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NEW HOPE FOR THE FISHER EFFECT?

A Reexamination Using Threshold Cointegration

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Abstract: This paper reassesses the long-run relation between nominal interest rates and inflation using German data. It shows that the empirical rejection of the strict Fisher effect in previous studies, i.e., the finding of interest rates not fully adjusting to changes in inflation, can be attributed to the particular time series behavior of inflation and interest rates which cannot be accounted for by standard non-stationary models. It is argued that the stochastic process governing the bivariate system of inflation and interest rates depends on the level of the variables and should be modeled as a threshold cointegration (TC) model. Contrary to the unit root hypothesis this model can be given an economic interpretation in terms of the opportunistic approach to disinflation. The full Fisher effect, even in its tax-adjusted form, cannot be rejected when a threshold cointegration model is estimated. The TC model not only explains the downward bias of the coefficient estimates, but also the sample and country sensitivity observed in previous studies. The TC model may prove useful in testing other long-run relations such as uncovered interest rate parity or purchasing power parity.

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1 Introduction

Ever since Irving Fisher's "The Theory of Interest," the conjecture that nominal interest rates vary, *ceteris paribus*, point-for-point with expected inflation has become one of the most studied topics in economics. The Fisher effect is a cornerstone of many theoretical models that generate monetary neutrality and is important for understanding movements in nominal interest rates.

Because changes in the value of money redistribute purchasing power between debtors and creditors, a unity response of nominal interest rates to changes in expected inflation is required to avoid such re-distributions and insulate the real rate of interest. This "full" or "strict" (point-for-point) Fisher effect, however, applies only to economies without taxes. Darby (1975), Feldstein (1976), and Tanzi (1976) have shown that because of taxation, nominal rates must change by more than the change in expected inflation if the real after-tax rate of interest is to be invariant to anticipated changes in the value of money. This effect produces an "augmented" Fisher effect. Darby (1975) suggests that the nominal rate should change by 1.3 to 1.5 times the change in expected inflation.

Generally empirical investigations, e.g., McDonald and Murphy (1989), Wallace and Warner (1993), Mishkin (1992), Phylaktis and Blake (1993), and Evans and Lewis (1995), tend to support the notion that movements in nominal interest rates primarily reflect fluctuations in expected inflation. However, support for the full Fisher effect has been scant. In line with Fisher's (1930) own results, changes in inflation generally seem to have less than a point-for-point effect on nominal rates, suggesting that expected inflation is non-neutral. Furthermore, the strength of this effect depends heavily on the period and country considered.

There have been several rationalizations of the apparent failure of the strict Fisher hypothesis. Fisher himself explained it by some form of money illusion, while Tobin (1969) stressed the negative effect of inflation rates on money demand. Mishkin (1984) argues that the failure of the full Fisher effect hypothesis is due to the negative correlation between inflation and the real interest rate.

This paper takes a completely different approach and extends the literature on the Fisher effect in a new direction. It argues that the rejection of the full Fisher effect is due to a downward bias of the coefficient estimates that results from a failure to adequately model the stochastic features of the data generating process. It suggests that this failure also accounts for the sample and country sensitivity of the results reported in previous studies. To model the particular time series behavior of inflation and interest rates, the threshold cointegration (TC) model is introduced and estimated. Contrary to the finding with conventional cointegration techniques, the conjecture that interest rates respond to inflation in the way the full Fisher effect suggests is confirmed. The contribution of the paper may be of relevance not only for the Fisher effect, but also for other long-run relations involving inflation and/or interest rates, e.g. uncovered interest rate parity or purchasing power parity.

The remainder of the paper consists of five sections. The next section presents the Fisher hypothesis more formally and reexamines standard findings in the literature that suggest the full Fisher effect does not hold. Section 3 provides a critical assessment of the variables' time series properties and questions the appropriateness of the unit root hypothesis. Then, section 4 introduces the threshold cointegration model and shows how the presence of threshold cointegration leads to a bias in coefficient estimates of standard cointegration regressions. In section 5 the Fisher effect is reexamined using a TC-model. Finally, section 6 concludes the paper.

2 Testing for the Fisher effect using cointegration techniques

Starting with Rose (1988), several empirical studies recently recognized the importance of accounting for non-stationarity when testing for the Fisher effect and pointed to the danger of spurious regressions when not taking the nonstationarity into account (see Granger and Newbold, 1974). The appropriate framework for the analysis of non-stationary variables is the cointegration theory put forward by Engle and Granger (1987). The cointegration analysis will detect the eventual long-run linkages between the nominal interest rate and the inflation rate.

More formally, if $i_t(m)$ denotes the *m*-period nominal interest rate at time t, $\pi_t^e(m)$ the expected rate of inflation from time t to $t+m$, and $r_t^e(m)$ the corresponding *ex-ante* real interest rate, then the Fisher equation reads

$$1+i_t(m)=[1+\pi_t^e(m)][1+r_t^e(m)],$$

which, for low rates of inflation, can be approximated by:

$$i_t(m)=\pi_t^e(m)+r_t^e(m). \tag{1a}$$

Accounting for tax effects implies that nominal interest rates adjust by more than expected inflation, i.e., $1/(1-\tau)$, where τ stands for an appropriate marginal tax rate.[1]

In equation (1) both $\pi_t^e(m)$ and $r_t^e(m)$ are unobservable and hence must be proxied. Assuming rational expectations $\pi_t^e(m)=\pi_t(m)+\varepsilon_t$, where ε_t is a mean-zero random disturbance orthogonal to any information available at time *t*. Since the Fisher equation is interpreted as a long-run equilibrium relation between integrated variables, the real rate has only to be assumed stationary.[2] Thus, testing for the Fisher effect requires estimating the following cointegration equation:

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[1] See also McCulloch (1977) for a critique of the supposition that the Fisher effect should work leveraged up by $1/1-\tau$.

[2] See Fisher (1930) and Summers (1983) for an explanation of why the Fisher hypothesis does not necessarily hold in the short-run. It is important to underscore that equation (1b) does not assume that the real rate is constant and equal to α. However and more precisely, in equation (1b) the real rate follows a stationary process with *mean* α.

$$i_t(m) = a + b p_t(m) + m_t \tag{1b}$$

Three cases are conceivable in this bivariate non-stationary context: First, the two variables are not cointegrated, that is there is no long-run relation between them at all. Second, there is a long-run relation between the two variables, but the cointegration vector does not correspond to the strict Fisher equation. This case corresponds to a "weak" Fisher effect. Third, a cointegration relation corresponding to the strict Fisher equation (b equals unity or $1/(1-t)$ in the tax-adjusted case) is observed between the two variables. Obviously, the monetary neutrality of expected inflation holds only in the third case.

Previous empirical studies have either not found cointegration between inflation and interest rates or obtained mixed results that are sensitive to the chosen time period and country. Rose (1988) analyzes the univariate time series properties of inflation and interest rates with standard univariate unit-root tests. He finds that nominal and real interest rates possess a unit root, whereas inflation does not. Straightforwardly, this implies that the Fisher effect must be rejected since changes in the real interest rate dominate changes in expected inflation as source of changes in the nominal interest rate which is contrary to what Fisher (1930) had in mind.

MacDonald and Murphy (1989) investigate the long-run relationship between inflation and interest rates using the Engle-Granger (1987) cointegration methodology. They obtain very different results depending on whether the sample covers the fixed or flexible exchange rate regime. Overall, they conclude that nominal interest rates and inflation tend to drift apart and therefore the Fisher effect has to be rejected. Applying the same econometric approach Mishkin (1992) reexamines the Fisher effect in the postwar United States. Although he finds that the two variables share a common stochastic trend, i.e., they are nonstationary but cointegrated, his coefficients are generally very imprecisely estimated. In addition, Mishkin observes that the validity of the Fisher effect depends heavily on the period considered, with the Fisher effect being most apparent in periods when there is strong evidence for stochastic trends.

Working on postwar US data as well, both Wallace and Warner (1993), and Crowder and Hoffman (1996) apply the fully-efficient Johansen (1988) cointegration estimator which has several advantages over the Engle and Granger two-step technique. Unlike the Engle-Granger specification, the long-run coefficient estimates do not depend on the essentially arbitrary choice of normalization, i.e., on the choice of the left-hand-side variable. Moreover, this approach does not suffer from the small sample bias of the Engle-Granger static regression. Generally, their results tend to confirm a one-to-one relationship between inflation and interest rates, but are also sensitive to the time period, country, and data frequency.

Evans and Lewis (1995) also corroborate a long-run (cointegration) relation between nominal interest rates and inflation. They show that nominal rates move less than one-for-one with inflation and attribute this finding to structural shifts in the inflation process as these may induce small sample serial correlation in forecast errors. Finally, Phylaktis and Blake (1993) observe drastic differences between low and high inflation economies. For high-inflation countries they find strong evidence of a full Fisher effect. The results are at best mixed for low-inflation countries, however.

In the next paragraphs, the empirical evidence of previous studies is reexamined, using conventional univariate unit-roots tests and the Johansen (1988) test for cointegration. The data are monthly observations of the German 12-month interest rate on T-bills and the consumer price index (CPI) from January 1967 to June 1996. The 12-month maturity was chosen because of its widespread use in the empirical literature. The inflation series is calculated as the annual percentage change in CPI shifted one year forward. Both series are taken from the German Bundesbank’s datatape. Figure 1 plots the two series.

[FIGURE 1 ABOUT HERE]

The univariate stationarity of the variables is examined using the Phillips-Perron (1988) test in a model excluding a deterministic trend. The nonstationarity of both series is clearly accepted at conventional significance levels.

Table 1. Unit Root and Cointegration Tests

	Unit Root Tests		Long-Run Fisher Equation Estimates			
	π	i	LR(0)	LR(1)	β	s.e.
PP(5)	1.94	2.38	21.42**	3.05	0.51	0.17

Notes: PP is the Phillips-Perron test statistic that allows for a constant with a 5-th order MA-correction. The long-run Fisher equation is estimated using the Johansen (1988) technique assuming no deterministic trend in the data. The vector error-correction-model (VECM) includes 12 lags. LR(p) is the likelihood-ratio test for p long-run relations. s.e. stands for the coefficient estimates' standard error. †, * and ** indicate significance at the 10, 5, and 1 percent level.

To test for cointegration between the two variables the maximum-likelihood procedure of Johansen (1988) is used. The evidence in Table 1 suggests the existence of one cointegration vector between the nominal interest rate and the inflation rate 12 months ahead. In accordance with previous studies, the cointegration vector does not correspond to a full Fisher effect, and even less to a tax-adjusted one. The estimate of the β-coefficient ($\hat{b}$) falls well below unity. The full Fisher effect can be formally tested by using a Wald test of the null hypothesis that $\hat{b}$ equals unity, which is asymptotically distributed as $c^2(1)$. The test statistic is 8.40 and thus the Fisher effect is rejected at the 1% significance level. Since the Johansen procedure has been shown to be sensitive to the number of lags included in the vector error correction model (VECM), I reexamined the results using various lag lengths ranging from 8 to 14 lags. However, the rejection of the full Fisher effect proves quite robust to variations in lag length.

3 The unit-root hypothesis revisited

There are two problems with representing nominal interest rates and inflation as unit root processes. First, standard unit root tests of the Dickey-Fuller (1979) or Phillips-Perron (1988) type are known to have low power against alternatives with roots close to but different from unity. Recently, Pippenger and Goering (1993) have shown that the power of these tests falls dramatically under threshold processes. Loosely speaking, this means that the researcher is likely to accept the null of a unit root even if the true process is a threshold process and therefore these tests should be interpreted with caution.

Second and more importantly, the unit root hypothesis is hard to reconcile with economic intuition and "stylized" facts. Saying that inflation and the nominal interest rate are non-stationary variables which are tied together in the long-run by a cointegration relation means that random shocks have permanent effects and the variables do not tend to return to their means. Thus, persistently high and negative realizations are entirely consistent with this representation. While theoretically under a fiat standard nothing assures monetary stability, a Central Bank committed to price stability will not allow inflation rates to become negative or persistently high. Generally, when inflation is "too" high there is likely to be a public pressure on the Central Bank to conduct a more restrictive monetary policy and to bring inflation down to "tolerable" regions. If this pressure prevails, like in Germany and the post-war United States, the inflation rate may become mean reverting for high levels of the series.[3] Besides, for nominal interest rates there is a clear lower bound since they cannot fall below zero. But also inflation rates seldom realize below zero, i.e., deflation is clearly the exception.

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[3] Sometimes this public pressure does not overcome, e.g. in high inflation countries and post World War I Germany, and inflation and interest rates are driven by a stochastic trend. Therefore, the upper trigger value is country specific and depends on the institutional framework as well as the society's preferences. While for some countries these factors may have changed over the last 30 years, for Germany they can be assumed constant. See Barsky (1987) for details on the dramatic differences in inflation persistence under different monetary regimes.

Therefore, in most industrial countries the random walk does not seem to be the best approximation of the stochastic process governing inflation and interest rates. A first visual comparison of the series in Figure 1 with a typical unit root realization already casts some doubt on the appropriateness of the unit root hypothesis.

Recently, Orphanides and Wilcox (1996) examined theoretically a regime dependent Central Bank behavior which is generally labeled the "opportunistic approach to disinflation." Depending on the convexity of the cost functions (costs of inflation versus costs of output deviations) and the capability of the Central Bank to fine-tune the inflation rate, the opportunistic approach is shown to be superior to the conventional approach. Since the Central Bank behavior implied by the opportunistic approach reflects in a path-dependent stochastic process for inflation and interest rates, i.e., a three regime threshold process, I think it useful to briefly review the approach. Contrary to the conventional policymaker, who pursues his goal of price stability $p*$ no matter the level of inflation, the opportunistic policymaker (though he has the same long-run objective) changes his behavior depending on the level of inflation. Whenever the inflation rate falls in a band of "tolerable" inflation the policymaker does not conduct an active policy, but merely engage in a policy of watchful waiting. Furthermore, Orphanides and Wilcox (1996) suppose that policymakers focus on stabilizing output and employment around their potential levels when inflation is inside the band. Consequently, in this region inflation and interest rates will be driven by the accumulation of random shocks, meaning that they are non-stationary and potentially cointegrated.

However, if the inflation rate realizes outside the band of "tolerable" inflation the opportunistic policymaker actively pursues a monetary policy that aims to bring back inflation inside the band. In turn, this means that inflation and interest rates are dominated by mean-reversion if the lower or upper threshold is violated. Note also, that this Central Bank behavior does not preclude the inflation rate from realizing outside the band, even during extended time intervals. The time the process spends outside the band depends on the error-variance and the strength of the mean reversion. Figure 2a depicts the optimal policy rules of the conventional and opportunistic policymaker and Figure 2b represents a linear approximation of the implied response function in the $(\Delta p, p_{-1})$ diagram. Evidently, the same holds true for the nominal interest rate.

[FIGURE 2A AND 2B ABOUT HERE]

Summing up, under the opportunistic view inflation and interest rates exhibit high persistence within a band of “tolerable” values. Outside the band, however, the time series properties of the variables change dramatically, becoming mean reverting. If policymakers follow an opportunistic approach, the technique used to estimate long-run relations between inflation and interest rates must take the level-dependence of the stochastic process into consideration. Traditional approaches to estimating the Fisher effect cannot cope with this peculiarity of the data generating process; however, the threshold cointegration model, introduced below, can.

4 Threshold cointegration

Threshold processes were first applied in biology and physics to model systems the behavior of which changes once they reach a point of saturation. The relationship between precipitation and river flow, for example, is best described by a threshold model. See Tong (1983) for a general discussion of these models. With respect to purchasing power parity Davutyan and Pippenger (1990) and Balke and Fomby (1995) argue that transaction costs may prevent agents from adjusting continuously which leads to discrete adjustment processes best approximated by a threshold model. Econometrically speaking, this implies that the error-correction mechanism is inactive inside a range determined by transaction costs and then becomes active once deviations from equilibrium exceed a critical threshold. Hence, the equilibrium error is modeled as a threshold autoregression that is mean-reverting outside a given range and has a unit root inside this range.

This paper differs in two important aspects. First, given the arguments outlined in the previous section, the variables themselves are modeled as threshold processes. Second, the emphasis of this study is on the ramifications of threshold processes on the long-run parameter estimates, i.e., the bias in estimating long-run relations induced by undetected threshold cointegration. I proceed with a brief description of the piecewise linear threshold cointegration model employed. The behavior of this bivariate TC model (and of course this extends in a straightforward manner to all multivariate generalizations) crucially depends on the value of a trigger, i.e. the level of the variables themselves. Therefore, these models are also known as self-exciting (SETAR) models.

In 'normal times' the long-run relation between the two non-stationary variables acts as an attractor. This means that the variables follow a non-stationary cointegrated process similar to the one described by a conventional error correction model. When the economy is in 'times of crises,' that is when agents perceive inflation and interest rates as being too high or too low, the variables themselves are targeted and the series exhibit mean-reversion.

The piecewise linear TC model can be represented by:

$$\begin{pmatrix}\Delta p_t\\ \Delta i_t\end{pmatrix}=\begin{pmatrix}c_1\\ c_2\end{pmatrix}+\begin{pmatrix}g_1\\ g_2\end{pmatrix}s_{t-1}(1-I(p_{t-1})-J(p_{t-1}))+\sum_{l=1}^{m}\Phi_l\begin{pmatrix}\Delta p_{t-l}\\ \Delta i_{t-l}\end{pmatrix}+$$
$$+\begin{pmatrix}k_{L,1}\\ k_{L,2}\end{pmatrix}(T_L-p_{t-1})I(p_{t-1})+\begin{pmatrix}k_{H,1}\\ k_{H,2}\end{pmatrix}(p_{t-1}-T_H)J(p_{t-1})+\begin{pmatrix}e_t^i\\ e_t^r\end{pmatrix} \tag{2}$$

with $I(p_{t-1})=\begin{cases}0 \text{ for } p_{t-1}\geq T_L\\ 1 \text{ for } p_{t-1}<T_L\end{cases}$,

$J(p_{t-1})=\begin{cases}0 \text{ for } p_{t-1}\leq T_H\\ 1 \text{ for } p_{t-1}>T_H\end{cases}$, and $s_t=i_t-a-bp_t$

where I and J represent indicator functions which take the value 0 if the corresponding inflation rate in the prior period is in the interval of “tolerable” values delimited by an upper (T_H) and lower threshold (T_L) and denoted $A^{\mathrm{p}} = (T_L^{\mathrm{p}}, T_H^{\mathrm{p}})$. In contrast, whenever the corresponding inflation rate is regarded as unusually high or low, that is, the realization one period ago lies outside the interval of “tolerable” values, the corresponding indicator function equals 1 and the process becomes mean-reverting. Or put differently, once the thresholds are violated, i_t and π_t behave like stationary series and are pulled towards their long-run means.[4]

The values of the κ-coefficients represent the strength of the mean-reversion. Given the different quality of the lower and upper threshold the behavior of the series below and above A may differ. In particular, I expect the k_H to be negative, while the k_L may well be positive, implying that the series are resolutely pushed away from zero and show exponential growth below the lower threshold.

Unfortunately, necessary and sufficient conditions for stationarity of the system (2) are still not well understood (see Balke and Fomby, 1995). Chan, Petrucelli, Tong, and Woolford (1985) have, however, derived necessary and sufficient conditions for the stationarity of a multiple threshold AR(1) model (SETAR[l;1,...,1]). At any rate, the stationarity of the system depends only on the nature of the process in the outer regimes. So long as in the outer regimes the system is pushed back towards the band it is stationary.

The observed behavior of the process depends on the time spent in between the thresholds. The more time the process stays inside the band, the

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[4] Note that the multiplication of the error correction term with 1 minus the indicator functions could be dropped without changing the subsequent results by much. The main arguments still hold. Furthermore, the negligence of the error correction term outside the band of “tolerable” inflation does not mean that inflation and interest rate will diverge, since they are both pulled back into the band.

more the process behaves non-stationary. Of course, the stronger the mean-reversion outside the thresholds, the less time the process spends outside the band. Another parameter of interest is the distance between the thresholds, i.e., the width of the band. Holding the other parameters constant, as $T_H - T_L$ rises the amount the process is outside the thresholds will decrease and the more the system will behave like a unit root process. Finally, the error-variance will also affect the behavior of the system. Holding the other parameters constant, an increase in the error-variance is tantamount to an increase in the frequency with which the process jumps outside the band and will make the system look more stationary.

Conventional approaches to estimate the long-run relationships between inflation and interest rates are misspecified under the threshold alternative; the estimates of a linear model would reflect the average across the three regimes and thus produce biased estimates of the long-run relation. The long-run coefficients would be unbiased only if the sample contains exclusively realizations inside the band. Whether a sample of given size is likely to fulfill this condition (and thus produces unbiased estimates) depends on the parameter values discussed above and especially on the level of the trigger variable. Moreover, the longer the observation period the more likely are we to find all three regimes and therefore the biased coefficients. Generally, the longer the sample period and the closer the level of the trigger is to the thresholds the more often the thresholds will be crossed and the more severe will be the bias.

Threshold cointegration is also consistent with the empirical findings of Mishkin (1992) who observes that the empirical validity of the Fisher effect is linked with the strength of evidence for stochastic trends. His b-estimates are smallest over the full sample (February 1964-December 1986) and the period from November 1979 to October 1982 where inflation reached postwar highs and thus the upper threshold should have bitten. Furthermore, in high inflation countries, the upper threshold is apparently not effective and conventional cointegration techniques should lead to less biased estimates. This might explain why there is more evidence in favor of the Fisher effect in high inflation countries than in low inflation countries (see Phylaktis and Blake, 1993).

To illustrate the potential dangers of applying conventional cointegration techniques to time series generated by a TC data generating process (DGP), 1000 observations of the series x and y according to the following simple model were generated:

$$\begin{pmatrix} \Delta x_t \\ \Delta y_t \end{pmatrix} = \begin{pmatrix} 0 \\ -0.1 \end{pmatrix} s_{t-1}(1 - I(y_{t-1}) - J(y_{t-1})) + \begin{pmatrix} 0.2 \\ 0.2 \end{pmatrix}(T_L - p_{t-1})I(y_{t-1}) + \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix}(p_{t-1} - T_H)J(y_{t-1}) + \begin{pmatrix} e_t^1 \\ e_t^2 \end{pmatrix}$$

$$\text{with } I(p_{t-1}) = \begin{cases} 0 \text{ for } y_{t-1} \geq 2 \\ 1 \text{ for } y_{t-1} < 2 \end{cases}, \quad s_t = y_t - 1.5x_t$$

$$J(p_{t-1}) = \begin{cases} 0 \text{ for } y_{t-1} \leq 6 \\ 1 \text{ for } y_{t-1} > 6 \end{cases}, \text{ and } \mathrm{Var}\begin{pmatrix} e_t^1 \\ e_t^2 \end{pmatrix} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}$$

The long-run relation between the two variables was estimated using three methods: First, the Engle-Granger (1987) method involving a simple regression of x_t on y_t ;[5] second, the simplest alternative dynamic single equation approach proposed by Sims, Stock, and Watson (1990) which consists in estimating a model like

$$\Delta y_t = c + d_y y_{t-1} + d_x x_{t-1} + f\,\Delta x_t + e_t \tag{3}$$

[5] It is well known, that the static OLS estimates of cointegrating vectors are subject to finite-sample biases. See, e.g., Banerjee *et al.* (1993) for more details. For the Monte-Carlo simulations below the sample size was chosen quite large (T=1000) to partly avoid this problem. However, since the dynamic regression leads to almost the same results and the bias becomes insignificant once one of the thresholds is ineffective, the conclusions do not depend on the choice of a particular regression method.

in which the long-run parameter β is implicitly equal to δ_x / δ_y ; third and finally, a TC-adjusted version of the second approach, i.e. the estimation of the following equation:

$$\Delta y_t = c + \delta_y y_{t-1} + \delta_x x_{t-1} + \phi \, \Delta x_t + \beta^I (4 - y_{t-1}) I(.) + \beta^J (y_{t-1} - 10) J(.) + \varepsilon_t . \tag{4}$$

Although in general, the single equation approaches are not efficient, in the particular application presented here x_t is weakly exogenous assuring efficiency. Note that, since all regressions include non-stationary variables, the standard asymptotic distribution theory does not apply to the parameter estimates of interest. Consequently, in the estimation exercises below the distribution of the parameters has to be proxied with resampling techniques such as the bootstrap.

Figures 3 to 5 depicts the frequency plots of the three estimators using 1000 Monte Carlo replications with the sample size, *T,* set to 1000. The estimation exercises neglecting the threshold term significantly underestimate the true long-run coefficient. The significant downward bias is robust to quite drastic changes in the DGP, including the abandon of the weak exogeneity assumption, an error correction mechanism that operates over the whole range of y_t , and negative values of κ_L . Figures 6 and 7 show the mean bias of the Engle-Granger method and the 90 percent confidence band for various values of the lower and upper threshold coefficients. It can be seen that, whenever the thresholds are effective, there is a relatively constant downward bias in the coefficient estimate. This holds for white noise processes outside the band, as well as for processes with an AR-coefficient as large as 0.9. The next section estimates a TC model and the long-run relationship between inflation and interest rates for German data.

[FIGURES 3 TO 7 ABOUT HERE]

5 A reevaluation

Iterative estimation of system (2) including 12 lags was carried out conditional on given lower and upper thresholds and then the thresholds were chosen to minimize the sum of squared errors.[6] For the thresholds a grid with a step size of 0.25 was used. Experiments taking alternatively inflation and interest rates as threshold variable revealed that the latter models generally performed better in terms of explanatory power; therefore in what follows the behavior of the system depends on level of the interest rate. If the thresholds were effective during the sample period the maximum likelihood should be achieved for thresholds lying above the minimum of the series and below maximum value of the series. Actually, a lower threshold of 5.01 and an upper threshold of 8.26 was retained, which seems plausible given the time series plot in Figure 1. Whenever the interest rate moves below or above the thresholds there appears to be a strong tendency to return into the band.

Assume that the real interest rate is constant and equals 3.5 percent (which corresponds to the mean difference between the interest rate and inflation). This suggests that when inflation is between 1.5 and 4.7 percent, the Bundesbank does not take deliberate action, rather it waits hoping that favorable shocks will bring inflation back towards its long run objective. When prices are

[6] See Balke and Fomby (1995) for a similar approach. According to Chan (1993), the estimated threshold value of a two regime autoregressive model is super-consistent and the asymptotic distribution of the estimates of the AR-parameters is independent of the estimated threshold values. Presumably, his results can be extended to the three regime case of model (2) and inference about the model's parameters can proceed as with known threshold values. For computational ease the long-run relation was assumed to equal the full Fisher effect in the grid search procedure. In subsequent computations (e.g. Table 2) all the parameters in the system are freely estimated; evidently b is restricted to be equal across equations. Technically speaking, the grid search was performed using seemingly unrelated regressions (SUR), whereas the results of Table 2 were obtained by multivariate non-linear SUR. True maximum likelihood techniques generally yield the similar results, but are very (computer-) time consuming.

rising rapidly, however, the Bundesbank engages in a restrictive policy to bring inflation back in the range of "tolerable" values.

I proceed with a somewhat less cumbersome version of (2) neglecting all insignificant lagged endogenous variables. Table 2 presents the parameter estimates and some regression diagnostics.

Table 2. Estimation of the Threshold Cointegration Model

Equation	b	k_L	k_H	Lags (Δp,Δi)	$\overline{R}^2$	LM(12)	ARCH
Δp		0.13	-0.10	1,8,10,12	0.32	7.36	0.05
	1.46	(0.05)	(0.02)	4,6,7,8			
Δi	(0.63)	0.06	-0.10	1,3,6,11	0.15	9.87	0.01
	[0.58,2.84]	(0.08)	(0.03)	1,5,9			

Note: Estimation by multivariate non-linear least squares. Standard errors in parentheses. The standard error of the b-estimate has been bootstrapped using 1,000 replications, the numbers in brackets correspond to the 5% and 95% fractiles of the bootstrapped distribution. LM(12) is a general LM test of 12-th order serial correlation distributed as $c^2(12)$, whereas ARCH is a test for first order autoregressive conditional heteroscedasticity (ARCH) distributed as $c^2(1)$.

Obviously, neither serial correlation nor autoregressive conditional heteroscedasticity (ARCH) seem to be present in the final specification retained. The threshold coefficients all show the expected sign, i.e. are positive for realizations below the band and negative when the interest rate rises above the band, and are significant except for k_L in the interest rate equation.

In contrast to the results neglecting threshold cointegration, the b-estimate now lies well above unity which is predicted when tax effects are considered. The parameter estimates presented in Table 2 are entirely consistent with the Fisher effect in either its full or tax-adjusted form. It is noteworthy, that these results are robust to changes in the lag length and slight variations of the thresholds.

6 Conclusion

In this paper, I reexamine the long-run relationship between inflation and interest rates using cointegration techniques. Based upon estimates using the Johansen (1988) approach, I show that the nominal rate moves less than point-for-point with inflation so that there appear to be permanent movements in ex post real interest rates. However, I attribute this rejection of the Fisher effect to the failure of conventional non-stationary cointegrated models to fully describe the time series behavior of inflation and interest rates.

Instead, I propose a threshold cointegration model which can account for the stylized fact that, at least in most industrialized countries, inflation and interest rates seldom occur outside some narrow band of “normal” values. Threshold cointegration not only explains the serious downward bias in parameter estimates, but also the sample sensitivity observed in previous studies. When a bivariate threshold cointegration model is estimated to reexamine the long-run relationship between interest rate and inflation one cannot reject that nominal interest rates vary one-for-one with inflation. Despite the imperfections of the present approach, it seems evident that the threshold model is a better approximation of the true underlying DGP than the conventional linear model. Finally, note that undetected threshold cointegration may not only affect estimates of the Fisher equation, but also of other long-run relations, e.g., uncovered interest rate parity or purchasing power parity. Obviously further research is needed.

GRAPHS AND FIGURES

Figure 1. German inflation and 12-month interest rate

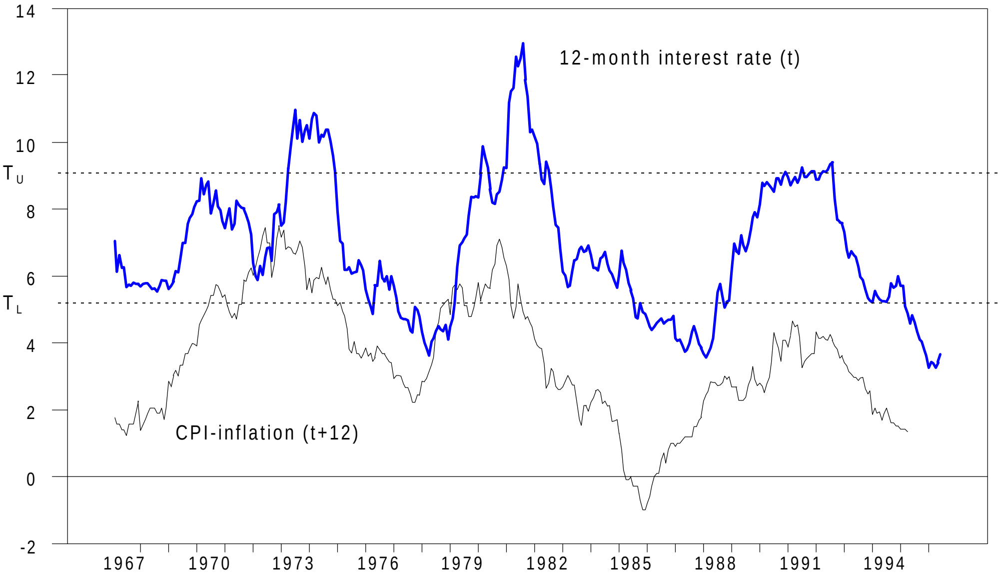
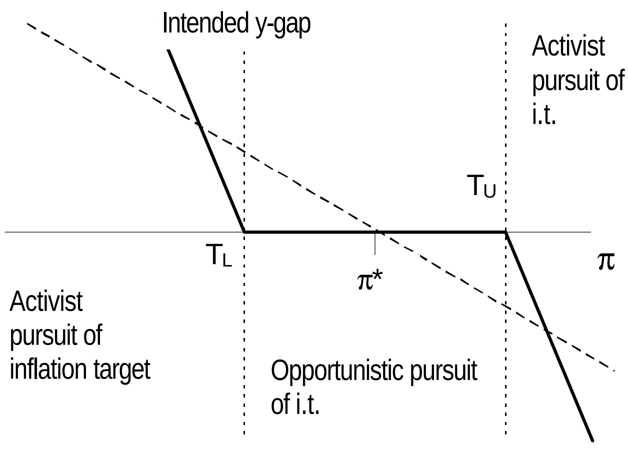


Figure 2a. Decision rules of the conventional and opportunistic policymaker



Note: The optimal decision rule of the conventional policymaker is drawn as a dashed line, whereas that of the opportunistic policymaker is drawn in bold.

Figure 2b. Response function under the opportunistic approach

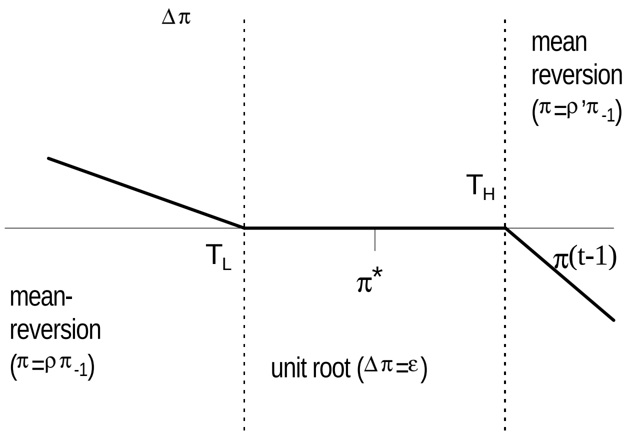


Figure 3. Distribution of LR coefficient with Engle-Granger

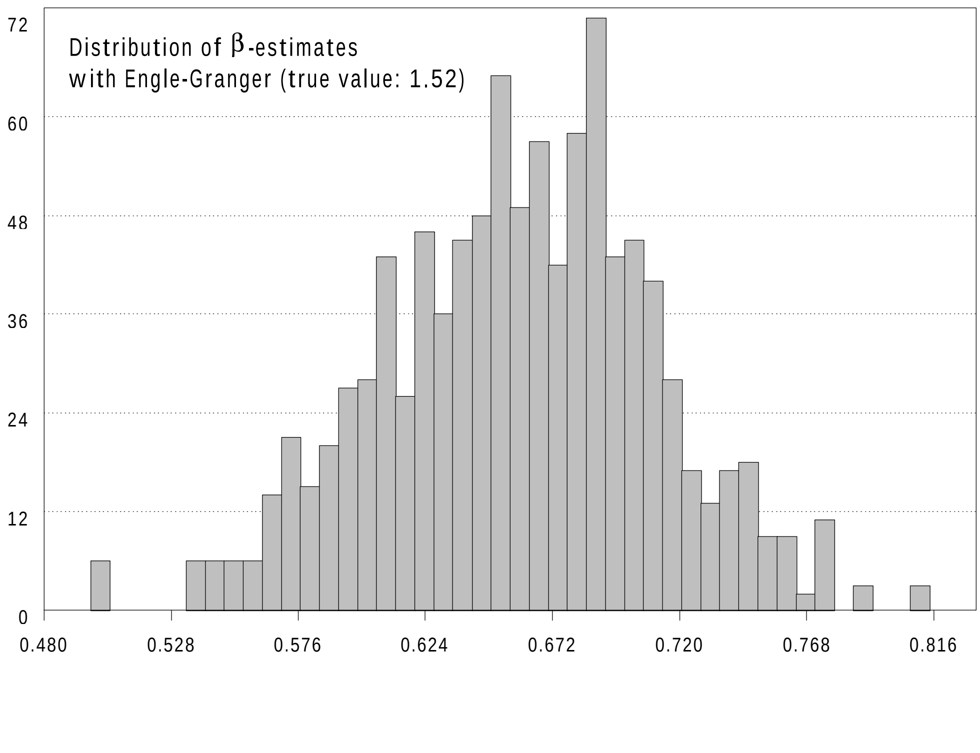


Figure 4. Distribution of LR coefficient with equation (3)

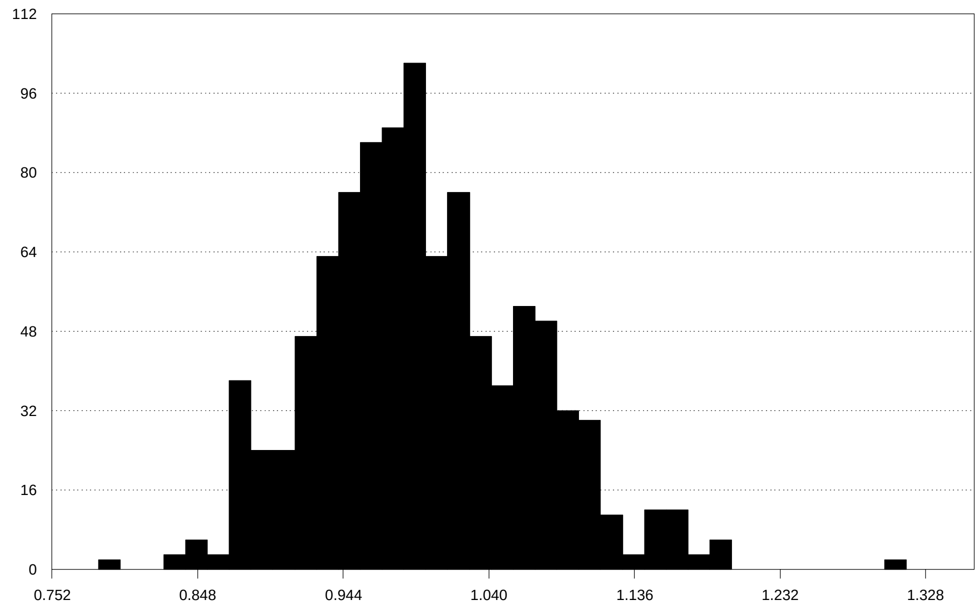


Figure 5. Distribution of LR coefficient with equation (4)

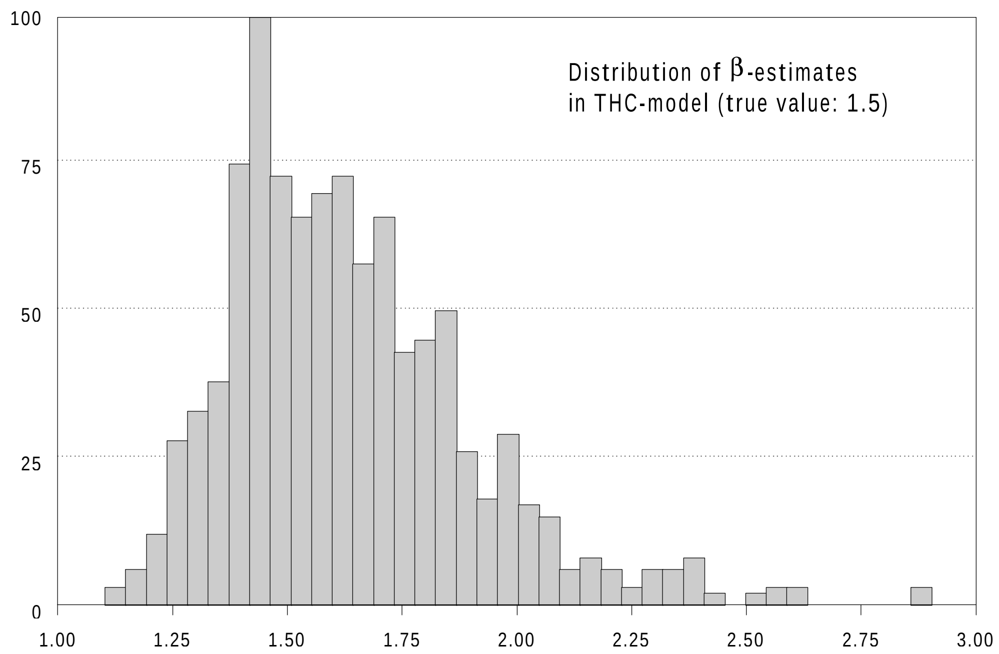


Figure 6. Bias of LR coefficient for various values of κ_L

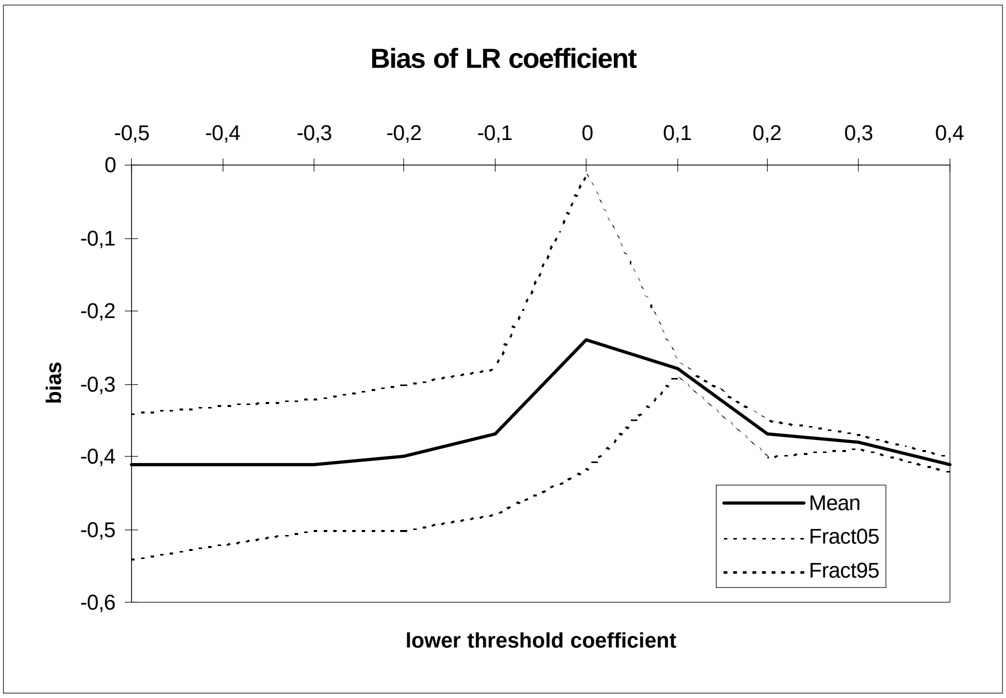
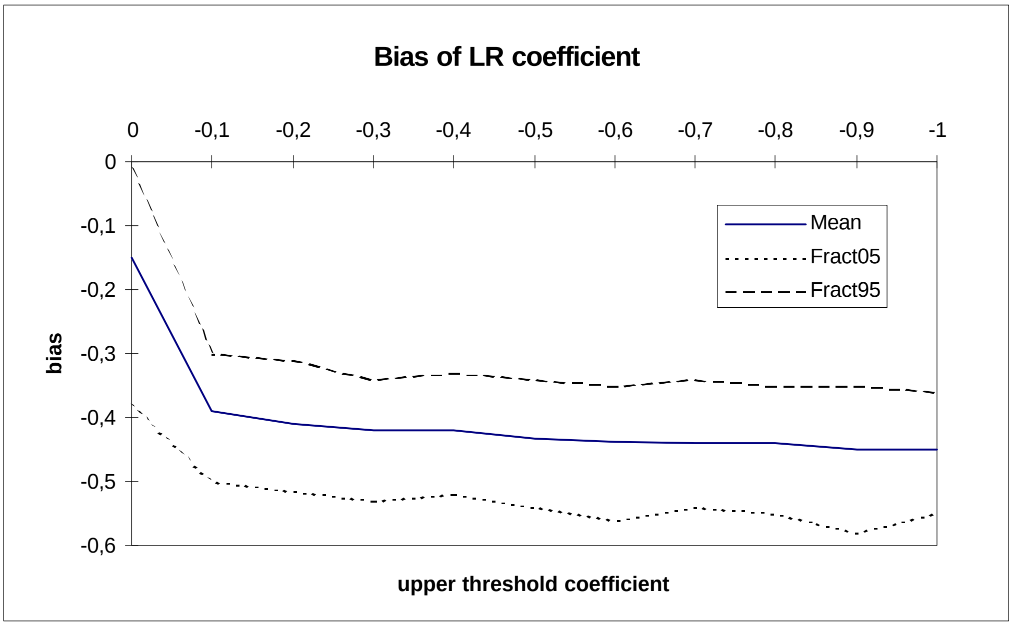


Figure 7. Bias of LR coefficient for various values of κ_H



References

Balke, N. and T. Fomby. 1995. "Threshold Cointegration." *Mimeo., Southern Methodist University.*

Banerjee, A., J. Dolado, D. Hendry and G. Smith. 1986. "Exploring Equilibrium Relationships in Econometric Theory Through Static Models: Some Monte Carlo Evidence." *Oxford Bulletin of Economics and Statistics,* 48, 253-277.

Banerjee, A., J. Dolado, J. Galbraith and D. Hendry. 1993. *Co-Integration, Error-Correction, and the Econometric Analysis of Non-Stationary Data.* (Oxford: Oxford University Press).

Barsky, R. 1987. "The Fisher Hypothesis and the Forecastability and Persistence of Inflation." *Journal of Monetary Economics,* 19, 3-24.

Chan, K. 1993. "Consistency and Limiting Distribution of the Least Squares Estimator of a Threshold Autoregressive Model." *The Annals of Statistics,* 21, 520-533.

Chan, K., J. Petrucelli, H. Tong, and S. Woolford. 1985. "A Multiple-Threshold AR(1) Model." *Journal of Applied Probability,* 22, 267-279.

Crowder W. and D. Hoffman. 1996. "The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited." *Journal of Money, Credit and Banking,* 28, 102-118.

Darby, M. 1975. "The Financial and Tax Effects of Monetary Policy on Interest Rates." *Economic Inquiry,* 33, 266-276.

Davutyan, N. and J. Pippenger. 1990. "Testing Purchasing Power Parity: Some Evidence of the Effects of Transaction Costs." *Econometric Reviews,* 9, 211-240.

Dickey, D. and W. Fuller. 1979. "Distribution of the Estimators for Autoregressive Time Series with a Unit Root." *Journal of the American Statistical Association,* 74, 427-431.

Engle, R. and C. Granger. 1987. "Co-integration and Error Correction: Representation, Estimation, and Testing." *Econometrica,* 55, 251-276.

Evans, M. and K. Lewis. 1995. "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?" *Journal of Finance,* 50, 225-253.

Feldstein, M. 1976. "Inflation, Income Taxes, and the Rate of Interest: A Theoretical Analysis." *American Economic Review,* 66, 809-820.

Fisher, I. 1930. *The Theory of Interest.* (New York: MacMillan).

Granger, C. and P. Newbold. 1974. "Spurious Regressions in Econometrics." *Journal of Econometrics,* 2, 111-120.

Johansen, S. 1988. "Statistical Analysis of Cointegration Vectors." *Journal of Economics Dynamics and Control,* 12, 231-254.

Johansen, S. and K. Juselius. 1990. “Maximum Likelihood Estimation and Inference on Cointegration—With an Application to the Demand for Money.” *Oxford Bulletin of Economics and Statistics,* 52, 169-210.

MacDonald, R. and P. Murphy. 1989. “Testing for the Long-Run Relationships between Nominal Interest Rates and Inflation Using Cointegration Techniques.” *Applied Economics,* 21, 439-447.

McCulloch, Huston. 1977. “Mises and Darby-Feldstein Reconciled: A Comment.” *Mimeo.* Boston College.

Mishkin, F. 1984. “The Real Interest Rate: A Multi-Country Empirical Study.” *Canadian Journal of Economics,* 17, 283-311.

Mishkin, F. 1992. “Is the Fisher Effect for Real?” *Journal of Monetary Economics,* 30, 195-215.

Orphanides, A. and D. Wilcox. 1996. “The Opportunistic Approach to Disinflation.” *Board of Governors of the Federal Reserve System,* Finance and Economics Discussion Paper 96-24.

Phillips, P. and P. Perron. 1988. “Testing for a Unit Root in Time Series Regressions.” *Biometrika,* 75, 335-346.

Phylaktis, K. and D. Blake. 1993. “The Fisher Hypothesis: Evidence from Three High Inflation Economies.” *Review of World Economics*, 129, 591-599.

Pippenger, M. and G. Goering. 1993. “A Note on the Empirical Power of Unit Root Tests under Threshold Processes.” *Oxford Bulletin of Economics and Statistics,* 55, 4, 473-481.

Rose, A. 1988. “Is the Real Interest Rate Stable?” *Journal of Finance,* 43, 1095-1112.

Schwert, W. 1989. “Testing for Unit Roots: A Monte Carlo Investigation.” *Journal of Business and Economics Statistics,* 7, 147-159.

Sims, C., J. Stock and M. Watson. 1990. “Inference in Linear Time Series Models with Some Unit Roots.” *Econometrica,* 58, 113-144.

Summers, L. 1983. “The Nonadjustment of Nominal Interest Rates: A Study of the Fisher Effect.” in: *Macroeconomics, Prices, and Quantities: Essays in Memory of Arthur M. Okun,* edited by J. Tobin (Washington: Brookings Institution).

Tanzi, V. 1976. “Inflation, Indexation, and Interest Income Taxation.” *Banca Nazionale del Lavoro Quarterly Review,* 29, 54-76.

Tobin, J. 1969. “A General Equilibrium Approach to Monetary Theory.” *Journal of Money, Credit and Banking,* 1, 15-29.

Tong, H. 1990. *Non-linear Time Series: A Dynamical System Approach.* Oxford University Press: Oxford.

Wallace, M. and J. Warner. 1993. "The Fisher Effect and the Term Structure of Interest Rates: Test of Cointegration." *Review of Economics and Statistics,* 320-324.