the market share of product i always stays strictly away from zero. □

An advertising campaign is usually associated with costs that depend on its reach. Let us therefore introduce advertising costs c_i^a for good i. Obviously, the derivative $\frac{\partial a_i}{\partial c_i^a}$ has to be non-negative. With this altered model at hand, various simulations can be performed for specific functions $\varphi_i(\xi_i)$, $s_i(\xi_i)$ and $a_i(c_i^a)$. One could for instance examine the product feasibility including advertising costs, whether advertising is profitable at all, how large a_i should optimally be, or whether there is a threshold value for x_i above which advertising is no longer beneficial. For illustration, we pick up example 3.2 and add advertising. We will compute the optimal affine pricing and advertising strategy. Before, however, we need to extend the definition of the normal form competition game.

With advertising, firms have a second strategic variable besides their product's price which represents the advertising expenses. Hence, the set $\mathfrak{S}$ of all possible strategy combinations now is a subset of the space of n-tuples over maps $(\xi_i, c_i^a) : \mathbb{R}_+ \to \mathbb{R}_+^2$, $t \mapsto (\xi_i(t), c_i^a(t))$, $\mathfrak{S} \subseteq \left[(\mathbb{R}_+^2)^{\mathbb{R}_+} \right]^n = (\mathbb{R}_+^2)^{\mathbb{R}_+} \times \ldots \times (\mathbb{R}_+^2)^{\mathbb{R}_+}$. The components of profit $\Pi : \mathfrak{S} \to \mathbb{R}_+^n$ now become $\Pi_i(\xi_1, \ldots, \xi_n, c_1^a, \ldots, c_n^a) = F\left[(\xi_i(t) - c_i) S_i \left(\vec{s}(\vec{\xi}(t)), \vec{\varphi}(\vec{\xi}(t)), \vec{a}(\vec{c^a}(t)), t \right) - \vec{c^a}(t) \right]$, where $\vec{S}(\vec{s}(\vec{\xi}(t)), \vec{\varphi}(\vec{\xi}(t)), \vec{a}(\vec{c^a}(t)), t)$ denotes the sales vector produced by the population game including advertising according to equations (12) and (13).

Example 4.1 (Cumulated discounted profit in a monopoly setting with advertising)**.** *For simplicity, let us assume* $\varphi_1 = s_1 = 1 - \frac{\xi_1}{\Xi_1}$ *and* $a_1 = \frac{1}{1+K/c_1^a}$*, and let us only allow for affine price and advertising cost functions* $\xi_1(\cdot), c_1^a(\cdot) \in \mathfrak{L}([0,T]) := \{f : [0,T] \to \mathbb{R} \,|\, \exists\, a, b : f(t) = a + bt\}$*. Consider the normal form competition game*

$$G = \left(1, \mathfrak{L}^2([0,T]), \int_0^T \exp[-rt] \left[(\xi_1(t) - c_1) S_1 \left(s_1(\xi_1(t)), \varphi_1(\xi_1(t)), a_1(c_1^a(t)), t \right) - c_1^a(t) \right] \mathrm{d}t \right).$$

For given parameters $R_1, T, r, c_1, x_1(0)$*, the optimal price paths* $\xi_1(t)$ *and* $c_1^a(t)$ *can*

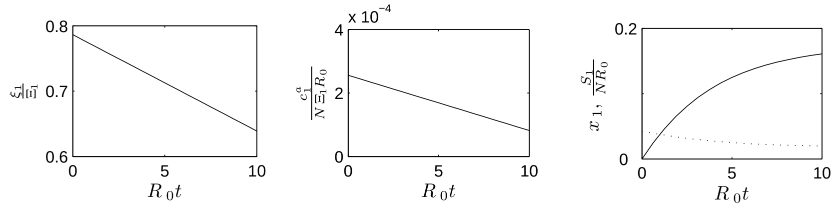

Figure 5: Optimal affine price evolution (left) and optimal affine advertising expenses (middle), as well as subpopulation (solid line) and sales (dotted line) evolution (right) for parameter values $R_1 = 0.1R_0$, $T = 10/R_0$, $r = 0$, $c_1 = 0.1/\Xi_1$, $K = 2 \cdot 10^{-6}$, $x_1(0) = 0$ (cf. example 4.1).

be found numerically. As a result, for a whole range of realistic parameters we obtain the reverse of example 3.2: The price decreases with time. One example calculation is depicted in figure 5.

Apparently, a firm is recommended to start an advertising campaign in parallel to the product launch and steadily decrease the product price as well as the advertising expenses during the lifespan of the product. Due to the initial advertising, the market share is rapidly increased with brute force. Via the subsequent price decrease, habit purchases can be kept on a high level, and reluctant customers are attracted. Thereby, the market is optimally exploited by initially letting customers with a high reservation price pay high prices and only later reducing the price to make people with low reservation prices buy the product (similarly to the concept of price discrimination). Advertising becomes less crucial when the market share has already reached a certain level (the product sells itself) and is therefore reduced.

5. Conclusion

We examined the optimal strategic pricing for firms when the demand evolution is generated by the behavior of boundedly rational consumers who follow a rule of thumb and base their decisions on imitation and habit. The demand dynamic is described within the framework of a population game with associated switching probabilities, and it serves as a basis for strategic pricing of a monopoly or oligopoly in a differential game. The optimal price paths correspond to Nash equilibria of a normal form competition game.

The modeling approach is supported by psychological and experimental studies, and the introduced methodology allows for broad applications and qualitative theoretical analysis.

We investigated product feasibility (i. e. the conditions under which firms operate profitably in the long-term) and expressed it with the help of the hypothetical popularity of the product if it was sold for a price equal to the marginal cost. Furthermore, we showed that markets with habitual imitative consumers are in a sense well-behaved: For a rising number of firms, the prices decrease, the prices of the weakest products (but not necessarily of all products) converge against marginal costs, and the welfare rises (at least for a symmetric market). Such results (despite the boundedly rational consumer behavior) prove once more the existence of some kind of efficiency in not totally rational markets.

We also proved for the monopoly that under certain conditions, Nash equilibria are found in the strategy space of all time-constant price paths so that a reduction of the (quite complex and untractable) strategy space of all possible price paths is at least sometimes sensible.

Finally, the assumed boundedly rational consumer behavior was shown to lead to observed market patterns such as product life cycles, and extensions to the model were proposed and examined such as an adequate definition of welfare, which allows for analysis of social implications, and the introduction of advertising, which allows to explore optimal advertising strategies.

A. Appendix

A.1. Criticality of a constant monopoly price (Proposition 3.2)

Using the relation (obtained via integration by parts and Fubini's theorem)

$$\int_0^t f(\tau)\int_0^\tau g(\theta)\mathrm{d}\theta\mathrm{d}\tau = \left[\int_0^t f(\tau)\mathrm{d}\tau\int_0^t g(\tau)\mathrm{d}\tau\right]_0^t - \int_0^t\int_0^\tau f(\theta)\mathrm{d}\theta g(\tau)\mathrm{d}\tau = \int_0^t g(\tau)\int_\tau^t f(\theta)\mathrm{d}\theta\mathrm{d}\tau,$$

we can write the variation of $x(t)$ with respect to ξ in some test direction ϑ as

$$\left\langle\frac{\partial x(t)}{\partial\xi},\vartheta\right\rangle = \frac{\exp(\varphi\Psi t)\left[\int_0^t(\varphi\Psi)'\vartheta\mathrm{d}\tau\left(\frac{\exp(\varphi\Psi t)-1}{\Psi}+\frac{1}{\Psi}\right)-\int_0^t\exp(\varphi\Psi\tau)\left(\varphi'\vartheta+\varphi\int_0^\tau(\varphi\Psi)'\vartheta(\theta)\mathrm{d}\theta\right)\mathrm{d}\tau-\frac{\int_0^T\exp(\varphi\Psi\tau)(\varphi'\vartheta+\varphi\int_0^\tau(\varphi\Psi)'\vartheta\mathrm{d}\theta)\mathrm{d}\tau(\exp(\varphi\Psi T)-1)-\int_0^T(\varphi\Psi)'\vartheta\mathrm{d}\tau\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\right]}{\left(\frac{\exp(\varphi\Psi t)-1}{\Psi}+\frac{1}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[(\varphi\Psi)'\int_0^t\vartheta\mathrm{d}\tau\frac{\exp(\varphi\Psi t)}{\Psi}-\int_0^t\exp(\varphi\Psi\tau)\left(\varphi'\vartheta+\varphi(\varphi\Psi)'\int_0^\tau\vartheta\mathrm{d}\theta\right)\mathrm{d}\tau-\frac{\int_0^T\exp(\varphi\Psi\tau)(\varphi'\vartheta+\varphi(\varphi\Psi)'\int_0^\tau\vartheta\mathrm{d}\theta)\mathrm{d}\tau(\exp(\varphi\Psi T)-1)-(\varphi\Psi)'\int_0^T\vartheta\mathrm{d}\tau\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[(\varphi\Psi)'\frac{\exp(\varphi\Psi t)}{\Psi}\int_0^t\vartheta\mathrm{d}\tau-\varphi'\int_0^t\exp(\varphi\Psi\tau)\vartheta\mathrm{d}\tau-\varphi(\varphi\Psi)'\int_0^t\exp(\varphi\Psi\tau)\int_0^\tau\vartheta\mathrm{d}\theta\mathrm{d}\tau-\frac{\int_0^T\exp(\varphi\Psi\tau)(\varphi'\vartheta+\varphi(\varphi\Psi)'\int_0^\tau\vartheta\mathrm{d}\theta)\mathrm{d}\tau(\exp(\varphi\Psi T)-1)-(\varphi\Psi)'\int_0^T\vartheta\mathrm{d}\tau\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[(\varphi\Psi)'\frac{\exp(\varphi\Psi t)}{\Psi}\int_0^t\vartheta\mathrm{d}\tau-\varphi'\int_0^t\exp(\varphi\Psi\tau)\vartheta\mathrm{d}\tau-\varphi(\varphi\Psi)'\int_0^t\exp(\varphi\Psi\tau)\int_0^\tau\vartheta\mathrm{d}\theta\mathrm{d}\tau-\frac{\int_0^T\exp(\varphi\Psi\tau)(\varphi'\vartheta+\varphi(\varphi\Psi)'\int_0^\tau\vartheta\mathrm{d}\theta)\mathrm{d}\tau(\exp(\varphi\Psi T)-1)-(\varphi\Psi)'\int_0^T\vartheta\mathrm{d}\tau\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[(\varphi\Psi)'\frac{\exp(\varphi\Psi t)}{\Psi}\int_0^t\vartheta\mathrm{d}\tau-\varphi'\int_0^t\exp(\varphi\Psi\tau)\vartheta\mathrm{d}\tau-\varphi(\varphi\Psi)'\int_0^t\vartheta\int_\theta^t\exp(\varphi\Psi\tau)\mathrm{d}\tau\mathrm{d}\theta-\frac{\int_0^T\exp(\varphi\Psi\tau)(\varphi'\vartheta+\varphi(\varphi\Psi)'\int_0^\tau\vartheta\mathrm{d}\theta)\mathrm{d}\tau(\exp(\varphi\Psi T)-1)-(\varphi\Psi)'\int_0^T\vartheta\mathrm{d}\tau\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[(\varphi\Psi)'\frac{\exp(\varphi\Psi t)}{\Psi}\int_0^t\vartheta\mathrm{d}\tau-\varphi'\int_0^t\exp(\varphi\Psi\theta)\vartheta\mathrm{d}\theta-\varphi(\varphi\Psi)'\int_0^t\vartheta\frac{\exp(\varphi\Psi t)-\exp(\varphi\Psi\theta)}{\varphi\Psi}\mathrm{d}\theta-\frac{\int_0^T\exp(\varphi\Psi\tau)(\varphi'\vartheta+\varphi(\varphi\Psi)'\int_0^\tau\vartheta\mathrm{d}\theta)\mathrm{d}\tau(\exp(\varphi\Psi T)-1)-(\varphi\Psi)'\int_0^T\vartheta\mathrm{d}\tau\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[-\varphi'\int_0^t\exp(\varphi\Psi\theta)\vartheta\mathrm{d}\theta+\frac{(\varphi\Psi)'}{\Psi}\int_0^t\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta-\frac{\int_0^T\exp(\varphi\Psi\tau)(\varphi'\vartheta+\varphi(\varphi\Psi)'\int_0^\tau\vartheta\mathrm{d}\theta)\mathrm{d}\tau(\exp(\varphi\Psi T)-1)-(\varphi\Psi)'\int_0^T\vartheta\mathrm{d}\tau\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[\frac{\varphi\Psi'}{\Psi}\int_0^t\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta-\frac{\varphi'\int_0^T\exp(\varphi\Psi\tau)\vartheta\mathrm{d}\tau+\varphi(\varphi\Psi)'\int_0^T\exp(\varphi\Psi\tau)\int_0^\tau\vartheta\mathrm{d}\theta\mathrm{d}\tau}{\exp(\varphi\Psi T)-1}+\frac{(\varphi\Psi)'\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau\int_0^T\vartheta\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[\frac{\varphi\Psi'}{\Psi}\int_0^t\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta-\frac{\varphi'}{\exp(\varphi\Psi T)-1}\int_0^T\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta-\frac{\varphi(\varphi\Psi)'}{\exp(\varphi\Psi T)-1}\int_0^T\exp(\varphi\Psi\tau)\int_0^\tau\vartheta\mathrm{d}\theta\mathrm{d}\tau+\frac{(\varphi\Psi)'\exp(\varphi\Psi T)\int_0^T\varphi\exp(\varphi\Psi\tau)\mathrm{d}\tau}{(\exp(\varphi\Psi T)-1)^2}\int_0^T\vartheta\mathrm{d}\tau\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2}$$

$$= \frac{\exp(\varphi\Psi t)\left[\frac{\varphi\Psi'}{\Psi}\int_0^t\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta-\frac{\varphi'\Psi}{\Psi(\exp(\varphi\Psi T)-1)}\int_0^T\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta+\frac{(\varphi\Psi)'}{\Psi(\exp(\varphi\Psi T)-1)}\int_0^T\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta\right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi}\right)^2} = \frac{\Psi\left[\varphi\Psi'\int_0^t\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta+\frac{\varphi\Psi'}{\exp(\varphi\Psi T)-1}\int_0^T\vartheta\exp(\varphi\Psi\theta)\mathrm{d}\theta\right]}{\exp(\varphi\Psi t)}.$$

Using the price ξ implicitly defined by $\xi - c = -\Psi(\xi)/\Psi'(\xi)$ (which follows from maximizing the profit for a constant price and a steady state, i. e. no periodic oscillations), the variation of the profit rate then is given as

$$
\begin{aligned}
&\left\langle \frac{\partial \Pi / R_0}{\partial \xi}, \vartheta \right\rangle = \frac{1}{R_0} \frac{d}{d\varepsilon} \Pi(\xi(\cdot) + \varepsilon \vartheta(\cdot)) \Big|_{\varepsilon=0} \\
&= \frac{N}{T} \int_0^T x(R + \varphi(\Psi - x))\vartheta + (\xi - c) \left(x(\varphi\Psi)'\vartheta - x^2 \varphi' \vartheta + \left\langle \frac{\partial x(t)}{\partial \xi}, \vartheta \right\rangle (R + \varphi\Psi - 2\varphi x) \right) \mathrm{d}t \\
&= \frac{N}{T} \int_0^T \Psi(R + \varphi(\Psi - \Psi))\vartheta - \frac{\Psi}{\Psi'} \left(\Psi(\varphi\Psi)'\vartheta - \Psi^2 \varphi' \vartheta + \left[\frac{\exp(\varphi\Psi t) \left[\frac{\varphi\Psi'}{\Psi} \int_0^t \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta + \frac{\varphi\Psi'}{\Psi(\exp(\varphi\Psi T)-1)} \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi} \right)^2} \right] (R + \varphi\Psi - 2\varphi\Psi) \right) \mathrm{d}t \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \int_0^T \vartheta \mathrm{d}t - \frac{N}{T} \int_0^T \frac{\Psi}{\Psi'}(R - \varphi\Psi) \left[\frac{\exp(\varphi\Psi t) \left[\frac{\varphi\Psi'}{\Psi} \int_0^t \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta + \frac{\varphi\Psi'}{\Psi(\exp(\varphi\Psi T)-1)} \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \right]}{\left(\frac{\exp(\varphi\Psi t)}{\Psi} \right)^2} \right] \mathrm{d}t \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \int_0^T \vartheta \mathrm{d}t - \frac{N}{T} \frac{\Psi^3}{\Psi'}(R - \varphi\Psi) \int_0^T \frac{\exp(\varphi\Psi t)}{(\exp(\varphi\Psi t))^2} \frac{\varphi\Psi'}{\Psi} \int_0^t \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \mathrm{d}t - \frac{N}{T} \frac{\Psi^3}{\Psi'}(R - \varphi\Psi) \int_0^T \frac{\exp(\varphi\Psi t)}{(\exp(\varphi\Psi t))^2} \frac{\varphi\Psi'}{\Psi\left(\exp(\varphi\Psi T) - 1\right)} \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \mathrm{d}t \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \int_0^T \vartheta \mathrm{d}t - \frac{N}{T} \Psi^2 \varphi(R - \varphi\Psi) \int_0^T \frac{1}{\exp(\varphi\Psi t)} \int_0^t \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \mathrm{d}t - \frac{N}{T} \Psi^2 \varphi(R - \varphi\Psi) \int_0^T \frac{1}{\exp(\varphi\Psi t)} \frac{1}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \mathrm{d}t \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \int_0^T \vartheta \mathrm{d}t - \frac{N}{T} \Psi^2 \varphi(R - \varphi\Psi) \int_0^T \vartheta \exp(\varphi\Psi\theta) \int_\theta^T \frac{1}{\exp(\varphi\Psi t)} \mathrm{d}t \mathrm{d}\theta - \frac{N}{T} \Psi^2 \varphi(R - \varphi\Psi) \int_0^T \frac{1}{\exp(\varphi\Psi t)} \frac{1}{\exp(\varphi\Psi T) - 1} \mathrm{d}t \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \int_0^T \vartheta \mathrm{d}t - \frac{N}{T} \Psi^2 \varphi(R - \varphi\Psi) \int_0^T \vartheta \exp(\varphi\Psi\theta) \left[-\frac{1}{\varphi\Psi \exp(\varphi\Psi T)} + \frac{1}{\varphi\Psi \exp(\varphi\Psi\theta)} \right] \mathrm{d}\theta - \frac{\frac{N}{T} \Psi^2 \varphi(R - \varphi\Psi)}{\exp(\varphi\Psi T) - 1} \left[-\frac{1}{\varphi\Psi \exp(\varphi\Psi T)} + \frac{1}{\varphi\Psi} \right] \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \int_0^T \vartheta \mathrm{d}t - \frac{N}{T} \Psi(R - \varphi\Psi) \int_0^T \vartheta \exp(\varphi\Psi\theta) \left[-\frac{1}{\exp(\varphi\Psi T)} + \frac{1}{\exp(\varphi\Psi\theta)} \right] \mathrm{d}\theta - \frac{\frac{N}{T} \Psi(R - \varphi\Psi)}{\exp(\varphi\Psi T) - 1} \left[1 - \frac{1}{\exp(\varphi\Psi T)} \right] \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \left[\int_0^T \vartheta \mathrm{d}t - \int_0^T \vartheta \exp(\varphi\Psi\theta) \left[-\frac{1}{\exp(\varphi\Psi T)} + \frac{1}{\exp(\varphi\Psi\theta)} \right] \mathrm{d}\theta - \frac{1}{\exp(\varphi\Psi T) - 1} \left[1 - \frac{1}{\exp(\varphi\Psi T)} \right] \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \right] \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \left[\int_0^T \vartheta \mathrm{d}t - \int_0^T \vartheta \exp(\varphi\Psi\theta) \left[-\frac{1}{\exp(\varphi\Psi T)} + \frac{1}{\exp(\varphi\Psi\theta)} \right] \mathrm{d}\theta - \frac{1}{\exp(\varphi\Psi T)} \int_0^T \vartheta \exp(\varphi\Psi\theta) \mathrm{d}\theta \right] \\
&= \frac{N}{T}(\Psi R - \Psi^2 \varphi) \left[\int_0^T \vartheta \mathrm{d}t - \int_0^T \vartheta \mathrm{d}\theta \right] \\
&= 0.
\end{aligned}
$$

A.2. Proof of lemma 3.3

Proof. For Lebesgue-integrable functions ϑ, θ, let us define the bilinear forms

$$\begin{aligned}\langle \vartheta, \theta \rangle_L &= \int_0^T \int_0^T \vartheta(t)\theta(\tau) \exp[\alpha(\tau - t + TH(t-\tau))] \mathrm{d}\tau \mathrm{d}t \\ \langle \vartheta, \theta \rangle_R &= \frac{\exp(\alpha T) - 1}{\alpha} \int_0^T \vartheta(t)\theta(t) \mathrm{d}t.\end{aligned}$$

For $\vartheta \in L^1([0,T]) \backslash L^2([0,T])$, $\langle \vartheta, \vartheta \rangle_R$ is unbounded so that $\langle \vartheta, \vartheta \rangle_L \leq \langle \vartheta, \vartheta \rangle_R$ trivially. Hence, let us assume $\vartheta \in L^2([0,T])$ from now on.

$\langle \cdot, \cdot \rangle_L$ and $\langle \cdot, \cdot \rangle_R$ are (sequentially) continuous in both arguments in $L^2([0,T])$: Let $\theta_k \overset{k\to\infty}{\longrightarrow} \theta$ in $L^2([0,T])$. By Hï£¡lder's inequality and the continuous Sobolev embedding $L^2([0,T]) \hookrightarrow L^1([0,T])$,

$$|\langle \vartheta, \theta \rangle_R - \langle \vartheta, \theta_k \rangle_R| = \left| \int_0^T \vartheta(\theta - \theta_k) \mathrm{d}t \right| \leq ||\vartheta||_{L^2} ||\theta - \theta_k||_{L^2} \overset{k\to\infty}{\longrightarrow} 0$$

$$\begin{aligned}|\langle \vartheta, \theta \rangle_L - \langle \vartheta, \theta_k \rangle_L| &= \left| \int_0^T \int_0^T \vartheta(\theta - \theta_k) \mathrm{e}^{\alpha(\tau - t + TH(t-\tau))} \mathrm{d}\tau \mathrm{d}t \right| \leq \mathrm{e}^{|\alpha T|} \int_0^T |\vartheta| \mathrm{d}t \int_0^T |\theta - \theta_k| \mathrm{d}t \\ &= \mathrm{e}^{|\alpha T|} ||\vartheta||_{L^1} ||\theta - \theta_k||_{L^1} \leq C ||\vartheta||_{L^2} ||\theta - \theta_k||_{L^2} \overset{k\to\infty}{\longrightarrow} 0\end{aligned}$$

for some constant C. Continuity in the first argument follows analogously. By

$$\left| \langle \theta, \theta \rangle_{L/R} - \langle \theta_k, \theta_k \rangle_{L/R} \right| \leq \left| \langle \theta, \theta \rangle_{L/R} - \langle \theta, \theta_k \rangle_{L/R} \right| + \left| \langle \theta, \theta_k \rangle_{L/R} - \langle \theta_k, \theta_k \rangle_{L/R} \right| \overset{k\to\infty}{\longrightarrow} 0$$

we immediately have continuity of $\vartheta \mapsto \langle \vartheta, \vartheta \rangle_{L/R}$.

We would like to show $\langle \vartheta, \vartheta \rangle_L \leq \langle \vartheta, \vartheta \rangle_R$ for regular step functions $\vartheta \in \mathcal{T} = \left\{ \sum_{i=0}^{N-1} a_i \chi_{[\frac{i}{N}T, \frac{i+1}{N}T]} \,\middle|\, N \in \mathbb{N}, a_i \in \mathbb{R} \right\}$. The lemma then follows by the above-shown continuity and the density of regular step functions in $L^2([0,T])$: It is well-known that step functions are dense in $L^2([0,T])$, so density of $\mathcal{T}$ in the space of step functions remains to be shown. Let $f = \sum_{j=0}^{n-1} b_j \chi_{[t_j, t_{j+1}]}$, $0 = t_0 \leq \cdots \leq t_n = T$. Set $f_k = \sum_{j=0}^{k-1} f(\frac{j}{k}T) \chi_{[\frac{j}{k}T, \frac{j+1}{k}T]} \in \mathcal{T}$. f_k equals f on all intervals $[\frac{j}{k}T, \frac{j+1}{k}T]$ which lie completely in an interval $[t_i, t_{i+1}]$. This is not the case for a maximum number of n intervals $[\frac{j}{k}T, \frac{j+1}{k}T]$. Hence, $||f_k - f||_{L^2} \leq n \frac{T}{k} (\max_{i,j} |b_i - b_j|)^2 \overset{k\to\infty}{\longrightarrow} 0$.

Now, let $\vartheta = \sum_{i=0}^{N-1} a_i \chi_{[\frac{i}{N}T, \frac{i+1}{N}T]}$ and assume $\alpha \neq 0$. Then,

$$\begin{aligned}\langle \vartheta, \vartheta \rangle_R &= \frac{e^{\alpha T} - 1}{\alpha} \sum_{j=0}^{N-1} a_j^2 \int_{\frac{j}{N}T}^{\frac{j+1}{N}T} 1 \mathrm{d}t = \frac{e^{\alpha T} - 1}{\alpha} \sum_{j=0}^{N-1} a_j^2 \frac{T}{N}, \\ \langle \vartheta, \vartheta \rangle_L &= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} a_i a_j \int_{\frac{i}{N}T}^{\frac{i+1}{N}T} \int_{\frac{j}{N}T}^{\frac{j+1}{N}T} \mathrm{e}^{\alpha(\tau - t + TH(t-\tau))} \mathrm{d}\tau \mathrm{d}t \\ &= \frac{\left(1 - e^{\alpha \frac{T}{N}}\right)\left(1 - e^{-\alpha \frac{T}{N}}\right)}{\alpha^2} a A a^{\mathrm{T}} + \frac{e^{\alpha T} - 1}{\alpha} \sum_{j=0}^{N-1} a_j^2 \frac{T}{N},\end{aligned}$$

where $a = (a_0, \ldots, a_{N-1})^{\mathrm{T}}$ and the matrix A is given by

$$A_{ij} = \begin{cases} -e^{\alpha \frac{i-j}{N} T}, & i > j, \\ -e^{\alpha \frac{N+i-j}{N} T}, & i < j, \\ \frac{e^{\alpha \frac{T}{N}} - e^{\alpha T}}{1-e^{\alpha \frac{T}{N}}}, & i = j. \end{cases}$$

Hence, $\langle \vartheta, \vartheta \rangle_L \leq \langle \vartheta, \vartheta \rangle_R$ is equivalent to

$$\big(1-e^{\alpha \frac{T}{N}}\big)\big(1-e^{-\alpha \frac{T}{N}}\big) a A a^T \leq 0 \quad \Leftrightarrow \quad a A a^{\mathrm{T}} \geq 0 \quad \Leftrightarrow \quad a B a^T := \frac{1}{2} a (A + A^{\mathrm{T}}) a^{\mathrm{T}} \geq 0.$$

However, this is true since B is weakly diagonally dominant with non-negative diagonal entries,

$$\sum_{j \neq i} |B_{ij}| = -\frac{1}{2} \left(\sum_{j \neq i} A_{ij} + \sum_{j \neq i} A_{ji} \right) = \frac{1}{2} 2 \sum_{j=1}^{N-1} e^{j \alpha \frac{T}{N}} = \frac{1 - e^{\alpha T}}{1 - e^{\alpha \frac{T}{N}}} - 1 = \frac{e^{\alpha \frac{T}{N}} - e^{\alpha T}}{1 - e^{\alpha \frac{T}{N}}} = B_{ii},$$

and thus by Gershgorin's theorem, B is positive semi-definite. For $\alpha = 0$, the inequality $\langle \vartheta, \vartheta \rangle_L \leq \langle \vartheta, \vartheta \rangle_R$ follows from continuity in $\alpha = 0$ (using de l'Hôpital's rule). □

A.3. Negative definite second variation of profit for constant price (Proposition 3.4)

$$
\begin{aligned}
&\left\langle \frac{\partial^2 \Pi/R_0}{\partial \xi^2}, \vartheta, \vartheta \right\rangle = \left\langle \frac{\partial \left\langle \frac{\partial \Pi/R_0}{\partial \xi}, \vartheta \right\rangle}{\partial \xi}, \vartheta \right\rangle = \frac{N}{T} \int_0^T \left\langle \frac{\partial x(t)}{\partial \xi}, \vartheta \right\rangle [R + \varphi(\Psi - 2x)]\vartheta + x[(\varphi\Psi)' - x\varphi']\vartheta\vartheta + \left[x(\varphi\Psi)'\vartheta - x^2\varphi'\vartheta + \left\langle \frac{\partial x(t)}{\partial \xi}, \vartheta \right\rangle (R + \varphi\Psi - 2\varphi x) \right] \vartheta \\
&+(\xi - c) \left[\left\langle \frac{\partial x(t)}{\partial \xi}, \vartheta \right\rangle (\varphi\Psi)'\vartheta + x(\varphi\Psi)''\vartheta\vartheta - 2x \left\langle \frac{\partial x(t)}{\partial \xi}, \vartheta \right\rangle \varphi'\vartheta - x^2\varphi''\vartheta\vartheta + \left\langle \frac{\partial^2 x(t)}{\partial \xi^2}, \vartheta, \vartheta \right\rangle (R + \varphi\Psi - 2\varphi x) + \left\langle \frac{\partial x(t)}{\partial \xi}, \vartheta \right\rangle \left((\varphi\Psi)'\vartheta - 2\varphi' x\vartheta - 2\varphi \left\langle \frac{\partial x(t)}{\partial \xi}, \vartheta \right\rangle \right) \right] \mathrm{d}t \\
&= \frac{N}{T} \int_0^T \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} [R + \varphi(\Psi - 2x)]\vartheta + x[(\varphi\Psi)' - x\varphi']\vartheta\vartheta \\
&\qquad + \left[x(\varphi\Psi)'\vartheta - x^2\varphi'\vartheta + \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} (R + \varphi\Psi - 2\varphi x) \right] \vartheta \\
&- \frac{\Psi}{\Psi'} \left[\frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} (\varphi\Psi)'\vartheta + x(\varphi\Psi)''\vartheta\vartheta - 2x \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \varphi'\vartheta - x^2\varphi''\vartheta\vartheta + \left\langle \frac{\partial^2 x(t)}{\partial \xi^2}, \vartheta, \vartheta \right\rangle (R + \varphi\Psi - 2\varphi x) \right. \\
&\left. + \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \left((\varphi\Psi)'\vartheta - 2\varphi' x\vartheta - 2\varphi \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \right) \right] \mathrm{d}t \\
&= \frac{N}{T} \int_0^T \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} (R - \varphi\Psi)\vartheta + \Psi\varphi\Psi'\vartheta\vartheta + \left[\Psi\varphi\Psi'\vartheta + \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} (R - \varphi\Psi) \right] \vartheta \\
&- \frac{\Psi}{\Psi'} \left[\frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} (\varphi\Psi)'\vartheta + \Psi(\varphi\Psi)''\vartheta\vartheta - 2\Psi \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \varphi'\vartheta - \Psi^2\varphi''\vartheta\vartheta + \left\langle \frac{\partial^2 x(t)}{\partial \xi^2}, \vartheta, \vartheta \right\rangle (R - \varphi\Psi) \right. \\
&\left. + \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \left(\varphi\Psi'\vartheta - \varphi'\Psi\vartheta - 2\varphi \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \right) \right] \mathrm{d}t \\
&= \frac{N}{T} \int_0^T \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} (R - \varphi\Psi)\vartheta + 2\Psi\varphi\Psi'\vartheta\vartheta \\
&+ \frac{\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \left[R\vartheta - 2\Psi\varphi\vartheta + \frac{\Psi^2}{\Psi'}\varphi'\vartheta + 2\varphi \frac{\Psi^2}{\Psi'} \frac{\left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \right] \\
&- \frac{\Psi^2}{\Psi'} \frac{\left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} (\varphi\Psi'\vartheta - \Psi\varphi'\vartheta) - 2\Psi^2\varphi'\vartheta\vartheta - \frac{\Psi^2}{\Psi'}\varphi\Psi''\vartheta\vartheta - \frac{\Psi}{\Psi'} \left\langle \frac{\partial^2 x(t)}{\partial \xi^2}, \vartheta, \vartheta \right\rangle (R - \varphi\Psi)\mathrm{d}t \\
&= \frac{N}{T} \int_0^T \frac{2\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \left(R\vartheta - 2\varphi\Psi\vartheta + \frac{\Psi^2}{\Psi'}\varphi'\vartheta + \frac{\varphi^2\Psi^2}{\exp(\varphi\Psi t)} \left[\int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{1}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right] \right) \\
&-2\Psi^2\varphi'\vartheta^2 - \frac{\Psi^2}{\Psi'}\varphi\Psi''\vartheta^2 + 2\Psi\varphi\Psi'\vartheta^2 - \frac{\Psi}{\Psi'}(R - \varphi\Psi) \left\langle \frac{\partial^2 x(t)}{\partial \xi^2}, \vartheta, \vartheta \right\rangle \mathrm{d}t \\
&= \frac{N}{T} \int_0^T \frac{2\Psi \left[\varphi\Psi' \int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{\varphi\Psi'}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right]}{\exp(\varphi\Psi t)} \left(R\vartheta - 2\varphi\Psi\vartheta + \frac{\Psi^2}{\Psi'}\varphi'\vartheta + \frac{\varphi^2\Psi^2}{\exp(\varphi\Psi t)} \left[\int_0^t \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta + \frac{1}{\exp(\varphi\Psi T) - 1} \int_0^T \vartheta \exp(\varphi\Psi\Theta)\mathrm{d}\Theta \right] \right) \mathrm{d}t \\
&+ \frac{N}{T} \left(-2\Psi^2\varphi' - \frac{\Psi^2}{\Psi'}\varphi\Psi'' + 2\Psi\varphi\Psi' \right) \int_0^T \vartheta^2 \mathrm{d}t - \frac{N}{T} \frac{\Psi}{\Psi'} (R - \varphi\Psi) \int_0^T \left\langle \frac{\partial^2 x(t)}{\partial \xi^2}, \vartheta, \vartheta \right\rangle \mathrm{d}t
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{N}{T}2\Psi\varphi\Psi'\int_0^T \frac{1}{\exp(\varphi\Psi t)}\int_0^t \vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\left[\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\vartheta+\frac{\varphi^2\Psi^2}{\exp(\varphi\Psi t)}\left(\int_0^t\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta+\frac{1}{\exp(\varphi\Psi T)-1}\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\right]\mathrm{d}t\\
&\quad+\frac{N}{T}\frac{2\Psi\varphi\Psi'}{\exp(\varphi\Psi T)-1}\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\left[\int_0^T\frac{1}{\exp(\varphi\Psi t)}\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\vartheta\mathrm{d}t+\int_0^T\frac{1}{\exp(\varphi\Psi t)}\frac{\varphi^2\Psi^2}{\exp(\varphi\Psi t)}\left(\int_0^t\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta+\frac{1}{\exp(\varphi\Psi T)-1}\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\mathrm{d}t\right]\\
&\quad+\frac{N}{T}\left(-2\Psi^2\varphi'-\frac{\Psi^2}{\Psi'}\varphi\Psi''+2\Psi\varphi\Psi'\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t\\
&= \frac{N}{T}2\Psi\varphi\Psi'\left[\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\frac{\vartheta}{\exp(\varphi\Psi t)}\int_0^t\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\mathrm{d}t+\varphi^2\Psi^2\int_0^T\frac{1}{\exp(\varphi\Psi t)^2}\left(\int_0^t\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)^2\mathrm{d}t+\frac{\varphi^2\Psi^2}{\exp(\varphi\Psi T)-1}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\int_0^T\frac{1}{\exp(\varphi\Psi t)^2}\int_0^t\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\mathrm{d}t\right]\\
&\quad+\frac{N}{T}\frac{2\Psi\varphi\Psi'}{\exp(\varphi\Psi T)-1}\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\left[\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t+\varphi^2\Psi^2\int_0^T\frac{1}{\exp(\varphi\Psi t)^2}\int_0^t\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\mathrm{d}t+\frac{\varphi^2\Psi^2}{\exp(\varphi\Psi T)-1}\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\int_0^T\frac{1}{\exp(\varphi\Psi t)^2}\mathrm{d}t\right]\\
&\quad+\frac{N}{T}\left(-2\Psi^2\varphi'-\frac{\Psi^2}{\Psi'}\varphi\Psi''+2\Psi\varphi\Psi'\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t\\
&= \frac{N}{T}2\Psi\varphi\Psi'\left[\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\int_\Theta^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\mathrm{d}\Theta+\frac{\varphi\Psi}{2}\int_0^T\vartheta\exp(\varphi\Psi\varpi)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\exp(-2\varphi\Psi\max\{\Theta,\varpi\})\mathrm{d}\Theta\mathrm{d}\varpi-\frac{\varphi\Psi}{2\exp(\varphi\Psi T)^2}\left(\int_0^T\vartheta\exp(\varphi\Psi\varpi)\mathrm{d}\varpi\right)^2\right.\\
&\quad\left.+\frac{\varphi\Psi}{2(\exp(\varphi\Psi T)-1)}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\int_0^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta-\frac{\varphi\Psi\exp(-2\varphi\Psi T)}{2(\exp(\varphi\Psi T)-1)}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)^2\right]+\frac{N}{T}\frac{2\Psi\varphi\Psi'}{\exp(\varphi\Psi T)-1}\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\left[\left(R-\frac{3}{2}\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\right.\\
&\quad\left.-\frac{\varphi\Psi}{2\exp(\varphi\Psi T)^2}\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta+\frac{\varphi\Psi}{2(\exp(\varphi\Psi T)-1)}\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\,(1-\exp(-2\varphi\Psi T))\right]+\frac{N}{T}\left(-2\Psi^2\varphi'-\frac{\Psi^2}{\Psi'}\varphi\Psi''+2\Psi\varphi\Psi'\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t\\
&= \frac{N}{T}2\Psi\varphi\Psi'\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\int_\Theta^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\mathrm{d}\Theta+\frac{N}{T}(\Psi'\varphi^2\Psi^2)\int_0^T\vartheta\exp(\varphi\Psi\varpi)\int_0^T\frac{\vartheta\exp(\varphi\Psi\Theta)}{\exp(\varphi\Psi\max\{\Theta,\varpi\})^2}\mathrm{d}\Theta\mathrm{d}\varpi-\frac{N}{T}\frac{\Psi'\varphi^2\Psi^2}{\exp(\varphi\Psi T)^2}\left(\int_0^T\vartheta\exp(\varphi\Psi\varpi)\mathrm{d}\varpi\right)^2\\
&\quad+\frac{N}{T}\frac{\Psi'\varphi^2\Psi^2}{(\exp(\varphi\Psi T)-1)}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\int_0^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta-\frac{N}{T}\frac{\Psi'\varphi^2\Psi^2}{\exp(\varphi\Psi T)^2(\exp(\varphi\Psi T)-1)}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)^2+\frac{N}{T}\frac{2\Psi\varphi\Psi'}{\exp(\varphi\Psi T)-1}\left(R-\frac{3}{2}\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\int_0^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\\
&\quad+\frac{N}{T}\frac{\Psi^2\varphi^2\Psi'}{\exp(\varphi\Psi T)^2-\exp(\varphi\Psi T)}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)^2+\frac{N}{T}\left(-2\Psi^2\varphi'-\frac{\Psi^2}{\Psi'}\varphi\Psi''+2\Psi\varphi\Psi'\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t\\
&= \frac{N}{T}2\Psi\varphi\Psi'\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\int_\Theta^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\mathrm{d}\Theta+\frac{N}{T}(\Psi'\varphi^2\Psi^2)\int_0^T\vartheta\exp(\varphi\Psi\varpi)\int_0^T\frac{\vartheta\exp(\varphi\Psi\Theta)}{\exp(\varphi\Psi\max\{\Theta,\varpi\})^2}\mathrm{d}\Theta\mathrm{d}\varpi\\
&\quad+\frac{N}{T}2\Psi\varphi\frac{\Psi'R-\Psi\varphi\Psi'+\Psi^2\varphi'}{\exp(\varphi\Psi T)-1}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\left(\int_0^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta\right)+\frac{N}{T}\left(-2\Psi^2\varphi'-\frac{\Psi^2}{\Psi'}\varphi\Psi''+2\Psi\varphi\Psi'\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t\\
&= \frac{N}{T}2\Psi\varphi\Psi'\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\int_\Theta^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\mathrm{d}\Theta+\frac{N}{T}(\Psi'\varphi^2\Psi^2)\int_0^T\vartheta\exp(\varphi\Psi\varpi)\left(\int_\varpi^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta+\int_0^\varpi\frac{\vartheta\exp(\varphi\Psi\Theta)}{\exp(2\varphi\Psi\varpi)}\mathrm{d}\Theta\right)\mathrm{d}\varpi\\
&\quad+\frac{N}{T}2\Psi\varphi\frac{\Psi'R-\Psi\varphi\Psi'+\Psi^2\varphi'}{\exp(\varphi\Psi T)-1}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\left(\int_0^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta\right)+\frac{N}{T}\left(-2\Psi^2\varphi'-\frac{\Psi^2}{\Psi'}\varphi\Psi''+2\Psi\varphi\Psi'\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t\\
&= \frac{N}{T}2\Psi\varphi\Psi'\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\int_\Theta^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\mathrm{d}\Theta+\frac{N}{T}(\Psi'\varphi^2\Psi^2)\int_0^T\vartheta\exp(\varphi\Psi\varpi)\int_\varpi^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta\mathrm{d}\varpi+\frac{N}{T}(\Psi'\varphi^2\Psi^2)\int_0^T\frac{\vartheta}{\exp(\varphi\Psi\varpi)}\int_0^\varpi\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\mathrm{d}\varpi\\
&\quad+\frac{N}{T}2\Psi\varphi\frac{\Psi'R-\Psi\varphi\Psi'+\Psi^2\varphi'}{\exp(\varphi\Psi T)-1}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\left(\int_0^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta\right)+\frac{N}{T}\left(-2\Psi^2\varphi'-\frac{\Psi^2}{\Psi'}\varphi\Psi''+2\Psi\varphi\Psi'\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t\\
&= \frac{N}{T}2\Psi\varphi\Psi'\left(R-2\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\int_\Theta^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\mathrm{d}\Theta+\frac{N}{T}(\Psi'\varphi^2\Psi^2)\int_0^T\vartheta\exp(\varphi\Psi\varpi)\int_\varpi^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta\mathrm{d}\varpi+\frac{N}{T}(\Psi'\varphi^2\Psi^2)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\int_\Theta^T\frac{\vartheta}{\exp(\varphi\Psi\varpi)}\mathrm{d}\varpi\mathrm{d}\Theta\\
&\quad+\frac{N}{T}2\Psi\varphi\frac{\Psi'R-\Psi\varphi\Psi'+\Psi^2\varphi'}{\exp(\varphi\Psi T)-1}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\left(\int_0^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta\right)+\frac{N}{T}\left(-2\Psi^2\varphi'-\frac{\Psi^2}{\Psi'}\varphi\Psi''+2\Psi\varphi\Psi'\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t\\
&= \frac{N}{T}2\Psi\varphi\Psi'\left(R-\varphi\Psi+\frac{\Psi^2}{\Psi'}\varphi'\right)\int_0^T\vartheta\exp(\varphi\Psi\Theta)\int_\Theta^T\frac{\vartheta}{\exp(\varphi\Psi t)}\mathrm{d}t\mathrm{d}\Theta+\frac{N}{T}2\Psi\varphi\Psi'\left(R-\Psi\varphi+\frac{\Psi^2}{\Psi'}\varphi'\right)\frac{1}{\exp(\varphi\Psi T)-1}\left(\int_0^T\vartheta\exp(\varphi\Psi\Theta)\mathrm{d}\Theta\right)\left(\int_0^T\frac{\vartheta}{\exp(\varphi\Psi\Theta)}\mathrm{d}\Theta\right)\\
&\quad+\frac{N}{T}2\Psi\varphi\Psi'\left(-\frac{\Psi\varphi'}{\varphi\Psi'}-\frac{\Psi\Psi''}{2(\Psi')^2}+1\right)\int_0^T\vartheta^2\mathrm{d}t-\frac{N}{T}\frac{\Psi}{\Psi'}(R-\varphi\Psi)\int_0^T\left\langle\frac{\partial^2x(t)}{\partial\xi^2},\vartheta,\vartheta\right\rangle\mathrm{d}t
\end{aligned}
$$

A.4. Oligopoly price for example 3.3

Proof. Let A_n and $\Lambda = (A_n^{-1})$ be as in example 3.3. Furthermore, let us write vectors $(a_i)_{i=1,\dots,n}$ as $\vec{a}$ and diagonal $n \times n$ matrices with diagonal entries $(a_i)_{i=1,\dots,n}$ as $\bar{a}$. If all n goods coexist on the market, then for the steady state the mean dynamic (7) can be rewritten as

$$\vec{\Psi} = A_n \vec{x}.$$

According to equation (3), the sales $\vec{S}$ and the profit $\vec{\Pi}$ then adopt the form

$$\vec{S} = N\bar{R}\vec{x} = N\bar{R}A_n^{-1}\vec{\Psi} \quad \text{and} \quad \vec{\Pi} = (\Xi - \bar{c})\vec{S} = (\Xi - \bar{c})N\bar{R}A_n^{-1}\vec{\Psi}.$$

Letting $(A_n^{-1})_i$ denote the ith row of matrix A_n^{-1}, the Nash equilibrium is found by solving the system of first order optimality conditions

$$0 = \frac{\mathrm{d}\Pi_i}{\mathrm{d}\xi_i} = NR_i \left(A_n^{-1}\right)_i \left[\vec{\Psi} + (\xi_i - c_i)\frac{\mathrm{d}\vec{\Psi}}{\mathrm{d}\xi_i}\right], \quad i = 1, \dots, n,$$

where $\frac{\mathrm{d}\vec{\Psi}}{\mathrm{d}\xi_i} = \left(0, \dots, 0, \frac{1-\Psi_i}{\varphi_i}, 0, \dots, 0\right)^T \frac{\mathrm{d}\varphi_i}{\mathrm{d}\xi_i}$. For the system to be analytically solvable, we chose $\varphi_i = \left(1 + \frac{\xi_i}{\Xi_i}\right)^{-1}$ and hence $\frac{d\varphi_i}{d\xi_i} = -\frac{\varphi_i^2}{\Xi_i}$. Then, for $i = 1, \dots, n$ the optimality condition can be equivalently transformed into

$$\Lambda_i \begin{pmatrix} 1 - \frac{1-\Phi_1}{\Xi_1}(\Xi_1 + \xi_1) \\ \vdots \\ 1 - \frac{1-\Phi_i}{\Xi_i}(\Xi_i - c_i + 2\xi_i) \\ 1 - \frac{1-\Phi_{i+1}}{\Xi_{i+1}}(\Xi_{i+1} + \xi_{i+1}) \\ \vdots \end{pmatrix} = 0,$$

or expressed as a sum,

$$\sum_{j=1}^{n} \Lambda_{ij} \left(\Phi_j - \frac{1 - \Phi_j}{\Xi_j}\xi_j\right) - \Lambda_{ii}\left(\frac{1 - \Phi_i}{\Xi_i}\xi_i - \frac{(1 - \Phi_i)c_i}{\Xi_i}\right) = 0, \quad i = 1, \dots, n.$$

In matrix notation this reads

$$\left[\Lambda + \begin{pmatrix} \Lambda_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Lambda_{nn} \end{pmatrix}\right] \begin{pmatrix} \frac{1-\Phi_1}{\Xi_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1-\Phi_n}{\Xi_n} \end{pmatrix} \vec{\xi} = \Lambda\vec{\Phi} + \begin{pmatrix} (1 - \Phi_1)\Lambda_{11}\frac{c_1}{\Xi_1} \\ \vdots \\ (1 - \Phi_n)\Lambda_{nn}\frac{c_n}{\Xi_n} \end{pmatrix},$$

which directly implies the solution for the optimal prices $\vec{\xi}^*$. □

A.5. Proof of lemma 3.10

Proof. In Matzke and Wirth (2008) we have shown

$$\det \begin{pmatrix} 1 & \alpha_2 & \cdots & \alpha_m \\ \alpha_1 & 1 & & \vdots \\ \vdots & \vdots & \ddots & \alpha_m \\ \alpha_1 & \alpha_2 & \dots & 1 \end{pmatrix} > 0 \tag{15}$$

for any $m > 0$ and $0 < \alpha_i < 1$, $i = 1, \dots, m$, which we will need later.

According to Cramer's rule

$$\Lambda_{ij} = \frac{\det(A_n)_{i \to e_j}}{\det(A_n)},$$

where $(A_n)_{i \to e_j}$ denotes matrix A_n with the ith column replaced by the jth unit vector e_j. For $i \neq j$, using Laplace expansion along the ith column and subsequent column and row interchanges of the (i, j)-minor matrix of A_n, we obtain

$$\det(A_n)_{i \to e_j} = -\det B_0,$$

where (without loss of generality assuming $i < j$)

$$B_0 = \left(\begin{array}{cccccccccc} \Phi_j & \Phi_1 & \cdots & \Phi_{i-1} & \Phi_{i+1} & \cdots & \Phi_{j-1} & \Phi_{j+1} & \cdots & \Phi_n \\ \Phi_j & 1 & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ \vdots & \Phi_1 & \ddots & \Phi_{i-1} & \vdots & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & 1 & \Phi_{i+1} & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \Phi_{i-1} & 1 & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots & \Phi_{i+1} & \ddots & \Phi_{j-1} & \vdots & & \vdots \\ \vdots & \vdots & & \vdots & \vdots & & 1 & \Phi_{j+1} & & \vdots \\ \vdots & \vdots & & \vdots & \vdots & & \Phi_{j-1} & 1 & & \vdots \\ \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \ddots & \Phi_n \\ \Phi_j & \Phi_1 & \cdots & \Phi_{i-1} & \Phi_{i+1} & \cdots & \Phi_{j-1} & \Phi_{j+1} & \cdots & 1 \end{array} \right).$$

In order to show $\det B_0 \leq 0$, let us define B_t for $0 \leq t \leq 1$ by replacing the $(1, 1)$-entry in B_0 by $\Phi_j + t(1 - \Phi_j)$. Now, $t \mapsto \det B_t$ is continuous with $\det B_1 > 0$ due to (15). Hence, if we had $\det B_0 < 0$, then by Rolle's theorem there would be some $t \in (0, 1)$ with $\det B_t = 0$. However,

$$\det B_t = (\Phi_j + t(1 - \Phi_j)) \det \begin{pmatrix} 1 & \Phi_1 & \cdots & \Phi_n \\ \frac{\Phi_j}{\Phi_j + t(1-\Phi_j)} & 1 & & \vdots \\ \vdots & \vdots & \ddots & \Phi_n \\ \frac{\Phi_j}{\Phi_j + t(1-\Phi_j)} & \Phi_1 & \cdots & 1 \end{pmatrix},$$

which due to (15) is larger than zero, since $0 < \frac{\Phi_j}{\Phi_j + t(1-\Phi_j)} < 1$. This contradicts $\det B_t = 0$ so that our initial assumption $\det B_0 < 0$ is wrong. Also, we have $\det A_n > 0$ due to (15) so that finally, $\Lambda_{ij} = \frac{-\det B_0}{\det A_n} \leq 0$. Moreover, from $1 =$

$\sum_{j=1}^{n} \Lambda_{ij}(A_n)_{ji} = \Lambda_{ii} + \sum_{j \neq i} \Lambda_{ij}(A_n)_{ji} \leq \Lambda_{ii}$ we obtain $\Lambda_{ii} \geq 1$. □

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