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INCENTIVES FOR INFORMATION ACQUISITION IN FIRMS

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Abstract: This paper investigates the effects of information acquisition in the light of two different organisational structures in various competitive settings. While the intuitive expectation that growing uncertainty raises the incentives to gather information can be confirmed, a changing organisational structure may also create a comparative tradeoff. A move from nonintegration to integration reflects less competition which tends to make information less valuable. Simultaneously, however, the decision power becomes more concentrated, implying that information is potentially more important. We analyse the interdependencies among organisational, informational and market structures and their impact on the relative incentives for information acquisition.

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1) INTRODUCTION

The relationship between organisational and informational structures of institutions has long been recognised as an important concern. Especially the contract-theoretic literature has emphasised that the distribution of private information affects the incentive mechanisms and consequently also the allocative decisions. Much work has also been devoted to examining the tradeoff of various organisational forms in order to derive the limits of a firm. The underlying informational structure, however, has generally been exogenously linked to a specific organisational mode. Typically it is postulated that the decision maker has full information on all variables of the integrated firm, but no information of nonintegrated firms[1]. These circumstances may often seem plausible and can be well motivated by interpreting the choice of an informational structure as a commitment device in case poor informativeness has strategic benefits (Riordan, 1990, and Schmidt, 1994) - a view that has taken its lead from Arrow's (1975) idea of linking information channels to property rights. Nevertheless, it is doubtful that an organisational alteration should necessarily and automatically lead to specific changes in the informational structure. The opposite (implicit) assumption of Grossman and Hart (1986) that integration has, in principle, no effect on the informational structure is just as problematic[2].

Envisage, for instance, a situation in which an entrepreneur buys a second firm and thereby takes over the entire staff. Whether the entrepreneur would be able to extract the same information of the manager upon integration would depend *inter alia* on a change of interests or property rights. There could, in fact, be many reasons. It is this observation of the *ad hoc* nature of assumptions specifying whether an organisational change *per se* affects the informational structures, that served as main motivation for this paper: any changes in the informational structures of firms in response to organisational changes should be derived endogenously to gain a better understanding of the interrelationship between organisational and informational features, and ultimately of comparative institutional theory.

Articles that focus on the organisation of firms in which the informational structure is endogenised are rare. This is not to say that informational structures are always exogenously given, especially not in contexts unrelated to organisational structures[3]. Considering

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[1] Refer for example to models of vertical integration, that by Riordan and Sappington (1987) and, in an oligopolistic setting, that by Gal-Or (1992).

[2] Grossman and Hart do, however, mention that integration may affect communication via changed incentive structures, but do not further pursue this issue.

[3] Milgrom (1981) and Matthews (1984), for example, have addressed the question of information acquisition in the framework of auctions. The optimal amount of research has also been analysed in the oligopoly models with uncertain demand of Li *et al* (1987), Vives (1988) and Hwang (1993), which mainly focus on efficiency aspects

exclusively articles that analyse the interrelationship between informational and organisational structures, it appears that Aoki (1986) was among the first to suggest that the degree of integration of firms may be explained in terms of the underlying informational structure (hierarchical vs horizontal flow of information). Within a different framework of vertical integration with complex uncertainty, Wiggins (1990) argues that integration is more flexible in absorbing cost and revenue shocks than long-term contracts. This result is clearly driven by his definition of institutional differences according to compensation arrangements such that a manager has no incentive to misrepresent private information as a fixed-wage earner, but well as a residual claimant. Nonintegration is shown to create opportunities for rent-seeking behaviour.

A very important contribution to endogenising both, the optimal informational and organisational structures is by Aghion and Tirole (1994). They investigate how the allocation of property rights affects the incentives to gather information by deriving a general tradeoff between the subordinate's initiative and the principal's loss of control. While their model captures the idea that information may have positive and negative effects, the principal's incentives to gather information become stronger with more decision rights. Consequently, the integrated structure endogenously improves the principal's information - in support of the traditional assumption.

The only paper (to my knowledge) that deals simultaneously with the endogenous determination of the organisational and informational structures and does not support the aforementioned traditional postulate is by Dewatripont and Tirole (1995). While their main purpose is to provide a rationale for advocacy in organisations, they show that decentralised information provision may or may not be preferred. The distinctive feature of their model is the two-dimensionality of information generation: their analysis relates to a specific class of agency problems in which two conflicting causes are to be investigated, such as the evaluation of pros and cons.

Departing from the contract-theoretic framework, this paper investigates the effects of information acquisition in the light of two different organisational structures in various competitive settings. For this purpose we rely on the premise that information is not automatically available (even though it may refer to "own" variables) but it can and will be acquired if its value exceeds the processing costs. While the most obvious and intuitive expectation that growing uncertainty raises the incentives to gather information can be confirmed, the effects of information acquisition are not always straightforward. A changing

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of information aggregation. Moreover, recent contributions to the principal-agent literature such as Crémer and Khalil (1994) and Kessler (1995) endogenise the agent's decision to gather pre-contractual information.

organisational structure may create a comparative tradeoff. A move from nonintegration to integration is accompanied by a reduction in the degree of competition which is intuitively expected to make information less valuable. Simultaneously, however, the decision power becomes more concentrated - the same information (data base), if available, serves as foundation for more decisions and hence is potentially more important. Despite maintaining that information is always valuable, the comparative organisational effects are not trivial and, depending on the structural parameters, the nonintegrated firm may - in contrast to the traditional postulate - be characterised by better information. This paper will analyse some factors that play a role in influencing the relative incentives for information acquisition to illuminate the interdependencies among the organisational, informational and market structures. In particular, we will emphasise the qualitative aspects of the impact of information generation, since they enable us to interpret the forces that drive the resulting effects.

The remainder of this article is organised as follows: in the context of the model introduced in Section 2, the subsequent Section 3 investigates the relative incentives for information acquisition under the two aforementioned organisational structures, provided that uncertainty pertains to production costs. Section 4 briefly shows that the effects of information differ considerably if, in contrast, a common value such as the level of demand is stochastic. Section 5 concludes.

2) THE MODEL

We compare two organisational modes. Under nonintegration, two Cournot duopolists i = 1, 2 are producing q_i units of good i. Their prices are given by the indirect linear industry demand curve $p_i = a - bq_i - dq_{-i}$ where the parameter $d \in [0, b]$ indicates the degree of product differentiation: a small value corresponds to a high degree of differentiation and vice versa. In particular, if d = b, the firms produce a homogeneous good, positive values indicate that the goods are substitutes. Assuming in addition constant returns to scale, the firms will choose production volumes to maximise their respective payoff functions $\pi_i = (a - bq_i - dq_{-i} - c_i)q_i$.

The second organisational form considered is that of integration which is characterised by one decision maker who maximises the joint profit $\pi = \pi_1 + \pi_2$. Despite being highly stylised, this setting is very general. It can be viewed as the outcome of an implicit contract between two plant owners where one of them has sold his decision right of choosing q_i in exchange for a lump-sum transfer.

Uncertainty is introduced in its simplest form where the stochastic variable assumes either a high or a low value according to a known probability distribution. In the main part of this paper we assume that firm 1 has constant production costs c_1, while firm 2 produces at low costs $\underline{c}$ with probability p and at high costs $\overline{c}$ with the complementary probability 1-p. Not only has this production cost uncertainty opposing effects on the duopolists, but its asymmetric setup with only firm 2 facing stochastic costs enables us to clearly distinguish between direct and indirect consequences of information provision.

With regard to the informational structure, both firms are initially assumed to be ignorant about the stochastic production costs c_2 in the sense that they do not automatically know the realised value. They are, however, aware of the probability of low costs and hence the expected average production costs $c_2^e = p\underline{c} + (1-p)\overline{c}$. Furthermore, we assume that both firms may invest in an information technology that produces a perfect signal of the actual production cost prevalent before they take any production decisions. The ex-post state of firm i's informativeness is denoted by $\tau_i \in \{p, c\}$, indicating whether it remains ignorant or becomes informed, respectively.

The postulated ignorance of firms can be easily motivated even though the literature commonly assumes that firms know their own production costs. It should not be difficult to imagine that - especially large - firms are not fully aware of all cost components. Or consider a mine, for example, where the actual production costs per unit of final output largely depends on the ore concentration underground. The mine could invest in some geological testing device to avoid the risk of mining low-concentration ore (i.e., high-cost production) before signing any supply contracts. The apparent discrepancy between this paper's postulated ignorance and the literature's frequently assumed informativeness lies in the analytical distinction between ex ante and ex post states of informativeness and can therefore easily be resolved: the typical assumption of known production costs implicitly assumes that the importance of knowledge justifies the information acquisition. This paper, in contrast, models explicitly the value of information based on the premise that information is not automatically present but needs to be gathered and analysed, even if this process is nearly costless. Ex ante all firms are in a state of ignorance; whether a firm will be informed ex post (i.e. invest in an information technology) will depend on the relative costs k and benefits v of information. We will focus on the latter.

The time structure is as follows. For a given organisational mode, each firm decides whether to acquire information or remain ignorant. In the following period the informativeness of

firms is observed[4], and, if informed, also the actual production costs. Thereafter output levels are chosen and profits realised.

For each organisational mode, we focus on the subgames in order to derive the incentives to acquire information, i.e., we compare the expected profits associated with all possible states of informativeness, $\tau = (\tau_1, \tau_2)$. The potential value of information is thus given by the difference between the expected profits of symmetrically informed and ignorant firms. Asymmetric informativeness with only one firm being informed, then serves as basis for a more qualitative analysis of the informational effects.

We will analyse the incentives for information acquisition over the entire region of product differentiation. At the one extreme we are considering the production of a homogeneous good, where the change from nonintegration to integration coincides with the change from a Cournot duopoly to a monopoly. This change in the organisational structure implies simultaneously a change in the degree of competition, which to some extent is always the case when two firms merge. Unless, of course, we consider the opposite limit case of complete product differentiation which involves the production of two goods whose demand curves are independent of each other. Here the impact of an organisational change on market power ultimately vanishes: whether these products are supplied by two monopolies or one does not matter in the limit of zero cross elasticity.

3) COST UNCERTAINTY

3.1 Nonintegration

For a given information structure τ (of four possible combinations of τ_i indicating ignorance or information; i = 1, 2), the optimal quantity choices maximise the expected profits subject to their rival's expected optimal production volume. The first order conditions are expressed by the reaction curves

$$\hat{q}_i = \tfrac{1}{2b}\left(a - dE_i\left[\hat{q}_{-i}|\tau\right] - E_i\left[c_i|\tau_i\right]\right) \qquad \text{for } i = 1, 2;$$

where $E_i\left[\hat{q}_{-i}|\tau\right]$ denotes firm i's belief of its rival's optimal production volume, depending on the informational state of both duopolists. Firm i's cost expectation $E_i\left[c_j|\tau_i\right]$, in contrast, depends only on the own informational status with[5] $E_i\left[c_2|\tau_i = p\right] = c_2^e$ if uninformed,

[4] This assumption that firms can observe who is informed merely simplifies the analysis.
[5] The subscripts indicating whose expectation formation is concerned will henceforth be omitted.

$E_i\left[c_2\middle|\tau_i = c\right] = c_2$ if informed, and $E_i\left[c_1\middle|\tau_i\right] \equiv c_1$ irrespective of the informational structure. The duopolists know who is informed, the information about the production cost, however, is private to the informed party. The optimal expected payoffs, given all different combinations of the firms' states of informativeness, as well as the corresponding values of information are summarised in *Table 1* in the Appendix. To ensure that the optimal quantities and expected payoffs are nonnegative, sufficient demand satisfying

$$a \geq \max\left\{\frac{2bc_1 - d\underline{c}}{2b-d}, \frac{2b\bar{c} - dc_1}{2b-d}\right\} \qquad [A1]$$

is assumed.

Define the total value of information for firm i

$$v_i \equiv TE_i := E\left[\pi_i^*\middle|\tau = (c,c)\right] - E\left[\pi_i^*\middle|\tau = (p,p)\right]$$

as the difference between the expected profits under symmetric information and ignorance. It thus captures i's additional expected profit if both firms acquire information. In addition, we analyse the partial effects of information to understand qualitatively the composition of its impact. The reaction curves indicate already that cost uncertainty enters firm 1's decision only indirectly via the opponent's expected quantity choice, while firm 2 is affected directly by the stochastic production costs in addition to their indirect effect. We therefore define

$$E\left[v_j\middle|i,\tau_{-i}\right] := E\left[\pi_j^*\middle|(\tau_i = c, \tau_{-i})\right] - E\left[\pi_j^*\middle|(\tau_i = p, \tau_{-i})\right] \qquad i, j = 1, 2$$

as firm j's value of i's information acquisition conditional on the rival's informativeness. Then, deriving these magnitudes for all states of informativeness[6], we can make

Observation 1: For nonintegrated firms
a) the value of information rises with growing uncertainty, and
b) information acquisition comprises a *direct* and a *strategic* effect.

The first part follows directly from the fact that all gains from information can be expressed as proportions of the variance $\gamma := p(1-p)(\bar{c} - \underline{c})^2$. The second part relates to the decomposition of the total effect of information acquisition that arises if only one firm gathers information, given the rival's informational state. A *direct* (or *precision*) *effect* arises if knowledge about the true state of nature enables a more efficient quantity choice while the opponent remains ignorant as defined by $DE_i := E\left[\pi_i^*\middle|(c,p)\right] - E\left[\pi_i^*\middle|(p,p)\right]$ for i = 1, 2 and $\tau = (\tau_i, \tau_{-i})$. A *strategic effect* arises, in contrast, if the acquired information enables firms to anticipate their rivals' behaviour more precisely and thus to choose a best response. This

[6] Refer to *Table 1* in the Appendix.

requires that both firms are informed. We can thus isolate the strategic effect by $SE_i := TE_i - DE_i$.

The fact that firm 1's production costs are constant rules out any direct adjustment in response to obtaining knowledge about the competitor's realised production costs. Hence $DE_1 \equiv 0$. Firm 1 can nevertheless gain strategically provided that firm 2 also becomes informed since the stochastic costs c_2 enter π_1 indirectly via 1's expectation of 2's quantity decision. On the other hand, firm 2's profit function is affected by the uncertain production costs c_2 directly via the realised values as well as indirectly via the quantity choices based upon the cost expectations. Provided firm 1 remains ignorant, it bases its quantity decision on c_2^e irrespective of 2's informativeness. Firm 2 will thus correctly anticipate its rival's action, any incremental profit that arises if firm 2 acquires information must therefore be direct, i.e., $DE_2 = \frac{1}{4b}\gamma$. The additional gain if firm 1 also becomes informed then obviously comprises the strategic value.

Proposition 1: Given nonintegration,

a) both, the direct and the strategic components of information are more valuable for the firm that faces stochastic production costs, in particular,

$$0 = DE_1 \leq SE_1 \leq SE_2 < DE_2 \qquad \forall\, d \in [0, b]$$

b) the value of information rises monotonically and at an increasing rate as the goods become more homogenised.

Proof:

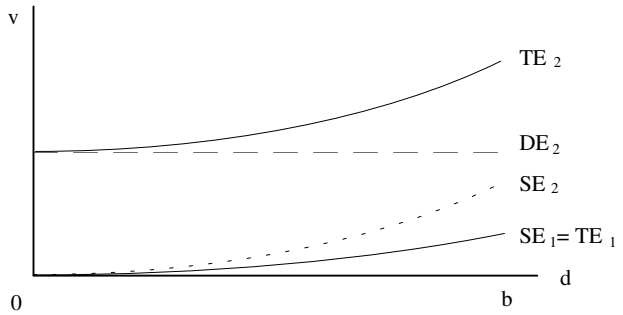
a) Compare the definitions of the direct and strategic effects (*Table 1*)

b) Both the first and second order derivatives of TE_i with respect to d are positive. Moreover TE_i is continuous for $d \in [0, b]$. □

The first part of *Proposition 1* is very intuitive. The second part is driven by the strategic effects, which, capturing the rising degree of competition, become more pronounced the more homogenised the products become. The direct effects, in contrast, are independent of the degree of product differentiation[7].

[7] This feature is specific to this particular model as it hinges on the constant unit costs of production which imply that the quantity produced has no effect on profitability.

Figure 1: Value of information for nonintegrated firms



Corollary: Information acquisition is complementary since all information effects are positive.

A duopolist's incentive to acquire information is larger if the opponent is informed rather than ignorant, because the strategic effects are positive and hence reinforce the direct effects. Consequently the informed firm has an ex ante incentive to share its information[8]. Once the actual production costs are realised, however, firm 2 would only benefit from announcing low costs. Unless it can commit itself to truthful information sharing (e.g. if the information were verifiable) its cost reports are not credible. Information sharing, though Pareto efficient, will therefore not generally be observed.

3.2 Integration

Consider now the integrated firm as a two-good monopoly that uses two different production plants i = 1, 2, where the decision maker compensates the other party with a lump-sum transfer. The decision maker compares the optimal expected profits under ignorance and information before determining whether to invest in the information technology. For each information structure $\tau \in \{p, c\}$, the optimisation problem becomes

$$\max_{q_1,q_2} \quad (a - bq_1 - dq_2 - c_1)q_1 + \left(a - bq_2 - dq_1 - E\left[c_2|\tau\right]\right)q_2 \quad \text{subject to} \quad q_1, q_2 \geq 0.$$

Compared with a nonintegrated firm, the integrated one operates with an enlarged decision set which enables the determination of the production volumes of both goods. This additional degree of freedom has far-reaching consequences on firm behaviour, because the optimal production decisions become particularly sensitive to the parameter values of both the cost variables and the degree of differentiation.

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[8] This result is well-known in the information-sharing literature of oligopoly in the case of private stochastic values and substitute goods. The tendency of oligopolists to share information translates into positive strategic effects in this model. Refer to Raith (1993) and Ziv (1993b).

To ensure that there exist quantities that fulfill the nonnegativity constraints we assume that

$$a \geq \max\{c_1, \bar{c}\}. \qquad [A2]$$

Moreover, the first and second order conditions

$$\begin{aligned} \hat{q}_1 &= \tfrac{1}{2b}\left(a - 2d\hat{q}_2 - c_1\right) && \geq 0 \\ \hat{q}_2 &= \tfrac{1}{2b}\left(a - 2d\hat{q}_1 - E\left[c_2|\tau\right]\right) && \geq 0 \\ 4b^2 - 4d^2 &> 0 \end{aligned} \qquad [1]$$

need to be satisfied for inner solutions to the maximisation problem. But whenever product differentiation is insufficient to warrant the cost difference and hence to ensure the profitability of producing both goods, the nonnegativity constraints become binding. In particular, the homogeneous production of only one good would be optimal for relatively little product differentiation. Consequently, the less cost-efficient plant remains idle with $q_{-i} \equiv 0$, and the optimisation problem reduces to

$$\max_{q_i} \quad \left(a - bq_i - E\left[c_i|\tau\right]\right)q_i \qquad \text{with} \qquad \tilde{q} = q_i = \tfrac{1}{2b}\left(a - E\left[c_i|\tau\right]\right).$$

The exact conditions that result in the optimal type of solution (i.e. whether an interior or corner solution) depend crucially on the degree of differentiation and the cost parameters as well as the firm's informational structure. Along the spectrum of increasing substitutability with perfect product differentiation (d = 0) and perfectly homogeneous goods (d = b) at the extremes, there exists one cutoff point $\delta\left(c_2^e\right)$ beyond which the ignorant firm switches from an inner solution to a corner solution. For the informed firm there exist two of these threshold degrees of differentiation, $\delta(\underline{c})$ and $\delta(\bar{c})$, which determine the optimality of an interior or corner solution according to the actual production costs c_2. In general, these threshold degrees of differentiation are derived from the first order conditions such that the smaller quantity equals zero; i.e.

$$\delta\left(E\left[c_2|\tau\right]\right) := b\,\frac{a - \max\left\{c_1, E\left[c_2|\tau\right]\right\}}{a - \min\left\{c_1, E\left[c_2|\tau\right]\right\}} \qquad \text{with} \quad E\left[c_2|\tau\right] \in \left\{c_2^e, \underline{c}, \bar{c}\right\}. \qquad [2]$$

It follows that an inner solution $\hat{q}(\cdot)$ is efficient if and only if $d \leq \delta(\cdot)$; a corner solution $\tilde{q}_1 = 0$ or $\tilde{q}_2 = 0$ should be chosen if $d > \delta(\cdot)$. The order of these cutoff levels depends exclusively on the parameter constellation.

Lemma: Define $\quad \underline{\delta} := \min_{\underline{c},\bar{c}} \delta\left(c_2\right)$ and $\bar{\delta} := \max_{\underline{c},\bar{c}} \delta\left(c_2\right)$

then the ranking of the threshold degrees of differentiation satisfies:

$$\text{i)} \quad (a-\underline{c})(a-\overline{c}) < (a-c_1)^2 \Leftrightarrow \quad \delta(\overline{c}) < \delta(\underline{c})$$

$$\left.\begin{array}{l} \text{ii)} \quad (a-c_1)^2 < (a-\underline{c})(a-c_2^e) \quad \wedge \quad c_1 < c_2^e \\ \quad \text{or} \\ \quad\quad (a-\overline{c})(a-c_2^e) < (a-c_1)^2 \quad \wedge \quad c_2^e < c_1 \end{array}\right\} \Leftrightarrow \quad \overline{\delta} < \delta(c_2^e)$$

$$\text{iii)} \quad \underline{\delta} < \delta(c_2^e)$$

This and all subsequent proofs are relegated to the Appendix.

The threshold degrees of differentiation separate four regions whose combinations of the optimal types of solutions for the two informational states differ:

Pure differentiation (region D): the products are substantially differentiated and it is always optimal for the integrated firm to produce both goods.

Two intermediate regions, *mixed differentiation* (region MD) and *alternating or mixed homogeneous production* (region AMH): the ignorant firm chooses a product range that is inefficient with a certain probability, p, 1-p, or 1.

Homogeneous production (region H): The goods are sufficiently similar, implying that the homogeneous production of only one good is efficient irrespective of the firm's informational status.

When examining the impact of information acquisition, we therefore distinguish between these regions. In addition, we identify four cases, whose distinctions depend on the size of the known production costs c_1 in relation to the parameters $\underline{c}$, $\overline{c}$ and c_2^e, as shown below. The remainder of this section is devoted to a detailed description of the effects of information provision, and the reader will frequently be referred to *Table 2* in the Appendix which summarises the technical results.

Case distinctions

	$c_1 < c_2^e$	$c_2^e < c_1$
$c_1 < \underline{c}$	I	
$\underline{c} < c_1 < \overline{c}$	II	III
$\overline{c} < c_1$		IV

3.2.1 Pure product differentiation (region D)

We investigate the integrated firm's incentives to acquire information in case the two goods are sufficiently differentiated to induce their simultaneous production irrespective of the firm's informational status, i.e., $0 \leq d < \underline{\delta}$. In this range there exist interior solutions to the above optimisation problem for $\tau = p$ and for $\tau = c$ as given by equation [1]; the associated value of information[9] DD indicates the gain in expected profits while inner solutions remain optimal in both informational states.

Observation 2: In case the integrated firm optimally to supplies both goods irrespective of its informativeness,
a) its incentive to acquire information rises with growing uncertainty and
b) information acquisition comprises a *precision* and a *correlation effect*.

The *precision effect* PE is analogous to the direct effect of nonintegration, because knowledge about the true state of nature enables a more efficient quantity choice. The *correlation effect* CE is essentially the integration counterpart of the duopoly's strategic effect. It arises if the acquired information enables the firm to correlate the two production volumes more precisely. Since the two products are substitutes to some degree, any quantity adjustment of one good will influence the revenue obtained from the other good. This correlation effect differs from the strategic effect of the nonintegration case, since here joint profits are maximised (instead of maximising own profit at the expense of the competitor). Nevertheless we can distinguish the precision and correlation effects from each other by interpreting integration as a cartel, where both firms choose their production volumes in order to maximise the team profit. For this purpose consider the case of asymmetric information where firm 2 is informed, firm 1 is ignorant and no information is shared[10]. In this case the resulting expected profit over and above that under ignorance resembles the precision effect $PE = \frac{1}{4b}\gamma$. Any informational value beyond this magnitude must therefore originate from correlating the two production volumes.

Proposition 2: Provided that the integrated firm produces positive amounts of both goods irrespective of the informational status, its incentive to acquire information (driven by the correlation effect) rises monotonically and at an increasing rate as the goods become closer substitutes.

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[9] See *Table 2* in the Appendix.
[10] This situation in which two firms who cooperate in a cartel but nevertheless withhold strategically valuable information from each other, corresponds to cheating in a cartel.

The rising value of information must be driven by the correlation effect, because the precision effect is independent of the degree of product differentiation. Since the latter is not affected by competition, it is furthermore independent of the organisational structure.

3.2.2 Homogeneous production (region H)

Let us now investigate the other extreme, where it is efficient to produce only one good while the cost-ineffcient plant remains idle. These corner solutions are optimal for (im)perfect substitutes as long as $\max\left\{\overline{\delta}, \delta\left(c_2^e\right)\right\} \leq d \leq b$. The integrated firm has thus another advantage over the nonintegrated one: apart from its ability to charge monopoly prices it can produce more efficiently by focusing on low-cost production; an additional degree of freedom that hinges on the absence of switching costs. In this region, the incentives to acquire information[11] are particularly sensitive to the parameter values, hence the case distinctions are crucial.

Whenever the constant production costs are particularly low or very high such that $c_1 < \underline{c} < \overline{c}$ or $\underline{c} < \overline{c} < c_1$ (case I or IV obtains), the plant choice can clearly be made ex ante, because it does not depend on the particular cost realisation. In the former case, the plant with uncertain costs will remain idle. Consequently, there is no element of uncertainty in the expected profit H1 irrespective of the informational status and the monopoly with known production costs has no incentive at all to acquire redundant information $HH1 \equiv 0$ about c_2. Given case IV, the monopoly's production plant 2 exhibits uncertain costs, leading to a value of information equal to the precision effect of product differentiation in region D.

Let us now consider cases II and III where $\underline{c} \leq c_1 \leq \overline{c}$ in order to focus on those parameter constellations where the optimal plant usage depends on the actual production costs. If the probability of low costs is relatively high such that $c_2^e < c_1$ (case III), then the uninformed firm favours plant 2 for production despite the associated uncertainty, yielding expected profit H2. Mutatis mutandis, if the probability of low costs is relatively small, implying that expected production costs c_2^e exceed the constant costs c_1 (case II), the chosen quantities will result in profit H1.

Once the firm learns c_2, it is able to employ plant 2 whenever the costs turn out to be low and plant 1 otherwise. The average production costs are thus lowered to $\tilde{c} := p\underline{c} + (1-p)c_1$ resulting in expected returns A. Information is therefore valuable as it allows the firm to

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[11] Refer to *Table 2* for all optimal output levels, expected returns and incentives of information acquisition of this section.

efficiently alternate between the homogeneous production of good 1 and good 2 depending on the cost realisation instead of only producing good 1 or only good 2 in cases II or III, respectively (as captured by H1A:= $\frac{1}{4b}(\alpha+\gamma_1)$ and H2A:= $\frac{1}{4b}(\beta+\gamma_1)$ in *Table 2*.)

Observation 3: Given integration in the region of homogeneous production, information acquisition comprises a *switching effect* (in case $\underline{c} \le c_1 \le \overline{c}$) in addition to the precision effect, but there is no correlation effect.

Obviously there can be no correlation effect if only one good is produced. But the precision effect can be identified, albeit in the slightly altered expression $PE = \frac{1}{4b}\gamma_1$ with $\gamma_1 := p(1-p)(c_1-\underline{c})^2$. The similarity of γ and γ_1 indicates the same source of gain due to information acquisition: adjusting the quantity choice to the realised and not to the average cost parameter. In the previous section, production levels were finetuned depending on the magnitude of c_2. Given that switching is optimal, however, the relevant costs are now c_1 and $\underline{c}$, because the firm will use plant 1 with probability 1-p, or more precisely, whenever $c_2 = \overline{c}$. Thus, the additional degree of freedom obtained by integration reduces the precision effect of information.

However, the ability to switch production to the low cost plant has itself a positive impact. By getting informed the firm can reduce its average production costs to $\tilde{c}$ if it uses plant 2 with probability p (i.e. whenever $c_2 = \underline{c}$) instead of always producing at constant costs c_1 or average production costs c_2^e in cases II or III, respectively. This *switching effect* (SWE) of information captured by $\frac{1}{4b}\alpha$ or $\frac{1}{4b}\beta$, respectively, arises, because the informed firm can always (i.e. without error) choose the more efficient plant for production, while an ignorant firm can do so only on average.

Proposition 3: The additional decision variable of plant choice that is induced by integration may have a positive (SWE) and a negative impact (reduced PE) on the value of information. Given any probability p, the precision effect is monotonically nondecreasing with the constant production costs c_1 while the switching effect is positive over the interval $c_1 \in (\underline{c}, \overline{c})$ and peaks at $c_1 = c_2^e$.

Figure 2: Value of information as a function of c_1

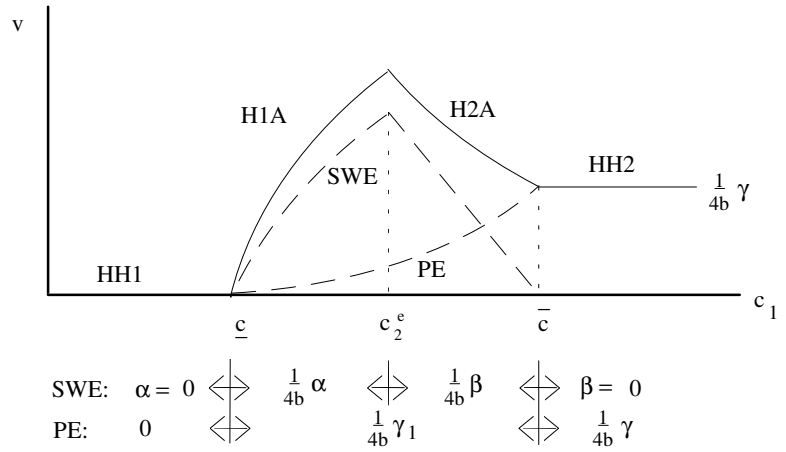

Figure 2 exhibits the value of information as a function of c_1 for a given probability p. The monotonicity of the precision effect

$$PE = \tfrac{1}{4b}\,p(1-p)\left(\min\{c_1,\overline{c}\} - \min\{c_1,\underline{c}\}\right)^2$$

can easily be explained: plant 2 will become more important with higher constant production costs c_1 despite its cost uncertainty; beyond the high costs $\overline{c}$ plant 1 will remain idle and the precision effect attains its normal magnitude compared to the pure differentiation region or nonintegration.

The zero switching effect for all $c_1 \le \underline{c}$ and $c_1 \ge \overline{c}$ needs no explanation. In the intermediate range $c_1 \in (\underline{c},\overline{c})$, the intuition behind the peaking switching effect is as follows: at $c_1 = c_2^e$ the ex ante plant choice does not matter, but once the costs have been realised, errors due to ignorance are particularly expensive. If on average it does not matter whether good 1 or 2 is produced, information yields the highest benefit because the opportunity cost of ignorance due to misjudgements attains a maximum of $\overline{p}(c_1 - \underline{c}) + (1-\overline{p})(\overline{c} - c_1)$ where $\overline{p} := \frac{\overline{c}-c_1}{\overline{c}-\underline{c}}$ is defined such that $c_2^e(\overline{p}) = c_1$. For c_1 closer to $\underline{c}$ or $\overline{c}$ these costly errors occur less frequently as the probability of low costs approaches either 0 or 1 and the overall shift towards the low expected cost production becomes less costly.

3.2.3 Intermediate regions of product differentiation (regions MD and AMH)

Having analysed the extreme cases where the choice between interior and corner solutions was independent of the informativeness of the firm, we now turn very briefly to the intermediate range where the efficient type of production - homogeneous or parallel - depends on the realised production cost variable.

The region of m*ixed differentiation* (MD) with a fairly large degree of product differentiation satisfying $d \in \left[\underline{\delta}, \min\left\{\overline{\delta}, \delta\left(c_2^e\right)\right\}\right]$ is characterised by the feature that the ignorant firm always supplies both goods to the market as in the adjacent region D, whereas a mixture of inner and corner solutions would indeed be optimal.

In the neighbouring region of *alternating or mixed homogeneous production* (AMH) the goods are more homogeneous, i.e., $d \in \left[\min\left\{\overline{\delta}, \delta\left(c_2^e\right)\right\}, \max\left\{\overline{\delta}, \delta\left(c_2^e\right)\right\}\right]$. As long as c_1 and c_2^e are sufficiently close[12] such that $\overline{\delta} < \delta\left(c_2^e\right)$, behaviour of the ignorant firm is similar to that in region MD: it inefficiently supplies the dual product range instead of streamlining its production by alternatively producing only good 1 (if $c_2 = \overline{c}$) or good 2 (if $c_2 = \underline{c}$). Consequently the gain of information captures the incremental profit associated with a definite change from inner to corner solutions. If, however, c_1 and c_2^e are sufficiently distinct as reflected by $\delta\left(c_2^e\right) < \overline{\delta}$, the ignorant firm consistently opts for the same corner solution instead of mixing inner and corner solutions conditional on the cost realisation. It could improve its efficiency by getting informed and differentiating production depending on the state of nature.

Despite the underlying complexity of the effects of information acquisition in these intermediate regions, we will summarise merely those features in the proposition below that allow statements on the entire spectrum of product differentiation[13].

Proposition 4: For all cost parameter constellations the value of information acquisition is a continuous and smooth function of the degree of homogenisation $d \in [0, b]$. The prevalence of considerable cost uncertainty under ignorance as reflected by similar constant and expected production costs such that $\overline{\delta} < \delta\left(c_2^e\right)$ is a necessary condition for monotonically nondecreasing incentives to acquire information as the goods become more substitutable. Little cost uncertainty such that $\delta\left(c_2^e\right) < \overline{\delta}$, in contrast, is sufficient to ensure a global maximum in the region of mixed differentiation.

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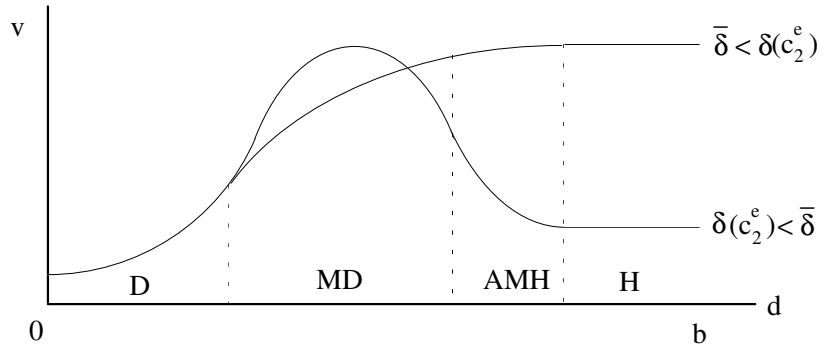
[12] Compare with Lemma.
[13] More details may be requested from the author.

Figure 3: Value of information under integration



3.3 Comparison of organisation structures

One of our main concerns relates to the comparative importance of information by of the two organisation structures[14]. For large degrees of product differentiation, the integrated firm has always stronger incentives to become informed than the nonintegrated firm. Its value of information, in fact, exceeds the combined benefit of information of both duopolists. Since the precision effect is independent of competition, in the sense that it is neither affected by the degree of product differentiation nor by the organisational structure[15], the divergence of incentives originates from strategic or, respectively, correlation considerations. The correlation effect of the integrated firm is particularly pronounced, because cooperation in the sense that the team profit is maximised is superior to the competitive firm structure. Moreover, it is intuitively plausible that the more concentrated decision power of the integrated structure is positively correlated with the incentives to acquire information.

On the opposite end of the differentiation spectrum where the goods are perfectly homogeneous, the relative incentives to acquire information are not as clear cut, but depend crucially on the parameter constellations. Nevertheless, one can identify certain tendencies: for very low constant production costs $c_1 < \underline{c}$ (case I), the monopoly considers information about c_2 superfluous. Consequently, both nonintegrated firms have stronger incentives to become informed than the integrated firm.

This result varies slightly for case IV where the known production costs are very high, $\bar{c} < c_1$. Only the duopoly that faces uncertain production costs finds information more valuable than the integrated firm, not anymore the duopoly which is only indirectly affected by its opponent's production cost uncertainty. This is a direct consequence of the relative importance of direct and strategic effects established in *Proposition 1*.

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[14] Casual observations can be made by comparing *Figures 1* and *3*.
[15] DE_2 under nonintegration equals PE under integration.

In the remaining cases II and III where $\underline{c} < c_1 < \overline{c}$, either organisational form may exhibit the stronger incentives to become informed. The higher relative value of information is maintained under integration if

$$1 - \frac{9\left[(a-\underline{c})^2 - (a-c_1)^2\right]}{16(\overline{c}-\underline{c})^2} < p < \frac{\overline{c}-c_1}{\overline{c}-\underline{c}} \qquad \text{in case II}$$

$$\text{or} \quad \frac{\overline{c}-c_1}{\overline{c}-\underline{c}} < p < \frac{9\left[(a-c_1)^2 - (a-\overline{c})^2\right]}{7(\overline{c}-\underline{c})^2} \qquad \text{in case III.} \qquad [3]$$

These inequalities - messy as they seem - reinforce the rather casual observation that can also be deduced from *Figure 2*: only if c_1 is in the vicinity of c_2^e and product demand is high, can the switching effect under integration be sufficiently large to exceed the strategic effect of the nonintegrated firm. The overriding factor clearly is the demand parameter a which boosts the switching effect but has absolutely no impact on the strategic effect under nonintegration.

The following theorem summarises the above to answer the question, under which organisational form is information more valuable?

Theorem: a) Given $\underline{c} < c_1 < \overline{c}$, then for all degrees of product differentiation d ∈ (0, b] the integrated firm has greater incentives to get informed than the nonintegrated duopoly in the presence of high cost uncertainty and sufficient product demand.
b) If $\underline{c} < \overline{c} < c_1$, then there exists a threshold degree of homogenisation $\delta_2 \in (0, b)$ beyond which the integrated firm has lower incentives to acquire information than the duopoly with uncertain costs.
c) If $c_1 < \underline{c} < \overline{c}$, then there exists another threshold degree of homogenisation $\delta_1 \in (0, b)$ in addition to δ_2 beyond which the monopoly benefits less from the information than the duopoly with known and constant production costs.

Thus, for weakly substitutable products, information is more valuable for the integrated two-good monopoly, because it can capitalise on the correlation effect. As the goods become more homogeneous, it is less likely that the integrated firm maintains its dual product range. When streamlining its production, the correlation effect vanishes. The only reason, why it might still benefit more from information than the nonintegrated firm lies in the potential switching effect, which exists if $\underline{c} < c_1 < \overline{c}$ (*Figure 2*). For high demand, this becomes particularly prevalent and may thus exceed the strategic effect of the nonintegrated firm. For extremely low or high certain production costs c_1, however, switching is no viable option under integration, and if at all, only the precision effect may prevail (case IV). Consequently, information may become more valuable for the nonintegrated firms beyond a certain degree of homogenisation.

Absent any legal restraints or other friction, if the organisational structure were endogenously determined, only (potentially two-good) monopolies would prevail. Since these integrated structures often have high incentives to acquire information, the question arises whether any informational benefits for the consumers offset their damages caused by the firm's monopoly power? A detailed analysis of the consumer surpluses induced by the various organisational and informational structures leads to the following conclusion. For sufficiently differentiated products the consumers may prefer a monopoly to a duopoly, provided the costs of acquiring information are too high for the duopolists but low enough to induce the monopoly to become informed.

Thus it can be demonstrated that there are exceptions to the view that monopolies always harm the consumers. Nevertheless, this does not constitute a very strong result. Casual observation suggests that these exceptions apply only for very low values of d where the goods are largely independent of each other. I have found no example where the cutoff is beyond 0.1 on a scale of $d \in [0, b=1]$. But it is the interpretation, not the relative size of the interval concerned (whose applicability would be a matter of empirical studies to determine) that dampens the significance of this statement. Its relevance for largely independent goods indicates that even the nonintegrated firms earn considerable monopolistic rents. In the limit of perfectly independent goods the consumers are indifferent to the prevailing market competition: it does not matter whether the nonsubstitutable goods are offered by one or by two monopolies. The consumers are thus unlikely to gain considerable spinoffs from the firms' benefits induced by merger activities. Expressed differently, the message is that consumers may in some instances suffer less from an informed two-good monopoly than from two uninformed nonintegrated near-monopolies.

4) DEMAND UNCERTAINTY

This model exhibits some parallels with the information-sharing literature of oligopoly, whose results are very sensitive to specifications such as the nature of competition and the locus of uncertainty concerned[16]. In particular, the distinction between a stochastic private or common value leading to opposing results in duopoly carries over to this paper's framework: when oligopolists tend to share information (e.g. when Cournot duopolists face cost uncertainty), the nonintegrated firms of this paper experience positive strategic value from

[16] Raith (1993) and Ziv (1993b) compare the assumptions and review the results of information sharing in oligopoly.

acquiring information. This effect is reversed when firms face a stochastic common value such as demand uncertainty instead.

Assuming that the demand intercept is high ($a = \bar{a}$) with probability p and low ($a = \underline{a} < \bar{a}$) otherwise, a firm intending to launch a new product could test the market by staging a pilot launch before making any capacity choice. But how useful will this information provision be? Following the same procedure as previously, it turns out that the incentives of nonintegrated firms to gather information again comprise two reasons: a constant direct effect and a strategic effect which becomes more important with growing homogenisation can be observed; the latter, however, is negative - in direct contrast of the cost uncertainty scenario.

Proposition 5: The value of information for a duopolist facing demand uncertainty falls monotonically though at a decreasing rate as the goods become more homogenised. Information acquisition, however, is secretive.

The intuition of secretive information gathering can be explained as follows. An informed firm knowing that demand is high can capture the gains (over and above the expected return under ignorance) for itself in case its rival is uninformed, but has to share this increment if both duopolists are aware of high demand and adjust their quantities accordingly.

Concerning the integration counterparts of informational effects, the shift to a stochastic common value also changes some results. The fact that plant choice as an additional dimension of decision making, apart from volume decisions, ultimately leads to four regions[17] across the spectrum of product differentiation, remains intact.

For substantially differentiated products, *pure product differentiation* (region D) is optimal. The associated value of information is monotonically decreasing and convex in the degree of homogenisation, and can be decomposed into a constant precision effect and a strictly positive correlation effect. Thus, if the joint profit is maximised, information contributes to better correlating the two production volumes. In contrast to CE of cost uncertainty and the SE_i (nonintegration) of demand uncertainty, this correlation effect vanishes as the goods become closer substitutes. The diminishing incentives of information acquisition are further dampened by a *homogenisation* effect in the adjacent regions of *mixed differentiation* (MD) and *mixed homogenisation* (MH), since the streamlining of production becomes increasingly

[17] These regions virtually resemble those under cost uncertainty; the subsequent analysis thus closely follows that of Section 3.2. apart that $\delta\left(E\left[a|\tau\right]\right) := b\frac{E\left[a|\tau\right]-c_{-i}}{E\left[a|\tau\right]-c_i}$ for $c_i < c_{-i}$ and $E\left[a|\tau\right] \in \left\{a^e, \underline{a}, \bar{a}\right\}$, hence $\delta(\underline{a}) < \delta(a^e) < \delta(\bar{a})$.

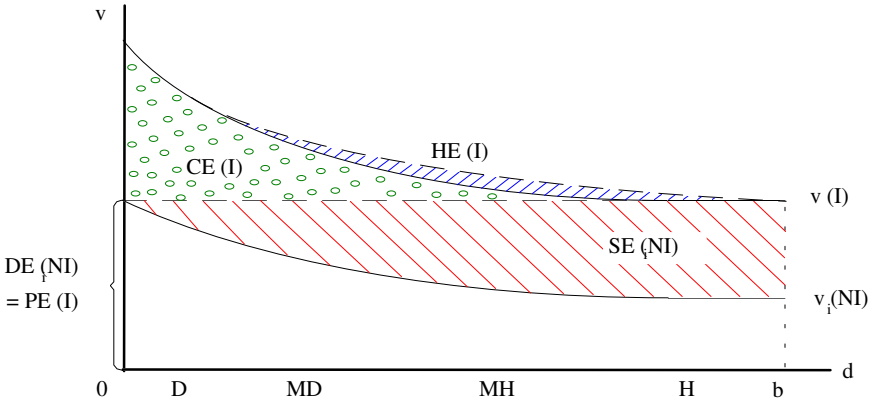
efficient with the goods' rising cross elasticity of demand. Information on the actual level of demand enables the optimal mix between differentiated and homogeneous production conditional on demand levels and thereby prevents inefficient choices under ignorance, i.e., supply of the dual product range whenever demand is low in region MD, and homogenous production in region MH if demand is high. In the last region H, *homogeneous production* is optimal irrespective of the firm's informational status and the actual level of demand. Only the low-cost good will be produced, hence the value of information reduces to the pure precision effect.

The role that the degree of product differentiation plays in determining the incentives to gather information is summarised in

Proposition 6: For the integrated firm facing demand uncertainty, the value of information acquisition is a continuous, smooth, monotonically decreasing and concave function of the degree of homogenisation $d \in [0, b]$.

Comparing the two organisation structures, it becomes obvious that the integrated firm has stronger incentives for information acquisition due to its concentrated decision and market power. The precision (= direct) effect serves as a floor for the monopoly but as a ceiling for the duopolists, as is exhibited by *Figure 4*.

Figure 4: Relative values of information for both organisation structures



5) CONCLUSION

This paper has analysed how the incentives to acquire information on production costs vary with the organisational structure along the entire spectrum of product differentiation. It has formalised the intuition of relative informational effects. On the one hand the concentration

of decision power implied by integration tends to make information more valuable compared to nonintegration. On the other hand, a shift along the dimension of product differentiation may have ambiguous effects on the degree of competition, depending on the underlying organisational structure. Growing product substitutability clearly raises the competitive pressure on nonintegrated duopolists and hence their incentives for information provision. For the monopoly, in contrast, increasingly homogenised goods may enhance the need for correlating the production volumes up to a certain point, beyond which the integrated firm would benefit from reducing its product line. The implied homogeneous production obviously involves no competitive forces at all, making information less valuable. In this range of highly substitutable goods the value of information is solely determined by cost efficiency considerations and the associated plant choice. To the extent that information helps to avoid unnecessary high-cost production it remains valuable.

We have shown that the often postulated better informativeness of integrated structures as opposed to nonintegrated ones can be confirmed in many, but certainly not in all, instances. In particular, this paper demonstrates that there can be interesting exceptions, especially when the products are fairly homogeneous, costs not too uncertain and demand relatively low. The mining business comes to mind again, and it would be interesting to test empirically whether large conglomerates or smaller mines invest more resources to yield more precise forecasts of ore concentration. And whether the sluggish demand of recent years had any effect on this relationship.

APPENDIX

Table 1: Expected profits and values of information under nonintegration

τ_2 / τ_1	**p**	**c**	$E[v_i \mid 2, \tau_1]$
p	$\pi_1 = \rho\left[2b(a-c_1) - d(a-c_2^e)\right]^2$ $\pi_2 = \rho\left[2b(a-c_2^e) - d(a-c_1)\right]^2$	$\pi_1 = \rho\left[2b(a-c_1) - d(a-c_2^e)\right]^2$ $\pi_2 = \rho\left[2b(a-c_2) - d(a-c_1)\right]^2 + \frac{1}{4b}\gamma$	$v_1 = 0$ $v_2 = \frac{(4b^2-d^2)^2}{4b^2}\rho\gamma \quad (DE_2)$
c	$\pi_1 = \rho\left[2b(a-c_1) - d(a-c_2^e)\right]^2$ $\pi_2 = \rho\left[2b(a-c_2^e) - d(a-c_1)\right]^2$	$\pi_1 = \rho\left[\left(2b(a-c_1) - d(a-c_2^e)\right)^2 + d^2\gamma\right]$ $\pi_2 = \rho\left[\left(2b(a-c_2^e) - d(a-c_1)\right)^2 + 4b^2\gamma\right]$	$v_1 = d^2\rho\gamma \quad (SE_1)$ $v_2 = 4b^2\rho\gamma$
$E[v_i \mid 1, \tau_2]$	$v_1 = 0 \quad (DE_1)$ $v_2 = 0$	$v_1 = d^2\rho\gamma$ $v_2 = \frac{d^2(8b^2-d^2)}{4b^2}\rho\gamma \quad (SE_2)$	$v_1 = d^2\rho\gamma \quad (TE_1)$ $v_2 = 4b^2\rho\gamma \quad (TE_2)$

where DE_i = direct effect
SE_i = strategic effect
TE_i = total effect

$\rho := \frac{b}{(4b^2-d^2)^2}$

$\gamma := p(1-p)(\bar{c}-\underline{c})^2$

Table 2: Expected profits and values of information under integration

Region D

	q_1	q_2	$E[\pi^* \mid \tau]$
τ = p	$\frac{1}{2(b^2-d^2)}\left[b(a-c_1)-d(a-c_2^e)\right]$	$\frac{1}{2(b^2-d^2)}\left[b(a-c_2^e)-d(a-c_1)\right]$	$D=\frac{1}{2(b+d)}(a-c_1)(a-c_2^e)+\frac{b}{4(b^2-d^2)}(c_1-c_2^e)^2$
τ = c	$\frac{1}{2(b^2-d^2)}\left[b(a-c_1)-d(a-c_2)\right]$	$\frac{1}{2(b^2-d^2)}\left[b(a-c_2)-d(a-c_1)\right]$	$D_c=\frac{1}{2(b+d)}(a-c_1)(a-c_2^e)+\frac{b}{4(b^2-d^2)}\left[(c_1-c_2^e)^2+\gamma\right]$
v			$DD=\frac{b}{4(b^2-d^2)}\gamma$

τ = (p,c)	$\frac{1}{2(b^2-d^2)}\left[b(a-c_1)-d(a-c_2^e)\right]$	$\frac{1}{2(b^2-d^2)}\left[b(a-c_2)-d(a-c_1)-\frac{d^2}{b}(c_2^e-c_2)\right]$	$D_{pc}=\frac{1}{2(b+d)}(a-c_1)(a-c_2^e)+\frac{b}{4(b^2-d^2)}(c_1-c_2^e)^2+\frac{1}{4b}\gamma$

Region H

Case I	q_1	q_2	$E[\pi^* \mid \tau]$
τ = p	$\frac{1}{2b}(a-c_1)$	0	$H1 = \frac{1}{4b}(a-c_1)^2$
τ = c	$\frac{1}{2b}(a-c_1)$	0	$H1 = \frac{1}{4b}(a-c_1)^2$
v			HH1 = 0

Case II	q_1	q_2		$E[\pi^* \mid \tau]$
τ = p	$\frac{1}{2b}(a-c_1)$	0		$H1 = \frac{1}{4b}(a-c_1)^2$
τ = c	0	$\frac{1}{2b}(a-\underline{c})$	if $c_2 = \underline{c}$	$A = \frac{1}{4b}\left[(a-\tilde{c})^2 + \gamma_1\right]$
	$\frac{1}{2b}(a-c_1)$	0	if $c_2 = \overline{c}$	
v				$H1A = \frac{1}{4b}[\alpha + \gamma_1] = \frac{1}{4b}p\left[(a-\underline{c})^2 - (a-c_1)^2\right]$

Region H

Case III	q_1	q_2	$E[\pi^* \mid \tau]$
τ = p	∩	$\frac{1}{2b}(a - c_2^e)$	$H2 = \frac{1}{4b}(a - c_2^e)^2$
τ = c	0	$\frac{1}{2b}(a - \underline{c})$ if $c_2 = \underline{c}$	$A = \frac{1}{4b}\left[(a - \tilde{c})^2 + \gamma_1\right]$
	$\frac{1}{2b}(a - c_1)$	0 if $c_2 = \overline{c}$	
v			$H2A = \frac{1}{4b}[\beta + \gamma_1] = \frac{1}{4b}\left[\gamma + (1-p)\left[(a - c_1)^2 - (a - \overline{c})^2\right]\right]$

Case IV	q_1	q_2	$E[\pi^* \mid \tau]$
τ = p	0	$\frac{1}{2b}(a - c_2^e)$	$H2 = \frac{1}{4b}(a - c_2^e)^2$
τ = c	0	$\frac{1}{2b}(a - c_2)$	$H2_c = \frac{1}{4b}\left[(a - c_2^e)^2 + \gamma\right]$
v			$HH2 = \frac{1}{4b}\gamma$

$\alpha = (a - \tilde{c})^2 - (a - c_1)^2$ $\quad$ $\beta = (a - \tilde{c})^2 - (a - c_2^e)^2$

$\gamma = p(1-p)(\overline{c} - \underline{c})^2$ $\quad$ $\gamma_1 = p(1-p)(c_1 - \underline{c})^2$

$\tilde{c} := p\underline{c} + (1-p)c_1$

Proof of *Lemma :* Recall that $\delta(\cdot) := b\,\frac{a-\max\{c_1,\cdot\}}{a-\min\{c_1,\cdot\}}$

i) $(a-\underline{c})(a-\overline{c}) < (a-c_1)^2$ can only be satisfied if $c_1 < \overline{c}$.

Case: $c_1 < \underline{c} < \overline{c} \quad \Rightarrow \quad \delta(\overline{c}) = b\,\frac{a-\overline{c}}{a-c_1} < \delta(\underline{c}) = b\,\frac{a-\underline{c}}{a-c_1}$

Case: $\underline{c} < c_1 < \overline{c} \quad \Rightarrow \quad \delta(\overline{c}) = b\,\frac{a-\overline{c}}{a-c_1} < \delta(\underline{c}) = b\,\frac{a-c_1}{a-\underline{c}}$

$\Leftrightarrow \quad (a-\underline{c})(a-\overline{c}) < (a-c_1)^2$

ii) $c_1 < c_2^e \quad \Rightarrow \quad c_1 < \overline{c}$

from i) $(a-\underline{c})(a-\overline{c}) < (a-c_1)^2 \quad \Rightarrow \quad \overline{\delta} = \delta(\underline{c}) = \begin{cases} b\,\frac{a-\underline{c}}{a-c_1} & \text{if} \quad c_1 < \underline{c} \\ b\,\frac{a-c_1}{a-\underline{c}} & \text{if} \quad \underline{c} < c_1 \end{cases}$

$(a-c_1)^2 < (a-\underline{c})(a-c_2^e) \quad \Rightarrow \quad \underline{c} < c_1$

$\Rightarrow \quad \overline{\delta} = \delta(\underline{c}) = b\,\frac{a-c_1}{a-\underline{c}} < \delta(c_2^e) = b\,\frac{a-c_2^e}{a-c_1}$

$c_2^e < c_1 \quad \Rightarrow \quad \underline{c} < c_1$

from i) $(a-c_1)^2 < (a-\underline{c})(a-\overline{c}) \quad \Rightarrow \quad \overline{\delta} = \delta(\overline{c}) = \begin{cases} b\,\frac{a-\overline{c}}{a-c_1} & \text{if} \quad c_1 < \overline{c} \\ b\,\frac{a-c_1}{a-\overline{c}} & \text{if} \quad \overline{c} < c_1 \end{cases}$

$(a-c_2^e)(a-\overline{c}) < (a-c_1)^2 \quad \Rightarrow \quad c_1 < \overline{c}$

$\Rightarrow \quad \overline{\delta} = \delta(\overline{c}) = b\,\frac{a-\overline{c}}{a-c_1} < \delta(c_2^e) = b\,\frac{a-c_1}{a-c_2^e}$

iii) follows from $\underline{c} < c_2^e < \overline{c}$ □

Proof of *Proposition 2:* $CE := DD - PE = \frac{d^2}{4b(b^2-d^2)}\gamma$

The first and second order derivatives

$$\frac{\partial DD}{\partial d} = \frac{\partial CE}{\partial d} = \frac{bd}{2(b^2-d^2)^2}\gamma \geq 0$$

$$\frac{\partial^2 DD}{\partial d^2} = \frac{\partial^2 CE}{\partial d^2} = \frac{b(b^2+3d^2)}{2(b^2-d^2)^3}\gamma > 0$$

are positive and DD is continuous for $d \in [0, \underline{d}]$. □

Proof of *Proposition 3:* Suppose $d > \left\{\overline{\delta}, \delta\left(c_2^e\right)\right\}$

$$\frac{\partial PE}{\partial c_1} = \begin{cases} 0 & c_1 \le \underline{c} \\ \frac{1}{2b} p(1-p)(c_1 - \underline{c}) > 0 & \underline{c} < c_1 < \overline{c} \\ 0 & \overline{c} < c_1 \end{cases}$$

$$\frac{\partial SWE}{\partial c_1} = \begin{cases} 0 & c_1 \le \underline{c} \\ \frac{1}{4b}\frac{\partial \alpha}{\partial c_1} = \frac{p}{2b}\left[a - c_1 - (1-p)(c_1 - \underline{c})\right] & \underline{c} < c_1 < c_2^e \\ \frac{1}{4b}\frac{\partial \beta}{\partial c_1} = -\frac{(1-p)}{2b}\left[a - c_2^e\right] < 0 & c_2^e < c_1 < \overline{c} \\ 0 & \overline{c} < c_1 \end{cases}$$

Since $\frac{\partial^2 \alpha}{\partial {c_1}^2} = -2p(2-p) < 0$ it suffices that

$$\left.\frac{\partial \alpha}{\partial c_1}\right|_{c_1 = c_2^e} > 0 \quad \Leftrightarrow \quad a - c_2^e - (1-p)\left(c_2^e - \underline{c}\right) > 0$$

$$\Leftrightarrow \quad a > \overline{c} - \left(p^2 - p + 1\right)(\overline{c} - \underline{c})$$

which is in view of [A2] satisfied $\forall\ p \in [0, 1]$. □

Proof of *Proposition 4:* We proceed in stages by first presenting the incentives of information acquisition in the intermediate regions and their partial derivatives.

Region MD:

If $\delta(\overline{c}) < \delta(\underline{c})$, the informed firm produces both goods if costs are low and only good 1 if costs are high, opposed to the ignorant firm which always supplies both goods:

$$v = DD(\underline{c}) = \frac{1}{4b\left(b^2 - d^2\right)}\left[b^2\gamma - (1-p)\left[b(a - \overline{c}) - d(a - c_1)\right]^2\right]$$

$$\frac{\partial DD(\underline{c})}{\partial d} = \frac{1}{2\left(b^2 - d^2\right)^2}\left[bd\gamma + (1-p)\left[b(a - \overline{c}) - d(a - c_1)\right]\left[b(a - c_1) - d(a - \overline{c})\right]\right]$$

If $\delta(\underline{c}) < \delta(\overline{c})$, the informed firm produces both goods if costs are high and only good 2 if costs are low, opposed to the ignorant firm which always supplies both goods:

$$v = DD(\overline{c}) = \frac{1}{4b\left(b^2 - d^2\right)}\left[b^2\gamma - p\left[b(a - c_1) - d(a - \underline{c})\right]^2\right]$$

$$\frac{\partial DD(\overline{c})}{\partial d} = \frac{1}{2\left(b^2 - d^2\right)^2}\left[bd\gamma + p\left[b(a - \underline{c}) - d(a - c_1)\right]\left[b(a - c_1) - d(a - \underline{c})\right]\right]$$

Region AMH:

If $\overline{\delta} < \delta(c_2^e)$, the informed firm alternates between the homogeneous production of good 1 and good 2 if costs are high and low, respectively, while the ignorant firm would still supply both goods:

$$v = DA = \frac{1}{4b(b^2-d^2)}\left[b^2\gamma - p\left[b(a-c_1)-d(a-\underline{c})\right]^2 - (1-p)\left[b(a-\overline{c})-d(a-c_1)\right]^2\right]$$

$$\frac{\partial DA}{\partial d} = \frac{1}{2(b^2-d^2)^2}\left[\begin{array}{l} bd\gamma + p\left[b(a-c_1)-d(a-\underline{c})\right]\left[b(a-\underline{c})-d(a-c_1)\right] \\ \quad + (1-p)\left[b(a-c_1)-d(a-\overline{c})\right]\left[b(a-\overline{c})-d(a-c_1)\right]\end{array}\right]$$

If $\delta(c_2^e) < \delta(\underline{c})$, the informed firm produces both goods if costs are low and only good 1 if costs are high, in contrast with the homogeneous production of good 1 by the ignorant firm:

$$v = H1D(\underline{c}) = \frac{1}{4b(b^2-d^2)}p\left[b(a-\underline{c})-d(a-c_1)\right]^2$$

$$\frac{\partial H1D(\underline{c})}{\partial d} = -\frac{p}{2(b^2-d^2)^2}\left[b(a-\underline{c})-d(a-c_1)\right]\left[b(a-c_1)-d(a-\underline{c})\right]$$

If $\delta(c_2^e) < \delta(\overline{c})$, the informed firm produces both goods if costs are high and only good 2 if costs are low, in contrast with the homogeneous production of good 2 by the ignorant firm:

$$H2D(\overline{c}) = \frac{1}{4b(b^2-d^2)}(1-p)\left[b(a-c_1)-d(a-\overline{c})\right]^2 + \frac{1}{4b}\gamma$$

$$\frac{\partial H2D(\overline{c})}{\partial d} = -\frac{(1-p)}{2(b^2-d^2)^2}\left[b(a-\overline{c})-d(a-c_1)\right]\left[b(a-c_1)-d(a-\overline{c})\right]$$

Regions D and H:

Recall *Proposition 2* and *Table 2*. Note that $\frac{\partial HH1}{\partial d} = \frac{\partial H1A}{\partial d} = \frac{\partial H2A}{\partial d} = \frac{\partial HH2}{\partial d} = 0$ in region H.

• Following the case distinctions and using the above as well as equation [2] it can be easily shown that the incentives of information acquisition of neighbouring regions coincide on the borders $\delta(E[c_2|\tau])$, as do their partial derivatives with respect to d - establishing thus the continuity and smoothness of v.

• $\overline{\delta} < \delta(c_2^e)$ is a necessary condition for monotonically nondecreasing incentives of information acquisition, because only DA has a positive partial derivative over region AMH: $\frac{\partial DA}{\partial d} \geq 0$ at the lower bound $\overline{\delta}$ and $\frac{\partial DA}{\partial d} = 0$ at the upper bound $\delta(c_2^e)$; where $\overline{\delta}$ may be $\delta(\underline{c}) = b\frac{a-c_1}{a-\underline{c}}$ or $\delta(\overline{c}) = b\frac{a-\overline{c}}{a-c_1}$, $\delta(c_2^e) = b\frac{a-c_2^e}{a-c_1}$ if $c_1 < c_2^e$ or $\delta(c_2^e) = b\frac{a-c_1}{a-c_2^e}$ if $c_2^e < c_1$. Moreover,

$$\frac{\partial^2 DA}{\partial d^2} = -\frac{1}{2\left(b^2-d^2\right)^3}\left[2(b-d)^3(a-c_1)\left(a-c_2^e\right)+b\left(b^2+3d^2\right)\left(c_2^e-c_1\right)^2\right]<0$$

• $\delta\left(c_2^e\right)<\overline{\delta}$ is sufficient for a global maximum in region MD, because the first partials are positive throughout region D including the lower bound of MD and negative at the upper border of MD as well as in region AMH, converging to 0 in region H:

Suppose $\delta(\overline{c})<\delta\left(c_2^e\right)<\delta(\underline{c})$

By the Lemma $\Rightarrow$ $c_1 \le c_2^e$ and $\left(a-c_2^e\right)(a-\underline{c}) \le (a-c_1)^2$; i.e. $DD(\underline{c})$ and $H1D(\underline{c})$ are relevant. Substituting:

1) $d=\delta(\overline{c})=b\frac{a-\overline{c}}{a-c_1}$ $\Rightarrow$ $\frac{\partial DD(\underline{c})}{\partial d}=\frac{bd\gamma}{2\left(b^2-d^2\right)^2}>0$

2) $d=\delta\left(c_2^e\right)=b\frac{a-c_2^e}{a-c_1}$ $\Rightarrow$ $\frac{\partial DD(\underline{c})}{\partial d}=\frac{\partial H1D(\underline{c})}{\partial d}=\frac{b^2p(1-p)(\overline{c}-\underline{c})}{2\left(b^2-d^2\right)^2(a-c_1)}\left[\left(a-c_2^e\right)(a-\underline{c})-(a-c_1)^2\right]<0$

3) $d=\delta(\underline{c})=b\frac{a-\underline{c}}{a-c_1}$ or $b\frac{a-c_1}{a-\underline{c}}$ $\Rightarrow$ $\frac{\partial H1D(\underline{c})}{\partial d}=0$

Furthermore $\frac{\partial^2 H1D(\underline{c})}{\partial d^2}=\frac{p}{2\left(b^2-d^2\right)^3}\left[b\left(b^2+3d^2\right)(c_1-\underline{c})^2+2(b-d)^3(a-\underline{c})(a-c_1)\right]>0.$

Suppose $\delta(\underline{c})<\delta\left(c_2^e\right)<\delta(\overline{c})$

By the Lemma $\Rightarrow$ $c_2^e \le c_1$ and $(a-c_1)^2 \le \left(a-c_2^e\right)(a-\overline{c})$; i.e. $DD(\overline{c})$ and $H2D(\overline{c})$ are relevant. Substituting:

1) $d=\delta(\underline{c})=b\frac{a-c_1}{a-\underline{c}}$ $\Rightarrow$ $\frac{\partial DD(\overline{c})}{\partial d}=\frac{bd\gamma}{2\left(b^2-d^2\right)^2}>0$

2) $d=\delta\left(c_2^e\right)=b\frac{a-c_1}{a-c_2^e}$ $\Rightarrow$ $\frac{\partial DD(\overline{c})}{\partial d}=\frac{\partial H2D(\overline{c})}{\partial d}=\frac{bdp(1-p)(\overline{c}-\underline{c})}{2\left(b^2-d^2\right)^2\left(a-c_2^e\right)}\left[(a-c_1)^2-\left(a-c_2^e\right)(a-\overline{c})\right]<0$

3) $d=\delta(\overline{c})=b\frac{a-\overline{c}}{a-c_1}$ or $b\frac{a-c_1}{a-\overline{c}}$ $\Rightarrow$ $\frac{\partial H2D(\overline{c})}{\partial d}=0$

Furthermore $\frac{\partial^2 H2D(\overline{c})}{\partial d^2}=\frac{(1-p)}{2\left(b^2-d^2\right)^3}\left[b\left(b^2+3d^2\right)(c_1-\overline{c})^2+2(b-d)^3(a-\overline{c})(a-c_1)\right]>0.$ □

Proof of *Theorem:* a) Compare the maximal value of information acquisition of the nonintegrated firms producing homogeneous goods, $TE_2 = \frac{4}{9b}\gamma$, to H1A and H2A if $c_2^e < c_1$ and $c_1 < c_2^e$, respectively. The two bounds of inequalities [3] diverge as a and $\underline{c}$ rise, and/or c_1 and $\overline{c}$ fall, while maintaining $\underline{c} < c_1 < \overline{c}$.

b) At d = b, $HH2 = \frac{1}{4b}\gamma < TE_2 = \frac{4}{9b}\gamma$. The existence then follows from the continuity of the curves.

c) At d = b, $HH1 = 0 < TE_1 = \frac{1}{9b}\gamma < TE_2 = \frac{4}{9b}\gamma$. Continuity again leads to the conclusion. □

Proof of *Proposition 5:* $TE_i = \frac{b}{(2b+d)^2}\gamma$ with $\gamma = p(1-p)(\overline{a}-\underline{a})^2$

$\frac{\partial TE_i}{\partial d} = -\frac{2b}{(2b+d)^3}\gamma < 0$ and $\frac{\partial^2 TE_i}{\partial d^2} = \frac{6b}{(2b+d)^4}\gamma > 0$

Secrecy follows from the negative strategic effect $SE_i = TE_i - DE_i = -\frac{(4b+d)d}{4b(2b+d)^2}\gamma$. □

Proof of *Proposition 6:* Noting that the value of information is given by

$DD = \frac{1}{2(b+d)}\gamma$ in Region D

$DD(\overline{a}) = \frac{1}{2(b+d)}\gamma - \frac{1-p}{4b(b^2-d^2)}\left[b(\underline{a}-c_{-i}) - d(\underline{a}-c_i)\right]^2$ in Region MD

$$HD(\overline{a}) = \frac{1}{2(b+d)}\gamma + \frac{1}{4b(b^2-d^2)}\begin{bmatrix}\left[b(a^e-c_{-i}) - d(a^e-c_i)\right]^2 \\ -(1-p)\left[b(\underline{a}-c_{-i}) - d(\underline{a}-c_i)\right]^2\end{bmatrix}$$ in Region MH

$HH = \frac{1}{4b}\gamma$ in Region H,

the values as well as first derivatives of information acquisition of neighbouring regions can easily be shown to coincide on the borders $\delta(E[a|\tau])$. Moreover

$\frac{\partial DD}{\partial d} = -\frac{1}{2(b+d)^2}\gamma < 0$

$\frac{\partial DD(\overline{a})}{\partial d} = -\frac{1}{2(b+d)^2}\gamma + \frac{(1-p)}{2(b^2-d^2)^2}\left[b(\underline{a}-c_{-i}) - d(\underline{a}-c_i)\right]\left[b(\underline{a}-c_i) - d(\underline{a}-c_{-i})\right] < 0$

because $b(\underline{a}-c_{-i}) < d(\underline{a}-c_i)$ for $\delta(\underline{a}) \le d \le \delta(a^e)$.

$$\frac{\partial HD(\overline{a})}{\partial d} = -\frac{1}{2(b+d)^2}\gamma + \frac{1}{2(b^2-d^2)^2}\begin{bmatrix}(1-p)\left[b(\underline{a}-c_{-i}) - d(\underline{a}-c_i)\right]\left[b(\underline{a}-c_i) - d(\underline{a}-c_{-i})\right] \\ -\left[b(a^e-c_{-i}) - d(a^e-c_i)\right]\left[b(a^e-c_i) - d(a^e-c_{-i})\right]\end{bmatrix} \le 0$$

because it is negative at $\delta(a^e)$, zero at $\delta(\overline{a})$ and

$\frac{\partial^2 HD(\bar{a})}{\partial d^2} = \frac{p}{2\left(b^2-d^2\right)^3}\begin{bmatrix} b\left(b^2-3d^2\right)\left(c_i-c_{-i}\right)^2 \\ +2\left(b-d\right)^3\left(\bar{a}-c_i\right)\left(\bar{a}-c_{-i}\right)\end{bmatrix} > 0$ for $\delta\left(a^e\right) \le d \le \delta(\bar{a})$; from the smoothness of the curve thus follows that $\frac{\partial HD(\bar{a})}{\partial d} \le 0$ for the entire region.

$\frac{\partial HH}{\partial d} = 0$ □

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