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A rationale for the coexistence of central and decentral marketing in team sports

by

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A rationale for the coexistence of central and decentral marketing in team

sports*

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Abstract

In some sports leagues, the sports association sells broadcasting rights centrally in order to

create competitive balance. In other ones, the market is decentral. As a result, there is

competitive imbalance. In this paper, the preferred kind of marketing of sports associations is

analysed. Distinctions are made between three cases. In case one, the sports association is

only interested in competitive balance. In the second case, it wishes to create a single high

performing team, and in the third, it maximises aggregate performance. It is found that,

depending on the preferences of the association, both kinds of marketing can be optimal.

Key words: central marketing, decentral marketing, collective tournament, complementarity.

JEL classification: D2, L1, L8, M5

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1. Introduction

In some sports leagues, the respective sports association restricts the possible actions of the clubs in order to create a balanced team structure. An example of this is German Major League Soccer. In Germany, the soccer clubs are limited to central marketing by the German Soccer Association (DFB). With this central marketing, the DFB wishes to create a higher competitive balance. It distributes the TV revenues, resulting from the central marketing, more or less equally to the soccer clubs¹ so that they are all able to invest in the quality of their respective team.²

According to Szymanski (2003) there are three core claims justifying these interventions:

- 1. Inequality of resources leads to uneven competition.
- 2. Fan interest declines when outcomes become more predictable.
- 3. Specific redistribution mechanisms produce more outcome uncertainty.³

However, in other sports leagues the respective sports association does not intervene in order to increase the degree of homogeneity between the teams. In Italian or Spanish Major League Soccer, e.g., marketing is completely decentral. Consequently, the clubs have the possibility to sell the rights to broadcast their home matches on their own. Due to the fact that very successful teams attract more viewers, the TV stations pay high prices to televise the home matches of the famous teams, while the less famous teams receive substantially lower payments.⁴ Since high payments entail the opportunity of increasing team quality, such decentral marketing leads to rather uneven teams. In the past, some small Italian soccer clubs

¹ In Germany, 50% of the TV revenues are shared equally, while the other 50% are distributed based on performance.

² The central marketing only affects the allocation of TV revenues. In all other items, e.g. sponsoring, the clubs are allowed to market decentrally. Since TV revenues represent the main part of the clubs' incomes, an equal allocation of these revenues decreases the differences in the clubs' incomes in a significant way.

³ In this paper, competitive balance and outcome uncertainty are used as synonyms. This is only for simplicity. In most team sports, the teams are more likely to win their home matches than their away matches. Therefore, total outcome uncertainty would be given if the away team was more able than the home team and not if the teams were equally able. See, e.g., Forrest and Simmons (2002) for a more detailed description of this item.

⁴ In the season 2002/2003 the Italian soccer club Juventus Turin received a payment of about 60 Million Euro for its television rights, while smaller clubs like Atalanta Bergamo or Chievo Verona received a payment of less than 10 Million Euro.

went on strike several times to get higher TV revenues. These mainly unsuccessful strikes caused the cancellation of several days of play in Italian Major League Soccer. Clearly, the question arises of why the Italian Soccer Association (FIGC) does not also install an instrument like central marketing to intensify competition between clubs.

An answer could be that competitive balance is not the only determinant of fan interest. For example, one could think that fans like to see top performances.⁵ A team consisting of several superstars might play outstanding soccer that enthuses a great many spectators. Although such a team would be a clear favourite in the championships, i.e., there would be only little outcome uncertainty, fan attendance could still be very high. The fans might enjoy the great performance of the superstar team. However, if all national clubs receive similar payments, no club might have enough money to engage a couple of national or international superstars and, therefore, fan interest might be very small. In this case, a mechanism like the central marketing is very inappropriate.⁶

In soccer, there is a second reason why decentral marketing could be preferred by the association. The best teams of the European soccer leagues qualify for the "European Champions League", a tournament arranged by the European Soccer Association (UEFA). A national soccer association might therefore be interested in creating several very strong teams, for these teams are more likely to be successful in the inter-country competition. Thus, the soccer association is not necessarily interested in creating competitive balance. It might well be interested in creating several high-performing teams or in maximising the aggregate quality of the league.

Most of the related literature analyses empirically whether fan attendance increases in outcome uncertainty or not (see, e.g., Hart et al., 1975; Jennett, 1984; Peel and Thomas, 1988,

⁵ See for a more general discussion of the importance of absolute performance in the sports context Hoehn and Szymanski (1999).

⁶ The point that fans are highly interested in outstanding performances is also of importance in other sports. In athletics, e.g., the organiser of a meeting tries to invite one or more outstanding teams (such as a relay) that might break a world record since fan enthusiasm in athletics significantly increases in the number of top performances.

1992, 1997; Forrest and Simmons 2002). Almost all papers support the hypothesis that a higher outcome certainty leads to a lower fan attendance. A normative analysis of central and decentral marketing in European team sports is offered by Falconieri et al. (2004). The authors determine the social welfare in both kinds of marketing and show the respective conditions under which each is socially optimal. In their analysis, they assume that the sports association will always install central marketing if it is in possession of the league's broadcasting rights. On the other hand, decentral marketing will only be possible if the soccer clubs possess these rights. The authors do not allow for the possibility that a sports association possessing TV rights installs decentral marketing voluntarily. Further, they do not analyse the association's preferences in detail. In contrast to their analysis, in this paper, it is assumed that there is a national sports association that is in possession of the league's broadcasting rights, and that is allowed to choose between central or decentral marketing. In this setting, I determine formally what kind of marketing is optimal for the association. I will distinguish between three situations. In situation (i), the association is interested only in competitive balance, in situation (ii), it is interested in creating one high-performing team, and in situation (iii), it wants to maximise the aggregate performance in the league. Several results will be derived in this paper. If the sports association is interested only in competitive balance, it will always choose central marketing. In the other two cases, the association's decision is influenced by three countervailing effects. Hence, its decision, whether to choose central or decentral marketing, is determined by which of the effects is dominant.

The paper is organised as follows: In section 2, the model is introduced. Section 3 contains the solution to the model and shows up the decision of the sports association. Additionally, the clubs' and players' preferred kind of marketing is analysed. Concluding remarks are offered in section 4.

2. Description of the model and notation

In the model, there are several parties: A sports association, the boards of two clubs (in what follows) referred to as "the clubs", and four players that form the two teams of the clubs.

The sports association: There is a sports association organising a league. It can either market the league's broadcasting rights centrally or allow the clubs to market decentrally. It is assumed that in each kind of marketing total TV revenue is the same, only its allocation between the two clubs differs. Total TV revenue is denoted by R. It is further assumed that, in case of central marketing, the sports association shares some part of the revenue equally between the two clubs, while the other part is shared according to the teams' previous season performances. In this context, we assume, without loss of generality, that the team of the first club has won the previous season league competition. Formally, the sports association determines some allocation parameter θ such that the first club receives $(1+\theta)\cdot R/2$ and the second $(1-\theta)\cdot R/2$. 8 If the clubs market decentrally, the revenue will be shared according to some rule $(\alpha, 1-\alpha)$, where the first club receives $\alpha \cdot R$ and the second one $(1-\alpha) \cdot R$. Suppose that, under central marketing, there is resistance against very uneven allocation rules, that is, the sports association is not allowed to choose θ in an arbitrary way. Particularly, let the association be restricted to choose θ from an interval $\left[\,0\,,\widetilde{\theta}\,\,\right]\!,$ where $\widetilde{\theta} < Max\{2 \cdot \alpha - 1, 1 - 2 \cdot \alpha\}$. The inequality means that resource allocation under decentral marketing is more uneven than under central marketing, which seems to map practice very well. Concerning the preferences of the association, it is assumed that it wishes to maximise the fan interest in its league. However, fan interest might increase for different reasons. I

⁷ Clearly, one could think that due to different bargaining positions, an association could achieve different total revenue from the clubs. However, in this paper, we focus on the use of central marketing as a redistribution mechanism. In particular, we ask whether the association always wants to redistribute resources within the league it organises or not. Assuming a fixed total TV revenue helps to eliminate effects distorting this analysis.

⁸ In practice, the installation of an allocation rule usually occurs before the season starts and not thereafter as assumed in the model. However, this is not very problematic. The model could be interpreted such that the association cannot influence the clubs' budgets and so the outcome in the next season, but in the season following the next. Hence, it tries to maximise fan interest in that season.

⁹ In each firm, it is problematic for the management when demand for the firm's products is very low. The management would be made responsible for the low demand and has to fear dismissals or wage reductions.

therefore distinguish between three situations. In situation (i), fan interest increases in the level of competitive balance, so the association wishes to minimise $|y_1 - y_2|$, where y_1 denotes the performance of team j (j=1,2). In situation (ii), fans are only interested in outstanding performances, the association then wishes to maximise $\max\{y_1, y_2\}$. In the last case (iii), the association maximises $y_1 + y_2$, since fan interest is determined by the aggregate quality of the league. Clearly, in practice, fan interest may be determined by all of these arguments. Yet, I prefer to firstly analyse situations (i) to (iii) in isolation, since, in this way, the effects influencing the association's decision can be made clearer. Subsequently, I shortly discuss what the association's preferred kind of marketing is when it cares for all three arguments.

The clubs: As mentioned before, there are two clubs. The first club is assumed to be quite famous, while the other is assumed to be less famous. That is, the first club is, e.g. due to historical successes, very popular and therefore attracts more fans than the second. Note that it is not important to our results that the team of the famous club has won the previous season league competition. Each club competes with one team in the league and so has to hire (and pay) two players, respectively. In this context, it is assumed that TV revenue is a club's only source of income. Hence, the budgets of the two clubs are $(1+\theta)\cdot R/2$ and $(1-\theta)\cdot R/2$ in the case of central marketing. In the case of decentral marketing, club 1 has a budget of $\alpha \cdot R$ and club 2 of $(1-\alpha) \cdot R$, where $\alpha > 1/2$. It is further assumed that a club is interested in maximising the winning probability of its team. 11 In the majority of clubs, boards do not gain from club profits and, hence, they do not care much about them. Moreover, if a club is very successful, the members of this club's board receive utility in terms of social prestige.

Hence, the management is interested in creating a high demand for the firm's products. Clearly, the same argumentation holds for the officials of an association. So they are also interested in maximising fan interest. ¹⁰ As already mentioned, the clubs have other sources of income. Since the TV revenues are of such high relevance, it is assumed for simplicity that all other incomes equal zero. ¹¹ See, for example, Dietl et al. (2003).

Consequently, they wish to maximise their expected social prestige and, on account of this, they wish to maximise the winning-probability of their team.

The teams: There are two teams competing against each other. Each team consists of two players, and each player has to handle a single task. ¹² As seems natural in team sports, assume that the performance of a team is characterised by the existence of strong complementarities between the single players' tasks, i.e., team performance will be constrained by the player who performs rather badly. A soccer team may have very good forwards, but if its goalkeeper is totally incapable, this team will not be very successful.¹³

Suppose that player i of team j chooses the effort level e_{ii} as a continuous variable from the interval $\left[\,0\,,k\,\right]$, where k is so high that the restriction $\,e_{_{ji}}\leq k\,$ never binds in equilibrium. To formally introduce the complementarities between the players' performances, it is assumed that the performance of team j (j=1,2) is:

(1)
$$y_j = e_{j1} \cdot e_{j2}^{14}$$

The two teams compete for a bonus b>0. This bonus is assumed to be exogenously given¹⁵ and equally distributed among the members of the winning team. 16 Moreover, as typical in sports economics, league competition is modelled as a logit-form contest (see, e.g., Hoehn and Szymanski, 1999, Szymanski, 2003, or for the use of logit form contests in other fields of economics Skaperdas, 1996; Gradstein and Konrad, 1999; Huck et al. 2001). Therefore, the contest success function of team 1, i.e., the probability of winning, is given by

¹² For example, each team could consist of one defender and one forward.

¹³ See, for a formal analysis of complementarities in production e.g. Milgrom/Roberts (1990), (1995) or Kremer

¹⁴ I admit that this is a very extreme assumption, since the function is characterised by the existence of very strong complementarities. However, the main effects to be derived in this paper are all qualitatively the same for an arbitrary, concave and supermodular function.

¹⁵ This assumption can be justified as follows: In the European sport leagues, the players receive winning bonuses from their clubs. However, these bonuses are negligibly small in comparison to the players' fixed wages. Hence, there must be another driving force that motivates the players. On the one hand, there is career concern. If a team presently performs very badly, the players of this team will have problems to sign a new contract with a soccer team in the future, since their present performance serves as a signal for future performance. Secondly, a player feels proud when his team wins a match. Therefore, he gets a non monetary gain from belonging to the winning team. These two elements are combined to an exogenously given bonus.

16 One could alternatively assume that the bonus possesses public-good characteristics. In this case, the bonus

need not be shared between the members of the winning team.

(2)
$$P_{1} = \begin{cases} \frac{e_{11} \cdot e_{12}}{e_{11} \cdot e_{12} + e_{21} \cdot e_{22}} & \text{for } e_{11} \cdot e_{12} + e_{21} \cdot e_{22} > 0 \text{ and} \\ 0.5 & \text{otherwise.} \end{cases}$$

The winning probability of team 2 is $P_2 = 1 - P_1$.¹⁷

The players: In the player market, there are $n_H \ge 4$ high-ability players with ability a_H and $n_L \ge 4$ low-ability players with ability $a_L (a_H > a_L)$. For simplicity and without loss of generality, I normalise a_L to 1. The players' abilities are common knowledge among all players as well as among the clubs and the association.¹⁸ A high-ability player is assumed to have a cost advantage, hence he exerts a certain effort more easily than a low-ability player. In order to introduce this cost advantage formally, I assume that costs, entailed by effort, are $C(e_{ji}) = \frac{1}{a_{ij}} \cdot e_{ji}^{\delta}, \delta > 1$, where a_{ji} denotes the ability of player i of team j and the parameter restriction is introduced to ensure the existence of all equilibria to be derived. The parameter δ determines the degree of convexity of this cost function and, for this reason, strongly influences the effective cost advantage of a high-ability player. Without modelling the market for players in detail, it is assumed that a low-ability player accepts a contract at a lower wage offer than a high-ability player. Let $\overline{w}_{_H}$ ($\overline{w}_{_L}$) denote the reservation wage that has to be paid for hiring a high-(low-)ability player. 19 These reservation wages indicate that there are clubs in other (usually foreign) leagues being interested in hiring the players. Further assume that $\alpha \cdot R \ge 2 \cdot \overline{w}_H > \overline{w}_H > (1 - \alpha) \cdot R$ and $\overline{w}_L = 0$ hold. The meaning of the inequalities becomes clear in the next section. The restriction on \overline{w}_L is made to ensure that each team is always

¹⁷ It should be noted that this paper is naturally also related to the literature on group rent-seeking contests. See, for this literature, e.g., Katz, Nitzan, Rosenberg (1990), Nitzan (1991), or Lee (1995). The main difference between these papers and the current one is that team performance in sports is usually characterised by the existence of strong complementarities between the single players' performances. In contrast, the contributions of the participants in a group contest are usually not complementary.

¹⁸ One could imagine that the players have played in the soccer league for a considerable period. As a consequence, their previous performances could be used as a good measure of their quality.

¹⁹ In proposition 5, it is shown that the players have an interest to self-select into teams in order to maximise the probability of receiving the bonus. It is in this context assumed that the bonus is relatively small in comparison to the reservation wages so that it cannot be used to hire high-ability players.

able to hire low-ability players. Lastly, all players are assumed to be risk-neutral. Hence, after being hired, player i of team j chooses his effort e_{ji} to maximise $EU_{ji} = P_j \cdot \frac{b}{2} - C(e_{ji})$.

The timing of the model is as follows: In the first stage, the association chooses whether to market centrally or decentrally. In case of central marketing, it also determines the allocation rule. In the second stage, the two clubs hire players for their teams, respectively. In the third stage, the players choose their optimal efforts.

3. Solution to the model

Efforts and player allocation to the teams:

As described in section 2, in the third stage of the model, player i of team j chooses his effort to maximise $EU_{ji} = P_j \cdot \frac{b}{2} - C(e_{ji})$. Inserting the winning probability of team j according to equation (2) and the cost function $C(e_{ji})$ yields the subsequent maximisation problem:

(3)
$$\text{Max EU}_{ji} = \frac{e_{ji} \cdot e_{jm}}{e_{ji} \cdot e_{jm} + e_{k1} \cdot e_{k2}} \cdot \frac{b}{2} - \frac{1}{a_{ji}} e_{ji}^{\delta},$$
with $i = 1, 2, j = 1, 2, k = 1, 2, m = 1, 2, i \neq m, k \neq j$.

This maximisation problem leads to the following first-order condition:

(4)
$$\frac{\partial EU_{ji}}{\partial e_{ii}} = \frac{e_{jm} \cdot e_{k1} \cdot e_{k2}}{\left(e_{ii} \cdot e_{im} + e_{k1} \cdot e_{k2}\right)^2} \cdot \frac{b}{2} - \frac{\delta}{a_{ii}} e_{ji}^{\delta - 1} \stackrel{!}{=} 0.20$$

The second-order condition is satisfied. The optimal players' efforts depend on the composition of the two teams. Since the clubs decide about which players to hire and, hence, about the team composition, I now turn to stage 2 of the model.

By hiring appropriate players, a club wants to maximise its team's winning-probability. For each kind of feasible team composition, one can use the first-order conditions to derive the

²⁰ We consider the Nash-equilibrium, where each player exerts positive effort. However, there exists a second trivial Nash-equilibrium, where each player exerts zero effort.

players' efforts and, from these, the teams' winning-probabilities. The winning-probabilities are given in the following matrix:²¹

Club 2

		Н,Н	H,L	L,L
	Н,Н	0.5, 0.5	$ (a_{\rm H})^{\frac{1}{\delta}} / (1 + (a_{\rm H})^{\frac{1}{\delta}}), $ $ 1 / (1 + (a_{\rm H})^{\frac{1}{\delta}}) $	$ \frac{1}{\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}} / \left(1 + \left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}\right), $ $ \frac{1}{\left(1 + \left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}\right)} $
Club 1	H,L	$\frac{1}{\left(1+\left(a_{H}\right)^{\frac{1}{\delta}}\right)},$ $\left(a_{H}\right)^{\frac{1}{\delta}}/\left(1+\left(a_{H}\right)^{\frac{1}{\delta}}\right)$	0.5, 0.5	$ \frac{\left(\mathbf{a}_{\mathrm{H}}\right)^{\frac{1}{\delta}} / \left(1 + \left(\mathbf{a}_{\mathrm{H}}\right)^{\frac{1}{\delta}}\right), }{1 / \left(1 + \left(\mathbf{a}_{\mathrm{H}}\right)^{\frac{1}{\delta}}\right) } $
	L,L	$\frac{1}{\left(1+\left(a_{H}\right)^{\frac{2}{\delta}}\right)},$ $\left(a_{H}\right)^{\frac{2}{\delta}}/\left(1+\left(a_{H}\right)^{\frac{2}{\delta}}\right)$	1 // 1)	0.5, 0.5

Figure 1. Winning-probabilities of the teams for different player allocations

From Figure 1, it is straightforward to derive the following proposition:

Proposition 1:

- (a) Each club wishes to hire as many high-ability players as possible.
- (b) Club 1 (club 2) weakly prefers decentral (central) marketing.

The intuition behind proposition 1 is obvious. Each club wishes to engage high-ability players in order to strengthen its team's quality. Therefore, decentral marketing leads to team

²¹ H,H means that a club hires two high-ability players. Similarly, H,L means that a club hires one high and one low-ability player, while L,L denotes the club's decision to hire two low-ability players.

formation $\{(a_H, a_H), (a_L, a_L)\}$, i.e., the first team consists of two high and the second of two low-ability players. This is the best outcome for the famous club and the worst outcome for the less famous club. So, the famous club prefers to market decentrally and, hence, to receive more TV revenue than the less famous club, while the less famous club naturally prefers a more egalitarian distribution rule for TV revenue. As described in the introduction, this is exactly what we observe in practice. Small (or less famous) clubs are highly interested in marketing centrally, while bigger clubs lobby for a decentral allocation rule.

Determination of marketing:

Let us now turn to stage 1 of the model. The sports association can influence the player allocation to the two teams in two ways, by determining the kind of marketing and, when choosing central marketing, by determining the parameter θ . In order to see what kind of marketing the association prefers, situations (i) to (iii) have to be analysed separately. Consider first situation (i):

• Situation (i):

Situation (i) is rather trivial. Here, the association is assumed to only be interested in competitive balance. In this case, the following proposition holds:

Proposition 2: If the association is interested only in competitive balance, it will always market centrally.

An association solely interested in competitive balance wishes to create two equally strong teams. Under central marketing, the association can perfectly achieve this aim by choosing $\theta = 0$, i.e., by equally sharing the TV revenue. On the other hand, under decentral marketing, the first team is stronger than the second one. Hence, if fan interest is only determined by the closeness and not by the quality of competition, the association will always be interested in distributing TV revenue equally in order to create competitive balance.

• Situation (ii):

In situation (ii), the decision of the association becomes more complex. In this case, the association wishes to create an extremely high-performing team. One might therefore think that the association allows decentral marketing in order to give the more famous club the possibility to increase its team's quality. However, this is not necessarily true as the following proposition shows.

Proposition 3: Let the association be interested in creating a very high-performing team.

- (a) For $R \ge 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \ge 2 \cdot \overline{w}_H$, the association will always market centrally.
- (b) For $R < 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \ge 2 \cdot \overline{w}_H$, central marketing is (weakly) preferred.
- (c) For $\widetilde{\theta} \cdot R < 2 \cdot \overline{w}_H$, there exists a cut-off value $\widetilde{\delta} > 1$ so that the following holds:
 - (i) For $\delta > \widetilde{\delta}$, the association always prefers decentral to central marketing.
 - (ii) For $\delta < \widetilde{\delta}$, there exists a value $\widetilde{a}_H > 1$ such that the association installs decentral marketing if and only if $a_H < \widetilde{a}_H$.

Proof: See Appendix.

If the association is interested in creating a very high-performing team, both kinds of marketing will be optimal for some parameter constellations. Let me explain this result in more detail. There exist three countervailing effects influencing the association's decision. On the one hand, there is a quality effect. For a given kind of team composition, a high-ability player always exerts a higher effort than a low-ability player. Hence, the association is interested in creating a team consisting of two high-ability players. Secondly, there is a complementarity effect. The complementarities in team performance (i.e. $\frac{\partial y_j^2}{\partial e_{jl}\partial e_{j2}} > 0$) make

it desirable for the association to collocate two players in a team exerting identical effort. This

can easily be demonstrated by an example: Using production function (1), a team with two players both exerting effort 0.5, is more successful than a team with one player exerting effort 0.6 and the other player exerting 0.4, even if aggregate effort equals 1 in both cases. Since in a homogeneous team the high-ability players always choose identical effort and the low-ability players also exert (lower) identical effort, the association wishes to place two equal able players in one team. Thirdly, there is also a competition effect. If homogeneity between the teams increases, i.e., if competition between the teams becomes more significant, the players' efforts will also increase.

With $R \geq 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \geq 2 \cdot \overline{w}_H$, central marketing allows the association to allocate TV revenue such that the first team consists of two high-ability players and the second of at least one. In this case, the quality effect and the complementarity effect are the same under central and decentral marketing. However, competition is more intense under central marketing so that it is preferred. For $R < 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \geq 2 \cdot \overline{w}_H$, central marketing can always replicate the outcome of decentral marketing. Moreover, it can achieve a different player allocation to the teams that is sometimes preferred. It is therefore (weakly) optimal.

The association's decision becomes more complex, when $\widetilde{\theta} \cdot R < 2 \cdot \overline{w}_H$ holds. In this case, central marketing will inevitable lead to two teams consisting of one high and one low-ability player. It is then necessary to better understand the single effects. The competition effect is very important when δ is small and a_H large. This is very intuitive. In case of a large δ , effort is extremely costly for the players. Hence, the players exert very low effort in both kinds of team composition. If δ becomes small, the players will increase their efforts. This increase in effort significantly depends on the degree of the high-ability players' cost advantage and the chosen kind of marketing. If marketing is decentral and, hence, the teams differ, the effort will only increase significantly in the case of a small a_H . Otherwise, the lowability players are discouraged, since they have almost no chance of winning the tournament,

and, therefore, reduce their efforts in order to save on costs. As a consequence, the high-ability players also reduce their efforts.²² However, in the case of central marketing, an increase in $a_{\rm H}$ does not serve to discourage, since both teams are affected by the ability change in the same way.

By understanding this competition effect, the results in proposition 3(c) become more plausible. In case of a large δ , the competition effect is negligible. On account of this, the association installs decentral marketing, since the disadvantage of this kind of marketing is low. If δ is quite small, the competition effect becomes more important. In this case, the association's decision depends on the cost parameter, a_H . If a_H becomes higher, the ability difference between the high and low-ability players increases. As a result, both, the quality effect and the complementarity effect become more relevant. However, the relevance of the competition effect rises disproportionately in a_H , and the association installs central marketing.

• Situation (iii):

In this situation, the association is interested in maximising aggregate performance in the league. Its choice of marketing is described in proposition 4:

Proposition 4: Let the association be interested in maximising aggregate performance in the league. Then,

- (a) For $R \ge 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \ge 2 \cdot \overline{w}_H$, the association will always market centrally.
- (b) For $R < 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \ge 2 \cdot \overline{w}_H$, central marketing is (weakly) preferred.

-

²² This behaviour of players has been observed in soccer many times. Consider a match between two heterogeneous teams, one favourite team and one "weak" team. When the favourite team scores very early in the match, often the following can be observed: the players of the weak team no longer believe that they can win the match and, for this reason, they reduce their efforts. Then, the members of the favourite team also reduce their efforts and just play to defend their advantage. As a consequence, the spectators neither see a top performance from the favourite team, nor from the weak team.

(c) For $\widetilde{\theta} \cdot R < 2 \cdot \overline{w}_H$, there exists a cut-off value $\hat{\delta} = 4$ such that the association installs central marketing if and only if $\delta < \hat{\delta}$.

Proof: See Appendix.

If the association wishes to maximise aggregate performance in the league, the three mentioned effects will again be present. In case (a) of proposition 4, central marketing allows the association to increase the strength of the second team relative to the decentral marketing outcome without decreasing the first team's strength. Hence, the competition effect and the quality effect make central marketing preferable. They dominate the complementarity effect that is optimally used under decentral marketing. In case (b), central marketing is able to replicate the decentral marketing outcome, but also to achieve a different outcome that is preferred for some parameter constellations. It is therefore (weakly) dominant. In case (c), the quality effect falls away. If the quality of team 1 is increased by installing decentral marketing, the quality of team 2 is automatically decreased and vice versa. Therefore, the association's trade-off is affected only by the competition effect and the complementarity effect only if δ lies below the cut-off $\hat{\delta} = 4$. This is intuitive. The competition effect will be significant and, therefore, dominant if δ is low. For a high δ , competition only induces a low increase in aggregate effort, and the complementarity effect dominates.

In practice, fan interest is usually determined by all the criteria depicted in situations (i) to (iii). There are fans that are mainly interested in intense competition with uncertain outcome, whereas other fans are more interested in high absolute performance. Hence, the objective function of the association usually is a combination of the three handled criteria. What does this mean for the optimal choice of marketing? Clearly, when $R \geq 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \geq 2 \cdot \overline{w}_H$,

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central marketing is always optimal since it entails higher absolute performance as well as

higher competitive balance. Note that these restrictions mean that the allocation of TV

revenue under decentral marketing is extremely unjust. Central marketing is then able to

redistribute TV revenue such that the aggregate number of high-ability players in the league

increases. On the other hand, for $\widetilde{\theta} \cdot R < 2 \cdot \overline{w}_H$, decentral marketing might be optimal. In case

of a large δ , decentral marketing is optimal from the viewpoint of absolute performance,

while central marketing creates competitive balance. The association's decision then depends

on the weights of the single criteria in its objective function.

Player preferences with respect to the two types of marketing:

Up to this point, the clubs' and the association's preferred kind of marketing has been

analysed. In order to complete the analysis we need to consider the players' preferences

concerning central and decentral marketing. This is done in proposition 5.

Proposition 5: If a high-ability player is hired under both kinds of marketing, he will prefer

decentral marketing. If a low-ability player is hired under both kinds of marketing, he will

prefer central marketing.

Proof: See Appendix.

Although at first sight not very surprising, this result is not as easily understood as it may

seem. A change in team composition has two effects on the players' utilities. On the one hand,

competition between the teams might change. Therefore, the players' efforts and, hence, the

costs of effort, are affected, too. On the other hand, there might be an alteration in the teams'

winning probabilities. For a high-ability player, both effects work in the same direction, that

is, he is most likely to win competition and he exerts lowest effort under decentral marketing.

Hence, a decentral allocation rule for TV revenue is optimal for him.

For a low-ability player, the two effects are countervailing.²³ If he is in a team with a high instead of a low-ability player, competition between the two teams will be intensified, and so the player suffers from higher effort costs. On the other hand, he will be more likely to win the bonus. For $\delta > 1$, the second effect always dominates the first one, so that a low-ability player prefers to be in a team with a high-ability player.

However, if the restriction $\delta > 1$ were cancelled, there are parameter constellations such that a low-ability player would wish to be in a team with another low-ability player. The reason is that for a rather small δ even the low-ability players choose high efforts. As a consequence, the winning probabilities in both cases do not differ significantly, and the first effect will be dominant. Since only $\delta > 1$ guarantees the existence of an equilibrium, proposition 5 does not consider the possibility that a low-ability player might prefer decentral marketing.

Lastly, the players achieve a rent from taking part in the league. Hence they prefer belonging to a team over not being hired at all, no matter what the player allocation to the two teams looks like.

Scarcity of high-ability players:

In the previous analysis, nothing has been said about the scarcity of high-ability players. The high-ability player reservation wage $\overline{w}_{\rm H}$ could be interpreted as a measure of scarcity. If there exist only a few high-ability players in the market, \overline{w}_{H} will be rather large, whereas it should be very small in case of many high-ability players. A small \overline{w}_{H} implies that TV revenue allocation under decentral marketing is very unjust. It is then very likely that central marketing is installed. However, one could think that \overline{w}_{H} would become so low that $\overline{w}_{_H} > \big(1-\alpha\big) \cdot R \quad \text{did not hold anymore. Decentral marketing could then also be optimal.}$ Moreover, it might also be optimal if high-ability players are rare in the market and their reservation wages are high.

²³ Note that both teams will never consist of two low-ability players.

4. Concluding remarks

The aim of this paper was to analyse under which conditions a national sports association will install central marketing and in which situations it will allow decentral marketing since both distribution rules can be observed in practice. It was found that a sports association that is only interested in competitive balance will always market centrally. In contrast, a sports association that is interested in one extremely high-performing team or that wishes to maximise the aggregate performance in the league for some parameter constellations chooses central marketing, while for others it chooses decentral marketing.

Usually, a sports association cares for both, intense competition between the clubs and high absolute performance. Central marketing is then always preferred when TV revenue allocation under decentral marketing is extremely unjust. Under central marketing, revenue could be redistributed such that the aggregate number of high-ability players in the league increases. In this case, competition becomes more intense and absolute performance gets higher. On the other hand, decentral marketing might be optimal, if the creation of a very strong team is desired that cannot be reached under central marketing.

Moreover, it was shown that we have extreme conflicts of interest between the more and less famous clubs and the more and less able players. The "weak" parties, i.e., the small clubs and less able players, always prefer central marketing, while the big clubs and more able players wish to exploit their superior position by means of decentral marketing.

Finally, it should be mentioned that the model naturally represents only a partial analysis of the market for broadcasting rights in professional soccer. Due to its complexity, not all market aspects could be treated in the model. However, in addition to its special application to professional soccer, the model could also be seen as a contribution to the literature on group rent-seeking contests.

Appendix

Proof of proposition 3:

Note that under decentral marketing the first team consists of two high and the second of two low-ability players. The two teams' performances are then $y_1^{dec} = \left((a_H)^{\frac{\delta+2}{\delta}} \cdot b \middle/ 2 \cdot \delta \cdot \left(1 + (a_H)^{\frac{2}{\delta}} \right)^2 \right)^{\frac{2}{\delta}} \quad \text{and} \quad y_2^{dec} = \left((a_H)^{\frac{2}{\delta}} \cdot b \middle/ 2 \cdot \delta \cdot \left(1 + (a_H)^{\frac{2}{\delta}} \right)^2 \right)^{\frac{2}{\delta}}. \quad \text{Their maximum is } y_1^{dec} \,. \quad \text{Think that } R \geq 4 \cdot \overline{w}_H \,. \quad \text{In this case, the sports association can install central marketing and allocate resources such that both teams consist of two high-ability players. The$

marketing and allocate resources such that both teams consist of two high-ability players. The two teams' performances are then $\overline{y}_1 = \overline{y}_2 = \left(b \cdot a_H/8 \cdot \delta\right)^{\frac{2}{\delta}}$. The inequality $\overline{y}_1 > y_1^{dec}$ simplifies to $\left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right)^{\frac{4}{\delta}} > 2^{\frac{4}{\delta}} \cdot \left(a_H\right)^{\frac{4}{\delta^2}} \Leftrightarrow \left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right) > 2 \cdot \left(a_H\right)^{\frac{1}{\delta}}$, which, using the second

binomial, can be shown to be satisfied.

Now assume that $4 \cdot \overline{w}_H > R \ge 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \ge 2 \cdot \overline{w}_H$. The association is then able to install central marketing and distribute revenue such that the first team consists of two highability players and the second of one low and one high-ability player. The two teams'

performances are then given by
$$\widetilde{y}_1 = \left(\left(a_H \right)^{\frac{\delta+1}{\delta}} \cdot b / 2 \cdot \delta \cdot \left(1 + \left(a_H \right)^{\frac{1}{\delta}} \right)^2 \right)^{\frac{2}{\delta}}$$
 and

$$\widetilde{y}_{2} = \left(\left(a_{H} \right)^{\frac{\delta+2}{2 \cdot \delta}} \cdot b / 2 \cdot \delta \cdot \left(1 + \left(a_{H} \right)^{\frac{1}{\delta}} \right)^{2} \right)^{\frac{2}{\delta}}. \quad \text{The condition} \quad \widetilde{y}_{1} > y_{1}^{\text{dec}} \quad \text{simplifies} \quad \text{to}$$

$$\left(1+\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}\right)^{\frac{2}{\delta}} > \left(1+\left(a_{\mathrm{H}}\right)^{\frac{1}{\delta}}\right)^{\frac{2}{\delta}} \cdot \left(a_{\mathrm{H}}\right)^{\frac{1}{\delta^{2}}}, \text{ which is equivalent to } 1+\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}} > \left(a_{\mathrm{H}}\right)^{\frac{1}{2\cdot\delta}} + \left(a_{\mathrm{H}}\right)^{\frac{3}{2\cdot\delta}} \text{ or }$$

$$\left(\left(a_{\mathrm{H}}\right)^{\!\!\frac{1}{\delta}}\right)^{\!0} + \left(\left(a_{\mathrm{H}}\right)^{\!\!\frac{1}{\delta}}\right)^{\!2} - \left(\left(a_{\mathrm{H}}\right)^{\!\!\frac{1}{\delta}}\right)^{\!0.5} - \left(\left(a_{\mathrm{H}}\right)^{\!\!\frac{1}{\delta}}\right)^{\!1.5} > 0 \;. \;\; \text{The function} \quad E(x) = K^x \quad \text{is, for } K > 1,$$

strictly increasing and convex. Hence, the difference $K^2 - K^{1.5}$ is bigger than $K^{0.5} - K^0$, so that the above inequality holds.

Suppose now that $R < 3 \cdot \overline{w}_H$ and $\widetilde{\theta} \cdot R \geq 2 \cdot \overline{w}_H$ hold. Under central marketing, the association is able to achieve - inter alia - the two following team compositions. The first is the same as under decentral marketing. In the second, each team consists of one high and one low-ability player. Since the second kind of team composition is sometimes preferred, as will be shown next, central marketing is weakly dominant.

Now assume that $\widetilde{\theta} \cdot R < 2 \cdot \overline{w}_H$. Under central marketing revenue can be allocated such that each team consists of one high and one low-ability player. The two teams' performances are then $\hat{y}_1 = \hat{y}_2 = (b/8 \cdot \delta)^{\frac{2}{\delta}} \cdot (a_H)^{\frac{1}{\delta}}$. The inequality $y_1^{dec} > \hat{y}_1 = \hat{y}_2$ can be rewritten as:

$$\left(\left(a_{\mathrm{H}}\right)^{\frac{\delta+4}{4\cdot\delta}}\cdot 2\right)^{\frac{4}{\delta}} > \left(1+\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}\right)^{\frac{4}{\delta}} \iff \left(a_{\mathrm{H}}\right)^{\frac{\delta+4}{4\cdot\delta}}\cdot 2-\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}-1>0.$$

I define the function $F(\delta, a_H) = (a_H)^{\frac{\delta+4}{4\delta}} \cdot 2 - (a_H)^{\frac{2}{\delta}} - 1$. The derivative of F with respect to δ is given by $\frac{\partial F(\delta, a_H)}{\partial \delta} = 2 \cdot \ln(a_H) \cdot (a_H)^{\frac{\delta+4}{4\delta}} \cdot \frac{-16}{16 \cdot \delta^2} - \ln(a_H) \cdot (a_H)^{\frac{2}{\delta}} \cdot \frac{-2}{\delta^2}$.

This derivative is positive if and only if the following condition holds:

$$2 \cdot ln\big(a_{_{\mathrm{H}}}\big) \cdot \big(a_{_{\mathrm{H}}}\big)^{\frac{\delta+4}{4 \cdot \delta}} \cdot \frac{-16}{16 \cdot \delta^2} - ln\big(a_{_{\mathrm{H}}}\big) \cdot \big(a_{_{\mathrm{H}}}\big)^{\frac{2}{\delta}} \cdot \frac{-2}{\delta^2} > 0 \Leftrightarrow 1 > \big(a_{_{\mathrm{H}}}\big)^{\frac{\delta-4}{4 \cdot \delta}} \Leftrightarrow \delta < 4.$$

Since $\lim_{\delta\to\infty} F(\delta,a_{_{\rm H}})>0$ holds, $F(\delta,a_{_{\rm H}})$ will always exceed zero if the condition $F(1,a_{_{\rm H}})=(a_{_{\rm H}})^{\frac{5}{4}}\cdot 2-(a_{_{\rm H}})^2-1\geq 0$ is satisfied. Since we have F(1,1)=0 and $\lim_{a_{_{\rm H}}\to\infty} F(1,a_{_{\rm H}})<0$, we consider the inequality $\frac{\partial F(1,a_{_{\rm H}})}{\partial a_{_{\rm H}}}=2.5\cdot(a_{_{\rm H}})^{\frac{1}{4}}-2\cdot(a_{_{\rm H}})>0\Leftrightarrow 1.25\cdot(a_{_{\rm H}})^{\frac{1}{4}}>(a_{_{\rm H}})$. This inequality can be transformed as follows: $\ln(1.25)>0.75\cdot\ln(a_{_{\rm H}})\Leftrightarrow a_{_{\rm H}}<1.25\cdot e^{\frac{4}{3}}$. Summarising, we see that for large δ , the condition $y_1^{\rm dec}>\hat{y}_1=\hat{y}_2$ always holds. For small δ , the condition $y_1^{\rm dec}>\hat{y}_1=\hat{y}_2$ only holds if $a_{_{\rm H}}$ is sufficiently small.

Finally, it has to be shown that the association never prefers to have one high and one low-ability player in the first team and two low-ability players in the second. The two teams'

performances in this case are
$$\ddot{y}_1 = \left(\left(a_H \right)^{\frac{\delta+2}{2 \cdot \delta}} \cdot b / 2 \cdot \delta \cdot \left(1 + \left(a_H \right)^{\frac{1}{\delta}} \right)^2 \right)^{\frac{2}{\delta}}$$
 and

$$\breve{y}_2 = \left(\left(a_H \right)^{\frac{1}{\delta}} \cdot b / 2 \cdot \delta \cdot \left(1 + \left(a_H \right)^{\frac{1}{\delta}} \right)^2 \right)^{\frac{2}{\delta}}.$$
 We prove this by showing that $\hat{y}_1 > \breve{y}_1$ always holds.

This inequality can be simplified to $\left(1+\left(a_{H}\right)^{\frac{1}{\delta}}\right)^{\frac{4}{\delta}}>2^{\frac{4}{\delta}}\cdot\left(a_{H}\right)^{\frac{2}{\delta^{2}}}\Leftrightarrow 1+\left(a_{H}\right)^{\frac{1}{\delta}}>2\cdot\left(a_{H}\right)^{\frac{1}{2\delta}}.$ Using the second binomial, one can show that the last condition is always satisfied.

Proof of proposition 4:

Note first that $2 \cdot \overline{y}_1 > 2 \cdot \hat{y}_1$. Because of part (c) of this proposition, one therefore only needs to show that $2 \cdot \overline{y}_1 > y_1^{\text{dec}} + y_2^{\text{dec}}$ holds for $\delta \geq 4$. The condition simplifies to $\left(2 \cdot \left(a_H\right)^{\frac{2 \cdot \delta - 4}{\delta \cdot (\delta - 4)}}\right)^{\delta - 4} > \left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right)^{\delta - 4}$. Hence, one has to show that $2 \cdot \left(a_H\right)^{\frac{2 \cdot \delta - 4}{\delta \cdot (\delta - 4)}} > 1 + \left(a_H\right)^{\frac{2}{\delta}}$ holds. For $\delta \geq 4$, the term $(2 \cdot \delta - 4)/(\delta - 4)$ is always bigger than 2 so that the last condition is always fulfilled.

The inequality $\widetilde{y}_1 + \widetilde{y}_2 > y_1^{\text{dec}} + y_2^{\text{dec}}$ can be rewritten as $\left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right)^{\frac{\delta-4}{\delta}} < \left(1 + \left(a_H\right)^{\frac{1}{\delta}}\right)^{\frac{\delta-4}{\delta}} \cdot \left(a_H\right)^{\frac{\delta-2}{\delta}} \cdot \left(a_H\right)^{\frac{\delta-2}{\delta^2}}. \text{ For } \delta = 4 \text{, this inequality holds. For } \delta > 4 \text{, it has to be shown that } 1 + \left(a_H\right)^{\frac{2}{\delta}} < \left(a_H\right)^{\frac{\delta-2}{\delta(\delta-4)}} + \left(a_H\right)^{\frac{2\cdot\delta-6}{\delta(\delta-4)}}, \text{ which clearly holds if } \left(a_H\right)^{\frac{2}{\delta}} < \left(a_H\right)^{\frac{2\cdot\delta-6}{\delta(\delta-4)}}, \text{ or } 1 < \left(a_H\right)^{\frac{2}{\delta}(\delta-4)}. \text{ For } \delta > 4 \text{, this condition is fulfilled. Assume now that } \delta < 4 \text{ holds. We then need to show that } 1 + \left(a_H\right)^{\frac{2}{\delta}} > \left(a_H\right)^{\frac{\delta-2}{\delta(\delta-4)}} + \left(a_H\right)^{\frac{2\cdot\delta-6}{\delta(\delta-4)}}. \text{ This condition can be rewritten as } \left(\left(a_H\right)^{\frac{1}{\delta}}\right)^0 + \left(\left(a_H\right)^{\frac{1}{\delta}}\right)^2 - \left(\left(a_H\right)^{\frac{1}{\delta}}\right)^{\frac{\delta-2}{\delta-4}} - \left(\left(a_H\right)^{\frac{1}{\delta}}\right)^{\frac{2\cdot\delta-6}{\delta-4}} > 0 \text{. As seen in the proof of proposition 3,}$

this condition holds if $2 - \frac{2 \cdot \delta - 6}{\delta - 4} \ge \frac{\delta - 2}{\delta - 4}$. Rearranging yields $0 \le \delta$, for $\delta < 4$, which is necessarily satisfied.

The condition $2 \cdot \hat{y}_1 > y_1^{\text{dec}} + y_2^{\text{dec}}$ can be rewritten as $(a_H)^{\frac{\delta-4}{\delta^2}} \cdot 2^{\frac{\delta-4}{\delta}} > \left(1 + (a_H)^{\frac{2}{\delta}}\right)^{\frac{\delta-4}{\delta}}$, or equivalently, as $\left((a_H)^{\frac{1}{\delta}} \cdot 2\right)^{\frac{\delta-4}{\delta}} > \left(1 + (a_H)^{\frac{2}{\delta}}\right)^{\frac{\delta-4}{\delta}}$. It is easy to see that for $\delta = 4$ the condition $2 \cdot \hat{y}_1 = y_1^{\text{dec}} + y_2^{\text{dec}}$ holds. Further, using the second binomial, we can see that $2 \cdot \hat{y}_1 > (<)y_1^{\text{dec}} + y_2^{\text{dec}}$ holds only if $\delta < (>)4$.

Lastly, it has to be shown that it is never optimal for the association to allocate resources such that the first team consists of one high and one low-ability player and the second of two low-ability players. To prove this, we show that $2 \cdot \hat{y}_1 > \bar{y}_1 + \bar{y}_2$ always holds. This condition simplifies to $2^{\frac{\delta-4}{\delta}} \cdot (a_H)^{\frac{\delta-2}{\delta^2}} > \left(1 + (a_H)^{\frac{1}{\delta}}\right)^{\frac{\delta-4}{\delta}}$. For $\delta = 4$, this inequality surely holds. Consider now the case $\delta > 4$. It then has to be shown that $2 \cdot (a_H)^{\frac{\delta-2}{\delta(\delta-4)}} > 1 + (a_H)^{\frac{1}{\delta}}$. Since, for $\delta > 4$, $(a_H)^{\frac{\delta-2}{\delta(\delta-4)}} > (a_H)^{\frac{1}{\delta}}$, this condition holds. Suppose now that $4 > \delta$. In this case, it has to be shown that $2 \cdot (a_H)^{\frac{\delta-2}{\delta(\delta-4)}} > 1 + (a_H)^{\frac{1}{\delta}}$. This condition always holds, when $(a_H)^{\frac{\delta-2}{\delta(\delta-4)}} < (a_H)^{\frac{1}{2-\delta}}$ or equivalently, $(a_H)^{\frac{0.5\delta}{\delta(\delta-4)}} < 1$, which is fulfilled for $4 > \delta$.

Proof of proposition 5:

From Figure 1 we know that a high-ability player's winning probability is highest under decentral marketing. It is therefore sufficient to show that his effort, and hence his effort costs, is lowest under decentral marketing. His effort is $e_H^{dec} = \left(\left(a_H \right)^{\frac{\delta+2}{\delta}} \cdot b \middle/ 2 \cdot \delta \cdot \left(1 + \left(a_H \right)^{\frac{2}{\delta}} \right)^2 \right)^{\frac{1}{\delta}}.$ In the possible cases under central marketing his

effort is $\overline{e}_H = \left(a_H \cdot b/8 \cdot \delta\right)^{\frac{1}{\delta}}$ (both teams consist of two high-ability players), $\widetilde{e}_H = \left(\left(a_H\right)^{\frac{1+\delta}{\delta}} \cdot b \middle/ 2 \cdot \delta \cdot \left(1 + \left(a_H\right)^{\frac{1}{\delta}}\right)^2\right)^{\frac{1}{\delta}}$ (the first team consists of two high-ability players and the second of one low and one high-ability player) and $\hat{e}_H = \left(a_H \cdot b/8 \cdot \delta\right)^{\frac{1}{\delta}}$ (each team consists of one high and one low-ability player). The condition $e_H^{dec} \le \hat{e}_H = \overline{e}_H$ simplifies to $2^{\frac{2}{\delta}} \cdot (a_H)^{\frac{2}{\delta^2}} \le \left(1 + (a_H)^{\frac{2}{\delta}}\right)^{\frac{2}{\delta}} \Leftrightarrow (a_H)^{\frac{2}{\delta}} - 2 \cdot (a_H)^{\frac{1}{\delta}} + 1 \ge 0$, which is always satisfied. The condition $e_H^{dec} \le \widetilde{e}_H$ simplifies to $\left(a_H\right)^{\frac{1}{\delta^2}} \cdot \left(1 + \left(a_H\right)^{\frac{1}{\delta}}\right)^{\frac{2}{\delta}} \le \left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right)^{\frac{2}{\delta}}$. As shown in the proof of proposition 3, this condition always holds.

A low-ability player may be hired under central marketing in two cases, in the case where both teams consist of one high and one low-ability player (and where his effort and his expected utility are $\hat{e}_L = \left(b/8 \cdot \delta\right)^{\frac{1}{\delta}}$ and $E\hat{U}_L$) and in the case where the first team consists of two high-ability players and the second of one high and one low-ability player. The low-ability player then chooses effort $\tilde{e}_L = \left(\left(a_H\right)^{\frac{1}{\delta}} \cdot b \middle/ 2 \cdot \delta \cdot \left(1 + \left(a_H\right)^{\frac{1}{\delta}}\right)^2\right)^{\frac{1}{\delta}}$ and has expected utility $E\tilde{U}_L$. Let EU_L^{dec} denote the low-ability player's expected utility under decentral marketing. The condition $E\hat{U}_L > EU_L^{dec}$ is $\frac{2 \cdot \delta - 1}{8 \cdot \delta} > \frac{1}{2 \cdot \left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right)} \cdot \left(1 - \frac{\left(a_H\right)^{\frac{2}{\delta}}}{\delta \cdot \left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right)}\right)$.

Simplifying this condition leads to:

$$\left(\left(2\cdot\delta-1\right)\cdot\left(1+\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}\right)\right)\cdot\left(1+\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}\right)>4\cdot\left(\delta\cdot\left(1+\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}\right)-\left(a_{\mathrm{H}}\right)^{\frac{2}{\delta}}\right)$$

$$\Leftrightarrow (2 \cdot \delta - 1) \cdot (\mathbf{a}_{H})^{\frac{4}{\delta}} > 2 \cdot \delta + 1 - 2 \cdot (\mathbf{a}_{H})^{\frac{2}{\delta}}.$$

For $\delta > 1$, the right-hand side of the last inequality is always smaller than $2 \cdot \delta - 1$. Therefore, the inequality is always fulfilled for $\delta > 1$.

$$\text{The condition } \widetilde{EU}_L > EU_L^{\text{dec}} \text{ is } \frac{\left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right)}{\left(1 + \left(a_H\right)^{\frac{1}{\delta}}\right)} \cdot \left(1 - \frac{\left(a_H\right)^{\frac{1}{\delta}}}{\delta \cdot \left(1 + \left(a_H\right)^{\frac{1}{\delta}}\right)}\right) > \left(1 - \frac{\left(a_H\right)^{\frac{2}{\delta}}}{\delta \cdot \left(1 + \left(a_H\right)^{\frac{2}{\delta}}\right)}\right). \text{ After }$$

some calculations, this condition simplifies to $\delta > \frac{\left(a_H\right)^{\frac{4}{\delta}} - \left(a_H\right)^{\frac{3}{\delta}} - \left(a_H\right)^{\frac{1}{\delta}} + 1}{\left(a_H\right)^{\frac{4}{\delta}} - 1}$. The left-hand-

side of this condition is always smaller than 1. So, the condition is always satisfied for $\delta > 1$.

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