Projektbereich A

Discussion Paper No. A-509

Dynamic Gains from Trade

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January 1996

I would like to thank Tilman Börgers, Martin Hellwig, Ulrich Kamecke, Urs Schweizer and Reinhard Selten for their help. Financial support by the Deutsche Forschungsgemeinschaft, SFB 303 at Bonn University, is gratefully acknowledged.

Abstract

This article examines the validity of a gains from trade proposition in a world in which the direction of technological change is determined endogenously. We first give an extreme example in which a part of the world that would smoothly develop under autarchy forever remains underdeveloped under free trade. An assumption is then introduced, which excludes the example and guarantees that development under free trade dominates development under autarchy in the long-run. The assumption is closely related to the assumption of irreducible markets in McKenzie [1959]. It requires the existence of a 'closed scarcity chain' connecting tastes and endowments of all types of consumers. The result complements the classical gains from trade proposition that assumes the state of technological knowledge to be given.
Keywords: Endogenous growth, gains from trade, underdevelopment.
JEL Classification Numbers: D50, F10, O10, O31.

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1 Introduction

The classical gains of trade proposition, in its simplest form, states that, *given* the state of technological knowledge, there are winners from free trade in each country in which there are losers (Samuelson [1939 and 1962], Kemp [1962], Grandmont and McFadden [1972]). This article is about a similar gains of trade proposition in a world in which the state of technological knowledge is determined endogenously. The simple model of develpment that we use reflects many features of Schumpeter's early theory of economic development (Schumpeter [1911]). Change is modeled as a sequence of temporary economies that are perfectly competitive in all traditional markets and monopolistically competitive in new markets. Which new markets are opened or which new technologies are introduced depends on the set of perceived potential innovations and on the expected profitability of these potential innovations. All resources – including the resources that are necessary for technological change – flow to those activities that generate sufficiently high profits. Our model is an endogenous growth model. However, in contrast to the literature (see (Romer [1990], Aghion and Howitt [1992], Grossman and Helpman [1991]), it is centers around the direction of change rather than the intensity of change.

We first provide an extreme example, in which part of the world that would be developed under autarchy, *forever* remains underdeveloped under free trade. All individuals of one country are better off under autarchy than under free trade. This contradicts the classical gains from trade proposition even in its weakest form.

Two crucial features of the examples are that not all consumers own all primary inputs and that consumers may be satiated in some commodities once they consume sufficiently high amounts of some other commodities. Although the counter-example should mainly illustrate in the simplest possible way what can go wrong with a gains from trade proposition, these features are rather natural. They are certainly more realistic than the corresponding standard assumptions about preferences and the distribution of endowments (*strict* monotonicity of preferences and the classical survival assumption). Thus, it seems important to identify weaker assumptions than the classical conditions about tastes and the distribution of endowments that allow to exclude persistent inequalities as those in the example. This is the aim of the present chapter.

In the second part of this article we introduce such an assumption on preferences and the distribution of endowments. The assumption is sufficient to exclude the persistence of underdevelopment in the present framework of development. In an international trade framework the assumption allows to reestablish a gains from trade proposition. The assumption is that there should exist a 'closed scarcity chain' that connects all types of consumers world-wide. Each member in a scarcity chain owns strictly positive endowments in a primary input that is 'scarce' in the production of a consumption commodity in which the next member of the chain not strongly satiable (that is, he may be satiable asymptotically, but not for finite quantities). The assumption weakens the 'survival assumption' of the classical existence theorem for Walras Equilibria and is closely related to the assumption of irreducible markets in McKenzie [1959]. The survival condition, which assumes that every consumer owns strictly positive amounts of all primary inputs, has been criticized in McKenzie [1959] already, on the grounds of lacking realism. It is particularly unrealistic in an international trade framework in which one wants to investigate into potential reasons of persistent underdevelopment.

Note that the classical gains of trade literature does *not* intend to show that free trade is better than autarchy for *everybody*. As Paul Samuelson noted

> "Practical men and economic theorists have always known that trade may help some people and hurt others. Our problem is to show that trade lovers are theoretically able to compensate trade haters for the harm done them, thereby making every body better off."[1]

We only show that the asymptotic state of knowledge under free trade dominates that of autarchy in each country. In no country there are only losers from free trade (If the citizens of a country all own strictly positive amounts of the same inputs, then nobody in that country loses from free trade). Whether or not domestic transfers given the asymptotic state of knowledge can make everybody better of (under free trade than under autarchy) is an issue that can then be addressed along the lines of the classical gains of trade literature.

The remaining of this article is organized as follows. In Section 2 the basic framework of development is introduced. Section 3 contains the example of underdevelopment (caused by free trade) is revisited. We give an extreme version of the example for which standard Walras equilibrium (given the state of knowledge) does not exist. We therefore define 'generalized Walras equilibrium' (given the state of knowledge) in terms of pair-wise terms of trade. In the extreme version underdevelopment under free trade persists almost independently of properties of the evolution of innovation possibilities (described by the

[1] Samuelson [1962], p 823.

innovation function). In Section 4 closed scarcity chains are defined and their existence is postulated. It is shown that this leads back to existence of temporary equilibrium and to a gains of trade proposition.

2 The Framework of Development

Free trade and autarchy. We will use the term 'free trade' for a fully integrated world economy with free trade of outputs and full factor mobility. Correspondingly, when introducing the general setup we do not need to differentiate between the cases of autarchic development and of free trade development. In both cases we define development for fully integrated closed economies (which under autarchy is any isolated national economies and under free trade is the world economy). This simplifies the presentation without essentially influencing the analysis.

The economy consists of an infinite sequence of periods with a new generation of individuals in every period. All generations are identical.

Consumers and commodities. In each generation there are finitely many types of consumers $i \in I$ and a continuum A_i of identical consumers of each type i.

There are finitely many perishable commodities, of which some are primary inputs, some are final consumption commodities and some are intermediate goods. We allow for intermediate commodities only to render more plausible the key assumption of this paper (Assumption (Chain)). Otherwise, the introduction of intermediate commodities does not much influence the analysis. A consumer $a \in A = \cup_i A_i$ has a continuous, differentiable, weakly monotone and convex utility function u^a defined over primary inputs and final consumption commodities and owns a vector of endowments in primary inputs ω^a. Total (per capita) resources are bounded, i.e. $0 < \int_{a \in A} \omega_h^a d\mu(a) = \omega_h < \infty$ for all primary inputs h.

State of knowledge and competitive equilibria. The **state of knowledge** in a given period t is defined as the (finite) set of technologies $\mathcal{Y}^t = \{Y_1^t, \cdots, Y_{\#\mathcal{Y}^t}^t\}$, $\#\mathcal{Y}^t < \infty$, that individual firms can use at t. A technology produces a single output with possibly multiple inputs. The **aggregate technology** corresponding to a given individual technology in $\mathcal{Y}^t$, is the smallest cone containing that individual technology. If k is a final consumption commodity we denote by $\hat{Y}_k$ the **compound aggregate technology** producing output k with primary inputs only. **Temporary equilibria given the state of knowledge** in a given period are defined as (temporary) competitive equilibria of the economy that operates with the aggregate technologies. The idea is that in a sufficiently large economy with free entry to all traditional technologies aggregates behave as if the economy were perfectly competitive given the appropriate cone technologies. In order to guarantee that individual firms are 'small' compared to the rest of the economy, it is assumed that the asymptotic cones of all individual technologies do not contain strictly positive elements.[2]

Innovation possibilites. Given the state of knowledge $\mathcal{Y}^t$ at t there is a set of perceived potential innovations. More generally, there is an exogenous **innovation-function** $\mathcal{I}$ that defines a set of potential innovations $\mathcal{I}(\mathcal{Y})$ for each state of knowledge $\mathcal{Y}$. Potential innovations have the same general properties as traditional technologies (i.e. they are 'small'). Because there are only finitely many commodities in our simple framework all innovatons are process-innovations once all commodities have been introduced.

Choosen innovations. There are scarce resources that can be used to improve upon existing technologies. For simplicity, it is assumed that in each period the resources in one country just suffice to implement one of the potential innovation of that period. Thus, under autarchy one of the potential innovations in $\mathcal{I}(\mathcal{Y})$ is chosen in each country. Under free trade several innovations can be chosen in principle.[3] **Equilibrium development** is defined as before.

Innovators can use their innovations as monopolists (or as few ologopolists) for one period. Since technologies are small compared to the aggregate technology of the competitive sector, one can neglect the question of how the profits of the innovators are distributed. Furthermore, one can assume that innovators takes as given all prices of the competitive sector and chooses the innovation which guarantees the highest profits. After the period of innovation, there is free entry to the new technology, as well, and profits are reduced to zero. Thus innovators choose the innovation that generates highest short-run profits given current prices.

Equilibrium developmet. The new aggregate technology in the industry of in which an innovation occurs is t he smallest cone containing the improved technology used by the innovator. Depending on the new state of knowledge there is a new set of potential innovations (defined by the innovation function) of which the most profitable is carried through,

[2]For a non-cooperative foundation of this equilibrium concept (given the state of knowledge) in a general equilibrium Cournot framework see Novshek and Sonnenschein [1980] and in a general equilibrium Bertrand framework see Funk [1996a].

[3]If there are several innovators that have access to the full set of potential innovations they will in general choose the same innovation. Since all individual technologies are small the presence of a second innovator will not affect the decision of a first innovator.

which in turn defines the state of knowledge of the next period. Thus, given the innovation function and given an initial state of knowledge, we get a sequence of states of knowledges, which we call **equilibrium development of knowledge**, and a correspoding sequence of temporary competitive allocations and prices, which we call **equilibrium development**.

In Funk [1996b] we explain how features of the innovation-function can cause inefficient dead locks of equilibrium development. There we assume that preferences and the distribution of endowments are well behaved. Here we want to concentrate on problems that arise due to properties of preferences and of the distribution of endowments that arise when we abolish the assumption of strictly monotone preferences and of strictly positive individual endowments, rather than on problems arising due to properties of the innovation-function. We will assume that given any feasible state of knowledge there are improving innovations in all industries (i.e. for all k). Inefficient dead ends can then only be caused by features of preferences and endowments. We make precise what is meant by 'improving innovations' in all industries given any feasible state of knowledge.

Innovation $I \in \mathcal{I}_k(\mathcal{Y})$ **improves upon** existing knowledge in industry k, if given any vector of primary inputs which produces a positive amount of k with the existing aggregate compound technology k of $\mathcal{Y}$ (i.e. with $\hat{Y}_k$) , the output produced by the new compound technology k of $\mathcal{Y} \cup I$ is larger than that produced with the existing compound technology.

Note that if innovation I improves upon $\hat{Y}_k$, then for any given vector of inputs, the marginal product in the compound aggregate technology of k rises for at least one input. We assume that for any k there is at least one primary input h, such that for all given primary vectors of inputs in k, all states of knowledge, and all improving innovations in k, input h belongs to the inputs with rising marginal productivity and, furthermore, that the corresponding marginal productivity is bounded away from zero. We call such a *primary input h essential for final output k.*

From these assumptions it follows that for any initial state of knowledge and any vector of primary inputs that produce some output in k given the initial state of knowledge, the marginal product of an essential input for k tends to infinity for some feasible development of knowledge. Note that we assume infinite productivity of essential factors in the Fully Developed Economy only for the sake of the exposition. We may allow for potential development in which some or all final consumption commodities remain scarce.

The main question of this article can then be framed as: *Under which assumptions on endowments and preferences will those commodities that in principle could become free commodities for a certain group of people (the owners of essential inputs for these com-*

modities, for instance the citizens of one country) will in fact become free commodities for that group of people under equilibrium development? Or, adapted to the international trade framework: *Under which assumptions on endowments and preferences does development under free trade asymptotically dominate development under autarchy for all trading countries, in the sense that the set of commodities that become asymptotically free for the citizens of any given country contains the set of commodities that become free under autarchy in this country?*

3 Free Trade and Persistent Underdevelopment.

In this section we briefly present an over-stylized example in which free trade prevents the development of one of the trading countries. In its present form the example merely serves to illustrate in the simplest possible way why a gains of trade proposition may fail in the present framework. We give two versions of the example. The first version is more convetional in that equilibria given the state of knowledge exist in every period. In the second version the terms of trades between low-quality and high-quality commodities are so extreme that Walras equilibria in the usual sense do not exist. We have to define Generalized Walras equilibria which do exist in the second version example. We include this version because it pushes the class of counter-examples to its logical limit. The example no longer depends on any additional specifications of the innovation function.

3.1 Version 1.

There are two continents A and B. Within continents, across continents and across generations individuals have an identical continuous and quasi-concave utility function $u(x, y_1, y_2)$, where x is the number of hours worked per day, y_1 is the amount of low-quality output consumed and y_2 is the amount of high-quality output consumed. The two commodities, y_1 and y_2, may be two varieties of food differing in quality, but identical in nutritional value. Consumers do not care for quality or leisure if $y_1 + y_2$ is less than some minimal level $\underline{y}$ (the hunger-line) and go for calories only. If $y_1 + y_2 > \underline{y}$ they are interested in leisure and high quality. They are satiated in the low quality if their total consumption of y_2 is sufficiently high given y_1, say $y_2 > \bar{y}_2(y_1)$ (the satiation-line). Thus, indifference curves for (y_1, y_2) iven x are straight lines with slope -1 for $y_1 + y_2 < \underline{y}$, and are horizontal for $y_2 > \bar{y}_2(y_1)$. An example for a utility function displaying these properties is given in the appendix to the present chapter.

The low-quality commodity is produced with unskilled labor, the high-quality commodity is produced with skilled labor. Individuals of the two continents differ with respect to their endowments. The workers of continent A own 'skilled' labor only ($\omega_1^A = 0$, $\omega_2^A > 0$) and the workers of continent B own 'unskilled' labor only ($\omega_1^B > 0$, $\omega_2^B = 0$).

Since all individual technologies use single inputs the aggregate production functions are linear. The productivity of the low-quality technology at t is denoted α_1^t and that of the high-quality technology α_2^t. Of course these productivities are also the marginal products of the corresponding factors. In accordance with the general framework there is a potential innovation in each period and in each industry, increasing the productivity of the aggregate technology. The increments are bounded away from zero.

Development under autarchy. Because there is only one type of potential innovation in each isolated country development under autarchy is trivial. Under autarchy the low-quality industry will be continuously developed in B and the high-quality technology will be continuously in A. All workers will eventually cross the hunger-line.

Development under free trade. We will show that under free trade the B-workers may remain below their hunger-line, while the A-workers become ever richer.

Suppose that at the initial state of knowledge ω_2^A is sufficiently large (given the utility function and given the productivity of the productivity α_2^0 of the initial aggregate high-quality technology of A) to ensure that $x_2^A \alpha_2^0 > \overline{y}$. Then, the A-workers do not buy y_1 if its price in terms of y_2 is strictly positive. Therefore, at temporary equilibrium the B-workers cannot purchase y_2 (this follows from the budget condition of the A-workers and the fact that $\alpha_2^0 = (w_2^0/p_2^0)$). If we make sure that the B-workers are really hungry by setting $\omega_1^B \alpha_1^0 < \underline{y}$ then they will certainly not demand y_2 if $p_2^0 > p_1^0$. Thus any price vector with $(w_1^0/p_1^0) = \alpha_1^0, (w_2^0/p_2^0) = \alpha_2^0, p_2^0 > p_1^0$ is an equilibrium (if ω_2^A is sufficiently large and ω_1^B sufficiently small). If the aggregate productivity, α_2^t, of the high-quality industry of A grows in the course of development and the productivity of the low-quality industry of B remains constant ($\alpha_1^t = \alpha_1^0 \forall t$), then any price vector with $(w_1^t/p_1^0) = \alpha_1^0, (w_2^t/p_2^0) = \alpha_2^t$ is a temporary equilibrium of the integrated world economy at t.

Now assume that the innovation function be such that $\lim(\pi_2^t/p_2^0) > (\pi_1^0/p_1^0)$, where π_h^t are the maximal profits an innovator in industry h can realize in period t (there is nothing pathological about this assumption as can be easily verified). Then the innovator in each period will in fact choose the second technology, i.e. $\alpha_2^t \to \infty$, $\alpha_1^t \not\to \infty$.

3.2 Version 2. Generalized Walras Equilibria

In the first version of the example the satiation of the A-citizens in low-quality leads to the existence of a continuum of temporary equilibrium prices. If we introduce an additional factor of production, (temporary) competitive equilibria in the usual sense may fail to exist quite generally. For this case a broader than usual definition of competitive equilibrium is called for. The persistence of underdevelopment can then be derived quite independently of assumptions of the innovation-function. In the case of strict satiation in low quality by the wealthy, the terms of trade are so unfavorable for the unskilled, that even very high profits in terms of low-quality commodities cannot attract innovators.

We therefore introduce land as an additional factor of production in the two countries to get the underdevelopment trap independently on any assumption on the innovation-function (except for the fact that there should be at least one potential process-improving innovations in each existing industry).

Suppose land is needed in the production of both qualities and that some of the B-land is owned by A-citizens (or non-labor B-citizens). Furthermore suppose that owners of B-land living in A do not derive direct utility from their B-land. In the example with strict satiation in low-quality of the wealthy, the A-citizens remain satiated in y_1. They buy the low-quality only if the price of the low-quality commodity in terms of the high-quality commodity, denoted $T^{\bar{t}}_{y_2 y_1}$, is zero, in which case they are indifferent. This also means that they consume all y_2 that is produced. B-workers produce the low-quality, only. Since unskilled labor in A and in B receives the same wage (and firms of the competitive sectors make zero profits) total income of B-citizens cannot buy the total production of y_1. Thus, at market-clearing equilibrium, $T^{\bar{t}}_{y_1 y_2}$ has to be zero, so that the A-workers will buy the remaining low-quality production. Therefore, the rent $T^{\bar{t}}_{l_A y_2}$ of A-land in terms of y_2 (the only output A-landlords are interested in) is zero. Thus, the terms of trade between any pair of commodities are now well defined (possibly infinite or zero for some pairs). We call the described situation a (temporary) 'Generalized Walras Equilibrium'.

More generally, a **Generalized Walras Equilibrium** is a matrix of terms of trades (one for each pair of commodities) and, for each economic agent, a matrix of pair-wise exchanges, such that the aggregate pair-wise demands and supplies are equalized for all pairs of commodities (a complete formal definition is given in the appendix). In our example with strong satiation (zero marginal utility at finite quantities) Walras Equilibria (given the state of knowledge) in the narrow sense do not exist: If the absolute price of y_1 were positive the A-citizens would not demand any y_1. If this price were zero the B-citizens would at least demand their satiation level quantities. In both cases the low-quality market does not clear. While a (temporary) Walras Equilibrium in the narrow sense is a (temporary) Generalized Walras Equilibrium the inverse is not always true, as the example shows. The distinction between the two concepts is of no importance in most of equilibrium theory since one rarely allows for the possibility that some consumers are satiable in certain commodities. If one does allow for satiation, the notion of a generalized WE is the natural extension of WE. In fact, it is the more elementary concept (if pair-wise barter is more elementary than exchange against a numéraire).

In the present example the satiation of the wealthy in low-quality commodities plays a central role. Together with the assumptions on resources this leads to extreme terms of trades. These extreme terms of trade, in turn, inevitably lead to a bias in development. In fact, innovators will choose the high-quality technology whatever are the innovation possibilities in the low-quality technology. The case in which Walras equilibria in the narrow sense do not exist allows best to illustrate a potential reason for unequal development.

As has been noted the example is meant to show a potential source of underdevelopment in a very stylized way. The aim of the present article is to give conditions that exclude this possibility. In Funk [1996b] we show how the assumptions causing underdevelopment can be weakened without altering the essential idea of the example.

4 Scarcity Chains and the Gains from Trade Proposition

The literature on Classical gains of trade not only shows that – given the state of knowledge – free trade is better than autarchy in the sense that *global* lump-sum transfers plus free trade can make every consumer better of as compared with autarchy. It also gives conditions that make sure that "Given an allocation achieved under autarchy, a system of world trade prices and *domestic* lump-sum transfers can be found for which a competitive equilibrium allocation will exist and will be at least as satisfactory as autarchy for every consumer."[4]

In the example of the previous section development under free trade does not lead (in the long-run) to states of knowledge that dominate the states of knowledge of development under autarchy. Under autarchy the high-quality commodity would become a free commodity also for the B-citizens. The winners of free trade could not compensate the losers

[4] Grandmont and McFadden [1972], p. 110.

in order to make everybody better off, if *local* lump-sum transfers *given* the asymptotic states of knowledge are feasible.

In this section a condition (Assumption Chain) is introduced, that excludes the persistent inequality of the previous section by making sure that all technologies that are developed under autarchy will also be developed under free trade (if this is desired). In this sense the states of knowledge under free trade development will asymptotically dominate the union of the states of knowledge under autarchy. Taking this for granted, one can then proceed along the lines of the 'Classical gains from trade' literature to show that free trade (at the asymptotic state of knowledge achieved through free trade development) together with domestic lump-sum transfers is Pareto better than autarchy (at the asymptotic state of knowledge achieved through autarchic development).

Assumption Chain is a combined assumption on preferences and endowments, which weakens the assumption of strongly monotone preferences as well as the 'classical' survival assumption (all consumers own positive endowments of all primary inputs). Obviously, the assumption of positive endowments is too restrictive in a framework of international trade, especially if one wants to investigate into the phenomenon of persistent inequality. For proving existence of Walras Equilibria it is much stronger than needed , as was shown in McKenzie [1959 and 1961].

As does the assumption of Irreducible Markets of McKenzie [1959] the assumption also guarantees existence of Walras Equilibria given the state of knowledge (and also that all Generalized Walras Equilibria are Walras Equilibria). Although existence of Walras Equilibria given all states of knowledge is not necessary for defining equilibrium development (existence of Generalized Walras Equilibria is sufficient), the fact that Walras Equilibria may not exist is closely related to the persistence of inequality. Assumption Chain not only guarantees existence of Walras Equilibria given the states of knowledge, but also makes sure that the establishment of free trade will not suppress the development of technologies that would be developed under autarchy. The condition will *not* guarantee that all commodities that become free for *some* type of consumers will become free for *all* types of consumers. It only makes sure that those commodities that *can* become free commodities for a consumer do so in equilibrium development under free trade.

Definition. Consumer a is **not strongly satiable** in consumption commodity k if for all primary inputs h with $\omega_h^a > 0$, the marginal rates of substitution $\frac{u_k^a}{|u_h^a|} \not\to 0$ if $y_k^a \not\to \infty$ and $\frac{u_k^a}{|u_h^a|} \to \infty$ if $y_k^a \not\to \infty$ and $y_h^a \to 0$. The sequence $(a_1, \cdots, a_N)$ of consumers is called **a scarcity chain** if for all $n \in \{1, \cdots N\}$ there exists a primary input h_n and a consumption commodity k_{n+1}, such that (1) input h_n is essential for commodity k_{n+1}, (2) input h_n is owned by consumer a_n (i.e. $\omega_{h_n}^{a_n} > 0$) and (3) consumer a_{n+1} is not strongly satiable in commodity k_{n+1}. Let $(h_1, \cdots, h_{N-1})$ and $(k_2, \cdots, k_N)$ be the corresponding sequences of inputs and outputs. A scarcity chain $(a_1, \cdots, a_N)$ is called **closed** if $a_1 = a_N$.

ASSUMPTION (CHAIN): *There exists a closed scarcity chain that includes each type of consumer at least once.*

Note that not all primary inputs or consumption commodities have to be included in the chain.

In order to keep the exposition as simple as possible it is assumed that all technologies produce a single output with a single input. The following propositions and the theorem are valid in the general setting, too. We denote the compound aggregate technology that produces final consumption good k with primary input h by $\hat{Y}_{kh}^t$ and the slope of this technology by α_t^{kh}. In this simplified framework h is essential for commodity k if h is the only primary input that can be used in compound technologies producing commodity k.

4.1 Existence of Temporary Equilibria

In this subsection it is shown that temporary Walras equilibria given any state of knowledge exist if Assumption Chain is satisfied. Since the state of knowledge will be kept constant in this subsection, the index t is suppressed. Because of the simple structure of technologies we need to consider compound (aggregate) technologies only, and primary inputs as well as final outputs (consumption commodities), only. For each of the remaining technologies there is a single firm (the number of firms and the ownership structure is irrelevant since the aggregate technologies are cones). Let the price system p be an element of the unit simplex in $\mathbb{R}^H$, where H is the number of primary inputs and final outputs. Let $y^i(p) \in \mathbb{R}^H$ be the vector of optimal supplies and demands given p of a consumer of type $i \in I$ (where supplies are negative and demands are positive) and let $z(p) \in \mathbb{R}^H$ be the sum of aggregate firms' price-taking optimal (per capita) demands and supplies given p (where demands are negative and supplies are positive). Then, $(p, (y^i(p))_{i \in I}, z(p))$ is a Walras Equilibrium if the excess supply, $z(p) - \sum_i \eta_i y^i(p)$, is zero in all components (where η_i is the relative number of consumers of type i).

Proposition 1 *Assume (Chain). Then the set of Walras equilibria (given any state of knowledge) is non-empty.*

Proof: See Appendix 6. ■

Of course, existence of Walras equilibrim given the state of knowledge would also follow if instead of Assumption Chain we would make McKenzie's assumption of Irreducible markets *for all* feasible states of knowledge.

4.2 Equilibrium Development: Free Trade Dominates Autarchy

Given a sequence of allocations, we say that consumer a is **asymptotically satiated** in final consumption commodity k if $[\frac{u_k^a}{|u_h^a|} \to 0$ for all primary inputs h with $\omega_h^a > 0]$ or if $[y_k^a \to \infty]$ (Remember that consumers may be strongly satiable in some outputs). For primary inputs h and h', with $\omega_h^a > 0, \omega_{h'}^a > 0$, it is assumed that $\frac{u_{h'}^a}{u_h^a} \not\to \infty$ if $y_h^a \not\to 0$.

Proposition 2 *Assume (Chain). Then, given any equilibrium development, each type of consumer is asymptotically satiated in all consumption commodities for which he owns essential primary inputs.*

Proof: See Appendix. ∎

Again, it should be noted that the proposition can be generalized to allow for potential development under which some or all commodities remain scarce.

Proposition 2 does *not* make sure that any commodity that becomes free for some consumers becomes free for all consumers. The fact that for all n the productivity of input h_n tends to infinity ($\alpha_{k_{n+1}h_n}^t \to \infty$), guarantees high factor incomes only in terms of k_{n+1}. The factor incomes generated by input h_n in terms of other inputs may still tend to zero ($\frac{p_{h_{n-1}}^t}{p_{h_n}^t} \to 0$ is not excluded). However, as is easily seen, the kind of persistent inequality of the previous sections *is* excluded by the lemma. In these examples the unskilled remained poor and hungry even in terms of the low-quality consumption good. This is excluded by the lemma since the unskilled own an essential input for the low-quality output.

Assume that there are several countries. To arrive at a simple gains of trade proposition, which does not depend on domestic or global transfers assume that the individuals within any given country all hold strictly positive endowments of the same resources. Then, the following theorem is a direct corollary of the more general Proposition 2.

Theorem 1 *Assume (Chain). Then, the limit of equilibrium development under free trade is not Pareto-dominated by the limit of equilibrium development under autarchy. If the set of consumption commodities is not the same in all countries, and if preferences are strictly convex, then the limit of equilibrium development under free trade Pareto-dominates the limit of equilibrium development under autarchy.*

Proof: Follows as a corollary to Proposition 2. Free trade cannot domonate autarchy, since under free trade the individuals of a given country are asymptotically satiated in all commodities that are producable under autarchy in that country (if the citizens of a given country can produce a commoditiy they must own an essential input for this commodity. If they own an essential input for this commodity they are asymptotically satiated in the commodity under free trade, because of Proposition 2). ■

If in the example of Section 3 the marginal rate of substitution between the low-quality and the high-quality commodity would tend to zero only if the consumed quantity of the low quality would tend to infinity, then the low-quality technology on continent B would be developed until the poor would reach the sufficient skill-level to work in the high-quality technology. Asymptotically, all commodities would become free for all consumers. If the high quality commodities of the two continents are not perfect substitutes, then all consumers will be better off in the long-run under free trade than under autarchy.

The case that all consumers in a country hold strictly positive endowments of the same resources is not very realistic. If there are more than one type of individuals in a country some transfers between winners and losers (given a state of knowledge after development under free trade) may be necessary to make every individual prefer free trade to autarchy. What has been shown here is that free trade development asymptotically leads to states of knowledge that dominate the states of knowledge which are reached under autarchy. The question of whether or not, given these states of knowledge, domestic transfers are sufficient to make everybody better off under free trade, can then be addressed in terms of the static gains of trade literature.

5 Comments and Extensions

We have first given a stylized example of development in which a country that would smoothly develop under autarchy remains underdeveloped if a free trade regime is established.

We have then introduced an assumption – the existence of a closed scarcity chain – that excluded this possibility and that made sure that free trade development dominates development under autarchy. The assumption is much weaker than the assumption of survival that is used in the classical existence theorems for Walras equilibria. It requires that every type of consumer owns some resources that are essential in the technological chain producing a final consumption good in which the next consumer in the chain is not strongly satiable.

Consider a world in which every type of consumer (or household) has some working capacity. Furthermore, assume that no final consumption commodity can be produced without labor as an essential *primary* factor of production (not necessarily as a *direct* input).

If there were only one type of labor in that world, Assumption Chain would be satisfied. In fact Assumption Chain would essentially hold if – in the long-run – each type of consumer had access to and could afford education and if by educating himself each type of consumer could in principle learn how to perform any given task. In this case all types of consumers would enjoy the fruits of development in the long term. The stylized counter-examples of section 3 would at best be relevant in the short or medium term.

If, in contrast, the level of skill and education is determined by some *rigid* sociological or other non-economic factors, then Assumption Chain and the conclusions may fail even in the long-run. The same is true if education is an endogenous variable that depends on past and present incomes and possibly on the education level of parents. This requires an imperfection of credit market which can in fact be observed in many less developed countries. Thus, although Assumption Chain considerably weakens the classical survival hypothesis it is not an innocuous assumption.

We have *not* shown in this article, that every citizens of every country is better off under free trade than under autarchy. As is known from the 'Classical gains from trade' literature there are several ways to make precise the intuition that free trade is better than autarchy if there may be losers and winners from trade in some countries. As we have explained it is not our objective to solve this question of the literature on classical gains from trade. Grandmont and McFadden [1972] have shown under certain conditions that given the state of knowledge and given an autarchic allocation one can always find domestic lump-sum transfers such that there is a free trade competitive equilibrium after transfers that Pareto dominates the autarchic allocation. The assumption on resource ownership and preferences (with respect to strict monotonicity) they need are stronger than Assumption Chain given the state of knowledge. In fact, Cordella, Minelli and Polemarchakis [1993] have shown that the Grandmont and McFadden's proposition may fail in economies in which Assumption (Chain) is satisfied. Thus, in order to show a more complete version of a gains of trade proposition one may have to restore to assumptions that go beyond Assumption (Chain).

6 Appendix

An example for the utility function in section 3.

$u(x,e,y_1,y_2) =$

$$\begin{cases} y_1 + y_2 & \text{for } y_1 + y_2 \leq \underline{y} \\ \left(\underline{y} + \frac{1}{R(y_1,y_2)}\right)^{\alpha} (\bar{x} - x)^{\beta} e^{\rho} & \text{for } y_1 + y_2 \geq \underline{y} \text{ and } y_2 \leq \bar{y}_2(y_1) \\ \left(\underline{y} + \frac{1}{R((\bar{y}_2)^{-1}(y_2),y_2)}\right)^{\alpha} (\bar{x} - x)^{\beta} e^{\rho} & \text{for } y_2 \geq \bar{y}_2(y_1) \text{ and } y_2 \leq y_2(0) \\ \left(\underline{y} + \frac{1}{R(0,\bar{y}_2(0))} + y_2 - \bar{y}_2(0)\right)^{\alpha} (\bar{x} - x)^{\beta} e^{\rho} & \text{for } y_2 \geq \bar{y}_2(0), \end{cases}$$

where $\alpha + \beta + \rho < 1$, $\bar{y}_2(y_1) = \underline{y} + (2 - \sqrt{2})a - \frac{1}{\sqrt{2}-1} y_1$ and $\frac{1}{R(y_1,y_2)} = \sqrt{2} \frac{y_1 + y_2 - \underline{y}}{(y_1+a)^2 + (y_2 - a - \underline{y})^2}$. The geometry of the indifference-curves in the $(y_1, y_2)-$plane is as follows: The region of hunger lies in the south-west of the line $y_1 + y_2 = \underline{y}$. Here quality does not matter. The indifference have slope -1. The region of satiation lies in the north-east of the satiation line $\bar{y}_2(y_1)$. Here consumers are satiated in y_1. The indifference curves are constant in y_1. These two regions are connected through segments of circles that are tangent to the line $y_1 + y_2 = \underline{y}$ at $y_1 = -a < 0$. The lowest point of the circle with radius $R(y_1, \bar{y}_2)$ is the point $(y_1, \bar{y}_2(y_1))$ on the satiation-line. Indifference-curves in the (y_1, y_2)-plane are continuous convex and differentiable. Considering (y_1, y_2) as a compound commodity, the utility function of consumption, leisure, and education is a standard Cobb-Douglas function.

Generalized Walras Equilibrium. The definition of Generalized Walras equilibrium in section 3 was rather informal. In particular we did not define the set of actions of an individual. We now give a precise definition.

Given a finite set H of commodities, consider any matrix $T = (t_{kh})_{k,h \in H}$ of terms of trades, with $t_{kh} = (1/t_{kh})$, where t_{kh} is the relative price of commoditiy k in terms of commodity h (i.e. the number of units of h that one gets for one unit of k).

Consumer problem. Given T consumer $a \in A$ defined by the utility function $u^a : I\!R^H \to I\!R$, endowments $\omega^a \in I\!R^H$, and consumption set $X^a \subset I\!R^H$, chooses a consumption plan $y^a = (y^a_h)_{h \in H}$ in X^a and a matrix of trades $z^a = (z^a_{kh})_{k,h \in H}$ such as to maximize $u^a(y^a)$ under the constraints

$$\begin{aligned} & 0 \leq y^a_h = \omega^a_h + \textstyle\sum_k z^a_{hk} \text{ for all } h \text{ and} \\ & z^a_{hk} = -t_{kh} z^a_{kh} \text{ for all } k, h. \end{aligned} \tag{1}$$

The first line of 1 describes the net trades of commodity h aggregated over all bilateral markets involving h, the second line describes the trading rule given T. Consumer a with trading plan z^a, exchanges z^a_{hk} units of commodity h aggainst z^a_{kh} of commodity k. Or he 'buys' z^a_{hk} units of h in terms of k at price of $t_k h$ per unit of h.

Producer problem. We remain in the setting with single-product firms. Given T, firm $j \in J$ defined by output h_j and technology $Y_j \subset I\!R^H$, chooses a production plan $y^j = (y^j_h)_{h \in H}$ in Y_j and a matrix of net trades $z^j = (z^j_{kh})_{k,h \in H}$ such as to maximize profits in terms of its output, $\sum_h t_{hh_j} y^j_h$ under the constraints

$$\begin{aligned} y_h &= \textstyle\sum_k z^j_{hk} \text{ for all } h \text{ and} \\ z^j_{hk} &= -t_{kh} z^j_{kh} \text{ for all } k, h. \end{aligned} \tag{2}$$

Definition. A matrix of net trades for all consumers and firms together with a matrix of terms of trade, $((z^a)_{a \in A}, (z^j)_{j \in J}, T)$, is a *Generalized Walras Equilibrium* if for all $a \in A$ z^a solves a's consumer problem given T, for all $j \in J$ z^i solves j's producer problem given T and if

$$\sum_{a \in A} z^a + \sum_{j \in J} z^j = 0. \tag{3}$$

Walras Equilibria and Generalized Walras Equilibria. If consumers are not strongly satiable in any commodity (marginal utility of a commoditiy not zero unless the amount consumed of this commodity is infinite), then at GWE we must have $t_{kr} t_{rh} = t_{kh}$, for any $r, h \in H$ and any k of which some consumers owns positve endowments or which is produced at the GWE. This follows from a simply no-arbitrage argument.

Consider the terms of trade aggainst k at a GWE, $(t_{kr})_r$ and assume that the no-arbitrage condition holds. If t_{kr} is neither 0 for two different r's nor ∞, then we can derive the full matrix T from the column $(t_{kr})_r$ (i.e. $t_{hs} = t_{hk} t_{ks} = (1/t_{kh}) t_{ks}$). In this case the GWE has a corresponding Walras equilibrium with prices $(p_r)_{r \in H} = \lambda (t_{kr})_{r \in H}$, for any $\lambda > 0$.

If, on the other hand, t_{kr} is either 0 for two different r's or ∞, then we cannot reduce the matrix T to a H-vector without losing information, even if the no-arbitrage condition would hold. In the second version of the example in section 3 the matrix T was of the following form (with 1= A-land, 2= B-land, 3= skilled labor, 4= unskilled labor, 5=high-quality commodity, 6= low-quality commodity).

$$\begin{pmatrix} 1 & \infty & t_{13} & \infty & t_{15} & \infty \\ 0 & 1 & 0 & t_{24} & 0 & t_{26} \\ (1/t_{13}) & \infty & 1 & \infty & t_{35} & \infty \\ 0 & (1/t_{24}) & 0 & 1 & 0 & t_{46} \\ (1/t_{15}) & \infty & (1/t_{35}) & \infty & 1 & \infty \\ 0 & (1/t_{26}) & 0 & (1/t_{46}) & 0 & 1 \end{pmatrix}.$$

Knowing the terms of trades between all world-A commodities (1,3, and 5) and between world-A and world-B commodities (these are always 0 or ∞) does not tell us anything about the terms of trades between the world-B commodities (2, 4, and 6).

The GWE of this example has no corresponding WE. In fact, as we saw, no WE exists in the example. On the other hand, it

Proof of the Proposition 1. We will first perturb the vector of endowments for each consumer such that the 'survival assumption' of Debreu [1959] is satisfied (assumption (c) on page 84 of Debreu [1959]). For all primary inputs and consumption commodities h define $e_h^{a\epsilon} = e_h^a + \epsilon$, with $\epsilon > 0$. Consider the corresponding perturbed economy. Since no consumer can be (globally) satiated all the assumptions of the existence theorem in Debreu [1959] are satisfied. Therefore, the set of equilibria in the perturbed economy given $\epsilon > 0$ is non-empty. Consider a sequence $(\epsilon^q)_{q \in I\!N}$ with $\epsilon^q \to 0$ for $q \to \infty$ and a corresponding sequence of Walras Equilibria (p^q, y^q, z^q), with $y^q = (y^{iq})_{i \in I}$. Since (per capita) resources are bounded the sequence is bounded. Consider any convergent subsequence (same notation). Let $(a_1, \cdots, a_{N-1}, a_1)$ be a closed scarcity chain including all types of consumers and let $(h_1, \cdots, h_{N-1})$ and $(k_2, k_3, \cdots, k_{N-1}, k_1)$ be the corresponding sequences of primary inputs and outputs (i.e. consumer a_n owns primary input h_n that can produce commodity k_{n+1} and a_n is not strongly satiable in consumption good k_n).

1. It is shown, that if there exists an $n \in \{1, \cdots, N-1\}$ with $p_{h_n}^q \not\to 0$, then for all $n' \in \{1, \cdots, N-1\}$ $p_{h_{n'}}^q \not\to 0$. Suppose that $p_{h_n}^q \not\to 0$. Then, $p_{k_n}^q \not\to 0$, since otherwise $y_{k_n}^{a_n} \to \infty$ (a_n owns h_n and is not strongly satiable in k_n), which is impossible since resources are bounded and $\alpha_{k_n h_{n-1}}$ is bounded. Furthermore $\frac{p_{h_{n-1}}^q}{p_{k_n}^q} = \alpha_{k_n h_{n-1}}$ is bounded away from zero since h_{n-1} is essential for k_n. Therefore, $p_{h_{n-1}}^q \not\to 0$. Thus, since the chain is closed the same is true for all n'.

2. It is shown, that there exists an n with $p_{h_n}^q \not\to 0$. Suppose not, i.e. suppose that for all n $p_{h_n}^q \to 0$. Then for all n $p_{k_{n+1}}^q \to 0$ (since $\frac{p_{h_n}^q}{p_{k_{n+1}}^q} = \alpha_{k_{n+1} h_n}$) is bounded away from zero. Therefore, and since all consumers are included in the chain, the incomes of all consumers

have to tend to zero with ϵ^q. (If the income of, say, consumer a_n would not tend to zero his demand for k_n would tend to infinity, since he is not strongly satiable in k_n. This is impossible since resources and $\alpha_{k_n h_{n-1}}$ are bounded). Thus, since each input is owned by at least one (type of) consumer the prices of all inputs have to tend to zero and, therefore, the prices of all outputs have to tend to zero as well. This is impossible, since the vector of prices p^q belongs to the unit simplex.

3. From 1. and 2. it follows that all consumers own an input which can generate factor incomes that are bounded away from zero. Therefore, the income of no consumer tends to zero when $\epsilon^q \to 0$. Thus, for all consumers, the budget correspondence is lower-hemi continuous in p at $p = \lim_{q\to\infty} p^q$. Therefore, $\lim y^q$ is a vector of optimal demands and supplies given $\lim p^q$ in the unperturbed economy. Hence $(\lim p^q, \lim y^q, \lim z^q)$ is a Walras Equilibrium in the unperturbed economy. ■

Proof of Proposition 2. Because of Lemma 1 Walras Equilibria exist in every period. Let $(p^t, y^t, z^t)_t$ be an equilibrium development.

(1) It is shown that if $\frac{p^t_{h_{n-1}}}{p^t_{h_n}} \to_t 0$, then $\alpha^t_{k_n h_{n-1}} \to \infty$. Suppose $\frac{p^t_{h_{n-1}}}{p^t_{h_n}} \to 0$. Then, since $\frac{p^t_{h_{n-1}}}{p^t_{k_n}} = \alpha^t_{k_n h_{n-1}} \not\to 0$, $\frac{p^t_{k_n}}{p^t_{h_n}} = \frac{p^t_{k_n}}{p^t_{h_{n-1}}}\frac{p^t_{h_{n-1}}}{p^t_{h_n}} \to 0$. Therefore, since a_n is not strongly satiable in k_n and since $\omega^{a_n}_{h_n} > 0$, $y^{a_n t}_{k_n} \to \infty$. Since h_{n-1} is essential for t k_n and since $\omega_{h_{n-1}} < \infty$, it follows that $\alpha^t_{k_n h_{n-1}} \to \infty$.

(2) It is shown that if $\frac{p^t_{h_{n-1}}}{p^t_{h_n}} \not\to 0$ and $\alpha^t_{k_n h_{n-1}} \not\to \infty$, then $\alpha^t_{k_{n+1} h_n} \not\to \infty$. Suppose that $\alpha^t_{k_n h_{n-1}} \not\to \infty$. Then, innovations in industry k_n are chosen only in finitely many periods, and $\frac{\pi^t_{k_n h_{n-1}}}{p^t_{h_{n-1}}} \not\to 0$, where π^t_{kh} is the maximal profit an innovator that chooses an innovation on the production path (h, k) can realize (This follows from the the assumption that there always are non-negligible potential innovations in industry k and from the fact that h_{n-1} is essential for k_n). Therefore, $\frac{\pi^t_{k_n h_{n-1}}}{p^t_{h_n}} = \frac{\pi^t_{k_n h_{n-1}}}{p^t_{h_{n-1}}}\frac{p^t_{h_{n-1}}}{p^t_{h_n}} \not\to 0$ and hence $\frac{\pi^t_{k_{n+1} h_n}}{p^t_{h_n}} \not\to 0$ ($\frac{\pi^t_{k_{n+1} h_n}}{p^t_{h_n}} \to 0$ is possible only if industry k_{n+1} is chosen in infinitely many periods. In this case industry k_n would rather be chosen, since $\frac{\pi^t_{k_n h_{n-1}}}{p^t_{h_n}}$ is larger than $\frac{\pi^t_{k_{n+1} h_n}}{p^t_{h_{n+1}}}$ for t sufficiently large. This is a contradiction). Therefore, $\alpha^t_{k_{n+1} h_n} \not\to \infty$.

(3) It is shown that if there exists an $\bar{n}$ with $\frac{p^t_{h_{\bar{n}-1}}}{p^t_{h_{\bar{n}}}} \not\to 0$ and $\alpha^t_{k_{\bar{n}} h_{\bar{n}-1}} \not\to \infty$, then the same is true for all n. Suppose that $\frac{p^t_{h_{\bar{n}-1}}}{p^t_{h_{\bar{n}}}} \not\to 0$ and $\alpha^t_{k_{\bar{n}} h_{\bar{n}-1}} \not\to \infty$. Then it follows from (2) that $\alpha^t_{k_{\bar{n}+1} h_{\bar{n}}} \not\to \infty$, and thus, it follows from (1) that $\frac{p^t_{h_{\bar{n}}}}{p^t_{h_{\bar{n}+1}}} \not\to 0$. By applying (2) again one gets $\alpha^t_{k_{\bar{n}+2} h_{\bar{n}+1}} \not\to \infty$. Since the chain is closed the same procedure can be repeated until all n

have been reached.

(4) It is shown that either $\alpha^t_{k_n h_{n-1}} \not\to \infty$ for all n or $\alpha^t_{k_n h_{n-1}} \to \infty$ for all n. From (3) it follows that if there exist n_1, n_2 with $\alpha^t_{k_{n_1} h_{n_1-1}} \to \infty$, $\alpha^t_{k_{n_2} h_{n_2-1}} \not\to \infty$, then $\frac{p^t_{h_{n_2-1}}}{p^t_{h_{n_2}}} \to 0$. Therefore, from (3). it follows that $\alpha^t_{k_{n_2} h_{n_2-1}} \to \infty$ which is a contradiction. Hence, there are no n_1 and n_2 with $\alpha^t_{k_{n_1} h_{n_1-1}} \to \infty$, $\alpha^t_{k_{n_2} h_{n_2-1}} \not\to \infty$.

(5) It is shown, that $\alpha^t_{k_n h_{n-1}} \to \infty$ for all n. Since there are finitely many production paths only, there must be at least one path, say (h,k) that will be improved upon infinitely many times (when t tends to infinity), where h is a primary input and k a consumption commodity. Therefore, $\frac{\pi^t_{kh}}{p^t_h} \to 0$ and hence $\frac{\pi^t_{k_n h_{n-1}}}{p^t_h} \to 0$ for all n (otherwise, (h,k) would not be chosen infinitely often).

Case 1. There exists an n, say $\bar{n}$, with $\frac{p^t_h}{p^t_{h_{\bar{n}-1}}} \not\to \infty$. Hence, $\frac{\pi^t_{k_{\bar{n}} h_{\bar{n}-1}}}{p^t_{h_{\bar{n}-1}}} = \frac{\pi^t_{k_{\bar{n}} h_{\bar{n}-1}}}{p^t_h} \frac{p^t_h}{p^t_{h_{\bar{n}-1}}} \to 0$. Therefore, $\alpha^t_{k_{\bar{n}} h_{\bar{n}-1}} \to \infty$. Thus, it follows from 4. that $\alpha^t_{k_n h_{n-1}} \to \infty$ for all n.

Case 2. For all n $\frac{p^t_h}{p^t_{h_{n-1}}} \to \infty$. Since $\omega_h > 0$, some consumer say a_1 owns h (i.e. $\omega^{a_1}_h > 0$). Since $\alpha^t_{k_1 h_{N-1}} \not\to 0$, $\frac{p^t_{h_{N-1}}}{p^t_{k_1}} \not\to 0$. Therefore, $\frac{p^t_h}{p^t_{k_1}} = \frac{p^t_h}{p^t_{h_{N-1}}} \frac{p^t_{h_{N-1}}}{p^t_{k_1}} \to \infty$. Therefore, consumer a_1 can afford $y^{a_n t}_{k_1} \to \infty$. Since he is not strongly satiable in k_1 he will do so. Hence, we need $\alpha^t_{k_1 h_{N-1}} \to \infty$. Thus, it follows from (4) that $\alpha^t_{k_n h_{n-1}} \to \infty$ for all n.

(6) The lemma is now proven. Take any n. Suppose , that $\omega^{a_n}_h > 0$ and that h is essential for the production of k.

First, suppose that $\alpha^t_{kh} \to \infty$. Then $y^{a_n}_k \to \infty$, since a_n is not strongly satiable in k_n and since $\omega^{a_n}_h > 0$. In this case he is asymptotically satiated.

Second, suppose that $\alpha^t_{kh} \to \infty$. Because of (5) $\alpha^t_{k_{n+1} h_n} \to \infty$. Therefore, $\frac{\pi^t_{k_{n+1} h_n}}{p^t_{h_n}} \to 0$ and hence $\frac{\pi^t_{kh}}{p^t_{h_n}} \to 0$. Thus, if $\frac{p^t_{h_n}}{p^t_h} \not\to \infty$, then $\frac{\pi^t_{kh}}{p^t_h} \to 0$ and therefore, $\frac{p^t_k}{p^t_h} = (1/\alpha^t_{hk}) \to 0$. Hence, $\frac{u^{a_n}_k}{|u^{a_n}_h|} \to 0$ if $\frac{p^t_{h_n}}{p^t_h} \not\to \infty$. Thus, it remains to be shown that $\frac{p^t_{h_n}}{p^t_h} \not\to \infty$ if $\alpha^t_{kh} \not\to \infty$. Suppose not, i.e. suppose $\frac{p^t_{h_n}}{p^t_h} \to \infty$. Then, $\frac{|u^{a_n}_{h_n}|}{|u^{a_n}_h|} \to \infty$ and, therefore, $y^{a_n}_h \to 0$. On the other hand, $\frac{u^{a_n}_k}{|u^{a_n}_h|} = (1/\alpha^t_{kh}) \not\to \infty$, since $\alpha^t_{kh} \not\to 0$. Therefore, $y^{a_n}_h \not\to 0$, since a_n is not strongly satiable in k and since $y^{a_n}_k \not\to \infty$ if $\alpha^t_{kh} \not\to \infty$. Thus, since $\alpha^t_{kh} \not\to \infty$ we have a contradiction and $\frac{p^t_{h_n}}{p^t_h} \not\to \infty$ follows. As we have shown, $\frac{u^{a_n}_k}{|u^{a_n}_h|} \to 0$. In this case a_n is asymptotically satiated in k. ■

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