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Splitting Leagues

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Abstract

Splitting leagues or tournaments seems to be puzzling when agents are homogeneous and splitting leads to a negative competition effect. However, it can be shown that the principal can nevertheless benefit from splitting. First, splitting can be used as a divide-and-rule strategy by the principal to create additional incentives when collusion among the agents is possible. Second, splitting leagues gives the principal the opportunity to introduce promotions and relegations between nested tournaments (i.e., tournaments that are intertemporally linked), which also enhances incentives.

JEL classification: J3, J4, M5.

Key words: collusion, leagues, promotion, relegation, tournaments

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1 Introduction

In practice, we find a lot of situations in which agents compete in tournaments for certain prizes that have been fixed in advance. The agent with the best performance receives the high winner prize, whereas the other agents only get lower loser prizes.[1] Examples for such rank-order tournaments can be found in sports (see, e.g., Ehrenberg and Bognanno [1990] on golf tournaments, Becker and Huselid [1992] on stock-car racing, Lynch and Zax [1998] on horse racing and Garicano and Palacios-Huerta [2000] on soccer) and in internal labor markets (see, e.g., Baker, Gibbs and Holmström [1994] and Treble, van Gameren, Bridges and Barmby [2001] on job-promotion tournaments, Mantrala, Krafft and Weitz [2000] on contests between salesmen, and Gibbons and Murphy [1990] on relative performance pay between managers).

Often we find different tournaments or leagues instead of one large league in which all agents would compete together against each other. For example, there are parallel sport contests, simultaneous job-promotion tournaments in the same firm (particularly, on different hierarchical levels, in different plants of a firm, and in the so-called "dual ladder"[2] between managers on the one hand and engineers on the other hand), and different leagues in nearly any sport. In addition, tournaments or leagues are often nested so that winning one tournament gives an agent the opportunity to compete in a higher tournament in the next period. Such nesting can be observed in sports as well as in corporate hierarchies.

[1] For rank-order tournaments see Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), Green and Stokey (1983), O'Keefe, Viscusi, and Zeckhauser (1984), Rosen (1986).

[2] See, e.g., Gunz (1980), Feuer (1986).

Some arguments for splitting leagues can be derived from the theory of rank-order tournaments: If agents are heterogeneous and the principal is able to observe an agent's type, he will prefer to organize a different tournament or league for each type to preserve overall incentives (Lazear and Rosen 1981, pp. 857–863). However, if the principal cannot distinguish between heterogeneous agents, he may still prefer different leagues as a self-selection device (O'Keefe, Viscusi and Zeckhauser 1984). Lazear (1989) discusses the problem of sabotage in tournaments.[3] Heterogeneity may even worsen the situation as the presumable losers may see no other way but sabotage to get back into the race. In such situations, organizing different leagues may mitigate the sabotage problem.

However, at least three aspects speak against these arguments: First, participants of tournaments are not always heterogeneous. Sometimes they are homogeneous, or there is symmetric uncertainty so that agents have to be treated as being homogeneous. Second, splitting leagues may not be the best response to solve the above mentioned problems. Incentives in tournaments with observed heterogeneity can also be restored by introducing a handicap system (Lazear and Rosen 1981, pp. 861–863). Different leagues are not necessarily the best (information revelation) mechanism to solve sorting and incentive problems in situations with asymmetric information about the agents' types. Sabotage problems may be better solved by adjusting the optimal prize spread (see Lazear 1989) or by announcing Draconian sanctions such as disqualification. Third, splitting leagues may be harmful from

[3] Perhaps the most spectacular case in sports has been the one with Tonya Harding, who had hired someone to injure her opponent Nancy Kerrigan during the Olympic Games 1994 in Lillehammer.

the viewpoint of competition. As is known from market competition, a large number of competitors results in stronger competition. In analogy, splitting leagues might lead to a negative competition effect so that overall incentives are weakened.

In this paper, I will show that despite a negative competition effect and despite homogeneity of the agents there may be a rationale for splitting leagues: As mentioned by Dye (1984), for instance, collusion among the agents can destroy the incentive effects of a tournament. However, if collusion is not correlated between different leagues, it can be shown that splitting leagues mitigates the collusion problem, since stable collusion in one league may not necessarily be accompanied by collusion in the other league (insurance effect). Moreover, the principal can design a superordinate tournament between the different leagues so that collusion in one league will shift the tournament prizes to the other leagues. Note that these additional incentives do not violate the self-commitment property of tournaments which has been emphasized by Malcomson (1984). Since the total amount of tournament prizes for the leagues are still fixed in advance, opportunistic behavior by the principal can be excluded which then ensures incentives for the agents. A second argument for splitting leagues comes from the possibility of relegation and promotion of agents between nested leagues. It can be shown that the negative competition effect of splitting leagues, which has been mentioned above, is dominated by the additional incentives from relegation and promotion.

The model discussed in the paper is related to the literature on logit-form rent-seeking contests (see, e.g., Tullock [1980], Hillman and Riley [1989, pp.

30-35], Baik and Kim [1997],Gradstein and Konrad [1999], Wärneryd [2000]). However, there are two important differences: Whereas the agents' cost function (i.e., the spent resources measured in monetary terms) is assumed to be linear in logit-form contests, in this paper costs are represented by a convex function as is usual in rank-order tournaments to describe an agent's disutility of effort. Second, prizes or rents are typically given exogenously in contests. In this paper, tournament prizes are endogenous. As typical for rank-order tournament models, the prizes are optimally chosen by the principal. There are also parallels to the paper of Rosen (1986), who analyzes nested elimination tournaments using a logit-form contest success function. However, the Rosen (1986) paper focuses on optimal prize structures and selection instead of splitting versus no-splitting. The paper is most strongly related to Kräkel (2002) who also considers such logit-form tournaments with convex costs and endogenous prizes. In Kräkel (2002), this tournament type is labeled "J-type tournament", because it is often found in Japanese firms. Kräkel then compares these J-type tournaments with the so-called "U-type tournaments" which are typical of US firms. However, the advantages of splitting leagues are not discussed in Kräkel (2002).

It is important to mention that the tournament type considered in this paper is not only relevant for Japanese firms. Since the logit-form contest success function can be endogenously derived in a rank-order tournament with exponentially distributed noise as has been shown by the literature on patent races (see, e.g. Loury [1979], and especially Baye and Hoppe [2003]), the model presented in this paper is relevant for a wide range of tournament situations: In a patent race, the agent with the shortest time to success wins. In analogy, we can think of tournaments (in sports or internal labor markets) in which the most swift agent (e.g., runner) or the first agent who successfully finishes his task or the agent with the least faults or the salesman with the least customer complaints, for example, is declared the winner of the tournament.

The paper is organized as follows. Section 2 describes the model and highlights the negative competition effect of splitting leagues. In section 3, the advantages of splitting leagues for mitigating collusion problems are discussed. Section 4 shows that additional incentives from relegation and promotion between nested leagues dominate the negative competition effect.

2 The Basic Model

A model with $n \geq 4$ risk neutral and homogeneous agents,[4] and a risk neutral principal is considered. It is assumed that n is an even integer, which simplifies the discussion of splitting leagues. The principal wants to organize a tournament competition between the agents. This tournament can be either directly interpreted as a one-shot competition between the agents or, alternatively, as a reduced form for a Round Robin tournament during a sport season, for example. First, the principal has to choose between splitting and no-splitting. At the second stage, the principal chooses the optimal prize structure that maximizes his objective function. It is assumed that

[4] Whenever possible, organizers of real tournaments try to match contestants of equal quality to induce maximal incentives. Often such matching can be realized quite easily by using publicly observable characteristics of the agents. Particularly in sports, only tournaments that exhibit high degrees of homogeneity can be attractive for the audience (Rosen and Sanderson 2001, p. F50). Hence, most real tournaments in both sports and internal labor markets are by construction quite homogeneous.

the principal wants to maximize total effort of the n agents minus the sum of the tournament prizes. At the third stage, the n agents compete for the given tournament prizes by exerting effort e_i $(i = 1, ..., n)$. Efforts are non-contractible. The winning probability of agent i can be described by

$$p(e_i) = \begin{cases} \dfrac{e_i}{e_i + \sum_{j \neq i} e_j} & \text{for } e_i + \sum\limits_{j \neq i} e_j > 0 \\ \dfrac{1}{m} & \text{otherwise} \end{cases}$$

with m as the number of contestants (logit-form contest success function). Without splitting we have $m = n$. If the n-player tournament is split into different leagues, we will have $m < n$.

The agents are assumed to have zero reservation utility. Furthermore, tournament prizes are not allowed to be negative. In other words, we consider a situation with limited liability, because otherwise the incentive problem would be trivial: The principal could always implement first-best efforts by choosing a certain prize spread and guarantee each agent his reservation utility by appropriately adjusting the loser prizes. It is assumed that effort entails costs on an agent which is described in monetary terms by the function $c(e_i) = \frac{c}{2} \cdot e_i^2$ with $c > 0$.[5]

As a benchmark result the outcome of the n-player tournament is considered. Suppose that the principal has only fixed a winner prize $w > 0$ at the previous stage, i.e. the winner receives w and the $n-1$ other agents get zero

[5]Note that the negative competition effect generally holds for any convex cost function and endogenous prizes. See Proposition 6(i) in Kräkel (2002), p. 625.

loser prizes.[6] At the third stage, agent i $(i = 1, ..., n)$ wants to maximize

$$EU_i(e_i; n) = \frac{e_i}{e_i + \sum_{j \neq i} e_j} w - \frac{c}{2} e_i^2. \tag{1}$$

The first-order condition yields[7]

$$\frac{w}{c \cdot \left(e_i + \sum_{j \neq i} e_j\right)^2} = \frac{e_i}{\sum_{j \neq i} e_j}. \tag{2}$$

Analogously, for any other agent $k \neq i, j$ we get $w / \left[c \cdot \left(e_i + \sum_{j \neq i} e_j\right)^2\right] = e_k / \sum_{j \neq k} e_j$.Together with Eq.(2) we obtain

$$e_i \cdot \left(\sum\nolimits_{j \neq k} e_j\right) = e_k \cdot \left(\sum\nolimits_{j \neq i} e_j\right),$$

which shows that the game has a unique symmetric Nash equilibrium $e_i^* = e_k^* = e^*(n)$ with

$$e^*(n) = \sqrt{\frac{(n-1)w}{cn^2}}. \tag{3}$$

Inserting into Eq. (1) gives

$$EU_i(e^*(n); n) = \frac{(n+1)w}{2n^2}. \tag{4}$$

Hence, each agent receives a positive rent compared to his zero outside option. Moreover, since prizes are not allowed to be negative (limited-liability assumption), the principal cannot extract this rent from the agents by charging an entrance fee or choosing negative loser prizes. The best the principal

[6] Later on, it will become obvious that this is indeed the optimal prize structure.
[7] Here and in the following, the second-order condition is always satisfied.

can do is choosing $n-1$ zero loser prizes and a positive winner prize that maximizes his objective function

$$\pi(n) = n \cdot e^*(n) - w = \sqrt{\frac{(n-1)w}{c}} - w. \tag{5}$$

The first-order condition for the optimal prize $w^*(n)$ leads to

$$w^*(n) = \pi^*(n) = \frac{n-1}{4c}. \tag{6}$$

Hence, any splitting of the n-player tournament into z leagues with n_L agents competing in league L and $\sum_{L=1}^{z} n_L = n$ yields profits

$$\sum_{L=1}^{z} \pi^*(n_L) = \sum_{L=1}^{z} \frac{n_L - 1}{4c} = \frac{n-z}{4c} < \pi^*(n). \tag{7}$$

The following proposition summarizes the benchmark result of Section 2:

Proposition 1 *(i) A tournament between $m \leq n$ agents has a unique symmetric equilibrium in which the principal chooses $m-1$ zero loser prizes and a winner prize $w^*(m)$ according to (6), and the m agents choose identical efforts $e^*(m)$ according to (3). (ii) Any splitting of the n-player tournament into different leagues results into lower profits than no-splitting.*

Proposition 1(i) shows that the optimal tournament prize structure consists of a positive winner prize which is linearly increasing in the number of contestants and $n-1$ zero loser prizes. Furthermore, result (ii) emphasizes the *negative competition effect*: Splitting the n-player tournament into different leagues would lead to decreased efforts and, therefore, decreased profits. According to Eq. (7), the larger the number of leagues the lower the principal's profits. For this reason, it is assumed for the rest of the paper that the principal will choose two leagues each with $n/2$ contestants, if he decides to split the n-player league. Altogether, the previous results strictly favor no-splitting. However, introducing the possibility of collusion and the possibility of relegation and promotion between different leagues will show that splitting may dominate no-splitting.

3 Collusion and Superordinate Tournaments

In this section, the basic model is modified by the additional assumption that the agents may be able to generate a stable collusion that leads to minimal efforts. Since efforts are non-contractible so that the principal cannot use legal sanctions to punish the agents, optimal collusive efforts are zero.[8] By this, again each agent has the same probability of winning – as in the symmetric equilibrium of Proposition 1 – but now his effort costs are zero. Note that stability of collusion is not derived endogenously here.[9] As collusion is illegal, endogenous stability would require a tournament supergame and appropriate supergame strategies so that collusion becomes self-enforcing. However, in the one-shot game of Section 2 we have a unique equilibrium with positive efforts. Hence, any collusive outcome cannot be stable in a static context.

In practice, besides dynamic interaction stable collusion also depends on other factors such as trust between the agents. All these factors should not

[8] All results will remain qualitatively the same, if the lowest possible effort in case of collusion is assumed to be a sufficiently low positive number.

[9] For a discussion of endogenous stability see, for example, Kräkel (2002).

be modelled explicitly in the paper. Instead, the following discussion focuses on the connection between splitting leagues and the collusion problem given that agents are able to form a stable collusive agreement with a certain exogenous probability. Since, there does not exist a standard model for this problem, three variants of the collusion model will be discussed to check the robustness of the results. In the first model, stable collusion is assumed to be equally likely in any league irrespective of the number of contestants. In other words, splitting does not change the likelihood of a stable collusion in a given league. However, as mentioned in the introduction, splitting leagues offers the opportunity for the principal to organize a superordinate tournament between the leagues. In the first model, there exists an exogenous probability δ for a stable collusion between the leagues in this superordinate tournament. Since splitting leagues typically leads to a separation of the agents (which implies less communication, trust and so on between agents of different leagues), usually δ should be lower than the probability of a stable collusion within a league. The two other models take into account that usually stable collusion is more difficult to sustain in large groups.[10] Therefore, it is assumed that splitting leagues has the additional disadvantage of stable collusion becoming more likely in a single league. In the second model, there are different exogenous probabilities for stable collusion in the splitting and the no-splitting case. In the third model, the collusion probability is assumed to be decreasing in the number of contestants.

The three models have nearly the same timing of events. At the first

[10] There are several arguments in favor of this assumption – for example, trust arguments, transaction-costs arguments, free-rider problems in sanctioning agents that deviate from the collusive agreement.

stage, the principal decides whether to split the n-player tournament into different leagues or not. Given his decision at the previous stage, the principal chooses the optimal tournament prize structure at the second stage. In the no-splitting case, he chooses $n-1$ zero loser prizes and one positive winner prize in analogy to Proposition 1. In the splitting case, the principal chooses an optimal prize structure for each league. In addition, he has the option to organize a superordinate tournament between the leagues and to choose an appropriate prize structure for this tournament. At the third stage, for each single tournament or league nature decides whether a stable collusion takes place (with probability γ) or not (with probability $1-\gamma$). To abstract from asymmetric information problems, it is assumed that each agent can observe whether a collusive climate has formed in the different leagues.[11] In the splitting case of the first model, however, first with probability δ $(1-\delta)$ stable collusion between the leagues is realized (not realized). If overall collusion is not possible, again for each single league nature decides whether there is stable collusion (with probability γ) or not (with probability $1-\gamma$).[12] At the fourth stage, each agent chooses his effort. If stable collusion has formed, the agents can coordinate their decisions.[13] Otherwise, they simultaneously choose those individual efforts that maximize their respective utilities.

In the **first model**, the collusion probability of each league is described by

[11] Note that we could introduce an additional stage where – given the possibility of a stable collusion – the agents can decide whether to coordinate their effort decisions (i.e., use the chance of collusion) or not. In the no-splitting case, this decision would be trivial. In the splitting case, it can easily be seen that – despite the superordinate tournament – coordinating efforts will dominate not coordinating.

[12] We can think of a two-step negotiation. First, the agents try to form a grand coalition, because it would be most attractive. If this coalition is not realized (with probability $1-\delta$), the agents will try to coordinate their decisions at least within the single leagues.

[13] Formally they write a binding contract that maximizes their collective utility.

γ and the collusion probability in the superordinate tournament by δ. If the principal has decided not to split the n-player league and if stable collusion is not possible, an agent's equilibrium effort is described by Eq. (3). In the collusive case, however, all agents agree to exert zero effort. Hence, the principal chooses the optimal prize that maximizes $\pi(n) = (1-\gamma)ne^*(n) - w$ which gives

$$w^*(n) = \pi^*(n) = \frac{(1-\gamma)^2(n-1)}{4c}. \tag{8}$$

If the principal has preferred to split the n-player league into two $n/2$-leagues A and B, he has the additional opportunity to organize a superordinate tournament between the two leagues. In particular, let w_L denote the winner prize[14] in league L $(L = A, B)$ and $\sum_i e_{iL}$ the sum of efforts exerted in league L. Then the principal can choose an optimal prize w which is split between the two leagues according to

$$w_A = \frac{\sum_i e_{iA}}{\sum_i e_{iA} + \sum_i e_{iB}} w \quad \text{and} \quad w_B = \frac{\sum_i e_{iB}}{\sum_i e_{iA} + \sum_i e_{iB}} w. \tag{9}$$

This design of the superordinate tournament has at least two advantages: On the one hand, the self-commitment property of tournaments that has been highlighted by Malcomson (1984) is preserved. On the other hand, additional incentives are induced so that now a collusive agreement of choosing zero efforts in one league cannot longer be optimal for the agents.

When choosing efforts at the last stage of the game, it is decisive whether a grand coalition has formed (with probability δ) or not (with probability $1-\delta$).

[14] Note that we know from the discussion in Section 2 that the principal optimally chooses $\frac{n}{2} - 1$ zero loser prizes in each league.

If it has formed, all agents will choose zero efforts; if not, four outcomes have to be distinguished – either collusion takes place (1) in both leagues (with probability γ^2), or (2) in neither league (with probability $(1-\gamma)^2$), or (3) only in league A (with probability $\gamma(1-\gamma)$), or (4) only in league B (with probability $(1-\gamma)\gamma$):

(1) Given stable collusion in both leagues, each league will choose identical efforts for its members, because agents are homogeneous and the cost function is convex. The optimal effort e_A^* of league A then maximizes

$$EU_A(e_A) = \frac{e_A}{\frac{n}{2}e_A}w_A - \frac{c}{2}e_A^2 = \frac{e_A}{\frac{n}{2}e_A} \cdot \frac{\frac{n}{2}e_A}{\frac{n}{2}e_A + \frac{n}{2}e_B}w - \frac{c}{2}e_A^2 = \frac{e_A}{e_A + e_B}\frac{2w}{n} - \frac{c}{2}e_A^2$$

and optimal effort e_B^* of league B maximizes

$$EU_B(e_B) = \frac{e_B}{e_A + e_B}\frac{2w}{n} - \frac{c}{2}e_B^2.$$

Both first-order conditions together yield

$$\frac{2w}{cn\left(e_A + e_B\right)^2} = \frac{e_A}{e_B} = \frac{e_B}{e_A}.$$

Hence, we have $e_A^* = e_B^* =: e^*$ and $EU_A(e_A^*) = EU_B(e_B^*) =: EU(e^*)$ with

$$e^* = \sqrt{\frac{w}{2cn}} \quad \text{and} \quad EU(e^*) = \frac{3w}{4n}. \tag{10}$$

(2) Given no collusion in both leagues, agent i of league A wants to

maximize

$$EU_{iA}(e_{iA}) = \frac{e_{iA}}{\sum_j e_{jA}} \cdot \frac{\sum_j e_{jA}}{\sum_j e_{jA} + \sum_j e_{jB}} w - \frac{c}{2} e_{iA}^2 = \frac{e_{iA}}{\sum_j e_{jA} + \sum_j e_{jB}} w - \frac{c}{2} e_{iA}^2.$$

From the first-order condition we obtain

$$\frac{w}{c \cdot \left(\sum_j e_{jA} + \sum_j e_{jB} \right)^2} = \frac{e_{iA}}{\sum_{j \neq i} e_{jA} + \sum_j e_{jB}}. \tag{11}$$

In analogy, for another member k of league A we get $w / \left[c \cdot \left(\sum_j e_{jA} + \sum_j e_{jB} \right)^2 \right] = e_{kA} / \left(\sum_{j \neq k} e_{jA} + \sum_j e_{jB} \right)$ and together with i's first-order condition we have

$$e_{iA} \cdot \left(\sum_{j \neq k} e_{jA} + \sum_j e_{jB} \right) = e_{kA} \cdot \left(\sum_{j \neq i} e_{jA} + \sum_j e_{jB} \right).$$

This equation shows that $e_{iA} = e_{kA} =: \hat{e}_A^*$, i.e. all members of league A choose the same optimal effort level. Interchanging the subscripts of the two leagues implies that in league B all agents also prefer identical efforts $\hat{e}_B^*$. Hence, Eq. (11) for league A together with the corresponding first-order condition for league B leads to

$$\frac{\hat{e}_A^*}{\left(\frac{n}{2} - 1 \right) \hat{e}_A^* + \frac{n}{2} \hat{e}_B^*} = \frac{\hat{e}_B^*}{\left(\frac{n}{2} - 1 \right) \hat{e}_B^* + \frac{n}{2} \hat{e}_A^*},$$

which shows that $\hat{e}_A^* = \hat{e}_B^* =: \hat{e}^*$. Inserting into Eq. (11) leads to

$$\hat{e}^* = \sqrt{\frac{w(n-1)}{cn^2}} \quad \text{and} \quad EU(\hat{e}^*) = w \frac{n+1}{2n^2}. \tag{12}$$

Comparing these results with (10) shows that $\hat{e}^* > e^*$ and $EU(\hat{e}^*) < EU(e^*)$. This finding is also intuitively plausible: Collusion implies lower efforts and, therefore, higher expected utilities for the agents. In addition, $\hat{e}^*$ is identical with $e^*(n)$ from Eq. (3), because without collusion we have the same incentives for the agents as in the n-player tournament due to the construction of the superordinate tournament.

(3) If collusion only takes place in league A, the agents of league A will coordinate their efforts to maximize

$$EU_A(e_A) = \frac{e_A}{\frac{n}{2}e_A} \cdot \frac{\frac{n}{2}e_A}{\frac{n}{2}e_A + \sum_j e_{jB}} w - \frac{c}{2}e_A^2 = \frac{e_A}{\frac{n}{2}e_A + \sum_j e_{jB}} w - \frac{c}{2}e_A^2.$$

The first-order condition for optimal effort $\tilde{e}_A^*$ yields

$$\frac{w}{c \cdot \left(\frac{n}{2}\tilde{e}_A^* + \sum_j e_{jB}\right)^2} = \frac{\tilde{e}_A^*}{\sum_j e_{jB}}. \tag{13}$$

In league B, however, agent i wants to maximize

$$EU_{iB}(e_{iB}) = \frac{e_{iB}}{\sum_j e_{jB}} \cdot \frac{\sum_j e_{jB}}{\frac{n}{2}e_A + \sum_j e_{jB}} w - \frac{c}{2}e_{iB}^2 = \frac{e_{iB}}{\frac{n}{2}e_A + \sum_j e_{jB}} w - \frac{c}{2}e_{iB}^2.$$

The first-order condition leads to

$$\frac{w}{c \cdot \left(\frac{n}{2}e_A + \sum_j e_{jB}\right)^2} = \frac{e_{iB}}{\frac{n}{2}e_A + \sum_{j \neq i} e_{jB}}. \tag{14}$$

Since the respective first-order condition for any other member k of league B is given by (14) with subscript "i" being replaced by "k", obviously $e_{iB} = e_{kB}$

holds. Hence, each agent of league B exerts the same optimal effort level $\tilde{e}_B^*$. Using the notation $\tilde{e}_A^*$ and $\tilde{e}_B^*$, the two conditions (13) and (14) together give

$$\frac{\tilde{e}_A^*}{\frac{n}{2}\tilde{e}_B^*} = \frac{\tilde{e}_B^*}{\frac{n}{2}\tilde{e}_A^* + \left(\frac{n}{2}-1\right)\tilde{e}_B^*} \qquad \Rightarrow \qquad \tilde{e}_B^* = \frac{n-2+\sqrt{5n^2-4n+4}}{2n}\tilde{e}_A^*. \quad (15)$$

Inserting into (13) yields

$$\tilde{e}_A^* = \frac{2\sqrt{w\left(n-2+\sqrt{5n^2-4n+4}\right)}}{\sqrt{c}\left(3n-2+\sqrt{5n^2-4n+4}\right)}, \tag{16}$$

which leads to

$$\tilde{e}_B^* = \sqrt{\frac{w}{cn^2}\frac{\left(n-2+\sqrt{5n^2-4n+4}\right)^{\frac{3}{2}}}{3n-2+\sqrt{5n^2-4n+4}}} \tag{17}$$

according to Eq. (15). By substituting for $\tilde{e}_A^*$ and $\tilde{e}_B^*$ in the agents' objective functions we obtain

$$EU_A(\tilde{e}_A^*) = \frac{2w\left(5n-2+\sqrt{5n^2-4n+4}\right)}{\left(3n-2+\sqrt{5n^2-4n+4}\right)^2} \qquad \text{and}$$

$$EU_B(\tilde{e}_B^*) = w\left(n-2+\sqrt{5n^2-4n+4}\right)\frac{3n^2-4+(n+2)\sqrt{5n^2-4n+4}}{n^2\left(3n-2+\sqrt{5n^2-4n+4}\right)^2}.$$

The solution for case (4), where collusion is only given in league B, can easily be calculated by interchanging the subscripts A and B in case (3). Note that in all four cases the agents have positive expected utilities, i.e. they realize positive rents compared with their zero outside options.

At the second stage, given the splitting case the principal has to choose the optimal tournament prize. Since at this time he does not know whether there will be collusion in the superordinate tournament or in any of the two leagues, he chooses w to maximize

$$\begin{aligned}
\pi &= (1-\delta)\left(\gamma^2 n e^* + (1-\gamma)^2 n\hat{e}^* + 2\gamma(1-\gamma)\left(\frac{n}{2}\tilde{e}_A^* + \frac{n}{2}\tilde{e}_B^*\right)\right) - w \\
&= (1-\delta)\left(\gamma^2\sqrt{\frac{n}{2}} + (1-\gamma)^2\sqrt{n-1}\right. \qquad (18) \\
&\qquad \left. +\gamma(1-\gamma)\sqrt{n-2+\sqrt{5n^2-4n+4}}\right)\sqrt{\frac{w}{c}} - w.
\end{aligned}$$

The solution of the maximization problem and the corresponding profits of the principal from splitting leagues are therefore given by

$$\begin{aligned}
w^* = \pi^* = \frac{(1-\delta)^2}{4c}&\left(\gamma^2\sqrt{\frac{n}{2}} + (1-\gamma)^2\sqrt{n-1}\right. \qquad (19) \\
&\left. +\gamma(1-\gamma)\sqrt{n-2+\sqrt{5n^2-4n+4}}\right)^2.
\end{aligned}$$

Comparing Eqs. (19) and (8) shows that splitting will be more favorable than no-splitting from the principal's viewpoint if

$$\begin{aligned}
(1-\delta)&\left(\gamma^2\sqrt{\frac{n}{2}} + (1-\gamma)^2\sqrt{n-1}\right. \qquad (20) \\
&\left. +\gamma(1-\gamma)\sqrt{n-2+\sqrt{5n^2-4n+4}}\right) > (1-\gamma)\sqrt{n-1}
\end{aligned}$$

$$\Leftrightarrow \delta < 1 - \frac{1}{1+\frac{\gamma^2}{1-\gamma}\sqrt{\frac{n}{2(n-1)}} + \gamma\underbrace{\left(\sqrt{\frac{n-2+\sqrt{5n^2-4n+4}}{n-1}} - 1\right)}_{>0}} \equiv \bar{\delta}(\gamma, n) \qquad (21)$$

Note that the denominator is monotonically increasing in γ and monotonically decreasing in n. These findings are summarized in the following proposition:

Proposition 2 *If the exogenous collusion probability* γ *is independent of the number of contestants, there will exist a cut-off value* $\bar{\delta}(\gamma, n)$ *so that the principal prefers splitting to no-splitting if and only if* $\delta < \bar{\delta}(\gamma, n)$ *with* $\partial\bar{\delta}(\gamma, n)/\partial\gamma > 0$ *and* $\partial\bar{\delta}(\gamma, n)/\partial n < 0$. *If* $\delta \to 0$, *from the principal's viewpoint splitting will strictly dominate no-splitting for any* $\gamma \in (0, 1]$.

The first result of Proposition 2 shows that the principal will choose splitting instead of no-splitting at the first stage of the game, if the probability δ of a grand coalition in the superordinate tournament is sufficiently small. Note that splitting leagues has two advantages with respect to collusion: On the one hand, the superordinate tournament *prevents the agents from choosing zero efforts* when there is stable collusion within a single league. The sign of the derivative $\partial\bar{\delta}(\gamma, n)/\partial\gamma$ points out that this comparative advantage against no-splitting becomes even more important the higher γ. In the limit, if $\gamma \to 1$, efforts will be zero in the no-splitting case but strictly positive with splitting. On the other hand, since stable collusion is stochastically independent between the leagues, we have a kind of *insurance effect* when splitting leagues. If collusion happens in one league the principal may have luck so that there is no collusion in the other league. Note that for simplicity it has been assumed that the principal chooses only two leagues in the splitting case. This assumption even seems to be plausible in the light of Eq. (7). However, for optimizing on the insurance effect perhaps more than two leagues may result in even better outcomes for splitting compared to no-splitting.

The comparative-static result $\partial\bar{\delta}(\gamma, n)/\partial n < 0$ shows that the principal will rather prefer no-splitting than splitting, if n becomes large. This result indicates that there is an additional competitive disadvantage from splitting leagues in the presence of collusion. This effect can be best seen from inequality (20) which can be rewritten as

$$(1-\delta)\left[\gamma^2\sqrt{\frac{n}{2}} + 2\gamma(1-\gamma)\sqrt{\frac{n}{4} - \frac{1}{2} + \sqrt{\frac{5}{16}n^2 - \frac{1}{4}n + \frac{1}{4}}} + (1-\gamma)^2\sqrt{n-1}\right]$$
$$> (1-\gamma)\sqrt{n-1}.$$

On the left-hand side, the expression in brackets consists of three terms, because we have to distinguish between three events when splitting leagues – there is collusion either in both leagues (with probability γ^2) or in only one league (with probability $2\gamma(1-\gamma)$) or in neither league (with probability $(1-\gamma)^2$). Hence, $\sqrt{\frac{n}{2}}$ indicates total profits for the collusion case, $\sqrt{\frac{n}{4} - \frac{1}{2} + \sqrt{\frac{5}{16}n^2 - \frac{1}{4}n + \frac{1}{4}}}$ for the semi-collusion case, and $\sqrt{n-1}$ for the no-collusion case. Analogously, $\sqrt{n-1}$ on the right-hand side indicates total profits in the no-collusion case under no-splitting. Note that both $\sqrt{\frac{n}{2}}$ and $\sqrt{\frac{n}{4} - \frac{1}{2} + \sqrt{\frac{5}{16}n^2 - \frac{1}{4}n + \frac{1}{4}}}$ are smaller than $\sqrt{n-1}$ and that in both terms the coefficient of n is smaller than 1. Note also that the realization of the large value $\sqrt{n-1}$ is less likely under splitting than under no-splitting, i.e. the coefficient of $\sqrt{n-1}$ is $(1-\gamma)^2$ under splitting, whereas it is $(1-\gamma) > (1-\gamma)^2$ under no-splitting. Altogether, increasing n leads to higher incentives and to higher total profits in either case, but the marginal effect is significantly stronger under no-splitting than under splitting because of the different coefficients. As inequality (20) and Proposition 2 show, this *additional negative competition effect* has to be outweighed by a lower δ for splitting still dominating no-splitting.

Perhaps, the comparison between splitting and no-splitting has been quite unfair in favor of no-splitting, because in the splitting case agents have two chances of realizing a stable collusion – first in the superordinate tournament, and, if the grand coalition fails, then in each single league. However, if we assume collusion only to be possible within and not between leagues (i.e., $\delta = 0$), then the second result of Proposition 2 shows that splitting will always dominate no-splitting for any arbitrary value of γ. In other words, in that case the additional negative competition effect of splitting leagues is strictly dominated by the additional incentive effect and the insurance effect of the superordinate tournament.

Starting from the last point, in the **second model** we assume $\delta = 0$. Nevertheless, splitting leagues is not assumed to be completely advantageous for preventing collusion. Since in practice, stable collusion is easier to sustain in smaller groups, let γ again denote the collusion probability in the $n/2$-player leagues (splitting case) and $\alpha \cdot \gamma$ the collusion probability in the n-player league (no-splitting case) with $\alpha \in (0, 1)$. Hence, by assumption collusion is more likely in a single league after splitting. The following proposition shows that despite this assumption, splitting leagues may still be favorable for the principal when collusion among the agents cannot be ruled out:

Proposition 3 *Let (a) $\delta = 0$, (b) the collusion probability in the $n/2$-player* *leagues be γ, and (c) that in the n-player league be $\alpha \cdot \gamma$ with $\alpha \in (0,1)$. There always exists a feasible cut-off value $\bar{\alpha}(\gamma, n)$ so that splitting will dominate (be dominated by) no-splitting from the principal's viewpoint, if and only if $\alpha > (<) \bar{\alpha}(\gamma, n)$. The cut-off value $\bar{\alpha}(\gamma, n)$ is monotonically increasing in both γ and n.*

Proof. See Appendix. ■

The proposition shows that for any parameter values c, n and γ, when splitting leagues there is always a trade-off between the competitive disadvantages on the one hand, and the additional incentive effect and the insurance effect on the other hand. In other words, there does not exist any possible parameter constellation for which the cut-off value $\bar{\alpha}(\gamma, n)$ becomes negative or greater than one so that splitting always or never dominates no-splitting. Moreover, like $\bar{\delta}(\gamma, n)$ from Proposition 2, the cut-off $\bar{\alpha}(\gamma, n)$ is also independent of c, but depends on γ and n. $\bar{\alpha}(\gamma, n)$ is strictly increasing in n because of the additional negative competition effect discussed above. In this situation, no-splitting can only be dominated by splitting, if the collusion probability is similar in both cases (i.e., if $\alpha \to 1$). The cut-off $\bar{\alpha}(\gamma, n)$ is also monotonically increasing in γ. This result can be explained as follows: If γ is large, the parameter α has a high impact on the relation between splitting and no-splitting concerning the collusion probability. For small values of α, collusion is much more likely in the splitting than in the no-splitting case which can hardly be compensated by the additional incentive effect and the insurance effect. Hence, for large values of γ, the parameter α has to be high enough so that splitting has still a chance to beat no-splitting from the principal's viewpoint.

In the **third model**, the collusion probability is assumed to be a decreasing function of the number of league members since, in practice, it is difficult to sustain a collusive agreement among a large number of agents. In particular, I assume that the probability of a stable collusion will be given by

$$\gamma(m) = \frac{\beta}{m} \qquad \text{with } 0 < \beta \leq \frac{n}{2}$$

if the league consists of m agents. The upper bound, $\frac{n}{2}$, guarantees that splitting the n-player league into two $n/2$-player leagues leads to a collusion probability that is not greater than one. Again, like in the second model, the collusion probability between leagues is assumed to be $\delta = 0$. The following result can be obtained:

Proposition 4 *If $\delta = 0$ and the collusion probability of a league with $m \leq n$ agents is given by $\gamma(m) = \beta/m$ with $0 < \beta \leq \frac{n}{2}$, splitting will always dominate no-splitting from the principal's viewpoint.*

Proof. See Appendix. ■

Interestingly, the principal will always prefer splitting to no-splitting independent of the parameters β and n. This result supports the findings of Proposition 2: If stable collusion is possible within but not between leagues ($\delta = 0$), splitting will *always* be beneficial for the principal, even if it leads to a significantly higher collusion probability within a single league and to serious competitive disadvantages.

To sum up, the results of Propositions 2–4 have shown that splitting leagues may dominate no-splitting in the presence of possible collusion between the agents. There is a trade-off, because splitting generates additional incentives and has an insurance effect for the principal in connection with collusion, but also leads to an additional negative competition effect. If stable collusion between leagues is rather impossible, the principal will nearly always prefer splitting to no-splitting.

4 Relegation and Promotion in Nested Tournaments

In this section, a second advantage of splitting leagues will be discussed. Although splitting leads to a negative competition effect, combining both leagues via nesting may yield additional intertemporal incentives. Here nesting means that the first-period outcomes of the two $n/2$-player leagues determine the composition of future tournaments. For example, the principal can choose the best player of each league to compete in a *major league* after the first period whereas the other $(n/2) - 1$ players of each league are relegated to a *minor league*. I will show that the additional incentives generated by nesting dominate the negative competition effect. In order to discuss nested tournaments, the static model of Section 2 has to be replaced by a dynamic one. For this reason, I consider a dynamic model without discounting that lasts 2 periods. In each period, there is a tournament that guarantees each agent his zero reservation utility by setting appropriate tournament prizes.

If the principal does not want to split the n-player league (**no-splitting**

case), his expected profits will be

$$2\pi^*(n) \overset{(6)}{=} \frac{(n-1)}{2c}. \tag{22}$$

In the **splitting case**, however, the principal has the opportunity to introduce a promotion-and-relegation rule within a nested-tournament setting. In the first period, there are two leagues, each with $n/2$ agents. Again, call these two leagues A and B, and let w_L denote the winner prize of league L ($L = A, B$).[15] Suppose that after the first period the principal promotes x ($1 \leq x \leq \frac{n}{2} - 1$) agents from each league to a high-prize or major league, and chooses $\frac{n}{2} - x$ agents from each league that are relegated to a low-prize or minor league. For the second period, let w_{MA} denote the winner prize in the major league and w_{MI} ($\leq w_{MA}$) the winner prize in the minor league.[16] I assume that the principal chooses the first-period winner and by random $x - 1$ of the $\frac{n}{2} - 1$ losers of each league L to play in the major league. The remaining $\frac{n}{2} - x$ losers of each league L are relegated to the minor league. To sum up, in the first period we have only the two leagues A and B each consisting of $n/2$ players, and in the second period there only exist a major league with $2x$ players and a minor league with $n - 2x$ players.

The random-selection part of the promotion-and-relegation rule is, of course, a simplification of rules that are used in practice. However, to calculate ex ante the probability of belonging to the x most ($\frac{n}{2} - x$ least) successful agents of each league, we must compute the respective order statistics and

[15] Again, the principal optimally chooses $(n/2) - 1$ zero loser prizes in each league.

[16] Again, it is optimal (and feasible) for the principal to choose zero loser prizes.

these calculations would not be tractable in the given setting.[17] A further justification for the random-selection rule can be seen in the fact that for the first period a symmetric equilibrium in which each agent chooses the same effort will be the most plausible one because of the assumption of homogeneous agents. But then ex post the agents that are relegated and promoted must indeed be chosen by random.[18] However, the most relevant justification for the random-selection rule is given by the main purpose of this section: I will show that there exist promotion-and-relegation rules that lead to splitting dominating no-splitting. As can be seen below, a random-selection rule is one of these rules. Hence, if a more exact discrimination among the $(n/2) - 1$ losers would lead to an even better result for the splitting case, the main result of this section would only be reinforced.

Contrary to the no-splitting case, the two periods have to be strictly distinguished when splitting leagues. In the first period, promotion and relegation can generate additional incentives, which is not possible in the second period. The game is now solved by backward induction. The solution for the second period is already given by Eqs. (3) and (4) when substituting

[17] In the introduction, I have mentioned that the logit-form model can be endogenously derived by assuming exponentially distributed noise; see, e.g., Loury (1979). Let s_i be the score of agent i $(i = 1, ..., n)$, which is distributed with cdf $\text{prob}\{s_i \leq s\} = F_i(s) = 1 - \exp\{-e_i s\}$ over $[0, \infty]$ with e_i as agent i's effort. Let the s_is be stochastically independent and assume that the agent with the lowest score s_i is declared the winner (e.g., the agent with the least faults). Then agent i's probability of winning is $\int_0^\infty \left[\exp\{-s \sum_{j \neq i}^{n/2} e_j\} e_i \exp\{-e_i s\} \right] ds = e_i / (e_i + \sum_{j \neq i}^{n/2} e_j)$, which yields the well-known logit-form contest success function. To see the intractability of calculating the exact order statistics for the relegation problem we simply have to look at the cdf of the lowest order statistic: This function will become relevant if, for example, only the least successful agent of each league is relegated. In this case, the relegation probability is given by $\text{prob}\{s_i > \max[s_j | j \neq i]\} = \int_0^\infty \left[\prod_{j \neq i} [1 - \exp\{-e_j s\}] \, e_i \exp\{-e_i s\} \right] ds$.

[18] Of course, assuming a random rule in advance leads to different incentives compared to a random rule which is derived endogenously.

for the respective number of contestants and the respective notation. We obtain

$$e_{MA}^{*}=\sqrt{\frac{(2x-1)\,w_{MA}}{4cx^{2}}}\quad\text{and}\quad EU_{MA}^{*}=\frac{(2x+1)\,w_{MA}}{8x^{2}}\tag{23}$$

for the major league, and

$$e_{MI}^{*}=\sqrt{\frac{(n-2x-1)\,w_{MI}}{c\,(n-2x)^{2}}}\quad\text{and}\quad EU_{MI}^{*}=\frac{(n-2x+1)\,w_{MI}}{2\,(n-2x)^{2}}\tag{24}$$

for the minor league.

At the end of the first period, the members of each league L ($L=A,B$) are either promoted to the major league or relegated to the minor league. The expected utility of member i choosing effort e_{iL} can be written as

$$\begin{aligned}EU_{iL}^{*}=&\frac{e_{iL}}{e_{iL}+\sum_{j\neq i}e_{jL}}(w_{L}+EU_{MA}^{*})\\&+\left(1-\frac{e_{iL}}{e_{iL}+\sum_{j\neq i}e_{jL}}\right)\frac{x-1}{\frac{n}{2}-1}EU_{MA}^{*}\\&+\left(1-\frac{e_{iL}}{e_{iL}+\sum_{j\neq i}e_{jL}}\right)\frac{\frac{n}{2}-x}{\frac{n}{2}-1}EU_{MI}^{*}-\frac{c}{2}e_{iL}^{2}.\end{aligned}$$

The first-order condition for the optimal effort choice yields

$$\frac{w_{L}+\frac{n-2x}{n-2}\left(EU_{MA}^{*}-EU_{MI}^{*}\right)}{c\cdot\left(e_{iL}+\sum_{j\neq i}e_{jL}\right)^{2}}=\frac{e_{iL}}{\sum_{j\neq i}e_{jL}}.\tag{25}$$

Analogously, we find for member $k\neq i,j$ that the left-hand side of Eq. (25) must equal $e_{kL}/\left(\sum_{j\neq k}e_{jL}\right)$. Hence, we have a symmetric equilibrium in which each agent chooses

$$e_L^* = \sqrt{\frac{2(n-2)\,w_L}{cn^2} + \frac{2(n-2x)}{cn^2}\left(EU_{MA}^* - EU_{MI}^*\right)} \qquad (L = A, B)\,. \quad (26)$$

Comparing Eq. (26) with the equilibrium effort in a static $n/2$-player tournament, i.e. $e^*(\frac{n}{2}) \overset{(3)}{=} \sqrt{\frac{2(n-2)w_L}{cn^2}}$, shows that nesting tournaments indeed creates additional incentives: The first term under the square root in (26) is identical with the one in the expression for $e^*(\frac{n}{2})$, but there is an additional term, which is positive. This term is monotonically increasing in the spread between expected utility in the major and the minor league, $EU_{MA}^* - EU_{MI}^*$, which can be interpreted as the "option value" of being promoted instead of relegated after the first period. The higher this value, the higher the incentives for each agent in the first period. In addition, the second term under the square root in (26) is monotonically decreasing in the number of promotions, x.[19] This finding is also intuitively plausible: The larger the number of promotions, the higher the probability that an agent is promoted by luck and, therefore, the lower the additional incentives. Hence, if all agents are promoted to the major league after the first period (i.e. if $x = \frac{n}{2}$), then the additional incentives from nesting will be zero. However, note that the overall impact of x on incentives also depends on the effect of x on e_{MA}^* and e_{MI}^*.[20]

At the beginning of the first period, the principal has to decide about x,

[19] Note that $\frac{\partial}{\partial x}\left(EU_{MA}^* - EU_{MI}^*\right) = -\frac{1}{4} w_{MA} \frac{x+1}{x^3} - w_{MI} \frac{n-2x+2}{(n-2x)^3} < 0$.

[20] By substituting (26) into the agents' objective function we obtain $EU_{iL}^* = w_L \frac{n+2}{n^2} + \frac{(2x-1)n+2x}{n^2} EU_{MA}^* + (n+1)\frac{n-2x}{n^2} EU_{MI}^*$, which is strictly positive since $x \leq \frac{n}{2}$.

w_A, w_B, w_{MI} and w_{MA}. He wants to maximize his objective function

$$\begin{aligned}\pi &= \frac{n}{2}e_A^* + \frac{n}{2}e_B^* + 2xe_{MA}^* + (n-2x)\,e_{MI}^* - (w_A + w_B + w_{MI} + w_{MA}) \qquad (27)\\ &= \sqrt{\frac{(n-2)\,w_A}{2c} + \frac{(n-2x)}{2c}\left(EU_{MA}^* - EU_{MI}^*\right)}\\ &+ \sqrt{\frac{(n-2)\,w_B}{2c} + \frac{(n-2x)}{2c}\left(EU_{MA}^* - EU_{MI}^*\right)}\\ &+ \sqrt{\frac{(2x-1)\,w_{MA}}{c}} + \sqrt{\frac{(n-2x-1)\,w_{MI}}{c}} - (w_A + w_B + w_{MI} + w_{MA}).\end{aligned}$$

We obtain the following result:

Proposition 5 *If tournaments can be nested, the principal will always prefer splitting to no-splitting.*

Proof. See Appendix. ■

The proposition shows that – in the given setting – the principal is always able to design nested tournaments with relegations and promotions so that the additional incentives dominate the negative competition effect of splitting leagues. Interestingly, this result holds although the principal faces an additional limited-liability problem when nesting tournaments: The proof in the Appendix (see Eq. (A3)) shows that optimal first-period prizes are given by

$$w_L^* = \frac{(n-2)}{8c} - \frac{(n-2x)}{(n-2)}\left(EU_{MA}^* - EU_{MI}^*\right) \qquad (L = A, B)\,.$$

Hence, if the prize spread between major and minor league and, therefore, the "option value" $EU_{MA}^* - EU_{MI}^*$ becomes very large, optimal first-period prizes may become negative to prevent excessive incentives in leagues A and B. However, this is not allowed because of the limited-liability assumption.

As can be seen in the proof of Proposition 5, splitting always dominates no-splitting despite this additional restriction.

Note that Proposition 5 applies to a wide range of sport events with nested tournaments which exhibit a certain knockout rule: In the first round, there are two groups of players, A and B, with identical prizes w_A^* and w_B^*. In the second round, the best x agents of each group compete for a high prize w_{MA}^*, whereas the remaining $n - 2x$ players only compete for a low prize w_{MI}^*. However, the results of Proposition 5 do not apply to regular leagues in sports which have a constant number of players. In the proof of Proposition 5, it is assumed that the number of relegations differs from the number of promotions ($x \neq n/4$). Hence, in the second period the two leagues will be of different size. But perhaps Proposition 5 can be applied to scenarios where the size of leagues is altered in time. Such events sometimes happen even in leagues with a constant number of players due to commercial considerations or statutory changes. In such cases, the results of Proposition 5 give some hints on the resulting incentive effects.

Applications of Proposition 5 can also be found in internal labor markets of hierarchical firms. In particular, job-promotion tournaments are often nested: Only those agents that have been successful in the past are promoted to compete with other successful agents on a higher rank of the hierarchy. For example, we can think of two divisions, A and B, and only the best x employees of each division are promoted to a higher rank where the $2x$ employees then compete for the high prize w_{MA}^*.

5 Conclusion

In this paper, two arguments in favor of splitting leagues have been discussed. First, splitting leagues may be beneficial for the principal in the presence of possible collusion between the agents. Splitting leads to additional incentives and has a kind of insurance effect for the principal given that stability of collusion is stochastically independent between leagues. However, splitting also suffers from a competitive disadvantage, which becomes even worse in connection with the collusion problem. If stable collusion between leagues is rather unlikely, splitting becomes dominant compared to no-splitting. Second, splitting yields additional incentives when introducing the possibility of promotion and relegation between nested tournaments. These additional incentives will always dominate the negative competition effect from splitting leagues in the given setting.

Of course, the analysis above is somewhat restrictive since a special type of tournament – logit-form tournament with endogenous prizes – and a special type of cost function – quadratic costs – have been discussed. Perhaps other settings would lead to different results. Unfortunately, by assuming a general convex cost function instead of quadratic costs explicit solutions cannot be derived any longer. However, the analysis above only wants to emphasize that there are situations in which splitting leagues can be profitable for the principal despite homogeneous agents and a negative competition effect.

Appendix

Proof of Proposition 3:

Using $\delta = 0$ for the superordinate tournament, γ for the splitting case and $\alpha\gamma$ for the no-splitting case, (20) can be written as

$$\gamma^2\sqrt{\frac{n}{2}}+(1-\gamma)^2\sqrt{n-1}+\gamma(1-\gamma)\sqrt{n-2+\sqrt{5n^2-4n+4}} > (1-\alpha\gamma)\sqrt{n-1}$$

$$\Leftrightarrow \alpha > (2-\gamma) - \gamma\sqrt{\frac{n}{2(n-1)}} - (1-\gamma)\sqrt{\frac{n-2+\sqrt{5n^2-4n+4}}{n-1}} \equiv \bar{\alpha}(\gamma, n). \tag{A1}$$

The cut-off $\bar{\alpha}(\gamma, n)$ will be feasible, if it is positive but smaller than one. $\bar{\alpha}(\gamma, n) > 0$ can be rearranged to

$$\underbrace{2-\sqrt{\frac{n-2+\sqrt{5n^2-4n+4}}{n-1}}}_{>0} > \gamma\cdot\underbrace{\left(1+\frac{\sqrt{\frac{n}{2}}-\sqrt{n-2+\sqrt{(5n^2-4n+4)}}}{\sqrt{n-1}}\right)}_{<0},$$

which is always satisfied: The left-hand side is positive, because $\sqrt{\frac{n-2+\sqrt{5n^2-4n+4}}{n-1}}$ is monotonically decreasing and $\sqrt{\frac{n-2+\sqrt{5n^2-4n+4}}{n-1}} = 1.8481$ for $n = 4$. The right-hand side is negative, since

$$1+\frac{\sqrt{\frac{n}{2}}-\sqrt{n-2+\sqrt{5n^2-4n+4}}}{\sqrt{n-1}} < 0$$

$$\Leftrightarrow \sqrt{\frac{n}{2}}+\sqrt{n-1}-\sqrt{n-2+\sqrt{5n^2-4n+4}} < 0 \tag{A2}$$

is true, because $\sqrt{\frac{n}{2}}+\sqrt{n-1}-\sqrt{n-2+\sqrt{5n^2-4n+4}}$ is monotonically decreasing for $n \geq 4$ and $\sqrt{\frac{n}{2}}+\sqrt{n-1}-\sqrt{n-2+\sqrt{5n^2-4n+4}} = -0.054706$

for $n = 4$. Furthermore, $\bar{\alpha}(\gamma, n) < 1$ can be rewritten as

$$\gamma < \underbrace{\frac{\sqrt{n-2+\sqrt{5n^2-4n+4}}-\sqrt{n-1}}{\sqrt{n-2+\sqrt{5n^2-4n+4}}-\sqrt{n-1}-\sqrt{\frac{n}{2}}}}_{>1},$$

which obviously is true.

In addition, we have

$$\frac{\partial \bar{\alpha}(\gamma, n)}{\partial n} = \frac{\gamma\sqrt{\frac{2(n-2+\Omega)}{n-1}}\Omega + 2\sqrt{\frac{n}{n-1}}\left(3n+2-\Omega\right)\left(1-\gamma\right)}{4\sqrt{\frac{n}{n-1}}\left(n-1\right)^2\sqrt{\frac{n-2+\Omega}{n-1}\Omega}} > 0$$

(with $\Omega = \sqrt{5n^2-4n+4}$) since $3n+2 > \Omega$ for all feasible n, and

$$\frac{\partial \bar{\alpha}(\gamma, n)}{\partial \gamma} = \frac{\sqrt{n-2+\sqrt{5n^2-4n+4}}-\sqrt{\frac{n}{2}}}{\sqrt{n-1}} - 1 \overset{(A2)}{>} 0.$$

Proof of Proposition 4:

Now (20) has to be rewritten as

$$\left(\frac{2\beta}{n}\right)^2\sqrt{\frac{n}{2}} + \left(1-\frac{2\beta}{n}\right)^2\sqrt{n-1} + \frac{2\beta}{n}\left(1-\frac{2\beta}{n}\right)\sqrt{n-2+\sqrt{5n^2-4n+4}}$$
$$> \left(1-\frac{\beta}{n}\right)\sqrt{n-1} \Leftrightarrow$$

$$\underbrace{\sqrt{n-2+\sqrt{5n^2-4n+4}}-\frac{3\sqrt{n-1}}{2}}_{>0}$$
$$> \frac{2\beta}{n}\underbrace{\left(\sqrt{n-2+\sqrt{5n^2-4n+4}}-\sqrt{\frac{n}{2}}-\sqrt{n-1}\right)}_{>0} \Leftrightarrow$$

$$\underbrace{\frac{\sqrt{n-2+\sqrt{5n^2-4n+4}}-\frac{3\sqrt{n-1}}{2}}{\sqrt{n-2+\sqrt{5n^2-4n+4}}-\sqrt{\frac{n}{2}}-\sqrt{n-1}}}_{>1} > \underbrace{\frac{2\beta}{n}}_{<1},$$

which is always satisfied.

Proof of Proposition 5:

Using Eq. (27) and the expressions for EU_{MA}^* and EU_{MI}^* (see (23) and (24)), the first-order conditions for w_A and w_B yield

$$w_L^* = \frac{(n-2)}{8c} - \frac{(n-2x)}{(n-2)}\left(\frac{(2x+1)\,w_{MA}}{8x^2} - \frac{(n-2x+1)\,w_{MI}}{2\,(n-2x)^2}\right) \qquad (L = A, B)\,. \tag{A3}$$

The first-order conditions for w_{MA} and w_{MI} are

$$\begin{aligned}&\frac{(2x+1)\,(n-2x)}{32cx^2\sqrt{\frac{(n-2)w_A}{2c}+\frac{(n-2x)}{2c}\left(\frac{(2x+1)w_{MA}}{8x^2}-\frac{(n-2x+1)w_{MI}}{2(n-2x)^2}\right)}}\\ &+\frac{(2x+1)\,(n-2x)}{32cx^2\sqrt{\frac{(n-2)w_B}{2c}+\frac{(n-2x)}{2c}\left(\frac{(2x+1)w_{MA}}{8x^2}-\frac{(n-2x+1)w_{MI}}{2(n-2x)^2}\right)}}\\ &+\frac{2x-1}{2\sqrt{c\,(2x-1)\,w_{MA}}}=1\end{aligned}$$

and

$$\begin{aligned}&-\frac{(n-2x+1)}{8c\,(n-2x)}\frac{1}{\sqrt{\frac{(n-2)w_A}{2c}+\frac{(n-2x)}{2c}\left(\frac{(2x+1)w_{MA}}{8x^2}-\frac{(n-2x+1)w_{MI}}{2(n-2x)^2}\right)}}\\ &-\frac{(n-2x+1)}{8c\,(n-2x)}\frac{1}{\sqrt{\frac{(n-2)w_B}{2c}+\frac{(n-2x)}{2c}\left(\frac{(2x+1)w_{MA}}{8x^2}-\frac{(n-2x+1)w_{MI}}{2(n-2x)^2}\right)}}\\ &+\frac{n-2x-1}{2\sqrt{c\,(n-2x-1)\,w_{MI}}}=1\end{aligned}$$

Substituting for w_A and w_B according to (A3) leads to

$$w_{MA}^{*} = \frac{4\,(n-2)^2\,(2x-1)\,x^4}{(2x\,(2x-1)\,(n-1)-n)^2\,c}$$

and

$$w_{MI}^{*} = \frac{(n-2)^2\,(n-2x-1)\,(n-2x)^2}{4\,((n-2x)\,(n-1)+1)^2\,c}.$$

Inserting the four optimal prizes into the principal's objective function gives

$$\pi = \frac{n-2}{4c} + \frac{(n-2)\,(2x-1)\,x^2}{c\,(2x\,(2x-1)\,(n-1)-n)} + \frac{(n-2x-1)\,(n-2)\,(n-2x)}{4c\,((n-2x)\,(n-1)+1)}.$$

Comparing this expression with Eq. (22) shows that the principal will prefer splitting to no-splitting if and only if

$$\frac{n-2}{4c} + \frac{(n-2)\,(2x-1)\,x^2}{c\,(2x\,(2x-1)\,(n-1)-n)} + \frac{(n-2x-1)\,(n-2)\,(n-2x)}{4c\,((n-2x)\,(n-1)+1)} > \frac{(n-1)}{2c}$$

$$\Leftrightarrow \Delta(x) := 8n\,(2n-3)\,x^3 - 8n\left(n^2-2\right)x^2 + 2n\left(3n^2-7n+3\right)x + n^2\,(2n-1) > 0. \tag{A4}$$

The first derivative[21]

$$\Delta'(x) = 24n\,(2n-3)\,x^2 - 16n\left(n^2-2\right)x + 2n\left(3n^2-7n+3\right)$$

[21] Note that for simplicity we abstract from the fact that x has to be a positive integer.

shows that the function $\Delta(x)$ has two relative extrema:

$$x_1^* = \frac{1}{24\,(2n-3)}\left(8n^2 - 16 + 4\sqrt{43 + 53n^2 + 4n^4 - 18n^3 - 81n}\right) \quad \text{and}$$
$$x_2^* = \frac{1}{24\,(2n-3)}\left(8n^2 - 16 - 4\sqrt{43 + 53n^2 + 4n^4 - 18n^3 - 81n}\right).$$

As $\Delta'(x)$ describes a parabola open to the top, x_1^* corresponds to a relative minimum and x_2^* to a relative maximum. Note that $x_2^* \in \left[0, \frac{n}{2}\right]$ for $n \geq 4$, and that x_2^* leads to positive prizes w_L^* in (A3). Hence, x_2^* is a feasible solution. Since $\Delta(0) = n^2\,(2n-1)$, $\Delta(\frac{n}{2}) = -n^3 + 2n^2$, and

$$\Delta(x_2^*) = n\frac{2\Psi^{\frac{3}{2}} - 2n^2\Lambda - 1215n + 452}{27\,(2n-3)^2}$$

(where $\Psi = 43 + 53n^2 + 4n^4 - 18n^3 - 81n$ and $\Lambda = -642 + 8n^4 + 51n^2 + 243n - 54n^3$) with $\Delta(x_2^*) > \Delta(0) > 0$, the solution x_2^* describes a global maximum which always satisfies condition (A4).

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