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Abstract

In multi-attribute procurement auctions with multiple objects, the auctioneer may care about the interplay of quality attributes that do not belong to the same item – like each item's delivery time, if all items are needed at once. This can influence the performance of the auction mechanism. We generalize the Ausubel-Milgrom ascending proxy auction to such an environment and show that the main properties still hold: Equilibria in profit-target strategies exist, the final allocation maximizes the surplus and the payoff vector is in the core.

Furthermore, the scoring rule used to evaluate the bids may contain valuable information about the auctioneer for his competitors, providing an incentive not to reveal it. In our setting, it is possible to keep the scoring rule secret without changing the outcome of the auction. Additionally, for additive scoring rules a close connection to the original proxy auction exists.

JEL: D44, D82

Keywords: Multi-object auction, multi-attribute auction, information revelation

1 Introduction

In a procurement auction, the buyer is usually not only interested in getting an object as cheap as possible, but also cares about its quality. Scoring auctions provide the opportunity to submit bids that specify prices and quality attribute levels. These bids are evaluated with the help of a scoring rule (a function of quality attributes and price) and ranked according to the resulting scores. If the bidders know the scoring rule, this procedure resembles a classical auction with bids being scores. This simple relationship can

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get lost if the buyer wants to acquire multiple objects: His perception of an object's quality may heavily depend on the quality attributes of the other objects. In this paper, the scoring rule can be an arbitrary increasing function of all quality attributes (but quasi-linear in price). Especially, the overall quality may depend on the attribute levels of *all* items in a non-trivial way. Consequently, the score that a supplier is able to generate with his bid may depend on the bids of the other suppliers. Such an interdependency of bids does not appear in price-only auctions. Suyama and Yokoo (2004) have shown that the presence of such quality interdependencies in the scoring rule is not innocuous: The Vickrey-Clarke-Groves mechanism may fail to achieve individual rationality. We analyze the properties of a different mechanism in the presence of an interdependent scoring rule: The Ausubel-Milgrom ascending proxy auction. In standard auctions, this mechanism does not suffer from certain weaknesses of the Vickrey-Clarke-Groves mechanism, e.g. regarding collusion. It is thus a suitable candidate to work well with an interdependent scoring rule.

To illustrate the role of the interdependency, think of a quality attribute like delivery time: The buyer may need several objects simultaneously. He thus only values a fast delivery time of one object if the other objects are delivered quickly as well – otherwise, the speed advantage of one supplier is worthless. If these preferences of the buyer are reflected in the scoring rule, it is difficult for the suppliers to estimate the impact of their bid on the overall quality in advance – it depends crucially on the bids of the other suppliers. Hence, a single bidder can be very influential, e.g. if he is the only one who can deliver a particular item very quickly.

Other problems arise if the buyer does not want to give out information on its scoring rule to the sellers, e.g. because he tries to avoid information spillovers to his competitors. This could be information about his preferences toward different suppliers which are reflected in the scoring rule, or information about the quality of an object he is able to produce out of the items he wants to buy in this scoring auction. Due to such reasons, the auctioneer may want to keep his scoring rule secret. In particular, we have an example like the following setting in mind: A manufacturing firm is facing two procurement situations. On the one hand, it wants to be the seller of a specific product, and has a competitor who is able to deliver a similar product. Revealing information about the firm's production abilities to the competitor would have a negative impact on the firm's revenue, because the other firm can profitably use this information in its pricing process. On the other hand, the firm wants to acquire the components to manufacture the product by means of an auction. Using a public scoring rule in this auction provides the competitor with an informative signal about the firm's production abilities. If the firm wants to avoid these signals, is it possible to adapt the Ausubel-Milgrom proxy auction to deal with secret scoring rules as well?

Our version of the Ausubel-Milgrom proxy auction works as follows: For each possible quality configuration for each package, a seller submits a minimum price at which he is willing to deliver. This can be interpreted as the seller's cost structure. With the help of this cost structure, the proxy bidder submits bids automatically on behalf of the seller. The proxy follows a simple bidding strategy: It bids on all possible quality configurations yielding the highest potential profits (with respect to the reported cost structure). Bidding is stopped in case this potential profit gets negative.

We show with direct proofs that main theorems for the Ausubel-Milgrom proxy auction extend to this mechanism. This includes, with respect to the reported preferences, surplus maximization and the core property for the final winning allocation, as well as existence of equilibria in profit-target strategies. The scoring rule can be kept secret without influencing the outcome.

Furthermore, we consider the special case of an additive scoring rule. A scoring rule is called additive if a score can be calculated for each item separately, and these scores are added up to generate the score for a package. Here, the auction procedure can stay essentially the same compared to the original price-only proxy auction, in case the scoring rule is public: Each bidder calculates the maximum score he is able to generate for each single item and submits these scores (not necessarily truthfully) to the proxy. Similar to the price-only proxy auction, the proxy then bids myopically on packages of items. Consequently, results stay the same compared to the price-only proxy auction. We extend the bidding procedure to secret scoring rules by using price-quality bids. Although the scoring rule is not known, the outcome of the auction with public scoring rule is replicable with this bidding procedure. Particularly, this enables us to directly carry over some theory on the Ausubel-Milgrom proxy auction to secret scoring rules.

The literature on scoring auctions is surprisingly scarce[1], if one thinks of the variety of procurement settings where price *and* quality matter. There is a first strand of literature looking at optimal scoring auctions by adapting the scoring rule (Che 1993; Branco 1997; David et al. 2002a). Contrary to this approach, the scoring rule is fixed in our environment – we assume that the decision on the scoring rule has already been made.

Mueller et al. (2007) generalize Asker and Cantillon (2008) to combinatorial auctions: They show that the set of equilibria can be transferred from multi-dimensional price-only auctions to the corresponding scoring auctions. Mueller et al. (2007) use scoring rules for every possible package, which are not necessarily the sum of the scoring rules for the single items. The winning allocation is then determined by an allocation rule over scores. The additive scoring rule we use in part of this paper is a special case of their setting. Our *general* scoring rule differs from their approach, as it allows for interdependencies of quality attributes for different items across bidders – there is just one single scoring rule for all objects. Such an interdependent scoring rule can also be found in Suyama and Yokoo (2004) and (2005) in the context of Vickrey-Clarke-Groves mechanisms. Which type of scoring rule one wants to use is a question of the context. Furthermore, mixed forms are possible as well.

[1]A good survey can be found in Strecker (2004).

The possible failure of individual rationality in the Vickrey-Clarke-Groves mechanism with general scoring rules was first shown by Suyama and Yokoo (2004), with a more detailed characterization in Suyama and Yokoo (2005). Even if the rule is publicly known, a score for bids on packages cannot be calculated – it depends on the bids submitted by the other bidders. Imagine a scoring rule that treats two quality attributes of different items as perfect complements. The score thus only increases if a bid raise is made on the attribute levels for *both* items. Hence, if this raise is due to two different suppliers in the winning allocation, both suppliers have to be paid for it in the VCG mechanism – the buyer has to pay the raise *twice*. This may make the outcome too expensive for him, generating a negative final score – he would have preferred not to conduct the auction at all. We thus have to be careful when transferring results to scoring auctions with general scoring rules. For example, Rieck (2006) shows that the Bernheim-Whinston first-price package auction (Bernheim and Whinston 1986) can be extended to the setting of a general scoring rule without complications.

Secret scoring rules in the context of single-unit English auctions go back to David et al. (2002b). In their setting only monotonicity properties of the scoring rule with respect to the attribute values are announced to the sellers. Bids consist of price-quality combinations. As bidders do not know the scoring rule, submitted bids may be rejected if the score they generate is lower than the standing high bid. The auction ends when no sufficiently high bids are submitted in a prespecified period of time or all bidders stop their bidding activity. The set of possible attribute levels is assumed to be finite.

David et al. (2002b) show that in this setting, it is a dominant strategy for the suppliers to follow a *bid list strategy*: Each bidder ranks the possible bids (there are only finitely many due to the finite attribute space) according to his own preferences (potential profit). Then, he submits his bids in order of decreasing profit. In case he is standing high bidder he suspends the submittance of bids. Otherwise, he submits bids until all bids on his list were submitted. We extend their approach to the multi-object case: A proxy bidder takes the role of submitting the multi-object counterpart of the bid list, a ranking of possible bids according to their potential profit. This is the extension of the Ausubel-Milgrom proxy auction to secret scoring rules.

Finally, Rezende (2009) makes use of *bias functions* instead of scoring rules to account for quality differences. In his setting, the release of information is always optimal. The ascending proxy auction that we use is introduced and discussed by Ausubel and Milgrom (2002). A final discount stage is added by Lamy (2007), ensuring that truthful bidding leads to a bidder-optimal point in the core. Ranger (2005) extends the proxy auction to a setting with externalities.

The paper is organized as follows: First, we introduce the general framework in section 2. The results on the proxy auction with a general scoring rule are developed in section 3. Finally, we discuss the properties of secret scoring rules in the general context and in the presence of an additive scoring rule in section 4, before we conclude in section 5.

2 The Model

We consider the set N of n suppliers in a procurement auction. They bid to provide all or some of the m indivisible goods in the set G. Each good $j \in G$ is specified by r_j quality attributes. The attributes may take different real-valued attribute levels. For a realization of these levels of good j, we denote the attribute vector by $q_j \in Q_j \subset \mathbb{R}_+^{r_j}$. Let $r = \sum_{j=1}^{m} r_j$ be the total number of attributes of all goods in G and $q = (q_1, q_2, \ldots, q_m) \in \bigtimes_{j=1}^{m} Q_j \subset \mathbb{R}_+^r$ the total attribute vector. Additionally, each bid on a good j specifies a price $p_j \in \mathbb{R}_+$. Let $p = \sum_{j=1}^{m} p_j$ be the total price of all goods. The set P of all possible prices and Q_j are assumed to be discrete and finite[2]. For a subset $G_j \subset G$, we denote the vector of attributes of the goods in G_j by q_{G_j}. r_{G_j} and p_{G_j} are defined analogously. The buyer has a valuation function v for the set G of goods with a quality vector q. We assume quasilinear utility for the buyer: If the set G is bought for a price p, the buyer has a total utility of $v(q) - p$.

In a *multi-object scoring auction*, the sellers submit bids (p_{G_j}, q_{G_j}) for sets G_j of goods. The evaluation of the bids is done on the basis of *scores* which are calculated by a scoring rule that does not necessarily need to match the buyers true valuation v.

We consider a quasilinear *scoring rule* $S : \mathbb{R}_+^{r+1} \to \mathbb{R}$, which takes the form

$$S\left(p, (q_1, \ldots, q_m)\right) = \Phi(q_1, \ldots, q_m) - p, \tag{1}$$

with an overall price of $p = \sum_{i=1}^{m} p_i$ and increasing in the vector of quality attributes $(q_1, \ldots, q_m)$.

$\Phi(q)$ represents the quality level that is achieved by the attribute vector q.

We denote the set of all possible allocations by $\mathcal{H}$. A (winning) allocation $q_H \in \mathcal{H}$ specifies a partition $H = \{H_1, \ldots, H_n\}$ in which every bidder i gets assigned a subset $H_i \subset G$ such that $\bigcup_{i=1}^{n} H_i = G$ and $H_i \cap H_j = \emptyset$ for all $i \neq j$. These are the items each supplier has to deliver. The set H_i may be empty. Additionally, q_H fixes the attribute levels for the items that each supplier got assigned. The winning allocation is the one that maximizes the overall score according to S with respect to the submitted bids. Note that we do not select a specific tie breaking rule; any rule will do for our purposes.

In our model, the suppliers differ with respect to their cost structure, which are private values. In general, cost functions are specified for each package. Thus, bidder i has a cost function $c_i\left(G_j, q_{G_j}\right)$ to produce quality q_{G_j} for a package G_j. We write in short $c_i\left(q_{G_j}\right)$ for this. The costs are assumed to be strictly increasing in each quality attribute, and each q_{G_j} may be delivered for some finite price. Furthermore, let $c_i(q_\emptyset) \equiv 0$.

The social surplus $W\left(q_H\right)$ that an allocation q_H achieves can thus be denoted as follows:

[2]This assumption is not very restrictive, as we will use an ascending auction procedure in the following, where it is common to use discrete bid increments. Furthermore, there are usually technical restrictions on possible attribute levels, and the auctioneer has a maximum price he is willing to pay. All in all, the attribute space is allowed to become very large, such that any realistic bid can be included.

$$W(q_H) := \Phi(q_H) - \sum_{i \in N} c_i(q_{H_i}).$$

We illustrate this model by the following example:

Example 1 Consider three bidders (1,2 and 3) and two objects (A and B). There is one quality attribute for each object, the negative[3] delivery time, represented by q_A and q_B, respectively. The buyer uses the scoring rule

$$S(p_A, p_B, q_A, q_B) := \underbrace{15 + \min\{q_A, q_B\}}_{=\Phi(q_A, q_B)} - p_A - p_B$$

We assume that bidders are able to produce the objects in either one or five days or not at all, according to the costs given in table 1.

	Object A		Object B		Objects A and B			
Quality	-1	-5	-1	-5	$(-1,-1)$	$(-1,-5)$	$(-5,-1)$	$(-5,-5)$
Costs bidder 1	$\underline{5}$	4	10	9	20	16	16	12
Costs bidder 2	10	9	$\underline{4}$	3	19	15	15	13
Costs bidder 3	11	5	11	5	30	20	20	14

Table 1: Costs of the bidders.

The underlined costs mark the efficient allocation: Bidder 1 delivers item A and bidder 2 delivers item B, both in time 1. We now look at a Vickrey-Clarke-Groves mechanism to determine the payments. Denote the winning allocation by q_H^* (maximizing $W(q_H)$) and the winning allocation if bidder i were not present by $q_{H_{-i}}^*$. Bidder i gets paid p_i according to the VCG payment rule:

$$p_i := W(q_H^*) - W\left(q_{H_{-i}}^*\right) + c_i\left(q_{H_i}^*\right)$$

Thus, each winning bidder gets paid for the surplus that he generates by his presence plus his costs. Bidder 1 has costs of 5 and generates a score of $((15-1)-(4+5)) - ((15-5)-(3+5)) = 3$ (if bidder 1 were not present, bidder 3 would deliver item A instead and delivery time would go up to 5). Hence, he gets paid $p_1 = 8$. Similarly, bidder 2 gets paid 9. With these payments we get a total score of $(15-1)-(8+9) = -3 < 0$. Consequently, the buyer would prefer not to buy the items at these payments – the VCG mechanism does not guarantee individual rationality.[4]

[3]We take the negative time to make the score increasing in quality.

[4]See Suyama and Yokoo (2005) for a more detailed discussion.

3 Ascending Proxy Scoring Auctions

The auction format we use in this paper is a generalization of the Ausubel-Milgrom ascending (proxy) package auction. We first describe how the ascending package auction extends to the scoring auction environment and then introduce the proxy bidder.
The auctioneer (buyer) publicly announces the items he wants to buy and the items' corresponding possible attributes and their levels. He decides on a scoring rule[5] S to evaluate the bids. In each round, a seller i bids according to the following general structure: First, he selects the packages of items he wants to bid on and decides for each package G_j which quality attribute levels q_{G_j} he wants to offer – he may offer different combinations of attribute levels for each package. For each of these offers, the bidder specifies a price bid $\beta_i\left(q_{G_j}\right)$ at which he is willing to sell. Then, he submits all of these bids $\left(\beta_i\left(q_{G_j}\right), q_{G_j}\right)$ simultaneously. Bids are treated as mutually exclusive – for each bidder, at most one bid will be selected by the auctioneer for the standing high bids, the winning allocation of each round. The auctioneer may also include bids of previous rounds in the standing high bids (e.g. of bidders that already stopped bidding).
For the first bidding round, the auctioneer specifies a maximum price $\bar{p}$ the sellers may ask for. For the following rounds, there is a (minimum) bid increment b by which the sellers have to lower their previous price offer on a particular package and attribute level configuration. We denote the maximum bid price by $m_i\left(q_{G_j}\right)$. A single bid is rejected if the maximum bid rule is not met, all bids are rejected if the resulting standing high bids would yield a negative score. Rejected bids are treated as a zero bid. Bidding ends if no new bids are submitted or all submitted bids violate the maximum bid rule.
In the proxy auction, the bidding process is automated with the help of a *proxy bidder*. It uses the following strategy, where asking for a price of ∞ corresponds to submitting no bid on this attribute level configuration:

Definition 2 The bidding strategy

$$\forall q_{G_j}: \quad \beta_i\left(q_{G_j}\right) := \begin{cases} m_i(q_{G_j}) & \text{if } q_{G_j} \in \arg\max_{q'_{G_j}} \left[m_i\left(q'_{G_j}\right) - c_i\left(q'_{G_j}\right)\right] \\ \infty & \text{if } q_{G_j} \notin \arg\max_{q'_{G_j}} \left[m_i\left(q'_{G_j}\right) - c_i\left(q'_{G_j}\right)\right] \end{cases}$$

is called *straightforward bidding strategy.* New bids are only submitted in case bidder i is not one of the standing high bidders.
Bidding stops in case $\arg\max_{q'_{G_j}} \left[m_i\left(q'_{G_j}\right) - c_i\left(q'_{G_j}\right)\right] < 0$.

According to this strategy, the bidder places the maximum bid on all attribute level configurations that yield the highest potential profit. Note that the straightforward bidding strategy does not depend on the scoring rule used by the auctioneer, but only on the cost structure of each bidder. Thus, using the straightforward bidding strategy is similar to

[5]The analysis of the decision process is not part of this paper – any decision process is fine for our purposes.

sorting all bids according to their potential profit into a bid list and submitting one after the other (and all bids with the same profit at the same time).

In the proxy auction, each seller reports a cost structure c_i (not necessarily truthfully) to the proxy bidder. Then, the proxy submits bids on behalf of the seller following the straightforward bidding strategy with respect to the reported cost structure.

Example 3 We use the setting of example 1 and apply the proxy scoring auction. Bidders costs are given by table 1. The maximum starting price has to be chosen high enough – a price of 15 will do for our purposes. Bidders start by bidding myopically on the attribute level configuration with the lowest production cost, yielding the highest possible profit. Table 2 shows the first set of bids submitted by the proxy bidders. We assume that the bidders reported their costs truthfully to the proxy bidder.

	Object A		Object B		Objects A and B			
Quality	-1	-5	-1	-5	$(-1,-1)$	$(-1,-5)$	$(-5,-1)$	$(-5,-5)$
Bids bidder 1	–	15	–	–	–	–	–	–
Bids bidder 2	–	–	–	15	–	–	–	–
Bids bidder 3	–	15	–	15	–	–	–	–

Table 2: First round bids.

As no combination of these bids generates a positive payoff, all bids are rejected by the buyer. The proxies submit a new set of bids, uniformly lowering the potential profit – we assume a bid increment of 1 here. Table 3 shows the second set of bids.

	Object A		Object B		Objects A and B			
Quality	-1	-5	-1	-5	$(-1,-1)$	$(-1,-5)$	$(-5,-1)$	$(-5,-5)$
Bids bidder 1	15	14	–	–	–	–	–	–
Bids bidder 2	–	–	15	14	–	–	–	–
Bids bidder 3	–	14	–	14	–	–	–	–

Table 3: Second round bids.

Again, bids are rejected because no positive score is generated by the submitted bids. In the following rounds, the proxies will continue to lower the potential profit until a nonnegative score is generated by the submitted bids. This is the case for the bids in table 4.

The winning bids are the underlined bids in table 4 – bidder 3 will submit one more set of bids with a potential profit of zero, but these bids are not high enough to outbid the other two. The winning allocation is the same as in example 1, but prices are lower – the outcome is individually rational.

	Object A		Object B		Objects A and B			
Quality	-1	-5	-1	-5	$(-1,-1)$	$(-1,-5)$	$(-5,-1)$	$(-5,-5)$
Bids bidder 1	$\underline{7}$	6	12	11	–	–	–	14
Bids bidder 2	13	12	$\underline{7}$	6	–	–	–	–
Bids bidder 3	12	6	12	6	–	–	–	15

Table 4: Bids leading to a nonnegative score.

For the following analysis, similar to Ausubel and Milgrom (2002), we assume that bid increments are negligibly small, such that we have a continuous price range. We think of bidding rounds as taking place at times $t \geq 0$.

To derive our main results, we need to make sure that all allocations that may theoretically win the auction are included in the bidding process. Particularly, bidding does not stop before e.g. bidder i starts submitting a bid on an attribute level configuration that is complementary to the others and would yield a higher score. This is established in the following lemma.

Lemma 4 *Consider any possible set of cost structures and any allocation q_H that possibly generates a positive score. Then, a sufficiently high starting price $\bar{p}$ exists, fulfilling the following: In each bidding round that yields a nonnegative score, every bidder who delivers one or more items in this allocation q_H submits a bid on his respective attribute level configuration.*

Proof See Appendix. □

In other words, the proxy starts bidding with a very high price, such that bids get rejected in the beginning of the auction. As soon as the score gets positive, and bids are not rejected any more, all allocations that are theoretically able to win the auction may be chosen by the auctioneer.

To analyze the properties of the proxy scoring auction, we take a look at the corresponding game in coalitional form. In particular, we want to show that the proxy auction leads to a core outcome of this game.

First, we denote the set of all participants in the auction by $N_s = N \cup \{0\}$. The buyer is player 0. A coalition is any subset $N_c \subset N_s$. The set of allocations for a coalition N_c is the set where all items get assigned to sellers in $N_c \setminus \{0\}$, denoted by $\mathcal{H}_c := \{q_H \in \mathcal{H} | \forall i \notin N_c \setminus \{0\} : q_{H_i} = 0\}$. The coalitional value function w represents the profit a coalition N_c can achieve by producing and trading all items only within its members. Note that only coalitions including the buyer can achieve positive profits as he is paying the bill for the delivered items. For such a coalition, an allocation in $\mathcal{H}_c$ is chosen and and w takes the following form:

$$\forall N_c \subset N_s : w(N_c) := \begin{cases} \max_{q_H \in \mathcal{H}_c} \left[\Phi(q_H) - \sum_{i \in N_c \setminus \{0\}} c_i(q_{H_i}) \right] & \text{if } 0 \in N_c \\ 0 & \text{if } 0 \notin N_c \end{cases}$$

A payoff vector $\pi = (\pi_0, \ldots, \pi_n)$ is called *feasible* if its aggregate payoff does not exceed the value achievable by the coalition of everyone. It is called *unblocked* if no coalition is able to improve the payoff of its members on its own. The *core* is the set of feasible and unblocked payoff vectors:

$$\text{Core}(N_s, w) := \left\{ \pi \,\middle|\, \sum_{i \in N_s} \pi_i = w(N_s) \wedge \forall N_c \subset N_s : \sum_{i \in N_c} \pi_i \geq w(N_c) \right\}$$

Let $\tilde{\pi}^t$ denote the intermediate payoff vector in the auction at time t. We can now derive the following connection between core and proxy scoring auction:

Theorem 5 *The surplus with respect to the scoring rule and the reported cost structures $\Phi(q_H^*) - \sum c_i(q_H^*) = w(N_s)$ is maximized in the final winning allocation q_H^* of a proxy scoring auction. The final payoff vector at time $\bar{t}$ is in the core, $\tilde{\pi}^{\bar{t}} \in$ Core(N_s, w).*

Proof See Appendix. □

The strategies of the sellers in the proxy scoring auction describe what kind of cost structures they submit to the proxy. One particular type of strategy is the π_i-*profit-target* or *semi-sincere* strategy. Such a strategy guarantees bidder i a profit of π_i in case he is one winner of the auction. The strategy can be realized by submitting a cost structure $\tilde{c}_i = c_i + \pi_i$. Let $\Pi_i(\tilde{c}_i, \tilde{c}_{-i})$ denote the profit bidder i makes in the proxy scoring auction if he reports $\tilde{c}_i$ and the others report $\tilde{c}_{-i}$. Generalizing the results of Ausubel and Milgrom (2002), we first show that there is always a best reply which is a profit-target strategy.

Theorem 6 *For any bidder i and any reports $\tilde{c}_{-i}$ to the proxy by the other bidders, let $\bar{\pi}_i = \max_{\tilde{c}_i} \Pi_i(\tilde{c}_i, \tilde{c}_{-i})$. Then the $\bar{\pi}_i$-profit-target strategy is a best reply for bidder i in the proxy scoring auction.*

Proof See Appendix. □

To characterize a set of equilibria of the proxy scoring auction, we need the following definition.

Definition 7 A payoff vector π is called *bidder-optimal* if $\pi \in \text{Core}(N_s, w)$ and there exists no $\pi' \in \text{Core}(N_s, w)$ with $\pi'_{-0} \geq \pi_{-0}$ and $\pi'_{-0} \neq \pi_{-0}$.

Bidder-optimal points in the core are associated with Nash equilibria of the proxy scoring auction:

Theorem 8 *Suppose that π is bidder-optimal. Then the corresponding π_i-profit-target-strategies constitute a Nash equilibrium of the proxy scoring auction. Conversely, the payoff vector in any Nash equilibrium in profit-target strategies at which losing bidders bid sincerely is bidder-optimal.*

Given Theorem 6, the proof of Theorem 8 is now identical to the proof of the corresponding theorem in Ausubel and Milgrom (2002) (Theorem 4).
In general, there may be several bidder-optimal points in the core, yielding several equilibria. A condition guaranteeing a unique bidder-optimal point in the core is *bidder-submodularity* of the coalitional value function:

Definition 9 A coalitional value function w is called *bidder-submodular*, if for any bidder i and all coalitions N_1, N_2 that include the seller, $N_1 \subset N_2$,

$$w(N_1 \cup \{i\}) - w(N_1) \geq w(N_2 \cup \{i\}) - w(N_2)$$

holds.

Bidder-submodularity of w also relates the outcome of the proxy scoring auction to the outcome of the VCG mechanism, π^V:

Theorem 10 *Suppose w is bidder-submodular. Then, the strategy profile where every bidder i reports c_i truthfully to the proxy bidder is an equilibrium of the proxy scoring auction. Its payoff vector π is the unique bidder-optimal point in $Core(N_s, w)$, and $\pi_i = \pi_i^V = w(N_s) - w(N_s \setminus \{i\}) = \max\{\pi_i | \pi \in Core(N_s, w)\}$.*

Given Theorem 5, the proofs of the corresponding theorems in Ausubel and Milgrom (2002) (Theorem 8) or Milgrom (2004) (Theorem 8.11) apply.
The theorem provides a sufficient condition for the proxy scoring to work well: With bidder-submodularity of the coalitional value function, truthtelling is an equilibrium and it is thus easy for the sellers to follow this strategy. Additionally, the unique bidder-optimal core point with respect to the true valuations is reached. Under these circumstances, the proxy scoring auction works as well as the VCG mechanism, as it reaches the same payoff vector. Note, however, that the proxy scoring auction additionally always guarantees individual rationality of the outcome, which the VCG mechanism does not.

4 Secret Scoring Rules

We now turn to the question of how far the auctioneer is able to keep the scoring rule secret. First, we discuss whether the analysis of the general model in section 3 can be extended to secret scoring rules. Then, we consider a specific type of scoring rules: additive scoring rules, where the total score can be calculated as the sum of the scores of the individual items. For this type of scoring rules, a general result connecting the Ausubel-Milgrom proxy auction and the proxy scoring auction can be derived.

4.1 General Scoring Rules

How does the proxy scoring auction work with a secret scoring rule? Note that the auction procedure did not specifically rely on the scoring rule being public. Without knowledge of the scoring rule, sellers submit a cost structure to the proxy. Its bidding behavior stays the same: The proxy submits the bids myopically in order of the respective *seller's* preferences – it does not need the scoring information to do so, but only the submitted cost structure. Then, bids get evaluated according to the scoring rule – this can be done by a proxy as well to ensure that the auctioneer sticks to the scoring rule and does not change it during the auction process. The submitted bids do not need to be publicly announced. The minimum information that is necessary to make the procedure work is to let every bidder know when he is standing high bidder. Announcing this publicly has no impact on the outcome of the auction. However, any information the auctioneer reveals contains information about his scoring rule. Consequently, the more concerned he is with keeping it secret, the less information on bids and their evaluation should be given out.
We start the theoretical analysis with the observation that Theorem 5 holds even with a secret scoring rule, as the bidding behavior of the proxy does not change.

Corollary 11 *The proxy scoring auction with a secret scoring rule reaches the same outcome as the proxy scoring auction with a public scoring rule. Particularly, the final winning allocation maximizes the surplus with respect to the scoring rule and the reported cost structures. The final payoff vector is in the corresponding core.*

The equilibrium analysis in theorems 6, 8 and 10 is in principle valid with a secret scoring rule as well: As the outcome stays the same, best replies are still best replies, no matter whether the scoring rule is secret or not. However, to enable the sellers to completely derive the equilibrium strategies themselves, they need to have full knowledge of the cost structures of their competitors (similar to bidders needing knowledge of all bidders valuations in the original Ausubel-Milgrom proxy auction). Additionally, to derive best replies and equilibria in the sense of theorems 6 and 8 they need knowledge of the scoring rule (to calculate their potential maximum profit and the core, respectively).
Nevertheless, if the coalitional value function is bidder-submodular, this knowledge is *not* needed: By Theorem 10, it is an equilibrium if all sellers report their cost structures truthfully. Thus, if the auctioneer had a way to (credibly) announce that his scoring rule is such that the coalitional value function is bidder-submodular, the sellers could play their equilibrium strategy without knowledge of the scoring rule. However, such an announcement can be difficult to make: The coalitional value function depends on the bidders' cost structures. If the auctioneer does not know these cost structures, a general characterization for bidder-submodularity would be needed, enabling the auctioneer to deduce bidder-submodularity using only the scoring rule and, if necessary, some regularity conditions on the cost structures.

4.2 Additive Scoring Rules

We now analyze the special case of an *additive* scoring rule. Then, the score for each item j can be calculated individually by

$$S_j(p_j, q_j) := \phi_j(q_j) - c_i(q_j).$$

The total score for a set G_j is then given according to

$$S_{G_j}(p_{G_j}, q_{G_j}) = \sum_{l \in G_j} S_l.$$

Suppose in a first step that the scoring rule is publicly known. Note that, in contrast to our previous analysis, an additive scoring rule enables the bidders to calculate the value of their bids in terms of the score they generate. The bidding procedure can be substantially simplified in this setting: Bidders need to submit only one score for each package they bid on. To show this, think of a bidder i who is one of the winners of the auction. He has to deliver the package G_j, and the auctioneer expects to get a score of t_j^i on this package. We suppose that bidder i has the freedom to provide a score of t_j^i in any way he likes. Analogously to statements in Asker and Cantillon (2008) and Mueller et al. (2007) we can formulate the following lemma:

Lemma 12 *The optimal level of quality $q^*_{G_j} \in Q_{G_j}$ that a supplier i with cost function c_{G_j} produces for a package G_j is independent of the score t_j^i he has to fulfill.*

Proof See Appendix. □

In our general setting, bidders needed to differentiate their bids by submitting different attribute levels and configurations for each package. Lemma 12 shows that, with an additive scoring rule, the suppliers are not interested in submitting different attribute level configurations for the same package – they produce the same configuration in any case. This leads us to the following corollary:

Corollary 13 *Consider a multi-object scoring auction with a publicly known additive scoring rule. Bidders have no restriction on how to deliver the requested score. Then, there is no difference between bidders submitting bids of price-quality combinations or bids of scores: Both mechanisms lead to the same outcome – the same quality is delivered and the same price is paid.*

Hence, it is sufficient to let bidders submit a single score for each package instead of a price-quality combination as long as they know the scoring rule. For this setting with an additive and public scoring rule we thus assume for the sequel that bids consist of scores. Furthermore, Lemma 12 shows that each bidder has a maximum of social surplus he can generate with respect to the scoring rule. This leads to the following definition.

Definition 14 Suppose that bidder i has a cost function c_{G_j} for each $G_j \subset G$. Then

$$k_{G_j} := \max_{q_{G_j}} \left[\phi_{G_j}(q_{G_j}) - c_{G_j}(q_{G_j}) \right] \tag{2}$$

is the *pseudotype* of bidder i for the package G_j.

Alternatively, the pseudotype can be interpreted as the maximum score a seller is able to deliver without losing any money. Regarding this interpretation, the pseudotype is similar to the valuation in price-only auctions – there, the valuation is the maximum amount of money a buyer can pay without obtaining an object at a loss.
In this context, the proxy scoring auction works as follows: The sellers submit a pseudotype vector (the pseudotype for each package) to the proxy. Then, the proxy submits mutually exclusive bids of scores according to the straightforward bidding strategy – he bids myopically on all packages promising the highest profit. The auctioneer selects the standing high bidder by selecting the allocation that maximizes the sum of submitted scores. We can compare this bidding procedure with the original Ausubel-Milgrom proxy auction:

Remark 15 The proxy scoring auction with a public additive scoring rule can be interpreted as the original Ausubel-Milgrom proxy auction with bidders submitting pseudotypes as valuation vectors and proxies submitting scores as bids. Particularly, if bidders types in the scoring auction are their pseudotypes and distributed as the types in the original scoring auction, all theorems that hold for the original proxy auction hold for the additive proxy scoring auction as well (in their corresponding reformulations).

Note that this is a general statement on the transferability of results to the scoring auction environment. Not only Theorems 5, 6, 8 and 10 hold, but all other statements that are true for the original proxy auction have their counterpart for the additive proxy scoring auction. For a general scoring rule we do not have this kind of general transferability – each theorem has to be proven in the new environment, as we did in section 3.
However, transferring results for the additive scoring auction can be a bit more complicated in an independent private values model with incomplete information: If bidders have multidimensional types that do not represent the pseudotypes (but can be reduced to get them), it is not directly obvious that the strategic bidding behavior of each participant is the same as in the price-only auction. Nevertheless, Mueller et al. (2007) show that under mild regularity assumptions the set of equilibria of a price-only auction and the corresponding scoring auction is basically the same.
We now turn to the analysis of secret scoring rules in the context of the additive proxy scoring auction. In this auction, bids are submitted as scores generated out of the pseudotypes vectors of the bidders. As bidders need to know the scoring rule to calculate their pseudotype, reducing bids to scores is not possible. Consequently, the bidding procedure needs to be transformed in the presence of a secret scoring rule. A suitable bidding procedure is the one used in section 3: Each bidder submits a cost structure to the proxy,

which generates bids on all possible attribute level configurations. The additive scoring rule imposes enough structure to connect the two different bidding procedures:

Theorem 16 *Consider a winning bidder and the associated winning package in any round of a proxy scoring auction with a secret additive scoring rule. Then, among the bids on the different attribute levels of that package by the bidder, the buyer chooses the bid using the optimal quality attribute level from the seller's perspective (if he knew the scoring rule).*

Proof See Appendix. □

What does this theorem tell us? Consider a seller i who decides to bid according to a cost structure c_i'. Suppose he submits this cost structure to the proxy bidder, it bids accordingly and the seller is a standing high bidder in one of the bidding rounds. Then, the auctioneer will select the *optimal* attribute level in this standing high bid – the one the seller would have chosen (according to his submitted cost structure) in case he would have only been forced to deliver a particular package and score, and not a particular attribute level configuration. Thus, the outcome of the auction is the same in case the scoring rule is public and each seller i uses similarly c_i' to calculate his pseudotype and decide on the attribute levels he will deliver.
Note one particular difference: If seller i decides to choose c_i' such that the relative costs of attribute levels for a specific package are changed, his true optimal quality might differ from the optimal quality implied by c_i'. In case of a public scoring rule, he would have an ex post incentive to supply his true optimal attribute levels after being told the package and score he has to deliver. With a secret scoring rule, he is forced to deliver the attribute levels chosen by the auctioneer. However, this distortion cannot appear when for each possible package G_j there is a π_{G_j} such that $c_i'(q_{G_j}) = c_i(q_{G_j}) + \pi_{G_j}$ – each bidder uses his true relative costs for a package. This kind of bidding behavior is a best reply: Suppose a seller would distort his costs for a specific package and win the auction on that package with a profit π'. Then, he always makes at least the same profit by not distorting and uniformly asking for a profit-target of π' on that package. In fact, he could possibly even raise the profit-target and still win the auction, as he is able to generate a (weakly) higher score on the same package if he does not distort. Consequently, we can conclude:

Corollary 17 *The outcome of the proxy scoring auction with a public additive scoring rule can be reproduced using a secret scoring rule.*

This is particularly interesting as bidding with public additive scoring rule relied on the bidders' knowledge of the scoring rule. Thus, we showed a way to transfer theory regarding the original Ausubel-Milgrom proxy auction to scoring auctions with secret additive scoring rule, using the public additive scoring rule as an intermediate step. Of course, the restrictions on the use of secret scoring rules as mentioned in section 4.1 still apply.

5 Conclusion

We showed that the Ausubel-Milgrom proxy auction can be extended to a combinatorial scoring auction setting. It is able to replicate the desirable outcome of the Vickrey-Clarke-Groves mechanism in case the coalitional value function is bidder-submodular, but does not suffer of the individual rationality problems that may appear in the context of a general scoring rule when the VCG mechanism is used.

Furthermore, we discussed the possibility of keeping the scoring rule secret: The outcome stays the same, best replies are still best replies, and if it is publicly known that the coalitional value function is bidder-submodular, sellers can submit the truthful equilibrium bid without further knowledge of the scoring rule. For an additive scoring rule we derived a close connection to the original Ausubel-Milgrom proxy auction.

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A Appendix: Proofs

Proof of Lemma 4

Let

$$\bar{p} := 2 \cdot \max_q S(0, q),$$

which is twice the maximum possible score (if all sellers would give away their items for free). Hence, bids get rejected at least until the first submitted prices reach $\frac{\bar{p}}{2}$. Now suppose we are in any bidding round after this point and consider any allocation q_H that generates a positive score with respect to the reported cost structures (these are the allocations that may theoretically win the auction). In particular, this means that the individual costs $c_i(q_{H_i})$ of each seller in this allocation are below $\frac{\bar{p}}{2}$. As this seller has already submitted at least one bid (on some attribute level configuration) with a price lower than $\frac{\bar{p}}{2}$, the maximum profit he may obtain in this round is lower than $\frac{\bar{p}}{2}$. He can make at least the same profit by asking for a price of $\bar{p}$ on q_{H_i}, because $\bar{p} - c_i(q_{H_i}) \geq \bar{p} - \frac{\bar{p}}{2}$. Hence, according to the straightforward bidding strategy, the proxy places a bid on q_{H_i} in this round. □

Proof of Theorem 5

We first show that at any time t the provisional payoff vector is unblocked by any coalition.

Among all submitted bids at[6] time t, the auctioneer selects the allocation $q_H^t \in \mathcal{H}$ that maximizes the score[7]:

$$q_H^t \in \arg\max_{q_H \in \mathcal{H}} S\left((\beta_1(t, q_{H_1}), q_{H_1}), \ldots, (\beta_n(t, q_{H_n}), q_{H_n})\right)$$

Now, we can rearrange this score:

$$\begin{aligned}
\tilde{\pi}_0^t &= \max_{q_H \in \mathcal{H}} S\left((c_1(q_{H_1}) + \tilde{\pi}_1^t, q_{H_1}), \ldots, (c_n(q_{H_n}) + \tilde{\pi}_n^t, q_{H_n})\right) \\
&= \max_{N_c \subset N_s} \max_{q_H \in \mathcal{H}_c} \left(\Phi(q_H) - \sum_{i \in N_c \setminus \{0\}} \left(c_i(q_{H_i}) + \tilde{\pi}_i^t\right) \right) \\
&= \max_{N_c \subset N_s} \left[\max_{q_H \in \mathcal{H}_c} \left(\Phi(q_H) - \sum_{i \in N_c \setminus \{0\}} c_i(q_{H_i}) \right) - \sum_{i \in N_c \setminus \{0\}} \tilde{\pi}_i^t \right] \\
&= \max_{N_c \subset N_s} \left[w(N_c) - \sum_{i \in N_c \setminus \{0\}} \tilde{\pi}_i^t \right] \qquad (3)
\end{aligned}$$

The second equality holds as bidders not in coalition N_c receive a payment of 0 and do not deliver an item. Using equation (3) we can directly see that the payoff vector is unblocked:

$$\begin{aligned}
\forall N_c \subset N_s : \quad & \tilde{\pi}_0^t \geq w(N_c) - \sum_{i \in N_c \setminus \{0\}} \tilde{\pi}_i^t \\
\Longleftrightarrow \forall N_c \subset N_s : \quad & \sum_{i \in N_c} \tilde{\pi}_i^t \geq w(N_c)
\end{aligned} \qquad (4)$$

We still need to show that the final payoff vector is indeed feasible. Denote the set of bidders in the final winning coalition at time $\bar{t}$ by W. Then, we get as final payoff vector $\tilde{\pi}^{\bar{t}}$:

$$\tilde{\pi}_i^{\bar{t}} = \begin{cases} \beta_i(\bar{t}, q^*_{H_i}) - c_i(q^*_{H_i}) & \text{if } i \in W \\ \Phi(q^*_H) - \sum_{j \in W} \beta_j(\bar{t}, q^*_{H_j}) & \text{if } i = 0 \\ 0 & \text{if } i \notin W \cup \{0\} \end{cases}$$

This payoff vector yields

$$w(N_s) \overset{(4)}{\leq} \sum_{i \in N_s} \tilde{\pi}_i^{\bar{t}} = \Phi(q^*_H) - \sum_{i \in W} c_i(q^*_H) \leq \max_{q_H \in \mathcal{H}_c} \left(\Phi(q_H) - \sum_{i \in N_s \setminus \{0\}} c_i(q_{H_i}) \right) = w(N_s).$$

Hence, feasibility and maximization of surplus with respect to the scoring rule and the reported cost structures are established. □

[6] Similar to the original Ausubel-Milgrom proxy auction, all bids up to time t can be included in the optimization problem of the auctioneer. However, as the proxy simultaneously lowers the price on all possible quality levels, the auctioneer will always prefer the latest bid submitted.

[7] Note that the auctioneer pays an amount of zero to every not winning bidder although the notation suggests something different. For the ease of a simple notation we stick to it.

Proof of Theorem 6

Suppose c'_i is a cost structure that yields a profit of $\bar{\pi}_i$ for bidder i if the others report $\tilde{c}_{-i}$. Denote the associated quality attribute allocation by q'. Then, bidder i sells q'_i for a price of $c_i(q'_i) + \bar{\pi}_i$, as $\bar{\pi}_i$ is the profit that he makes with respect to his true cost structure. We can then slightly change this strategy without altering the outcome of the auction: Let $c''_i(q_i) = c'_i(q_i) - c'_i(q'_i) + c_i(q'_i) + \bar{\pi}_i$. The report c''_i shifts the report c'_i such that the winning quality allocation q' makes a profit of zero with respect to the new cost structure c''_i. Especially, bidding behavior by the proxy is not changed with this alteration of the reported cost structure: The cost minimal quality allocation $\hat{q}$ stays the same, $\hat{p}$ stays the same and relative reported costs stay the same as well. The shift can only affect the potential profit, which is not visible for the auctioneer. Thus, decisions by the auctioneer stay the same in every round, and the final decision q' will stay the final decision with a report of c''_i as well – only that the internal profit in the calculations of the proxy for this allocation is reduced to 0.

Theorem 5 showed that q' is surplus maximizing with respect to the scoring rule and cost structures $(c''_i, \tilde{c}_{-i})$. Hence, surplus cannot be increased by choosing an allocation excluding i. Thus, keeping the reports of the others fixed, with any cost structure $\tilde{c}_i$ that specifies $\tilde{c}_i(q'_i) = c''_i(q'_i)$ a quality allocation of q' is feasible and bidder i will be included in the winning allocation (either q' or some other allocation including i).

Now note that the $\bar{\pi}_i$-profit-target-strategy specifies a bid of $c_i(q'_i) + \bar{\pi}_i = c''_i(q'_i)$ for q'. So from our considerations above we know that bidder i will be included in the winning allocation using this strategy. Furthermore, as the $\bar{\pi}_i$-profit-target-strategy guarantees a profit of $\bar{\pi}_i$ in case i is in the winning allocation, the maximum possible profit of $\bar{\pi}_i$ is realized with this strategy. It is thus a best reply. □

Proof of Lemma 12

The bidder chooses to supply the price-quality combination $(p^*_{G_j}, q^*_{G_j})$ that maximizes his profit. Thus, his objective is

$$\max_{(p_{G_j}, q_{G_j})} \left[p_{G_j} - c(q_{G_j})\right] \quad \text{s.t.} \quad \phi_{G_j}(q_{G_j}) - p_{G_j} = t^i_j.$$

This can be rewritten as

$$\begin{aligned} &\max_{q_{G_j}} \left[\phi_{G_j}(q_{G_j}) - t^i_j - c(q_{G_j})\right] \\ =&\max_{q_{G_j}} \left[\phi_{G_j}(q_{G_j}) - c(q_{G_j})\right] - t^i_j. \end{aligned} \tag{5}$$

Note that the maximum exists and is unique: $\phi_{G_j}(q_{G_j}) - c(q_{G_j})$ is a concave and continuous function on a compact set. Furthermore, in (5) we can see that the optimal quality does not depend on the score to fulfill. □

Proof of Theorem 16

Consider the bids of a bidder i. For each possible attribute level configuration q_{G_j} for each possible package, a price exists such that a potential profit value π_i is realized. As the proxy continuously decreases the profit value during the auction process, for every quality level and package the proxy will simply set the bid price $\beta_i(q_{G_j})$ such that $\beta_i(q_{G_j}) := c_i(q_{G_j}) + \pi_i$. Hence, in each round the buyer receives a set of bids by each seller including every attribute configuration and the corresponding prices, all leading to the same profit for the seller. Every level q_{G_j} thus generates a score $\phi_{G_j}(q_{G_j}) - \beta_i(q_{G_j})$. As the buyer is rational, for every π_i he will prefer the bid $(\beta_i(q_{G_j}), q_{G_j})$ out of the bids of seller i that maximizes his score for a certain package. This is the bid that maximizes $\phi_{G_j}(q_{G_j}) - \beta_i(q_{G_j}) + \beta_i(q_{G_j}) - c_i(q_{G_j}) = \phi_{G_j}(q_{G_j}) - c_i(q_{G_j})$, as $\beta_i(q_{G_j}) - c_i(q_{G_j}) = \pi_i$ is a constant. But the maximum of $\phi_{G_j}(q_{G_j}) - c_i(q_{G_j})$ for a certain package is the bidder's pseudotype for this package. Hence, the optimal quality is chosen by the buyer. $\square$