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On the Dynamic Efficiency of the Market System

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Abstract

We introduce a framework of development in which the direction of change is determined endogenously. Which new products, which new qualities and which new techniques are introduced in the course of development is determined by the profitability of different potential innovations. We define a concept of long-run efficiency of development which formalizes a widespread notion of 'dynamic efficiency'. The concept merely excludes *persistent* inefficiencies. We finally give conditions that guarantee long-run efficiency of laissez-faire development. This formalizes a popular claim about the dynamic efficiency of the market system, and, at the same time, makes more precise the limits to the claim.

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1 Introduction

Popular opinions about the performance of the free market system stress dynamic aspects much more than static efficiency. The optimistic view of the advocates of the laissez-faire can be paraphrased as follows:

> *Useful* improvements upon existing products, qualities or techniques are profitable to those who first implement these improvements. *Profitable* improvements in turn, if they are within the horizon of perception of the agents in the economy, will eventually be carried through by some of these agents. Thus, innovations that are at the same time useful for consumers and feasible for innovators will not be *persistently* neglected in a system that uses profits as private incentives for improving upon existing routines and existing knowledge. In the long-run the incentives of a 'laissez-faire' economy select and stimulate that kind of change which most appeals to consumers' tastes.

In a (perhaps less) popular reply to such a view critics of a 'blind' laissez-faire hold that

> there is no automatism in the system which guarantees that the *most* useful innovations also are the most profitable innovations, in particular when the profits are only short lived. Desirable innovations need not be successful in the struggle for scarce resources. The market system, left to its own, tends to continuously develop into wrong directions, persistently neglecting feasible and desirable improvements and missing unique opportunities.

Both views focus on the 'dynamic' performance of the system, in the sense that they center around its capability of *endogenously determining change*;[1] both views are aware of the 'multidimensionality' of the allocation problem, in that they are statements about the

[1] In the spirit of Schumpeter [1911], our use of the term 'dynamic' refers to endogenous changes in the state of technological knowledge.

right kind of change; and both views are concerned with the *long-run* performance of the system or with *persistent* market failures.

Of course we only state these popular[2] views to announce next that we provide a formal framework which allows to take an impartial view on the issue and which helps resolving the dispute.

Before we write more about our own framework, we take a short look on what conventional theory has to say to the issue.

Although an outsider would probably expect to find the issue at hand – the dynamic performance of the market system – to belong to the heart of welfare economics, no formal theory seems to exist, which provides a general framework allowing to systematically define the issue, to check the opposing arguments, and to derive precise conditions necessary for long-run efficient change in a laissez-faire economy.

First, existing formal welfare theory is incapable to capture the discussion either because it is static by nature (as is Walrasian theory), or because it is essentially one-dimensional (as is most of growth theory). Second, economic theory does not provide any precise *notion* of long-run efficiency which formalizes the vague notion of efficiency, implicite in the popular views. Such a notion should primarily exclude *persistent* inefficiencies. We briefly comment on both these points.

Traditional Walrasian theory, even where it explicitly includes time, has little to say about the allocation of those resources that are necessary for the production of new knowledge. The only prediction about a perfectly competitive market economy with free entry in a dynamic framework is that there will be no endogenous 'production' of new knowledge at all. New knowledge, once produced, is a non-rival commodity. If frictionless perfect competition means that any knowledge can be used immediately and by everybody (non-excludability) and that competition evades all profits because of free entry, then no private agent would engage in costly search for new knowledge. Therefore, models of frictionless perfect competition with free entry cannot endogenously explain the intentional

[2]Possibly we have arranged the 'popular' opinions a bit, but we have not invented them. The first reflects a typical 'conservative' opinion, the second reflects a 'liberal' opinion (in the North-American terminology).

'production' of new knowledge. The production of new knowledge, however, is crucial in our 'popular claims' about the dynamic performance of the market system.

Modern growth theory (Romer [1990], Aghion and Howitt [1992], Grossman and Helpman [1991]) endogenously explains the production of new knowledge (why it is usually called 'Endogenous Growth Theory'). It is widely accepted in this literature that "technological change arises in large part because of intentional actions taken by people who respond to market incentives"[3]. The theory solves the problem caused by the public good character of knowledge by dismissing with the hypothesis of price-taking perfect competition with free entry. However, growth in this literature, as in almost all growth theories, is essentially one-dimensional.[4] The scope of this literature is limited to studying the rate or the intensity of change. It does not analyse the *direction* of change. Consequently, the issue of whether the *right* kind of change arises endogenously, i.e. whether the *right* products and the *right* techniques are introduced in the long term, cannot be addressed.

Pareto-efficiency is too strong. Both modern Walrasian welfare theory as well as modern normative growth theory are forged around the concept of Pareto-efficiency. This concept is by far too strong a concept to help formalizing the wide spread but vague notion of dynamic efficiency which is mainly concerned with *persistent* inefficiencies. No practical woman or man, not even the most ardent advocate of the laissez-faire, does seriously expect that actual development in a non-stationary dynamic world like ours comes close to achieving (overall) Pareto-efficiency. And nobody would much bother about a critique of the laissez-faire if it were merely to assert its Pareto-inefficiency. The excessive strength of the efficiency notion implies excessively strong assumptions for any intertemporal welfare theorem using this notion. Whenever conventional welfare theorems show (Pareto-)efficiency of (dynamic) equilibrium, they implicitly or explicitly rely on utterly unrealistic assumptions on rationality, foresight, and on the coordination of beliefs. Even

[3]Romer [1990].

[4]An important exception is the literature on induced technical change of the sixties (Fellner [1961], Kennedy [1964], Samuelson [1965], von Weizsäcker [1966]). A central question in that literature is whether the deepening of capital relative to labor induces a bias towards labor-augmenting technical progress. This issue is not addressed in the present study, as capital cannot be accumulated.

as a bench-mark, Pareto-efficiency is too strong.

The aim of the present article is threefold:

1. Provide a simple framework of development that allows to formalize the discussion about the dynamic efficiency of the market system. Correspondingly, the evolution of the set of produced and consumed commodities and of used production processes is explained endogenously in our framework. The model and the definition of *equilibrium* development do not depend on too stringent requirements on consumers' foresight and coordination of future plans. Although the model is introduced in a completely neoclassical dress, it allows for a direct evolutionary reinterpretation.
2. Introduce a definition of efficiency which makes precise the intuitive concept of dynamic efficiency that merely excludes *persistent* inefficiencies.
3. Give a bench-mark theorem stating conditions under which equilibrium development is efficient, formalizing the claim about the dynamic efficieny of the market system and, at the same time, making precise the limits to the claim.

Thus, the present article centers around the three features we have distilled from the conflicting views about the performance of the market system: (1) In contrast to Walrasian welfare economics, it concentrates on the *'dynamic'* part of the problem of resource allocation in endogenously explaining the evolution of the technological state of knowledge. (2) In contrast to most of traditional and modern growth theory, the present article emphasizes the *multidimensionality* of the allocation problem. (3) Finally, in contrast both to Walrasian theory as well as to modern normative growth theory, it is concerned with *persistent* inefficiencies, rather than with Pareto-inefficiencies.

2 The Model

We first give an informal description of the model. The economy is modeled as a sequence of temporary economies that are perfectly competitive in all traditional markets and monopolistically competitive in new markets. *Which* new markets are opened or *which* technologies are introduced in each period is determined endogenously. Depending on the *state of knowledge* of a given period new technological possibilities emerge. This defines a set of *potential innovations*. In order to turn a potential innovation into a real technology that can be used for production some resources have to be spent. These resources are scarce. Therefore, only those potential innovations that guarantee sufficiently high profits are implemented. These profits are short-run as they are competed away by imitators after a transition period. The mapping that defines a set of potential innovations for each state of knowledge (the innovation-function) is exogenously given. The indeterminacy that remains open for economic explanation is removed endogenously. To this extent the short-run profits determine the direction of change.

Generations. The economy consists of an infinite sequence of periods. In each period there is a continuum of consumers with only finitely many types of different consumers $i \in I$. The consumer-sector is assumed to be identical in all periods. There are new generations of consumers in each period each living for one period (alternatively, one may imagine a single generation of immortals for which transactions between different periods are not possible). This disconnects the periods in the most drastical way and reduces consumers *interest* in the future to nil. In a more elaborated and realistic version of the model individuals live longer than a period and trade between periods is allowed (See Funk [1996b]). The present extreme separation of periods not only allows to do without modelling expectations, but also strengthens the conflict between short-run interests and long-run efficiency. This constitutes the hardest environment for testing the dynamic efficiency of laissez-faire development.

Commodities and preferences. In order to concentrate on a simple framework that allows to discuss the issue of dynamic efficiency as outlined above, we assume that there are finitely many primary inputs and types of output only, and that each type of output can be vertically differentiated. Each type $i \in I$ of consumers owns strictly positive endowments of all primary inputs. A consumption plan of consumer i in a given period is a vector of inputs $x^i = (x^i_1, \cdots, x^i_H)$ and a vector $\tilde{y}^i = (\tilde{y}^i_1, \cdots, \tilde{y}^i_H)$ of consumption functions, where $\tilde{y}^i_h$ describes the list of the quality differentiated commodities of group h consumed by i. The general utility function of type i is $u^i : \mathcal{C} = I\!R^H_+ \times \mathcal{Q}^H \mapsto I\!R$, $(x, \tilde{y}) \mapsto u^i(x, \tilde{y})$, where $\mathcal{Q}$ is the set of functions from $I\!R_+$ to $I\!R_+$. The utility function is assumed to be continuously differentiable (point-wise) and quasi-concave. Preferences are monotone in the sense that the marginal utility u^i_{hq} from quality q of commodity h is strictly positive if i's total consumption of commodity h is finite ($y^i_h = \sum_{q'} y^i_{hq'} < \infty$). Furthermore, the marginal disutility of selling input h, $|u^i_{x_h}|$, is finite for all $|x^i_h| \leq e^i_x$, where $e^i_x \geq 0$ is i's endowment of input h.

A class of utility functions satisfying all assumed features is the class[5] $B - \sum_{h=1}^{H}[\gamma_{x_h}(x_h)^{\rho_{x_h}} - \gamma_{y_h}(\hat{y}_h)^{\rho_{y_h}}]$, where all γ's are strictly positive, all ρ_x's are larger than one, and all ρ_y's are smaller than zero and where $\hat{y}_h = \sum_j q_j y_{hj}$. Bliss ($B$) can only be reached by a consumer who has much of all commodities (and sells little of all inputs). If a consumer is satiated in one component he still has a desire for others. Also note that if a consumer has a utility function of this type, then he always weakly prefers to consume a single quality in each group (unless he is rationed). For the sake of exposition we assume in the main text that this last feature is generally satisfied, i.e. that given the prices for all qualities every individual weakly prefers to consumes one quality of each commodity at most. This will allow to define convergence of a sequence of allocations in a simple way. In an appendix we will extent this to a general class of vertical product differentiation. All assumptions are expressed for the general case.

The state of knowledge in period t. The state of knowledge in a given period $t \geq 0$ is the set of technologies that are known in period t, $\mathcal{Y}^t = \{Y^t_1, \cdots, Y^t_{K_t}\}$, $K_t < \infty$. Each technology k can produce a single output y_k with a single input x_k only (possibly different for different technologies), and there are strictly positive fixed costs. Thus, in period t, at most $2K_t$ commodities in $\mathcal{C}$ are used. Typically, the number of used commodities will be less than $2K_t$, since many technologies may use the same inputs or produce the same outputs. Each technology $Y^t_k \subset \mathbb{R}^2_+$ can be represented by a production function, that is denoted by f^t_k. For all t and all $k \in \{1, \cdots, K_t\}$ the average product $\frac{f^t_k(x)}{x}$ tends to zero if x tends to infinity. We denote $\alpha(Y_k)$ the maximal average productivity of technology Y_k. The symbol Y for a technology also includes the information about which output is

[5] A more general class is $B - \sum_{h=1}{}^{H} \gamma_{x_h}(x_h)^{\rho_{x_h}} + \sum_{h=1}^{H} \gamma_{y_h}(y_h)^{\rho_{y_h}}$, where $\gamma_{x_h}\rho_{x_h} > 0$ and $\rho_{x_h} > 1$ for all h, and $\gamma_{y_h}\rho_{y_h} > 0$ and $\rho_{y_h} < 1$ for all h.

produced with which input[6].

The perfectly competitive temporary economy in period t. There is an infinite number of *potential* firms, each of which can use at most one technology. For sufficiently large economies, the assumed shape of technologies and the assumption of free entry allow to apply the classical intuition for the case of perfect competition in which firms take prices as given and produce at minimal average costs.[7] Here, the equilibria of the perfectly competitive period at t are simply defined as Walras Equilibria of the economy with technologies $\hat{\mathcal{Y}}^t = \{\hat{Y}_1^t, \cdots, \hat{Y}_{K_t}^t\}$ where $\hat{Y}_k^t = CY_k^t$, i.e. where, for all k, Y_k^t has been replaced by the smallest cone containing Y_k^t (see Figure 1). Because these are the usual macroeconomic constant returns to scale technologies it does not matter which firms are active. The technology $\hat{Y}_k^t$ is fully characterized by the slope of its boundary, which is $\alpha_k^t = \alpha(Y_k^t)$. The economy with this aggregate technology $\hat{\mathcal{Y}}^t$ is denoted $\mathcal{E}(\mathcal{Y}^t)$. Note that due to the short live of consumers, demand and supply on all markets given the state of knowledge is independent on future prices. Since at any finite period t, at most finitely many commodities can be produced, the equilibria given the state of knowledge are standard static Walras equilibria of an economy with constant returns to scale technologies and finitely many commodities. Our assumptions on preferences guarantee that such equilibria exist in every period.

[6]One can allow for much more general technologies with multiple and overlapping factors and products. In this case the assumption on the average product has to be replaced by the assumption that the asymptotic cone of a technology does not contain stricly positive amounts of outputs. These assumptions are crucial for a non-cooperative foundation. Convexity (up to fixed costs) of technologies is not necessary.

[7]A non-cooperative justification for the equilibrium concept introduced in this section can be given, see Funk [1996a] for a founadation in a Bertrand model or Novshek and Sonnenschein [1980] for a foundation within a Cournot model. 'Large economy' in these articles means that the technologies used by individual firms are 'small' as compared to the size of the consumer sector. Talking of 'small' technologies makes sense only for technologies that satisfy our assumption on the average productivity. Given the consumer sector and given a list of technologies Y that satisfy this assumption, the technologies ϵY are small if ϵ is small. In Figure 1 technology Y_k^t and technology ϵY_k^t, $\epsilon > 0$ generate the same aggregate 'macro' technology $\hat{Y}_k^t$. While a sound foundation of the underlying concept of competition depends on the smallness of individual firms' technologies, our definitions of equilibrium do not formally depend on it. The ϵ plays no formal role in the present model.

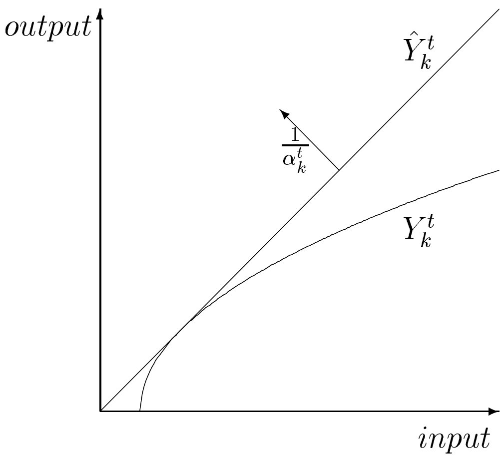

Figure 1

The innovations in period t. Given the state of knowledge, $\mathcal{Y}^t$, there is a *set of potential innovations*, denoted by $\mathcal{I}(\mathcal{Y}^t)$. The general restrictions on the shapes of a potential innovation are the same as those on the existing technologies (mainly the assumption on average productivity (or on the asymptotic cone)). A potential innovation cannot directly be used for production. Some resources have first to be spent. These resources are scarce and not sufficient to implement *all* new possibilities of period t. In order to concentrate on the simplest case that allows to ask the question *which* innovation is implemented, i.e., that allows to study the *direction* of development it is assumed that the resources for change are supplied in a fixed amount that suffices to implement exactly one innovation in each period. The relevant feature for the present purpose is that the resources for change are sufficiently scarce, so that not all potential innovations are profitable enough to be implemented.

Let $I^t \in \mathcal{I}(\mathcal{Y}^t)$ be the chosen innovation at t. One may For a while, which may be called the transition period, the innovation is used by a single firm (the innovator). The choice of I^t is determined by the profits of the innovator as described below. Since the size of the innovation is small compared to the aggregate economy this choice will not affect the Walras Equilibria of period t.

The state of knowledge in period t+1. After the monopolistically competitive transition period the new knowledge becomes public, i.e. there is free entry to the new technology as well. The choice of the innovation I^t leads to a new state of knowledge $\mathcal{Y}^{t+1} = \mathcal{Y}^t \cup \{I^t\}$ and a new economy $\mathcal{E}(\mathcal{Y}^{t+1})$ with aggregate technology $\hat{\mathcal{Y}}^{t+1}$. If I^t is a pure process-innovation the set of used commodities is the same in $\mathcal{E}(\mathcal{Y}^{t+1})$ as in $\mathcal{E}(\mathcal{Y}^t)$. If I_t is a product-innovation one or two (input and output) new commodities are added to those already activated. At the same time, depending on the state of knowledge $\mathcal{Y}^{t+1}$, further new possibilities $\mathcal{I}(\mathcal{Y}^{t+1})$ become known.

Note that, because the innovator is small, it does not matter whether we take the 'transition period' litarally or not. In an interpretation close to Schumpeter [1911] there is a perfectly competitve period at t without innovators, a transition period whith monopolistic competition for new technologies and perfect competition for others connecting two perfectly competitive periods, and then the next perfectly competitive period at $t+1$. Equivalently, one can do without the transition period. In this case, innovations occur at the beginning of each period. At each period, there simply is perfect competition for all traditional technologies and monopolistic competition for new technologies.

The profits of the innovator. During period t the innovator of period t is the only producer that can use the innovation I^t. After this period there is free entry to I^t and the profits of any firm using I^t will be zero. Knowing this, the innovator of period t will choose the technology in $\mathcal{I}(\mathcal{Y}^t)$, that maximizes the monopoly profits in the current period t. Since the size of his technology is small, he can take as given all prices of the WE for the traditional commodities. Thus, he sets prices and quantities on the markets of his inputs and outputs that maximize profits given the equilibrium prices of the competitive sector. If the innovation uses an already activated input the innovator will in fact be a price-taker for that input. Or, if the chosen innovation is purely process-innovating, then he is a price-taker even for his own output. If, on the other hand, he introduces a new product or a new quality of an existing product, he will face less than fully elastic demand.

The question of who is the innovator and who appropriates the transitory monopoly profits is irrelevant, as the new sector is negligible in the period decisive for the direction of change. In fact, the profits are 'quasi rents' that become the factor-incomes for the resources that are necessary to implement innovations, possibly including an entrepreneurial factor (if entrepreneurs are scarce). Again, in the present setting, the fact that the innovator is sufficiently small allows to neglect these incomes in the determination of the Walras Equilibria at t.

The data of the economy. Given the consumer-sector, the exogenous data of the economy are fully specified by the pair $(\mathcal{I}, \mathcal{Y}^0)$, where $\mathcal{I}$ is the *innovation function* that maps any potential state of knowledge $\mathcal{Y}$ into a finite set of potential innovations $\mathcal{I}(\mathcal{Y})$ and where $\mathcal{Y}^0$ is the initial state of knowledge (including finitely many technologies, only).

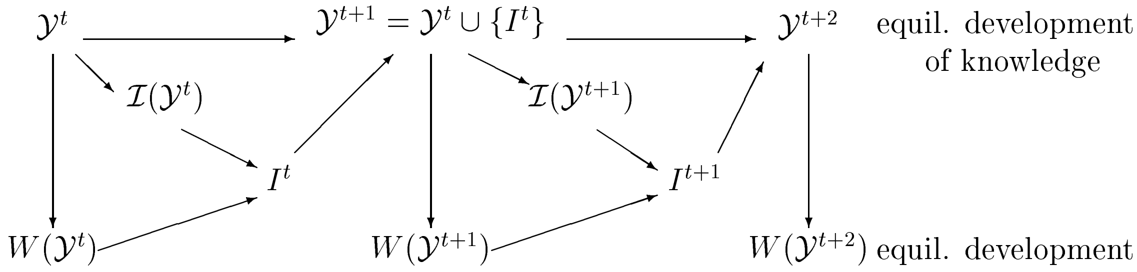

Figure 2

The unfolding of development is summarized in Figure 2: Given the state of knowledge, $\mathcal{Y}^t$, equilibria in period t are the (temporary) Walras Equilibria of the economy $\mathcal{E}(\mathcal{Y}^t)$ with technology $\hat{\mathcal{Y}}^t$. Given a particular Walras Equilibrium, $W(\mathcal{Y}^t)$, of $\mathcal{E}(\mathcal{Y}^t)$ a profit-maximizing innovation $I^t \in \mathcal{I}(\mathcal{Y}^t)$ is chosen. This defines a new state of knowledge $\mathcal{Y}^{t+1} = \mathcal{Y}^t \cup \{I^t\}$ and a new economy $\mathcal{E}(\mathcal{Y}^{t+1})$ with technology $\hat{\mathcal{Y}}^{t+1}$. At the same time, depending on the state of knowledge $\mathcal{Y}^{t+1}$, further new possibilities $\mathcal{I}(\mathcal{Y}^{t+1})$ become known, a new innovation,I^{t+1}, is chosen, $\cdots$.

Feasible development. The sequence $(\mathcal{Y}^t)_{t>0}$ is called a *feasible development of knowledge* if the state of knowledge at t, $\mathcal{Y}^t$, can be reached by a potential innovation given the previous state of knowledge, $\mathcal{Y}^{t-1}$, i.e. if $\mathcal{Y}^t \setminus \mathcal{Y}^{t-1} \in \mathcal{I}(\mathcal{Y}^{t-1})$ for all $t > 0$. A sequence $(z^t)_t$ of production plans is called a *feasible development* if there exists a feasible development of knowledge, $(\mathcal{Y}^t)_t$, with $z^t \in \hat{\mathcal{Y}}^t$ for all t. Given an economy $(\mathcal{I}, \mathcal{Y}^0)$, $\mathcal{Y}$ is a *feasible state* *of knowledge at t* if and only if there exists a sequence $(I^0, I^1, \cdots, I^{t-1})$ of innovations such that $\mathcal{Y} = \mathcal{Y}^0 \cup \{I^0, I^1, \cdots, I^{t-1}\}$, $I^0 \in \mathcal{I}(\mathcal{Y}^0)$, and $I^\tau \in \mathcal{I}(\mathcal{Y}^0 \cup \{I^0, I^1, \cdots, I^{\tau-1}\})$ for all $\tau \in \{1, \cdots, t-1\}$. Note that a sequence of feasible states of knowledge is not necessarily a feasible development of knowledge.

We will often use the richer sequence of hypothetical states of knowledge that one would get if in all periods all potential innovations would be carried through. In fact, the strong notion of efficiency defined in the present paper will label a given development efficient only if it does well as compared to developments that were feasible given this richer 'potential' development of knowledge.

The *set of feasible states of knowledge at t* is denoted by $\mathcal{P}^t(\mathcal{I}, \mathcal{Y}^0)$. The *potential state of knowledge* is $\mathcal{Y}^t_{\mathcal{P}} = \{Y \mid Y \in \mathcal{Y}, \mathcal{Y} \in \mathcal{P}^t(\mathcal{I}, \mathcal{Y}^0)\}$ and the set of potential aggregate technologies at t is the set $\hat{\mathcal{Y}}^t_{\mathcal{P}} = \{\hat{Y} \mid Y \in \mathcal{Y}^t_{\mathcal{P}}$ and $\nexists Y' \in \mathcal{Y}^t_{\mathcal{P}}$, with Y and Y' produce the same output with the same input and $\hat{Y} \subset \hat{Y}'\}$. The sequence of potential states of knowledge is called *potential development of knowledge*.

For the specified commodity set, the set of aggregate technologies that can produce commodities in group h, $\hat{\mathcal{Y}}^t_h$, is fully represented by the function $\alpha^t_h : \mathbb{R}_{+\infty} \mapsto \mathbb{R}_{+\infty}$, $q \mapsto \alpha^t_h(q) = \max_{q' \geq q, Y \in \mathcal{Y}^t_{hq'}} \alpha(Y)$ (where $\mathbb{R}_{+\infty} = \mathbb{R}_+ \cup \{\infty\}$), and $\hat{\mathcal{Y}}^t$ is represented by the function $\alpha^t = (\alpha^t_h)_h$. Similarly, $\hat{\mathcal{Y}}^t_{\mathcal{P}h}$ is represented by the function $\alpha^t_{\mathcal{P}h} : \mathbb{R}_{+\infty} \to \mathbb{R}_{+\infty}$, $q \mapsto \alpha^t_{\mathcal{P}h}(q) = \max_{q' \geq q, Y \in \mathcal{Y}^t_{\mathcal{P}hq'}} \alpha(Y)$ and $\hat{\mathcal{Y}}^t_{\mathcal{P}}$ is represented by $\alpha^t_{\mathcal{P}} = (\alpha^t_{\mathcal{P}h})_h$.

Equilibrium development. Let $W(\cdot)$ be a WE selection function that chooses equilibrium $W(\mathcal{Y})$ among the Walras equilibria of $\mathcal{E}(\mathcal{Y})$ and let $I(W(\mathcal{Y}), \mathcal{I}(\mathcal{Y}))$ be a profit-maximizing innovation given $(W(\mathcal{Y}), \mathcal{I}(\mathcal{Y}))$. An *equilibrium development of knowledge* is a feasible development of knowledge $(\mathcal{Y}^t)_t$ with $\mathcal{Y}^{t+1} \setminus \mathcal{Y}^t = I(W(\mathcal{Y}), \mathcal{I}(\mathcal{Y}^t))$ given the WE selection rule $W(\cdot)$. The sequence $(W(\mathcal{Y}^t))_t$ is an *equilibrium development* if $(\mathcal{Y}^t)_t$ is an equilibrium development of knowledge given W. We often write I^t instead of $I(W(\mathcal{Y}^t), \mathcal{I}(\mathcal{Y}^t))$.

Note that an equilibrium development always exists (since equilibria given the state of knowledge exist in all finite periods). Equilibrium development need not be unique since nothing in our analysis guarantees that equilibria given the state of knowledges are unique.

3 Pareto-inefficient Change

It should not be surprising that the (all) WE of the economy $\mathcal{E}(\mathcal{Y}^{t+1})$ can be Pareto-inferior to the (all) WE of the economy $\mathcal{E}(\mathcal{Y}^t \cup \{I\})$, where I is any feasible innovation other than the chosen one, i.e. $I \neq I^t$ and $I \in \mathcal{I}(\mathcal{Y}^t)$. The monopoly profits, decisive for the actual direction of change, fall short of the full economic surplus of the given period. In addition, the ('small') technology used by the innovator on the one hand and the constant returns to scale technology relevant for efficiency on the other hand have very little resemblance. Clearly, the possibility that all Walras equilibria of the economy $\mathcal{E}(\mathcal{Y}^{t+1})$ are $\mathcal{E}(\mathcal{Y}^t \cup \{I\})$, where $I \in \mathcal{I}(\mathcal{Y}^t)$ is not a pathological exception. In this case we say that development at t is short-run inefficient. In fact, equilibrium development will typically go wrong from time to time in the sense that in many periods people will not be happy with the allocation of the preceding period's resources.[8] The fact that the choice of an innovation is short-run inefficient in the above sense not necessarily implies that equilibrium development (i.e. the sequence) is Pareto-inefficient in these cases (i.e. that there exists an alternative feasible development which is better for some consumers of some generations and worse for no consumer of any generation). What appears to be undesirable in the short-run may benefit later generations (or the same individuals in later periods). A weak assumption (namely, that the innovation-function is dominance-preserving) has to be added to conclude that equilibrium development (i.e. the full sequence) is almost surely Pareto-inefficient (Funk [1996b]).

These results are not surprising and we will not state and derive them here, as this would require much additional notation and concepts that are not relevant for the understanding of the core of the paper. However, as a motivation for our weaker notion of efficiency it is important to realize the generic Pareto-inefficiency of equilibrium development. It shows that one cannot reasonably hope equilibrium development to satisfy much stronger efficiency conditions than the long-run efficiency to which we will turn in the next sections.

[8]Several formalizations are possible for what is meant by saying "equilibrium development will typically go wrong". In Funk [1996b] it is shown for a suitable probability measure, that, with probability one, an inefficient innovation will be chosen in infinitely many periods.

4 Long-run efficiency

Realizing that the *order of succession* in which different innovations are chosen can not be expected to be efficient, a natural question is: Can the economy *persist* to develop into 'wrong' directions, can it continuously neglect innovations that could and should be introduced? In this section we introduce a notion of dynamic efficiency which excludes such persistent inefficiencies. In the next section we give conditions on the data of the economy that guarantee that equilibrium development is efficient.

The fully developed economy. We first define the 'Fully Developed Economy' as the hypothetical economy in which all potential gains of development are exhausted, i.e. as the economy that would be approached if all potential innovations of each period would be carried through. Formally, the *fully developed economy (FDE)* given the exogenous data, $(\mathcal{I}, \mathcal{Y}^0)$, is the economy with technology $\hat{\mathcal{Y}}_{\mathcal{P}} = \cup_{t>0}\hat{\mathcal{Y}}^t_{\mathcal{P}}$. Thus, the FDE is the economy with the constant returns to scale technology characterized by the function $\alpha_{\mathcal{P}} := \lim_t \alpha^t_{\mathcal{P}}$.

Note that, depending on the innovation-function, the productivities of aggregate technologies for certain qualities in any given industry may in principle tend to infinity in potential development. In the FDE the corresponding qualities will than be free commodities. Similarly, the quality of any commodity may or may not tend to infinity in potential development. However, we assume that there exists a commodity which remains scarce for everybody even in potential development, i.e. there exists a h with $\alpha_{\mathcal{P}h}(q) < \infty$ for all $q > 0$ (This assumption can be replaced by the alternative assumption that all consumer own something of all inputs, i.e. $e^i_h > 0$ for all i and h. These are two different ways to make sure that if all commodities become free for some consumer in the limit of equilibrium development, then they become free for all consumers. Note that the first alternative is quite realistic, while the second is extremely unrealistic).

Output h of quality q is called a *potentially free commodity* if, in the FDE, it can be produced without any input. Output h is said to be potentially completely free if it is potentially free for all qualities. Output h is said to be potentially of free quality if any quality can be produced in the FDE.

Since we have assumed that a consumer weakly preferes to consume a single quality at most in each group of commodities an allocation at t is fully described by a $3HI-$dimensional vector $(x^t, y^t, q^t)=$ $((x_h^{it}, y_h^{it}, q_h^{it})_h)_i)$, where the q^{it}'s are the qualities consumed by i and where the y^{it}'s are the corresponding quantities.

Long-run efficient development. Development is long-run efficient if all temporary equilibrium sequences of allocations tend to allocations that are Pareto-efficient in the FDE. Formally, development $(x^t, y^t, q^t)_t$ is called *long-run efficient* if any limit point of $(x^t, y^t, q^t)_t$ is a Pareto-efficient allocation in the FDE.

This definition is much weaker than its reference to Pareto-efficiency in the FDE may suggest at a first glance. First, it is an asymptotic concept only. A general condition merely excluding *persistent* inefficiencies *has* to be an asymptotic condition. In fact, the only *general* way to say 'persistently neglected' is to say 'neglected for ever'. Second, as in the definition of the FDE, 'limits' may be infinite in principle. Efficient development converges to the limits of potential development in (desirable) directions in which potential development converges, and is unbounded in (desirable) directions in which potential development is unbounded. Commodities that remain 'scarce' in potential development eventually have to be allocated efficiently, commodities that become free under potential development have to become free under actual development.

In what follows we introduce general conditions on the innovation-function and on preferences under which any equilibrium development is long-run efficient. The conditions are tight in the sense that for any one that is taken away a class of persistent market failures can be given (see Funk [1996b]). We will indicate the role each assumption will play in the proof of the efficiency theorem.

The two first assumptions will be used to make sure that whenever the state of knowledge of the FDE dominates the present state of knowledge in a certain direction, it is possible in principle to move in this direction. Thus, these assumptions exclude technical dead-locks. A consequence will be that potential profits of doing the next step in a persistently neglected direction are bounded away from zero.

(1) *Loss of opportunities.* By construction of the model loss of *knowledge* has been excluded. There may be loss of *opportunities*, however. The premature development of one

technology may obstruct innovators view on the possibility to develop another technology, which had already been a potential innovation in previous periods. The following assumption excludes this possibility. It essentially amounts to exclude negative external effects of the research in one direction on the productivity of research into another direction.

Assumption. No loss of opportunities (NLO). If $I \in \mathcal{I}(\mathcal{Y})$ and $\mathcal{Y} \subset \mathcal{Y}'$, then $I \in \mathcal{I}(\mathcal{Y}')$.

(2) *Lack of convexity.* More importantly, equilibrium development may fail to exhaust potential gains from development because of lack of a certain type of convexity of the innovation function. As an illustration consider Figure 3. There are two industries each producing a single non-quality differentiated output (y_1 and y_2) with a common input (x). There are no product-innovations. In industry 2 there is a new innovation, strictly improving upon existing technologies, each time the industry has been chosen by innovators. In industry 1, in contrast, in order to move from technology Y_1^0 to technology Y_1^2 (which reduces variable costs) innovators first have to practice with increased fixed costs (technology Y_1^1).

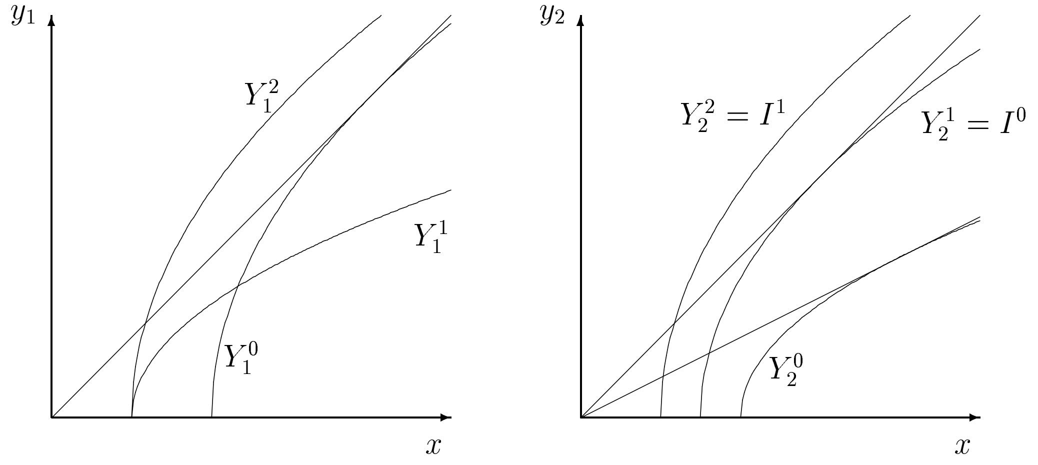

Figure 3a   Figure 3b

Then, it is not possible to make profits with an innovation in the first industry. No innovator will ever turn to the first industry, although profits (in terms of inputs) in the continuously improved industry (the second industry) generally tend to zero. Thus, y_2 will become cheaper and cheaper in terms of labour units ($\alpha_2^t \to \infty$), whereas y_1 remains expensive ($\alpha_1^t \not\to \infty$), even though the production costs of y_1 could be reduced to zero ($\alpha_{\mathcal{P}1}^t \to \infty$). In a loose interpretation of the model one may interprete the investment into innovation Y_1^1 as an investment in the market, such as advertisment, habit forming expenses, or investments in networks.

In order to exclude such examples we assume that it should be possible to at least slightly improve upon the initial knowledge in just one period if there is a path from the initial state of knowledge $\mathcal{Y}$ to a 'better' state of knowledge $\mathcal{Y}'$. The necessary improvement may be a very minor one (i.e. it has to be bounded away from zero).

Assumption. Convexity (CONV). Given the innovation function $\mathcal{I}$ there exists a $\beta \in [0,1)$ such that for all states of knowledge $\mathcal{Y}^0$, $\mathcal{Y}$, all qualities $q^0, q \in I\!R_+$, any class of commodities $h \in H$ and any natural number $\tau \geq 1$ with $\mathcal{Y} \in \mathcal{P}^\tau(\mathcal{I}, \mathcal{Y}^0)$ there exists a $\mathcal{Y}' \in \mathcal{P}^1(\mathcal{I}, \mathcal{Y}^0)$ and a $q' \in I\!R_+$ such that $\beta(\alpha_{h,q^0}(\mathcal{Y}^0), q^0) + (1-\beta)(\alpha_{h,q}(\mathcal{Y}), q) \leq (\alpha_{h,q'}(\mathcal{Y}'), q')$.

When (CONV) is assumed the function $\alpha_{\mathcal{P}h}$ can take one of the four following shapes only. In Case 1 commodity h is not potentially free for any quality. In this case $\alpha_{\mathcal{P}h}$ is a decreasing function and zero for all sufficiently large q. Denote by H_1 the commodities in this group. In Case 2 the commodity h is potentially of free quality. In this case $\alpha_{\mathcal{P}h}$ is constant in q. Denote the constant value by $\bar{\alpha}_h$ and denote by H_2 the set of commodities in this second group. The commodities $h \in H_3$ of Case 3 are potentially free for all qualities smaller than or equal to a constant $\bar{q}_h$. The commodities $h \in H_4$ of Case 4 are potentially completely free ($(\bar{q}_h, \bar{\alpha}_h) = (\infty, \infty)$).

The two next assumptions will imply that profits of innovations in a direction that is chosen again and again in the course of development become smaller and smaller. Since profits in a direction that is consitently neglected are bounded away from zero this will essentially suffice to prove a simple efficiency theorem. In section 5 we indicate how these assumptions can be relaxed to allow for bounded away from zero profits also into continuously chosen directions.

(3) *Unbounded monopoly quantities.* The first of these two assumptions is a further

condition on the innovation function. It makes sure that the monopoly quantities of a sequence of innovations does not tend to infinity. Since relative prices of a commodity which is continuously process-improved tend to zero, this will imply that innovative profits in a repeatedly chosen industry tend to zero. The assumption prevents an unbouded increase in monopoly quantities that may offset the unavoidable decrease of prices.

Assumption. No explosion of quantities. (NEQN). The asymptotic cone of $\lim_{t\to\infty} \cup_{Y \in \mathcal{Y}^t_{\mathcal{P}h}(\mathcal{I},\mathcal{Y}^0)} Y$ does not contain strictly positive amounts of any output.

This does essentially amount to assume that the average product of the production function representing technologies in the FDE tend to zero if the amount of input employed tends to infinity.

(4) *Exploding quality.* The previous assumption prevents unbounded growth of monopoly quantities. This will imply that the profits of process-innovations in an industry that is continuously process-improved peter out in the long-run. We also need an assumption which has the same effect for product-innovations.

In Funk [1996b] an example is given in which innovators in each period can choose either to increase the quality of the only consumption commodity in the economy or to improve upon the process that produces the best existing quality of this commodity. In equilibrium development q^t tends to infinity and $\alpha^t(q^t)$ remains bounded, though it is possible for q^t as well as for $\alpha^t(q^t)$ to grow without bound (i.e. $\alpha_{\mathcal{P}}(\infty) = \infty$). Equilibrium development induces people to work a lot, even asymptotically, whereas in optimal development hours worked remain constant at a moderate level. All MRS in this example are 'well behaved' including those involving qualities, i.e. $\frac{v_q}{v_x}$ and $\frac{v_q}{v_y}$ both tend to 0 if q tends to infinity, where v is the reduced utility function which depends on *consumed* qualities and on the corresponding quantities. In fact, assuming $\frac{v_q}{v_x} \to 0$ with $q \to \infty$ is not necessarily very effective if the quality increments of single innovations explode. What is required is a combined assumption on the innovation-function and on preferences.

Assumption. No explosion of quality. (NEQL). The marginal rate of substitution between two qualities q^τ and $q^\tau + \Delta q^\tau$ of commodity h, $\frac{u^i_{y_{hq^\tau}}}{u^i_{y_{h(q^\tau+\Delta q^\tau)}}}$, tends to 1 for all sequences $(q^\tau, \Delta q^\tau)_\tau$ such that $q^\tau \to_\tau \infty$ and such that Δq^τ is a feasible quality increment (with respect to $\mathcal{I}$) given q^τ for all τ.

The efficiency theorem is now easily stated. The proof is given in the Appendix.

Theorem *Under assumptions (No loss of knowledge), (Convexity), (No explosion of quantities), and (No explosion of qualities) any equilibrium development is long-run efficient.*

We briefly list the essential steps of the proof (which is given in detail in the Appendix):

It is first shown that at equilibrium development the profits of innovations in an industry that is chosen infinitely often (and measured in terms of the inputs in that industry) tend to zero in the course of development (Assumptions (NEQN) and (NEQL), Step 1).

Since the number of industries is finite there must be at least one industry which is chosen infinitely often, say industry 1. It follows from Step 1 that the profits in terms of input 1 in industry 1 tend to zero. Because neither potential profits in neglected industries nor the profits in other continuously chosen industries can remain higher than the profits in a continuously chosen industry (otherwise innovators would switch), the profits from innovations in all industries in terms of input 1 have to tend to zero (Step 2).

Using this fact it is shown in Step 3 and 4 that also the profits in terms of *own* inputs tend to zero in all industries (Assumption (Monotonicity)).

From this one can show with the help of Assumptions (CONV) and (NLO) that asymptotically all *consumed* qualities are produced as efficiently as they could be produced in the Fully Developed Economy (Step 5). In order to prove the theorem it mainly remains to show that the right qualities are introduced in the long-run. This is done in Step 6.

The result of Step 6 is interesting in its own right. Using Step 4 and 5 (as well as Assumptions (CONV) and (NLO)), we show that *any limit allocation of equilibrium development is a Walras equilibrium in the Fully Developed Economy*.

Because no consumer reaches a point of global satiation (Step 7), it follows from Assumption (Monotonicity) that any limit allocation of equilibrium development is Pareto-efficient in the Fully Developed Economy (Step 8).

5 Extensions and concluding comments

Informational requirements. What does the individual firm need to know in order to be able to behave as it has been assumed to behave? As in the 'pure circular flow' of Schumpeter's theory of economic development or as in the traditional intuition for (static) perfect competition the typical firm or consumer in our temporary economy given the state of knowledge need to know very little. They only know their individual technology or preferences and observe their own prices and excess demand or excess supply.[9]

The requirements on those firms that operate outside the routine activity of the circular flow are not much more demanding. In the extreme models analyzed in the present study investors and innovators can take as given the prices of the rest of the economy. In the model of the present chapter they do not have to know the future path of development, since their profits are transitory. They do not even have to compute the new equilibrium that emerges because of their innovations. They do not have to know the innovation function. Neither is it necessary that any (potential) innovator knows the set of *all* potential innovations given the present state of knowledge. One has to require that each element of the set of potential innovations is known by at least one (potential) innovator. In a sense this is the definition of the set of potential innovations (If one wants to make sure that innovators make zero-profits, then each potential innovation has to be known by sufficiently many (potential) innovators).

An Evolutionary Reinterpretation.

The way we have presented the basic model it is completely 'neoclassical'. All agents are perfectly rational and take into account all the information that is relevant for them. This does not require any froward looking rationality because of the extremely simple structure of the model. In the given neoclassical version of the model the innovation-function describes the horizon of perception of the agents in the economy. Innovators implement part of this potential knowledge when they belief this to be sufficiently profitable.

[9]Admittedly, this is not quite the framework in which a formal foundation of the Walrasian assumption of perfect competition has been furnished so far. Most explicit non-cooperative foundations still assume perfect information.

Not much reformulation of the basic model is needed to allow for an evolutionary reinterpretation. Firstly, suppose that *each* of the (finitely many) potential innovations is *actually* carried out by some entrepreneur who believes in this particular innovation, irrespective of its objective profitability. The innovation-function then defines the set of mutations for each state of the economy. Many of these innovations (mutations) will not be profitable, some will realize higher profits than others. Secondly, assume that only sufficiently profitable innovations are selected by the market, i.e., copied by imitators. In the simplest case, only one innovation (the most profitable one) is imitated in every period. The most profitable innovation is copied by the market, the others do not survive. Thirdly, redefine the state of knowledge as the set of technologies that has at least once been used by the market. This defines a simple evolutionary model which is formally equivalent to the model of the present chapter. In every period *all* of the finitely many potential innovations are carried through, rather than only the most profitable one. Since they are all negligable (as long as they are not copied) this does not affect equilibria. In the following period only the most profitable is copied by the market and we end up with a new state of knowledge and the a Walras equilibrium given the state of knowledge (Schumpeter's circular flow) which are identical to those reached in the neoclasical version.

If we stick to the notion of preferences and of short-run utility maximization laissez-faire development remains long-run efficient under the assumptions of the efficiency theorem. If we stick to the notion of preferences but not necessarily to that of short-run utility maximization, assuming however that consumers do not persistently ignore beneficial trades, then the long-run efficiency theorem still goes through.

Note that the assumption that only a single innovation is carried through in each period is easily dispensed with in the analyzis of the previous sections. In fact,for an evolutionary reinterpretation it is more natural to assume that all mutation-innovations that are *sufficiently* successful are selected by the market.

Sustained growth. In the proof of the theorem we made use of the fact that the profits of innovators in an industry which is continuously chosen tend to zero in the course of development. This does not exclude the possibility of sustained growth largely because the resources necessary for change in the simple model do not compete with other utilizations. If these resources were 'researchers' for instance, and if a researcher may either do research or else perform a non-resarch task, then development may cease before all gains from potential development have been exhausted. Sustained growth then mainly depends on researchers' preference over research jobs and non-research jobs when they do very little research (see Funk [1996b]). If we do not want the possibility of sustained growth to depend on the fact that relative profits from innovations to tend to zero, then assumptions (NEQN) or (NEQL) have to be violated. This may also endanger our result about the dynamic efficiency of development. However, assumptions (NEQN) and (NEQL) can be somewhat weakend to allow both for sustained growth and long-run eficiency. Assumptions (NEQN) and (NEQL) can be replaced by the following (informal) assumptions:

Assumption. Minimal Spillovers. (MinSpill). *If efficient scale quantities of innovations in a first industry tend to infinity, then there are spillovers to all other types of potential innovations (process-innovations in other industries and quality-innovations in all industries). The size of these spillovers is bounded away from zero and they improve non-chosen potential innovations each time an innovation in the first industry is chosen. Similarly, if the best known quality in an industry in a first industry q^τ tends to infinity, while the MRS between q^τ and $q^\tau + \Delta q^\tau$ remains bounded away from one, then there are bounded away from zero spillovers to all other types of potential innovations (process-innovations in other industries and quality-innovations in all industries) improving potential innovations each time a quality innovation in the first industry is chosen.*

Even (MinSpill) remains a strong assumption. In Funk [1995b] we give plausible examples of persistent inefficiencies that arise because consumers are not 'satiable in quality'. In this examples there are no spillovers from quality innovations on the productivity of research on process-innovations.

Appendix

Appendix A. Vertical Product Differentiation

The class of utility functions allowed for in section 2 was very specific with respect to the type of vertical product differentiation. We had assumed that at any vector of prices a consumer weakly prefers to consume a sinlge quality i each group of commodities. This restriction was not necessary. We now allow for a more general class of utility functions.

Consider the class of continuously differentiable and quasi-concave utility functions $u^i : \mathcal{C} = \mathbb{R}_+^H \times \mathcal{Q}^H \mapsto \mathbb{R}$, $(x, \tilde{y}) \mapsto u^i(x, \tilde{y})$, as defined in section 4. As before we want to assume vertical product differentiation within each group of commodities. To this end we assume that for all i, all h, all $\beta \in [0, 1]$, and all $(x^i, \tilde{y}^i)$

$$u^i_{h\dot{q}} \geq \beta u^i_{hq} + (1 - \beta) u^i_{hq^*} \text{ if } \dot{q} \geq \beta q + (1 - \beta) q^*.$$

(A weaker form of vertical product innovation would assume this for $\beta = 1$, only).

The reason for working with the specific class of utilities in the main text is that in order to be able to define limit allocations with respect to standard topologies we have to make sure that the number of commodities consumed by individual consumers does not tend to infinity. We first explain why we do want to have a bound on the number of consumed qualities by a single consumer. In the full commodity space a consumption plan of a consumer at t is a infinite sequence of quantities (of which almost all will be zero). We explain why it is not natural in our framework to define limits of sequences of consumption plans with respect to point-wise convergence. Let the number of different qualities of h that can be produced in the FDE be countably infinite and index these qualities by $n \in \mathbb{N}$, where, as always, we consider $n = \infty$ as a natural number. Then at any t $\tilde{y}^{it}_h$ is a sequence $y^{it}_h = (y^{it}_{hn})_{n \in \mathbb{N}}$. Now consider the following simple example. Suppose that at equilibrium development $y^{it}_{hn} = 1$ if $n = t$ and 0 otherwise. If one new quality of h is introduced by innovators in each period, then type i consumer buys one unit of the latest quality in each period. Then, in the limit of development the type i consumer should, one may think, consume one unit of quality $n = \infty$. However, if we consider point wise convergence, the limit of the sequence $(y_h^{it})_t$ has zeros everywhere, also at $n = \infty$. Thus, pointwise convergence is not the right convergence concept for our needs.

Rather than to refer to non standard topologies, we require that there is a number $N < \infty$ such that each consumer weakly prefers to consume less than N qualities of each type, given the prices and given that he is not rationed. A consumption plan for i at t then is as a finite vector $(x^{it}, y^{it}, q^{it}) \in I\!R^{3(1+2N)H}$. This allows us to stick to the notation of the main text, where N was 1. The definition of long-run efficieny is unchanged.

Since the requirement that, for all vector of prices, there is a common bound N, such that no consumer strictly preferes to to consume more than N qualities in any group of commodities, involves prices which are only determined endogenously, we do not know in general whether the assumption is binding or not. We offer two alternative assumptions on exogenous concepts that will imply what is needed. These assumptions make sure that consumers weakly prefer to buy finitely many qualities of h *in the limit* of development. Therefore, at equilibrium development the number of commodities consumed by each consumer is bounded.

The first possibility is to assume that *if* $(1/\alpha_{\mathcal{P}h}(q))$ *as a function of* q *lies (weakly) above* $(u^i_{hq}/u^i_{x_h})$ *given* $(x^i, \tilde{y}^i)$ *as a function of* q *for some* $(x^i, \tilde{y}^i)$ *, then the two functions do not take the same values for infinitely many* q*'s (the graphes are tangent at most finitely often).* This property is generic in the space of allowed preferences and innovation-functions.

The second possibility is to strengthen Assumption (CONV) by the following requirement: *If technologies with parameters* $(q', \alpha_h^{'}(q'))$ *and* $(q'', \alpha_h^{''}(q''))$ *are known at* t*, then there is a potential innovation with parameter* $(q, \alpha_h(q)) > \beta(q', \alpha_h^{'}(q')) + (1-\beta)(q'', \alpha_h^{''}(q''))$ *for all* $\beta \in (0, 1)$. This assumption guarantees that in the limit of development a consumer consumes a single quality of each h for all utility functions (in the class with vertical differentiation within each group of commodities).

5.1 Appendix B. Proof of the Welfare Theorem

Given $(\mathcal{I}, \mathcal{Y}^0)$ consider any sequence of equilibrium development $(x^t, y^t, q^t)_t$ and $(p^t, w^t)_t$ and the corresponding sequences of states of knowledge$(\mathcal{Y}^t)_t$, (where p^t is the list of active prices t and w^t is the vector of the H input prices). Let π_h^t be the maximal profit that can be realized when choosing to innovate in industry h in period t (i.e. when choosing an innovation in $\mathcal{I}_h(\mathcal{Y}^t)$ and using it as a monopolist).

Step 1. It is first shown that $(\pi_h^t/w_h^t) \to 0$ if industry h is chosen infinitely often. Let $(\hat{p}_h^t, \hat{x}_h^t, \hat{y}_h^t, \hat{q}_h^t)$ be the innovator's price, quantities and quality if he chooses h. Let q_h^t be the actually consumed quality of h at t that is closest to $\hat{q}_h^t$, i.e. define $q_h^t := \arg\max_{q' \le \hat{q}_h, y_{hq'}^t > 0}(\hat{q}_h^t - q')$. Denote by $p_h^t := p_{hq_h^t}^t$ the competitive price of this quality, by $y_h^t := y_{hq_h^t}^t$ the corresponding competitive quantity, and denote by $\hat{f}_h^t := f_{h\hat{q}_h^t}^t$ the technology used by the innovator. Recalling that $(p_h^t/w_h^t) = (1/\alpha_h^t(q_h^t))$, we can write $(\pi_h^t/w_h) = (\hat{p}_h^t/w_h^t)\hat{f}_h^t(\hat{x}_h^t) - \hat{x}_h^t = \frac{\hat{p}_h^t}{w_h^t}\frac{p_h^t}{p_h^t}\hat{f}_h^t(\hat{x}_h^t) - \hat{x}_h^t = \frac{\hat{p}_h^t}{p_h^t}\frac{\hat{f}_h^t(\hat{x}_h^t)}{\alpha_h^t(q_h^t)} - \hat{x}_h^t$. Furthermore, $(\hat{p}_h^t/p_h^t) = \max_{i\in I} \frac{u^i_{y_{h\hat{q}_h^t}}}{u^i_{y_{hq_h^t}}}|_{y^i_{hq_h^t}=0} \to 1$ if $t \to \infty$ (because of the continuous differentiability of u^i if $(\hat{q}_h^t - q_h^t) \to 0$ and because of Assumption (NEQL) if $\hat{q}_h^t \to \infty$). Therefore, $\lim_t(\pi_h^t/w_h) = \lim_t\left(\frac{\hat{f}_h^t(\hat{x}_h^t)}{\alpha_h^t(q_h^t)} - \hat{x}_h^t\right)$. Since $\hat{x}_h^t > (\hat{y}_h^t/\hat{\alpha}_h^t(\hat{q}_h^t))\forall t$ it follows that

$$\lim_t(\pi_h^t/w_h) \le \lim_t\left(\frac{f_h^t(\hat{x}_h^t)}{\alpha_h^t(q_h^t)} - \frac{f_h^t(\hat{x}_h^t)}{\hat{\alpha}_h^t(\hat{q}_h^t)}\right) = \lim_t \hat{f}_h^t(\hat{x}_h^t)\frac{\hat{\alpha}_h^t(\hat{q}_h^t) - \alpha_h^t(q_h^t)}{\hat{\alpha}_h^t(\hat{q}_h^t)\alpha_h^t(q_h^t)}.$$

Firstly, the term $\frac{\hat{\alpha}_h^t(\hat{q}_h^t) - \alpha_h^t(q_h^t)}{\hat{\alpha}_h^t(\hat{q}_h^t)\alpha_h^t(q_h^t)}$ tends to 0 (either $(\alpha_h^t(q_h^t)/\hat{\alpha}_h^t(\hat{q}_h^t))$ tends to 1 or $\alpha_h^t(q_h^t)$ tends to infinity). Secondly, we show that $\hat{f}_h^t$ is bounded. Since the maximal profits are non-negative $((\pi_h^t/w_h^t) \ge 0)$ $\hat{f}_h^t/\hat{x}_h^t \ge \hat{\alpha}_h^t(\hat{q}_h^t)$. Since $\hat{\alpha}_h^t(\hat{q}_h^t) \not\to 0$, it follows that $\lim_t \hat{f}_h^t/\hat{x}_h^t > 0$. Because of (NEQN) this is possible only if $\hat{x}_h^t$ is bounded (From (NEQN) it follows, that $\frac{f_h(x_h^\tau)}{x_h^\tau} \to 0$ for any unbounded sequence $(x_h^\tau)_\tau$, where f_h is the production function corresponding to the technology $\lim_{t\to\infty} \cup_{Y\in\mathcal{Y}^t_{\mathcal{P}h}(\mathcal{I},\mathcal{Y}^0)} Y$. Since $Y^t_{h\hat{q}_h^t} \in \cup_{Y\in\mathcal{Y}^t_{\mathcal{P}h}(\mathcal{I},\mathcal{Y}^0)} Y \subset \lim_{t\to\infty} \cup_{Y\in\mathcal{Y}^t_{\mathcal{P}h}(\mathcal{I},\mathcal{Y}^0)} Y$ it follows that $f_{h\hat{q}_h^t}/\hat{x}_h^t \to 0$ if $\hat{x}_h^t \to \infty$. This would contradict $\hat{f}_h^t/\hat{x}_h^t \ge \hat{\alpha}_h^t(q_h^t) \not\to 0$. Therefore, $\hat{x}_h^t$ is bounded). Therefore, $\lim_t(\pi_h^t/w_h^t) = 0$ if industry h is chosen infinitely often.

Step 2. Since H is finite at least one industry, say 1, must be chosen infinitely often in the course of development. It follows from Step 1 that $\lim_t(\pi_1^t/w_1^t) = 0$. Hence also $\lim_t(\pi_h^t/w_1^t) = 0$, otherwise 1 would not be chosen infinitely often (since $\pi_h^t > \pi_1^t$ for t sufficiently large).

Step 3. It is shown that $(w_1^t/w_h^t) \not\to \infty$ if there exists a type of consumers, say $i = 1$, with

$e_1^1 > 0$ such that $\alpha_h^t(q^t) \not\to \infty$, for any sequence $(q^t)_t$, $q^t \in Q_h^{it}$, where Q_h^{it} is the set of qualities of commodity h consumed by i at t. Suppose $(w_1^t/w_h^t) \to \infty$. Because $\alpha_h^t(q^t) = (w_h^t/p_{hq_h^{1t}}^t) \not\to 0$ it follows that $\frac{u_{x_1}^1}{u_{hq^t}^1} = (w_1^t/p_{hq^t}^t) = (w_1^t/w_h^t)(w_h^t/p_{hq^t}^t) \to \infty$. Therefore, since $u_{x_1}^1 \not\to \infty$ (because preferences are monotone), it follows that $u_{hq^t}^1 \to 0$. Again form the monotonicity of preferences, it follows that there is a sequence $(q'^t)_t$, $q'^t \in Q_h^{it}$, $q'^t \geq q^t$, such that $y_{hq'}^{1t} \to \infty$ and therefore $\alpha_h^t(q'^t) \to \infty$ (since mean endowments are bounded). Hence, $(p_{hq'}^t/p_{hq}^t) \to 0$ (since the qualities are produced in the same industry). Therefore, for t sufficiently large $y_{hq'}^i = 0$ (since $q' > q$ and q' cheaper than q). This contradicts $q^t \in Q_h^{it}$.

Step 4. It is shown that from Step 1, 2, 3 follows $(\pi_{hq^t}^t/w_h) \to 0$ for all h and all sequences $(q^t)_t$, $q^t \in Q_h^{it}$. If $\alpha_h^t(q^t) \to \infty$ for some i and some sequence $(q^t)_t$, $q^t \in Q_h^{it}$, then h is chosen infinitely often and the claim follows from Step 1. If $\alpha_h^t(q^t) \not\to \infty$ for all i and all sequences $(q^t)_t$, $q^t \in Q_h^{it}$, then there exists some i, say $i = 1$ with $e_1^i > 0$ and some sequence $(q^t)_t$, $q^t \in Q_h^{it}$, with $\alpha_h^t(q^t) \not\to \infty$. In this case it follows from Step 3 that $(w_1^t/w_h^t) \not\to \infty$ and hence from Step 2 that $(\pi_h^t/w_h^t) = (\pi_h^t/w_1^t)(w_1^t/w_h^t) \to 0$.

Step 5. It is shown that for $h \in H \setminus H_1$ the only limit point of any sequence $(q^t, \alpha_h^t(q^t))_t$, $q^t \in Q_h^{it}$ is $(\bar{q}_h, \bar{\alpha}_h)$ (possibly ∞) for all i and that for $h \in H_1$ the set of limit points of any sequence $(q^t, \alpha_h^t(q^t))_t$, $q^t \in Q_h^{it}$ is a subset of the graph of $\alpha_{\mathcal{P}h}$ for all i. Suppose not i.e. suppose that the claim is false for some i and some sequence $(q^t, \alpha_h^t(q^t))_t$, $q^t \in Q_h^{it}$. Consider any convergent subsequence (same notation). Let $(q, \alpha_h(q))$ be the corresponding limit. Since $(q, \alpha_h(q))$ does not belong to the graph of $\alpha_{\mathcal{P}h}$ it follows that $\alpha_{\mathcal{P}h}(q) > \alpha_h(q)$ and, therefore, by the definition of $\alpha_{\mathcal{P}h}$, there exists a q^* with $(q^*, \alpha_{\mathcal{P}h}(q^*)) > (q, \alpha_h(q))$ (possibly equality for the first component) such that $(q^*, \alpha_{\mathcal{P}h}(q^*))$ is the limit of some feasible path of development. From (NLO) it follows that $(q^*, \alpha_{\mathcal{P}h}(q^*))$ also is the limit of some feasible continuation of development starting from $\mathcal{Y}^t$, say $(q^{*t+\tau}, \alpha_h^{*t+\tau}(q^{*t+\tau}))_{\tau>0}$.

Then, because of (CONV), there exists a $\beta > 0$ and an element of $\mathcal{I}(\mathcal{Y}^t)$ that produces quality $\dot{q}$ of commodity h with minimal average costs $\dot{\alpha}_h(\dot{q})$ such that given t, for any τ, $(\dot{q}, \dot{\alpha}_h(\dot{q})) \geq \beta(q^t, \alpha_h^t(q^t)) + (1-\beta)(q^{*t+\tau}, \alpha_h^{*t+\tau}(q^{*t+\tau}))$. Since $(q^*, \alpha_{\mathcal{P}h}(q^*)) > (q, \alpha_h(q))$ it follows that for t and τ sufficiently large that $(\dot{q}, \dot{\alpha}_h(\dot{q})) > (q, \alpha_h(q))$. Thus, an innovator choosing $(\dot{q}, \dot{\alpha}_h(\dot{q}))$ at large $\bar{t}$ makes positive profits in terms of input h even at prices $1/\alpha_h(q)$. Because of (NLO) the innovation remains a feasible innovation in all later periods $t > \bar{t}$. Thus, the normalized profits $(\dot{\pi}_h^t / w_h^t)$ that could be realized when choosing this innovation do not tend to zero. This contradicts Step 4.

Step 6. We show that any limit allocation of any equilibrium development is a WE in the FDE. For each t normalize the set of input prices, such that the vector of input prices always belongs to the unit simplex. Consider an equilibrium development, $(x^t, y^t, q^t)_t$ and $(p^t, w^t)_t$. Consider any convergent subsequence of equilibrium development allocations and prices. The limit allocation (x, y, q) is a WE in the FDE at the following price system. Input price $w_h = \lim w_h^t$ and $p_{hq} = (w_h / \alpha_{\mathcal{P}h}(q))$ for all hq that can be produced in the FDE. First note that because of Step 5 the sequence of prices on active markets in fact converges to the corresponding limit prices. Since (x, y, q) maximizes profits at these prices (firms are indifferent), at least one type of consumer (say i) must be rationed in at least one market. Because of differentiability he has to be rationed on some market given the constraints on the other markets. This must be a market, say for commodity hq^*, that is not open in the limit of equilibrium development. Thus, $(u^i_{hq^*} / u^i_{x_h}) > (1/\alpha_{\mathcal{P}h}(q^*))$ at (x^i, y^i, q^i).

Let $q \in Q^i_h = \lim Q^{it}_h$. We first show that for all $\beta \in (0, 1]$ and $\dot{q} = \beta q + (1 - \beta) q^*$ we also have

$$(u^i_{h\dot{q}} / u^i_{x_h}) > \beta((1/\alpha_{\mathcal{P}h}(q)) + (1 - \beta)((1/\alpha_{\mathcal{P}h}(q^*)). \tag{1}$$

Because of vertical quality differentiation within groups it follows that $u^i_{h\dot{q}} \geq \beta u^i_{hq} + (1 - \beta) u^i_{hq^*}$. Therefore, $(u^i_{h\dot{q}} / u^i_{x_h}) \geq \beta(u^i_{hq} / u^i_{x_h}) + (1 - \beta)(u^i_{hq^*} / u^i_{x_h})$. Therefore, equation (1) follows at (x^i, y^i, q^i), since $(u^i_{hq} / u^i_{x_h}) = (1/\alpha_{\mathcal{P}h}(q))$ and $(u^i_{hq^*} / u^i_{x_h}) > (1/\alpha_{\mathcal{P}h}(q^*))$ at (x^i, y^i, q^i).

We need to show that there is an innovation which is feasible for all sufficiently large t and makes bounded away from zero profits. From (CONV) it follows that there exists a $\beta \in (0, 1]$ such that for $\bar{t}$ sufficiently large there is an innovation $\dot{I}$ with parameters $(\dot{q}, \alpha_h(\dot{q}))$, $\dot{q} = \beta q + (1 - \beta) q^*$ such that $(1/\alpha_h(\dot{q}))< \beta(1/\alpha_{\mathcal{P}h}(q)) + (1 - \beta)(1/\alpha_h(q^*))$. Because of equation (1) we get that $(u^i_{h\dot{q}} / u^i_{x_h}) > (1/\alpha_h(\dot{q}))$. Therefore, by continuous differentiability $(u^{it}_{h\dot{q}}/u^{it}_{x_h}) > (1/\alpha_{h(\dot{q})})$ for t sufficiently large. Thus consumer i would pay at least $(w^t_h/\alpha_h(\dot{q}))$ per unit of $h\dot{q}$ at sufficiently large t. Therefore, the innovation $\dot{I}$ is profitable at $\bar{t}$. Because of (NLO) the innovation remains available at any larger t. Its profit $(\dot{\pi}^t_h/w^t_h)$ does not tend to zero. This contradicts Step 4.

Step 7. The limit allocation (x, y, q) is a satiation point either for no consumer, or for all consumers. This follows from the assumption that either $e^i_h > 0$ for all i and h (in this case all commodities that become free for some consumer become free for all) or that there exists a h with $\alpha_{\mathcal{P}h}(q) < \infty$ for all $q > 0$ (in which case h remains scarce for all consumers).

Step 8. To prove the theorem it remains to observe that any WE in the FDE, at which no consumer reaches a point of satiation is Pareto-efficient in the FDE. Note that for this claim the monotonicity of preferences is essential.

6 References

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