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Experimental Investigation of a Cyclic Duopoly Game

by

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Abstract

The notion of a cyclic game has been introduced by Selten and Wooders (2001). They illustrate the concept by the analysis of a cyclic duopoly game. The experiments reported concern this game. The game was played by eleven matching groups of six players each. The observed choice frequencies were compared with the predictions of Nash equilibrium, impulse balance equilibrium (Selten, Abbink and Cox (2005), Selten and Chmura (2007)) and two-sample equilbrium (Osborne and Rubinstein(1998)). Pairwise comparisons by the Wilcoxon Signed-rank test show that impulse balance equilibrium as well as two-sample equilibrium have a significantly better predictive success than Nash equilibrium. The difference between impulse balance equilibrium and two-sample equilibrium is not significant. In each matching group three players acted only in uneven periods and the other three only in even periods. This game has two pure strategy equilibria in which both types of players behave differently. The data exhibit a weak but significant tendency in the direction of coordination at a pure strategy equilibrium.

Keywords: cyclic game duopoly experiment, impulse balance equilibrium, two-sample equilibrium
JEL classification: C73, D43, C90

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1 Introduction

In a paper by Selten and Wooders (2001) the notion of a cyclic game is defined and illustrated by the example of a duopoly game. In this model a supplier enters a market and stays there for two periods and then exits. This means that a firm exists for exactly two periods. One may think of taxi concessions in a small town, which last for a limited time. The market is too narrow to permit more than two taxis at the same time.

A cyclical game may be looked upon as a condensed description of an infinite game. In the following we shall describe the infinite game structure underlying our experiment. The game runs over periods $t = 1, 2, ...$ In each period t a potential entrant t has to decide wether he enters the market or not in period t. If he enters he stays in the market for periods t and $t + 1$ and exits then. A potential entrant t may face an empty market, this is always true for t=1. For $t = 2, 3, ...$ the market is empty at period t if the potential entrant $t - 1$ did not enter the market. The market is occupied for t with $t = 2, 3, ..$ if the potential entrant $t - 1$ did enter the market.

		Potential Entrant *t+1*	
		Enter	Not Enter
Potential entrant t	Enter	$U + V$	$2U$
	Not Enter	W	W

Table 1: *Possible payoffs for the situation of an empty market*

		Potential entrant *t+1*	
		Enter	Not Enter
Potential entrant t	Enter	$2V$	$V + U$
	Not Enter	W	W

Table 2: *Payoffs for the situation of an occupied market*

Tables 1 and 2 describe the payoffs of a potential entrant t with $t = 1, 2, ...$ for the two cases of an empty or occupied market at period t. In both cases not entering the market yields a payoff W for the two periods together by an alternative use of capital. The payoff for entering depends on the choice of the next potential entrant $t + 1$. In the case of an empty market the payoff for period t is U and the payoff for period $t + 1$ is V if the potential entrant enters the market and U if he does not. Similarly, if the market is occupied in period t, the payoff is V for period t. The payoff for period $t + 1$ is V or U depending wether the potential entrant $t + 1$ enters or not.

It is assumed, that the parameters U, V and W satisfy the following inequality:

$$U + V > W > 2V$$

This inequality implies $U > V$. It can be seen by table 1, that in the situation of an empty market entry is a dominant strategy of the potential entrant. However, in the situation of an occupied market no strategy of the potential entrant is dominated.

The game situation described above extends over an infinite number of time periods. However, in an experiment one cannot play for infinite time. Therefore, our experiments run over 200 periods. The calculation of somebody's payoff, who enters in the last period requires the decision of a potential entrant in the next period. This creates an "end-problem". We solved this problem by substituting a randomly chosen decision of an earlier entrant facing an occupied market for the decision in the next round.

In the underlying game model one thinks of every potential entrant as a different player. Of course it is not practicable to recruit 200 participants for one observation. This is also not necessary. In our experiments an independent subject group consisted of 6 players involved in three markets. In the beginning the odd numbered players $1, 3$ and 5 were randomly assigned to the three markets A, B and C as potential entrants. In period two the even numbered players $2, 4$ and 6 were also randomly assigned as potential entrants to the three markets. In the next period the same procedure was applied to the odd numbered players again, etc.. . The players did not know which market they would enter, they were only told that the assignment to the markets was randomly made.

The concept of a cyclical game permits a condensed description of game situations like the one investigated in our experiment. In our case, the cyclical game has only two players. These two players are roles of the potential entrants. The odd numbered members of an independent subject group are in the role of player 1 and the even numbered are in the role of player 2.

The transition from the game played in our experiment to the cyclical game involves a reduction of the strategy space. Every participant is a potential entrant hundred times. Therefore, behavior may depend on prior experience. However, the stationary equilibria of the experimental game coincide with the equilibria of the cyclical game.

We shall not only look at stationary equilibria, but also at 2 stationary behavioral concepts. One of these concepts is impulse balance equilibrium (Selten, Abbink, Cox (2005) and Selten and Chmura (2007)). The other concept was proposed by Osborne and Rubinstein (1998). These two behavioral concepts turned out to be especially successful in the experimental comparison by Selten and Chmura (2007).

Impulse balance equilibrium is based on learning direction theory (Selten and Buchta (1999)). This theory concerns situations in which a player selects a value of one decision parameter out of an interval. According to learning direction theory, this parameter is increased after a period in which a higher parameter value would have yielded a greater payoff. Similarly, the parameter is decreased if a lower value would have yielded a higher payoff.

In our case this theory is not applied to a decision parameter but to the probability of entry which may be described as a behavioral tendency. A positive difference between the payoffs for entry and the one for non-entry is called an impulse in favor of entry. Similarly, a positive difference between the payoffs for non-entry and for entry is called an impulse for non-entry.

In 2×2 games each strategy has a minimal payoff and the maximum of the two minimal payoffs is called the pure strategy maximin. This pure strategy maximin is a natural aspiration level. If a lower payoff is obtained, then the difference to the pure strategy maximin is perceived as a loss. In 2×2 games a loss is always connected with an impulse towards the other strategy.

Impulse balance equilibrium is reached at a point in which the expected impulses in both directions are equal. In the paper by Selten and Chmura (2007) losses were counted double in the computation of impulse balance. This was reasonable in the context of ordinary 2×2 games. However in this paper the double counting of losses will not be applied since there is a important difference between the 2×2 games of Selten and Chmura (2007) and the cyclical game explored here. Whereas in these 2×2 games a player always receives immediate feedback about the other player's choice, in the cyclical game there is no such feedback in the case of staying out of the market. In this case a player never learns what he would have earned if he had entered. Entering is the only way of obtaining feedback about other players' behavior.

In 2×2 games with full feedback about the other players' choice there are two reasons for playing a strategy and both are connected to experiences with playing the other strategy. When the other strategy has been played one may on the one hand have received an impulse away from it and on the other hand one may have experienced a loss. Of course the second reason is present only when there was a loss. Therefore losses are counted double. In cyclical games there are still two reasons not to enter if entering has resulted in a loss. However there is also a reason to enter since only by entering information can be obtained. One may say that one of the reasons for not entering is canceled by this reason for entering. Therefore counting losses double is not adequate for a cyclical game. This argument is summarized by Table 3

	reasons in favor of	
	entering	not entering
next player does not enter	1. gain 2. information *(no impulse)*	-
next player does enter	1. information .	1. forgone payoff 2. loss

Table 3: Reasons for entering and not entering

The probability to enter is looked upon as a behavioral tendency which is shaped by prior impulses. The player needs this feedback and is actively seeking for it. Somebody who does not enter does not get any feedback and therefore receives no impulse. If after entering the next player does not enter then no impulse is received because nothing better could have been done. A player experiences an impulse only if the next player does enter. In this case there are two reasons for not entering but also one for entering.

We shall use the name two-sample equilibrium for the concept proposed by Osborne and Rubinstein (1998). Two-sample equilibrium depends on a parameter, the sample size n. Imagine a stationary equilibrium in which a player takes two samples of n prior periods, one of periods in which the first strategy was played by this player and another one in which he or she played the second strategy. Suppose one of the samples yields a higher total payoff, then the player plays this strategy. If both total payoffs are equal, each of the two strategies is chosen with probability $\frac{1}{2}$.

The rule for equal sample payoffs is not part of the original concept of Osborne and Rubinstein. They did not specify behavior in the case of equal payoffs. We added the rule in order to obtain a unique equilibrium prediction.

In a study about twelve 2×2 games (Selten and Chmura (2007)) the best fitting value of the parameter n for the two-sample equilibrium was $n = 6$. In the case of the duopoly game, investigated here, $n = 5$ yields a better fit to the data.

It turns out, that the two-sample equilibrium, with $n = 5$ yields the best fit to the data compared with the other concepts. The impulse balance equilibrium is almost as successful and no significant difference between these two stationary concepts can be found. Both concepts are significantly more successful than the Nash equilibrium.

In principle, our experimental setup also permits asymmetric equilibria in which one of the two players, say player one, always enters and the other player never enters. This is the case since players who enter in odd periods are always confronted with potential entrants in an even period. Therefore the two groups, those who decide in odd periods and those who decide in even periods, may have different learning histories over time. In fact the data reveal a significant tendency towards convergence to pure strategies. This finding throws some doubt on the comparison of the three stationary concepts.

2 The cyclic game

In our experiment the parameters have the following values:

$$U = 10 \qquad V = 2 \qquad W = 5$$

The method of graphical presentation of cyclical games has been explained in Selten and Wooders (2001). There the duopoly game experimented here has been used as an example with abstract parameters U, V and W. Figure 1 shows the same drawing but with the experimented parameter values shown instead of U, V and W.

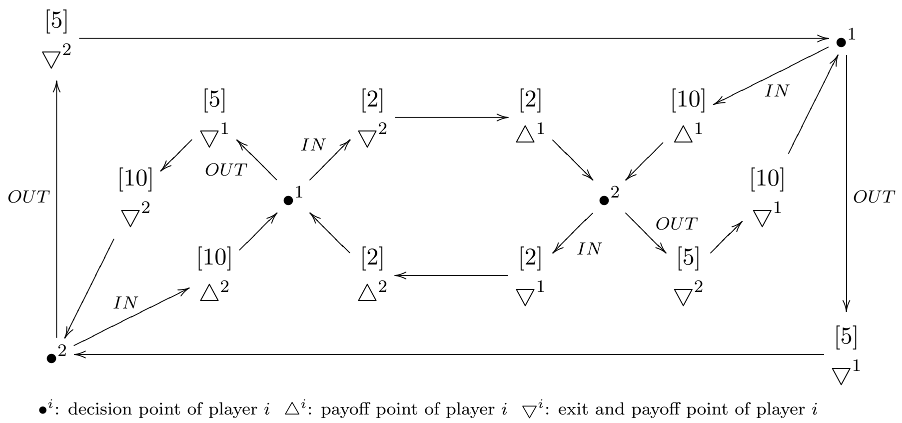

Figure 1: Structure of the cyclic duopoly game

The figure has the structure of a directed graph with 3 types of nodes and additional information regarding player and payoff. Each point describes a situation of either player 1 or player 2. At a decision point a player has to decide between two alternatives in our case IN or OUT. Here IN means entering and OUT means not entering. At a payoff point or an exit and payoff point the payoff of the concerning player is shown in rectangular brackets above this point. The arrows show the direction in which the game moves from one situation to the next.

At the upper right corner player 1 can decide between IN and OUT in a situation of an empty market. In the case OUT he receives a payoff of 5 at a payoff and exit point. If he chooses IN he receives a payoff of 10 in this period and the game moves to a decision point of player 2. If than player 2 chooses OUT first a payoff and exit point is reached at which player 2 receives 5 and then a payoff and exit point of player 1 where he receives 10. From there the game moves back to the upper right corner. The other part of the figure are understood in the same way.

3 Three stationary concepts for the cyclic game

We shall look at three stationary concepts Nash equilibrium, impulse balance equilibrium and two-sample equilibrium. As has been shown in Selten and Wooders (2001) the cyclic game has three **Nash equilibria**. A symmetric mixed strategy equilibrium and two pure strategy equilibria. In the symmetric mixed equilibrium the probability α for entering if the market is occupied is as follows:

$$\alpha = \frac{U + V - W}{U - V} = 0.875$$

If the market is empty the probability of entering is one. This is also true for the two pure equilibria. However, there the probabilities α_1 and α_2 are

$$\alpha_1 = 1 \text{ and } \alpha_2 = 0$$

in the first pure strategy equilibrium and

$$\alpha_1 = 0 \text{ and } \alpha_2 = 1$$

in the second pure strategy equilibrium.

We now turn our attention to the **impulse balance equilibrium**. Learning direction theory looks at probabilities of decisions as behavioral tendencies. The probabilities are adjusted in the light of experiences after a decision has been made. After a decision a player thinks about wether the decision was the right one. If he comes to the conclusion that the decision could have been improved he receives an impulse towards the better choice or more precisely the choice which would have been the better one, ex-post. Applied to the cyclical game this picture of the mental process meets the difficulty that after not entering no feedback is received. The question arises how in this case the player can find a substitute for the missing feedback. We shall discuss this problem below.

Let α be the probability of entering at impulse balance equilibrium. If he enters and the next player also enters he receives an impulse of $W - 2V$ towards not entering. He receives no impulse if the next player does not enter since in this case entering ex-post proofed to be the best choice. If the other player also enters he gets an impulse of $W - 2V$. Therefore the probability of getting an impulse for not entering is α^2.

We now consider the case that the player does not enter. In this case he does not receive any feedback. Impulse balance equilibrium is based on the picture of a mental process which substitutes this lack of feedback by imagination. This mental process does not involve any ex-ante deliberation but only ex-post reflections. This also applies to the cognitive process of imagining the behavior of the hypothetical next player. This hypothetical player finds himself in the same situation of an occupied market as he himself did. Therefore he applies the same mental process he used to make his own decision. In this way he forms a definite assumption about how the hypothetical next player would have acted. At equilibrium this mental process leads to entering with probability α and not entering with probability $1 - \alpha$. Therefore with probability α the player arrives at the assumption that the hypothetical next player did enter and with probability $1 - \alpha$ he imagines that the hypothetical player did not enter. If he assumes that the hypothetical next player did enter then he chose the ex-post optimal alternative and therefore receives no imagined impulse. If he imagines that the hypothetical next player did enter he receives an imagined impulse of $U + V - W$ in the direction of entering.

The player first chooses not to enter with probability $1 - \alpha$ and then with the same probability $1 - \alpha$ he assumes that the next hypothetical player would not have entered. The probability of these two successive events is $(1 - \alpha)^2$. This leads to an imagined impulse of $U + V - W$ with the probability of $(1 - \alpha)^2$.

Figure 2 describes this situation and gives the actual or imagined impulses depending on the decision of the next player or on the assumption about the behavior of the hypothetical next player.

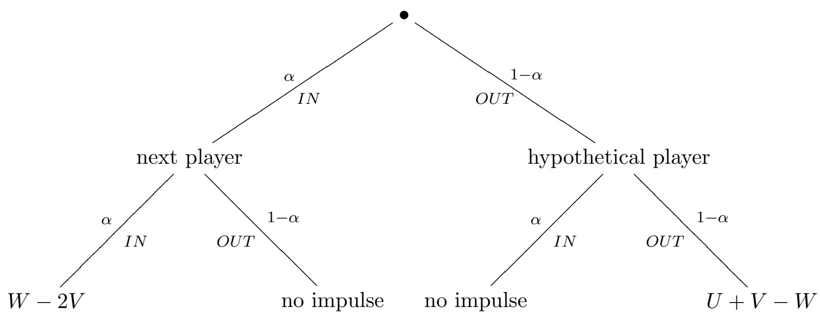

Figure 2: Impulses in the direction of the other strategy

At equilibrium the mathematical expectation of impulses in the direction of not entering is equal to the one towards entering. This is expressed by the following impulse balance equation:

$$\alpha^2(W-2V) = (1-\alpha)^2(V+U-W)$$

Thus, the probability of entering an occupied market can be stated as:

$$\alpha = \frac{\sqrt{V+U-W}}{\sqrt{W-2V}+\sqrt{V+U-W}} = 0.726$$

Pure strategy Nash equilibria are also impulse balance equilibria. In a pure strategy Nash equilibrium nobody receives any impulse.

The last stationary concept we will report on is the **two-sample equilibrium**. Osborne and Rubinstein (1998) introduced the idea behind this concept which is as follows: The player draws two samples of equal sizes n, one for the strategy IN and one for the strategy OUT. He then forms the payoff sums in the two samples and compares them and plays the strategy with the highest payoff sum. If both payoff sums are equal he flips a coin and thus choses a pure strategy with probability $\frac{1}{2}$. The sample size n is a parameter.

We now describe how the probability α for entering is determined in the case of two-sample equilibrium. Consider a sample of n cases in which the player has played IN. Let k be the number of cases in this sample in which the next player entered. Let S_k be the payoff sum of the sample we have

$$S_k = 12(n-k) + 4k.$$

This payoff sum S_k must be compared to the payoff sum $5n$ obtained for not entering n-times. The player does not enter if the payoff sum difference

$$D_k = 5n - S_k = 8k - 7n$$

is positive. In the case $D_k = 0$ the probability of **not entering** is $\frac{1}{2}$. The conditional probability of not entering if there are k cases of next players entering in the sample for IN is as follows

$$\eta(n,k) = \begin{cases} 0 & \text{for } 7n > 8k \\ \frac{1}{2} & \text{for } 7n = 8k \\ 1 & \text{for } 7n < 8k \end{cases}$$

With the help of this notation we now can derive an equation for the entry probability α:

$$\alpha = 1 - \sum_{k=0}^{n} \eta(n,k) \binom{n}{k} \alpha^k$$

The sum on the right hand side of this equation is the total probability of not entering. The probability of not entering if there are exactly k cases with next players entering in the sample for IN is $\eta(n,k)$ times the binomial probability for k out of n players entering.

We shall now show that the equation for α has exactly one solution with $0 \leq \alpha \leq 1$. In order to see this one can look at the left and the right hand side of the equation as functions of α and imagine that they are shown in a diagram. The left hand side is the 45 degree line. The right hand side begins with the value of 1 for $\alpha = 1$ and than decreases in view of $\eta(n,n) = 1$. Therefore the two curves have exactly one intersection at a value of $0 \leq \alpha \leq 1$.

It can be seen that without difficulty that for $n = 1,..,10$ the equation for α takes the following forms:

$$\begin{array}{ll} \alpha = 1 - \alpha^n & \text{, for } n = 1,..,7 \\ \alpha = 1 - 4\alpha^7 - \alpha^8 & \text{, for } n = 8 \\ \alpha = 1 - n\alpha^{n-1} - \alpha^n & \text{, for } n = 9, 10 \end{array}$$

For the purposes of this paper it is not necessary to compute the two-sample probability α for larger samples.

It is not immediately clear whether a strict pure strategy equilibrium can be considered a two-sample equilibrium. If really always only the equilibrium strategy has been played in the past there is no sample for the other strategy.

4 Experimental design

The experiment was carried out in 2005 at the Laboratory for Experimental Economics, University of Bonn. The participants, all students, were invited via the ORSEE[1] database of the laboratory.

Four sessions were conducted, in three sessions there were 18 participants and in one session only twelve. The participants did not know that they were subdivided into independent subject groups of six. Therefore they were let to belief that there are nine or six markets to which they could be randomly assigned. In our experiment we had 11 independent matching groups of 6 players each. This means there were altogether 66 participants.

[1] Greiner (2004)

At the beginning of the experiment the participants were briefed with a written instruction[2], which was read out to them. Afterwards the participants were separated into cabins with computer terminals and the experiment started[3].

The payoffs in the game were given in the fictitious currency *Taler* and at the end of the game transferred in to Euro with an exchange rate of 1 Taler equals 1 EuroCent. In addition to the cumulated payoffs subjects received a show-up fee of 5 Euro. To guarantee anonymity of the decisions participants were separately payed. Overall one session lasted about one hour and the payoffs were between 10 and 16 Euros.

5 The experimental results

The three concepts serve as predictions for the frequencies for entering a free market and an occupied market. As in the already mentioned study by Selten and Chmura (2007), we do not assume that the theories can predict the behavior of a single player. We compare the predictive power of the three concepts for the average behavior in independent subject groups. In each independent subject group i we will use the quadratic distance

$$Q_i = (\alpha - f_i)^2$$

between the theoretical probabilities α and the observed mean relative frequency f_i as the measure of predictive success. The overall predictive success is measured by the mean of all theses quadratic distances over the elven subject groups.

$$Q = \frac{1}{11}\sum_{i=1}^{11} Q_i$$

In the following we will first search for the sample size n of the two-sample equilibrium with the best fit to the data. Then we will compare the predictive success of the three theories. We will compare the overall predictive success and wether this success measure changes over time. Afterwards we will analyze tendencies of convergences towards the pure strategy equilibria.

5.1 Comparison of Sample Sizes for Two-Sample Equilibrium

In the study by Selten and Chmura (2007) the sample size 6 yielded to the best fit for the data in the named 2×2-games. However, it is not clear whether this

[2] See Appendix C for the translated instruction

[3] See Appendix B for screenshots of the experiment

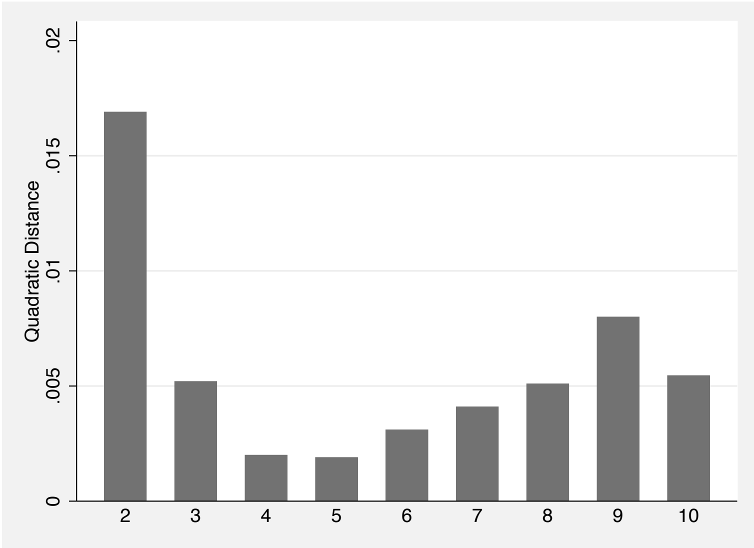

Figure 3: Mean quadratic distance for different sample sizes

sample size would lead to the predictions with the best fit to the data in our cyclical game. Therefore, we compared the predictive success of different sample sizes. Figure 3 shows the quadratic distances for the two-sample equilibrium with the sample sizes n=2,...,10. It can be seen that the sample size 5 yields the best fit to the data, while a sample size of 6 would have lead to weaker results. We will base our comparison of the stationary concepts on the two-sample equilibria with both sample sizes.

5.2 Predicted and Observed relative Frequencies

We will start with the comparison of the relative frequencies obtained in our experiment with the predictions of the three concepts in the case of an empty market. In this case all three concepts predict a frequency of 1 for entry. In three of the eleven observations this prediction is correct and all potential entrants join the markets when they are empty. In the other eight observations the relative entry frequencies are very high, too. The smallest entry rate is 0.9506. It is not surprising that the participants realize that the strategy of not entering the market is dominated in this case by the strategy of entering the market. The numerical values for all observations are shown in table 8 in the appendix.

In the case of an occupied market the three stationary concepts predict different relative frequencies of entry. Table 4 shows the observed relative frequency and the quadratic distance to each of the three concepts for all eleven observations. We will use the quadratic distance as our measurement for the predictive power of the three concepts.

The mean quadratic distance of the two-sample equilibrium with $n = 5$ to the observations is marginally smaller than the mean quadratic distance of the impulse balance equilibrium, while the mean quadratic distance of the two-sample equilibrium with $n = 6$ is bigger than the mean quadratic distance of the impulse balance equilibrium. The mean quadratic distance of the Nash equilibrium to the data is the biggest in comparison with the other two concepts.

Figure 4 gives the quadratic distances of each of the three stationary concepts to the data in the eleven observations.

Observed Relative Frequencies	Nash Equilibrium	Impulse Balance Equilibrium	Two - Sample Equilibrium $n=5$	Two - Sample Equilibrium $n=6$
0.8068	0.0046	0.0066	0.0027	0.0008
0.7841	0.0083	0.0034	0.0008	0.0000
0.7292	0.0213	0.0000	0.0007	0.0024
0.7325	0.0203	0.0000	0.0005	0.0021
0.7479	0.0162	0.0005	0.0001	0.0009
0.7271	0.0219	0.0000	0.0008	0.0026
0.7447	0.0170	0.0004	0.0001	0.0011
0.7065	0.0284	0.0050	0.0024	0.0051
0.7964	0.0062	0.0000	0.0017	0.0003
0.7238	0.0229	0.0053	0.0010	0.0029
0.6530	0.0493	0.0156	0.0104	0.0156
	0.0197	0.0020	0.0019	0.0031

Table 4: *Quadratic distances in full markets*

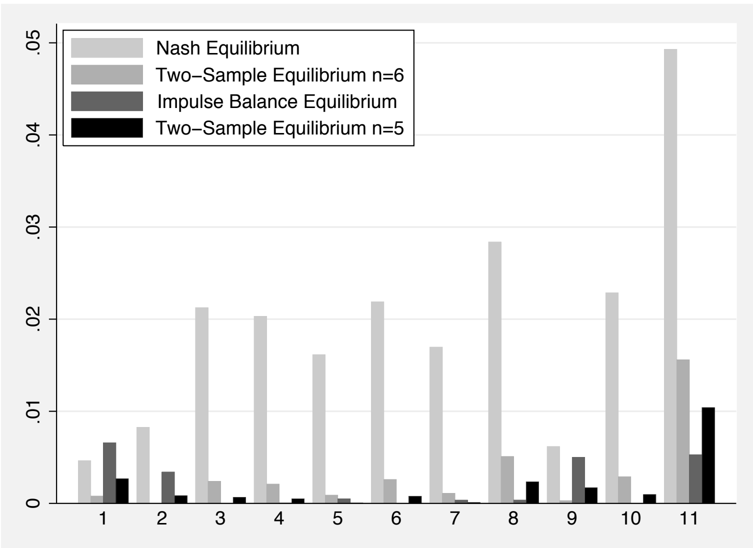

Figure 4: Quadratic distances in full markets

In six observations the quadratic distance to impulse balance equilibrium is the smallest, in two observation the quadratic distance to two-sample equilibrium with sample size 5 is the smallest and in three observations the quadratic distance to the two-sample equilibrium with sample size 6 is the smallest. The quadratic distance to the Nash equilibrium is in one observation smaller than the quadratic distance to the impulse balance equilibrium, but nevertheless larger than the quadratic distances to the 2 two-sample equilibria. In the other ten observations the quadratic distance to the Nash equilibrium is the largest of all three theories.

Testing the quadratic distances to the data of the three concepts with the two-tailed Wilcoxon signed-rank test, we obtain the results given in table 5. The significances are in favor of the row concept. No statistically significant difference between the predictive power of the 2 two-sample equilibria and the impulse balance equilibrium can be observed. But both concepts fit the data highly significantly better than the Nash equilibrium does. The two-sample equilibrium with the smaller sample size performs significantly better than the two-sample equilibrium with the higher sample size does.

	Nash Equilibrium	Two-Sample Equilibrium n=6	Impulse Balance Equilibrium
Two-Sample Equilibrium n=5	0.0033	0.0754	n.s.
Impulse Balance Equilibrium	0.0058	n.s.	-
Two-Sample Equilibrium n=6	0.0033	-	-

Table 5: *Two-tailed Wilcoxon signed-rank test for the quadratic distances to the data in favor of the row concept*

5.3 Changes over Time

Up to here we have analyzed the data on an aggregated basis and could elicit an order for the predictive success for 200 periods. We now investigate whether this order changes over time. In our experiment players had to decide whether they enter or do not enter the market 100 times. Thus, our analyses are limited to this 100 decisions. We compare the first 50 decisions with the second 50 decisions. Figure 5 gives the mean quadratic distance of the three theories to the observed frequency of entering the market for the first 50 decisions, for the second 50 decisions and for all 100 decisions.

The mean quadratic distance of the Nash equilibrium to the data decreases over time, while the mean quadratic distances of the impulse balance equilib-

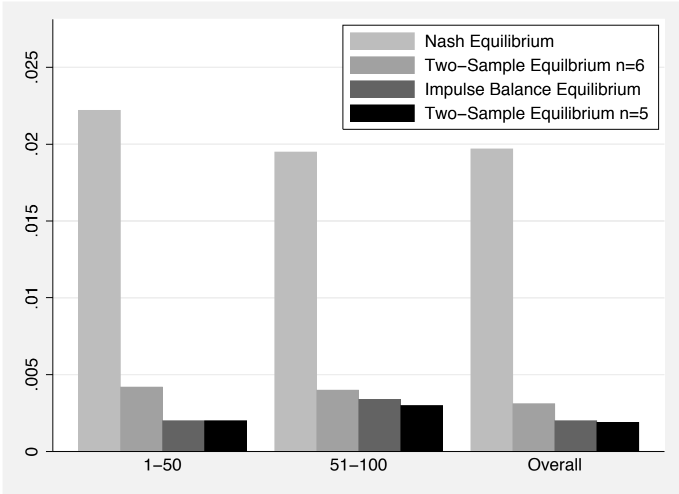

Figure 5: Quadratic distances to the data for the first 50 decisions, the second 50 decisions and overall

rium and of the two-sample equilibrium with a sample sizes of $n = 5$ are slightly increasing. Neither the improvement of the performance of the Nash equilibrium nor the disimprovement of impulse balance equilibrium and the two-sample equilibrium are statistically significant (Wilcoxon Signed Rank test).

Table 6 shows the two tailed significances of the Wilcoxon Signed-Rank test in favor of the row concept. The first value in a cell is the level of significance in the first 50 decisions and the second value is the level of significance in the second 50 decisions. Comparing the behavioral concepts with the Nash equilibrium, the significance levels are a little bit weaker in the second 50 decisions than in the first. Nevertheless, the tendencies are the same as in the overall comparison: The prediction of the Nash equilibrium has a significantly weaker predictive success than the predictions of impulse balance equilibrium and of the two-sample equilibria with sample sizes of $n = 5$ and $n = 6$. No significant difference can be found for the comparison of the predictive success of impulse balance equilibrium and both two-sample equilibria. The comparison between the 2 two-sample equilibria is significant only for the first 50 rounds.

5.4 Convergence to Pure Strategies?

In the above analysis we compared the mean entrance frequencies with the prediction of the three concepts in mixed strategies. However there are also further asymmetric equilibria in pure strategies. In these equilibria one player always enters an occupied market whereas the other never does this. Of course at equilibrium the first type of player never gets an opportunity to enter an

	Nash Equilibrium	Two-Sample Equilibrium n=6	Impulse Balance Equilibrium
Two-Sample Equilibrium n=5	0.0033 0.0099	0.0616 n.s.	n.s. n.s.
Impulse Balance Equilibrium	0.0044 0.0164	n.s. n.s.	-
Two-Sample Equilibrium n=6	0.0033 0.0099	-	-

Table 6: *Two-tailed Wilcoxon signed-rank test for the quadratic distances of the data in round 1-50 (top) and rounds 51-100 (bottom) in favor of the row concept*

occupied market but nevertheless it is his strategy to do this if he can. In our experimental setup coordination at an asymmetric pure equilibrium is possible since one type of players decides to enter a market or not in odd periods and the other one in even periods. It is quite possible that learning processes produce a tendency towards a coordination at an asymmetric pure equilibrium. In the following we shall argue that a weak tendency in this direction can be observed in our data.

Of course it can not be predicted how the roles in the asymmetric equilibrium will be distributed to the players deciding in odd and even rounds. However we can compare the relative frequencies of entries into occupied markets for players deciding in odd and even periods in the first 50 and the second 50 decisions. Therefore we form the quadratic distance between the entrance rates in even and odd periods. Table 7 gives these quadratic distances for the first and second 50 rounds per observation.

Observation	Quadratic Distance in 1-50	Quadratic Distance in 51-100
1	0.0007	0.0059
2	0.0002	0.1444
3	0.0097	0.0940
4	0.0025	0.1182
5	0.0125	0.0072
6	0.0465	0.1764
7	0.0312	0.0933
8	0.1044	0.0614
9	0.0021	0.0982
10	0.0044	0.0281
11	0.0405	0.0045
Mean	0.0232	0.0756

Table 7: *Quadratic distance between entry in odd and even periods*

In 8 of the 11 observations the quadratic distance of entry in odd and even rounds increases over time and the mean distance over all observations increases. A tendency towards coordination to a pure strategy equilibrium would lead to such an increase. The two-sided Wilcoxon signed-rank test between the quadratic distances in the first and second 50 decisions reveals a significant difference with $p = 0.0505$.

6 Summary and Discussion

In this paper three stationary concepts, namely mixed Nash equilibrium, Impulse balance equilibrium, and two-sample equilibrium, have been compared in an experimental setting. The game played in the experiment was a market entry game based on the structure of a cyclical game introduced by Selten and Wooders (2001).

Eleven independent subject groups participated in the experiment. Each independent subject group consisted of 6 participants, three deciding in odd rounds and three deciding in even rounds. Each subject group played over 200 rounds with random matching.

In the case of an empty market all three theories predicted entry and nearly all participants acted accordingly.

In the case of an occupied market we used the mean squared distance between predicted and observed frequencies as the measurement of the predictive success of a theory. The comparison of the mean squared distances reveals the following order, from best to worst: two-sample equilibrium with a sample size of $n = 5$, impulse balance equilibrium, two-sample equilibrium with a sample size of $n = 6$ and mixed Nash equilibrium.

To test whether the observed order is statistically reliable we applied a pairwise comparison with the Wilcoxon signed-rank test. The test confirmed that the mixed Nash equilibrium is the worst fitting concept and that the two-sample equilibrium with the smaller sample size fits the data better than the two-sample equilibrium with the higher sample size does. The rank of the impulse balance could not be confirmed. There is no significant difference between the predictive success of impulse balance equilibrium on one side and each of the two-sample equilibria on the other side. The performance rank two of impulse balance equilibrium may be due to random fluctuations.

The best sample size for the two-sample equilibrium was five. However, since only one probability had to be predicted, adjusting the sample size to the data gives an unfair advantage to two-sample equilibrium over the two non-parametric equilibria, impulse balance equilibrium and mixed Nash equilibrium. In order to see it is sufficient to look at a very simple theory with one parameter: This theory just says that the relative frequencies of entry are described by a symmetric distribution around an unknown mean. The unknown mean is the parameter to be estimated from the data. Of course this theory with one parameter provides the best possible fit. Therefore a comparison of impulse balance equilibrium with the two-sample equilibrium with the sample size $n = 5$ estimated from the data is heavily biased in favor of the two-sample equilibrium. The comparison of two-sample equilibrium with sample size estimated from the data presented here shows a better performance than impulse balance equilibrium but despite of the bias produced by the free parameter the difference is not statistically significant.

To give the comparison a broader basis with more observations we also applied the two-sample equilibrium with sample size $n = 6$, which yielded the best fit over 108 independent subject groups in 12 different 2×2-games (Selten and Chmura (2007)). Compared to impulse balance equilibrium the two-sample equilibrium with sample size $n = 6$ has a worse performance, but it is not statistically significant.

These findings give support to the results of Selten and Chmura (2007) where the Impulse balance equilibrium and the two-sample equilibrium outperformed the mixed Nash equilibrium and no significant difference between the two-sample equilibrium and the impulse balance equilibrium could be observed. Regarding the comparison of impulse balance equilibrium and mixed Nash equilibrium our results are in line with Avrahami, Kareev and Güth (2005) .

The comparison of the behavioral theories with the mixed Nash equilibrium is highly significant and stable over time when the quadratic distances in the first 50 decisions and the quadratic distances in the second 50 decisions are tested. Pairwise comparison of the behavioral concepts reveals a significant difference between the 2 two-sample equilibria in favor of the equilibrium with the smaller sample size but only for the first 50 decisions. Comparisons with the impulse balance equilibrium reveals no significant difference.

In addition to the mixed equilibrium the game also has two pure equilibria. The observed frequencies are far from those predicted by the pure equilibria. Nevertheless it is not excluded that in the long run learning processes would result in players entering in odd periods playing one pure equilibrium strategy and those entering in even periods the other one. The quadratic distance between the relative frequencies of entering in odd and even periods is significantly higher in the second half of the experiment than in the first one. One may interpret this as a weak tendency towards a pure strategy equilibrium.

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Appendix

A. Table

Observation	Market Empty	Market Entries	Relative Frequency
1	103	99	0,9612
2	109	109	1
3	131	130	0,9924
4	129	129	1
5	124	123	0,9919
6	131	131	1
7	126	121	0,9603
8	140	138	0,9857
9	104	103	0,9904
10	133	131	0,9850
11	162	154	0,9506
Mean:			0,9798

Table 8: *Frequency of entries into an empty market*

B. Screenshots

Figure 6: *Screenshot decision screen*

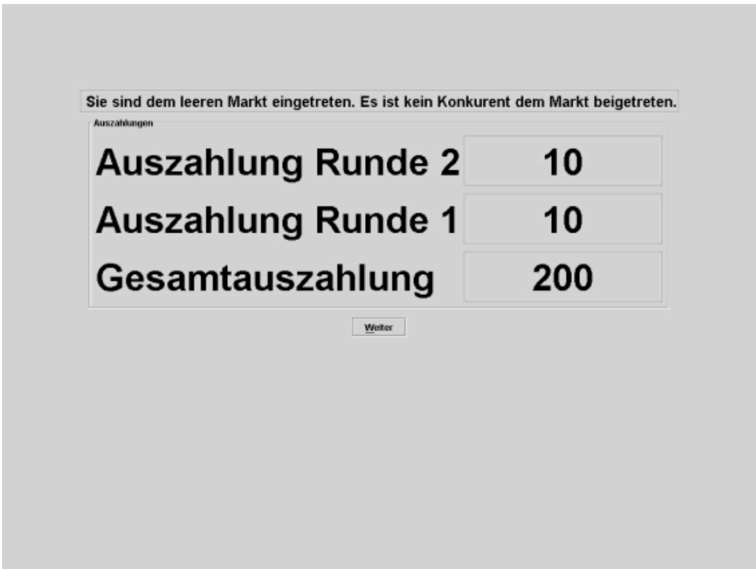

Figure 7: *Screenshot payoff screen*

C. Instructions

Thank you very much for participating in todays decision experiment. Please read the following instruction carefully. If you do not understand something and have some questions you can ask them at the end of the this introduction. As soon as the experiment has started no more questions will be answered. If you still have questions please take a look at this instructions. For the conduction of this experiment it is necessary that you do not communicate with other participants. Please do not talk with the other participants.

In this experiment you can earn money. Your payoff depends on your decisions and other participant's decisions.

The Experiment

The experiment consists out of 200 rounds. Decisions are done alternating, that means that everyone decides every two rounds. To which half of the participants you belong is only known to you.

In the rounds in which you make your decisions you are assigned to a market. Then you will receive the status of the market, as the market is either free or occupied. Then you can decide whether you want to enter the market or not. If you enter the market you stay in the market for two rounds. If you do not enter the market you stay outside the market for two rounds. After these two rounds and while the 200 rounds of the experiment are not exhausted you can decide again. After your decision the first round is over and the second round begins in which players from the other half of participants decide. After this player have decided the second round is over and you will again be allocated to a market (mostly a new one). This is repeated 100 times, thus there are 200 rounds to play.

Payoffs in the Experiment

In every round you receive a payoff.

If you do not enter the market you receive 5 Taler in the first round and 0 Taler in the second round.

If you enter the market you receive an amount in the first round which depends on the status of the market. If the market is empty you receive in the first round a payoff of 10 Taler. If the market is occupied you receive a payoff in the first round of 2 Taler. In the second round your payoff depends on the decision made by the next participant from the other group randomly allocated to this market. If this participant enters the market you receive a payoff of 2 Taler, if he does not enter you receive a payoff of 10 Taler.

The payoffs from this rounds are summed up and form your round payoffs. This round payoffs are summed up over all rounds and form your total payoff at the end of the experiment. Your payoff of this experiment is payed to you in Euro, where 1 Taler is 1 EuroCent.

The following tables should illustrate your payoffs:

Your payoffs in an empty market:

		Next player	
		Enter	Not Enter
You	Enter	$10 + 2$	$10 + 10$
	Not Enter	5	5

Your payoffs in an occupied market:

		Next player	
		Enter	Not Enter
You	Enter	$2 + 2$	$2 + 10$
	Not Enter	5	5