Different Dynamical Specifications of the Term Structure of Interest Rates and their Implications*

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Abstract

Alternative ways of introducing uncertainty to the term structure of interest rates are considered. They correspond to the different expectation hypotheses. The dynamics of the term structure is analysed in a convenient framework of stochastic equations in infinite dimensions.

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1. Introduction.

The interest rate level depends on the length of time on which it applies. For example, short term borrowing and lending is done at a different rate than for intermediate and long term loans. Such a differentiation of interest rates is called the term structure of interest rates or the yield curve.

Assume for a moment that there is no uncertainty about the future behaviour of interest rates. Then any dynamical equilibrium is characterized by the no arbitrage condition which builds the relationships between the consecutive yield curves. For example, the forward curve tomorrow must be the same as the forward curve today but shifted to the left by one day.

The uncertainty about the future interest rates, reflected by their volatility, is responsible for changes in the shape of the yield curve. There are many ways to introduce the uncertainty into the term structure dynamics. Several approaches have been suggested in the literature (cf. [1,2,3,4,6,7,8,9] and the references therein).

In this paper we introduce uncertainty to the processes of forward returns and to the corresponding discount factors. It turns out that the dynamics obtained that way lead to the different expectation hypotheses formulated and analysed in the very lucid paper by Cox–Ingersoll–Ross [1]. Contrary to their finding that only the Local Expectation Hypothesis is compatible with a Rational Expectation Equilibrium we show in this paper that also the Unbiased Expectation and the Return to Maturity Hypotheses can be used to describe the arbitrage–free dynamics of the term structure, depending on whether we look at the dynamics of the spot or the x–forward economy. The relations between these dynamics are studied in section 3 using the FX–analogy.

2. Expectation hypotheses.

We adopt the notation of the term structure model proposed by Musiela [10]. Let $r(t,x)$ denote the spot forward rate at time t for time x forward, i.e., $r(t,x)$ is the continuously compounded rate prevailing at time t for a forward contract over the time interval $[t+x, t+x+dt]$[1)]. On date $t=0$ the initial term structure is given by the forward curve $r(0,x) = r_0(x)$. If there is no volatility on the time interval $[0,t]$, then on date t the term structure will be given by $r(t,x) = r_0(x+t)$. Otherwise an investor could realize an arbitrage profit through trading in bonds. Therefore, assuming r_0 is a C^1 function, in a

1) Note that the traditionally analysed spot forward rate at time t for time x, denoted

world without uncertainty the development of the term structure over time is completely described by the initial forward curve r_0 and the condition

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right) r(t,x) = 0 \quad , \quad \forall t \geq 0 \quad , \quad \forall x \geq 0 \ . \tag{1}$$

To introduce uncertainty into the term structure movements Musiela [10] assumes that the family of rates $\{r(t,x);\ t \geq 0,\ x \geq 0\}$ is defined on the probability space $(\Omega, \{\mathrm{F}_t;\ t \geq 0\}, \mathrm{P})$ and that, for each $x \geq 0$,

$$dr(t,x) = \alpha(t,x)dt + \tau(t,x) \cdot dW(t) \ , \tag{2}$$

where W is an $n-$dimensional Brownian motion and the dot denotes scalar product. The filtration $\{\mathrm{F}_t;\ t \geq 0\}$ is the P$-$augmentation of the natural filtration of W. The process $\{\alpha(t,x);\ t,x \geq 0\}$ is locally bounded F_t-adaped with values in R. The volatility process $\{\tau(t,x);\ t,x \geq 0\}$ indexed by the time variable t and the space variable x represents the risk related to borrowing and lending at various maturities along the yield curve. It is F_t-adapted bounded on $\mathrm{R}^2_+ \times \Omega$ with values in R^n.

It is difficult to assert however that one can observe the instantaneous rates $r(t,x)$, and their risk parameters $\tau(t,x)$ in the market place. What at best we may observe are forward returns based on forward rates for some intervals $[t, t+x]$, which we denote by

$$R(t,x) = \exp\left(\int_0^x r(t,u)du\right) \ . \tag{3}$$

Alternatively we can look at the discount factors (or zero coupon bond prices)

$$D(t,x) = \exp\left(-\int_0^x r(t,u)du\right) = R(t,x)^{-1} \ , \tag{4}$$

which reflect the same observations. It seems therefore more natural to describe the dynamics and uncertainty of the term structure in terms of the observables $R(t,x)$ or $D(t,x)$. Clearly, by Ito's lemma there will always be a one-to-one correspondence between the dynamics of the processes r, R and D. But some approaches are more natural or give more economic insight than others. The aim of this paper is to clarify the implications of different dynamical specifications for r, R and D and analyse their relationships.

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by $f(t,x)$, and our $r(t,x)$ are not equal. However, there is the following obvious relationship $r(t,x) = f(t,t+x)$.

Under certainty (1) and (3) imply that, for any $x \geq 0$,

$$d \log R(t,x) = \frac{dR(t,x)}{R(t,x)} = \Big(r(t,x) - r(t,0) \Big) dt$$

or, equivalently,

$$d \log D(t,x) = \frac{dD(t,x)}{D(t,x)} = \Big(r(t,0) - r(t,x) \Big) dt \ .$$

There are now three different ways to introduce uncertainty to the observables $R(t,x)$ and $D(t,x)$ by means of the volatility process $\{\sigma(t,x);\ t,x \geq 0\}$ in analogy to the rate process (2). Depending which of the above (so far equivalent) processes we consider we obtain the following dynamics:

(5i)
$$\begin{aligned} d \log R(t,x) &= \Big(r(t,x) - r(t,0) \Big) dt + \sigma(t,x) \cdot dW(t) \ , \\ d \log D(t,x) &= \Big(r(t,0) - r(t,x) \Big) dt - \sigma(t,x) \cdot dW(t) \ , \end{aligned}$$

(5ii)
$$\frac{dR(t,x)}{R(t,x)} = \Big(r(t,x) - r(t,0) \Big) dt + \sigma(t,x) \cdot dW(t) \ ,$$

and

(5iii)
$$\frac{dD(t,x)}{D(t,x)} = \Big(r(t,0) - r(t,x) \Big) dt - \sigma(t,x) \cdot dW(t) \ .$$

As before the volatility process $\{\sigma(t,x);\ t,x \geq 0\}$ describes the risk related to borrowing and lending for various time intervals of length x along the yield curve, but now it refers to the observables $R(t,x)$ and $D(t,x)$. For all $x \geq 0$ the processes

$$\Big\{ \sigma(t,x);\ t \geq 0 \Big\},\ \Big\{ \frac{\partial}{\partial x}\sigma(t,x);\ t \geq 0 \Big\}\ ,\ \Big\{ \frac{\partial^2}{\partial x^2}\sigma(t,x,);\ t \geq 0 \Big\}$$

are assumed F_t-adapted and locally square integrable. Moreover, for all $t \geq 0$

$$\sigma(t,0) = 0 \ .$$

Proposition 1: The dynamics ($5i - 5iii$) imply that $r(t,x)$ satisfies

(6)
$$dr(t,x) = \alpha(t,x)dt + \frac{\partial}{\partial x}\sigma(t,x) \cdot dW(t) \ ,$$

where the drift term is given, respectively, by

$$(6i) \qquad \alpha(t,x) = \frac{\partial}{\partial x} r(t,x) \qquad \text{for } (5i)\ ,$$

$$(6ii) \qquad \alpha(t,x) = \frac{\partial}{\partial x}\left(r(t,x) - \frac{1}{2}|\sigma(t,x)|^2\right) \qquad \text{for } (5ii)\ ,$$

$$(6iii) \qquad \alpha(t,x) = \frac{\partial}{\partial x}\left(r(t,x) + \frac{1}{2}|\sigma(t,x)|^2\right) \qquad \text{for } (5iii)\ .$$

Proof. Relation (3) implies that $r(t,x) = \frac{\partial}{\partial x}\log R(t,x)$ what immediately gives

$$dr(t,x) = \frac{\partial}{\partial x}(r(t,x)dt + \sigma(t,x)\cdot dW(t))$$

for dynamics $(5i)$. By Ito's lemma

$$\begin{aligned} dr(t,x) = \frac{\partial}{\partial x} d\log R(t,x) &= \frac{\partial}{\partial x}\left(\frac{dR}{R} - \frac{1}{2}\ \frac{1}{R^2} d < R >\right) \\ &= \frac{\partial}{\partial x}\left(r(t,x)dt + \sigma(t,x)\cdot dW(t) - \frac{1}{2}|\sigma(t,x)|^2\, dt\right) \end{aligned}$$

and $(6ii)$ follows. The relation $r(t,x) = -\frac{\partial}{\partial x}\log D(t,x)$ leads to $(6iii)$.

Comment: Comparing the coefficients in dynamics (2) and (6) leads to

$$\sigma(t,x) = \int_0^x \tau(t,u)du\ .$$

That has an obvious interpretation. The volatility of the return on forward rates over the interval $[t, t+x]$ is the sum of the volatilities of the instantaneous forward rates. Proposition 1 shows that from a mathematical point of view there is no difference between modelling the forward rates as in the HJM-approach [4] or modelling the returns on forward rates, but the latter seem to be more natural market parameters. Moreover looking at the market rates may lead to significant simplification of the term structure models. In the HJM framework, even if we have exposure to one rate only, say over a six month period, we still need to consider all the intantaneous forward rates covering that period. Therefore to model a "one-dimensional" situation we need to use an "infinite dimensional" family of rates (cf. also Musiela [9,10]).

Proposition 1 shows that the difference in the dynamics $(5i-5iii)$ lies in the drifts of the processes $r(t,x)$. Dynamics (iii) gives "higher" forward rates as dynamics (ii), with dynamics (i) lying in between.

Proposition 2: Processes $(5i-5iii)$ satisfy, respectively,

$$(7i) \qquad \log R(t,x) = \int_t^{t+x} E\Big[r(s,0)|\mathrm{F}_t\Big]ds \ ,$$

$$(7ii) \qquad R(t,x) = E\left[\exp\left(\int_t^{t+x} r(s,0)ds\right)|\mathrm{F}_t\right] \ ,$$

$$(7iii) \qquad D(t,x) = E\left[\exp\left(-\int_t^{t+x} r(s,0)ds\right)|\mathrm{F}_t\right] \ .$$

Remark: Using the familiar notation $B(t,T) = D(t,T-t)$ for a zero-bond maturing at T, $f(t,T) = r(t,T-t)$ for the forward rate and $r_t = r(t,0)$ for the spot rate, it is easily seen that relations $(7i-7iii)$ are equivalent to the following relations:

$$(8i) \qquad B(t,T) = \exp\left(-\int_t^T E[r_s|\mathrm{F}_t]ds\right)$$

$$\Leftrightarrow \qquad f(t,s) = E[r_s|\mathrm{F}_t] \qquad \forall t \leq s$$

(= Unbiased Expectation Hypothesis),

$$(8ii) \qquad B(t,T) = \left(E\left[\exp\left(\int_t^T r_s ds\right)|\mathrm{F}_t\right]\right)^{-1}$$

(= Return to Maturity Expectation Hypothesis),

$$(8iii) \qquad B(t,T) = E\left[\exp\left(-\int_t^T r_s ds\right)|\mathrm{F}_t\right]$$

(= Local Expectation Hypothesis).

Hence the three different dynamics correspond to the three different expectation hypotheses for the continuous time term structure models (cf. Cox, Ingersoll, Ross [1], Ingersoll [5], ch. 18). As is well known the three hypotheses are not compatible under uncertainty, since Jensen's inequality implies

$$B(t,T)_{(iii)} > B(t,T)_{(i)} > B(t,T)_{(ii)} \ .$$

Proof of Proposition 2: Assume $(5i)$ holds, then by Proposition 1

$$dr(t,x) = \frac{\partial}{\partial x}\Big(r(t,x)dt + \sigma(t,x)\cdot dW(t) \Big) \ ,$$

and thus also (cf. [3])

$$r(t,x) = r(0,x+t) + \int_0^t \frac{\partial}{\partial x}\sigma(s,x+t-s)\cdot dW(s) \ .$$

For each $T>0$ the process $f(t,T) = r(t,T-t), 0 \le t \le T$, satisfies

$$f(t,T) = r(t,T-t) = f(0,T) + \int_0^t \frac{\partial}{\partial x}\sigma(s,T-s)\cdot dW(s) \ ,$$

and therefore it is a martingale. This in turn leads to $(8i)$ and hence also to $(7i)$.
For fixed x, define $T = t+x$ and consider the process $B(t,T) = D(t,T-t), 0 \le t \le T$. Then the dynamics $(5iii)$ implies that

$$\frac{dB(t,T)}{B(t,T)} = r(t,0)dt - \sigma(t,T-t)\cdot dW(t) \ .$$

Define for $0 \le t \le T$

$$Y(t,T) = B(t,T)\exp\left(-\int_0^t r(s,0)ds\right) \ .$$

Then

$$\frac{dY(t,T)}{Y(t,T)} = -\sigma(t,T-t)\cdot dW(t) \ ,$$

what implies that $Y(t,T), 0 \le t \le T$, is a martingale and hence $Y(t,T) = E(Y(T,T)|\mathrm{F}_t)$. Substituting for $Y(T,T)$ gives

$$B(t,T)\exp\left(-\int_0^t r(s,0)ds\right) = E\left[\exp\left(-\int_0^T r(s,0)ds\right)|\mathrm{F}_t\right] \ .$$

Consequently

$$D(t,x) = E\left[\exp\left(-\int_t^{t+x} r(s,0)ds\right)|\mathrm{F}_t\right] ,$$

what proves $(7iii)$.

A similar argument applied to the processes $R(t,T-t), 0 \leq t \leq T$, proves $(7ii)$.

Remark: The idea of the proof of $(7iii)$ stems from Jamshidian [6], Lemma 1, Appendix 1. See also Cox, Ingersoll, Ross [1], p. 775, footnote 10.

Proposition 2 clarifies the relationship between different intuitively plausible dynamics for a term structure model and their underlying expectation hypotheses. As it is well known, only the Local Expectation Hypothesis is consistent with an arbitrage-free model of the term structure, and hence only the dynamics $(5iii)$ is acceptable, thus the "correct" drift in (6) is

$$\alpha(t,x) = \frac{\partial}{\partial x}\left(r(t,x) + \frac{1}{2}|\sigma(t,x)|^2\right) ,$$

which was already shown by Musiela [10] using a different approach. If $(8iii)$ holds for all $0 \leq t \leq T < \infty$ then there is no arbitrage possible between the zero coupon bonds $B(\cdot, T)$ of all maturities $T > 0$ and the savings account (cf. [4,10]). The martingale representation theorem allows to represent every integrable contingent claim as a sum of its expectation and of a stochastic integral with respect to the Brownian motion W. The integrand in the stochastic integral (i.e., derivative of the claim with respect to W in the Ito integral sense) permits to derive a self financing hedging strategy. Thus the expectation is the arbitrage free price of the claim.

While there is nothing wrong with working under other expectation hypotheses[1)], one must be aware that the prices obtained by such models are not consistent across maturities[2)] and may give rise to arbitrage opportunities.

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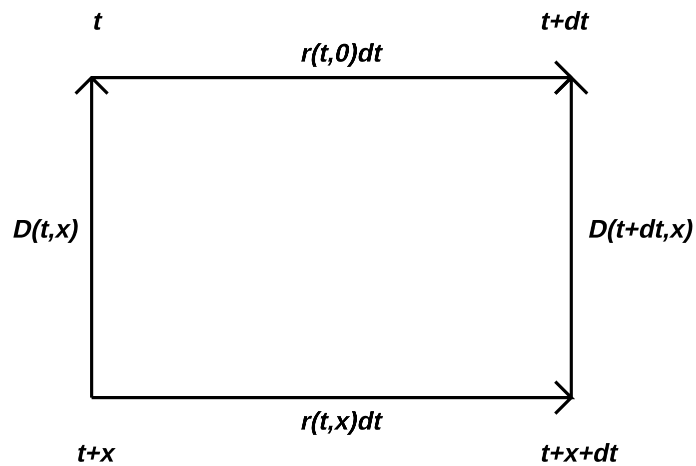
1) For a recent study of a bond price dynamics corresponding to the Return-to-Maturity Hypothesis see Platen [11]

2) Such models are still quite popular, see e.g. the Black model in pricing caplets and bond options and its many variants used by practitioners. For a review of such models see Rady-Sandmann [12].

3. FX analogy.

Note that $D(t,x)$ can be interpreted as the "spot" price at time t of \$1 available at time $t+x$. Thus, for any x, the process $D(t,x)$ behaves like an exchange rate between the spot market (dollars delivered at time t), and the $x-$forward market (dollars delivered at time $t+x$), which is illustrated by the following diagram.



Hence $D(t,x)$ is t–time price of the asset "one dollar at time $t+x$". But this asset has the continuous dividend yield $r(t,x)$. Assume that $D(t,x)$ is an Itô–process with drift $\mu(t,x)$, i.e.

$$\frac{d\ D(t,x)}{D(t,x)} = \mu(t,x)dt - \sigma(t,x)\cdot dW(t)$$

Then under the risk–neutral measure the "asset" $D(t,x)$ must have the same rate of return as the riskless asset "one dollar at time t", namely $r(t,0)$. Hence no arbitrage implies

$$\mu(t,x) + r(t,x) = r(t,0)$$

or

(9) $$\frac{d\ D(t,x)}{D(t,x)} = (r(t,0) - r(t,x))\,dt - \sigma(t,x)\cdot dW(t).$$

This argument, which is analogous to that used in pricing FX–options or options on stocks with continous dividend yields, leads directly to the dynamics $(5iii)$ resp. $(6iii)$ as the arbitrage–free dynamics for our term structure model. It also shows that the term structure model can be viewed as the classical foreign exchange rate model (with an infinite dimensional manifold of exchange rates).

For all $x \geq 0$ let P^x be the probability measure defined by

$$P^x(A) = \int_A \mathcal{E}(-N^x(T))dP$$

where for $0 \leq t \leq T$

$$\mathcal{E}\left(-N^x(t)\right) = \exp\left(-N^x(t) - \frac{1}{2} < N^x > (t)\right);$$

$$N^x(t) = \int_0^t \sigma(s,x) \cdot dW(s).$$

From the Girsanov theorem it follows that the process

$$W^x(t) = W(t) + \int_0^t \sigma(s,x)ds$$

is a Brownian motion under P^x.

Let $R(t,x) = D(t,x)^{-1}$ be the price of 1 dollar in the spot market in dollars of the x–forward market (i.e. the exchange rate where the domestic economy is the x–forward market and the foreign is the spot market). From (9) and Ito's formula it follows that

$$dR(t,x) = R(t,x)\left(\left(r(t,x) - r(t,0)\right)dt + \sigma(t,x) \cdot dW^x(t)\right) \tag{10}$$

Hence the measure P^x is the arbitrage free measure or the "foreign economy", i.e. of the x–forward market.

We could have obtained the arbitrage–free dynamics (10) of $R(t,x)$ directly by the same FX–analogy used before, but now looking at the "foreign economy" under the martingale measure P^x, observing that in this economy $r(t,x)$ plays the role of the "domestic" interest rate, whereas $r(t,0)$ is now the "foreign" rate. Note, however, that according to Proposition (2ii) the solution of (10) is

$$R(t,x) = E^x[\exp\left(\int_t^{t+x} r(s,0)ds\right)|\mathcal{F}]$$

where E^x denotes expectation under P^x. But this means that No–Arbitrage (NA) in the "foreign economy" is equivalent to the Return to Maturity Hypothesis (RTMH), and no longer to the Local Expectation Hypothesis (LEH). This is an important observation since it shows that on has to be quite careful by claiming that NA is equivalent to LEH. In fact LEH in the "spot" market is equivalent to RTMH in the "x–forward" market when viewed under the corresponding martingale measures P and P^x.

Let $\sigma(t,x) = \int_0^x \tau(t,u)du$, then the arbitrage free dynamics (6iii) of $\{r(t,x), t \geq 0, x \geq 0\}$ is given by

$$dr(t,x) = \left(\frac{\partial}{\partial x} r(t,x) + \tau(t,x) \cdot \sigma(t,x)\right) dt + \tau(t,x) \cdot W(t)$$

Define the process

$$r^y(t,x) = S(y)r(t,x) = r(t,x+y),$$

where $S(y)$ is a semigroup of left shifts. Then

$$dr^y(t,x) = d\left(S(y)r(t,x)\right) = S(y)dr(t,x)$$

$$= S(y)\left[\left(\frac{\partial}{\partial x} r(t,x) + \tau(t,x) \cdot \sigma(t,x)\right) dt + \tau(t,x) \cdot dW(t)\right]$$

$$= \left(\frac{\partial}{\partial x} r^y(t,x) + \tau(t,x+y) \cdot \sigma(t,x+y)\right) dt + \tau(t,x+y) \cdot dW(t)$$

$$= \left(\frac{\partial}{\partial x} r^y(t,x) + \tau(t,x+y) \cdot (\sigma(t,x+y) - \sigma(t,y))\right) dt$$

$$+\tau(t,x+y) \cdot (dW(t) + \sigma(t,y)dt).$$

But the process

$$W^y(t) = W(t) + \int_0^t \sigma(s,y)ds$$

is a Brownian motion under P^y and

$$\sigma(t,x+y) - \sigma(t,y) = \int_y^{x+y} \tau(t,u)du = \int_0^x \tau(t,u+y)du.$$

Therefore under P^y we can write that

$$dr^y(t,x) = \left[\frac{\partial}{\partial x} r^y(t,x) + (S(y)\tau(t,x)) \cdot \int_0^x (S(y)\tau(t,u))\, du\right] dt$$

$$+ (S(y)\tau(t,x)) \cdot dW^y(t),$$

which describes the arbitrage free dynamics of $r^y(t,x)$.

We have shown that for all $y \geq 0$ in the y–forward market the forward rates $r^y(t,x)$ and their volatilities $r^y(t,x)$ are given by

$$r^y(t,x) = S(y)r(t,x) = r(t,x+y)$$

$$\tau^y(t,x) = S(y)\tau(t,x) = \tau(t,x+y).$$

In particular it follows for $r(t,x) = r^x(t,0)$ that

$$\begin{aligned} dr(t,x) = S(x)dr(t,0) &= \frac{\partial}{\partial x} r(t,x)dt + \tau(t,x) \cdot dW^x(t) \\ &= \frac{\partial}{\partial x} r(t,x)dt + \frac{\partial}{\partial x}\sigma(t,x) \cdot dW^x(t) \end{aligned}$$

Thus the dynamics of $r(t,x)$ in the x–forward market corresponds to the dynamics $(5i)$ in the spot market. But as shown before this dynamics corresponds to the Unbiased Expectation Hypothesis. Indeed the proof of Proposition $(2i)$ gives immediately that

$$r(t,x) = E^x[r(t+x,0) \mid \mathcal{F}_t] \ , \tag{11}$$

i.e. the x–forward rate is equal to the expected spot rate in the x–forward economy. In this sense also the Unbiased Expectation Hypothesis is compatible with No–arbitrage, a result already observed for the "forward" measure by EL Karoui [2] and Jamshidian [7]. Note, however, that (11) is no longer equivalent to $(7i)$, which is the Yield–to–Maturity Expectation Hypothesis (YTM–EH), since integrating (11) leads to

$$log \ R(t,x) = \int_t^{t+x} E^s[r(s,0) \mid \mathcal{F}_t] ds \ ,$$

i.e. expected spot rates now depend on forward time s. (cp. also Miltersen [8]).

4. Stochastic equations in infinite dimensions.

One of the main advantages of the new parametrization (i.e., $r(t,x)$ rather than $f(t,T)$ of HJM) is the possibility to analyse equations $(6i - 6iii)$ describing the term structure dynamics as equations in infinite dimensions. The state space in which the processes evolve is the space of all forward curves. If we write $r(t)$ and $\sigma(t)$ to represent $\{r(t,x); \ x \geq 0\}$

and $\{\sigma(t,x);\ x\geq 0\}$ (i.e., random vectors with values in appropriate function spaces), then equations $(6i-6iii)$ take the following form

$$(9i)\qquad dr(t)=A\Big(r(t)dt+\sigma(t)\cdot dW(t)\Big)$$

$$(9ii)\qquad dr(t)=A\left((r(t)-\frac{1}{2}|\sigma(t)|^2)dt+\sigma(t)\cdot dW(t)\right)$$

$$(9iii)\qquad dr(t)=A\left((r(t)+\frac{1}{2}|\sigma(t)|^2)dt+\sigma(t)\cdot dW(t)\right),$$

where $A=\frac{\partial}{\partial x}$. Equations $(9i-9iii)$ are analysed in greater detail in [3].

REFERENCES

[1] Cox, J.C., J.E. Ingersoll, JR., and S.A. Ross (1981): 'A Re–Examination of Traditional Hypotheses about the Term Structure of Interest Rates', *The Journal of Finance*, XXXVI(4): 769–799.

[2] El Karoui, N. and Rochet, J.–C. (1989): 'A Pricing Formula for Options on Coupon–Bonds'. Working paper, Université Paris 6.

[3] Goldys, B. and Musiela, M. (1993) 'On stochastic partial differential equations describing the yield curve evolution'. in preparation.

[4] Heath, D., Jarrow, R. and Morton, A. (1992) 'Bond pricing and the term structure of interest rates: a new methodology'. *Econometrica*, **60(1)**, 77-105.

[5] Ingersoll, J. (1987) Theory of financial decision making. Totowa, NJ: Rowman and Littlefield.

[6] Jamshidian, F. (1992) 'Options and futures evaluation with deterministic volatilities' Working paper, August 1992.

[7] Jamshidian, F. (1990): 'Bond and Option Evaluation in the Gaussian Interest Rate Model'; working paper Merrill Lynch.

[8] Miltersen, K.R. (1993): 'An Arbitrage Theory of the Term Structure of Interest Rates', Working Paper, Odense University, to appear in *Annals of Applied Probability.*

[9] Musiela, M. (1991) 'General framework for pricing derivative securities'. Conference on Derivative Securities. Center for Finance Technology. University of Technology, Sydney.

[10] Musiela, M. (1993) 'Stochastic PDEs and term structure models' Miniconference on Stochastics and Finance, Canberra, Statistics Technical Reports, CSIR-002-93, SMS-017-93.

[11] Platen, E. (1993) 'An approach to bond pricing'. Miniconference on Stochastics and Finance, Canberra, Statistics Technical Reports CSIR-002-93, SMS-017-93.

[12] Rady, S. and Sandmann, K. (1994) 'The direct approach to debt option pricing' Review of Futures Markets, Vol. 13/2, 461–514.