Monetary Policy and Business Cycles with Endogenous Entry and Product Variety*

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Abstract

This paper studies the role of endogenous producer entry and product creation for monetary policy analysis and business cycle dynamics in a general equilibrium model with imperfect price adjustment. Optimal monetary policy stabilizes producer prices, but lets the consumer price index vary to accommodate changes in the number of available products. The free entry condition links the price of equity (the value of products) with marginal cost and markups, and hence with inflation dynamics. No-arbitrage between bonds and equity links the expected return on shares, and thus the financing of product creation, with the return on bonds, affected by monetary policy via interest rate setting. This new channel of monetary policy transmission through asset prices restores the Taylor Principle in the presence of capital accumulation (in the form of new production lines) and forward-looking interest rate setting, unlike in models with traditional physical capital. We also study the implications of endogenous variety for the New Keynesian Phillips curve and business cycle dynamics more generally, and we document the effects of technology, deregulation, and monetary policy shocks, as well as the second moment properties of our model, by means of numerical examples.

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1 Introduction

A large body of literature has grown since the mid 1980s that analyzes monetary policy in microfounded, dynamic, stochastic, general equilibrium (DSGE) models of the business cycle with monopolistic competition and nominal rigidity. The importance of this 'New Keynesian' literature (summarized, for instance, by Woodford, 2003) for policymaking is testified by the current use of such models by many central banks or international institutions as input for policy decisions.[1] Most of this literature, however, relies on monopolistic competition merely as a vehicle to introduce price (or wage) setting power and then assume that price (or wage) setting is not frictionless, resulting in nominal rigidity and a role for monetary policy. The overwhelming majority of models abstracts from free entry and assumes a constant number of producers. The joint assumption of monopolistic competition and no entry raises two questions: one pertaining to modeling and the other to empirical evidence. First, absent either properly designed markup-offsetting subsidies or increasing returns of appropriate degree, monopolistic competition in these models results in permanent (i.e., steady-state) positive profits, casting doubts on the theoretical appeal of the zero-entry assumption.[2] Furthermore, recent empirical evidence for the U.S. has substantiated the endogenous fluctuations in the number of producers and the range of available goods that take place over the typical length of a business cycle. A previous literature documented the strong procyclical behavior of net producer entry (measured either as incorporated firms or as production establishments).[3] Bernard, Redding, and Schott (2006) document how existing establishments devote a substantial portion of their production to goods that they did not previously produce. Axarloglou (2003) and Broda and Weinstein (2007) directly measure the introduction of new varieties in the U.S. economy and document a strong correlation with the business cycle. These theoretical and empirical observations suggest that there is scope for introducing producer entry and product creation in models with monopolistic competition and imperfect price adjustment, and studying the consequences of endogenous product variety for business cycle propagation and policy in these models.

This paper takes an initial step in this direction by re-introducing the endogenous link between product creation (firm entry) and monopolistic competition in a DSGE model with imperfect price

[1] See, for instance, the IMF's GEM model (illustrated by Laxton and Pesenti, 2003, among others) and the Federal Reserve Board's SIGMA model (illustrated by Erceg, Guerrieri, and Gust, 2005, among others).

[2] Rotemberg and Woodford (1995) addressed the implausibility of positive steady-state profits by assuming increasing returns to scale induced by fixed, per-period costs. However, under this assumption, any shock that causes profits to fall below zero should generate exit and induce a non-linearity in firm decisions.

[3] See Campbell (1998), Chatterjee and Cooper (1993), and Devereux, Head, and Lapham (1996a,b). We illustrate similar evidence in Bilbiie, Ghironi, and Melitz (2005).

adjustment. We explore the positive and normative consequences of endogenous producer entry and product variety over the business cycle in a version of the model developed by Bilbiie, Ghironi, and Melitz (2005 – henceforth, BGM) featuring nominal rigidity of a standard form often used in the recent New Keynesian literature – a quadratic cost of price adjustment as in Rotemberg (1982).[4]. Producer entry and product creation subject to sunk entry costs are at the heart of the model, with producer entry under monopolistic competition tied to household saving decisions via the household's choices of share holdings in the portfolio of firms that operate in the economy.[5] In BGM, we show that a benchmark version of the flexible-price model with C.E.S. preferences and labor as the only factor of production performs similarly to the standard real business cycle (RBC) framework with regard to the cyclicality of U.S. macroeconomic aggregates that are normally the focus of RBC studies, but it additionally allows us to explain such features of the data as the procyclicality of firm entry and profits. When preferences are of the translog form, the model additionally generates a countercyclical markup, owing to the positive relation between substitutability and the number of products with such preferences, and it matches the time profile of the correlation between the markup and GDP thanks to the slow adjustment of the stock of producers over time. Importantly, the model generates a countercyclical markup while still preserving the procyclicality of profits – a well known challenge for the benchmark New Keynesian model with sticky prices.[6]

The introduction of endogenous product variety in a sticky-price model of the business cycle allows us to address issues that are absent in existing, fixed-variety models, as well as to qualify some of the results of those models in the presence of this new margin. To start with, one-sector, fixed-variety models usually do not contain a meaningful distinction between producer and consumer price indices (and inflation rates). Therefore, those models are silent as to what is the inflation rate that the central bank should stabilize. In a model with endogenous variety such as ours, a meaningful distinction between the two price indices arises, because the welfare-relevant

[4] We choose the Rotemberg model over the familiar Calvo (1983)-Yun (1996) setup to avoid heterogeneity in prices within and across cohorts of price setters that entered at different dates. Earlier flexible-price, business cycle models with monopolistic competition and endogenous entry include also Ambler and Cardia (1998) and Cook (2001). Comin and Gertler (2006), Jaimovich (2004), Jovanovic (2006), and Stebunovs (2006) are more recent contributions to the theoretical literature. See BGM for a discussion of the relation with our model.

[5] There is a one-to-one identification between a product, a producer, and a firm in our model. For consistency with much literature, we routinely use the word firm to refer to an individual productive unit. But the latter is best thought of as a production line associated to the specific good it produces, potentially introduced at a pre-existing, multi-product firm whose managers are each in charge of a specific production line. The setup of our model is such that we can remain agnostic as to where the boundaries of firms are.

[6] When we augment the model to include physical capital in production of existing goods and creation of new production lines, the model does better than the standard RBC framework at matching volatility and persistence of U.S. GDP. However, a high rate of capital depreciation is required for local determinacy and stability.

consumer price index varies with the number of varieties (it is cheaper to satisfy a given level of demand with more varieties) for given producer price level. Otherwise put, the relative price of each good is increasing with the number of varieties. We show that, when price rigidity concerns price setting for individual goods, optimal policy should stabilize producer prices (the average price of output) rather than the welfare-consistent consumer price index. The issue of what inflation rate should be targeted by policy is also related to an empirically relevant measurement problem that occurs because CPI data do not account for the introduction of new goods in the welfare-consistent manner prescribed by the model. As a consequence, the observed CPI is a biased measure of the welfare-based cost-of-living index, as documented by a recent and growing literature – see e.g. Broda and Weinstein (2006). Any implementation of normative prescriptions needs to address this issue: If the central bank were to target the cost of living as measured by CPI inflation and there were substantial bias in its measurement, interest rate setting should reflect that bias (this point is made for example by Broda, 2004). However, since we find that in the presence of endogenous product creation (precisely of the form that would induce a bias in CPI data) the central bank should not aim to stabilize CPI inflation, the measurement bias seems to be rather innocuous from a normative perspective as long as the central bank stabilizes the appropriate price index – unlike in Broda (2004), who argues for stabilizing CPI inflation as a policy prescription. Moreover, we show that even if the central bank were obliviously targeting measured CPI inflation, the bias would paradoxically work in the 'right' direction of inducing the central bank to target an index that is in fact closer to accounting for producer prices than to welfare-consistent cost of living.

Our framework also suggest a new motive for price stability as a desirable policy prescription. Since, as in Rotemberg (1982), price adjustment costs are deducted from firm profits, and these costs are proportional to (squared) producer price inflation, the latter acts as a distortionary tax on firm profits in our model. This tax distorts the allocation of resources to product creation (versus production of existing varieties) and induces a suboptimal amount of product variety in each period. This is an intuitive explanation for why the central bank should pursue producer price stability in our model, and an extra argument for price stability absent from fixed-variety models.

Turning to implications of our model that qualify results from fixed-variety models, but remaining in the area of policy prescriptions, it is by now conventional wisdom from the benchmark fixed-variety model without physical capital that the central bank should follow what has become known as the Taylor Principle. This policy prescription requires that the central bank be 'active,' in the sense of increasing the nominal interest rate more than one-to-one in response to increases in inflation.[7] Perhaps surprisingly, however, the introduction of physical capital in the fixed-variety model changes this prescription dramatically, as shown by Dupor (2001) in a continuous-time model and further developed by Carlstrom and Fuerst (2005) in discrete time and in the presence of adjustment costs. Dupor shows that 'passive' interest rate setting (a less than proportional response to inflation) is necessary and sufficient for local determinacy and stability, while Carlstrom and Fuerst conclude that it is essentially impossible to achieve determinacy with forward-looking interest rate setting. In contrast to these results, the Taylor Principle holds in our economy in which capital accumulation takes the form of creating new production lines, regardless of whether the monetary authority responds to expected or current producer price inflation.[8]

The Taylor Principle is restored with our form of capital accumulation precisely because our framework features an endogenous price of capital that plays a crucial role in monetary policy transmission. Indeed, we show that free entry implies that the price of equity shares (the value of the firm) appears in the New Keynesian Phillips curve that governs the dynamics of inflation. Moreover, a no-arbitrage condition links the real return on bonds (which the central bank affects by setting the nominal interest rate) to the real return on equity – the ratio of next period's dividends and share price to the current price of equity. This identifies a novel channel of monetary policy transmission that links interest rate setting to equity prices and, through the free entry condition, inflation. In a nutshell, a temporary interest rate cut reduces the real return on bonds, inducing the household to consume more today and the price of equity to rise today relative to tomorrow. Since the price of equity (the value of the firm) is related to marginal cost by the free entry condition in our model, marginal cost also rises, inducing a fall in the markup and, by the Phillips curve, an increase in inflation. This transmission of monetary policy through the price of equity is absent in standard, fixed-variety models even when those models do feature an endogenous price of capital due to adjustment costs (see Carlstrom and Fuerst, 2005). The validity of the Taylor Principle with current or forward-looking interest rate setting is an important result (in the light of findings by Dupor and Carlstrom and Fuerst), which could be interpreted as a relative advantage of modeling capital accumulation as investment toward the creation of new product lines associated to the introduction of new varieties (the extensive margin), rather than reproducing the same good (intensive margin).

[7] Kerr and King (1996) and Clarida, Galí, and Gertler (2000) were the first to derive this result in the now standard New Keynesian framework. Leeper (1991) has a related discussion.

[8] The same holds for welfare-consistent CPI inflation, subject to the caveat implied by our normative analysis – that monetary policy should not target welfare-consistent consumer prices in our model.

Further implications of explicitly modeling endogenous product creation pertain to inflation and markup dynamics. As in the standard fixed-variety model, a New Keynesian Phillips curve relating producer price inflation to its expected value and the current markup holds in our model. However, endogenous product creation has important consequences for empirical exercises that estimate Phillips curves. First, in the presence of endogenous variety, the markup is not simply the inverse of the labor share of income, as in Sbordone (2002) or Galí and Gertler (1999). In our model, the markup can be expressed as the inverse of a labor share in consumption output, controlling for labor used to set up new production lines (labor which is 'overhead' from an aggregate perspective). A close proxy for this labor share has been estimated by Rotemberg and Woodford (1999), and it is the relevant variable that should be used to estimate the Phillips curve in the presence of endogenous variety. Moreover, we propose an alternative proxy for the markup based on the inverse of the share of profits in consumption, which is 'model-free', in the sense that it could be used regardless of one's stand on product creation. Furthermore, we identify an 'endogeneity bias' in the identification of what the literature commonly labels 'cost-push shocks' (see e.g. Clarida, Galí, and Gertler, 1999): In the presence of endogenous variety, the Phillips curve features an extra term that depends on the number of available varieties. This term would be attributed to cost-push shocks by a researcher using a markup proxy that does not account for variety when estimating the Phillips curve. Finally, it has been pointed out (e.g. Fuhrer and Moore, 1995) that one of the main drawbacks of the forward-looking New Keynesian Phillips curve is its failure to generate endogenous inflation persistence. We show that our version of the Phillips curve can potentially alleviate this problem, because the number of varieties featured in the Phillips curve is a state variable, and hence it induces extra persistence in inflation.

Numerical examples show that the responses to aggregate productivity and deregulation shocks under simple, but plausible specifications of interest rate setting are very close to the flexible-price responses. Exogenous interest rate cuts induce the economy to expand, but reduce entry because of the unfavorable effect of inflation on firm profits. With productivity shocks as the source of fluctuations and an empirically plausible, simple rule for interest rate setting involving interest rate smoothing and a response to expected producer price inflation, the cyclical properties of endogenous variables are very close to those of the flexible-price counterpart and, in turn, to those of the benchmark RBC model, as documented by BGM. In contrast to the flexible-price model with translog preferences studied in BGM, sticky prices with C.E.S. preferences yield too much markup countercyclicality and a highly counterfactual time profile of this cyclicality. This happens because the markup is no longer tied to the number of producers as in BGM-translog. On the bright side, aggregate profits remain procyclical (consistent with stylized facts) even in the presence of a very countercyclical markup, and the model remains able to explain the procyclicality of business creation.

Producer entry and product creation pose an interesting question for the modeling of nominal rigidity. In our model, a new entrant in period t starts producing (and setting prices) only in period $t+1$, capturing a realistic time-to-build lag in the development of new production lines. When a new entrant makes its first price setting decision, we must take a stand on whether it operates as all pre-existing producers do, subject to the same nominal rigidity – thus preserving the symmetry across producers that is a feature of the Rotemberg (1982) model –, or whether it sets its price in flexible fashion, but knowing that it will face a cost of adjusting its price in all subsequent periods. We begin our analysis by assuming that new entrants inherit the same price rigidity as pre-existing firms. This considerably simplifies the model and allows us to obtain an initial set of analytical and numerical results. We then turn to the model in which new entrants set prices in flexible fashion, but knowing that they will be subject to a cost of price adjustment from the following period on.[9] In this case, nominal rigidity results in heterogeneity in price levels across cohorts of producers that entered the economy at different points in time, and the aggregate degree of nominal rigidity is endogenous: Expansions are associated with lower aggregate rigidity because the number of new entrants whose decision is not influenced by past price setting increases. We show that the log-linear version of this extended model can still be solved in tractable fashion, and we explore the consequences of endogeneity in aggregate rigidity by means of numerical examples. Plausible parameter values imply responses to shocks that are virtually indistinguishable from those of the benchmark model. Since we assume that average product turnover is realistically small at quarterly frequency, small changes in the fraction of firms that set prices in more flexible fashion triggered by shocks have negligible aggregate consequences, and the benchmark model in which new entrants inherit the same price adjustment cost as incumbents yields robust conclusions.

Finally, we explore the consequences of non-C.E.S. preferences by replacing the familiar Dixit-Stiglitz (1977) variety aggregator with a general, homothetic specification of symmetric preferences – parametrized in translog form for model solution purposes. This implies that the elasticity of substitution across products increases with the number of producers, introducing an additional

[9] For completeness of comparison, we also consider a version of the model in which new price setters simply set their initial price as a constant markup over marginal cost.

effect of the number of available goods on inflation in the New Keynesian Phillips curve. In our numerical examples, this extension yields conclusions that are similar to those of the benchmark model, although it further improves the performance of the model on the inflation persistence front.

Lewis (2006) and Mancini-Griffoli and Elkhoury (2006) develop models with nominal rigidity that are closest to the one studied here. Lewis introduces monopoly power in the labor market and sticky wages in familiar Calvo (1983)-Yun (1996) fashion into the BGM model. She documents VAR evidence that monetary policy expansions result in increased firm entry by boosting aggregate demand, and she shows that the sticky-wage model reproduces this evidence. Mancini-Griffoli and Elkhoury assume that entry costs in the BGM model take the form of fees paid to lawyers with monopoly power. Under nominal rigidity, the lawyers set the entry fees in Calvo-Yun fashion and, as in Lewis, a monetary expansion that boosts the economy results in increased firm entry. The positive effect of inflationary shocks on firm entry in these models follows from the assumption that inflation has no direct, distortionary effect on entry decisions. Bergin and Corsetti (2005) document VAR evidence on the consequences of exogenous changes in monetary policy for entry similar to that in Lewis' paper. They set up a model with entry and one-period price rigidity that replicates this evidence, and they characterize optimal monetary policy and the properties of shock transmission in their model. As in Lewis' and Mancini-Griffoli and Elkhoury's models, Bergin and Corsetti's result that monetary expansions induce increased firm entry stems from a modeling of nominal rigidity that implies no direct costs of inflation to the firms, other than the inefficiency of pre-set prices. In our setup with costs of price adjustment, flexible wages, and exogenous entry costs, inflation taxes firm profits, and an exogenous decrease in the interest rate generally reduces producer entry.[10,11]

The rest of the paper is organized as follows. Section 2 presents our benchmark model. Section 3 obtains the results on optimal monetary policy in the benchmark setup. Section 4 discusses the implications of endogenous entry and product variety for the New Keynesian Phillips curve. Section 5 studies monetary policy through interest rate setting in our model. Section 6 illustrates the business cycle properties of the model. Section 7 discusses the main results of the extensions we explore: the alternative assumption on initial price setting by new entrants and non-C.E.S.

[10] Depending on the response of policy to endogenous variables, this negative effect of an inflationary shock can be limited to the impact period if labor supply is sufficiently elastic, but a negative contemporaneous effect is a robust feature of the scenarios we consider.

[11] The models in Lewis (2006) and Mancini-Griffoli and Elkhoury (2006) are subject to one of the main problems that our approach aims to address: They rely on monopoly power as a stepping stone for nominal rigidity, but they abstract from entry (by workers or lawyers) in the presence of monopoly profits.

preferences. Section 8 concludes.

2 The Model

Household Preferences and the Intratemporal Consumption Choice

We consider a cashless economy as in Woodford (2003). The economy is populated by a unit mass of atomistic, identical households. The representative household supplies L_t hours of work in each period t in a competitive labor market for the nominal wage rate W_t and maximizes expected intertemporal utility $E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}U\left(C_s, L_s\right)\right]$, where C_t is consumption and $\beta \in (0,1)$ the subjective discount factor. The period utility function takes the form $U\left(C_t, L_t\right) = \ln C_t - \chi\left(L_t\right)^{1+1/\varphi}/\left(1+1/\varphi\right)$, $\chi > 0$, where $\varphi \geq 0$ is the Frisch elasticity of labor supply to wages, and the intertemporal elasticity of substitution in labor supply.

At time t, the household consumes the basket of goods C_t, defined over a continuum of goods Ω: $C_t = \left(\int_{\omega\in\Omega} c_t\left(\omega\right)^{\theta-1/\theta} d\omega\right)^{\theta/(\theta-1)}$, where $\theta > 1$ is the symmetric elasticity of substitution across goods. At any given time t, only a subset of goods $\Omega_t \subset \Omega$ is available. Let $p_t\left(\omega\right)$ denote the nominal price of a good $\omega \in \Omega_t$. The consumption-based price index for the home economy is then $P_t = \left(\int_{\omega\in\Omega_t} p_t\left(\omega\right)^{1-\theta} d\omega\right)^{1/(1-\theta)}$, and the household's demand for each individual good ω is $c_t\left(\omega\right) = \left(p_t\left(\omega\right)/P_t\right)^{-\theta} C_t$.

Firms

There is a continuum of monopolistically competitive firms, each producing a different variety $\omega \in \Omega$. Production requires only one factor, labor. Aggregate labor productivity is indexed by Z_t, which represents the effectiveness of one unit of labor. Productivity is exogenous and follows an $AR(1)$ process in percent deviation from its steady-state level. Output supplied by firm ω is $y_t\left(\omega\right) = Z_t l_t\left(\omega\right)$, where $l_t\left(\omega\right)$ is the firm's labor demand for productive purposes. The unit cost of production, in units of the consumption good C_t, is w_t/Z_t, where $w_t \equiv W_t/P_t$ is the real wage.

Prior to entry, firms face a sunk entry cost of $f_{E,t}$ effective labor units, equal to $w_t f_{E,t}/Z_t$ units of the consumption good. There are no fixed production costs. Hence, all firms that enter the economy produce in every period, until they are hit with a "death" shock, which occurs with probability $\delta \in (0,1)$ in every period. We assume that the entry cost $f_{E,t}$ is exogenous and treat changes in $f_{E,t}$ as changes in market regulation.

Firms face nominal rigidity in the form of a quadratic cost of adjusting prices over time (Rotemberg, 1982). Specifically, the real cost (in units of the composite basket) of output-price inflation volatility around a steady-state level of inflation equal to 0 facing firm ω is:

$$PAC_t(\omega) \equiv \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2 \frac{p_t(\omega)}{P_t} y_t^D(\omega).$$

This expression is interpreted as the amount of marketing materials that the firm must purchase when implementing a price change. We assume that this basket has the same composition as the consumption basket. The cost of adjusting prices is proportional to the real revenue from output sales, $(p_t(\omega)/P_t)\, y_t^D(\omega)$, where $y_t^D(\omega)$ is firm ω's output demand.

Firms face demand for their output from consumers and firms themselves when they change prices. In each period, there is a mass N_t of firms producing and setting prices in the economy. When a new firm sets the price of its output for the first time, we appeal to symmetry across firms and interpret the $t-1$ price in the expression of the price adjustment cost for that firm as the notional price that the firm would have set at time $t-1$ if it had been producing in that period. An intuition for this simplifying assumption is that all firms (even those that are setting the price for the first time) must buy the bundle of goods $PAC_t(\omega)$ when implementing a price decision.[12] It should be noted, however, that this assumption is entirely consistent both with the original Rotemberg (1982) setup and with our timing assumption. Specifically, new entrants behave as the (constant number of) price-setters do in Rotemberg's framework, where an initial condition for the individual price is dictated by nature. In our framework, new entrants at any time t who start producing and setting prices at $t+1$ are subject to precisely the same assumption as price setters in Rotemberg's original setup. Moreover, the assumption that a new entrant, at the time of its first price setting decision, knows the average market price last period is consistent with the timing assumption that an entrant starts producing only one period after entry, hence being able to 'learn' the average market price during the entry period.

The total demand for the output of firm ω is thus

$$y_t^D(\omega) \equiv \left(\frac{p_t(\omega)}{P_t}\right)^{-\theta} (C_t + PAC_t),$$

where $PAC_t \equiv N_t PAC_t(\omega)$, and we used symmetry across firms in the definition of the aggregate demand of the consumption basket for price adjustment purposes PAC_t.

Let $\rho_t(\omega) \equiv p_t(\omega)/P_t$ denote the real price if firm ω's output. Then, firm ω's profit in period

[12] We relax this assumption below.

t (distributed to households as dividend) can be written as

$$d_t(\omega) = \rho_t(\omega)\, y_t^D(\omega) - w_t l_t(\omega) - \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2 \rho_t(\omega)\, y_t^D(\omega)\,.$$

The real value of the firm at time t (in units of consumption) is the expected present discounted value of future profits from $t+1$ on, discounted with the household's stochastic discount factor (see below):

$$v_t(\omega) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s(\omega)\,, \tag{1}$$

where $\Lambda_{t,s} \equiv [\beta(1-\delta)]^{s-t}\, U_C(C_s, L_s)/U_C(C_t, L_t)$ is the discount factor applied by households to future profits from firm ω (which faces a probability δ of being hit with the "death" shock in each period).

At time t, firm ω chooses $l_t(\omega)$ and $p_t(\omega)$ to maximize $d_t(\omega) + v_t(\omega)$ subject to $y_t(\omega) = y_t^D(\omega)$, taking w_t, P_t, C_t, PAC_t, and Z_t as given. Letting $\lambda_t(\omega)$ denote the Lagrange multiplier on the constraint $y_t(\omega) = y_t^D(\omega)$, the first-order condition with respect to $l_t(\omega)$ yields:

$$\lambda_t(\omega) = \frac{w_t}{Z_t}.$$

The shadow value of an extra unit of output is simply the firm's marginal cost, common across all firms in the economy.

The first-order condition with respect to $p_t(\omega)$ yields:

$$p_t(\omega) = \mu_t(\omega)\, P_t \lambda_t(\omega)\,.$$

Firm ω sets the price as a markup ($\mu_t(\omega)$) over nominal marginal cost, where the markup $\mu_t(\omega)$ is given by

$$\mu_t(\omega) \equiv \frac{\theta y_t(\omega)}{(\theta-1)\, y_t(\omega)\left[1 - \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2\right] + \kappa \Upsilon_t},$$

$$\Upsilon_t \equiv y_t(\omega)\frac{p_t(\omega)}{p_{t-1}(\omega)}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right) - E_t\left[\Lambda_{t,t+1} y_{t+1}(\omega)\frac{P_t}{P_{t+1}}\left(\frac{p_{t+1}(\omega)}{p_t(\omega)}\right)^2\left(\frac{p_{t+1}(\omega)}{p_t(\omega)} - 1\right)\right].$$

As expected, the markup reduces to $\theta/(\theta-1)$ in the absence of nominal rigidity ($\kappa = 0$) or if the price $p_t(\omega)$ is constant.

Firm Entry and Exit

In every period, there is an unbounded mass of prospective entrants. These entrants are forward looking, and correctly anticipate their future expected profits $d_t(\omega)$ in every period as well as the probability δ (in every period) of incurring the exit-inducing shock. We assume that entrants at time t only start producing at time $t+1$, which introduces a one-period time-to-build lag in the model. The exogenous exit shock occurs at the very end of the time period (after production and entry). A proportion δ of new entrants will therefore never produce. Prospective entrants in period t compute their expected post-entry value given by the present discounted value of their expected stream of profits $v_t(\omega)$. This also represents the average value of incumbent firms *after* production has occurred (since both new entrants and incumbents then face the same probability $1-\delta$ of survival and production in the subsequent period). Entry occurs until firm value is equalized with the entry cost, leading to the free entry condition $v_t(\omega) = w_t f_{E,t}/Z_t$. This condition holds so long as the mass $N_{E,t}$ of entrants is positive. We assume that macroeconomic shocks are small enough for this condition to hold in every period. Finally, the timing of entry and production we have assumed implies that the number of producing firms during period t is given by $N_t = (1-\delta)(N_{t-1} + N_{E,t-1})$.

Symmetric Firm Equilibrium

In equilibrium, all firms make identical choices. Hence, $\lambda_t(\omega) = \lambda_t$, $p_t(\omega) = p_t$, $\mu_t(\omega) = \mu_t$, $\rho_t(\omega) = \rho_t$, $l_t(\omega) = l_t$, $y_t(\omega) = y_t$, $d_t(\omega) = d_t$, and $v_t(\omega) = v_t$. The aggregate output of the consumption basket (used for consumption and to pay price adjustment costs) is

$$Y_t^C \equiv C_t + PAC_t = N_t \rho_t y_t = N_t \rho_t Z_t l_t.$$

The expression of the price index P_t implies that the relative price ρ_t and the number of producing firms N_t are tied by the "variety effect" equation $\rho_t = p_t/P_t = (N_t)^{\frac{1}{\theta-1}}$.

Let π_t denote inflation in producer prices: $\pi_t \equiv p_t/p_{t-1} - 1$. Then, we can write:

$$\mu_t = \frac{\theta}{(\theta-1)\left[1-\frac{\kappa}{2}(\pi_t)^2\right] + \kappa\left\{(1+\pi_t)\pi_t - \beta(1-\delta)E_t\left[\frac{C_t}{C_{t+1}}\frac{N_t}{N_{t+1}}\frac{Y_{t+1}^C}{Y_t^C}(1+\pi_{t+1})\pi_{t+1}\right]\right\}}.$$

This can be simplified further by noting that $PAC_t = \kappa(\pi_t)^2 Y_t^C/2$, so that $C_t = \left[1-\kappa(\pi_t)^2/2\right] Y_t^C$, to obtain:

$$\mu_t = \frac{\theta}{(\theta-1)\left[1-\frac{\kappa}{2}(\pi_t)^2\right]+\kappa\left\{(1+\pi_t)\pi_t-\beta(1-\delta)E_t\left[\frac{1-\frac{\kappa}{2}(\pi_t)^2}{1-\frac{\kappa}{2}(\pi_{t+1})^2}\frac{N_t}{N_{t+1}}(1+\pi_{t+1})\pi_{t+1}\right]\right\}}. \quad (2)$$

Log-linearization of this equation delivers our model's New Keynesian Phillips curve incorporating the effect of endogenous product variety, which we discuss in detail in Section 4.

Household Budget Constraint, Saving, and Labor Supply

Households hold two types of assets: shares in a mutual fund of firms and bonds. Let x_t be the share in the mutual fund of firms held by the representative household *entering* period t. The mutual fund pays a total profit in each period (in units of currency) that is equal to the total profit of all firms that produce in that period, $P_t N_t d_t$. During period t, the representative household buys x_{t+1} shares in a mutual fund of $N_{H,t} \equiv N_t + N_{E,t}$ firms (those already operating at time t and the new entrants). Only $N_{t+1} = (1-\delta) N_{H,t}$ firms will produce and pay dividends at time $t+1$. Since the household does not know which firms will be hit by the exogenous exit shock δ at the *very end* of period t, it finances the continuing operation of all pre-existing firms and all new entrants during period t. The date t price of a claim to the future profit stream of the mutual fund of $N_{H,t}$ firms is equal to the average nominal price of claims to future profits of home firms, $V_t \equiv P_t v_t$.

The household enters period t with nominal bond holdings $B_{N,t}$ and mutual fund share holdings x_t. It receives gross interest income on bond holdings, dividend income on mutual fund share holdings and the value of selling its initial share position, and labor income. The household allocates these resources between purchases of bonds and shares to be carried into next period and consumption. The period budget constraint (in units of currency) is:

$$B_{N,t+1} + V_t N_{H,t} x_{t+1} + P_t C_t = (1+i_{t-1})B_{N,t} + (D_t + V_t) N_t x_t + \left(1+\tau_t^L\right) W_t L_t + T_t^L,$$

where i_{t-1} denotes the nominal interest rate on holdings of bonds between $t-1$ and t, D_t denotes nominal dividends ($D_t \equiv P_t d_t$), τ_t^L is a labor subsidy whose role we discuss below, and T_t^L is a lump-sum tax satisfying the constraint $T_t^L = -\tau_t^L W_t L_t$ in equilibrium. Dividing both sides by P_t and denoting holdings of bonds in units of consumption with $B_{t+1} \equiv B_{N,t+1}/P_t$, we can write

$$B_{t+1} + v_t N_{H,t} x_{t+1} + C_t = (1+r_t)B_t + (d_t + v_t) N_t x_t + \left(1+\tau_t^L\right) w_t L_t + t_t^L, \quad (3)$$

where $1+ r_t$ is the gross, consumption-based, real interest rate on holdings of bonds between $t-1$ and t, defined by $1+r_t \equiv (1+i_{t-1}) / \left(1+\pi_t^C\right)$, with $\pi_t^C \equiv P_t/P_{t-1} - 1$, and $t_t^L \equiv T_t^L/P_t$. The home household maximizes its expected intertemporal utility subject to this budget constraint.

The Euler equations for bond and share holdings are:

$$(C_t)^{-1} = \beta E_t \left[\frac{1+i_t}{1+\pi_{t+1}^C} (C_{t+1})^{-1} \right] \quad \text{and} \quad v_t = \beta (1-\delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} (v_{t+1} + d_{t+1}) \right].$$

As expected, forward iteration of the equation for share holdings and absence of speculative bubbles yield the asset price solution in equation (1).[13]

The first-order condition for the optimal choice of labor effort requires that the marginal disutility of labor be equal to the marginal utility from consuming the real wage received for an additional unit of labor:

$$\chi (L_t)^{\frac{1}{\varphi}} = \left(1+\tau_t^L\right) \frac{w_t}{C_t}.$$

Aggregate Accounting and Equilibrium

Aggregating the budget constraint (3) across households and imposing the equilibrium conditions $B_{t+1} = B_t = 0$ and $x_{t+1} = x_t = 1$, $\forall t$, yields the aggregate accounting identity $Y_t \equiv C_t + N_{E,t} v_t = w_t L_t + N_t d_t$, where we defined GDP, Y_t: Consumption plus investment (in new firms) must be equal to income (labor income plus dividend income).

Labor market equilibrium requires $N_t l_t + N_{E,t} f_{E,t}/Z_t = L_t$: The total amount of labor used in production and to set up the new entrants' plants must equal aggregate labor supply. (Of course, this condition is redundant once equilibrium in goods and asset markets is imposed.) The equilibrium conditions of our benchmark model are summarized in Table 1.

[13] We omit the transversality conditions for bonds and shares that must be satisfied to ensure optimality.

Table 1. Benchmark Model, Summary

Pricing	$\rho_t = \mu_t \frac{w_t}{Z_t}$
Markup	$\mu_t = \frac{\theta}{(\theta-1)\left[1-\frac{\kappa}{2}(\pi_t)^2\right]+\kappa\left\{(1+\pi_t)\pi_t-\beta(1-\delta)E_t\left[\frac{1-\frac{\kappa}{2}(\pi_t)^2}{1-\frac{\kappa}{2}(\pi_{t+1})^2}\frac{N_t}{N_{t+1}}(1+\pi_{t+1})\pi_{t+1}\right]\right\}}$
Variety effect	$\rho_t = (N_t)^{\frac{1}{\theta-1}}$
Profits	$d_t = \left(1-\frac{1}{\mu_t}-\frac{\kappa}{2}(\pi_t)^2\right)\frac{Y_t^C}{N_t}$
Free entry	$v_t = w_t \frac{f_{E,t}}{Z_t}$
Number of firms	$N_t = (1-\delta)(N_{t-1}+N_{E,t-1})$
Intratemporal optimality	$\chi (L_t)^{\frac{1}{\varphi}} = \left(1+\tau_t^L\right)\frac{w_t}{C_t}$
Euler equation (shares)	$v_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}(v_{t+1}+d_{t+1})\right]$
Euler equation (bonds)	$(C_t)^{-1} = \beta E_t\left[\frac{1+i_t}{1+\pi_{t+1}^C}(C_{t+1})^{-1}\right]$
Output of consumption sector	$C_t = \left[1-\frac{\kappa}{2}(\pi_t)^2\right]Y_t^C$
Aggregate accounting	$C_t + N_{E,t}v_t = w_t L_t + N_t d_t$
CPI inflation	$\frac{1+\pi_t}{1+\pi_t^C} = \frac{\rho_t}{\rho_{t-1}}$

The model is closed by specifying a rule for nominal interest rate setting by the monetary authority, the setting of the labor subsidy τ_t^L, and processes for the exogenous entry cost $f_{E,t}$ and productivity Z_t.

3 Price Stability with Endogenous Entry and Product Variety

The flexible-price analysis of BGM leaves inflation in consumer and producer prices indeterminate because monetary policy has no real effect in the flexible-price, cashless economy, and we need not concern ourselves with the paths of nominal variables in order to solve for real ones. When prices are sticky, this is no longer the case. Specifically, given a change in the number of producers N_t and the associated movement in the relative price ρ_t implied by the variety effect equation $\rho_t = p_t/P_t = (N_t)^{\frac{1}{\theta-1}}$, the allocation of this relative price movement to changes in producer or consumer prices is important for the dynamics of real variables and welfare. In turn, producer price inflation is a determinant of firm entry – and thus N_t – via its impact on firm profits. This section studies optimal monetary policy in our model and the optimal allocation of variety effects to producer versus consumer prices.

Our analysis of optimal monetary policy builds on results in Bilbiie, Ghironi, and Melitz (2006). We show there that the flexible-price version of the economy described above is efficient – the competitive equilibrium coincides with the social planner's optimum – if labor supply is inelastic ($\varphi = 0$) and $L_t = 1 \ \forall t$. The reason is that, with C.E.S. Dixit-Stiglitz preferences, the profit destruction externality generated by producer entry (which reduces demand for each individual firm) is exactly matched by the consumer's love for variety – both determined by the elasticity of substitution θ. The flexible-price economy is inefficient if $\varphi > 0$ because there is a misalignment of markups across the items the consumer cares about (consumption, priced at a markup over marginal cost, and leisure, priced competitively), but efficiency is restored if the labor subsidy τ_t^L is equal to the net markup of pricing over marginal cost, $1/(\theta - 1)$ in all periods. This subsidy aligns markups across consumption goods and leisure while preserving the expected profitability of firm entry, thus inducing the efficient equilibrium. We assume that $\tau_t^L = 1/(\theta - 1) \ \forall t$ below.

Sticky prices imply a time-varying markup whenever producer prices are changing over time. As shown in Bilbiie, Ghironi, and Melitz (2006), markup non-synchronization across periods (as well as across states and arguments of the utility function) generates inefficiency compared to the planner's optimum. Since in this particular model time variation of the markup in the competitive equilibrium is due to producer price inflation, we expect a zero rate of inflation in producer prices to be the optimal monetary policy chosen by a planner. The following proposition confirms that this is indeed the case. To isolate our main result, we prove the proposition for the case of inelastic labor and then briefly discuss the elastic labor case.

Proposition 1 *The optimal rate of producer price inflation π_t chosen by a social planner is zero.*

Proof. See the Appendix. ■

The intuition for Proposition 1 is straightforward: Producer price inflation acts as a tax on firm profits in our model. It distorts firm entry decisions and the allocation of labor to creation of new firms versus production of existing goods, resulting in suboptimal consumption and lower welfare. Optimal policy, therefore, aims to stabilize producer price inflation at zero. Importantly, however, while producer prices must be stabilized, the optimal rate of consumer price inflation must move freely to accommodate changes in the number of varieties:

$$1 + \pi_t^{C*} = \left(\frac{\rho_t^*}{\rho_{t-1}^*} \right)^{-1} = \left(\frac{N_t^*}{N_{t-1}^*} \right)^{-\frac{1}{\theta-1}},$$

where a star denotes variables in the efficient equilibrium. Given the evidence of bias in the measurement of CPI inflation (precisely due to poor accounting for new varieties) convincingly documented by Broda and Weinstein (2006a), we view this normative implication of our model as "good news." The central bank should target inflation in producer prices rather than (mismeasured) CPI inflation.

When labor supply is elastic, the subsidy $\tau_t^L = 1/(\theta - 1)$ ensures that the flexible-price equilibrium is efficient, removing the wedge otherwise present between the marginal rates of substitution and transformation between consumption and leisure. In this case, price stickiness distorts both the total amount of labor supplied and its allocation to creation of new firms and production of existing goods. It is easy to verify that a zero rate of inflation in producer prices is still the optimal monetary policy.

The optimality of producer price stability with inelastic labor supply highlights a new argument for price stability (at the producer level) implied by endogenous entry and product variety. In a model with exogenously fixed number of firms and inelastic labor supply, time variation in the markup would have no impact on the equilibrium path of consumption and welfare: Consumption would be simply determined by the exogenous productivity and labor supply regardless of markup dynamics. Endogenous entry and product variety imply that markup variation reduces welfare by distorting entry decisions and the allocation of the fixed amount of labor to firm creation versus production of existing goods. This introduces a role for monetary policy in welfare maximization by stabilizing producer price inflation at zero – and the markup at its flexible-price level. We discuss implementation of the optimal monetary policy by setting the nominal interest rate below.

4 The New Keynesian Phillips Curve and the Log-Linear Model

This section describes the implications of endogenous entry and product variety for the New Keynesian Phillips curve and presents the key log-linear equations of the model.

The New Keynesian Phillips Curve

To study the propagation of shocks and compute second moments of the endogenous variables implied by assumptions on the processes for exogenous shocks, we log-linearize the model around the efficient steady state with zero inflation under assumptions of log-normality and homoskedasticity, and denote percent deviations from steady state with sans serif fonts. Our model's version of the

New Keynesian Phillips curve follows from log-linearizing equation (2):

$$\pi_t = \beta (1 - \delta) E_t \pi_{t+1} - \frac{\theta - 1}{\kappa} \mu_t, \tag{4}$$

where π_t and μ_t now denote percent deviations from steady state (of gross inflation in the case of π_t).

Since $\rho_t = p_t / P_t = (N_t)^{\frac{1}{\theta - 1}}$ and optimal firm pricing implies $\mu_t = \rho_t / \lambda_t = \rho_t Z_t / w_t$, it follows that $\mu_t = (N_t)^{\frac{1}{\theta - 1}} Z_t / w_t$, or, in log-linear terms:

$$\mu_t = \frac{1}{\theta - 1} \mathsf{N}_t - (\mathsf{w}_t - \mathsf{Z}_t). \tag{5}$$

(With a constant number of firms, this relation reduces to the familiar negative relation between markup and marginal cost of the benchmark New Keynesian model.) Substituting (5) into (4) yields:

$$\pi_t = \beta (1 - \delta) E_t \pi_{t+1} + \frac{\theta - 1}{\kappa} (\mathsf{w}_t - \mathsf{Z}_t) - \frac{1}{\kappa} \mathsf{N}_t. \tag{6}$$

Equation (6) is a New Keynesian Phillips curve relation that ties firm-level inflation dynamics to marginal cost in a standard fashion. Importantly, the effect of marginal cost is adjusted to reflect the number of producers that operate in the economy. This is a predetermined, state variable, which introduces directly a degree of endogenous persistence in the dynamics of firm-level inflation in the Phillips curve.

Furthermore, our model links the dynamics of inflation to asset prices in an endogenous way, as can be seen by combining (6) with the log-linear free entry condition to obtain:

$$\pi_t = \beta (1 - \delta) E_t \pi_{t+1} + \frac{\theta - 1}{\kappa} (\mathsf{v}_t - \mathsf{f}_{E,t}) - \frac{1}{\kappa} \mathsf{N}_t. \tag{7}$$

This equation ties inflation dynamics to the relative price of investment in new firms. It stipulates that, for given expected inflation and number of firms, inflation is positively related to equity prices. Together with the no-arbitrage condition between bonds and equity implied by optimal household behavior, this connection between inflation and equity prices (and thus capital accumulation in our model) plays a crucial role for the determinacy and stability properties of interest rate setting that we discuss below.

Finally, using the definition of CPI inflation, we can write the New Keynesian Phillips curve for consumption-based inflation:

$$\pi_t^C = \beta(1-\delta) E_t \pi_{t+1}^C + \frac{\theta-1}{\kappa}(\mathsf{w}_t - \mathsf{Z}_t) - \frac{1}{\kappa}\mathsf{N}_t - \frac{1}{\theta-1}\left[\mathsf{N}_t - \mathsf{N}_{t-1} - \beta(1-\delta)(\mathsf{N}_{t+1} - \mathsf{N}_t)\right]. \quad (8)$$

Consumption-based inflation displays an additional degree of endogenous persistence relative to firm-level inflation in that it depends directly on the number of firms that produced at time $t-1$, which was determined in period $t-2$.

Implications for Empirical Exercises

Existing empirical studies dealing with estimation of the New Keynesian Phillips curve (4), such as Sbordone (1998) and Galí and Gertler (1999), proxy the (unobservable) markup variable with the inverse of the labor share. This is an approximation that holds exactly in a model without endogenous variety. In our model with endogenous variety, however, this relationship no longer holds. Indeed, if one believes product variety to be important for business cycles, the proxy for the markup that one should use is the inverse of the share (in consumption output) of labor beyond the 'overhead' quantity (from an aggregate perspective) used to set up new product lines, $\mu_t = Y_t^C / \left[w_t \left(L_t - L_{E,t}\right)\right]$. This markup measure corresponds closely to the labor share measure used by Rotemberg and Woodford (1999) that takes into account overhead labor (reported in column 2 of their Table 2, page 1066), which we reproduce in Figure 4 below. Log-linearization of this equation, when replaced into (4), delivers a relationship that is testable empirically. Alternatively, exploiting the equation for profits, one could use as a proxy for markups as the inverse of (one minus) the profit share, $\mu_t = \left(1 - D_t^G / Y_t^C\right)^{-1}$, where $D_t^G \equiv d_t N_t + \frac{\kappa}{2}(\pi_t)^2 Y_t^C$ are profits gross of the costs of price adjustment. Note that since when log-linearizing around a zero-inflation steady-state, these costs are zero (and hence consumption is equal to consumption output and gross profits are equal to net profits), the empirically usable equation will feature only observable variables, i.e., consumption and total profit receipts (or dividends).[14]

A further implication of our framework for empirical exercises comes from the natural distinction between consumer and producer price inflation in our model: Our framework implies that, in order to overcome measurement issues inherent in using CPI inflation, empirical studies of the Phillips curve should concentrate on producer price inflation (which is also the relevant objective for monetary policy). Construction of CPI data by statistical agencies does not adjust for availabil-

[14] We leave estimation of Phillips curve using these alternative proxies for the markup for future research.

ity of new varieties in the specific functional form dictated by the welfare-consistent price index. Furthermore, adjustment for variety, when it happens, certainly does not happen at the frequency represented by periods in our model. Actual CPI data are closer to p_t (the average price level in our economy) than P_t. For this reason, when investigating the properties of the model in relation to the data (for instance, when computing second moments below or in the specification of policy rules that allow for reaction to measured real quantities), one should focus on real variables deflated by a data-consistent price index. For any variable X_t in units of the consumption basket, such data-consistent counterpart is obtained as $X_{R,t} \equiv P_t X_t / p_t = X_t / \rho_t$.

Related to this measurement issue, our framework implies an 'endogeneity bias' in cost-push shocks in much empirical literature on the New Keynesian Phillips curve. An endogenous term that depends on N_t (due the measurement bias from not accounting for variety) is attributed to exogenous cost-push shocks when estimating the Phillips curve equation (6) using a proxy for marginal cost without variety.

When the variety effect is removed from the welfare-consistent equity price, the Phillips curve (7) becomes:

$$\pi_t = \beta (1 - \delta) E_t \pi_{t+1} + \frac{\theta - 1}{\kappa} (\mathsf{v}_{R,t} - \mathsf{f}_{E,t}), \tag{9}$$

where $v_{R,t}$ is the value of the firm/price of shares net of the variety effect. For given expectations of future inflation, actual inflation is increasing in the data-consistent price of equity.

The Log-Linear Model

The log-linear counterparts of the equations in Table 1 are in the appendix. The system can be reduced to the following equations (plus the New Keynesian Phillips curve (4)):

$$\mathsf{N}_{t+1} = [1 + r + \psi] \mathsf{N}_t - (r + \delta + \psi)(\theta - 1) \mathsf{C}_t - \psi (\theta - 1) \mu_t + ((r + \delta + \psi)(\theta - 1) + \delta) \mathsf{Z}_t - \delta \mathsf{f}_{E,t}, \tag{10}$$

$$\begin{aligned} \mathsf{C}_t = {} & \frac{1-\delta}{1+r} E_t \mathsf{C}_{t+1} - \left[\frac{1-\delta}{1+r} \frac{1}{\theta - 1} - \frac{r+\delta}{1+r} \right] \mathsf{N}_{t+1} + \frac{1}{\theta - 1} \mathsf{N}_t \\ & + \left[\frac{1-\delta}{1+r} - \frac{r+\delta}{1+r} (\theta - 1) \right] \mu_{t+1} - \mu_t - \frac{1-\delta}{1+r} E_t \mathsf{f}_{E,t+1} + \mathsf{f}_{E,t}, \end{aligned} \tag{11}$$

$$E_t \mathsf{C}_{t+1} = \mathsf{C}_t + \mathsf{i}_t - E_t \pi_{t+1} + \frac{1}{\theta - 1} \mathsf{N}_{t+1} - \frac{1}{\theta - 1} \mathsf{N}_t, \tag{12}$$

where we defined $\psi \equiv \varphi \left[(r+\delta)(\theta-1)+\delta \right] / (\theta-1)$, which is zero when labor supply is inelastic. The model is closed by specifying the conduct of monetary policy (via the setting of the nominal interest rate i_t) over the business cycle, which we discuss below.

5 Monetary Policy over the Business Cycle

In this section, we discuss determinacy and stability properties of simple rules for nominal interest rate setting over the business cycle and the implementation of the optimal policy of producer price stability.

Simple Policy Rules

We consider the following class of simple inflation-targeting rules for interest rate setting:

$$\mathsf{i}_t = \tau_i \mathsf{i}_{t-1} + \tau E_t \pi_{t+s} + \tau_C E_t \pi_{t+s}^C + \xi_t^i, \quad 1 > \tau_i \geq 0,\ \tau \geq 0,\ \tau_C \geq 0,\ s = 0, 1. \tag{13}$$

where ξ_t^i is an i.i.d. shock capturing the non-systematic component of monetary policy. We assume that $\tau_C = 0$ when $\tau > 0$ and vice versa, restricting the central bank to reacting to either producer or consumer price inflation. For the reasons we discussed above, a response to welfare-based CPI inflation is clearly suboptimal (and not feasible in reality due to the measurement problems we mentioned). In considering this scenario below, we abstract from the clear normative prescription of not targeting welfare-based CPI inflation and from measurement issues; rather, we ask the question: What would the response of the economy to various shocks be if the central bank could monitor movements in welfare-consistent CPI inflation and followed a rule involving the latter?

Determinacy and Stability

In this section we study the determinacy and stability properties of our model under different monetary policy rules. To analyze local determinacy and stability of the rational expectation equilibrium, we can focus on the perfect foresight version of the system formed by (4), (10), (11) and the equation obtained by substituting the monetary policy rule (13) into the Euler equation for bonds (12). To begin with, consider the simple rule in which the central bank is responding to expected producer price inflation with no smoothing: $\mathsf{i}_t = \tau E_t \pi_{t+1}$. The following Proposition establishes that the Taylor Principle holds in our model economy for all plausible combinations of parameter values.

Proposition 2 *Let* $\gamma \equiv [1-\beta(1-\delta)]/[\beta(1-\delta)]$. *Assume that* $\varphi = 0$, *and* β, δ, *and* θ *are such that* $1-\gamma(\theta-1) > 0$, $\beta > 1/2$, $\theta > 2$, *and* $\tau < \bar{\tau} = (\kappa+\theta-1)/(\theta-1)$. *Then* $\tau > 1$ *is necessary and sufficient for local determinacy and stability.*

Proof. See the Appendix. ■

We remark that the parameter restrictions in the Proposition 2 are *sufficient* conditions for the Taylor Principle to hold, and they are extremely weak conditions. For instance, the values of κ and θ that we consider below ($\kappa = 77$ and $\theta = 3.8$) imply $\bar{\tau} = 28.5$: The sufficient condition $\tau < 28.5$ is satisfied by any realistic parametrization of interest rate setting. Moreover, while we cannot prove it analytically, we verify numerically that determinacy and stability hold for values of τ well above the threshold $\bar{\tau}$ for the parameter values we consider.

Validity of the Taylor Principle is an important result given the debate on the Taylor Principle in models with physical capital accumulation. Dupor (2001) shows that passive interest rate setting ($\tau < 1$) is necessary and sufficient for local determinacy and stability in a continuous-time model with physical capital. Carlstrom and Fuerst (2005) study the issue in a discrete-time model with capital and conclude that it is essentially impossible to achieve determinacy with forward-looking interest rate setting. Our result shows that the standard Taylor Principle is restored when capital accumulation takes the form of the endogenous creation of new production lines.

Intuition: The Role of Asset Prices in Monetary Policy Transmission

Since the validity of the Taylor principle in our setup is in striking contrast to results of models with traditional physical capital, an intuitive explanation of this difference is in order. Indeed, the mechanism for this result in our model is centered precisely on the role of the endogenous price of equity – the value of the firm – in our New Keynesian model with free entry. As we anticipated, the explanation relies on one hand on the Phillips curve (9) above that relates inflation and asset prices (net of the variety effect) $v_{R,t}$ and on the other hand on the no-arbitrage condition implied by the Euler equations for bonds and shares. This condition can be written as:

$$\mathsf{i}_t - \mathsf{E}_t\pi_{t+1} = -\mathsf{v}_{R,t} + \frac{1-\delta}{1+r}\mathsf{E}_t\mathsf{v}_{R,t+1} + \frac{r+\delta}{1+r}\mathsf{E}_t\mathsf{d}_{R,t+1}. \tag{14}$$

Focus first on the policy rule studied in Proposition 2, where the relevant inflation objective is expected PPI inflation, and run the following mental experiment. Suppose that a sunspot shock unrelated to any fundamentals hits the economy, and that (without losing generality) it is located in inflationary expectations, so that all other expected values are taken as given. We wish to show that if the policy rule is passive (the Taylor Principle is violated), this sunspot shock will have real effects, whereas if the policy rule is active the sunspot has no effect. When the Taylor Principle is violated, an increase in expected inflation triggers a fall in the real interest rate. From the no-arbitrage condition (14), this implies that the data-consistent price of shares must rise (a fall in the real return on bonds must be matched by a fall in the real return on shares, which, for fixed expected dividend and future price, means an increase in the share price today). But an increase in the share price implies, by (9), that actual inflation today will rise, and hence that the sunspot is self-fulfilling. When the Taylor Principle is satisfied, the opposite holds: The sunspot triggers an increase in the real interest rate, a fall in today's share price by no-arbitrage, and a fall in today's inflation by the Phillips curve, making the sunspot vanish.[15]

The same mechanism can be easily verified to hold for a policy rule responding to contemporaneous producer price inflation, and indeed to (contemporaneous or expected) inflation in consumer prices. Therefore, we omit the formal statements and proofs of the Taylor Principle for those cases to save space.[16]

A comparison of our results and intuition with those of Carlstrom and Fuerst (2005) allows to further emphasize the crucial role of the different type of capital at the core of our model. Carlstrom and Fuerst show that indeterminacy occurs in a discrete-time model with physical capital when the central bank responds to expected future inflation because the no-arbitrage condition between bonds and capital contains no variable dated at time t. This happens because the expected return on 'shares' (claims to return on physical capital) depends only on future variables determining the marginal product of capital at time $t + 1$. In turn, this implies that there is a zero root in the system, and indeterminacy.[17] Instead, in our model, the expected return on shares depends on the price of shares today (an endogenous variable), hence alleviating this 'zero-root' problem. Indeed, through the price of today's price of equity, our model provides a novel, intrinsic link between the no-arbitrage condition and the Phillips curve that is absent in models that do not feature endogenous variety and free entry.

[15]This argument does not hinge on having removed the variety effect from equity prices. The same argument can be made by using the Phillips curve equation (7) and the no-arbitrage condition with welfare-consistent equity prices.

[16]Details are available on request.

[17]The problem is only partially solved by the introduction of capital adjustment costs (introduced in order to endogenize the price of capital). Carlstrom and Fuerst show that the Taylor Principle is restored for forward-looking rules only for empirically implausible parameterizations of the adjustment cost parameter.

Implementing Price Stability with Endogenous Entry and Product Variety

The efficient, flexible-price equilibrium requires the nominal interest rate to be equal to the 'Wicksellian' interest rate (in Woodford's, 2003, terminology), i.e., the interest rate i_t^* that prevails when prices are flexible and producer price inflation is zero. In log-linear terms, the Wicksellian interest rate is:[18]

$$\mathsf{i}_t^* = E_t\mathsf{C}_{t+1}^* - \mathsf{C}_t^* - \frac{1}{\theta - 1}\left(\mathsf{N}_{t+1}^* - \mathsf{N}_t^*\right) = E_t\mathsf{C}_{t+1}^* - \mathsf{C}_t^* + \pi_{t+1}^{C*},$$

where $E_t\mathsf{C}_{t+1}^* - \mathsf{C}_t^*$ is the risk-free, real interest rate of BGM and π_{t+1}^{C*} is the optimal consumer price inflation that accommodates changes in variety between t and $t+1$ (known at time t). Note, however, that commitment to the policy rule $\mathsf{i}_t = \mathsf{i}_t^*$ would result in equilibrium indeterminacy, as in the standard model with a fixed number of producers discussed in Woodford (2003), because nominal interest rate setting would contain no feedback to variables that are endogenous in the sticky-price equilibrium.

A simple interest rate rule that implements the efficient, flexible-price equilibrium is

$$\hat{\imath}_t = \tau\pi_t + E_t\tilde{\mathsf{Y}}_{R,t+1}^C - \tilde{\mathsf{Y}}_{R,t}^C, \quad \tau > 1, \tag{15}$$

where $\hat{\imath}_t \equiv \mathsf{i}_t - \mathsf{i}_t^*$ is the interest rate gap relative to the Wicksellian interest rate, $\tilde{\mathsf{Y}}_{R,t}^C = \hat{\mathsf{C}}_{R,t} \equiv \mathsf{C}_t - [1/(\theta-1)]\,\mathsf{N}_t - \{\mathsf{C}_t^* - [1/(\theta-1)]\,\mathsf{N}_t^*\} = \mathsf{C}_{R,t} - \mathsf{C}_{R,t}^*$ is the gap between measured consumption output and its flexible-price level, and we use a subscript R to denote real variables from which we have removed the pure variety effect that is not accounted for in measured data following BGM. The interest rate rule (15) requires the monetary authority to track changes in the Wicksellian interest rate and in expected growth of the consumption output gap, and to respond more than proportionally to inflation. As we show in Appendix A, the following equation holds for the dynamics of the consumption output gap:

$$E_t\tilde{\mathsf{Y}}_{R,t+1}^C - \tilde{\mathsf{Y}}_{R,t}^C = \hat{\imath}_t - E_t\pi_{t+1}.$$

Substituting the interest rate rule (15) into this equation yields $\tau\pi_t = E_t\pi_{t+1}$, which has unique solution $\pi_t = 0\ \forall t$ since the Taylor Principle is satisfied. In turn, zero producer price inflation in all periods implies $\tilde{\mathsf{Y}}_{R,t}^C = 0$, and, therefore, $\mathsf{i}_t = \mathsf{i}_t^*\ \forall t$.[19]

[18] See Appendix A for details.

[19] Rule (15) is by no means the only interest rate rule that implements the optimal monetary policy. It is of course possible to design alternative rules that achieve this goal.

6 Business Cycles: Propagation and Second Moments

In this section we explore the properties of our benchmark model by means of numerical examples. We compute impulse responses to productivity, deregulation, and monetary policy shocks. Then, we compute second moments of our artificial economy and compare them to second moments in the data and those produced by the baseline BGM model with flexible prices and C.E.S. preferences. As shown in BGM, these moments (which also corresponds to those under the optimal monetary policy in the sticky-price economy) are very close to those generated by the standard RBC model.

Calibration

In our baseline calibration, we interpret periods as quarters and set $\beta = 0.99$ – a standard choice for quarterly business cycle models. We set the size of the exogenous firm exit shock $\delta = .025$ to match the U.S. empirical level of 10 percent job destruction per year.[20] We use the value of θ from Bernard, Eaton, Jensen, and Kortum (2003) and set $\theta = 3.8$, which was calibrated to fit U.S. plant and macro trade data.[21] We set initial productivity to $Z = 1$. The initial steady-state entry cost f_E does not affect any impulse response; we therefore set $f_E = 1$ without loss of generality. We consider different values for the elasticity of labor supply, φ, and we set the weight of the disutility of labor in the period utility function, χ, so that the steady-state level of labor effort is 1 – and steady-state levels of all variables are the same – regardless of φ.[22] We set the price stickiness parameter $\kappa = 77$, the value estimated by Ireland (2001). Although Ireland obtained this estimate using a different model, without entry and endogenous variety, our results are not sensitive to changes in the value of this parameter within a plausible range.

[20] Empirically, job destruction is induced by both firm exit and contraction. In our model, the "death" shock δ takes place at the product level. In a multi-product firm, the disappearance of a product generates job destruction without firm exit. Since we abstract from the explicit modeling of multi-product firms, we include this portion of job destruction in δ. As a higher δ implies less persistent dynamics, our choice of δ is also consistent with not overstating the ability of the model to generate persistence.

[21] It may be argued that the value of θ results in a steady-state markup that is too high relative to the evidence. However, it is important to observe that, in models without any fixed cost, $\theta/(\theta - 1)$ is a measure of both markup over marginal cost and average cost. In our model with entry costs, free entry ensures that firms earn zero profits *net* of the entry cost. This means that firms price at average cost (inclusive of the entry cost). Thus, although $\theta = 3.8$ implies a fairly high markup over marginal cost, our parametrization delivers reasonable results with respect to pricing and average costs. The main qualitative features of the impulse responses below are not affected if we set $\theta = 6$, resulting in a 20 percent markup of price over marginal cost as in Rotemberg and Woodford (1992) and several other studies.

[22] This requires $\chi = .924271$.

Impulse Responses

Productivity

Figure 1 shows the responses (percent deviations from steady state) to a temporary, 1 percent increase in productivity (with persistence 0.9) for the inelastic labor case, comparing the efficient flexible-price equilibrium obtained under optimal monetary policy (blue, round markers) with three alternative parametrizations of the monetary policy rule (13). The first is a simple rule responding to expected producer price inflation, $\mathsf{i}_t = 1.5E_t\pi_{t+1}$ (red, cross markers); the second is a rule involving interest rate smoothing, $\mathsf{i}_t = 0.8\mathsf{i}_{t-1} + 0.3E_t\pi_{t+1}$ (green, square markers), which features the same long-run response to expected inflation (1.5) as the previous rule; and the third is a rule responding to expected welfare-consistent CPI inflation, $\mathsf{i}_t = 1.5E_t\pi^C_{t+1}$ (pink, star markers). Note that the difference between the responses under each of the simple rules and the optimal policy measures the gap relative to the flexible-price equilibrium under the alternative rules. Periods are interpreted as quarters; the number of years after the shock is on the horizontal axis; and responses are normalized so that 0.3 (for instance) denotes 0.3 percent.

Focus on the responses under the optimal policy. The increase in productivity makes the business environment temporarily more attractive, drawing a higher number of entrants ($N_{E,t}$), which translates a gradual increase in the number of producers (N_t) before entry and the stock of production lines return to the steady state. The larger number of producers induces marginal cost (w_t/Z_t – not shown) and the relative price of each product ρ_t to be higher with unchanged markup. GDP (Y_t) and consumption (C_t) increase, and so does investment in new firms ($v^E_t = v_t N_{E,t}$) as the fixed labor supply is reallocated toward creation of new products. Interestingly, firm-level output (y_t) is below the steady state during most of the transition, except for a short-lived initial expansion. The effect of a higher relative price prevails on the expansion in consumption demand to push individual firm output below the steady state for most of the transition, with expansion in the number of producers and investment in new firms responsible for GDP remaining above the steady state throughout the transition. Notably, the dynamics of firm entry result in responses that persist beyond the duration of the exogenous shock, and, for some key variables, display a hump-shaped pattern.[23]

When comparing responses across policy rules, a remarkable feature of the results is that the

[23] The responses of several key macroeconomic variables deflated by average prices (the producer price level p) rather than with the consumption-based price index are qualitatively similar to those in Figure 1. For instance, this is the case for $C_{R,t}$ and $Y_{R,t}$.

dynamics of macroeconomic aggregates under the first two simple policy rules are strikingly similar to those in the flexible-price equilibrium. Indeed, the responses of consumption, output, number of producers, and the real interest rate are hardly distinguishable. Equivalently, the changes in producer price inflation and the markup induced by technology shocks under these policy rules are small. It is worth stressing that this is in contrast with responses in the fixed-variety, benchmark New Keynesian model, where there are quantitatively significant deviations from the flexible-price equilibrium under such simple policy rules. In our model, instead, a simple rule such as $\mathfrak{i}_t = 1.5E_t\pi_{t+1}$, despite not being overly aggressive in terms of responding to inflation, manages to bring the economy very close to its first-best optimum. This is no longer true when monetary policy responds to welfare-consistent CPI inflation: There is an evident difference in the responses of consumption, the real interest rate, and the number of producers, stemming from the sub-optimal response of the central bank to movements in welfare-based CPI inflation that reflect optimal fluctuations in the number of products.[24]

Deregulation

Figure 2 shows the responses to a 1 percent, permanent deregulation shock (a lowering of the entry cost $f_{E,t}$) with inelastic labor supply under the same policy scenarios as above.

Focus again on responses under the optimal policy. Deregulation attracts new entrants and firm value decreases (the relative price of the investment good falls). Since investment is relatively more attractive than consumption, there is intersectoral labor reallocation from the latter to the former. Consumption falls initially as households now postpone consumption to invest more in firms whose productivity has not increased. The number of firms starts increasing, but GDP initially falls as the decline in consumption dominates the increase in investment. All variables then move monotonically toward their new steady-state levels. A consequence of C.E.S. preferences is that the long run expansion of consumption is entirely driven by the extensive margin (the long-run increase in the number of producers), with output per firm back at the initial steady-state level.

As in the case of a productivity shock, the responses for the first two alternative policy rules, where the targeted measure of inflation is producer-price inflation, are again very similar to the flexible-price responses. As for productivity, the difference is larger when the monetary authority responds to welfare-based CPI inflation, for the same reasons discussed above. The most notable

[24] Another sub-optimal rule that can be studied features a response to movements in GDP (as opposed to the GDP gap, i.e., deviations of GDP from the flexible-price level, which is the relevant welfare benchmark). Results for this scenario are available upon request.

difference with respect to the flexible-price case in all responses (but more so when the rule responds to welfare-based CPI inflation) concerns the dynamics of consumer prices. Under the optimal policy, deregulation induces deflation in the welfare-based CPI (at a decreasing rate in absolute terms) precisely because there is an increase in the number of products (at a decreasing rate). Under the alternative rules, this response changes sign, positive inflation in the welfare-based CPI occurs, because the increase in producer price inflation is high enough to compensate the effect of the increase in the number of available varieties. This effect is strongest when the central bank responds to welfare-based CPI inflation.

Monetary Policy

The last set of responses, plotted in Figure 3, shows the effects of a purely transitory shock to interest rate setting – a 1 percent decrease with zero exogenous persistence. Because of the assumption of zero exogenous persistence, all responses are plotted for the policy rule involving interest rate smoothing (otherwise, the effect of the shock is very short-lived), but for different values of the labor supply elasticity, $\varphi = 0$, 2, and 4, respectively.[25] An interest rate cut generates inflation and a positive response of GDP (as measured both by Y_t and the data-consistent counterpart, $Y_{R,t}$), consistent with conventional wisdom and the bulk of empirical evidence for the post-1980 U.S. However, the expansionary effect on GDP is combined with a contractionary effect on entry and the number of producers that may at first sight appear counterintuitive – and conflicts with the evidence presented by Bergin and Corsetti (2005) and Lewis (2006).[26] The fall in the number of producers comes from two sources. First, from the consumers' standpoint, a fall in the real interest rate signals that consumption should increase today. Since the value of shares is high today compared to the future, investment in shares to finance new product creation falls under its steady-state value (the combination of prices that the household faces makes it relatively more attractive to consume rather than invest). From the firms' standpoint, the increase in inflation implies a fall in markups and profits, and hence less profit incentives for new product creation. Both these effects (on consumers and firms) change sign after a few years, when real interest rates reach positive values and share prices are under-valued with respect to steady state, making it attractive to save and invest. For all values of the labor supply elasticity in Figure 3, the number

[25] The inelastic labor case is in blue (round markers); $\varphi = 2$ is in red (cross markers); and $\varphi = 4$ is in green (square markers).

[26] Note, however, that Bergin and Corsetti find that unconditional correlations between a measure of expansionary monetary policy and measures of entry (gross or net) are negative.

of firms falls, and then recovers over time after an expansionary monetary policy shock.

Second Moments

To further evaluate the properties of the sticky-price model, we compute the implied second moments of our artificial economy for some key macroeconomic variables and compare them to those of the data and those produced by the BGM flexible-price model with C.E.S. preferences. In this exercise, we focus on random shocks to Z_t as the source of business cycle fluctuations, assuming that sunk entry costs are constant at $f_{E,t} = 1$ and abstracting from exogenous monetary policy shocks.[27] To start with, we compute moments of GDP, consumption, investment, hours worked, and producer price inflation. We use the same productivity process as King and Rebelo (1999), with persistence 0.979 and a standard deviation of innovations equal to 0.0072, to facilitate comparison of results with the baseline RBC setup and BGM. As in King and Rebelo's benchmark calibration, we set $\varphi = 2$.[28] Under sticky prices, we assume that monetary policy follows the rule $\mathsf{i}_t = 0.8\mathsf{i}_{t-1} + 0.3E_t\pi_{t+1}$. This rule is empirically plausible based on the findings of a large empirical literature, which documents the importance of interest rate smoothing in Federal Reserve policy, its focus on inflation targeting since the 1980s, and the marginal significance of GDP responses. Table 2 presents the results. For each moment, the first number (bold fonts) is the empirical moment implied by the U.S. data reported in King and Rebelo (1999), the second number (normal fonts) is the moment generated by the flexible-price model, and the third number (italics) is the moment generated by the sticky-price model. We compute model-implied second moments for HP-filtered variables for consistency with data and standard RBC practice, and we measure investment in our model with the real value of household investment in new firms ($v_R N_E$).

[27] The empirical literature has downplayed the role of exogenous monetary policy as a source of fluctuations, focusing instead on the role of systematic policy in response to economic conditions as a mechanism for propagation of the cycle. See, for instance, Leeper, Sims, and Zha (1996).

[28] The period utility function is defined over leisure $(1 - L_t)$ in King and Rebelo (1999), where the endowment of time in each period is normalized to 1. The elasticity of labor supply is then the risk aversion to variations in leisure (set to 1 in their benchmark calibration) multiplied by $(1 - L)/L$, where L is steady-state effort, calibrated to 1/3. This yields $\varphi = 2$ in our specification.

Table 2. Moments for: Data, BGM C.E.S. Model , and Sticky Prices

Variable X	σ_X			σ_X/σ_{YR}			$E[X_t X_{t-1}]$			$corr(X, Y_R)$		
Y_R	**1.81**	1.34	*1.36*	**1.00**			**0.84**	0.70	*0.70*	**1.00**		
C_R	**1.35**	0.65	*0.66*	**0.74**	0.48	*0.48*	**0.80**	0.75	*0.74*	**0.88**	0.97	*0.98*
Investment, $v_R N_E$	**5.30**	5.23	*5.20*	**2.93**	3.90	*3.82*	**0.87**	0.69	*0.69*	**0.80**	0.99	*0.99*
L	**1.79**	0.63	*0.63*	**0.99**	0.47	*0.46*	**0.88**	0.69	*0.69*	**0.88**	0.98	*0.98*
π			*0.01*			*0.01*			*0.82*			*–0.87*

Source for **data** moments: King and Rebelo (1999)

The performance of the sticky-price model is virtually indistinguishable from that of the flexible-price economy, which – in turn – is remarkably close to that of a benchmark RBC model in reproducing some key features of U.S. business cycles as documented in BGM. The similarity in performance across sticky- and flexible-price models is not surprising in the light of the similarity of impulse responses between the rule we are considering and the optimal policy that we discussed above. An empirically plausible degree of nominal rigidity does not do much to improve the performance of the model relative to the flexible-price counterpart once one takes into account that Federal Reserve policy appears not to have been too distant from optimal in the recent past. On the other hand, the baseline sticky-price framework (as the flexible-price one) faces the same well-known difficulties of the standard RBC model: Consumption and hours are too smooth relative to output; there is not enough endogenous persistence (as indicated by the first-order autocorrelations); and all real variables are too procyclical relative to the data.

In addition, the sticky-price model predicts a highly counterfactual pattern of markup cyclicality, as documented in Figure 4. The figure shows the model-generated correlation of the markup with GDP at various lags and leads, comparing it to that documented by Rotemberg and Woodford (1999) and that generated by the BGM model with translog preferences.[29] The flexible-price model with translog preferences almost perfectly reproduces the contemporaneous countercyclicality of the markup; furthermore, the time profile of its correlation with the business cycle is very similar to that documented by Rotemberg and Woodford. The markup is countercyclical with translog preferences

[29] Of the various labor share-based empirical measures of the markup considered by Rotemberg and Woodford, the one that is most closely related to the markup in our model is the version with overhead labor (reported in column 2 of their Table 2, page 1066), which we reproduce in Figure 4. That is because markups in our model can be written as the inverse of the share (in consumption output) of labor beyond the 'overhead' quantity used to set up new product lines, $\mu_t = Y_t^C / [w_t (L_t - L_{E,t})]$. There is of course an additional issue: This measure is specified as a share of consumption output, not GDP as in Rotemberg and Woodford. For issues pertaining to cyclicality, however, this makes little difference, since the share of consumption in GDP is relatively acyclical.

because the elasticity of substitution across goods is tied to the number of producers, which increases during expansions. The time profile of the correlation is due to the slow response of the stock of producers to shocks, with GDP increasing on impact, and the number of producers responding gradually and with a lag. The sticky-price model with C.E.S. preferences generates excessive contemporaneous countercyclicality and fails to replicate the time profile of the correlation because the markup – now determined by the dynamics of producer price inflation – is no longer tied to the number of producers. On the bright side, the sticky-price model with endogenous entry and product variety generates procyclical producer entry, qualitatively in line with empirical evidence, and procyclical aggregate profits: The contemporaneous correlation between $D_R \equiv N d_R$ and Y_R is .95. Even if the markup falls during expansions, aggregate profits increase due to the expansion in the number of producers.[30]

In sum, given plausible nominal rigidity and policy behavior for inflation-sensitive policymakers, the performance of the sticky-price model at replicating key business cycle moments is – not surprisingly – close to the flexible-price counterpart. The sticky price model fails to match the cyclicality of the markup, though endogenous variety generates procyclical profits. Interestingly, and consistent with the presence of an endogenous state variable in the New Keynesian Phillips curve (6), the model delivers a persistent inflation rate. This goes in the direction of ameliorating the inability of the standard setup to generate sufficient persistence highlighted by Fuhrer and Moore (1995).

7 Extensions

In this section, we discuss the implications of two extensions of the benchmark model above. First, we remove the assumption that new entrants inherit the same degree of price stickiness as incumbents and we allow new entrants to take their first price setting decision in flexible fashion. We consider two alternative assumptions: In one case, new entrants set their initial price flexibly, but taking into account that they will face a cost of price adjustment from next period on. In the other case, we simply assume that new entrants charge a constant markup over marginal cost. We show that these versions of the model deliver dynamic responses to shocks that are virtually identical to those of the benchmark model for plausible parameter values. Next, returning to the benchmark assumption on the cost of price adjustment, we explore the implications of departing

[30] Firm-level real profits d_R increase on impact following a favorable productivity shock with persistence .979, but quickly drop below the steady state and return to it from below. It is expansion in N that boosts D_R above the steady state throughout the transition, with a hump-shaped response. The figure is available on request.

from C.E.S. preferences, extending the benchmark model to a general, homothetic specification of consumption preferences. We parametrize this specification in translog form and show that also this extension leaves the key properties of the model roughly unchanged.

Endogenous Aggregate Stickiness and Producer Entry

So far, we assumed that new entrants are subject to the same nominal rigidity as incumbent firms. It is plausible, however, that new entrants in period t will make their first price setting decision in period $t+1$ without having to pay a cost of price adjustment relative to a past price setting decision they did not make. In this case, heterogeneity in price levels arises across cohorts of firms that entered at different points in time, as their price level decisions will differ depending on the marginal cost conditions at the time of entry, thus affecting price setting decisions in subsequent periods. The degree of aggregate price rigidity in the economy becomes endogenous, as the number of new price setters that face no cost of adjusting relative to a past price decision varies with the business cycle.[31]

We present the extended model in Appendix E.[32] Prior to log-linearization, the model features an infinite number of state variables (we assume that the economy has existed since the infinite past; thus, the set of currently producing firms, N_t, includes representatives of an infinite number of entrant cohorts). However, we show that in log-linear terms, the time-t price setting decisions of firms that entered in period $t-2$ and further in the past are identical.[33] As we verify in the appendix, this allows us to characterize the log-linearized behavior of producing firms in terms of the representative members of only two sets of firms: those that are one-period old at time t (and thus are taking their first price setting decision, given our assumptions on the timing of entry and production) and those who are two or more periods old.

Under the assumption that new price setters take into account that they will be subject to a cost of adjusting prices relative to their previous choice from their second period of price setting

[31] Fabiani, Gattulli, and Sabbatini (2004), Gautier (2006), and Hoeberichts and Stockman (2004) find evidence of higher price flexibility in more competitive sectors of the economies they consider. To the extent that entry is more prevalent in more competitive sectors, this evidence may be suggestive of a connection between entry and price stickiness. Hoffmann and Kurz-Kim (2006) analyze consumer prices in Germany over the period 1998-2003, taking into account the effect of product replacements. They report that the incidence of price changes increases when replacements are taken into account (although it is not clear that replacements are truly new products or just newly adopted products in a particular outlet).

[32] As a by-product, the model in Appendix E also extends Rotemberg's (1982) original model by removing the assumption of a nature-given initial price.

[33] We log-linearize the model around the same steady state with zero inflation in all prices as the benchmark model to facilitate the comparison of responses to shocks.

on, optimal behavior does not result in a constant markup over marginal cost in the first period of price setting, since new price setters incorporate the incentive to smooth price movements between the initial choice and next period's price implied by the expectation of future adjustment costs. For completeness of comparison, we consider also the scenario in which we assume that new price setters simply charge the constant elasticity markup $\theta/(\theta-1)$ over marginal cost.

Figure 5 presents the responses to a one percent productivity increase with persistence 0.9 for the benchmark model (blue, round markers), the model in which new entrants do not pay a cost of price adjustment but take into account future costs optimally (red, cross markers), and the model in which new entrants charge a constant markup over marginal cost (green, square markers). We keep the same parameter values as in the exercises above and we assume that labor supply is inelastic. For simplicity, we assume a policy rule in which the central bank responds with coefficient 1.5 to expected inflation in producer prices in the benchmark model. In the alternative (log-linearized) models, there are two producer price inflation rates: one that measures the change in the initial price set for time t by firms that entered at $t-1$ relative to the initial choice at $t-1$ by those that entered at $t-2$, and the other measuring inflation in producer prices by older firms. However, responding to producer prices in the benchmark model amounts to responding to the empirically consistent measure of consumer price inflation in the context of that model (since producer price inflation is equal to welfare-consistent consumer price inflation minus the product variety effect that is not captured by available CPI data). For this reason, we assume that in the alternative models, the central bank is responding with coefficient 1.5 to inflation in an average consumer price level $\tilde{P}_t$ that removes the pure product variety effect from the welfare-consistent price index P_t: $\tilde{P}_t \equiv (N_t)^{\frac{1}{\theta-1}} P_t$.[34] Under all scenarios, the central bank is thus responding to the empirically relevant measure of expected consumer price inflation in the context of the relevant model. Figure 5 focuses on aggregate quantities, the nominal and real interest rates, inflation in the welfare consistent price index, inflation in producer prices in the benchmark model, inflation in $\tilde{P}_t$ (denoted $\pi^C_{A,t}$) in the alternative models, and the real wage. The responses of non-model-specific variables are virtually identical across models. In addition, the response of $\pi^C_{A,t}$ is virtually identical to that of π_t in the benchmark model. To explore the intuition for this result, Figure 6 further presents the responses of variables that are specific to cohorts of firms. For all variables

[34] The price index $\tilde{P}_t$ is closer than P_t to empirical CPI data for the reason discussed above that data do not account for availability of new products at the frequency relevant for our model and in the form tied to our preference specification. Given any variable X_t in units of consumption, the data-consistent counterpart in the extended models is thus defined as: $X_{R,t} \equiv P_t X_t / \tilde{P}_t$. See Ghironi and Melitz (2005) for further discussion.

other than firm values, variables indexed by a superscript 1 refer to one-period-old firms in the alternative models, and variables without superscript refer to older firms in the alternative models and the representative firm in the benchmark model.[35] The response of v_t is the response of firm value in the benchmark model. The response of v_t^t is the response of the value of new entrants in the alternative models (the asset price that determines the allocation of resources to creation of firms versus production of existing goods). Although the responses point to heterogeneity of behavior across new price setters and incumbents in the extended models, the behavior of the representative firm of the benchmark model is virtually indistinguishable from that of incumbents in the alternative models – and v_t is essentially identical to v_t^t. Given the assumption of a small steady-state rate of product turnover implied by $\delta = 0.025$, the virtual identity of behavior across the representative firm of the benchmark model and incumbents in the alternatives implies that small departures of the number of new entrants (and new price setters) from the steady state have negligible consequences for aggregate dynamics relative to the benchmark model.

The role of δ for the differences across nominal rigidity assumptions is best illustrated by the extreme example of Figure 7. There, we present the responses of the same variables as in Figure 5 to a permanent decrease in the nominal interest rate with the rate of product destruction set to the unrealistically high value of $\delta = 0.25$. The shock causes a permanent increase in inflation, and thus a permanent drop in the number of producers, and a permanent reallocation of labor from firm creation to production of incumbent goods. Consistent with intuition, the real consequences of the shock become smaller as we move from the benchmark model to the model in which new price setters take into account the future cost of price adjustment, and from this to the model in which new price setters charge a constant markup. This is in line with decreasing aggregate nominal rigidity as we move from one model to the next. Nevertheless, unrealistically large average product turnover (and extremely high shock persistence) are required in our model for any noticeable difference to emerge in shock transmission as a consequence of more flexible price setting behavior by new entrants.

Non-C.E.S. Preferences

Having verified that our benchmark assumption on price stickiness yields results that are robust to alternative specifications of pricing behavior by new entrants, we return to the benchmark assumption on pricing, and we study the consequences of extending the model in a different direction

[35] The inflation rate π_t^1 measures the change in the initial price set by those that entered at $t-1$ relative to those that entered at $t-2$.

– allowing for non-C.E.S. preferences. Suppose the consumption basket takes a general, symmetric, homothetic form with elasticity of substitution across individual products $\theta(N_t)$ increasing in the number of available goods ($\theta'(N_t) > 0$). This is the assumption of BGM. A derivation mirroring that for C.E.S. preferences delivers a markup equation similar to (2), with $\theta(N_t)$ replacing θ. The only other equation from Table 1 that is affected is the one governing the variety effect, which now becomes $\rho_t = \rho(N_t)$, with elasticity $\epsilon(N_t) \equiv \rho'(N_t)N_t/\rho(N_t)$.

We prove in Appendix B that the (first-best) optimal rate of producer price inflation remains zero under this general preference specification. The same policy rule (15) as in the C.E.S case implements the optimal allocation, when combined with appropriately designed (and lump-sum-financed) fiscal instruments studied in detail in BGM (2006).[36] We log-linearize the markup equation for this general preference specification around the steady state with zero inflation, and parametrize preferences in the translog form introduced by Feenstra (2003) and explored by BGM (with symmetric price elasticity of demand $-(1+\sigma N_t)$). Assuming the calibration scheme $\theta(N) = 1 + \sigma N = \theta$ that ensures equality of the steady state across C.E.S. and translog preferences, we obtain the New Keynesian Phillips curve for producer price inflation under translog preferences:[37]

$$\pi_t = \beta(1-\delta)E_t\pi_{t+1} + \frac{\theta-1}{\kappa}(\mathsf{w}_t - \mathsf{Z}_t) - \left(\frac{1}{2\kappa} + \frac{\theta-1}{\theta\kappa}\right)\mathsf{N}_t. \tag{16}$$

Notice the difference from the Phillips curve with C.E.S. preferences (6): In that Phillips curve, once the variety effect is removed from w_t and the relation is written in terms of data-consistent marginal cost, the effect of the endogenous state variable N_t on inflation disappears. This is no longer the case under translog preferences, where variation in the number of firms has an independent effect on the flexible-price markup via its effect on the elasticity of substitution, and the steady-state benefit of additional variety is half of its C.E.S. counterpart.

Figure 8 shows the impulse responses to a one percent productivity increase with persistence 0.9 under C.E.S. (blue, round markers) and translog (red, cross markers) preferences for the benchmark parameter values. We assume that policy responds to expected inflation in producer prices with coefficient 1.5. Most responses are qualitatively similar across preference specifications, the most pronounced difference being in the markup response. The markup is below the steady state

[36] The (Ramsey) optimal rate of inflation would be non-zero in a second-best environment in which lump-sum instruments are unavailable. The monetary authority would trade the welfare costs of inflation against the welfare costs of markup variation coming from both a time-varying elasticity of substitution and the misalignment of the benefit of extra variety with the profit incentive provided by the markup. We leave this extension for future research.

[37] See Appendix F for details.

throughout the horizon of the response under translog preferences due to the effect of a larger number of producing firms on the elasticity of substitution. At the same time, the welfare benefit of product variety is smaller under translog preferences, and so the response of the number of producers to the shock is muted relative to the C.E.S. scenario.

To verify whether translog preferences have noticeable quantitative implications, we repeated the experiment of Table 2 under the translog specification. Table 3 replaces the model generated moments of Table 2 with the results of the flexible-price model with translog preferences (BGM Translog) and its sticky-price version. The conclusions are largely unchanged relative to Table 2, although – consistent with what we noted above – translog preferences noticeably increase the persistence of producer price inflation.

Table 3. Moments for: Data, BGM Translog Model , and Sticky Prices

Variable X	σ_X			σ_X/σ_{YR}			$E[X_t X_{t-1}]$			$corr(X, Y_R)$		
Y_R	**1.81**	1.25	*1.29*	**1.00**			**0.84**	0.70	*0.70*	**1.00**		
C_R	**1.35**	0.75	*0.81*	**0.74**	0.60	*0.63*	**0.80**	0.78	*0.74*	**0.88**	0.95	*0.98*
Investment, $v_R N_E$	**5.30**	4.27	*4.01*	**2.93**	3.42	*3.11*	**0.87**	0.66	*0.69*	**0.80**	0.96	*0.98*
L	**1.79**	0.49	*0.49*	**0.99**	0.39	*0.38*	**0.88**	0.66	*0.68*	**0.88**	0.95	*0.97*
π			*0.02*			*0.01*			*0.94*			*–0.61*

Source for **data** moments: King and Rebelo (1999)

Finally, Figure 9 augments Figure 4 by including the model-based markup cyclicality in the translog model with sticky prices. Introducing translog preferences shifts the correlation between markup and GDP at leads and lags in the "right" direction through the effect of variation in the number of producers. However, the contemporaneous correlation becomes even more excessively negative. The flexible-price translog model remains the best (among those we considered) at reproducing the cyclicality of the markup.

8 Conclusions

This paper studied the implications of introducing endogenous product creation in a sticky-price model of the business cycle suitable for monetary policy analysis. When variety is endogenous and the price setting distortion pertains to individual producer prices, first-best optimal monetary policy should aim at stabilizing producer price inflation and let the welfare-relevant consumer price index fluctuate to accommodate changes in the number of products. Our model highlights a novel motive for price stability, which occurs because inflation acts as a distortionary tax on firm profits, and profits provide incentives to firms for product creation.

Our model also identifies a new channel for monetary policy transmission through the price of equity (the value of a firm, or product). This price is featured in the inflation dynamics equation in a way that is absent from standard fixed-variety models, precisely due to the connection between the markup (and marginal cost) and the price of equity via the free entry condition. Moreover, since our model embeds a portfolio decision between holding equity and bonds, monetary policy influences the price of equity through a no-arbitrage condition that relates the real return on bonds (which the central bank influences) to the expected real return on equity. This link between inflation dynamics and monetary policy through assets prices is central to the validity of the Taylor Principle in our endogenous-variety model with capital accumulation in the form of new production lines. This is unlike results from fixed-variety models with physical capital.

Endogenous product variety has implications for inflation dynamics and the estimation of New Keynesian Phillips curve equations. Empirical proxies for (unobservable) markups need to be amended in order to estimate Phillips curves in the presence of product creation. Moreover, we show that the Phillips curve in the presence of endogenous variety features an extra term (with respect to its fixed-variety counterpart) that depends on the number of available varieties, a state variable. This goes in the direction of alleviating the notorious difficulty of New Keynesian models in accounting for inflation persistence with forward-looking price-setting. Finally, we identify an 'endogeneity bias' that is present whenever estimates of the Phillips curve ignore product variety and hence attribute the endogenous component coming from its impact on inflation dynamics to exogenous 'cost-push' shocks.

Numerical exercises show that the sticky-price model performs similarly to the flexible-price counterpart in terms of matching several features of U.S. business cycle fluctuations given a policy specification that is plausible for inflation-sensitive policymakers. Consistent with the presence of an endogenous state variable in the Phillips curve, the model generates fairly persistent inflation dynamics. These results are confirmed by two extensions of the benchmark setup, studying alternative assumptions for the initial price setting decision by new entrants and the consequences of non-C.E.S. (specifically, translog) preferences.

Recent empirical literature has documented the pervasiveness of product creation and destruction at a frequency that is relevant for business cycle propagation. This paper provides a starting point for incorporation of this phenomenon in monetary models of the business cycle suitable for policy analysis. Like the benchmark New Keynesian model with fixed product variety, the model of this paper has clear shortcomings from an empirical, quantitative perspective. However, it highlights realistic consequences of product creation subject to sunk costs (inflation persistence), a new motive for price stability, and a new connection between monetary policy and equity prices that is not featured in the previous New Keynesian literature. We view this as a promising stepping stone for future research on a variety of positive and normative questions in potentially richer, monetary models with endogenous variety.

References

[1] Ambler, S., and E. Cardia (1998): "The Cyclical Behaviour of Wages and Profits under Imperfect Competition," *Canadian Journal of Economics* 31: 148-164.

[2] Axarloglou, K. (2003): "The Cyclicality of New Product Introductions," *Journal of Business* 76: 29-48.

[3] Bergin, P. R., and G. Corsetti (2005): "Towards a Theory of Firm Entry and Stabilization Policy," NBER WP 11821.

[4] Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003): "Plants and Productivity in International Trade," *American Economic Review* 93: 1268-1290.

[5] Bernard, A. B., S. J. Redding, and P. K. Schott (2006): "Multi-Product Firms and Product Switching," NBER WP 12293.

[6] Bilbiie, F. O., F. Ghironi, and M. J. Melitz (2005): "Endogenous Entry, Product Variety, and Business Cycles," manuscript, University of Oxford, Boston College, and Princeton University.

[7] Bilbiie, F. O., F. Ghironi, and M. J. Melitz (2006): "Monopoly Power and Endogenous Variety in Dynamic Stochastic General Equilibrium: Distortions and Remedies," manuscript, University of Oxford, Boston College, and Princeton University.

[8] Broda, C. (2004): "CPI Biases and their Impact on Policy", *International Perspectives* 32, Federal Reserve Bank of New York.

[9] Broda, C., and D. E. Weinstein (2006): "Globalization and the Gains from Variety," *Quarterly Journal of Economics* 121: 541-585.

[10] Broda, C., and D. E. Weinstein (2007): "Product Creation and Destruction: Evidence and Price Implications," manuscript, University of Chicago.

[11] Calvo, G. A. (1983): "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12: 983-998.

[12] Carlstrom, C. T., and T. S. Fuerst (2005): "Investment and Interest Rate Policy: A Discrete Time Analysis," *Journal of Economic Theory* 123: 4-20.

[13] Carlstrom, C. T., T. S. Fuerst, and F. Ghironi (2006): "Does It Matter (for Equilibrium Determinacy) What Price Index the Central Bank Targets?" *Journal of Economic Theory* 128: 214-231.

[14] Chatterjee, S., and R. Cooper (1993): "Entry and Exit, Product Variety and the Business Cycle," NBER WP 4562.

[15] Clarida, R., J. Galí, and M. Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature* 37: 1661-1707.

[16] Clarida, R., J. Galí, and M. Gertler (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115: 147-180.

[17] Comin, D., and M. Gertler (2006): "Medium Term Business Cycles," *American Economic Review*, forthcoming.

[18] Cook, D., (2001): "Time to Enter and Business Cycles," *Journal of Economic Dynamics and Control* 25 (8): 1241-1261.

[19] Cooley, T. F., and E. C. Prescott (1995): "Economic Growth and Business Cycles," in Cooley, T.F., ed., *Frontiers of Business Cycle Research*, 1-38, Princeton University Press, Princeton.

[20] Devereux, M. B., A. C. Head, and B. J. Lapham (1996a): "Aggregate Fluctuations with Increasing Returns to Specialization and Scale," *Journal of Economic Dynamics and Control* 20: 627-656.

[21] Devereux, M. B., A. C. Head, and B. J. Lapham (1996b): "Monopolistic Competition, Increasing Returns, and the Effects of Government Spending," *Journal of Money, Credit, and Banking* 28: 233-254.

[22] Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review* 67: 297-308.

[23] Dupor, W. D. (2001): "Investment and Interest Rate Policy," *Journal of Economic Theory* 98: 85-113.

[24] Erceg, C. J., L. Guerrieri, and C. Gust (2005): "Expansionary Fiscal Shocks and the U.S. Trade Deficit," *International Finance* 8: 363-397.

[25] Fabiani, S., Gattulli, A., and R. Sabbatini (2004): "The Pricing Behaviour of Italian Firms: New Survey Evidence on Price Stickiness," ECB WP 333.

[26] Feenstra, R. C. (2003): "A Homothetic Utility Function for Monopolistic Competition Models, without Constant Price Elasticity," *Economics Letters* 78: 79-86.

[27] Fuhrer, J. and Moore, J., (1995): "Inflation Persistence," *The Quarterly Journal of Economics*, 110(1): 127-59

[28] Galí, J., and M. Gertler (1999): "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics* 44: 195-222.

[29] Gautier, E. (2006): "The Behaviour of Producer Prices: Some Evidence from French PPI Data," ECB WP 699.

[30] Ghironi, F., and M. J. Melitz (2005): "International Trade and Macroeconomic Dynamics with Heterogeneous Firms," *Quarterly Journal of Economics* 120: 865-915.

[31] Hall, R. E. (1988): "The Relation between Price and Marginal Cost in U.S. Industry," *Journal of Political Economy* 96: 921-947.

[32] Hoeberichts, M. M., and A. Stockman (2004): "Pricing Behavior of Dutch Companies: Main Results from a Survey," manuscript, De Nederlandsche Bank.

[33] Hoffmann, J., and J.-R. Kurz-Kim (2006): "Consumer Price Adjustment under the Microscope: Germany in a Period of Low Inflation," Discussion Paper 16/2006, Deutsche Bundesbank.

[34] Ireland, P. (2001): "Sticky-Price Models of the Business Cycle: Specification and Stability," *Journal of Monetary Economics* 47: 3-18.

[35] Jaimovich, N. (2004): “Firm Dynamics, Markup Variations, and the Business Cycle,” manuscript, University of California, San Diego.

[36] Kerr, W., and R. King (1996): “Limits on Interest Rate Rules in the IS-LM Model,” *Federal Reserve Bank of Richmond Economic Quarterly*, Spring.

[37] King, R. G., and S. T. Rebelo (1999): “Resuscitating Real Business Cycles,” in Taylor, J. B., and M. Woodford, eds., *Handbook of Macroeconomics*, vol. 1B, 927-1007, Elsevier, Amsterdam.

[38] Laxton, D., and P. Pesenti (2003): “Monetary Rules for Small, Open, Emerging Economies,” *Journal of Monetary Economics* 50: 1109-1146.

[39] Leeper, E. (1991): “Equilibria under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies,” *Journal of Monetary Economics* 27: 129-147.

[40] Leeper, E. M., C. A. Sims, and T. Zha (1996): “What Does Monetary Policy Do?” *Brookings Papers on Economic Activity* 2: 1-63

[41] Lewis, V. (2006): “Macroeconomic Fluctuations and Firm Entry: Theory and Evidence,” manuscript, Catholic University Leuven.

[42] Mancini-Griffoli, T., and M. Elkhoury (2006): “Monetary Policy with Firm Entry,” manuscript, Graduate Institute of International Studies, Geneva.

[43] Rotemberg, J. J. (1982): “Monopolistic Price Adjustment and Aggregate Output,” *Review of Economic Studies* 49: 517-531.

[44] Rotemberg, J. J., and M. Woodford (1992): “Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity,” *Journal of Political Economy* 100: 1153-1207.

[45] Rotemberg, J. and Woodford, M. (1995): ”Dynamic General equilibrium Models and Imperfect Product Markets’, in Cooley, T.F.., ed., *Frontiers of Business Cycle Research*, Princeton University Press.

[46] Rotemberg, J. J., and M. Woodford (1999): “The Cyclical Behavior of Prices and Costs,” in Taylor, J. B., and M. Woodford, eds., *Handbook of Macroeconomics*, vol. 1B, 1051-1135, Elsevier, Amsterdam.

[47] Sbordone, A. (2002): "Prices and Unit Labor Costs: A New Test of Price Stickiness," *Journal of Monetary Economics* 49: 265-292.

[48] Stebunovs, V. (2006): "Finance as a Barrier to Entry in Dynamic Stochastic General Equilibrium," Ph.D. Dissertation, Boston College.

[49] Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press.

[50] Yun, T. (1996): "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37: 345-370.

A The Reduced Benchmark Model and the Efficient Equilibrium

The system of equilibrium conditions in Table 1 can be further reduced by substituting all the equations that hold *within* a given period. The reduced system is shown in Table A.1 for the case $\varphi = 0$.

Table A.1. Reduced Benchmark Model ($\varphi = 0$)

Markup	$\mu_t = \frac{\theta}{(\theta-1)\left[1-\frac{\kappa}{2}\pi_t^2\right]+\kappa\left\{(1+\pi_t)\pi_t-\beta(1-\delta)E_t\left[\frac{1-\frac{\kappa}{2}\pi_t^2}{1-\frac{\kappa}{2}\pi_{t+1}^2}\frac{N_t}{N_{t+1}}(1+\pi_{t+1})\pi_{t+1}\right]\right\}}$
Number of firms	$N_{t+1} = (1-\delta)\left(N_t + \frac{1}{f_{E,t}}\left[Z_t - Y_t^C N_t^{\frac{1}{1-\theta}}\right]\right)$
Euler equation (shares)	$\frac{f_{E,t}}{\mu_t} N_t^{\frac{1}{\theta-1}} = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\frac{f_{E,t+1}}{\mu_{t+1}}N_{t+1}^{\frac{1}{\theta-1}} + \left(1-\frac{1}{\mu_{t+1}}\right)\frac{\left[1-\frac{\kappa}{2}\pi_{t+1}^2\right]C_{t+1}}{N_{t+1}}\right)\right]$
Euler equation (bonds)	$(C_t)^{-1} = \beta E_t\left[\frac{1+i_t}{1+\pi_{t+1}}\left(\frac{N_{t+1}}{N_t}\right)^{\frac{1}{\theta-1}}(C_{t+1})^{-1}\right]$

As for the system in Table 1, the reduced system is closed by specifying a rule for nominal interest rate setting, the setting of the labor subsidy τ_t^L, and processes for the exogenous entry cost $f_{E,t}$ and productivity Z_t.

Based on the first-best policy exercise in the text, the efficient equilibrium of this economy is obtained when policy mimics the flexible-price equilibrium through producer price stability ($\pi_t = 0$ $\forall t$). Denoting variables in this equilibrium with a star, the solution for the efficient equilibrium is obtained from the system in Table A.2.

Table A.2. Benchmark Model ($\varphi = 0$): The Efficient Equilibrium

Markup	$\mu_t^* = \frac{\theta}{\theta-1}$
Number of firms	$N_{t+1}^* = (1-\delta)\left(N_t^* + \frac{1}{f_{E,t}}\left[Z_t - C_t^*\left(N_t^*\right)^{\frac{1}{1-\theta}}\right]\right)$
Euler equation (shares)	$\frac{f_{E,t}}{\mu}\left(N_t^*\right)^{\frac{1}{\theta-1}} = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-1}\left(\frac{f_{E,t+1}}{\mu}\left(N_{t+1}^*\right)^{\frac{1}{\theta-1}} + \left(1-\frac{1}{\mu}\right)\frac{C_{t+1}^*}{N_{t+1}^*}\right)\right]$
Euler equation (bonds)	$(C_t^*)^{-1} = \beta E_t\left[(1+i_t^*)\left(\frac{N_{t+1}^*}{N_t^*}\right)^{\frac{1}{\theta-1}}\left(C_{t+1}^*\right)^{-1}\right]$

Log-linearizing under assumptions of log-normality and homoskedasticity, the last three equations in this table are:

$$\mathsf{N}^*_{t+1} = (1+r)\,\mathsf{N}^*_t - (r+\delta)(\theta-1)\,\mathsf{C}^*_t + [(r+\delta)(\theta-1)+\delta]\,\mathsf{Z}_t - \delta \mathsf{f}_{E,t}, \tag{17}$$

$$\mathsf{C}^*_t = \frac{1-\delta}{1+r} E_t \mathsf{C}^*_{t+1} - \left(\frac{1-\delta}{1+r}\frac{1}{\theta-1} - \frac{r+\delta}{1+r}\right)\mathsf{N}^*_{t+1} + \frac{1}{\theta-1}\mathsf{N}^*_t - \frac{1-\delta}{1+r}E_t \mathsf{f}_{E,t+1} + \mathsf{f}_{E,t}, \tag{18}$$

$$E_t \mathsf{C}^*_{t+1} = \mathsf{C}^*_t + \mathsf{i}^*_t + \frac{1}{\theta-1}\mathsf{N}^*_{t+1} - \frac{1}{\theta-1}\mathsf{N}^*_t. \tag{19}$$

Equations (17)-(18) fully determine consumption and the number of producers as a function of exogenous shocks (as in BGM). Equation (19) can then be used to obtain the Wicksellian interest rate i^*_t. Subtracting this equation from the log-linear version of the Euler equation for bond holdings in the sticky-price equilibrium and using $\tilde{\mathsf{Y}}^C_{R,t} = \hat{\mathsf{C}}_{R,t} \equiv \mathsf{C}_t - [1/(\theta-1)]\,\mathsf{N}_t - \{\mathsf{C}^*_t - [1/(\theta-1)]\,\mathsf{N}^*_t\} = \mathsf{C}_{R,t} - \mathsf{C}^*_{R,t}$ and $\hat{\imath}_t \equiv \mathsf{i}_t - \mathsf{i}^*_t$ yields $E_t \tilde{\mathsf{Y}}^C_{R,t+1} - \tilde{\mathsf{Y}}^C_{R,t} = \hat{\imath}_t - E_t \pi_{t+1}$, which we use in the derivation of the interest rate rule that supports the efficient, flexible-price allocation.

B Proof of Proposition 1

We study a hypothetical scenario in which a benevolent planner maximizes lifetime utility of the representative household by choosing quantities directly. The "production function" for aggregate consumption output is $Y^C_t = Z_t N_t^{\frac{1}{\theta-1}} L^C_t$, implying that consumption is given by $C_t = \left[1 - \frac{\kappa}{2}(\pi_t)^2\right] Z_t N_t^{\frac{1}{\theta-1}} L^C_t$. Hence, the problem solved by the planner can be written as:

$$\max_{\{L^C_s,\pi_s\}_{s=t}^{\infty}} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} U\left(\left[1-\frac{\kappa}{2}(\pi_s)^2\right] Z_s N_s^{\frac{1}{\theta-1}} L^C_s\right)\right],$$
$$s.t.\ N_{t+1} = (1-\delta)\,N_t + (1-\delta)\frac{\left(L - L^C_t\right) Z_t}{f_{E,t}}, \tag{20}$$

or, substituting the constraint into the utility function and treating next period's state as the choice variable:

$$\max_{\{N_{s+1},\pi_s\}_{s=t}^{\infty}} E_t \left\{\sum_{s=t}^{\infty} \beta^{s-t} U\left[\left[1-\frac{\kappa}{2}(\pi_s)^2\right] Z_s N_s^{\frac{1}{\theta-1}} \left(L - \frac{1}{(1-\delta)}\frac{f_{E,s}}{Z_s}N_{s+1} + \frac{f_{E,s}}{Z_s}N_s\right)\right]\right\}.$$

The first-order condition with respect to inflation is simply:

$$\pi_t = 0\ \forall t, \tag{21}$$

whereas the first order condition with respect to N_{t+1} for a given level of inflation is:

$$U'(C_t)\left[1-\frac{\kappa}{2}(\pi_t)^2\right] Z_t N_t^{\frac{1}{\theta-1}} \frac{1}{1-\delta}\frac{f_{E,t}}{Z_t}$$
$$=\beta E_t\left\{\begin{array}{c} U'(C_{t+1})\left[1-\frac{\kappa}{2}(\pi_{t+1})^2\right] Z_{t+1}\frac{1}{\theta-1}N_{t+1}^{\frac{1}{\theta-1}-1} \\ \cdot\left[L-\frac{1}{(1-\delta)}\frac{f_{E,t+1}}{Z_{t+1}}N_{t+2}+\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}+(\theta-1)\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}\right]\end{array}\right\}.$$

The term in square brackets in the right-hand side of this equation is:

$$L-\frac{1}{(1-\delta)}\frac{f_{E,t+1}}{Z_{t+1}}N_{t+2}+\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}+(\theta-1)\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}=L_{t+1}^C+(\theta-1)\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}.$$

Hence, the first-order condition becomes:

$$U'(C_t)\left[1-\frac{\kappa}{2}(\pi_t)^2\right] N_t^{\frac{1}{\theta-1}} f_{E,t}=\beta(1-\delta)E_t\left\{\begin{array}{c} U'(C_{t+1})\left[1-\frac{\kappa}{2}(\pi_{t+1})^2\right] Z_{t+1}\frac{1}{\theta-1}N_{t+1}^{\frac{1}{\theta-1}-1} \\ \cdot\left[L_{t+1}^C+(\theta-1)\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}\right]\end{array}\right\},$$

leading to

$$U'(C_t)\left[1-\frac{\kappa}{2}(\pi_t)^2\right] N_t^{\frac{1}{\theta-1}} f_{E,t}=\beta(1-\delta)E_t\left\{U'(C_{t+1})\left[\frac{1}{\theta-1}\frac{C_{t+1}}{N_{t+1}}+\left[1-\frac{\kappa}{2}(\pi_{t+1})^2\right]N_{t+1}^{\frac{1}{\theta-1}}f_{E,t+1}\right]\right\}. \tag{22}$$

Combining (22) and (21) we obtain the equation that, together with the dynamic constraint (20) fully determines the planner equilibrium:

$$U'(C_t)N_t^{\frac{1}{\theta-1}}f_{E,t}=\beta(1-\delta)E_t\left\{U'(C_{t+1})\left[\frac{1}{\theta-1}\frac{C_{t+1}}{N_{t+1}}+N_{t+1}^{\frac{1}{\theta-1}}f_{E,t+1}\right]\right\}. \tag{23}$$

General Homothetic Preferences

The proof is identical to that of Proposition 1, with $\rho(N_t)$ replacing $N_t^{\frac{1}{\theta-1}}$ and $\epsilon(N_{t+1})$ replacing $1/(\theta-1)$ in (23). Implementation of this first-best optimum in the decentralized economy is ensured by (15), together with one of the optimal subsidies studied in BGM (2006) to induce markup equalization across states and over time and balance the benefit from variety with the profit incentives for entry. Finally, in order to ensure optimality of the steady-state with zero inflation when we log-linearize around such a steady-state in the translog case, we only need to impose a subsidy in steady state. We impose an entry subsidy/tax that, as show in BGM (2006), is equivalent to doubling the entry cost in the steady state of the translog model, i.e., entrants need to pay $2f_{E,t}$ rather than $f_{E,t}$. None of the steady state ratios (and hence none of the log-linearized equations) is modified by this, since the entry cost is irrelevant for all of them.

C The Log-Linear Model

Table C.1 Log-Linear, Benchmark Model, Summary

Pricing	$\rho_t = \mu_t + \mathsf{w}_t - \mathsf{Z}_t$
Markup	$\pi_t = \beta(1-\delta) E_t \pi_{t+1} - \frac{\theta-1}{\kappa}\mu_t$
Variety effect	$\rho_t = \frac{1}{\theta-1}\mathsf{N}_t$
Profits	$\mathsf{d}_t = \mathsf{Y}_{C,t} - \mathsf{N}_t + (\theta-1)\mu_t$
Free entry	$\mathsf{v}_t = \mathsf{f}_{E,t} + \mathsf{w}_t - \mathsf{Z}_t$
Number of firms	$\mathsf{N}_{t+1} = (1-\delta)\mathsf{N}_t + \delta \mathsf{N}_{E,t}$
Intratemporal optimality	$\mathsf{L}_t = \varphi(\mathsf{w}_t - \mathsf{C}_t)$
Euler equation (bonds)	$\mathsf{E}_t\mathsf{C}_{t+1} = \mathsf{C}_t + \mathsf{i}_t - \mathsf{E}_t\pi^C_{t+1}$
Euler equation (shares)	$\mathsf{E}_t\mathsf{C}_{t+1} = \mathsf{C}_t + \frac{1-\delta}{1+r}\mathsf{E}_t\mathsf{v}_{t+1} - \mathsf{v}_t + \frac{r+\delta}{1+r}\mathsf{E}_t\mathsf{d}_{t+1}$
Aggregate accounting	$\mathsf{Y}_{C,t} + \frac{vN_E}{Y_C}\mathsf{v}_t + \frac{vN_E}{Y_C}\mathsf{N}_{E,t} = \frac{wL}{Y_C}\mathsf{w}_t + \frac{wL}{Y_C}\mathsf{L}_t + \frac{dN}{Y_C}\mathsf{d}_t + \frac{dN}{Y_C}\mathsf{N}_t$
Output of consumption sector	$\mathsf{Y}_{C,t} = \mathsf{C}_t$
CPI inflation	$\pi_t - \pi^C_t = \rho_t - \rho_{t-1}$
Nominal interest rate	$\mathsf{i}_t = \phi_\pi \mathsf{E}_t\pi^C_{t+1} + \varepsilon_t$

Since we log-linearize around a steady state with zero inflation, the steady-state ratios needed above are as in BGM:

$$\frac{vN_E}{Y_C} = \frac{\delta}{r+\delta}\frac{1}{\theta}, \quad \frac{dN}{Y_C} = \frac{1}{\theta}, \quad \frac{wL}{Y_C} = \frac{\theta-1}{\theta} + \frac{\delta}{r+\delta}\frac{1}{\theta}.$$

For the case of translog preferences, the only equations that change in the Table above are the second and third. The markup equation is replaced by (16) in the main text, and the variety effect is $\rho_t = [2(\theta-1)]^{-1}\mathsf{N}_t$.

D Proof of Proposition 2

Focus on the rule $i_t = \tau E_t \pi_{t+1}$ in the inelastic labor case. The proof of determinacy is similar to that in Carlstrom, Fuerst, and Ghironi (2006).

Recall that the steady state of the model is such that $1 + d/v = (1+r)/(1-\delta) = 1/[\beta(1-\delta)]$.

Define $\gamma \equiv d/v$. Note that, for plausible parameter values (β close to 1, δ small), γ is very small (0.036001 when $\beta = 0.99$ and $\delta = 0.025$). In particular, assume that β, δ, and θ are such that $1 - \gamma(\theta - 1) > 0$. This condition is satisfied by all plausible values of β, δ, and θ.

Omitting shocks and focusing on perfect foresight for the purposes of analyzing determinacy, we can rewrite the system as

$$A_1 \begin{bmatrix} \pi_{t+1} \\ \mathsf{C}_{t+1} \\ \mu_{t+1} \\ \mathsf{N}_{t+1} \end{bmatrix} = A_0 \begin{bmatrix} \pi_t \\ \mathsf{C}_t \\ \mu_t \\ \mathsf{N}_t \end{bmatrix},$$

where:

$$A_1 \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -(\tau - 1) & 1 & 0 & -\frac{1}{\theta-1} \\ 0 & 1 & 1-\gamma(\theta-1) & -\frac{1-\gamma(\theta-1)}{\theta-1} \end{bmatrix},$$

$$A_0 \equiv \begin{bmatrix} 1+\gamma & 0 & \frac{(\theta-1)(1+\gamma)}{\kappa} & 0 \\ 0 & -\frac{\gamma(\theta-1)}{\beta(1+\gamma)} & 0 & \frac{1}{\beta} \\ 0 & 1 & 0 & -\frac{1}{\theta-1} \\ 0 & 1+\gamma & 1+\gamma & -\frac{1+\gamma}{\theta-1} \end{bmatrix}.$$

Our interest is in the eigenvalues of the matrix $A \equiv A_1^{-1} A_0$.

The characteristic equation has the form:

$$J(e) = J_4 e^4 + J_3 e^3 + J_2 e^2 + J_1 e + J_0 = 0,$$

where:

$$
\begin{aligned}
&J_4 \equiv -\beta\kappa\left(1+\gamma\right)\left[1-\gamma\left(\theta-1\right)\right],\\
&J_3 \equiv \beta\left(1+\gamma\right)^2\left[\kappa-\left(\theta-1\right)\left(\tau-1\right)\right]+\kappa\left[1-\gamma\left(\theta-1\right)\right]\left[1+\beta\left(1+\gamma\right)\left(2+\gamma\right)\right],\\
&J_2 \equiv \left(\theta-1\right)\left(\tau-1\right)\left[\gamma^2\left(1+\gamma\right)\left(\theta-1\right)+\left(1+\gamma\right)\left(1+3\beta\gamma\right)+\beta\left(1+\gamma^3\right)\right]\\
&\quad -\kappa\left\{3\left(1+\beta\right)-\beta\gamma^3\left(\theta-2\right)-\gamma^2\left[\left(\theta-1\right)\left(1+2\beta\right)-5\beta\right]-\gamma\left[2\left(\theta-2\right)\left(1+\beta\right)-\beta\left(\theta+4\right)\right]\right\}\\
&J_1 \equiv \left(1+\gamma\right)\left\{\left(1+\gamma\right)\left[\kappa-\left(\theta-1\right)\left(\tau-1\right)\right]+\kappa\left[2+\beta\left(1+\gamma\left(2+\gamma\right)\right)-\gamma\left(\theta-1\right)\right]\right\}\\
&J_0 \equiv -\kappa\left(1+\gamma\right)^2.
\end{aligned}
$$

For determinacy (and stability), three roots of J must be outside the unit circle and one root must be within the unit circle. The condition $1-\gamma\left(\theta-1\right)>0$ ensures $J_4<0$. Plausible values of the price stickiness coefficient κ, the policy parameter τ, and substitutability θ also imply $\kappa > \left(\theta-1\right)\left(\tau-1\right)$, hence $J_3>0$.[38] The same plausible parameter values (and small γ) imply $J_2<0$ and $J_1>0$. Finally, $J_0<0$. Since $J(0)<0$, $J'(0)>0$, $J''(0)<0$, and $J'''(0)>0$, all the roots of J have positive real parts. The product of the four roots is equal to $J_0/J_4>1$. Furthermore,

$$
J(1)=\gamma\left(1+\gamma\right)\left(\theta-1\right)\left(\tau-1\right)\left[\beta\left(1+\gamma\right)+\gamma\left(\theta-1\right)-1\right].
$$

Since it is always $\theta>\left(1+\gamma\right)\left(1-\beta\right)/\gamma$, $J(1)$ has the sign of $\tau-1$. Therefore, if $\tau<1$, there are either 0 or 2 roots in $(0,1)$, so that we can never have determinacy. Hence, $\tau>1$ is necessary for determinacy.

We now turn to sufficiency.

Since $J(0)<0$ and $J(1)>0$ for $\tau>1$, we know that J has (at least) two real roots, one in the unit circle and one outside. Let us refer to these two real roots as $e_1<1$ and $e_2>1$. Our task is to examine the remaining two roots of J and demonstrate that they are outside the unit circle if $\tau>1$. The strategy is to examine these two roots in the neighborhood of $\tau=1$ defined by $\tau=1+\varepsilon$, with $\varepsilon>0$ and arbitrarily small. We can show that we have determinacy in this neighborhood whether the remaining roots are real or complex. In addition, we will show that as τ increases, these roots cannot pass back into the unit circle.

Focus on the case in which the two remaining roots are real

We first demonstrate that, in this case, J must have three roots outside the unit circle.

[38] For all plausible estimates, κ is much larger than θ and τ.

Define the function $h(x) \equiv J(e)$ where $x \equiv e-1$. The function h is also a quartic with coefficients h_0, h_1, h_2, h_3, and h_4. Note that $h_0 = J(1)$, $h1 = J'(1)$, $h_2 = J''(1)/2$, $h_3 = J'''(1)/3!$, etc. Inspection of the J function implies that $h_0 > 0$ and $h_4 < 0$. Also,

$$h_3 \equiv 6 \left\{ \begin{array}{c} \kappa \left[\beta (1+\gamma)^2 + (1+\beta\gamma(1+\gamma))(1-\gamma(\theta-1))\right] \\ -\beta(1+\gamma)\left[(1+\gamma)(\theta-1)(\tau-1) + 2\kappa(1-\gamma(\theta-1))\right] \end{array} \right\}.$$

If $\tau \to 1$ and $\beta \to 1$, $h_3 > 0$. The same holds for any plausible value of τ and β. Hence, Descartes' Rule of Signs implies that there is indeterminacy if and only if $h_1 > 0$ and $h_2 > 0$. In the neighborhood of $\tau = 1$, both $J'(1)$ and $J''(1)$, however, cannot be greater than or equal to zero since

$$(1-\gamma) J'(1|\tau=1) + \gamma J''(1|\tau=1) = -\kappa\gamma^2 \left[\beta\gamma^2 (3\theta-2) + \gamma(2\beta-1)(\theta-2) + (1-\beta)(\theta+2)\right] < 0,$$

under the weak, sufficient conditions $\beta > 1/2$ and $\theta > 2$. This then implies that $h(x)$ $(J(e))$ has three roots greater than zero (unity). Hence, we have determinacy (and stability) for τ just slightly greater than unity for any plausible parametrization.

As long as these two roots remain real, they must remain outside the unit circle for larger values of τ. This is true because $J(0) < 0$ and $J(1) > 0$ for all $\tau > 1$, so that the only way for there to be indeterminacy is to have three roots within the unit circle. This can never be the case without the roots first becoming complex. Therefore, as we increase τ out of the neighborhood $1+\varepsilon$, we must continue to have exactly one root in the unit circle.

The proof for the case of complex roots is similar to that in Carlstrom, Fuerst, and Ghironi (2006). We omit it to save space, but it is available on request.

E Endogenous Aggregate Stickiness and Producer Entry: The Model

This appendix develops the model in which new entrants do not pay a cost of price adjustment relative to a previous period's price.

The Price Index

Recall that a new entrant in period t starts producing (and setting prices) in period $t+1$. We can write the price index at time t as:

$$P_t = \left\{ \left[N_t - (1-\delta) N_{E,t-1} \right] (\tilde{p}_t)^{1-\theta} + (1-\delta) N_{E,t-1} \left(p_t^{t-1} \right)^{1-\theta} \right\}^{\frac{1}{1-\theta}}, \tag{24}$$

where p_t^{t-1} is the price chosen for period t by firms that entered in period $t-1$ and $\tilde{p}_t$ is an average price for firms that entered in periods $t-2$, $t-3$, and beyond, defined by:

$$\tilde{p}_t \equiv \left[\frac{1}{N_{t-1}} \sum_{s=2}^{\infty} (1-\delta)^{s-1} N_{E,t-s} \left(p_t^{t-s} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

Assume $\lim_{T\to\infty} (1-\delta)^{T-1} N_{t-T} = 0$. Then, using the law of motion $N_t = (1-\delta)(N_{t-1} + N_{E,t-1})$, it is possible to rewrite the price index as:

$$P_t = \left[\sum_{s=1}^{\infty} (1-\delta)^{s} N_{E,t-s} \left(p_t^{t-s} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{25}$$

This is a compact expression for P_t as weighted average of the prices chosen for that period by firms that entered in all past periods, weighted by their probability of survival. Equation (25) involves an infinite number of state variables (all the lags of the number of entrants). However, we show below that it can be reduced to a finite, small number of state variables in log-linear form.

Firms

A firm ω that entered in period $\upsilon \leq t-1$ produces output according to $y_t^{\upsilon,S}(\omega) = Z_t l_t^{\upsilon}(\omega)$ and faces demand

$$y_t^{\upsilon,D}(\omega) = \left(\frac{p_t^{\upsilon}(\omega)}{P_t} \right)^{-\theta} (C_t + PAC_t),$$

where:

$$PAC_t \equiv \sum_{s=2}^{\infty} (1-\delta)^{s} N_{E,t-s} PAC_t^{t-s}(\omega).$$

Note that the aggregate cost of price adjustment at time t aggregates only the costs of price adjustment borne by firms that entered in periods $t-2$, $t-3$, and beyond, since firms that entered in period $t-1$ pay no cost of price adjustment in period t. The firm-level cost of price adjustment takes the same form as in the benchmark model:

$$PAC_t^{\upsilon}(\omega) \equiv \frac{\kappa}{2}\left(\frac{p_t^{\upsilon}(\omega)}{p_{t-1}^{\upsilon}(\omega)} - 1\right)^2 \rho_t^{\upsilon}(\omega)\, y_t^{\upsilon,D}(\omega), \quad \upsilon \leq t-2,$$

where $\rho_t^{\upsilon}(\omega) \equiv p_t^{\upsilon}(\omega)/P_t$.

The value of a firm that entered in any period $\upsilon \leq t$ is given by the expected present discounted value of the stream of profits it will generate from period $t+1$ on:

$$v_t^{\upsilon}(\omega) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s^{\upsilon}(\omega), \quad \upsilon \leq t.$$

In particular, the value of a new entrant is:

$$v_t^{t}(\omega) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s^{t}(\omega),$$

and the free entry condition will now require equalization of this value to the sunk entry cost: $v_t^t(\omega) = w_t f_{E,t}/Z_t = v_t^t$ (identity of marginal cost across firms ensures symmetry of the equilibrium within cohorts of firms).

A new price setter in period t now maximizes

$$d_t^{t-1}(\omega) + E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s^{t-1}(\omega),$$

where

$$d_t^{t-1}(\omega) = \rho_t^{t-1}(\omega)\, y_t^{t-1,D}(\omega) - w_t l_t^{t-1}(\omega),$$
$$d_s^{t-1}(\omega) = \rho_s^{t-1}(\omega)\, y_s^{t-1,D}(\omega) - w_s l_s^{t-1}(\omega) - \frac{\kappa}{2}\left(\frac{p_s^{t-1}(\omega)}{p_{s-1}^{t-1}(\omega)} - 1\right)^2 \rho_s^{t-1}(\omega)\, y_s^{t-1,D}(\omega), \quad s \geq t+1.$$

The initial price setting decision yields:

$$p_t^{t-1}(\omega) = \mu_t^{t-1}(\omega)\, P_t \frac{w_t}{Z_t}, \tag{26}$$

with

$$\mu_t^{t-1}(\omega) = \frac{\theta}{(\theta-1) - \kappa E_t\left[\Lambda_{t,t+1}\frac{N_t}{N_{t+1}}\frac{Y_{t+1}^C}{Y_t^C}\left(1+\pi_{t+1}^{t-1}(\omega)\right)\pi_{t+1}^{t-1}(\omega)\right]}. \tag{27}$$

Because the firm knows that it will face a cost of price adjustment in the future, its first price setting decision is not just the flexible-price markup $\theta/(\theta-1)$ over marginal cost, but it incorporates the incentive to smooth price changes between the initial choice and the following one.[39]

Price setting decisions in the following periods – and the implied markup equation – and price setting decisions by firms that entered prior to $t-1$ take the same form as in the benchmark model.

The Household's Budget Constraint and Portfolio Decision

We assume that the household now decides at time t how many shares to hold in each cohort of firms that entered the economy up to and including period t. In real terms, exploit symmetry within cohorts, the budget constraint is now:

$$
\begin{aligned}
& B_{t+1} + \sum_{s=0}^{\infty} (1-\delta)^s v_t^{t-s} N_{E,t-s} x_{t+1}^{t-s} + C_t \\
& = (1+r_t) B_t + \sum_{s=0}^{\infty} (1-\delta)^{s+1} \left(d_t^{t-s-1} + v_t^{t-s-1}\right) N_{E,t-s-1} x_t^{t-s-1} + \left(1+\tau_t^L\right) w_t L_t + t_t^L .
\end{aligned}
$$

In period t, the household receives dividends from its holdings of shares in firms that entered in periods $t-1$, $t-2$, and beyond, and the value of selling its share holdings. It then buys holdings of shares to be carried into $t+1$ in all producing firms at time t plus the new entrants in period t (as in the benchmark model). The Euler equation for share holdings is:

$$
v_t^{t-s} = \beta (1-\delta) E_t \left[\frac{C_t}{C_{t+1}} \left(d_{t+1}^{t-s} + v_{t+1}^{t-s} \right) \right], \quad s \geq 0.
$$

As in the benchmark model, iteration of this equation yields the value of the firm. In particular, the Euler equation for investment in new firms is:

$$
v_t^t = \beta (1-\delta) E_t \left[\frac{C_t}{C_{t+1}} \left(d_{t+1}^t + v_{t+1}^t \right) \right]. \tag{28}
$$

[39] The version of the model in which new entrants charge a constant markup over marginal cost is obtained by setting the scaling parameter for the cost of price adjustment (κ) to zero in the markup equation (27) for new price setters.

Aggregate Accounting

Imposing the equilibrium conditions $B_{t+1} = B_t = 0$ and $x_{t+1}^{t-s} = x_t^{t-s-1} = 1$, and the government budget constraint $t_t^L = -\tau_t^L w_t L_t$, we have the aggregate accounting relation:

$$v_t^t N_{E,t} + C_t = w_t L_t + \sum_{s=0}^{\infty} (1-\delta)^{s+1} d_t^{t-s-1} N_{E,t-s-1}. \qquad (29)$$

The sum of investment in new firms and consumption must be equal to total income (labor income and dividend income paid by all cohorts of firms that produce in period t). As for the price index equation, the aggregate accounting equation (29) involves an infinite number of state variables, but it can be reduced to a small number of states in log-linear form. Note that the value of new entrants, determined by the Euler equation (28), is now the asset price that determines the allocation of resources to consumption or investment in new firms, obeying the free-entry condition $v_t^t = w_t f_{E,t}/Z_t$.

Some Log-Linear Relations

We log-linearize the model around the same steady state with zero inflation in all nominal prices as the benchmark model to facilitate comparison of the two setups in terms of their implications for the propagation of shocks. This sub-section shows how log-linearization makes it possible to reduce the number of state variables in solving the model and reports the log-linear versions of equations that feature an infinite number of state variables in level form.

Pricing and Price Index Dynamics

In log-linear terms , the price and markup equations (26) and (27) yield:

$$\mathsf{p}_t^{t-1} = \mu_t^{t-1} + \mathsf{P}_t + \mathsf{w}_t - \mathsf{Z}_t, \qquad (30)$$

$$\mu_t^{t-1} = \frac{\kappa\beta(1-\delta)}{\theta-1} E_t \pi_{t+1}^{t-1}, \qquad (31)$$

where we used symmetry of the equilibrium across firms in the same cohort, and μ_t^{t-1} and π_{t+1}^{t-1} now denote percent deviations from steady state (of gross inflation in the case of π_{t+1}^{t-1}).

Observe that price setting for any firm after its first price setting choice is such that:

$$\mathsf{p}_t^{\upsilon} = \mu_t^{\upsilon} + \mathsf{P}_t + \mathsf{w}_t - \mathsf{Z}_t, \tag{32}$$

$$\mu_t^{\upsilon} = -\frac{\kappa}{\theta - 1}\left[\pi_t^{\upsilon} - \beta(1-\delta)E_t\pi_{t+1}^{\upsilon}\right], \quad \upsilon < t-1 \tag{33}$$

where υ is the date of entry. Therefore, considering any two cohorts υ and $\upsilon - 1$,

$$\mathsf{p}_t^{\upsilon} - \mathsf{p}_t^{\upsilon-1} = \mu_t^{\upsilon} - \mu_t^{\upsilon-1}.$$

Combining this with the Phillips curves for the two cohorts and using the definitions of cohort-specific inflation rates implies that the price differential between cohorts obeys the difference equation:

$$\kappa\beta(1-\delta)E_t\left(\mathsf{p}_{t+1}^{\upsilon} - \mathsf{p}_{t+1}^{\upsilon-1}\right) - \{\theta - 1 + \kappa[1+\beta(1-\delta)]\}\left(\mathsf{p}_t^{\upsilon} - \mathsf{p}_t^{\upsilon-1}\right) + \kappa\left(\mathsf{p}_{t-1}^{\upsilon} - \mathsf{p}_{t-1}^{\upsilon-1}\right) = 0.$$

The characteristic polynomial for this equation is a convex parabola with one root inside the unit circle and one outside. Therefore, the equation has unique solution $\mathsf{p}_t^{\upsilon} - \mathsf{p}_t^{\upsilon-1} = 0$, or $\mathsf{p}_t^{\upsilon} = \mathsf{p}_t^{\upsilon-1}$. To a first-order approximation, firms that are in their second (or higher) period of price setting choose the same price, and thus the same producer price inflation rate: Given the initial condition $\mathsf{p}_{-1}^{\upsilon} = \mathsf{p}_{-1}^{\upsilon-1}$ if a shock happens at time 0, this implies $\pi_t^{\upsilon} = \pi_t^{\upsilon-1}$ and, in turn, $\mu_t^{\upsilon} = \mu_t^{\upsilon-1}$ for any cohorts υ and $\upsilon - 1$ that are not in the first period of price setting.

By exploiting this property of log-linearized price setting, it is possible to verify that log-linearized, welfare-consistent consumer price inflation depends negatively on variety growth and positively on a weighted average of inflation in the first pricing of new entrants at $t-1$ relative to new entrants at $t-2$ and inflation in the pricing of the "representative" cohort υ that entered in period $t-3$ or further in the past:

$$\pi_t^C = -\frac{1}{\theta - 1}(\mathsf{N}_t - \mathsf{N}_{t-1}) + \delta\left(\mathsf{p}_t^{t-1} - \mathsf{p}_{t-1}^{t-2}\right) + (1-\delta)\pi_t^{\upsilon}. \tag{34}$$

The equations above make it possible to fully determine the dynamics of all prices and price indexes of interest. The inflation rate π_t^{υ} is determined by the generic cohort υ's pricing and Phillip's curve. The time t price chosen by firms that entered at $t-1$ is determined by the price and markup equations (30) and (31). Note that these two equations together imply:

$$\mathsf{p}_t^{t-1} = \frac{\kappa\beta(1-\delta)}{\theta - 1 + \kappa\beta(1-\delta)} E_t \mathsf{p}_{t+1}^{t-1} + \frac{\theta - 1}{\theta - 1 + \kappa\beta(1-\delta)} (\mathsf{P}_t + \mathsf{w}_t - \mathsf{Z}_t). \tag{35}$$

From the perspective of period t, a firm that entered at $t-1$ and is setting the price for $t+1$ is no longer in its first period of price setting. Hence, the result obtained above applies and we may rewrite (35) as:

$$\mathsf{p}_t^{t-1} = \frac{\kappa\beta(1-\delta)}{\theta - 1 + \kappa\beta(1-\delta)} E_t \mathsf{p}_{t+1}^{\upsilon} + \frac{\theta - 1}{\theta - 1 + \kappa\beta(1-\delta)} (\mathsf{P}_t + \mathsf{w}_t - \mathsf{Z}_t), \quad \upsilon \leq t-1. \tag{36}$$

Finally, the time $t-1$ price chosen by entrants at $t-2$ is a state variable. At the time of a shock ($t = 0$), p_{t-1}^{t-2} is zero, and consumer price inflation is simply:

$$\pi_0^C = \mathsf{P}_0 = \delta \mathsf{p}_0^{-1} + (1-\delta)\mathsf{p}_0^{\upsilon},$$

where

$$\mathsf{p}_0^{-1} = \frac{\kappa\beta(1-\delta)}{\theta - 1 + \kappa\beta(1-\delta)} E_t \mathsf{p}_1^{\upsilon} + \frac{\theta - 1}{\theta - 1 + \kappa\beta(1-\delta)} (\mathsf{P}_0 + \mathsf{w}_0 - \mathsf{Z}_0),$$

and υ is the representative cohort that entered prior to period 0. In period 1,

$$\pi_1^C = -\frac{1}{\theta - 1}\mathsf{N}_1 + \delta\left(\mathsf{p}_1^0 - \mathsf{p}_0^{-1}\right) + (1-\delta)\pi_1^{\upsilon},$$

where p_0^{-1} was determined above and p_1^0 is determined by (36). And so on.

Aggregate Accounting, GDP, and the Labor Market

Exploiting symmetry of (log-linearized) behavior across cohorts of firms that are not in their first period of price setting, the log-linear version of the aggregate accounting identity (29) is:

$$\frac{vN_E}{C}\left(\mathsf{v}_t^t + \mathsf{N}_{E,t}\right) + \mathsf{C}_t = \frac{wL}{C}(\mathsf{w}_t + \mathsf{L}_t) + \frac{1}{\theta}\left[\mathsf{N}_t + \delta \mathsf{d}_t^{t-1} + (1-\delta)\mathsf{d}_t^{\upsilon}\right],$$

with

$$\mathsf{d}_t^{t-1} = \mathsf{y}_t^{t-1} + \theta\mu_t^{t-1} + \mathsf{w}_t - \mathsf{Z}_t, \quad \mathsf{y}_t^{t-1} = -\theta\rho_t^{t-1} + \mathsf{C}_t,$$
$$\mathsf{d}_t^{\upsilon} = \mathsf{y}_t^{\upsilon} + \theta\mu_t^{\upsilon} + \mathsf{w}_t - \mathsf{Z}_t, \quad \mathsf{y}_t^{\upsilon} = -\theta\rho_t^{\upsilon} + \mathsf{C}_t.$$

GDP is:

$$\mathsf{Y}_t = \frac{wL}{Y}\left(\mathsf{w}_t + \mathsf{L}_t\right) + \frac{1}{\theta}\frac{C}{Y}\left[\mathsf{N}_t + \delta \mathsf{d}_t^{t-1} + (1-\delta)\,\mathsf{d}_t^{\upsilon}\right],$$

Finally, labor market equilibrium requires:

$$\mathsf{L}_t = N_E\left(\mathsf{N}_{E,t} + \mathsf{f}_{E,t}\right) + (1-N_E)\left[\mathsf{N}_t + \delta \mathsf{y}_t^{t-1} + (1-\delta)\,\mathsf{y}_t^{\upsilon}\right] - \mathsf{Z}_t,$$

with sectoral labor allocation:

$$\mathsf{L}_t^E = \mathsf{N}_{E,t} + \mathsf{f}_{E,t} - \mathsf{Z}_t,$$
$$\mathsf{L}_t^C = \mathsf{N}_t + \delta \mathsf{y}_t^{t-1} + (1-\delta)\,\mathsf{y}_t^{\upsilon} - \mathsf{Z}_t.$$

Analytical Details

Derivation of Equation (25)

Observe that

$$N_t - (1-\delta)\,N_{E,t-1} = (1-\delta)\,N_{t-1},$$

so that

$$P_t = (1-\delta)^{\frac{1}{1-\theta}}\left[N_{t-1}\left(\tilde{p}_t\right)^{1-\theta} + N_{E,t-1}\left(p_t^{t-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{37}$$

Using the law of motion for N_t and the assumption $\lim_{T\to\infty}(1-\delta)^{T-1}N_{t-T} = 0$,

$$N_{t-1} = \sum_{s=2}^{\infty}(1-\delta)^{s-1}N_{E,t-s}. \tag{38}$$

From the definition of $\tilde{p}_t$, we have:

$$\tilde{p}_t \equiv \left[\frac{(1-\delta)\,N_{E,t-2}}{N_{t-1}}\left(p_t^{t-2}\right)^{1-\theta} + \frac{(1-\delta)^2 N_{E,t-3}}{N_{t-1}}\left(p_t^{t-3}\right)^{1-\theta} + \frac{(1-\delta)^3 N_{E,t-4}}{N_{t-1}}\left(p_t^{t-4}\right)^{1-\theta} + ...\right]^{\frac{1}{1-\theta}}$$
$$= \left(N_{t-1}\right)^{-\frac{1}{1-\theta}}\left[\sum_{s=2}^{\infty}(1-\delta)^{s-1}N_{E,t-s}\left(p_t^{t-s}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$
$$= (1-\delta)^{-\frac{1}{1-\theta}}\left(N_{t-1}\right)^{-\frac{1}{1-\theta}}\left[\sum_{s=2}^{\infty}(1-\delta)^{s}N_{E,t-s}\left(p_t^{t-s}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

Finally, substituting into (37) yields (25):

$$
\begin{aligned}
P_t &= (1-\delta)^{\frac{1}{1-\theta}} \left[\sum_{s=2}^{\infty} (1-\delta)^{s-1} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} + N_{E,t-1} \left(p_t^{t-1}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \\
&= (1-\delta)^{\frac{1}{1-\theta}} \left[\sum_{s=1}^{\infty} (1-\delta)^{s-1} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \\
&= \left[\sum_{s=1}^{\infty} (1-\delta)^{s} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} .
\end{aligned}
$$

Derivation of Equation (34)

First, observe that log-linearizing (38) yields:

$$
\mathsf{N}_{t-1} = \frac{\delta}{1-\delta} \sum_{s=2}^{\infty} (1-\delta)^{s-1} \mathsf{N}_{E,t-s}. \tag{39}
$$

Log-linearizing (25) yields:

$$
\begin{aligned}
\mathsf{P}_t &= \frac{\delta}{1-\theta} \left[\sum_{s=1}^{\infty} (1-\delta)^{s-1} \mathsf{N}_{E,t-s} + (1-\theta) \sum_{s=1}^{\infty} (1-\delta)^{s-1} \mathsf{p}_t^{t-s} \right] \\
&= \frac{\delta}{1-\theta} \left[\mathsf{N}_{E,t-1} + \sum_{s=2}^{\infty} (1-\delta)^{s-1} \mathsf{N}_{E,t-s} + (1-\theta) \sum_{s=1}^{\infty} (1-\delta)^{s-1} \mathsf{p}_t^{t-s} \right] \\
&= \frac{\delta}{1-\theta} \left[\mathsf{N}_{E,t-1} + \frac{1-\delta}{\delta} \mathsf{N}_{t-1} + (1-\theta) \sum_{s=1}^{\infty} (1-\delta)^{s-1} \mathsf{p}_t^{t-s} \right] \\
&= \frac{1}{1-\theta} \mathsf{N}_t + \delta \left[\mathsf{p}_t^{t-1} + \sum_{s=2}^{\infty} (1-\delta)^{s-1} \mathsf{p}_t^{t-s} \right],
\end{aligned} \tag{40}
$$

where the third line used (39) and the fourth line used $\mathsf{N}_t = (1-\delta)\,\mathsf{N}_{t-1} + \delta \mathsf{N}_{E,t-1}$ (the log-linear law of motion for N_t). Importantly, (40) features only one state variable (N_t), by exploiting the log-linear solution of the law of motion for the number of firms (39).

Consider now the first difference of (40):

$$
\begin{aligned}
\pi_t^C &\equiv \mathsf{P}_t - \mathsf{P}_{t-1} \\
&= \frac{1}{1-\theta} \left(\mathsf{N}_t - \mathsf{N}_{t-1}\right) + \delta \left[\mathsf{p}_t^{t-1} - \mathsf{p}_{t-1}^{t-2} + \sum_{s=2}^{\infty} (1-\delta)^{s-1} \left(\mathsf{p}_t^{t-s} - \mathsf{p}_{t-1}^{t-s-1} \right) \right].
\end{aligned} \tag{41}
$$

From the result we obtained above that $\mathsf{p}_t^{v} - \mathsf{p}_t^{v-1} = 0$ for all firms that entered prior to period

$t-1$, it follows that $\mathsf{p}_t^{t-s} - \mathsf{p}_{t-1}^{t-s-1}$ in (41) can be written independently of s as $\mathsf{p}_t^{\upsilon} - \mathsf{p}_{t-1}^{\upsilon}$, where υ now simply denotes a representative cohort of firms that entered before $t-2$. Hence,

$$
\begin{aligned}
\pi_t^C &= -\frac{1}{\theta-1}\left(\mathsf{N}_t - \mathsf{N}_{t-1}\right) + \delta\left[\mathsf{p}_t^{t-1} - \mathsf{p}_{t-1}^{t-2} + \left(\mathsf{p}_t^{\upsilon} - \mathsf{p}_{t-1}^{\upsilon}\right)\sum_{s=2}^{\infty}(1-\delta)^{s-1}\right] \\
&= -\frac{1}{\theta-1}\left(\mathsf{N}_t - \mathsf{N}_{t-1}\right) + \delta\left(\mathsf{p}_t^{t-1} - \mathsf{p}_{t-1}^{t-2} + \frac{1-\delta}{\delta}\pi_t^{\upsilon}\right) \\
&= -\frac{1}{\theta-1}\left(\mathsf{N}_t - \mathsf{N}_{t-1}\right) + \delta\left(\mathsf{p}_t^{t-1} - \mathsf{p}_{t-1}^{t-2}\right) + (1-\delta)\,\pi_t^{\upsilon}.
\end{aligned}
$$

F The New Keynesian Phillips Curve with Non-C.E.S. Preferences

Log-linearizing the markup equation around a zero-inflation steady state under the usual assumptions of lognormality and homoskedasticity yields:

$$
\pi_t = \beta(1-\delta)E_t\pi_{t+1} - \frac{\theta(N)-1}{\kappa}\mu_t - \frac{\theta'(N)\,N}{\theta(N)\,\kappa}\mathsf{N}_t. \tag{42}
$$

For the special case of translog preferences with $\theta(N) = 1 + \sigma N$, $\sigma > 0$, we obtain:

$$
\pi_t = \beta(1-\delta)E_t\pi_{t+1} - \frac{\sigma N}{\kappa}\mu_t - \frac{\sigma N}{1+\sigma N}\frac{1}{\kappa}\mathsf{N}_t.
$$

Finally, we impose the calibration scheme $\theta(N) = 1 + \sigma N = \theta$, where the latter is the elasticity of substitution in the C.E.S. case.[40] Then, the Phillips curve becomes:

$$
\pi_t = \beta(1-\delta)E_t\pi_{t+1} - \frac{\theta-1}{\kappa}\mu_t - \frac{\theta-1}{\theta\kappa}\mathsf{N}_t. \tag{43}
$$

Regarded as a markup equation, (43) implies that markup variation comes from two sources: changes in the inflation rate and product variety. Regarded as an equation for inflation dynamics, (43) implies an extra degree of persistence compared to the C.E.S. case (4), coming from the presence of the state variable N_t via its impact on the elasticity of substitution across products.

This extra persistence survives when writing the inflation equation in terms of marginal cost in data consistent terms. Since the benefit of product variety in log-linear terms is now $\rho_t = \epsilon(N)\,\mathsf{N}_t$, the markup is related to marginal cost by $\mu_t = \epsilon(N)\,\mathsf{N}_t - (\mathsf{w}_t - \mathsf{Z}_t)$, which for translog preferences

[40] This is achieved, in the translog case, by finding the implicit unique value $\sigma^* = (\theta-1)/N^{CES}$, where N^{CES} is the steady-state value of N under C.E.S. preferences.

(under the calibration scheme above) yields:

$$\mu_t = \frac{1}{2(\theta - 1)} \mathsf{N}_t - (\mathsf{w}_t - \mathsf{Z}_t).$$

Substituting this into (43) yields:

$$\pi_t = \beta(1 - \delta) E_t \pi_{t+1} + \frac{\theta - 1}{\kappa} (\mathsf{w}_t - \mathsf{Z}_t) - \left(\frac{1}{2\kappa} + \frac{\theta - 1}{\theta \kappa} \right) \mathsf{N}_t.$$

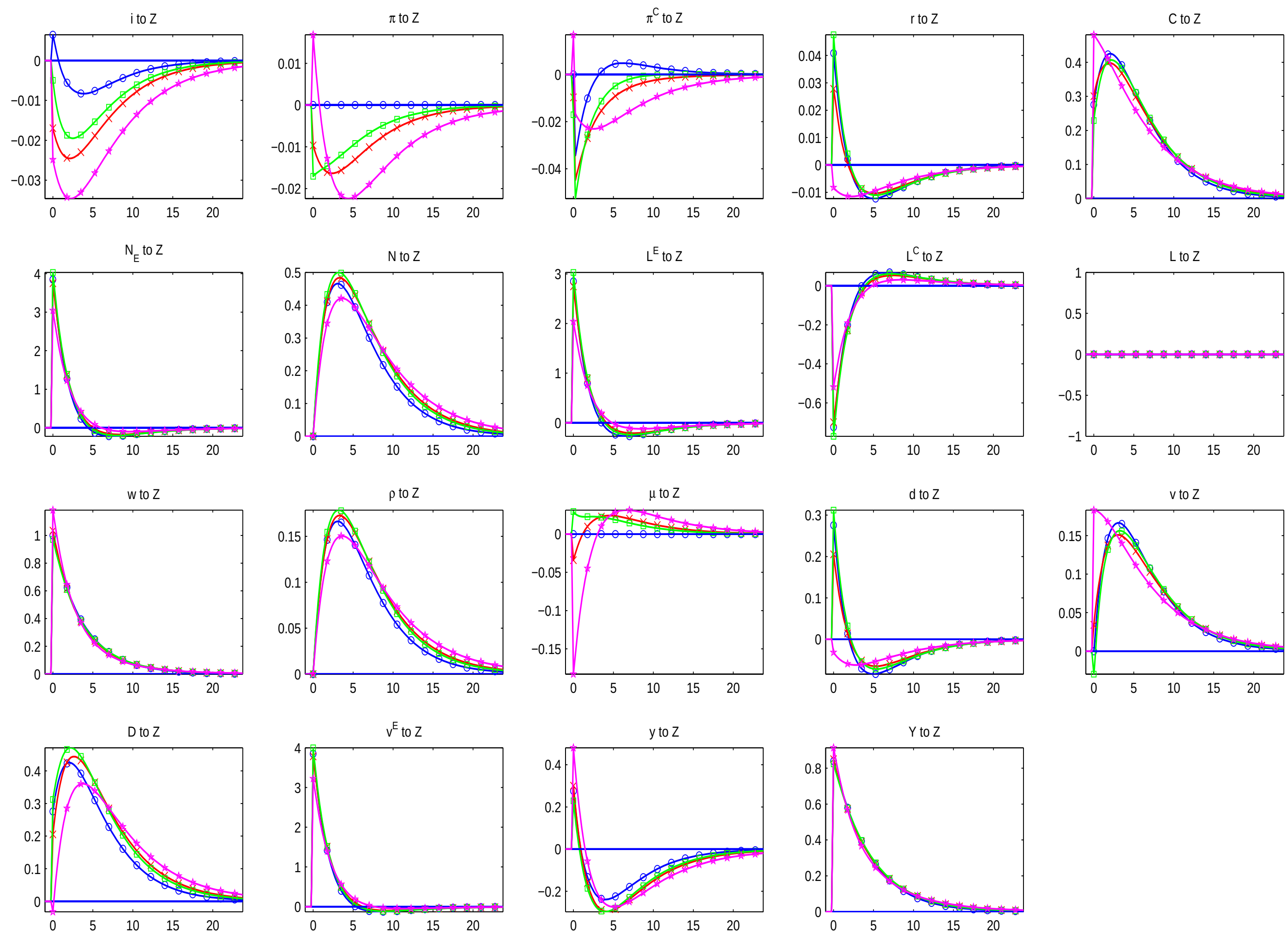


Figure 1. Impulse Responses: Productivity Shock, Persistence .9*

Round markers (blue): Optimal policy; Cross markers (red): $i_t = 1.5E_t\pi_{t+1}$; Square markers (green): $i_t = .8\ i_{t-1}+.3E_t\pi_{t+1}$; Star markers (pink): $i_t = 1.5E_t\pi^C_{t+1}$.

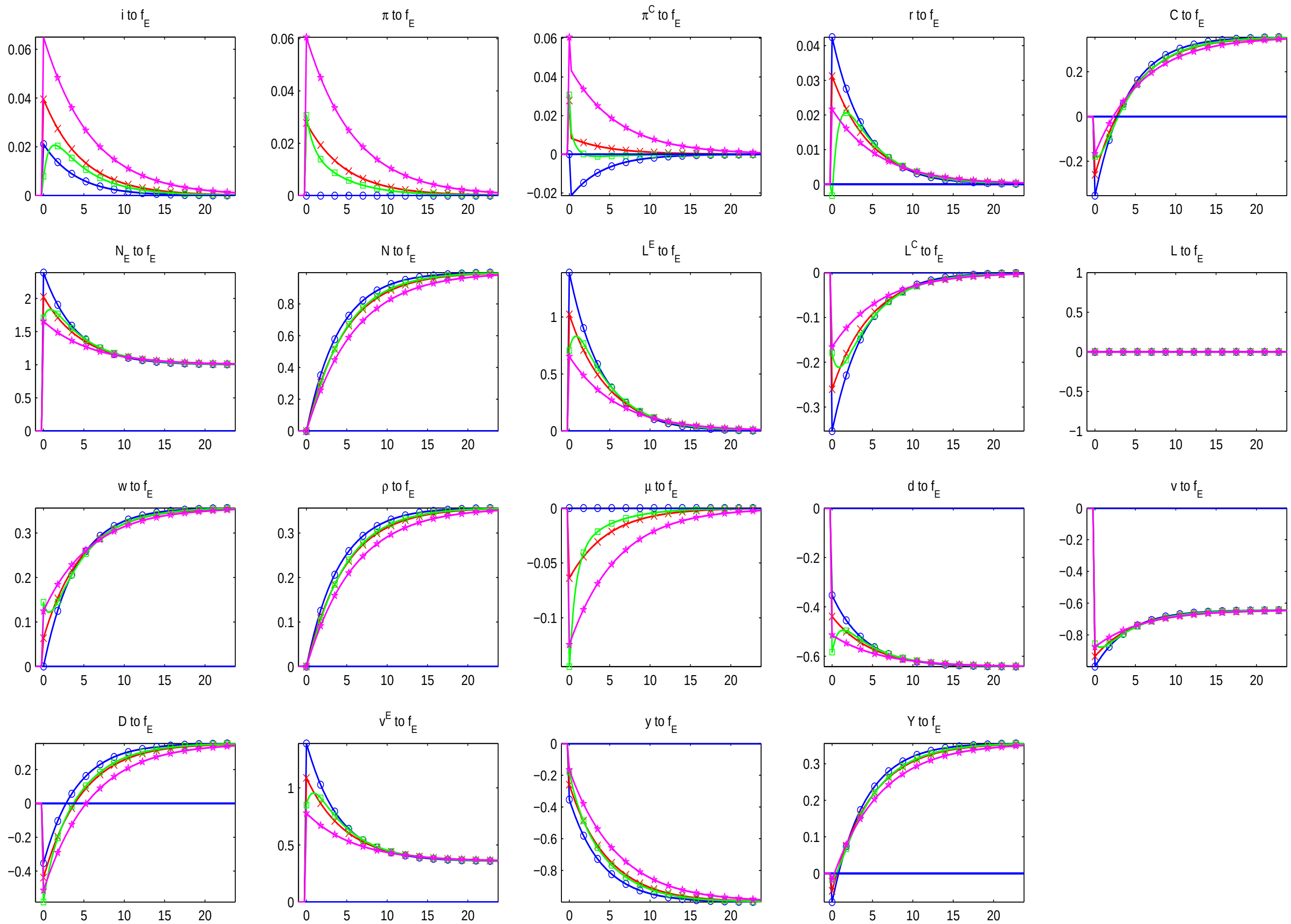


Figure 2. Impulse Responses: Permanent Deregulation*

Round markers (blue): Optimal policy; Cross markers (red): $i_t = 1.5E_t\pi_{t+1}$; Square markers (green): $i_t = .8\, i_{t-1}+.3E_t\pi_{t+1}$; Star markers (pink): $i_t = 1.5E_t\pi^C_{t+1}$.

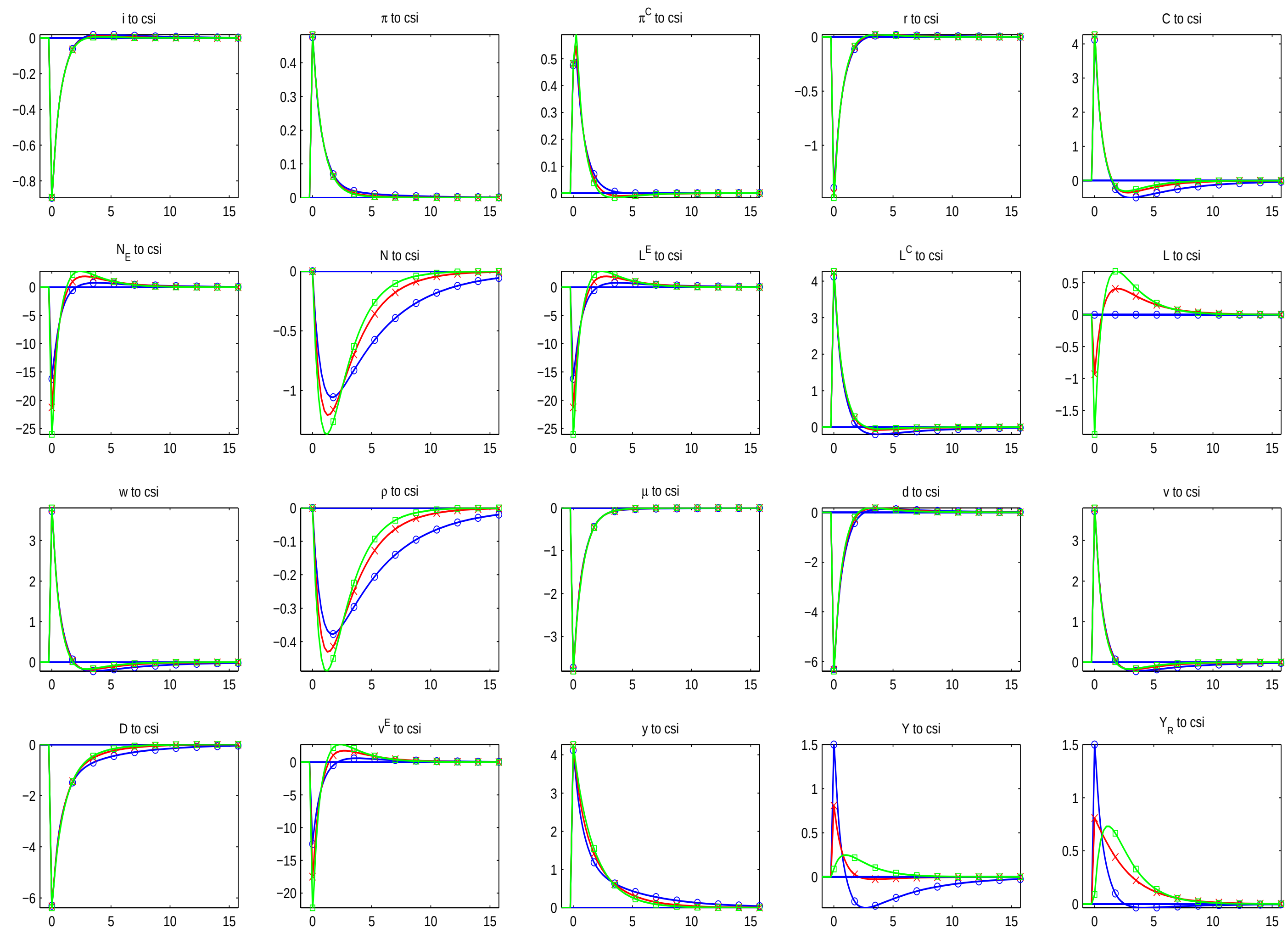
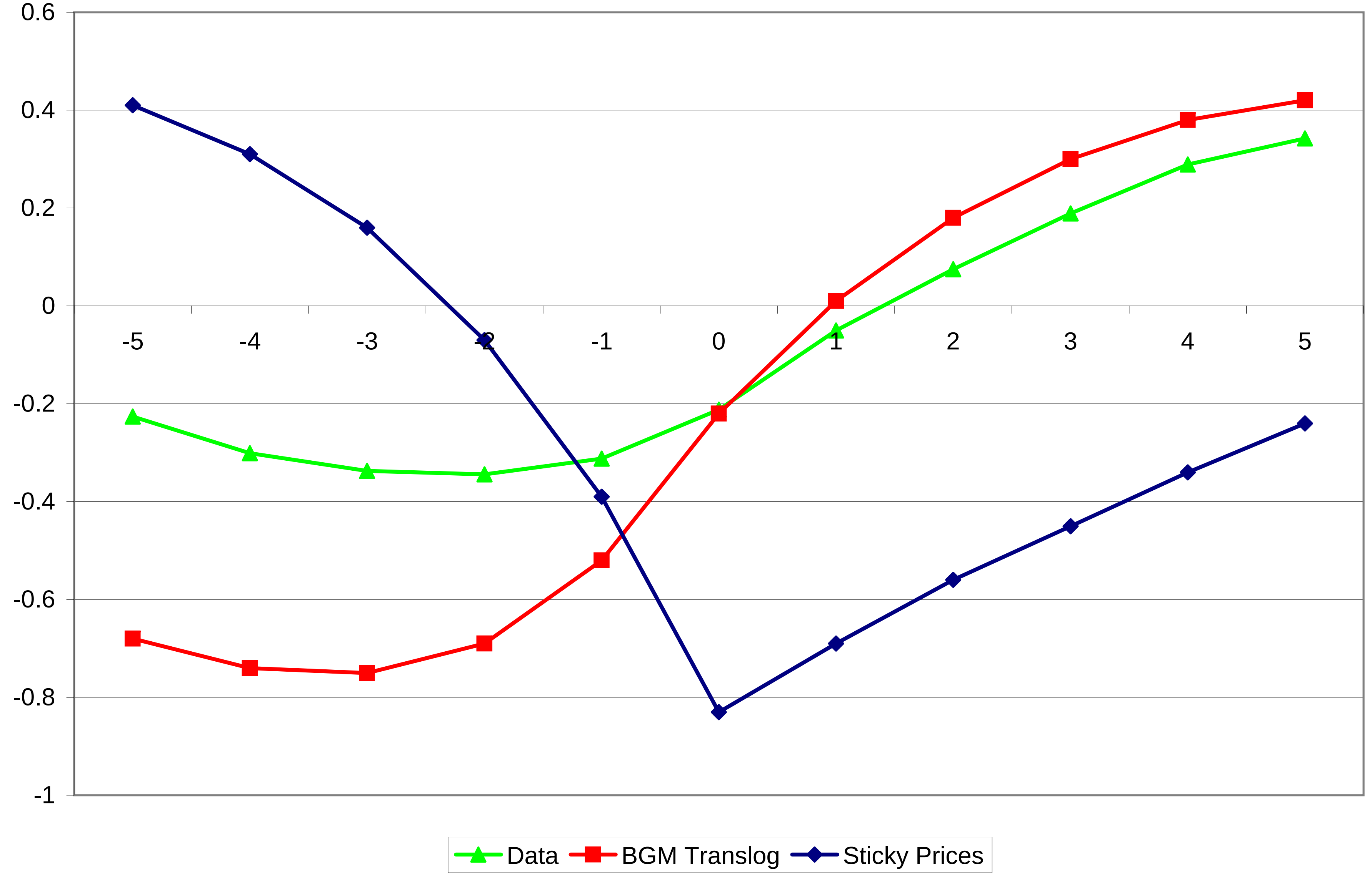


Figure 3. Impulse Responses: Interest Rate Shock, Persistence 0*

Round markers (blue): Inelastic labor; Cross markers (red): Labor supply elasticity = 2; Square markers (green): Labor supply elasticity = 4.

Figure 4. The Cyclicality of the Markup



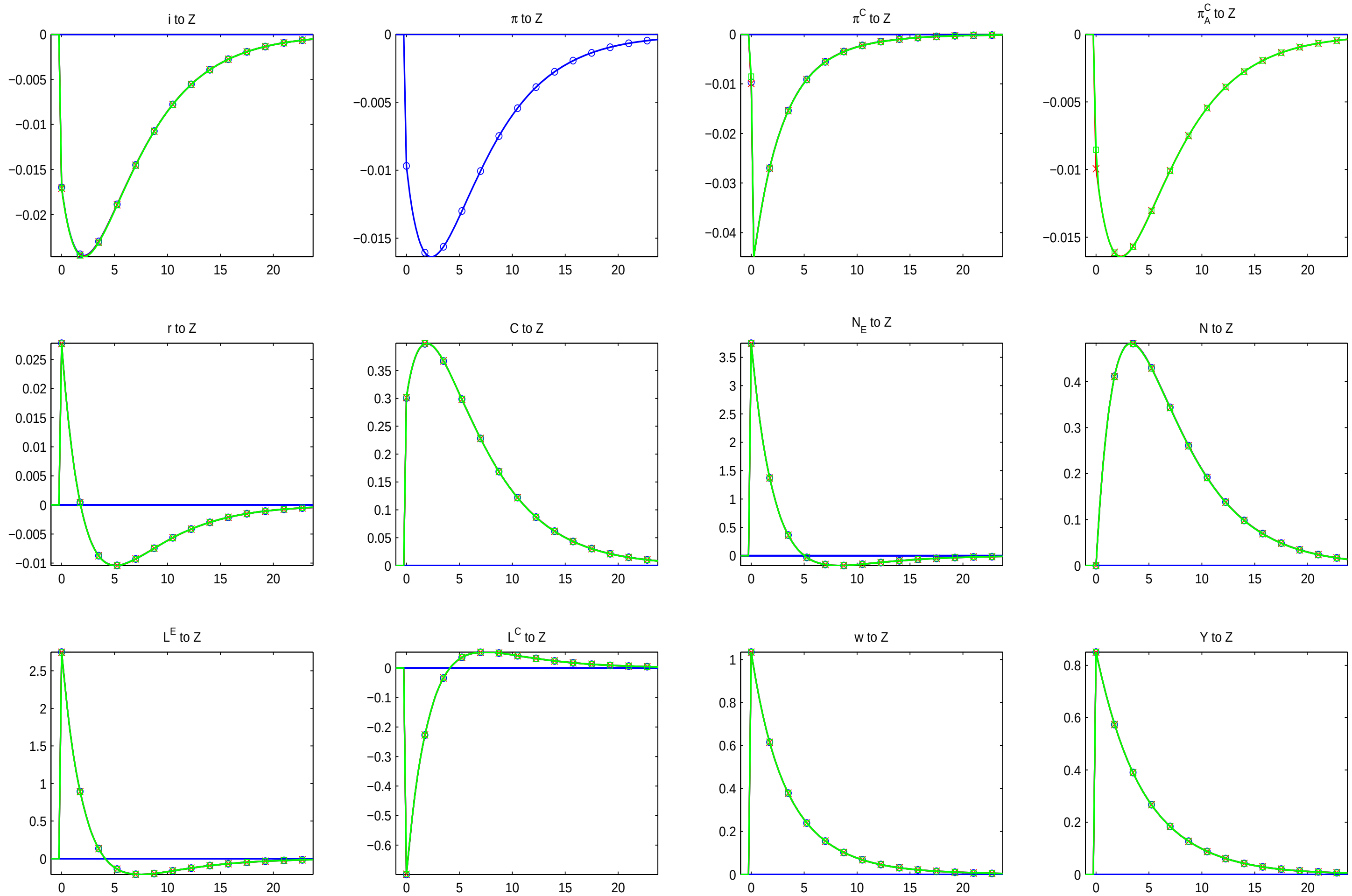


Figure 5. Alternative Assumptions on Initial Price Setting: Impulse Responses to a Productivity Shock, Aggregate Variables

*Round markers (blue): Benchmark; Cross markers (red): Entrants take future cost into account; Square markers (green): Entrants charge constant markups.

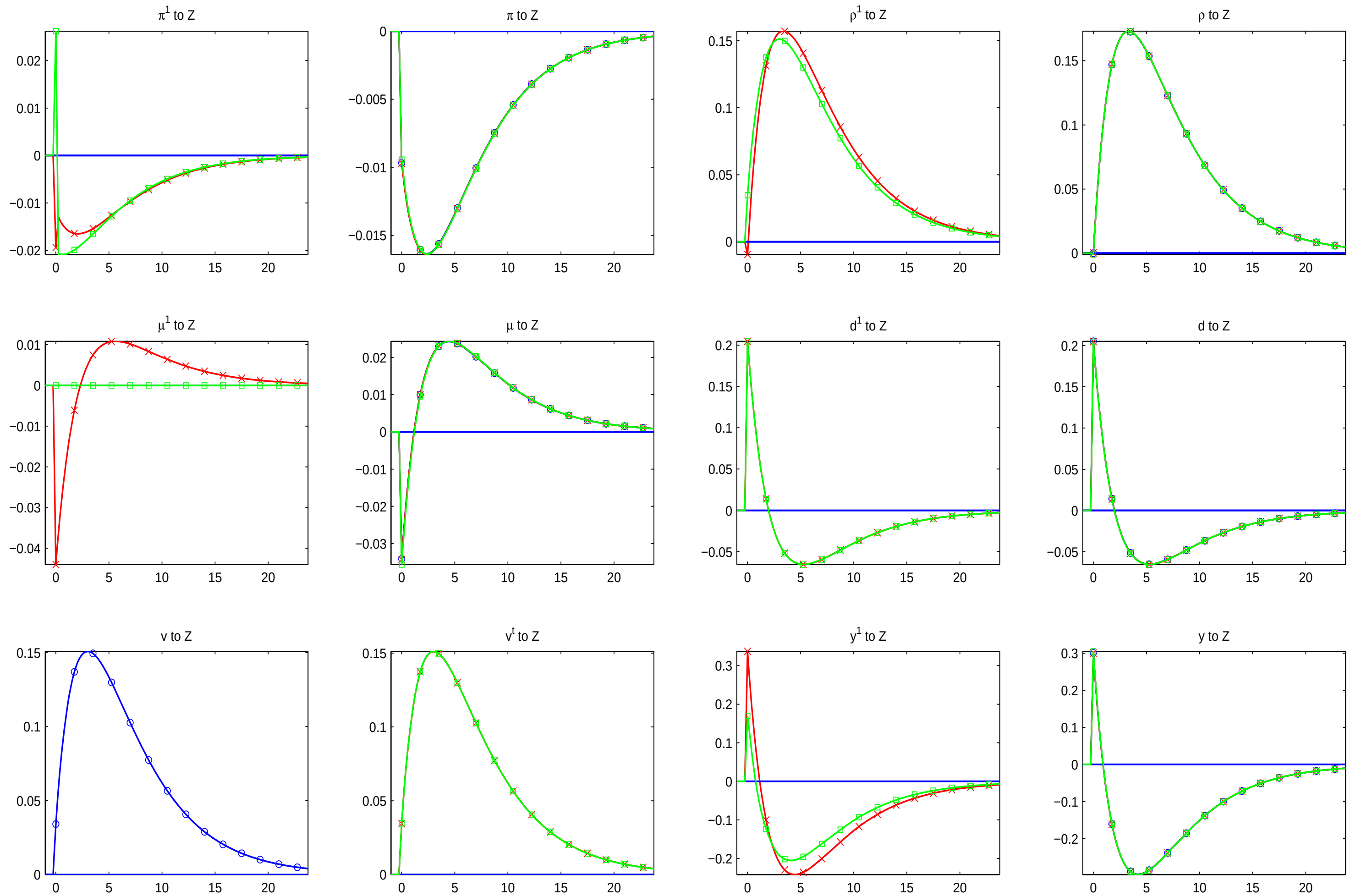


Figure 6. Alternative Assumptions on Initial Price Setting: Impulse Responses to a Productivity Shock, Firm-Level Variables

*Round markers (blue): Benchmark; Cross markers (red): Entrants take future cost into account; Square markers (green): Entrants charge constant markups.

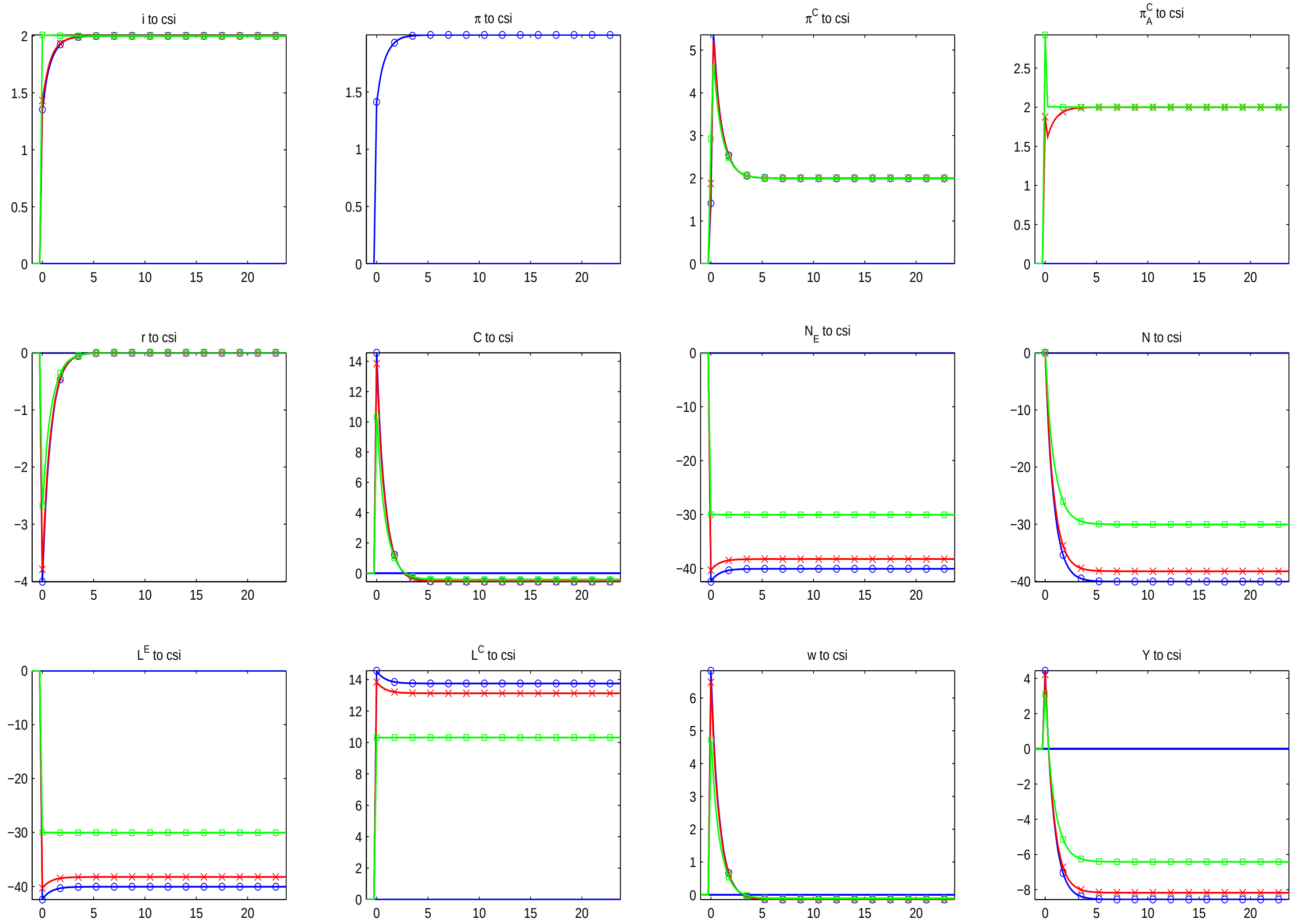


Figure 7. Alternative Assumptions on Initial Price Setting: Impulse Responses to a Permanent Interest Rate Shock*

*Round markers (blue): Benchmark; Cross markers (red): Entrants take future cost into account; Square markers (green): Entrants charge constant markups.

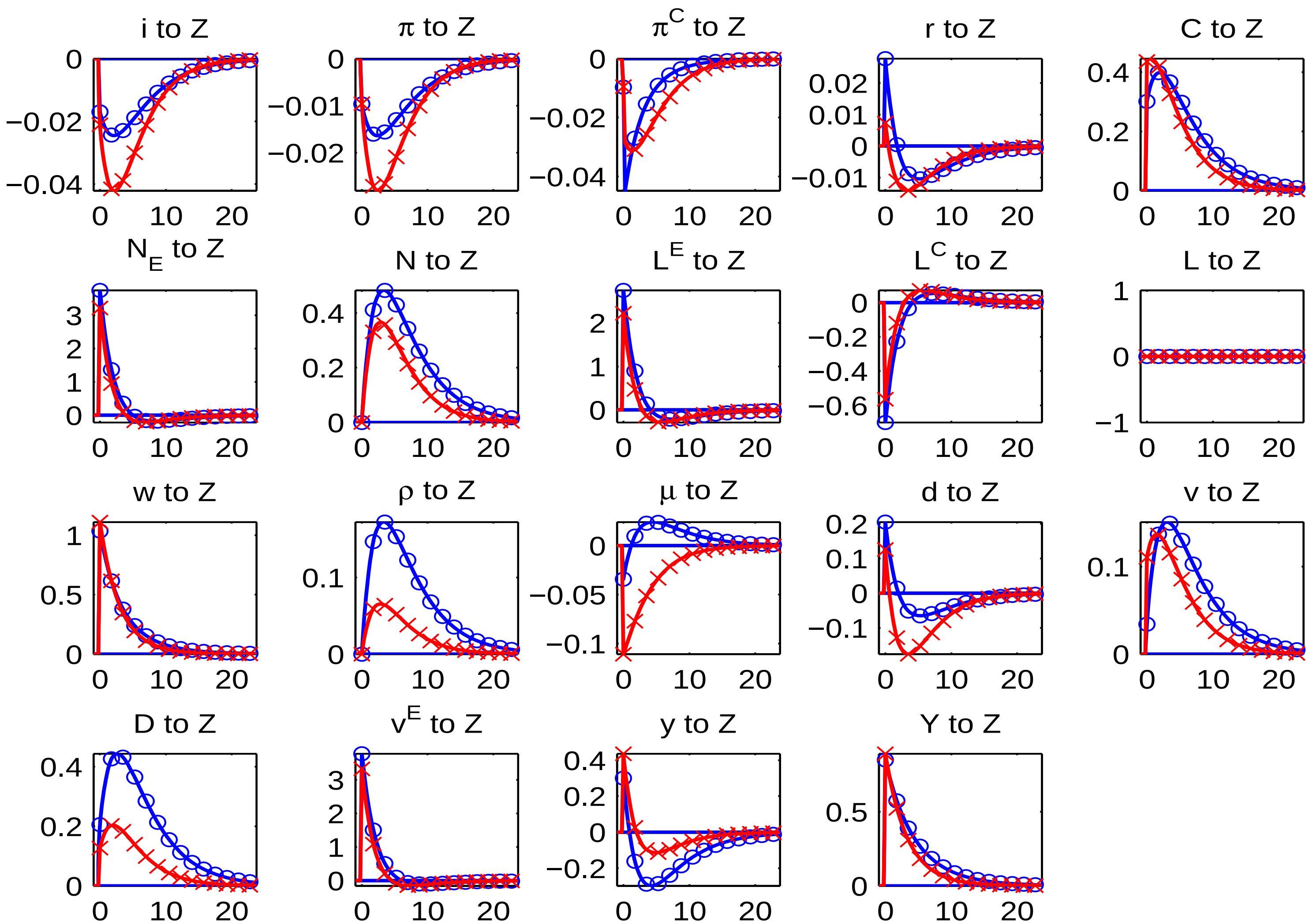


Figure 8. C.E.S. v.s. Translog: Impulse Responses to a Productivity Shock, Persistence .9*

Round markers (blue): C.E.S.; Cross markers (red): Translog; Policy: $i_t = 1.5E_t\pi_{t+1}$.

Figure 9. The Cyclicality of the Markup