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Inequality and Growth: A Joint Analysis of Demand and Supply

by

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Inequality and Growth:
A Joint Analysis of Demand and Supply

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Abstract

Empirical evidence on the relationship between a country's wealth inequality and economic growth is ambiguous. This paper provides reasonable explanations of this ambiguity. We investigate the implications which the shape of wealth distribution has for economic growth in a framework combining the Schumpeterian quality improvement model and the neoclassic production function. Since two types of individuals are assumed, the poor and the rich, the Gini-coefficient is decomposed in two variables, namely the relative wealth of the poor and the population share of the poor, each having a different effect on economic performance. Particularly in the separating equilibrium, an improvement in the relative wealth of the poor impedes economic growth, but a decline in the population share of the poor enhances economic growth. This suggests that empirical research on the base of the Gini-coefficient cannot generate a general relationship between wealth inequality and economic growth. Moreover, the impact of wealth inequality on economic growth is through the supply of human capital as well as the demand for better quality goods. Hence, the relationship between wealth inequality and economic growth is non-linear.

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1. Introduction

The relationship between a country's wealth inequality and its economic growth has been a major concern of economists for more than a century. Yet it is far from being well understood. In theoretical modelling, the distribution of wealth is the relevant inequality source. However, most empirical studies use income inequality data as a proxy for wealth inequality because of the scarcity of available data on the distribution of wealth.[2] "It is generally argued that this is unlikely to be a major problem since both measures of inequality generally vary together in cross-sections." (Aghion et al. 1999) In the current paper, initial wealth inequality coincides with income inequality through human capital investment.

Empirical evidence regarding this relationship is ambiguous. Some cross-country studies (e.g., Berg and Sachs 1988, Persson and Tabellini 1994, Alesina and Rodrik 1994, Clarke 1995) show that income inequality, as a proxy for wealth inequality, negatively impacts long run growth rates. Nonetheless, there also is evidence that income inequality has a positive impact on short or medium run growth rates (Forbes 2000), and that the relationship between income distribution and the long run growth rate is non-linear (Chen 2003, Banerjee and Duflo 2003). The ambiguous empirical results imply that there is not a clear relationship between income inequality and economic growth (Barro 2000). Hence, it is important for economists to develop models which illustrate the possible different effects of inequality on economic growth under different circumstances. The existing theoretical wisdom has proposed either a negative or a positive relationship between initial wealth inequality and economic growth. Here it will be shown that both are extreme cases in an integrating simple model. We further the analysis of the relationship between wealth inequality and economic growth in two directions.

First, in a simple model with two types of individuals, the poor and the rich, the distribution of wealth comprises two variables, namely the relative wealth of the poor and the population share of the poor. We argue these variables may have different, even opposite effects on economic growth under certain conditions. Hence, cross-country evidence which is based on the simple regression of the Gini-coefficient on the economic growth rate can be ambiguous. In particular, we may be unable to obtain from such empirical studies recommendations on

[2] There also are studies using other proxies. For instance, Alesina and Rodrik (1994) and Deininger and Squire (1998) include land inequality along with income inequality, Castelló and Doménech (2002) investigate human capital inequality.

redistribution policies for achieving a higher economic growth rate as well as a more equal distribution.

Second, we combine the supply of production factors and the demand for the new quality goods in a general equilibrium model. Thus, wealth inequality in two areas can affect the economic performance: the supply side and the demand side. Most of the literature maintains that wealth inequality reduces the aggregate human capital investment, given a neoclassical production function of investment and imperfect capital market. Consequently, inequality has a negative effect on the supply of consumption goods. We name this effect "the supply-side effect". The main arguments of the supply-side effect are included in the survey of Benabou 1996. On the other hand, following the literature on endogenous growth with quality-improving innovation (Aghion and Howitt 1998, Zweimüller et al. 2000) we argue that innovation is the engine that drives economic growth. This can improve the quality of goods and, in turn, increase the utility of consumers. The innovation cost is compensated by the monopolistic profit after successful innovation. Thus, the incentive of innovation is the monopolistic profit. Wealth distribution can affect the demand for the newly invented goods, and subsequently the price and profit of monopolist. We name this "the demand-side effect".

As we assume that there are only two types of individuals, the monopolistic supplier of newly invented goods can set the price either at the separating level, i.e. only the rich are able to buy it, or at the pooling level that even the poor can afford. Because wealth distribution has different effects on the profit in both cases, the relationship between inequality and economic growth is non-linear. Inequality may give rise to a higher incentive for firms to innovate because rich consumers can pay more than the poor for high quality goods. However, on the other hand, the relatively small market share of high quality goods implied by inequality impedes the spread of better quality goods.

This paper shows that in a separating equilibrium, a lower relative wealth of the poor is good for innovation, and a larger population share of the poor is bad for innovation. This result is consistent with Foellmi and Zweimüller (2002) and Shen (2004). In Foellmi et al. (2002) hierarchic preferences[3] are introduced, and innovation induces new goods but does not improve quality. Shen (2004) considers the interdependent relationship between the relative wealth of the poor and the population share of the poor. In the pooling equilibrium, the lower

[3] "A hierarchy of wants implies that goods can be ranked according to their priority in consumption" (Foellmi and Zweimüller 2002)

relative wealth of the poor is bad for innovation, and the population of the poor has no effect on innovation. The threshold value which distinguishes between these two equilibria depends on the strength of the supply-side effect. These findings imply that two nations with the same Gini-coefficient could have different economic growth rates if their wealth inequality is reached for different reasons (e.g., low relative wealth of the poor or large population share of the poor). Hence, it is important to decompose the Gini-coefficient in empirical research.

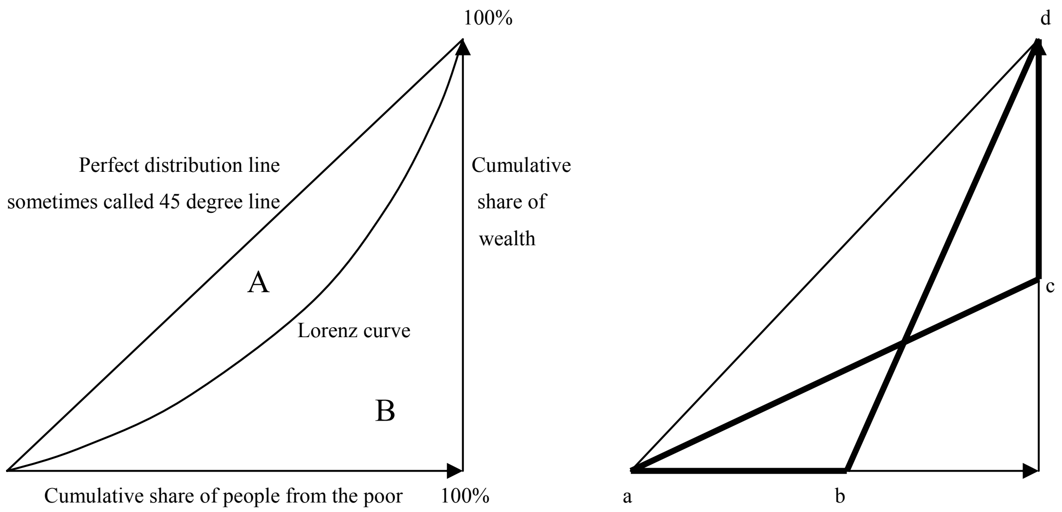
This paper integrates two main streams of theory relating growth and inequality. Recent surveys of the supply-side effect are by Benabou (1996) and Aghion et al. (1999), where three broad categories corresponding to the main feature are stressed: imperfect financial market, political economy and social unrest. The demand-side effect is illustrated by Murphy et al. (1989), Foellmi et al. (2002) and Zweimüller et al. (2000 and 2005).

The rest of this paper is organized as follows: Section 2 discusses briefly the measurement of inequality. Section 3 lays out the basic framework. In section 4 we analyze the equilibrium and in Section 5 we give an example and present some empirical implications with section 6 concluding.

2. The Measurement of Inequality

Since Corrado Gini, the Italian statistician, published his paper "Variabilità e mutabilità" in 1912, the Gini coefficient is widely used as a measurement of inequality. It is a number between 0 and 1, where 0 corresponds with perfect equality (everyone has the same wealth) and 1 means perfect inequality (one person has all the wealth; everyone else has nothing). The Gini index is the Gini coefficient expressed in percentage form, and is equal to the Gini coefficient multiplied by 100.

The Gini coefficient is calculated as a ratio of the areas on the Lorenz curve diagram. (see Figure 1(a)). If the area between the line of perfect equality and the Lorenz curve is A, and the area beneath the Lorenz curve is B, then the Gini coefficient is $\frac{A}{A+B}$. The advantages of using the Gini coefficient are clear: It is both scale and population-independent, hence, it can be compared across countries and is easily interpreted; by retaining anonymity it doesn't matter who the high and low earners are; last but not least, it is simple. However, economies with similar wealth and Gini coefficients can still have very different distributions. This is because the Lorenz curves may have different shapes and yet yield the same Gini coefficient. As an extreme example, an economy where half the households have no wealth, and half share the wealth equally has a Gini coefficient of 0.5 (Lorenz curve *abd* in Figure 1(b)); but an economy with complete wealth equality except for one wealthy household that has half the total wealth also has a Gini coefficient of 0.5 (Lorenz curve *acd* in Figure 1(b)). In this paper, we address the question: Does the shape of Lorenz curve having the same Gini coefficient matter?



(a) Lorenz curve

(b) different shapes of Lorenz curve with the same Gini coefficient

Figure 1: Lorenz curve and the Gini coefficient

3. The Model

We consider a closed economy with two types of individuals: the poor and the rich. They work for firms and consume products of firms. There are two kinds of goods: standard goods and quality goods. The quality improves over time due to innovation. Hence, the innovation rate represents the growth rate of quality, but not the growth rate of quantity. In turn, the economic growth is the growth of the consumers' utility, not the output.

3.1 The Environment

This is an overlapping-generations model. Time is discrete, indexed by $t = 1,2,...,$ and at each point in time there is a continuum of individuals who live for two periods, young and old. The population size of each generation is constant over time and normalized to 1. Individuals, within as well as across generations, are identical in their preferences. However, they may differ in their family wealth and thus, due to the absence of perfect financial markets, in their capacity to invest in human capital. For simplicity, we assume that there are two kinds of individuals: the poor and the rich, their population shares being β ($0 < \beta < 1$) and $1-\beta$, respectively. The average wealth of the whole society is denoted by V, which is the value of firms. Firms earn a flow profit and the value of firms equals the present value of this flow profit.[4] The poor individual has wealth $A_p = dV$, where d ($0 < d < 1$) means the wealth of the poor relative to the average level V. As a result the rich have $A_r = \frac{1-d\beta}{1-\beta}V$.[5] For simplicity, we assume that wealth should not be eaten and can be transferred from generation to generation. Thus, there is no social mobility in this simple model. At birth, a young individual i receives an amount of dividend θA_i, where θ is a constant dividend rate. Therefore, the wealth distribution is equivalent to the distribution of the initial income of the young people.

Figure 2 shows the resulting Lorenz-curve. Given our assumptions, the Lorenz-curve is piecewise linear. The Gini-coefficient of wealth, as well as that of the income of the young is $(1-d)\beta$, (see Appendix 1). Both an increase in the population share of the poor and a decrease in relative wealth of the poor can increase the inequality level of wealth. However, we claim that they have different effects on the economic growth.

There are four sectors in the economy. The education sector produces skilled labor which is the only production factor and is expressed by the efficient labor unit denoted by L. The education sector is run as a non-profit organization. It collects an education fee H from young individuals and hires S efficient labor units from old individuals to teach. The more that young individuals invest in the education sector, the more efficient labor units of the old

[4] See section 3.3 and 3.4.

[5] According to the definition of the average wealth, $V = A_p\beta + A_r(1-\beta)$. After substituting $A_p = dV$ and rearranging, we have $A_r = \frac{1-d\beta}{1-\beta}V$.

generation will be employed as teachers. As a result, more efficient labor units can be produced for the next period. Following education, young individuals have L units of efficient labor, which can be used in four sectors when individuals are old.

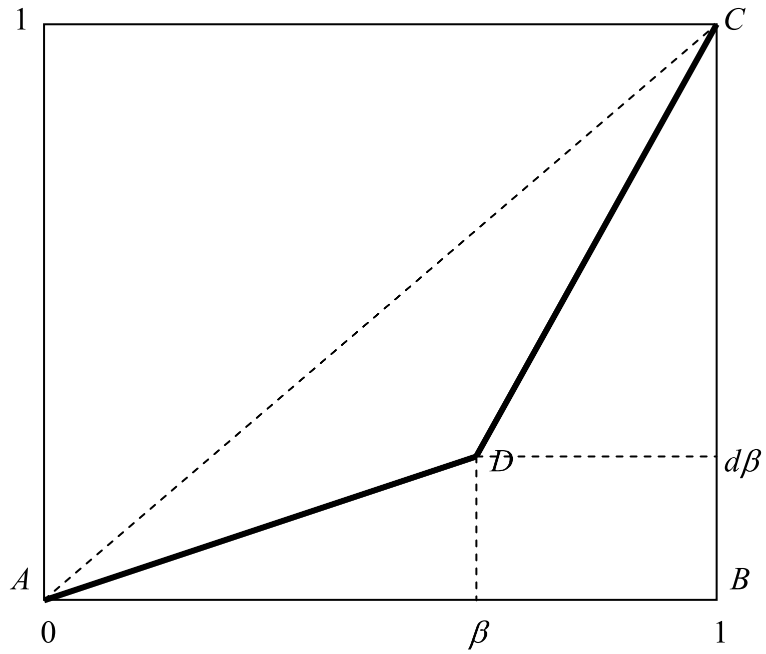


Figure 2: The wealth distribution

Two production sectors produce two kinds of goods, referred to as standard goods and quality goods, respectively. Let x be the quantity of the standard goods, which has a constant quality (normalized to 1) and is traded in a competitive market. Hence, the price P_x is equal to its marginal cost which is also normalized to 1. The marginal cost of the standard goods can be expressed as wb, where the unit labor demand is b. This determines how many units of efficient labor are needed to produce one unit of standard goods. w is the wage rate of the efficient labor unit. We get $P_x = wb = 1$.

In the quality goods sector, one monopolist produces the best quality goods. Anyone is allowed to produce competitively any other quality goods. We denote the quality level as $q_j \quad j = 0,-1,-2,...$, where q_0 is the best quality, q_{-1} is the second best one and so on. Furthermore, we assume for simplicity: $q_j = kq_{j-1} \quad k > 1$. Despite the different qualities, the quality goods have the same marginal cost wa, where a is again the unit labor demand. Since all quality goods except the best quality are sold on a competitive market, they have the same price $P_j = wa, \quad j = -1, -2,...$, and the monopolist sets P_0 to maximize her profit. For convenience, we assume that every consumer can consume one and only one unit of quality goods.

The new quality good is invented by the research sector. The research sector needs n units of efficient labor to achieve the innovation rate, ϕ, which is the probability of success. Each successful innovation introduces a k-times better quality good than the existing best one in the next period. The authority to produce this best quality will be sold to one monopolist. After successful innovation the current best quality becomes the second best quality in the next period. Hence, any competitor can produce it. Since in equilibrium the amount of consumption is constant, the economic growth throughout this model is not the growth of quantity, but of quality. Here the innovation rate ϕ represents this growth rate of quality. Later on, we will see that the growth rate of quality coincides with that of the consumers' utility.

The assumption of two kinds of goods, one with constant quality and the other with constant quantity, is an abstract of two dimensions of consumption. In reality the quality of each of the goods can be improved and there is no limit that consumers can only consume one unit. However, we can still find goods whose quality consumers readily appraise: automobiles for instance. Normally we have one car. However, we sometimes buy a new, better quality car, and sell the old one in order to improve our utility. There are other goods about whose quantity consumers also readily appraise, for example, leisure.

3.2 The Household's Decision Problem

As we assumed in last section, a young individual i has initial income θA_i at birth which can be used to buy standard goods[6] x_i^1 at the price 1 and invest in education H_i. The production function of efficient labor is $L(H_i) = l + H_i^{\alpha}, \quad (\alpha < 1)$[7], where l ($l > 0$) is constant and

[6] This assumption ensures only two kinds of consumers in the quality goods market. Allowing young people to be able to buy quality goods will not change our result qualitatively, but complicate the analysis, because then there are four types of consumers in the quality goods market. This assumption also is reasonable. For example, we can imagine the quality goods to be automobile, alcohol and/or cigarette, which are prohibited for young people.

[7] It is a closed form of $L = l + l' S^{\alpha}$, where l' is a constant parameter and S the labor units of teachers. Since the education sector has no profit, H is totally paid for teaching. Hence, $H = Sw$. For simplicity, we assume $l' = w^{\alpha}$. Thus, the closed form for the production function of labor is $L = l + H^{\alpha}$.

represents the basic supply of labor without any education. The H_i^{α} are the efficient labor units produced by the human capital investment, H_i, which is the choice variable of the young individual i. This production function is a strictly concave increasing function satisfying the neoclassical boundary conditions, and $L(0)=l$. For simplicity, we assume that l is equal to the unit labor demand of quality good: $l=a$, which simplifies the calculation without loss of generality. Hence,

$$L(H_i)=a+H_i^{\alpha}, \quad (\alpha<1) \tag{1}$$

In the second period, old individual i has L_i units of efficient labor. We assume a simplistic view regarding the production of consumption goods. Efficient labor is the single productive factor, and every individual inelastically supplies all of her efficient labor units to the competitive labour market. As a result, incomes of the poor and the rich in the second period are respectively: $y_i=wL_i, \quad i=p,r$. In section 4, we will show that the poor invest less than the rich. Hence, $L_p<L_r$, in turn, $y_p<y_r$. It means that there is no social mobility in our simple model.

We assume the instantaneous utility function in the first period to be $u_i^1=\ln x_i^1$. Because the standard good is the single good which young people can consume. Substituting the budget constraint in the first period $\theta A_i=x_i^1+H_i$, we have:

$$u_i^1=\ln(\theta A_i-H_i) \tag{2}$$

There is no saving for the old individual. All income is spent both on the consumption of the standard good and the quality good. Every individual can consume one and only one unit of the quality good q_j. There is no limit to the consumption of the standard good x_i^2 except for the budget constraint, i.e., $y_i=1\cdot x_i^2+P_j\cdot 1 \quad j=0,-1,\ldots$, where the price of standard goods is 1, the quantity of standard goods is x_i^2 and the price of the quality j is denoted by P_j. The preference for consumption of the standard good and the quality good is given by the following utility function:

$$u_i^2(x_i^2, q_j) = \ln x_i^2 + \ln q_j \qquad i = p, r \text{ and } j = 0, -1 \tag{3}$$

By substituting the budget constraint in the second period, (3) can be expressed as:

$$u_i^2 = \ln(y_i - P_j) + \ln q_j \tag{4}$$

The life-time utility function of individual i is assumed to be:

$$U_i = u_i^1 + \rho u_i^2 \tag{5}$$

where ρ is the subjective discount factor. It can, but need not necessarily, be equal to $\frac{1}{1+\theta}$, where θ is the dividend rate. The old individual i chooses the quality level q_j to maximize u_i^2, given income y_i being constant. By backward induction, when the subject is young she chooses H_i to maximize her life-time utility (5) with the rational expectation that q_j will be optimally determined in the second period. Hence, in order to solve the household's decision problem, we need to know the price of the quality good.

3.3 The Pricing Decision of the Monopolist

Firms have all the above information but are unable to distinguish between individuals based on income. The strategy which firms can pursue is to choose a price while quality is fixed. We concentrate only on the steady state where prices are constant over time. First of all, only the most recent old quality good (q_{-1}) can be sold at the price wa in the competitive market of quality goods q_j $j < 0$. Hence, the price that the monopolist can offer has to satisfy the condition:

$$\ln(y_i - P_0) + \ln q_0 \geq \ln(y_i - wa) + \ln q_{-1} \tag{6}$$

The left hand side of (6) is the utility when individuals buy the best quality good q_0 and the right hand side is the utility when they consume the second best quality good q_{-1}. Further, we assume that the consumer prefers better quality goods if both quality goods yield the same utility. Substituting $q_0 = kq_{-1}$ and rearranging (6), we get the highest price $\overline{P}_0$ of the best quality good:

$$\overline{P}_0 = (1-\frac{1}{k})y_i + \frac{wa}{k} \quad i = p,r \tag{7}$$

The monopolist thereby has two possible price strategies -- either to set the price high, to attract only the rich consumers (separating price), or, low to occupy the entire market (pooling price). The instantaneous profits are as follows:

$$\pi^{sep} = (1-\beta)(1-\frac{1}{k})(y_r^{sep} - wa) \tag{8}$$

$$\pi^{pool} = (1-\frac{1}{k})(y_p^{pool} - wa) \tag{9}$$

The monopolist sets the separating price in steady state, if 1) given the separating strategy before, she has no incentive to deviate, which means:

$$(1-\beta)(H_r^{sep})^{\alpha} \geq (H_p^{sep})^{\alpha} \tag{10}$$

2) the profit of separating strategy in steady state is larger than that of the pooling, viz.:

$$(1-\beta)(H_r^{sep})^{\alpha} \geq (H_p^{pool})^{\alpha} \tag{11}$$

Since the supplier of the best quality goods is monopolistic, it has a positive flow profit. All other firms have zero profit and their value also is zero. All firms are owned by individuals. Hence, the value of this monopolistic firm is equal to V.

3.4 Innovation

As mentioned earlier, the quality improves over time due to innovation. Following the work by Aghion and Howitt (1992), we assume that the innovation is random and arrives according to a Poisson process with parameter ϕ. The researcher can employ n units of efficient labor to reach the Poisson arrival rate ϕ, i.e., $\phi = \lambda n$, where λ is the productivity of efficient labor in research. Hence, the flow of research cost is wn. This assumption of innovation means that the success of research depends only on current input, not upon past research.

Once innovation succeeds, a new quality good is invented. This newly invented good is k-times better than the current best quality good, and can be produced by one monopolist in the next period. The authority to produce this new best quality good is sold to the monopolist via a simple auction. We assume that researchers prefer to sell the authority to the incumbent as long as its offer is at least the same as that of others. In order to keep this priority, the incumbent has to buy the new innovation from researchers at a price which is equal to the present value of the future monopolistic profit. Thus, we have a single monopolist who produces the best quality in every period. The price paid by the monopolist to the research sector is the flow of research benefit, ϕB, where ϕ is the probability of success and B is the present value of the future monopolistic profit:

$$B = \sum_{t=1}^{\infty}\left(\frac{\pi}{(1+\theta)^t} prob\{no\ innovation\ before\ t\}\right) = \sum_{t=1}^{\infty}\left(\frac{\pi(1-\phi^e)^{t-1}}{(1+\theta)^t}\right)$$

Leading to

$$B = \frac{\pi}{\phi^e + \theta} \qquad (12)$$

where t is a time index, $\phi^e = \lambda n^e$ is the expected future arrival rate of innovation, n^e is the expected future number of efficient labor units in the research sector, and θ is interest rate. The sum of the interest rate and the innovation rate is the discount factor of the monopolistic profit. In steady state, all agents have perfect foresight. Consequently, $\phi = \phi^e$ (or, $n = n^e$).

We are now in a position to define the average wealth of the whole society V. As we mentioned before, the average wealth is the value of the monopolistic firm, which can generate dividends θV in each period. Hence, the per period increase in the average wealth is the monopolistic profit net of the dividend and the payment to the researcher.

$$\Delta V = \pi - \theta V - \phi B \qquad (13)$$

4. Equilibrium

According to section 3.3 there are two possible equilibria, namely separating and pooling respectively. If the monopolist chooses the separating strategy, then the poor buy q_{-1} and the rich consume q_0. Hence, the rich young people maximize their life-time utility as follows:

$$\max_{H_r} U_r = \ln(\theta A_r - H_r) + \rho(\ln(y_r(H_r) - P_0) + \ln q_0)$$

substituting (7) in this equation and solving the first order condition, we have:

$$H_r^{sep} = \frac{\alpha\rho}{1+\alpha\rho}\theta A_r^{sep} \quad (14)$$

Similarly,

$$H_p^{sep} = \frac{\alpha\rho}{1+\alpha\rho}\theta A_p^{sep} \quad (15)$$

If the monopolist chooses the pooling price, then the poor set the optimal investment at:

$$H_p^{pool} = \frac{\alpha\rho}{1+\alpha\rho}\theta A_p^{pool} \quad (16)$$

$\frac{\alpha\rho}{1+\alpha\rho}$ is the saving rate of the young people. The results (14) - (16), consistent with Bénabou (1996), reflect the fact that the poor invest less in human capital than the rich. Due to the neoclassical production function of human capital investment, (1), the marginal productivity of the human capital investment of the poor is higher than that of the rich. Hence, the inequality of initial wealth reduces the aggregate supply of efficient labor units. This is the negative supply-side effect.

Substituting (14) and (16) in (8) and (9), we have

$$\pi^{sep} = (1-\beta)(1-\frac{1}{k})w(\frac{\alpha\rho}{1+\alpha\rho}\theta A_r^{sep})^{\alpha} \quad (17)$$

$$\pi^{pool} = (1-\frac{1}{k})w(\frac{\alpha\rho}{1+\alpha\rho}\theta A_p^{pool})^{\alpha} \quad (18)$$

The instantaneous profit of the monopolist in separating equilibrium depends on the initial wealth of the rich young people. Analogously, the profit in pooling equilibrium depends on the wealth of the poor.

Furthermore, we assume free entry in the research sector, which is the traditional assumption of the quality-improving model, to obtain the research arbitrage equation (Aghion and Howitt 2004). Hence, $wn = \phi B$, where wn is the flow cost of the research sector and ϕB is the flow benefit (see section 3.4). This leads to:

$$\frac{w}{\lambda} = \frac{\pi}{\phi + \theta} \tag{19}$$

The underlying intuition is similar to Aghion and Howitt (1992). The left hand side of equation (19) represents the flow cost of research in order to achieve a successful innovation, which decreases in the productivity of research workers λ. The effect of λ on ϕ is positive because the researcher is able to achieve a higher innovation rate with the same number of efficient labor units if their productivity increases. The effect of the interest rate is ambiguous. First, it is a discount factor. Hence, the higher θ, the lower the research benefit. Therefore, the innovation rate decreases in the interest rate. The other way through in which the interest rate can affect the innovation rate is, by raising the initial income of individuals. Hence, the higher θ, the larger the human capital and consequently, the larger the monopolistic profit. It has a positive effect on the innovation rate.

As the single production factor, the supply of efficient labor units should be equal to the demand for efficient labor units in equilibrium. The total efficient labor supply is $\beta L_p + (1-\beta)L_r$, which is equal to $a + \beta H_p{}^{\alpha} + (1-\beta)H_r{}^{\alpha}$. The demand for efficient labor consists of four parts. First, the research sector needs n. Second, the quality goods sector needs a because every consumer consumes one unit of quality good. Third, the standard goods sector needs $b(x^1 + \beta x_p^2 + (1-\beta)x_r^2)$. And finally, the education sector needs S. Hence, the total demand for efficient labor units is $n + a + b(\beta x_p + (1-\beta)x_r) + S$. In equilibrium, the labor market clearing condition is as follows:

$$\beta L_p + (1-\beta)L_r = n + a + b(\beta x_p + (1-\beta)x_r) + S \quad (20)$$

Solving (20) yields $$\pi = wn + \theta V \quad (21)$$

Proof: see Appendix 2.

From (19), (21) and $\phi = \lambda n$ we know that the average wealth $V^* = \frac{w}{\lambda}$ in equilibrium regardless of the price strategy of the monopolist. The higher the wage rate, the greater is the average wealth. This is because the high wage rate involves the rich consumer (recalling that the old people are the consumers of quality goods, their income is given by $y_i = wL_i \quad i = p, r$, which depends on the wage rate). Then the monopolist can set a high price and earn more profit (see equations 17 and 18). The larger λ, the higher is the innovation rate. Thus, the value of the monopolistic firm is less.

After substituting (14), (15) and (16) into (10) and (11), and using $V^* = \frac{w}{\lambda}$, we get the unique condition of the separating price in equilibrium:

$$(1-\beta)^{1-\alpha}(\frac{1}{d} - \beta)^{\alpha} \geq 1 \quad (22)$$

This condition shows that the larger the population share of the poor, and/or the richer the poor, the less probable will the monopolist choose the separating price strategy. The larger the α, the bigger the difference of income of old individuals. Hence, more probably will the separating price be chosen.

Rearranging (19) and substituting (17) and (18), we have two possible innovation rates in the separating and the pooling equilibria respectively:

$$\phi^{sep} = \lambda(1-\frac{1}{k})(1-\beta)(\frac{\alpha\rho}{1+\alpha\rho}\theta A_r)^{\alpha} - \theta \quad (23)$$

$$\phi^{pool} = \lambda(1-\frac{1}{k})(\frac{\alpha\rho}{1+\alpha\rho}\theta A_p)^{\alpha} - \theta \quad (24)$$

where $A_r = \dfrac{(1-d\beta)w}{(1-\beta)\lambda}$, $\quad A_p = \dfrac{dw}{\lambda}$.

Proposition 1

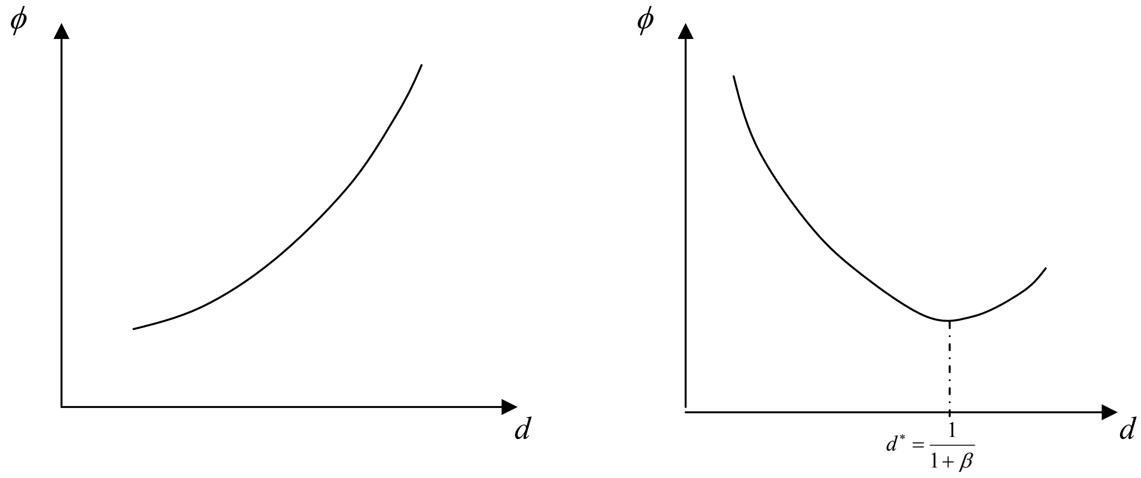
The effect of wealth inequality on the innovation rate is non-linear and ambiguous:

1) *Given* β *constant, the effect of* d *on* ϕ *is negative for* $d \in [0, d^*]$ *and positive for* $d \in [d^*, 1]$. *The threshold value* $d^* \in (0,1)$ *satisfies* $(1-\beta)^{1-\alpha}(\frac{1}{d^*} - \beta)^{\alpha} = 1$.

2) *Given* d *constant, the effect of* β *on* ϕ *is negative for* $\beta \in [0, \beta^*]$. *In the pooling case* $\beta \in [\beta^*, 1]$, β *has no effect on* ϕ. *The threshold value* $\beta^* \in (0,1)$ *satisfies* $(1-\beta^*)^{1-\alpha}(\frac{1}{d} - \beta^*)^{\alpha} = 1$.

The non-linear relationship between initial income inequality and economic growth has two interpretations in the current model: For one, d and β have different effects on the innovation rate. For the other, both the effect of d and that of β on ϕ are non-linear. Inequality can affect the innovation rate not only through the supply of the production factor (here, efficient labor units) but also the demand for the new better quality. The supply-side effect is discussed by most economists. Here, we assume the strictly concave increasing production function of the efficient labor units and an imperfect capital market as in the literature; hence, the negative effect of inequality on growth is not surprising (see Appendix 3). The parameter α is a measure of the strength of the supply-side effect.

Figure 3 shows different effects of the relative wealth of the poor on the innovation rate in two extreme cases. Both are the examples where the supply-side effect disappears. Suppose $\alpha \to 0$, then the saving rate of the young people ($\frac{\alpha\rho}{1+\alpha\rho}$) approaches zero. Both the poor and the rich young people have little incentive to invest in human capital. Hence, the difference in income for the old people approaches zero. The threshold value $d^* \to 0$. The monopolist faces a more equally distributed society and thus sets the pooling price. Consequently, the income of the poor is crucial for the price of the quality good. In this case, if the poor have more income, then the price of the quality good increases. Finally, the innovation rate increases. The effect of d on ϕ is overall positive in the case of (a).



$(a)\quad \alpha \to 0$ $\qquad$ $(b)\quad \alpha \to 1$

Figure 3: The pure demand-side effect of d on ϕ

The picture is reversed, if $\alpha \to 1$, $d^* \to \frac{1}{1+\beta}$, i.e., $Gini^* \to \frac{\beta^2}{1+\beta}$. Since $\frac{1}{1+\beta} > \frac{1}{2}$, we can argue that d has negative effect on the innovation rate over the most range through the demand side. If the condition (22) holds, then the poor are too poor and/or the population of the poor is too small. Hence, the monopolist sets the separating price to sell the best quality good only to the rich. In this case, if the rich become poorer and the poor become richer (d increases), i.e., if the Gini-coefficient decreases given constant β, then this inequality brings about less incentive for the researcher to innovate because of falling profits. If d increases further and exceeds the threshold value $\frac{1}{1+\beta}$, the monopolist sets the pooling price and then d has a positive effect on the innovation rate ϕ. This is case (b).

Contrary to d, the population of the poor β has a different effect on the innovation rate. In the case of the separating price, if the Gini-coefficient increases because the population of the poor β increases given constant d, then the inequality leads to a small market size for the quality good. Hence, the monopolistic firm has less profit, and the innovation rate decreases. If a country has a relatively even initial income distribution ($\beta \in [\beta^*, 1]$) then the monopolist sets the pooling price. Since the market of the quality good is the whole society, the population share of the poor does not affect the innovation rate.

What is the impact of wealth inequality (through β or d, respectively) on utility? From (2) and (3) we have:

$$\Delta u^1 = \frac{\Delta x^1}{x^1}$$
$$\Delta u^2 = \frac{\Delta x^2}{x^2} + \frac{\Delta q}{q} \qquad (25)$$

In a steady state, the consumption of standard goods is constant ($\Delta x = 0$), and $\Delta q = \phi(k-1)q$. Hence, we have $\Delta U = \rho\phi(k-1)$. The higher the innovation rate, the larger is the increase in the utility. Redistribution from the rich to the poor (d increases) decreases the wealth inequality, hence, the aggregate supply of efficient labor increases. This is the supply-side effect. What is the demand-side effect of this redistribution? If $d < d^*$, the monopolist sets a separating price. Redistribution leads to a decrease in the initial wealth of the rich, in turn, a less monopolistic profit. Consequently, the research sector employs less efficient labor units. Recalling that the quality good sector always needs a units of efficient labor and the education sector requires the same efficient labor as long as the aggregate investment of human capital remains constant, more efficient labor units are shifted from the research sector to the standard goods sector. This reallocation of efficient labor among different sectors is the demand-side effect. Summing up, consumers enjoy a higher utility level in the short run, but the long run growth rate of utility is lower than before because of a lower innovation rate. If $d > d^*$, we have a pooling equilibrium. In contrast to the separating case, redistribution from the rich to the poor can induce a higher price of quality goods and more monopolistic profits. Consequently, the research sector has a higher incentive to employ more efficient labor units and a higher innovation rate will be achieved. It is not a priori clear whether consumers have more or less consumption of standard goods in a new pooling equilibrium. It depends on which effect is dominant, the supply-side effect or the demand-side effect.

5. An example: China

Because d and β have different effects on the innovation rate, in particular, their effects offset each other in the separating equilibrium, the Gini-coefficient has no overall effect on economic growth. In this sense, it is important for us to decompose the Gini-coefficient in the empirical research. The different effects of the relative poorness and the population share of the poor imply the different policy recommendation. In a country where the separating equilibrium is overwhelming and the goal of government policy is to achieve both an increase in economic growth and a decrease in inequality, one should consider decreasing the population share of the poor but not redistributing from the rich to the poor.

Chinese experience in the last two decades bears witness to this prediction. In China, the disparity between urban and rural residents is assured by the Chinese household registration (Hukou) system, (Yang and Zhou 1999). Lacking free migration between urban and rural areas, the Chinese government has invested more in public goods such as education, social insurance and infrastructure, in the cities than in the rural areas since 1949. This can be stylized by assuming V to be the public social wealth.[8] The government implements an urban-biased redistribution policy, (Yang 1999). Hence, the urban resident is rich and the rural resident is poor. The goal of Chinese reform above all is to have a high economic growth rate. Government can control both the population share of the poor through the Hukou system and the relative poorness of the poor through the redistribution policy.

After the 1980s, this Hukou system was relaxed. However, it has never been abandoned. As a result the urban population (the rich) increased dramatically from 21% in 1982 to 36% in 2000. At the same time, the relative income of rural residents (y_p/y) decreased from 0.76 (1980) to 0.61 (2000), (China Statistical Yearbook 2002). Combining these results, Chinese firms have a great incentive to invent better quality goods and set prices at the separating level. The evidence for separating price strategy lies in the fact that the most new and better quality goods are sold in Chinese cities. According to the China Statistical Yearbook 2002, Chinese average growth rate of GDP per capita was approximately 9% over the last 20 years. Although there are many reasons for the rapid growth, we cannot deny that one of them is the high demand for the better quality goods.

[8] It reflects the central planning economy in China before reform, at which time almost all firms were state-owned. Hence, the Chinese government did have the power to distribute V between urban and rural. Since 1980, the power of this distribution diminishes as more and more firms went private. However, many state-owned firms remain, particularly, in the monopolistic branches and capital intensive industries.

6. Conclusion

This paper investigates the ambiguous relationship between wealth inequality and economic growth in a framework of a quality-improving growth model. Our contribution is to enhance the analysis of this relationship in two ways. First, we argue that the Gini-coefficient, used by most empirical research in this branch, can include too many variables which have diverse effects on economic growth. Therefore, we need to decompose the Gini-coefficient into different variables. The current model supplies an example that divides the Gini-coefficient into the relative wealth of the poor and the population share of the poor. We have shown that they induce a contradictory effect under certain conditions. This result indicates that we need to investigate not only the Gini-coefficient, but also the shape of wealth distribution. The empirical research on the base of the Gini-coefficient cannot generate a clear relationship between wealth distribution and economic growth. In particular, we may be unable to draw from such simple empirical studies recommendations on redistribution policies for achieving a higher economic growth rate as well as a more equal income distribution.

Additionally, we have combined two sides of the market within one simple model: the supply of production factors and the demand for the consumption goods. Thus, in this model, there are two different channels, by which wealth distribution can affect economic performance. Whereas the supply-side effect of wealth inequality is negative on economic performance, the demand-side effect could be positive under certain conditions. Hence, there is non-linear relationship between the wealth inequality and economic growth. This result is partly consistent with the empirical evidence (Chen 2003), although he uses the Gini coefficient, but not other variables which we investigate.

Appendix

Appendix 1

According to the definition of the Gini-coefficient, it is equal to the ratio of the areas ACD and ABC. As we normalized AB and BC to 1, we have:

$$\begin{aligned} Gini &= 2 \bullet \textit{the area of } ACD = 1 - 2 \bullet \textit{the area of } ABCD \\ &= 1 - \beta \bullet d\beta - (d\beta + 1) \bullet (1 - \beta) \\ &= (1 - d)\beta \end{aligned}$$

Appendix 2

The labor market clearing condition is $\beta L_p + (1-\beta)L_r = n + a + b(\beta x_p + (1-\beta)x_r) + S$

Substituting (1) and budget constraint equations of both periods, we have two possible cases:

1) if the monopolist sets the price at the pooling level:

$$\begin{aligned} a + \beta H_p^\alpha + (1-\beta)H_r^\alpha &= n + a + b(\beta(\theta A_p - H_p) + (1-\beta)(\theta A_r - H_r) + \beta(y_p - \overline{P}_0) + \\ &(1-\beta)(y_r - \overline{P}_0)) + S \end{aligned}$$

recall $wb = 1$, $wS = \beta H_p + (1-\beta)H_r$ and $\pi^{pool} = \overline{P}_0 - wa$:

$$\begin{aligned} w(\beta H_p^\alpha + (1-\beta)H_r^\alpha) &= wn + \beta(\theta A_p - H_p) + (1-\beta)(\theta A_r - H_r) + \beta(y_p - \overline{P}_0) + (1-\beta)(y_r - \overline{P}_0) \\ &+ \beta H_p + (1-\beta)H_r \\ \Leftrightarrow 0 &= wn + \theta V - \pi^{pool} \end{aligned}$$

2) if the monopolist sets the price at the separating level:

$$\begin{aligned} a + \beta H_p^\alpha + (1-\beta)H_r^\alpha &= n + a + b(\beta(\theta A_p - H_p) + (1-\beta)(\theta A_r - H_r) + \beta(y_p - wa) + \\ &(1-\beta)(y_r - \overline{P}_0)) + S \\ \Leftrightarrow 0 &= wn + \theta V - \pi^{sep} \end{aligned}$$

Summing, we have $\pi = wn + \theta V$.

Appendix 3

Here we show that the effect of d on L is positive. Hence, the redistribution from the rich to the poor can increase the supply of labor; in turn, the innovation rate increases.

$$L = \beta L_p + (1-\beta)L_r = a + \beta H_p^\alpha + (1-\beta)H_r^\alpha$$

From (14) (15) (16) and $V^* = \frac{w}{\lambda}$, we know $H_p = \frac{\alpha\rho}{1+\alpha\rho} d \frac{w}{\lambda}$, $H_r = \frac{\alpha\rho}{1+\alpha\rho} \frac{1-d\beta}{1-\beta} \frac{w}{\lambda}$. Hence,

$$\begin{aligned} \frac{\partial L}{\partial d} &= \frac{\alpha\rho w}{(1+\alpha\rho)\lambda}\left[\beta\alpha d^{\alpha-1} + (1-\beta)\alpha\left(\frac{1-d\beta}{1-\beta}\right)^{\alpha-1}\left(\frac{-\beta}{1-\beta}\right)\right] \\ &= \frac{\beta\alpha^2\rho w}{(1+\alpha\rho)\lambda}\left[d^{\alpha-1} - \left(\frac{1-d\beta}{1-\beta}\right)^{\alpha-1}\right] > 0 \end{aligned}.$$

In both extreme cases $\alpha \to 0$ and $\alpha \to 1$, the supply-side effect of inequality on growth disappears, i.e., $\frac{\partial L}{\partial d} \to 0$. α reveals the strength of this supply-side effect.

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